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MEMORIA PARA OPTAR AL TÍTULO DE INGENIERO CIVIL INDUSTRIAL

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AUCTIONS, NEGOTIATIONS AND INFORMATION ACQUISITION

En esta tesis se estudian remates en los cuales los oferentes compiten por el derecho a negociar con un agente. Estos remates están presentes en adquisiciones deportivas y licitaciones con alto impacto en las comunidades con intereses particulares distintos al vendedor de los derechos. En particular, se estudia cómo afectan diferentes estructuras de remates al excedente social, la utilidad de los oferentes, de los agentes y los incentivos de los agentes a invertir en información. Se consideró un modelo con tres tipos de agentes: el rematador, los oferentes, y el agente con que el ganador de la subasta negocia. En una primera etapa a cada oferente se le revela una señal con información sobre cuanto valora los servicios del agente. Con esto se realizan las ofertas. Se anuncia el ganador y cuanto tiene que pagar de forma inmediata y cuanto en caso que la negociación con el agente sea exitosa. Después, al ganador se le revela su verdadera valoración (afiliada con la señal). El agente le hace una oferta al ganador y éste decide si aceptar o no. En caso de aceptar debe pagar lo acordado al agente, y de acuerdo a la reglas del remate, lo acordado al rematador. Para distintos remates se buscaron equilibrios simétricos en los cuales las funciones de oferta fueran crecientes en la señal recibida. Se demuestra que para cierta familia de remates se sigue cumpliendo el teorema de Equivalencia de Ingresos. Además, que para una familia amplia es posible ordenar los remates de acuerdo al excedente social y la utilidad del agente negociador. En efecto, es mejor en términos sociales que siempre pague su oferta al rematador a que sólo pague en caso de que la negociación sea exitosa. Por otro lado, para ciertas distribuciones de la señal y la valoración, se demuestra que las funciones de oferta en equilibrio son lineales, al igual que en un remate clásico con distribución uniforme. Por último, la tesis estudia los incentivos a adquirir información. En este modelo adquirir información tiene también un efecto negativo pues el agente puede extraer más renta del ganador del remate. Se demuestra que en ciertos ambientes no existe adquisición de información, aún cuando es socialmente óptimo hacerlo. Como futura investigación se plantea el estudio de la monotonía de las ofertas al aumentar el pago que se hace en forma inmediata, lo que implicaría un orden total en el excedente social. Además, explorar cuales son los supuestos que condicionan la existencia de equilibrio en este tipo de juegos, y por último, investigar la clase general en la cual el efecto negativo de adquirir información es mayor al positivo.

Contents

List of Figures

Chapter 1

Introduction

1.1 Motivation

In 2005, the Japanese baseball player Daisuke Matsuzaka was all over the news. After convincing his team, the Seibu Lions, that it was time for him to play at the Major Leagues, a procedure known as the "posting system" started to operate. First, all interested American teams bid in a first price sealed bid auction. The winner (the Boston Red Sox), with a bid of US$51.1 millions, won the exclusive rights to negotiate with Matsuzaka. If an agreement was not reached with him, the Red Sox would not have to pay anything to the Seibu Lions. This non-standard system generated a lot of controversy in the baseball community. Some people thought that with that kind of transaction teams could over bid for the right to block other teams, and then negotiating with the player in bad terms to him. Other critique was directed to the percentage of the transaction that the Seibu Lions could get, and that through this system the player was getting a bad deal. In fact, Matsuzaka's manager directly criticized the system for reducing the amount of money that Mr. Matsuzaka could have received. Finally, after the period of negotiation, Mr. Matsuzaka signed a six year, US$ 52 million contract. [1]

This situation, an auction to get the "rights to negotiate" with a third party, can arise in many contexts. For example, consider a government that wants to build an hydroelectric plant. The problem is that, in general, this kind of projects involve not

[1] This "Posting System" is the only way in which a Japanese player can be transferred to an American team, and it was supposedly created to protect the Japanese team and the Japanese player in a transfer. The argument is that without this system, a team could transfer a player without notifying him, and the final agreement could leave him very unhappy. On the other hand, a player could leave the Japanese league, and sign a contract with an American team, leaving the Japanese team uncompensated for the formation costs.

only the government or the firm, but almost always an interest group. There could be some communities that live in the areas that are going to be flooded, activists who do not want the environment to be modified, etc. Probably the winning firm will have to pay some compensations to them in order to carry out the project. Also, in the process of a firm acquisition, if there is a group with veto power, after making the highest bid some compensation to that group must be needed. How much of the initial bid should the government charge the electric firm if it decides not to finish the project? What auction format maximizes revenue? What about social welfare? and finally, how does different types of auctions affect the information that the firms have before it?

In this paper we introduce a model to study this phenomenon and analyze the effect that different auctions have on the result, the expected payoff of different agents and the incentives to acquire information. Our model is as follows. First, bidders (American teams) receive a signal which is informative about their valuation for Matsuzaka's services. Then they participate in a first price auction where only a part of the bid is paid upfront. The winner then know its true valuation (signal and valuation are affiliated) and Matzusaka makes a "take it or leave it" offer. If accepted the remaining part of the bid must be paid. We concentrate on what we have called β-auctions, which are first price auctions where the winner pays its bid if the negotiation succeeds, or a fraction β of its bid if it does not. Our main results are :

- In equilibrium, increasing the amount paid regardless of the negotiation result (ie. increasing β) implies that the negotiation will be more successful.
- Increases in β will also increase the utility of the agent who negotiates (in the posting system that is Matsuzaka), the expected payoff of bidders in the auction and social welfare.
- If the information structure is such that the final valuation is the signal to the bidder times some random variable (which we interpret as noise), and signals are distributed uniformly, then the bidding strategies are linear.
- The strategic value given to the information can even diminish the incentives to acquire information in any auction format.

In fact, increasing β makes more of the cost in the auction a "sunk cost" at the moment of the negotiation process. This in fact makes the last stage negotiator more aggressive, but this is more than compensated by the bidder's willingness to agree. This effect is obviously positive for both Matsuzaka and social welfare, since non-agreement implies a net loss of value and it is also detrimental for Matsuzaka.

However, this immediately shows that the effect of β on the Japanese team is ambiguous. Bigger β makes bidders less aggressive in the auction, knowing that they have to pay regardless of the final value of the good to them. This is bad for for the Japanese team but but must be weighted against the good news: higher probability of agreement also implies more rent.

For a particular information structure, where valuations are basically a signal (known at the moment of bidding) times some noise, we can actually characterize the equilibrium, and we provide some interesting comparative statics. In fact, for the examples presented, the Japanese team never prefers a standard auction (where everything is paid upfront), and in one case the "posting system" ($\beta = 0$) optimal. This could help us explain why such a system was chosen in the first place.

Finally, we turn our attention to the problem of endogenous information acquisition. In general, bidders have incentives to acquire information, since a successful bid in the auction is only useful if the value of the player to the team is below the total amount to pay. Acquiring better information, that is correlating the original signal with the valuation better, makes for better bids and therefore a better expected payoff. However, there is another effect in our model. A better correlation between the signal and the bid makes Matsuzaka better informed too! Since he infers from the bid the original signal, the correlation between both the signal and the valuation makes it easier for him to extract rent from the winning team. We show that many cases this leads American teams to not acquire any information, which is detrimental to social welfare.

The paper is organized as follows. First we present the general model. In the next section we analyze the case when the information structure is exogenous. Then we present the case when the information structure is endogenous.

1.2 Related Literature

In his classical article, Vickrey (1961) started auction literature. He was the first that gave a Game Theory approach to auctions. In the context of private values, he showed a strong equivalence[2] between an English and a first-price sealed auction, and a somehow

[2]Not only in equilibrium outcomes, but also in the whole game, strategies and the mapping from strategies to outcomes.

weaker equivalence[3] for the Dutch and second-price sealed auction. Also, for last auctions, he showed that the equilibrium allocation is always Pareto optimal.[4] Finally, he demonstrated that the four auctions lead to the same expected utility for the seller of the object, result that was later known as "Revenue Equivalence" theorem.

Myerson (1981), generalized that theorem. In his classical paper he showed that, again in a private values model, for two auctions with the same allocation rule[5] in which all the bidders have the same utility if their estimated value is the lowest possible, then all equilibria of both auctions lead to the same expected utility for the seller. Myerson went further, and showed that even when people might think that the classical auctions studied by Vickrey were the best thing to do for the seller, it was not the case. Sometimes giving the object to a bidder with not the biggest bid could make that in equilibrium the bids were higher, and therefore increase the expected utility of the seller.

Later, Milgrom and Weber (1982) analyzed the case in which agents are symmetric, but valuation are interdependent[6]. They showed that the English auction is not equivalent to the first price auction, generating a higher expected revenue. They also show than if agents are risk neutral, the second price auction has a higher expected revenue than the Dutch auction and the first price auction. Therefore, they showed that the "Revenue Equivalence" was a special case for the private values case.

Because of the ex-post bargaining procedure, the payoffs from the auction are not necessarily well behaved, certainly wore behaved than in a classical one. Therefore a problem of existence of equilibrium arise. Related to that, Athey (2001) shows conditions for some games of incomplete information to have equilibria. This basically depends on a single crossing property, which says that if whenever the players are using a non-decreasing strategy, the best response for a player is also non-decreasing. She also extend the result to cases when the payoffs are not continuous (such as auctions).

Later, Reny and Zamir (2004) find conditions for a first price auction to have an equilibrium in non decreasing pure strategies in an environment of interdependent valuations and asymmetric bidders. They allow for single dimensional and affiliated signals, and show that if there is a single crossing property (somewhat different that the classical one), there exists an equilibrium with the desired properties. What they do is to generalize the

[3]Both auctions have a unique dominant-strategy equilibrium, telling the truth, which conducts to the same outcome

[4]For the English and first-price auction that result holds if we are in a symmetric environment, it that is not the case, it could be that the equilibrium allocation is not Pareto optimal

[5]For example, two auctions in which the highest bid wins

[6]The information received by each bidder could affect the valuation of other bidders.

previous result of Athey, but restricting it only to first-price auctions.

Finally, Reny (2009) also extends Athey's results for general Bayesian games. Using another techniques (basically, a different fixed point theorem), he gets a weaker single crossing property to assure monotone pure strategies equilibria in those games. Even thought these papers lead to somehow general conditions, we could not adapt it to our problem. The main problem is that our game is a dynamic Bayesian game, thus beliefs arise, and to prove existence with the previous theorems we must fix such beliefs, use the theorems, and then show that the beliefs used where right. The last thing involves knowing more properties from the strategies than the theorem gives us.

Related to information acquisition in auctions, we can mention Hausch y Li (1991), who show that if valuations are independent, and the agents symmetric, first and second price auctions lead to the same information acquisition incentives.

Persico (2000) analyzed the incentives to acquire information when valuations are interdependent. He showed that in a general context, a first price auction induces more information acquisition than a second price auction. For that he gives a definition on what is in general something more informative.

Bergemann y Valimaki (2002) studied the problem of information acquisition in a context of mechanism design, specifically addressing the question of efficiency. They found that if valuations are interdependent the information acquired by agents is in general different to the socially efficient amount if we insist on a mechanism that is ex-post efficient. If we are in a private value model then if the information is acquired in a non cooperative game, a Nash equilibria will be Pareto optimal.

The information acquisition literature is a relatively new one, and there are some issues that have not been solved. For example, there is not a common definition of what a more informative structure means. We used such literature to have an idea of how to deal with endogenous information, and to focus on the problems that could arise. We did not give a general theoretic treatment as in those papers to the problem, but it was enough to have some interesting results.

Chapter 2

Model

To fix ideas we will concentrate on the "posting system". The agents who participate are the American teams and the player. Indirectly there is the Japanese team, but its participation is passive, he only receives money for being the player's owner.

There are $n \in \mathbb{N}$ teams that compete for the rights to negotiate with the Japanese player. First each team receives a signal, then with that information they bid, after that the winner is announced, the real valuation is revealed and it has to negotiate with the player. After the negotiation the winner team has to pay the amount accorded to the Japanese team, and depending on how the negotiation was, something to the player.

2.1 Information Structure

Each team has a valuation in $X_i \subset \mathbb{R}$ of the player that is unknown before the auction. But they have a signal of how this valuation might be. These signals have support in $V_i \subset \mathbb{R}$ and are informative because are linked to the real valuation by the joint distribution $g_i(.,.)$ in $V_i \times X_i$, that allows us to calculate how is the valuation distributed given a realization of the signal. This information is known by all the participants (teams, player, player's owner).

Definition 1. *The set* $\{V_i, X_i g_i(.,.)\}_{i=1}^n$ *is called* information structure.

In general, we will assume that the signal and the valuation are affiliated.

Definition 2. [1]*Given two random variables* $\{X_1, X_2\}$ *in* $X \subset \mathbb{R}^2$ *and their joint density distribution* f *, we say that both are* ***affiliated*** *if and only if for all* $x, x' \in X$*:*

[1]As in Krishna (2002)

$$f(x' \wedge x)f(x' \vee x) \geq f(x')f(x)$$

Where

$$x' \wedge x = (\max x_1', x_1, \max x_1', x_1)$$

$$x' \vee x = (\min x_1', x_1, \min x_1', x_1)$$

[2]

There are a lot of properties for this kind of random variable. We show some for those that are not familiar with this concept.

Proposition 1. *Given two random variables $\{X_1, X_2\}$ in $X \subset \mathbb{R}^2$ and their joint density distribution f such that $f : \chi \to \mathbb{R}^+$ is strictly positive and twice continuously differentiable, then both random variables are affiliated if and only if $\forall i \neq j$:*

$$\frac{\partial^2 f}{\partial x_i \partial x_j} \ln \quad f \geq 0$$

The proof of this proposition can be found in Topkis (1978).

Proposition 2. *Given two affiliated random variables $\{X_1, X_2\}$ and f, g two increasing (or both decreasing) functions, then $Cov\{f(X_1, X_2), g(X_1, X_2)\}$*

The proof is in Karlin and Rinot 1980. Both results say that affiliation is some kind of correlation in which bigger values of one variable are somehow associated to big values of the other, something that should happen in this environment.

Finally, even though many results presented on this paper holds for many situations, for simplicity we are going to assume independence between the signals and that all of them are identically distributed.

2.2 The Auction

So far, the schema used is not to different with a classical approach to auctions. So, what is different here? The main difference is that now the auctions are more complex, since the payment rules may depend not only on the bids, but also on the outcome of the negotiation. Then, an auction will consist in allocation rules, payment rules and the amount of information that is public after the winner is announced.

[2]We are saying that affiliation occurs when the natural logarithmic of the density function is super modular

Definition 3. *In this environment, an* ***auction*** *is define by a payment rule*

$$t : \mathbb{R}^n \times S \longrightarrow \mathbb{R}^n$$

, where $S = \{0, 1\}$ *is the set with the possible results of the negotiation (in this case,* $S = \{0, 1\}$*), the assignment rule*

$$p : \mathbb{R}^n \longrightarrow \Delta^3$$

and the rule of how much information is transmitted after the auction.

For example, imagine a pure common value auction, that is, when there is a single valuation of the player and the signals are estimations of that. If when the winner is announced the player knows all the bids, he will have a better starting point to negotiate than in the case where he only knows the offer of the winner. The same applies to a second price auction. Knowing the winner bid and how much he has to pay (that is, the second highest bid), is better for him than knowing only the second highest bid.

2.3 Negotiation

The last stage of the game studied on this paper involves the negotiation between the player and the winner of the auction. This can me model in many ways. In a framework of incomplete information one can use Rubinstein's bargaining approach, mechanism design, etc. In order to take this simple we consider a *take it or leave it* offer. This means that, after the winner is announced, the player makes an offer to the team. Knowing its valuation, and the payments it must make to the Japanese team and the player, the winner of the auction

With all the information in hand, it chooses to accept or reject it, and the game finish.

2.4 Strategies

Each team has to chose two things. First, its bid, and then, if it wins, either to accept or reject the player's offer.

Definition 4. *A* ***strategy of a team*** *will consist on a pair* $\{b_i(.), d_i(.)\}$*, with:*

[3] in general, Δ is the simplex in $\mathbb{R}^n$, but because we are in an auction, $p_i = 1$ if the bid i is the biggest one, and we will mix in case of ties

$$b_i : V_i \longrightarrow \mathbb{R}$$

$$d_i : X_i \times \mathbb{R} \times \mathbb{R} \times I \longrightarrow S^4$$

Where, $b_i(.)$ is its bid function, that depends on the signal received, and d_i is a decision function, which specifies if accept or not the player's offer, depending on the valuation, the bid made before, the offer made by the player and the information revealed after the auction

Definition 5. *A* ***strategy for the player*** *will consist in a function:*

$$u : I \longrightarrow \mathbb{R}$$

Where u is the amount that the player ask from the winner in order to accept the transfer

It is important to remark that, the player must form some beliefs about the winner's valuation. The player knows how is distributed, but after the auction some information is revealed, an he updates its information accordingly. Thus, an important part of his strategy is how he updates his beliefs about the signal received by the winner team, given the information released after the auction.[5]

2.5 Payoffs

We are going to assume that the utilities of the agents are lineal on the money. Given an auction, $\{p, v\}$ a valuation v', a result of the negotiation s, an offer of the player u, and bids $\{b_i\}_{i=1...N}$ the utility of a team i is given by:

$$U_i(b_i, b_{-i}, u, v', s) = sp_i(b)(v' - u) - t_i(b, s)$$

Hidden in that utility form is the assumption of a private value environment, since the utility of the teams depends only on their own valuation. This is a strong assumption, but allow us to simplify the problem. For the player it payoff is given by:

[4] I represents the information available after the auction. For example, if we are in a second price auction, it could be the first and second highest bids, or just the second.

[5] A belief is a distribution of then unknown parameter given the information available

$$V(s,u) = su$$

For the Japanese team, the utility will be the payments of the auction, that means:

$$J(b) = \sum_{i=1 \ldots N} t_i(b,s)$$

Chapter 3

Exogenous Structure

Now we analyze the case in which the information structure is exogenous ($\{V_i, X_i, g_i\}_{i=1}^N$. To do that we first define an equilibrium.

Definition 6. *Given an auction,* $\{\{b_i^*(.), d_i^*(.)\}_{i\leq n}, \{u(.), B(.)\}\}$ *is an equilibrium if and only if:*

1. $b_i^*(v_i) \in \arg\max_{b\in\mathbb{R}} \mathbb{E}[U_i(b, b_{-i}^*(v_{-i}), v', u^*(w), d_i^*(v', b, u^*(w), w))|v_i], \forall v_i \in V_i$
2. $u^*(w) \in \arg\max_{u\in\mathbb{R}} \mathbb{E}[V(d_{win}^*(v', b_{win}^*(v_{win})), u^*(w), w), u)|w], \quad \forall w \in I$[1]
3. *Beliefs are updated according to Bayes Rule whenever possible.* [2] *q*
4. $d_i^*(v', b, u, w) \in \arg\max_{d\in S} \mathbb{E}[U_i(b, b_{-i}^*(v_{-i}), v', d)|v', w], \forall w \in I; \quad \forall b, u \in \mathbb{R}, \quad \forall v' \in X_i$

Condition 4 states that given an offer by the player, the winner of the auction chooses optimally. Given the assumptions over the payoff functions we have:

$$d_i^*(v', b, u, w) = \begin{cases} 1 & u + t_i(b,1) - t_i(b,0) < v' \\ 0 & v' < u + t_i(b,1) - t_i(b,0) \\ x \in [0,1] & u + t_i(b,1) - t_i(b,0) = v' \end{cases}$$

It is just a matter of comparing $v' - u - t_i(b,1)$ (the utility of the team in case it accepts the offer), with $-t_i(b,0)$ (its utility if it does not accept the offer).

[1] "win" is the index of the winner

[2] This means that in the equilibrium the player knows the strategy of the team, and with this information estimates the signal received on the first stage.

Condition 3 is standard in dynamic games of incomplete information. In the auctions that will analyze, the auctioneer reveal the bid made by the winner, and because is our main interest in such auctions to find equilibria with increasing bid strategies, the beliefs in equilibrium will have to be a pointwise distribution (that is, for any winning bid, it will assigns a value of the signal, such that in equilibrium is correct). [3]

Condition 3 is very intuitive, indicates that the player maximizes its expected revenue with his take-it-or-leave-it offer. With our specification it becomes:

$$u^*(w) \in \arg\max_{u \in \mathbb{R}} u \times \mathbb{P}[d^*_{win}(.) = 1|\text{beliefs updated according to } w]$$

First, Matzusaka will only choose positive values. In zero, the objective function is zero, and if the valuation of the winning team is bounded, it is easy to check that $u^*(.)$ will exist for any w. [4]

Finally, the first condition is very similar to he one stating the optimality of the bid function in most auctions. However the bid made by a team could change the value that it assigns to the object (in this case, Matzusaka). Why is that? Because a change in the bid changes Matsuzaka's reaction, and therefore, the expected surplus of the winning team. It is formally equivalent to have an auction in which the value of the object depends on the signal and on the bid itself.

Even though at this point it seems that the environment used is very similar to the classic one (just replacing Matzusaka's strategies on the utility of an American team, and then solving a strange kind of auction) it is important to say that this is not the case. Introducing a negotiation stage , with incomplete information, makes things very hard. [5].

[3]For example, if we have an equilibrium with symmetrical bidding strategies, $b(.)$, then after seeing a bid b, in equilibrium Matzusaka will believe that the signal of such team as $b^{-1}(b)$.

[4]In case the valuation of an American team is not bounded, we need to assume that the probability that is multiplying u goes to zero quickly as u increases. We know that the probability goes to zero as u increases. In fact, $1 - \hat{F}(u)$ goes to zero as u increases, but we need it to go to zero quickly enough.

[5]Some equilibria that are not particularly interesting may arise. For example, if Matzusaka believes that everyone bids always the same amount $\bar{b} = 0$, and if someone does not do that, he punish it by believing it has the highest type (ie, asking for the biggest amount of money possible). Then, it will be an equilibrium to always offer zero. (for any auction in which the bid of the winner is announced). This kind of equilibria are not particularly interesting, but will always exist. Because of that, we will focus on looking for equilibria with increasing bidding strategies. But in that case, we may be being to restrictive. The theorems of existence in dynamic games of incomplete information are not strong enough to ensure the existence of such equilibria. See Athey (2001), Reny and Zamir(2004) and Reny (2006)

Now, we will study the properties of equilibria with increasing bidding strategies in case they exist.

Lemma 1. *Consider an environment with symmetric bidders, and two auction formats such that*

- *In both auctions the winning team has to pay its bid regardless of the result of the negotiation.*
- *The marginal payment that must be made to the Japanese team in case the negotiations are successful is the same and does not depend on the bid made by the winner team.*
- *In both auction formats the equilibrium involves a symmetric equilibrium with strictly increasing bid functions. Moreover, the worst type obtains the same utility in both formats.*

Then the expected revenue for the Japanese team is the same.

The proof is in the appendix, and it is a version of the typical Revenue Equivalence Theorem. This Lemma says that even though there is a negotiation stage, in this context the revenue will be the same, for example, between an all pay auction and a first price auction in which the amount bid must be paid regardless of the negotiation result.

As it might be expected, in this kind of auctions there's no general Revenue Equivalence theorem. Different auction formats modify the behavior of the player at the bargaining stage, therefore changing the team's problem at the auction stage. However, if the set of auction formats is narrowed, a similar result can be proved.

What is critical is that the marginal payment to be made to the Japanese team if the negotiation is successful is the same. If this marginal payment is different, then the behavior of the winner in the negotiation will also be different, affecting the valuations and therefore the auction revenue.

The lemma allows us to prove something stronger. It is possible, for a class of auctions with the above characteristics, to prove that the utilities for the Japanese player and the American teams are all the same.

Corollary 1. *Consider two auctions as in Lemma* 1*. Then the expected utility for the Japanese team, the Japanese player and the American teams are the same.*

Even though the previous result is a strong one, the set in which is valid is not big enough to make all the relevant comparison. For example, if we would want to compare the "Posting System" with an auction in which the biggest bid is always paid to the Japanese team[6], the previous result does not help, since th e marginal payment is different.

However, in general it is not easy to find equilibria and we are not even sure if for any auction there will be one with increasing bidding strategies. We have to narrow our search, and focus on a particular family of auctions that include, as special cases, the standard first price auction and the posting system.

3.1 Analysis of $\beta-$auctions

Definition 7. *A $\beta-$**auctions** is given by:*

- *The highest bid wins, and that bid is announced after the auction. Losers do not pay.*
- *If the negotiation reaches an agreement, the winner of the auction has to paid its bid to the Japanese team, and the result of the negotiation to Matsuzaka.*
- *If the negotiation does not succeed, the winner pays only a fraction β of its bid to the Japanese team, and nothing to Matzusaka.*

With this we can study a wide range of auction, including the "posting system" ($\beta = 0$), and the more classical or intuitive "standard auction" ($\beta = 1$). Our study will focus on what we can say about the expected utilities of the agents for different $\beta-$auction.

Before presenting you our main results, we discuss a little the intuition behind changes in β, the amount that the winning team is forced to pay regardless the result of the negotiation. A change in β will affect the equilibrium in two ways:. trough changes in Matzusaka's strategy and trough change in bidding strategies.

But, how are this changes, when β is increased?. We were tempted to say that is a trivial question. Increases in β should make the American teams to bid less. And for Matzusaka, for any winning bid, an increase in beta should make him to do more agrees

Let's analyze how the changes in β affect the problem of the American teams. Assuming that in equilibrium Matzusaka plays $u^*(.)$ and we the bidding strategies are symmetric and increasing, the bidding function of an American team satisfies:

[6]that we will call "standard auction"

$$b^*_\beta(v) \in \arg\max_{b\in\mathbb{R}^+}\{ \int\limits_{v'\geq u^*(b)+(1-\beta)b} v' - u^*_\beta(b) - (1-\beta)bdv' - \beta b\} F^{n-1}(b^{*-1}_\beta(b))$$

and Matsuzaka's offer satisfies:

$$u^*_\beta(b) \in \arg\max_{u\in\mathbb{R}^+} u\mathbb{P}[X \geq u + (1-\beta)b|B(b)^7]$$

In equilibrium, the bidding strategy of an American team balances three effects. First, by making bigger offers the American team has more probabilities of winning. Second, by making bigger offers, the American team will have to pay more regardless of the success of the negotiation. And third, for given beliefs, bigger offers make Matzusaka to change the amount offered. This effect can be decomposed in two mini effects. On the one hand, because the bid was bigger, the American team could say "I paid a lot before, so I can't pay you a lot now". In other words, the size of the prize that Matzusaka could take becomes smaller. On the other hand, if we are in an eq. with increasing strategies, Matzusaka's beliefs will make him to assume that the type of the winner is higher, and therefore, he will ask for more money. The sum of these effects have no clear sign.

Now, what happens with these effects if we increase β? In equilibrium, the first effect will change depending on how the slope of the eq. strategy changes. For example, if it is lower, then it will become stronger, because a little change on the bid will be equal to say it lot different signal. It is clear that the second effect will become stronger. But for the third effect we do not know what happens. Then, it is not direct that how the bidding strategies change when we increase β.

These are not good news. Changes in β appear to make unknown changes in the equilibrium strategies, and therefore on the utilities of the agents and expected welfare. So what can we say ?

After studying the problem, we found that we do not need monotonicity on β over the bidding strategies, we need something less strong to be able to compare the social welfare and Matzusaka's utility in two different $\beta-$auctions. The next lemma helps us to do that.

Lemma 2. *If the winning team made an offer of b, and we have that,* $\lim_{x\to\infty} x\mathbb{P}[v' \geq x|B(b)] = 0$, *and* $\mathbb{P}[v' \geq x|v]$ *is* $C^2[\mathbb{R}]$, *in equilibrium;*

[7] The probability is given the updated beliefs, but as we explained earlier, in this case the beliefs are updated according to a function that maps the bidding space to the signal space. Therefore $B(b)$ are the beliefs of Matzuzaka about the winners type.

i. Matzusaka will make an offer in $(0, \infty)$

ii. In equilibrium, Matzuzaka's strategy will satisfy:

$$\frac{1 - F(u^*_\beta(b) + (1-\beta)b|B(b))}{f(u^*_\beta(b) + (1-\beta)b|B(b))} = u^*_\beta(b)$$

With $F(.)$ *the distribution of the valuation.*

Proof. Direct from the FOC. □

The lemma allows us to find some condition under which(hopefully) we can rank the $\beta-$auctions.

Proposition 3. *Suppose you have two* $\beta-$*auction* (β_1 *and* β_2)*, such that for both* β*s there is a symmetric equilibrium with increasing bidding strategies. If that* $\bar{b}(\beta, v) = (1-\beta)b^*_\beta(v)$ (*with* b^* *the equilibrium bidding strategy*) *is higher for* β_1 (*for any* v) *, then we have that Social Welfare and Matzusaka's expected payoff is higher, or equal, in* β_1 *than in* β_2*.*

Proof. See appendix. The sketch of the proof is this. First, we show that in equilibrium, given the signal, the social welfare (and also the probability of agreement) is decreasing on $u^*_\beta(b^*_\beta(v)) + (1-\beta)b^*_\beta(v)$. Then, using the previous lemma, and because under affiliation we have decreasing inverse hazard rate,we show that if $\bar{b}(\beta_1, v) \geq \bar{b}(\beta_2, v)$ we have the expected result. Using the same argument, we show that also $u^*_{\beta_1}(b^*_\beta(v)) \geq u^*_{\beta_2}(b^*_\beta(v))$. Therefore Matzusaka's utility is greater for β_1.

□

Matzusaka faces two effects. If he increases his offer, when an agreement is reached, he will get more money. But, if he increases it, the probability of an agreement will be lower . Now, after a change in β, our assumption implies that the marginal payment will weakly decrease. If that is the case, for the same offer by Matzusaka there will be a higher probability of acceptation, thus in equilibrium, his expected utility will be higher. This is a result that only depends on the monotonicity of the marginal payment.

The social welfare depends basically on the probability that the negotiation reaches an agreement. What we are saying is, even if Matzusaka increases his offer, because of the affiliation that implies a non increasing inverse hazard rate, Matzusaka won't increase it more than the amount in which the marginal payment was reduced.

Now, lets discuss how real are these assumptions. Affiliation is a very weak form of correlation between the signal and the valuation. In general it is well accepted in auction theory. Not that obvious is the one related to the marginal payments. As we discussed earlier, we do not know how the increasing bidding strategies will react to changes in β. Intuition says that it should decrease, and if you check the next section you will see that for the specific cases studied that is what happens.

We have a corollary, however, that dispenses with these problems. One thing we can do is compare each $\beta-$auction with the standard auction (ie. $\beta = 1$).

Corollary 2. *Suppose that with $\beta = 1$ ("standard auction") there exist an equilibrium with increasing bidding strategies. Then, for any other $\beta-$auction with an equilibrium in increasing bidding strategies, the social welfare and Matzusaka's utilities will be smaller.*

Proof. This comes from the fact that $B(\beta, v) = (1-\beta)b^*_\beta(v)$ equals zero when $\beta = 1$, thus in any of the auctions being studied, we will have at least that $B(\beta, v) \geq 0$, then using the previous proposition, we have that the expected social welfare will be maximized in $\beta = 1$ □

This theorem shows the drawback of the "posting system". It induce lower rents to the player, and overall, to the society. Then, the only reason why this system could be implemented is because it could increase the utility of the American or Japanese teams. In fact, in the cases studied in the next section we show that for the Japanese team it does not appear to be a general rule of the behavior of its expected utility when β is raised. On the other hand, for the American teams, their payoffs are increasing in β.

3.2 Linear Strategies

The previous analysis assumes the existence of an equilibrium in increasing bidding strategies. While this is not in general guaranteed, there exist environments where it is possible to find conditions under which we can assure the existence of such strategies. Moreover, we will show that such optimal strategies exist and, moreover, are linear.

Definition 8. *Let X and Y be two random variables. We will say that X and Y have the property A if for every $K \in \mathbb{R}^+$ we have:*

$$F(Kx|y) = H(K)$$

With $F(.|y)$ the distribution of X given a realization y of Y, and $H : \mathbb{R}^+ \to [0,1]$ non decreasing.

The property A is not uncommon. For example if $X = Y * Z$ with Z independent from Y, the property holds. For our purpose, Z could be a random variable that makes the signal noisy with respect to the valuation.

For the specific case in which the signal is uniform, we have that if the previous property holds, then it does not matter how the valuation is distributed, there will be no equilibrium with increasing bidding strategies, or a linear one. In other words, for those familiar with auction theory, given the property, the negotiation stage preserves the linearity of the auction.

Lemma 3. *If property A holds for the signal V (which is uniform in $[0,1]$) and the valuation X and both are affiliated, so $F(Kv'|v) = H(k)$, denoting H' by h, we have that, a*

$$
\begin{aligned}
k &= \frac{1-H(.)}{h(.)} && (3.1)\\
0 &= \{[u' + (1-\beta)][1-H(.)] - (1-\beta)\} + && (3.2)\\
&\quad \{\int_{\bar{k}}^{\bar{x}} x \cdot h(x)dx - (k + (1-\beta)c)H(.) - (1-\beta)c\}(n-1)\frac{1}{c} && (3.3)\\
u' &= -\frac{(1-\beta)(h(.) + k\cdot h'(.)) - c^{-1}(c\cdot h(.) + kh(.) + k\cdot c\cdot h'(.))}{k\cdot h'(.) + 2h(.)} && (3.4)
\end{aligned}
$$

can be solved in c and k, an equilibrium with linear bidding strategies exists. In this case, H, h, h' are evaluated on $\bar{k} = k + (1-\beta)c$.

Proof. See appendix. □

For $\beta = 1$, we can actually prove existence and characterize the bidding strategies.:

Corollary 3. *Given the assumptions of the previous lemma, for $\beta = 1$ there exist a symmetric equilibrium with linear increasing bidding strategies*

Proof. See Appendix. □

3.3 Equilibrium

We now compute the equilibrium in two environments. We assume that signals (v)are independents across teams, identically distributed and uniform in $[0, 1]$. For the valuation (v') either:

1. The valuation v' given the signal is distributed according to $G(v'|v) = 1-\frac{1}{v}\exp[-\frac{v'}{v}]$, that is, exponential of mean v. We are saying that the valuation is the product between the signal and a random variable with exponential distribution of mean 1.
2. The valuation, given the signal, is distributed uniform in $[(1-\alpha)v, (1+\alpha)v]$, with $0 < \alpha < 1$. In other words, the valuation is the signal times a random variable with uniform distribution in $[1-\alpha, 1+\alpha]$

In each of these formats we also computed the expected utilities of the agents. We were able to find the equilibrium conditions to every $\beta-$auction, and even more, we could prove than in both cases the bidding strategies are linear in the signal (as we predict in the previous section), for every β. The result is the following:

Lemma 4. *In both the exponential and the uniform case, for any $\beta-$auction there exist an equilibrium with linear bidding strategies ($b_\beta(v) = C^x(\beta)v$, with $x = u$ or e), with the slope satisfying for the exponential case:*

$$[n-2-(1-\beta)C^e(\beta)]\exp[-1-(1-\beta)C^e(\beta)] - n\beta C^e(\beta) = 0$$

and for the uniform case:

$$\{(1+\alpha)-(1-\beta)C(\beta)\}\{(n-3)(1+\alpha)-(n+1)(1-\beta)C(\beta)\} - 16n\beta\alpha C(\beta) = 0$$

[8]

depending on α being higher or lower than $\frac{1}{3}$

Proof. The existence of a linear strategy comes from the previous proposition. For the exponential case the equation comes from replacing in the system of equation obtained

[8]Note that this is a quadratic equation, thus there are two solutions, but for a particular case, one will be discarded. Also, the solution has to be such that if $C(\beta)(1-\beta) \geq (1-3\alpha)$ in other case there is no solution. If α is greater than one third, there won't be any problem.

	Posting system ($\beta = 0$)	Standard Auction ($\beta = 1$)
American Team	$\frac{1}{(n+1)e^{(n-1)}}$	$\frac{2}{n(n+1)e}$
Matzusaka	$\frac{n}{e^{(n-1)}(n+1)}$	$\frac{n}{(n+1)e}$
Japanese Team	$\frac{n(n-2)}{e^{(n-1)}(n+1)}$	$\frac{(n-2)}{(n+1)e}$
Social Welfare	$\frac{n^2-n+1}{e^{(n-1)}(n+1)}$	$\frac{n^2-n+1}{n(n+1)e}$

Table 3.1: Utilities, Exponential Case

	Posting system ($\beta = 0$)	Standard Auction ($\beta = 1$)
American Team	$\frac{(1+\alpha)^2}{\alpha}\frac{1}{(n+1)^3}$	$\frac{3(1+\alpha)^2}{16\alpha n(n+1)}$
Matzusaka	$\frac{2(1+\alpha)^2}{\alpha}\frac{n}{(n+1)^3}$	$\frac{(1+\alpha)^2}{8\alpha}\frac{n}{n+1}$
Japanese Team	$\frac{n(n-3)(1+\alpha)}{(n+1)^2}$	$\frac{(1+\alpha)^2(n-3)}{16\alpha(n+1)}$
Social Welfare	$\frac{n^2-n+1}{e^{(n-1)}(n+1)}$	$\frac{n^2-n+1}{n(n+1)e}$

Table 3.2: Utilities, Uniform Case, $\alpha \geq \frac{1}{3}$

before with the density function associated to such exponential. For the uniform is the same, but to ensure that all the agents have strategies in the interior of their maximization problems, we have impose that $\alpha \geq \frac{1}{3}$. The details are in the appendix. $\square$

For the cases with $\beta = 1$ or $\beta = 0$ it is possible to compute the strategies explicitly, and also it is possible to check that the only equilibrium with increasing bidding strategies is one satisfying the conditions of lemma 4. In the next tables we show the expected utilities for each agent in these auctions.

There are some interesting facts to note. First, for both cases revenue equivalence holds (so an all pay auction is equivalent to a first price standard auction). Second, the social welfare and Matzusaka's utility are greater when the winner of the auction is forced to pay (standard auction) than in the "posting system", as we showed previously. Moreover, the utility of the American teams is also greater when the winner is forced to pay. For the Japanese team, however, we do not have such a result. Depending on n, in the exponential case, it can be the case that it is better off in one of the two extreme mechanism, with $\beta = 1$ or $\beta = 0$. For the uniform case, it will depend on n and α with which β the Japanese team is better.[9] Table 3.1 shows these results for the exponential case. Table 3.2 for the uniform case.

This last result is not that unexpected. When the winner of the auction is forced to pay, it receives a bigger fraction of the total amount paid. But that amount may decrease because knowing that, the American Teams will make lower bids. That's why sometimes the posting system may (or may not) have advantages for the Japanese team. The dependence on n for the exponential case has to do with how sensible is the bidding function to changes on that value. With low n there is not much competence, and then the American Team bid less than with a lot of competitors, because it is more probable to win the auction. In the uniform case, is must be something like that.

For the general $\beta-$auctions, as we said, the strategies are linear, and satisfy specific equations. For the exponential case we can go further, and prove some interesting things.

Proposition 4. *In the exponential case, for equilibria with linear bidding strategies ($b_\beta(v) = C(\beta)v$), the slope $C(\beta)$ is decreasing on β. Therefore Matzusaka's utility and social welfare are increasing on β*

Proof. See Appendix. □

What can we say anything for the other agents? If we compute the expected utilities and put them on a graph, the resulting figures showed on Fig. 1.

Numerically, the expected utilities change with β as it's shown on the fig. 3.1. Corroborating the previous proposition, we have a monotone function for Matzusaka's and the American teams' expected payoffs. On the other hand, the utility of the Japanese team has an interior maximum, different from zero or one. This is due to the interaction of the effect explained before (as β increases, $b(.)$ decreases) and also the changes in the probability of agreement, which obviously affect the Japanese team when part of the payment is made only if the negotiation is successful.

More rigorously, it can be proved in this particular setting that:

Proposition 5. *In the family of auctions described before, the American team and the player's utility, and the total welfare are increasing on β. The Japanese team's utility is increasing at $\beta = 0$, but decreasing at $\beta = 1$, therefore there is a global interior maximum.*

[9] We also have no existence of equilibrium with increasing bidding strategies if $n \leq 2$ for both cases, and when $\alpha \leq \frac{1}{n}$ in the uniform case

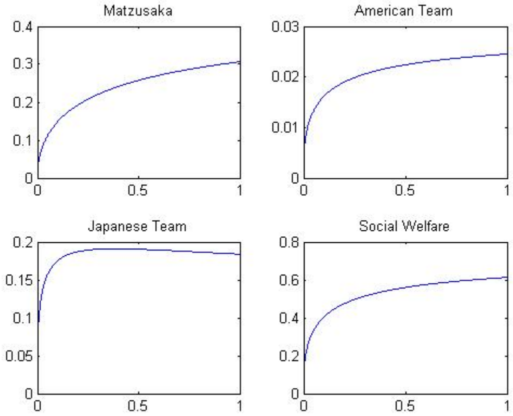

Figure 3.1: Utilities vs β, Exponential Case

Proof. See Appendix. □

This proposition says that almost everyone benefits with an increase in β. In fact, for everyone but the Japanese team, the optimal mechanism is the standard auction ($\beta = 1$). However, for the Japanese team its utility is not maximized neither at zero nor at one. It is not that the "posting system" is optimal, in fact a small increase on β has always a positive impact on the Japanese team's utility. We conclude that there is no rationale why the "posting system" should be implemented.

For the Uniform case, we have the same results, except for the Japanese team. Figure 2 illustrates that.

In general, if the values of α are bigger than $\frac{1}{3}$, or n is big enough, the Japanese team's utility is decreasing on β. If this distributions were the real ones, we could explain the existence of the "Posting System´´, because the Japanese team has all the incentives to use it.

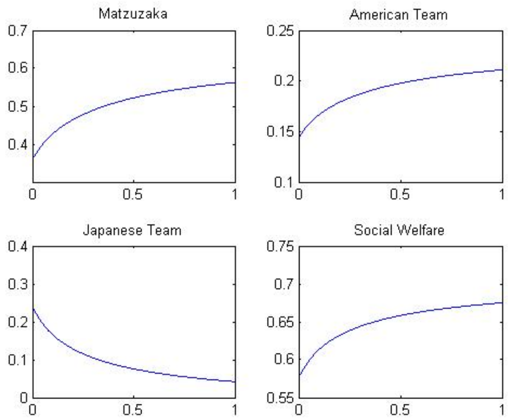

Figure 3.2: Utilities vs β, Uniform Case

Chapter 4

Endogenous Structure

So far we have assumed that the information structure is given. However, in this kind of environments this is not always true. Choosing different kind of auctions might change the incentives that agents have to acquire information, thus the auction mechanism will sometimes endogenously determine the information structure of the auction. In this particular example, someone may think that in the Posting System and the Standard Auction create different incentives to acquire information, for the American teams, since in the posting system they do not have to pay if later they find out that the valuation is not what they thought it was.

We now try to find out what kind of things can happen when the information structure is no longer exogenous, and try to reconcile those results with the existing literature of information acquisition in auctions. First we will define what is in this context endogenous information structure, and the environment in which we are going to work. We will study what happens in that particular setting if the information is acquired only by the American teams if they can coordinate, and if the can not. In both cases we will assume that the information is costly to obtain.

4.1 Setting

We will consider the second example used on the previous section, in which the signal is a uniform random variable in $[0, 1]$ and the valuation conditional on a realization of the signal(v) is uniform in $[(1-\alpha)v, (1+\alpha)v]$, with α in $[0, 1)$. The information will be represented in α, with bigger α representing more information. We consider a cost function $C : [0, 1] \to \mathbb{R}^+$ which represents the monetary cost of obtaining an amount α of information. Because less α makes a more informative signal, the cost of acquiring should be always decreasing, and to avoid problems, we are going to assume that the cost are convex.

Is this a good way to model the information in the problem studied? We can say that it is at least a pretty intuitive one, because the quality of the information is directly related with the noise of the signal. Moreover, this way to model the information structure satisfies the order of Persico (2000). [1]

4.2 Teams can coordinate

First, we will analyze what happens if the American teams can coordinate to acquire information. This means that they can agree to buy a given amount of information, which improves the correlation between each individual signal ant the true valuation.

Proposition 6. *If teams can coordinate to acquire information, and there exists an equilibrium, then the "Posting System" auction induces more information acquisition than the "Standard Auction".*

Proof. The problem that we have to solve is:

$$\max_{\alpha} \mathbb{E}(U|\alpha)_j - C(\alpha)$$

Where $C(\alpha)$ is the cost of α, and j is an index for the auction, with $j = 1$ the posting system, when $j = 2$ the standard auction.

First of all, we have to force that α_j^* (the $\arg\max$ of the problem) be bigger than $\frac{1}{n}$, because for values smaller than that we don't have an equilibrium. Second, for the case *winner pays*, α_2^* can't be smaller than $\frac{1}{3}$. If it is smaller, then the team will pay more than $C(1/3)$, but its utility will be the same than $\mathbb{E}(U|\alpha = \frac{1}{3})_2$.

In general, $C(.)$ can induce a small α_1^*, so there will be situations where the first auction will make the teams to buy more information. But, are there any cases that induce more information acquisition in the second auction? You might think that it must be, because if you have to pay whenever you win, the information has a bigger value, and the intuition says that this gives more incentives to buy information. So lets check if there's any. Suppose that $\alpha_1^* > \frac{1}{3}$. Then the problem solved by the team in the auction j is:

[1] But not the one of Bergemann and Valimaki (2002)

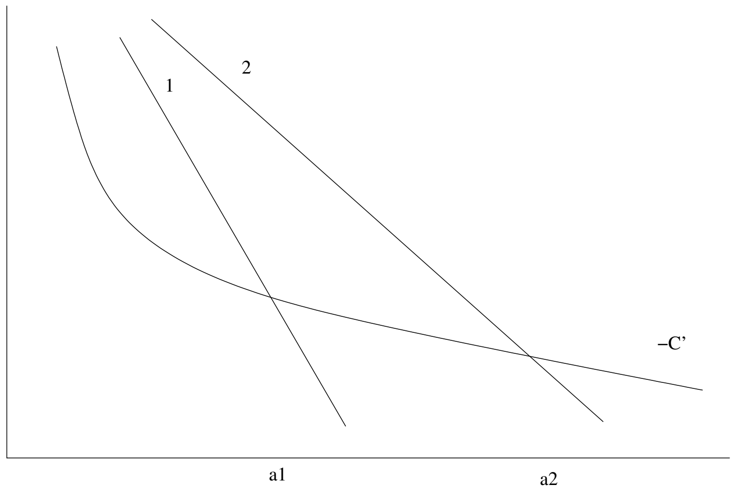

Figure 4.1: Coordination

$$\max_{\alpha} K_j \frac{(1+\alpha)^2}{\alpha^2} - C(\alpha)$$

The FOC is:

$$K_j 2 \frac{1+\alpha}{\alpha^2}(1 - \frac{1+\alpha}{\alpha}) = C'(\alpha)$$

$$-C'(\alpha_j^*) = 2K_j \frac{1+\alpha_j^*}{\alpha_j^{*3}}$$

For the assumptions that we have made, $-C'(.)$ decreasing, and $C\frac{1+x}{x^3}$ too. So for each j we are intersecting two decreasing functions. For the other hand, we know that $K_1 > K_2$ (easy to check). So it must be that in this case, $\alpha_1^* < \alpha_2^*$, (if the two exist).

□

4.3 Teams buy information

We now consider the more interesting case, where each individually chooses the amount of information to buy before the auction takes place. This requires that we add a stage of information acquisition, and the game changes a little. A strategy for a team now consist on α, $b(.)$ and a function to decide to accept or not the offer. For the player, we have to add beliefs of the amount of information acquired by each team (and for each team, the beliefs of how much information did the other teams acquire).

We look for a symmetric equilibrium, that will include the ones calculated on the previous section, and consider those strategies in the information acquisition stage where nobody has incentives to deviate.

The main problem that arises is that now, more information is not necessarily better. Without the negotiation process, one can rank different experiments and then conclude that in some good orders, better information means greater expected utility (see Persico 2001). But now, acquiring information affects the American team's utility in two ways. First, with better information they can bid better, therefore, if nothing changes it is logical to assume that its expected payoff will increase. But, on the other hand, in an equilibrium, the Japanese player knows that the team has better information, and then in the negotiation process he will be in a much better position to extract rent. To illustrate this, think in the information structure we are using. With better information, a team has a better estimator of what his valuation will be. But, at the moment of the negotiation, in equilibrium, the Japanese player has correct beliefs of that, and therefore, knows that the final valuation will be in a narrower interval. This leaves the winning team with less surplus than in a case with less information.

This is because now the information has not only a statistical value, but also a strategic one. It modifies the expected utility and the action taken by the other agents. This is one of the main differences between the literature of information acquisition in auctions (and mechanism design as well) and our case of study, and is what makes this problem quite difficult.

Having said so, we have some interesting results about the amount of information acquired in this case. Now, we want to compare the posting system with other auction formats. First we study the case in which the winner is forced to pay its bid regardless of the negotiation results, what we have called the standard auction. It is not obvious what happens, because intuitively someone might think that because of the payment, there are stronger incentives to be more informed. But, more information means that the negotiation is tougher. In the end, the second effect dominates, and it is not possible to find an symmetric equilibrium in which there is positive information acquisition. We present this result in two propositions.

Proposition 7. *In this environment, any equilibrium in symmetric strategies in the*

"Posting System" induces no information acquisition.

Proof. Suppose that all the teams are playing α, and $b(.) = \frac{n-3}{n+1}(1+\alpha)v$ [2], and the player knows that (this means that he will play $\frac{(1+\alpha)b^{-1}(b)+b}{2}$. A deviation consist on acquiring $\bar{\alpha}$ different to α, and having a different bidding function of the one that everyone else is playing. The optimal deviation can be calculated as:

$$\bar{b}(v) \in \arg\max_{b<(1+\bar{\alpha})} \frac{1}{4\bar{\alpha}v}[(1+\bar{\alpha})v - u(b)]^2 b^{-1}(b)^{n-1}$$

The objective function can be written as:

$$\frac{1}{4\bar{\alpha}v}[\frac{n+1}{(n-3)(1+\alpha)}]^{n-1} b^{n-1}[(1+\bar{\alpha})v - \frac{n-1}{n-3}b]^2$$

It's easy to check that the best response for a team is playing:

$$\bar{b}(v) = \frac{n-3}{n+1}(1+\bar{\alpha})v$$

Then, if a team decide to deviate, choosing a different α, its utility will be:

$$\mathbb{E}[\bar{U}|v] = \frac{1}{4\bar{\alpha}}\frac{(1+\bar{\alpha})^{n+1}}{(1+\alpha)^{n-1}}[\frac{2}{n+1}]^2 v^n$$

And the utility is:

$$\mathbb{E}[\bar{U}] = \frac{1}{(1+\alpha)^{n-1}}\frac{1}{(n+1)^3}\frac{(1+\bar{\alpha})^{n+1}}{\bar{\alpha}}$$

Now, each team will choose the $\bar{\alpha}$ that maximize $\mathbb{E}[U]$. Let's analyze how does the expected utility change when we move $\bar{\alpha}$.

$$\frac{d\mathbb{E}[U]}{d\bar{\alpha}} = \frac{1}{(1+\alpha)^{n-1}}\frac{1}{(n+1)^3}\frac{(1+\bar{\alpha})^n}{\bar{\alpha}}\{n\bar{\alpha} - 1\}$$

We don't want to exist incentives to deviate, so we force that every team choose $\bar{\alpha} = \alpha$, in other words, we want that:

$$\alpha \in \arg\max_{\bar{\alpha}\in A} \mathbb{E}[U] - C(\bar{\alpha})$$

[2]This is the equilibrium bid strategy if everyone have the same amount of information, calculated on the previous section

Where $C(.)$ (the cost of acquiring $\bar{\alpha}$) is decreasing, and convex[3], and A is the set of possible information.[4] But because in the set that we are looking for equilibrium less information is better for a team, we only have two options. The first is that the optimal deviation for a team is buying an α lower than $\frac{1}{n}$. In that case there is no equilibrium in symmetric strategies. This could happen if the cost on buying information is not very high for small α. And second, a symmetric equilibrium could emerge for some cost functions that induce to not acquire any information. In effect, if we have an upper bound, greater than $\frac{1}{n}$, we have a local maximum in there, because both functions are increasing in that value.

□

Proposition 8. *In the "Standard Auction", there is no symmetric equilibrium in which the information has values in the interval* $(\frac{1}{n}, 1)$.

Proof. See appendix.

□

Basically, what we have proved is that in a symmetric equilibrium, there are incentives to acquire less information ex ante. In other words, the strategic effect of the information dominates.

[3]Less α represents more information

[4]In this case, we need that $\alpha > \frac{1}{n}$, because for values lower than that there was not any equilibrium, and we can force $\alpha < 1$ if we said that that's public information.

Chapter 5

Conclusions

There are three main conclusions of this work that can be saved. First, we have shown that forcing to pay the bids before the negotiation makes it more successful. When this happens, the bidders seems to make lower bids and therefore there is a much greater set in which there will be an agreement in the negotiation stage.

Second, we have shown that in general, forcing to pay the bids before a negotiation dominates not doing that in terms of social welfare, and is also better for the negotiator (in this case, Matsuzaka). This comes directly from the bigger quantity of agreements that paying the bid does. In a more specific environment, we show that this relation holds also for the bidders, but not always for the auctioneer. In terms of the Posting System, this could be explaining why it exists. We have shown that for Matsuzaka and the sum of the agents it is not good. But maybe for the Japanese team its better to wait until the negotiation reaches an agreement to collect the bid.

Finally, we have shown that the stage of negotiation adds a problem to the already difficult problem of information acquisition. In general, the literature has only given a statistical value to the information, but in this case this is no longer true. Being better informed could make that the negotiator be harder on me if I reach that stage, thus sometimes could be better to be not informed at all. Our conclusions in this work probably have a lot to do with the model on negotiation chosen. However, this is a very simple model, and with little complications we have no information acquisition at all, thus something must be happening with the stage of negotiation that in a normal does not arise.

Chapter 6

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Chapter 7

Appendix

7.1 Examples

7.1.1 Non existence of equilibrium with increasing bidding strategies

Consider for example a second price auction in which the winner team is forced to pay its bid only if it accepts Matzusaka's offer, and that the designer only announce the second highest bid. Then, the strategy used by Matzusaka will only depend on that value (and on the beliefs that he has about the winners type). Therefore, the problem that a team will solve to chose its bid will be:

$$\max_b \int_0^b L(v,x) f_v(x) dx$$

Where x represents the second highest bid, and v the type of the team. $L(.,.)$ is the expected utility given the second highest and $f_v()$ is the density function of x given v. In this case, because the winner pays its bid only when it accepts the offer of the player, the expected utility $H(v,x)$ will alway be greater than zero, therefore the optimum bid in this case is the maximum b possible. That value could be any b such that $\forall x \geq b, f_v(x) = 0$. If the support of $x|v$ is $[\underline{v}, \infty)$ there won't be any equilibrium. If the support is $[\underline{v}, \bar{v}]$ there would be multiple equilibrium that will consist on bidding anything greater than $\bar{v}$ given any v.

7.2 Proofs

7.2.1 Revenue Equivalence

As it might be expected, in this kind of auctions there's no general Revenue Equivalence theorem. Different auction formats modify the behavior of the player at the bargaining stage, therefore changing the team's problem at the auction stage. However, if the set of auction formats is narrowed, a similar result can be proved.

Basically, if the marginal payment from an American team to the Japanese team is the same in two auction formats, and the equilibrium involves strictly increasing strategies in both, such a result exists. Intuitively, this is not direct. First, because the utility of the bidders does not depend only on his type (in this case, the signal about its valuation), but also on the type they say they are. And in other cases, it also depend on the particular setting of the auctions. The idea of the result obtained is to find a family of auction in which the format used does not strongly affect the behavior of the bidders, and then show that even when the marginal utility of a team depends also on what they say its type is, the Revenue Equivalence holds.

Lemma 5 (1)**.** *Consider an environment with symmetric bidders, and two auction formats such that*

- *The marginal payment that must be made to the Japanese team in case the negotiations are successful is the same and equal to the bid made by the winner team (whom also get's the right to negotiate).*
- *In both auction formats the equilibrium involves a symmetric equilibrium with strictly increasing bid functions. Moreover, the worst type obtains the same utility in both formats.*

Then the expected revenue for the Japanese team is the same.

Proof. The problem solved by the player in the bargaining stage depends only on the beliefs generated by the winning bid. If the symmetric equilibrium involves strategies $b(\cdot)$, the player solves given a winning bid b,

$$\max_u u\mathbb{P}_{b^{-1}(b)}[v' > u + K]$$

If the optimal offer is u^*, then the team that wins the auction can anticipate an expected profit of $E_v[\max\{0, v' - u - k\}]$. Since u^* depends on the beliefs induced by the winning bid b through the function b^{-1}, we can write this expected utility as $U(v, b^{-1}(b))$.

Therefore, at the auction stage, assuming that the beliefs of the Japanese team are that everybody plays strategy $b(\cdot)$, an American team chooses $b_i(\cdot)$ to solve

$$b_i(v) \in \arg\max_b P(b)U(v, b^{-1}(b)) - t^a(b)$$

where $P(b)$ is the probability of winning, in this case $F^{n-1}(b^{-1}(b))$, and $t^a(b)$ is the expected payoff in the auction a given the bid. If we consider that choosing $b_i(v)$ is equivalent to misrepresent the type, and bid as type $\bar{v}$, the problem of the team can be written as:

$$\max_{\bar{v}} P(b(\bar{v}))U(v, \bar{v}) - t^a(b(\bar{v}))$$

Since the probability of winning depends only on representing the highest type, $P(b(\bar{v})) = F^{n-1}(\bar{v})$, so finally the problem can be written as

$$\max_{\bar{v}} \bar{P}(\bar{v})U(v, \bar{v}) - t^a(b(\bar{v}))$$

The FOC is:

$$\bar{P}'(\bar{v})U(v, \bar{v}) + \bar{P}U_2(v, \bar{v}) - \frac{dt^a(b(\bar{v}))}{d\bar{v}} = 0$$

And in the equilibrium, we will have:

$$\bar{P}'(\bar{v})U(v, v) + \bar{P}U_2(v, v) - \frac{dt^a(b(v))}{dv} = 0$$

Thus,

$$t^a(b(v)) = t^a(b(0)) + \int_0^b \bar{P}'(b)U(v, v) + \bar{P}U_2(v, v)dv$$

which does not depend on the auction.

□

□

This lemma, allows us to prove something stronger. It is possible, for a class of auctions with the above characteristics, to prove that the utilities for the Japanese player and the American teams are all the same.

Corollary 4 (1)**.** *Consider two auctions as in Lemma* 1*. Then the expected utility for the Japanese team, the Japanese player and the American teams are the same.*

Proof. By the previous lemma, we know that the expected utility for the Japanese team is the same. Moreover, since the player's problem only depends on the (always correct) beliefs about the winning team's signal, and not on the auction format, it is also the same. The same argument applies to total surplus, since for a given winning team's valuation it only depends on the success of the negotiation, which does not depend on the mechanism and, moreover, the valuation of the winning team does not on the auction format (the equilibrium bidding strategies are increasing). The three previous facts imply that the expected utility of an American team is also the same.

□

□

7.2.2 Proposition 3

First, the Social Welfare in this context equals to:

$$SW = \int_0^1 \int_{u(b(v))+(1-\beta)b(v)} v' dF(v'|v) dF_m(v)$$

Where F_m is the distribution of the $V_m = \max\{v_i, i \in N\}$.

Then, the social welfare will be increased.

7.2.3 Lemma 3

If we have the property A for the valuation and the signal, $C \in \mathbb{R}^+$, and $F(Cv|v) = H(C)$, with $F(.|v)$ the distribution of the valuation given a signal realization, the following identities holds:

- $f(Cv|v) = \frac{h(C)}{v}$
- $F_2(Cv, v) = -C\frac{h(c)}{v}$

- $f_1(Cv|v) = \frac{h'(C)}{v^2}$
- $f_2(Cv|v) = -\frac{h(c)+Ch'(C)}{v^2}$

To prove the lemma we will first assume that the expected results holds (this is, $b(v) = Cv$ and $u(b(v)) = Kv$, with $b(.)$ the bidding strategy and $u(.)$ the strategy of Matzusaka) and then we will find the equations that give us C and K.

Given a winning bid and the amount of money payed in any case β, Matzusaka solves;

$$u(b) \in \arg\max_{u \in \mathbb{R}^+} u\mathbb{P}[v' \geq u + (1-\beta)b|B(b)]$$

with FOC:

$$1 - F(u(b) + (1-\beta)b|B(b)) = u(b)f(u(b) + (1-\beta)b|B(b))$$

where $B(b)$ are the Matzusaka's beliefs. The FOC helps us to understand changes in Matzusaka's offer after changes in the winning bid. Calling the inverse hazard rate given the beliefs $IHR(.,.)$, where the first argument is where the hazard rate is evaluated and the second the beliefs, we can write:

$$\frac{\partial u}{\partial b} = \frac{(1-\beta)IHR_1(.,.)}{1 - IHR_2(.,.)} + \frac{IHR_2(.,.)B'(b)}{1 - IHR_1(.,.)}$$

We wrote it in that way because it shows the two effects present on changes of the winning bid. The first effect refers to changes in the amount payed. Because under affiliation the inverse hazard rate is non increasing, the first term is negative. It says that if the winning bid increases, it is more difficult for Matzusaka to reach an agreement with the winning team, therefore if the beliefs stay constant, Matzusaka will try to ask for less money. On the other hand, if we assume that bigger bids make Matzusaka think on bigger signals received by the team, because of the affiliation, a big signal in general should mean a more probable high valuation. Therefore, induces Matzusaka to ask more money for the bid. Both effects live simultaneously except for $\beta = 1$, when only the second survives.

Also, we can write that identity in terms of the parameters as:

$$\frac{\partial u}{\partial b} = -\frac{(1-\beta)(f(.|.) + u(b)f_1(.|.)) + B'(F_2(.|.) + u(b)f_2(.|.))}{u(b)f_1(.|.) + 2f(.|.)}$$

In both ways we have omitted the argument inside the function, but those are always $u(b) + (1-\beta)b$ and $B(b)$. Evaluating both identities in $b(v) = Cv$ and $u(b(v)) = Kv$ we have:

$$K = \frac{1-H(.)}{H'(.)} \tag{7.1}$$

$$u'(Cv) = -\frac{(1-\beta)(h(.)+Kh'(.)) - C^{-1}(Ch(.)+Kh(.)+KCh'(.))}{Kh'(.)+2h(.)} \tag{7.2}$$

With H, h y h' being evaluated in $K + C(1-\beta)$.

Now, lets do the same in the problem of an American team. Each of one faces the problem:

$$b(v) \in \arg\max_{b\in\mathbb{R}^+}\{\mu(b,v) - \beta b\}\mathbb{P}[v \leq b^{-1}(b)]^{n-1}$$

with

$$\mu(b,v) = \int_{v' \geq u(b)+(1-\beta)b} \{v' - [u(b) + (1-\beta)b]\}dF(v'|v)$$

When signals are uniform the problem is;

$$b(v) \in \arg\max_{b\in\mathbb{R}^+}\{\mu(b,v) - (1-\beta)b\}b^{-1}(b)^{n-1}$$

And the respective FOC:

$$\{\mu_1(b(v),v) - (1-\beta)\}v + \{\mu(b(v),v) - (1-\beta)b(v)\}(n-1)\frac{1}{b'(v)} = 0$$

with

$$\mu_1(b(v),v) = \{\frac{\partial u}{\partial b} + (1-\beta)\}\{1 - F[u(b(v)) + (1-\beta)b(v)|v]\}$$

Assuming that we are in an equilibrium with linear strategies, the FOC changes to

$$\{[u'(Cv) + (1-\beta)][1-H] - (1-\beta)\}v + \{\mu(b(v),v) - (1-\beta)Cv\}(n-1)\frac{1}{C} = 0$$

That after some manipulations changes to

$$\{[u'(Cv)+(1-\beta)][1-H]-(1-\beta)\}+\{\int_{K+(1-\beta)C}^{\bar{x}} xh(x)dx-(K+(1-\beta)C)H-(1-\beta)C\}(n-1)$$

As we have seen, all the equations does not depend on the signal, therefore the functional form is correct, and by replacing (2) in (3) the existence of such equilibrium depends only in if there is a solution for the system of equations formed between (1) and (2).

7.2.4 Proposition 4 and 5

Consider the case in which the winner team is forced to pay only a fraction (we will call it β) of it bid in case that the negotiation does not result. Lets find the equilibrium in this general schema.

The player has to maximize expected utility given the winner bid. This is:

$$u(b) \in \arg\max_u u\mathbb{P}[\text{team accepts the offer}]$$

But now, because of the "fine" that the team has to pay in some cases, the team will accept any offer that leaves it better than paying the fine. This is, given a realization v' of the valuation, the team will accept if $v'-u-b>-\beta b$, or $v'>u+(1-\beta)b$. Thus, the problem faced by Matsuzaka is:

$$u(b) \in \arg\max_u u\mathbb{P}[v'>u+(1-\beta)b]$$

In our first case, when the valuation is distributed as an exponential with mean on the signal, this is:

$$u(b) \in \arg\max_u u\exp[-\frac{u+(1-\beta)b}{v(b)}]$$

Where $v(b)$ is the type that Matsuzaka believes the American team has. It is easy to check that $u(b)=v(b)$ maximizes Matuzaka's utility.

For a team, the problem will be:

$$b(v) \in \arg\max_b\{\int_{v'>u(b)+(1-\beta)b}(v'-(u(b)+b))\frac{1}{v}\exp[-\frac{v'}{v}]dv'-\int_{v'\leq u(b)+(1-\beta)b}\beta b\frac{1}{v}\exp[-\frac{v'}{v}]dv'\}F^{n-1}(b^{-1}(b))$$

Doing some algebra, this equals to:

$$b(v) \in \arg\max_b \{ \int\limits_{v' > u(b) + (1-\beta)b} (v' - (u(b) + (1-\beta)b)) \frac{1}{v} \exp[-\frac{v'}{v}] dv' - \beta b \} F^{n-1}(b^{-1}(b))$$

$$b(v) \in \arg\max_b \{ v \exp[-\frac{u(b) + (1-\beta)b}{v}] - \beta b \} F^{n-1}(b^{-1}(b))$$

To have an equilibrium with increasing strategies, where nobody has incentives to deviate it must hold:

$$[n - 2 - (1-\beta)b'(b)] \exp[-1 - (1-\beta)\frac{b}{v}] - (n-1)\beta \frac{b}{v} - \beta b'(b) = 0$$

Our guess is that the strategy of the American team is linear on the signal or type v, that is, $b(v) = Cv$, where C depends on β and n. This makes a lot easier to find the equilibrium, because it is reduced to find a solution to:

$$[n - 2 - (1-\beta)C] \exp[-1 - (1-\beta)C] - n\beta C = 0$$

This conditions tell us that C is decreasing on β and increasing on n[1]

It is important to acknowledge how β changes the utility of the agents. So we will calculate those utilities to see what happens.

For Matsuzaka; his utility on the equilibrium will be $v \exp[-1 - (1-\beta)C]$, where v is the winners type. Therefore, his expected utility will be:

$$\mathbb{E}[U] = \frac{n}{n+1} \exp[-1 - (1-\beta)C]$$

C is decreasing on β, so $-(1-\beta)C$ increasing, thus the expected utility of Matsuzaka grows with β. For any team, the expected utility given its type will be:

$${}^2\mathbb{E}[V|v] = \{\exp[-1 - (1-\beta)C] - \beta C\} v^n = \exp[-1 - (1-\beta)C] \frac{1}{n} \{n - n + 2 + (1-\beta)C\} v^n$$

and the expected utility will be:

[1]This is easy to check, first if the conditions is written as $F(C, \beta) = 0$, and if the partial derivatives are calculated, both are negative. Therefore, $C'(\beta)$ is decreasing. For the second result, notice that the partial derivative on n is positive, so $C(n)$ must be increasing on n.

[2]We used the relation of β, C and n in the equilibrium.

$$\mathbb{E}[V] = \exp[-1-(1-\beta)C]\frac{1}{n(n+1)}\{2+(1-\beta)C\}$$

We will use $f(\beta) = (1-\beta)C(\beta)$. As you might notice, $f > 0$ and $f' < 0$, and $\mathbb{E}[U] = K(n)\{2+f(\beta)\}\exp[-f(\beta)]$. And the derivative of the expected utility is $K(n)\{f' - (2+f)f'\}\exp[-f(\beta)]$, which is always greater than zero.

The expected utility of the Japanese team given the winner's type is

$$\mathbb{E}[B|v] = \int\limits_{v' > v+(1-\beta)Cv} CvdF(v') + \int\limits_{v' < v+(1-\beta)Cv} \beta CvdF(v')$$

Equal to:

$$\mathbb{E}[B|v] = (1-\beta)Cv\exp[-1-(1-\beta)C] + \beta Cv$$

And the expected utility before the auction:

$$\mathbb{E}[B] = \frac{n}{n+1}\{(1-\beta)C\exp[-1-(1-\beta)C] + \beta C\}$$

$$\mathbb{E}[B] = \frac{1}{n+1}\{(n-1)(1-\beta)C + n - 2\}\exp[-1-(1-\beta)C]$$

Calling again $f = (1-\beta)C$, The derivative is :

$$\mathbb{E}[B]' = K'(n)\{(n-1)f' - f'(n-2+(n-1)f)\}\exp[-1-(1-\beta)C]$$

$$\mathbb{E}[B]' = K'(n)\{f'(1-(n-1)f)\}\exp[-1-(1-\beta)C]$$

But, in the previous section we demonstrated that $f(0)$ is $n-2$, then at zero the expected utility is increasing (f' is less than zero). On the other hand, $f(1) = 0$, then at one the expected utility is decreasing. Therefore for the Japanese team there is a $\beta*$ that maximizes its utility.

Finally, the total welfare is the sum of all the utilities calculated, then:

$$\mathbb{E}[W] = [\frac{n}{n+1} + n\frac{1}{n(n+1)}\{2+(1-\beta)C\} + \frac{1}{n+1}\{(n-1)(1-\beta)C + n - 2\}]\exp[-1-(1-\beta)C]$$

$$\mathbb{E}[W] = \frac{1}{n+1}[n+2+(1-\beta)C+(n-1)(1-\beta)C+n-2]\exp[-1-(1-\beta)C]$$

$$\mathbb{E}[W = \frac{n}{n+1}[(2)+(1-\beta)C]\exp[-1-(1-\beta)C]$$

Which is increasing on β.

7.2.5 Proposition 7

For the game in which the agents can buy information, we want to find an amount of info that arise in equilibrium, greater than $\frac{1}{n}$ and smaller than 1. If that were the case, we could be sure that an auction of this kind induces more information acquisition than the original of the "posting system".

Si $\alpha \geq \frac{1}{3}$

$b_\alpha(v) = \frac{(1+\alpha)^2(n-3)}{16\alpha n}v = K(\alpha^*)v$

$u_\alpha(b) = (1+\alpha)\frac{v(b)}{2}$

We want to find the α^* such that no one has incentives to deviate.

Suppose $\alpha^* \geq \frac{1}{3}$ and everyone is playing $(\alpha^*, K(\alpha^*)v)$ as a strategy. If we want to check that is an equilibrium, we have to assume that a single player deviate, playing $(\alpha, b_\alpha(v))$.

The problem faced by a team is:

$$b_\alpha(v) \in \arg\max_b\{\int\limits_{u(b)}^{(1+\alpha)v} \frac{v'-u(b)}{2\alpha n}dv' - b\}[\frac{b}{K(\alpha^*)}]^{n-1}$$

Doing some algebra, equals to:

$$b_\alpha(v) \in \arg\max_b\{\frac{1}{4\alpha v}[(1+\alpha)v - \frac{1+\alpha^*}{2}\frac{b}{K(\alpha^*)}]^2 - b\}[\frac{b}{K(\alpha^*)}]^{n-1}$$

Using the first order conditions, it is easy to check that $b_\alpha(v) = K(\alpha,\alpha^*)v$, with $K(\alpha,\alpha^*)$ satisfying:

$$(n-1)\frac{1}{4\alpha}[(1+\alpha)v - \frac{1+\alpha^*}{2}\frac{K(\alpha,\alpha^*)}{K(\alpha^*)}]^2 - nK(\alpha,\alpha^*) = \frac{1}{2\alpha}[(1+\alpha)v - \frac{1+\alpha^*}{2}\frac{K(\alpha,\alpha^*)}{K(\alpha^*)}]\frac{1+\alpha^*}{2}\frac{K(\alpha,\alpha^*)}{K(\alpha^*)}$$

Now, the expected utility of a team can be written as:

$$U(\alpha,\alpha^*) = \frac{1}{n+1}[\frac{1}{4\alpha}[(1+\alpha)v - \frac{1+\alpha^*}{2}\frac{K(\alpha,\alpha^*)}{K(\alpha^*)}]^2 - K(\alpha,\alpha^*)]\frac{K(\alpha,\alpha^*)}{K(\alpha^*)}^{n-1}$$

Now, to see that if everyone is with the same information, regardless of the cost, there will be incentives to deviate. To do that we only have to differentiate the utility and the evaluate in $\alpha = \alpha^*$. The result is:

$$U_\alpha(\alpha^*,\alpha^*) = \frac{1}{n+1}\frac{1+\alpha^*}{4\alpha^*}[1 - \frac{1+\alpha^*}{4\alpha^*}]$$

Which is always greater than zero for $\alpha^* \geq \frac{1}{3}$

For $\alpha^* \leq \frac{1}{3}$ we use the same. The bid functions change, but they are always linear. following the steps made before, finally we can demonstrate that:

$$U_\alpha(\alpha^*,\alpha^*) = 0$$

Now, the problem faced by a team is like this:

$$\max_{\alpha\in A} U(\alpha,\alpha^*) - C(\alpha)$$

And the cost is decreasing, we have that if the team choses the same amount of information, it has strong incentives to acquire a little less, therefore, those α's are not equilibriums.

7.3 Exponential distribution

7.3.1 Posting System

To actually find an equilibrium, and compare its properties with other mechanisms, we will consider that initially a player receives a signal v_i distributed uniformly in $[0,1]$, and then the true valuation is distributed according to $G(v'|v) = 1-\exp[-\frac{v'}{v}]$. We also assume that all the signals are independent and identically distributed with $F(v) = v$. We look for a symmetric equilibrium with strictly increasing strategies. This makes thing a lo simpler, and the condition that an equilibrium must satisfy are:

1.
$$b(v_i) \in \arg\max_b \int_{v'>u(b)} [v' - u(b)]\frac{1}{v}\exp[-\frac{v'}{v}]dv' F(b^{-1}(b))^{n-1}$$

2. Beliefs has mass in only one point, $v(b) = b^{-1}(b)$. This is implied by the fact that $b(.)$ is increasing, thus in the equilibrium, Matzusaka can invert this function to compute the signal received by the team.

3. $$u(b) \in \arg\max_u [u - b] \exp[\frac{u}{v(b)}]$$

Now lets find it explicitly. In the last stage, Matzusaka chooses $u(\cdot)$ as in (3). The first order conditions is:

$$\exp[-\frac{u(b)}{v(b)}](1 - \frac{u(b) - b}{v(b)}) = 0 \Leftrightarrow u(b) = b + v(b)$$

In the previous phase, when teams participate in the auction, they maximize their expected payoff as in (1). That equals to:

$$b(v_i) \in \arg\max F^{n-1}(b^{-1}(b)) v_i \exp[-\frac{u(b)}{v_i}]$$

And the respective FOC is:

$$(n-1)F^{n-2}(b^{-1}(b))\frac{f(b^{-1}(b))}{b'(b^{-1}(b))} v_i \exp[-\frac{u(b)}{v_i}] - F^{n-1}(b^{-1}(b)) \exp[-\frac{u(b)}{v_i}] u'(b) = 0$$

Since in a symmetric equilibrium $b^{-1}(b) = v_i$ we obtain the differential equation:

$$(n-1)\frac{f(v_i)}{b'(v_i)} v_i - F(v_i) u'(b) = 0$$

Using that $u(b) = b + v(b) = b + b^{-1}(b)$, we get:

$$b'(v_i) = (n-1)\frac{f(v_i)}{F(v_i)} v_i - 1 = (n-2)$$

Which implies, because of the uniform distribution, that $b(v_i) = (n-2)v_i + C, C \in \mathbb{R}$, which is increasing if $n > 2$. We choose $C = 0$, to force small bids when the signal is low.

With the equilibrium computed above, we can compute the expected utility obtained by each participant: the Japanese team, an American team, and Mr. Matsuzaka.

If an American team wins, it gets:

$$\int\limits_{v'>u(b)} [v' - u(b)]\frac{1}{v}\exp[-\frac{v'}{v}]dv' = \int\limits_{v'>u(b)} \exp[-\frac{v'}{v}]dv' = v\exp[-\frac{u(b)}{v}] = ve^{-(n-1)}$$

Therefore, its ex-ante expected utility, before receiving a signal, is:

$$\mathbb{E}(U) = \int_0^1 ve^{-(n-1)}v^{(n-1)}dv = \frac{e^{-(n-1)}}{n+1}$$

Mr. Matzusaka's utility, if the auction's winner offered b, is given by

$$\int\limits_{v'>u(b)} v_{win}\frac{1}{v_{win}}\exp[-\frac{v'}{v_{win}}]dv' = v_{win}e^{-(n-1)}$$

So, his ex-ante expected utility is:

$$\mathbb{E}(V) = \int_0^1 ve^{-(n-1)}nv^{(n-1)}dv = \frac{e^{-(n-1)}(n)}{n+1}$$

Finally, the Japanese team gets the highest bid b only if Mr. Matzusaka and the winner reach an agreement, therefore if the winner offered b it gets:

$$\int\limits_{v'>u(b)} (n-2)v_{win}\frac{1}{v_{win}}\exp[-\frac{v'}{v_{win}}]dv' = (n-2)v_{win}e^{-(n-1)}$$

Its ex-ante expected utility is given by:

$$\mathbb{E}(B) = \frac{e^{-(n-1)}n(n-2)}{n+1}$$

7.3.2 Standard Auction

We now analyze an environment where the winner in the auction must pay its bid regardless of the result of the negotiation with the player. Using the same assumptions used in the previous section, the equations that characterize the equilibrium in this case are a little different. First, notice that now the player solves

$$\max_u (u-b)\mathbb{P}[v' - u > -b|b]$$

, since the winner's bid is now a sunk cost. So with all the assumptions, Mr. Matsuzaka's strategy satisfies

$$u(b) \in \arg\max_u (u-b)\exp[-(u-b)v(b)^{-1}]$$

Which led us to the same strategy used before, $u(b) = v(b) + b$. Again, $v(b)$ are the beliefs, and in an equilibrium with increasing functions we have $v(b) = b^{-1}(b)$.

For an American team, the utility can take three values, $v' - u(b) = v' - v(b) - b$ if it wins the auction and it accepts the offer, $-b$ if it wins but does not accept, and zero in any other case. Given a signal v, the expected utility of a team is:

$$\mathbb{E}[U|v] = \{ \int\limits_{v'>u(b)-b=v(b)} (v' - v(b))\frac{1}{v}\exp[-\frac{v'}{v}]dv' - b\}\mathbb{P}[b > b(v_j)\forall j \neq i]$$

$$\mathbb{E}[U|v] = \{v\exp[-\frac{v(b)}{v}] - b\}F(b^{-1}(b))^{n-1}$$

Since in equilibrium $b(\cdot)$ must be in the $\arg\max_b \mathbb{E}(U|v)$, it must satisfy the FOC:

$$F(b^{-1}(b))^{n-1}\{-v'(b)\exp[-\frac{v(b)}{v}]-1\}+\{v\exp[-\frac{v(b)}{v}]-b\}(n-1)F(b^{-1}(b))^{n-2}f(b^{-1}(b))\frac{1}{b'(b^{-1}(b))} = 0$$

Which can be rewritten, in a symmetric equilibrium, as:

$$F(v)\{-\frac{1}{b'(v)}e^{-1} - 1\} + \{ve^{-1} - b\}(n-1)f(v)\frac{1}{b'(v)} = 0$$

And we have a differential equation:

$$b'(v) + (n-1)\frac{b(v)}{v} = (n-2)e^{-1}$$

The solution to this equation is $b(v) = \frac{(n-2)}{ne}v + Cv^{1-n}$, with $C \in \mathbb{R}$. As in the previous case, we choose $C = 0$ to have an increasing bidding function in the whole range. Therefore we have $b(v) = \frac{(n-2)}{ne}v$.

Having solved the game, lets calculate the utilities:
If an American team wins, it gets:

$$\{v\exp[-\frac{v(b)}{v}] - b\} = v(1 - \frac{n-2}{n})e^{-1} = v\frac{2}{n}e^{-1}$$

Therefore, its ex-ante expected utility, before receiving a signal, is:

$$\mathbb{E}[U] = \int_0^1 v\frac{2}{n}e^{-1}v^{(n-1)}dv = \frac{2}{n(n+1)e}$$

Mr. Matzusaka's utility, if the auction's winner offered b, is given by

$$(u(b) - b)\exp[-(u(b) - b)v(b)^{-1}] = v_{win}e^{-1}$$

So, his ex-ante expected utility is:

$$\mathbb{E}[V] = \int_0^1 ve^{-1}nv^{n-1}dv = \frac{n}{(n+1)e}$$

Finally, the Japanese team gets always the highest bid b so his expected utility, is:

$$\mathbb{E}(B) = \int \_0^1 b(v_{win})dF_{win}(v_{win}) = \int \_0^1 \frac{n-2}{ne}vnv^{n-1}dv = \frac{(n-2)}{(n+1)e}$$

	Posting System	Standard Auction
Team	$\frac{1}{(n+1)e^{(n-1)}}$	$\frac{2}{n(n+1)e}$
Player	$\frac{n}{e^{(n-1)}(n+1)}$	$\frac{n}{(n+1)e}$
Owner	$\frac{n(n-2)}{e^{(n-1)}(n+1)}$	$\frac{(n-2)}{(n+1)e}$

Looking at this, it's easy to check that for $n > 3$, everyone is better with the second case, in other words, using a *Standard Auction* auction is Pareto improving (for $n > 3$).

7.3.3 Everyone pays

The last kind of auction that I'm going to study is one where everyone pays, even when someone lose. Using the same assumptions made in the previous cases, the second stage is the same that in the case *Standard Auction*, because the decision of the player is not affected by this new auction, so he is going to use $u(b) = v(b) + b$.

On the other hand, every team will maximize a different function, because they will have to always pay the bids. Their strategy must be:

$$b(v_i) \in \arg\max \mathbb{P}(b > b_j(v_j), \forall j) \int\limits_{v' > v(b)} [v_i' - v(b)] dF_v(v') - b = v \exp[-\frac{v(b)}{v}] F(b^{-1}(b))^{n-1} - b$$

With FOC:

$$-v'(b) exp[-\frac{v(b)}{v}] F(b^{-1}(b))^{n-1} + (n-1) v \exp[-\frac{v(b)}{v}] F(b^{-1}(b))^{n-2} f(b^{-1}(b)) \frac{1}{b'(b^{-1}(b))} - 1 = 0$$

In the equilibrium:

$$(n-2) \exp[-1] v^{n-1} = b'(v)$$

And the solution is.

$$b(v) = \frac{(n-2)}{ne} v^n + C$$

I chose $C = 0$, (as v goes to zero, they should bid small amounts). An the utilities can be calculated as follows:

First, for the team:

$$\mathbb{E}[U|v] = \frac{1}{e} v^n - \frac{(n-2)}{ne} v^n = \frac{2}{ne} v^n$$

And the expected utility is:

$$E[U] = \frac{2}{n(n+1)e}$$

For the player, after the auction:

$$\mathbb{E}[V|v_{winner}] = v_{winner} e^{-1}$$

And before:

$$\mathbb{E}[V] = \frac{(n)}{(n+1)e}$$

Finally, the player's owner wins always $\sum b(v)$, taking expectations:

$$\mathbb{E}[\sum b(v)] = \sum \mathbb{E}[b(v)] = n\mathbb{E}[b(v)] = n \int\limits_{[0,1]} \frac{(n-2)}{ne} v^n dv = \frac{(n-2)}{(n+1)e}$$

This values are the same that in the second case.

7.4 Uniform Case

Suppose that now the signal received on the first stage has more information than just the expectation. In this case, lets assume that the valuation of the player its distributed uniform in $[v(1-\alpha), v(1+\alpha)]$ where v is the signal received on the first stage, and α represents the information available to the team (which can be bought).

7.4.1 Posting System

In this case, as before, the one who wins the auction only pays if he accepts the offer from the player. With this new schema, the player will maximize:

$$\max_{u \geq b}(u-b)\mathbb{P}[v' > u]$$

But now, the probability is defined by:

$$\mathbb{P}[v' > u] = \begin{cases} 0 & (1+\alpha)v(b) < u \\ \dfrac{(1+\alpha)v(b) - u}{2\alpha v(b)} & u \in [(1-\alpha)v(b), (1+\alpha)v(b)] \\ 1 & u < (1-\alpha)v(b)) \end{cases}$$

This implies that the objective functions can be written as:

$$H_{b,v(.)}(u) = \begin{cases} 0 & (1+\alpha)v(b) < u \\ (u-b)\dfrac{(1+\alpha)v(b) - u}{2\alpha v(b)} & u \in [(1-\alpha)v(b), (1+\alpha)v(b)] \\ u-b & u < (1-\alpha)v(b)) \end{cases}$$

For $u \in [(1-\alpha)v(b), (1+\alpha)v(b)]$ this is a parabola. It easy to check that is continuous, and increasing before the interval, and non-increasing after that. Therefore the strategy used by the player is not difficult to compute:

$$u(b) = \begin{cases} b & (1+\alpha)v(b) < b \\ \dfrac{((1+\alpha)v(b) + b)}{2} & b \in [(1-3\alpha)v(b), (1+\alpha)v(b)] \\ (1-\alpha)v(b) & b < (1-3\alpha)v(b) \end{cases}$$

In the first case he utility of the player is zero, the same than not to offer. Thus, the strategy is equivalent to [3]:

$$u(b) = \begin{cases} \text{not to offer} & (1+\alpha)v(b) < b \\ \dfrac{((1+\alpha)v(b)+b)}{2} & b \in [(1-3\alpha)v(b), (1+\alpha)v(b)] \\ (1-\alpha)v(b) & b < (1-3\alpha)v(b) \end{cases}$$

Even thought the strategies are equivalent to the player, they induce different strategies. For example, when the player does not offer, the teams utility is always zero, but if the player uses the other strategy, the winner team, after offering, could have a positive utility even when his bid large enough. This changes the problem faced by a team, and therefore the strategies that it will use in an equilibrium. Notice that the two strategies depend not only in the bid, they also depend on the beliefs (correctly updated are the inverse of the strategy used by a team, assuming all the good properties of those strategies) in a non-trivial way. This difficult finding an equilibrium, because for some strategies of a team, the strategy of the player could be sometimes $\frac{(1+\alpha)v(b)+b}{2}$, $(1-\alpha)v(b)$ or b, and this could allow the existence of multiple equilibrium.[4] For simplicity, let's assume that the strategies of the teams force the player to be always on the same interval.

Again, we are looking for symmetric equilibrium, with increasing bidding strategies, and with uniforms, independent signals.

Case 1

Suppose that the strategies chosen by the teams implies that $u(b) = \frac{((1+\alpha)v(b)+b)}{2}$. To find the strategy of the teams, we have to maximize utility expectation.

$$\max_b \int_{u(b)}^{(1+\alpha)v} \frac{v' - u(b)}{2\alpha v} dv' \; b^{-1}(b)^{n-1}$$

The last term is the probability of winning if the bid is b, and the first term is the utility conditional on winning. It starts from $u(b)$ because if the valuation is less than that, the team does not accept the offer. Solving the integral, the objective function can be written as:

[3] In terms of the player.

[4] For example, suppose that $b(v) = v^2$, then the condition for choosing the lower bound is $b < (1-3\alpha)^2$, thus he will alternate between the two values.

$$\frac{1}{4\alpha v}[(1+\alpha)v - u(b)]^2 b^{-1}(b)^{n-1}$$

The FOC is:

$$\begin{aligned}
0 &= \frac{1}{4\alpha v}\{-2u'(b)[(1+\alpha)v - u(b)]b^{-1}(b)^{n-1} + [(1+\alpha)v - u(b)]^2(n-1)b^{-1}(b)^{n-2}\frac{1}{b'(b^{-1}(b)}\} \\
0 &= \frac{[(1+\alpha)v - u(b)]}{4\alpha v}b^{-1}(b)^{n-2}\{-2u'(b)b^{-1}(b) + [(1+\alpha)v - u(b)](n-1)\frac{1}{b'(b^{-1}(b)}\} \\
0 &= -2u'(b)b^{-1}(b) + [(1+\alpha)v - u(b)](n-1)\frac{1}{b'(b^{-1}(b)}
\end{aligned}$$

The final expression can be written, assuming that we are in an equilibrium, as:

$$2u'(b)vb'(v) = [(1+\alpha)v - u(b)](n-1)$$

For $u(b) = \frac{((1+\alpha)v(b)+b)}{2}$, the FOC turns into a differential equation:

$$[\frac{1+\alpha}{b'(b^{-1}(b))} + 1]vb'(v) = 0.5[(1+\alpha)v - b](n-1)$$

$$b'(v) + \frac{(n-1)}{2v}b = (1+\alpha)\frac{(n-3)}{2}$$

The solution of this is $b = \frac{n-3}{n+1}(1+\alpha)v + Cv^{0.5[1-n]}$, with $C \in \mathbb{R}$. We chose $C = 0$, to assure $b(.)$ is increasing and that low types make close to zero bids. But, does this strategy induce $u(b) = \frac{((1+\alpha)v(b)+b)}{2}$? We assumed that, but this only happens when $b \in [(1-3\alpha)v(b), (1+\alpha)v(b)]$. For $n > 3$, $b < (1+\alpha)v(b)$. And the cases where $b \geq (1-3\alpha)v(b)$ are:

$$b \geq (1-3\alpha)v(b) \Leftrightarrow \frac{n-3}{n+1}(1+\alpha) \geq (1-3\alpha) \Leftrightarrow \alpha > \frac{1}{n}$$

In this case, the utilities of the agents are as follows. For an American team, given the signal, we have:

$$\mathbb{E}[U|v] = \frac{(1+\alpha)^2}{\alpha}\frac{v^n}{(n+1)^2}$$

And the expected utility before the auction:

$$\mathbb{E}[U] = \frac{(1+\alpha)^2}{\alpha}\frac{1}{(n+1)^3}$$

For the player, given the winner of the auction:

$$\mathbb{E}[V|v_{win}] = 2\frac{(1+\alpha)^2}{\alpha}\frac{v_{win}}{(n+1)^2}$$

Before the auction:

$$\mathbb{E}[V] = 2\frac{(1+\alpha)^2}{\alpha}\frac{n}{(n+1)^3}$$

Finally, the owner of the player gets:

$$\mathbb{E}[b_win] = \frac{n-3}{n+1}(1+\alpha)\frac{n}{n+1}$$

Case 2

Now, suppose that $u(b) = (1-\alpha)v(b)$. The FOC for a team is:

$$u'(b)vb'(v) = [(1+\alpha)v - u(b)](n-2)$$

Replacing we have:

$$(1-\alpha)v = (n-2)2\alpha v$$

So the only case where there is an equilibrium is when $\alpha = \frac{1}{2n-3}$.

7.4.2 Standard Auction

Lets analyze another auction. Suppose that now the winner is forced to pay, even when he does not reach an agreement with the player. The player will be aware of this, and will maximize:

$$\max_u (u-b)\mathbb{P}[v' > u-b]$$

The probability is:

$$\mathbb{P}[v' > u - b] = \begin{cases} \dfrac{((1+\alpha)v(b) - u)}{2\alpha v(b)} & u \in [(1-\alpha)v(b) + b, (1+\alpha)v(b) + b] \\ 0 & u > (1+\alpha)v(b) + b \\ 1 & u < (1-\alpha)v(b) + b \end{cases}$$

As in the previous section, the objective functions is a parabola in an interval, that sometimes has a maximum in the interior, and sometimes in the bounds. The strategy of the player must be:

$$u(b) = \begin{cases} b + (1+\alpha)\frac{v(b)}{2} & \alpha \geq \frac{1}{3} \\ (1-\alpha)v(b) + b & \alpha < \frac{1}{3} \end{cases}$$

The strategy used by the player depends on α.

Case 1: $\alpha \geq \frac{1}{3}$

The utility of a team can be written, after some manipulation, as:

$$\mathbb{E}[U|v] = \{\int_{u(b)-b}^{(1+\alpha)v} \frac{v' - [u(b) - b]}{2\alpha v} dv' - b\}[b^{-1}(b)]^{n-1} = \{\frac{1}{4\alpha v}[(1+\alpha)v - [u(b) - b]]^2 - b\}[b^{-1}(b)]^{n-1}$$

When $\alpha \geq \frac{1}{3}$, $u(b) = b + (1+\alpha)\frac{v(b)}{2}$. Thus, replacing,

$$\mathbb{E}[U|v] = \{\frac{(1+\alpha)^2}{4\alpha v}[(v - \frac{v(b)}{2}]^2 - b\}[b^{-1}(b)]^{n-1}$$

The strategy of a team in an equilibrium will be in the $\arg\max \mathbb{E}[U|v]$, so lets check the FOC:

$$0 = \{-v'(b)\frac{(1+\alpha)^2}{4\alpha v}[v - \frac{v(b)}{2}] - 1\}[b^{-1}(b)]^{n-1} + \{\frac{(1+\alpha)^2}{4\alpha v}[v - \frac{v(b)}{2}]^2 - b\}(n-1)[b^{-1}(b)]^{n-2}\frac{1}{b'(b^{-1}(b))}$$

In the equilibrium, this turns into a differential equation:

$$0 = \{-\frac{(1+\alpha)^2}{4\alpha v}[\frac{v}{2}] - b'(v)\}v + \{\frac{(1+\alpha)^2}{4\alpha v}[\frac{v}{2}]^2 - b\}(n-1)$$

$$b'(v) + \frac{n-1}{v}b(v) = \frac{(1+\alpha)^2}{16\alpha}(n-3)$$

The solution to this equation is $b = \frac{(1+\alpha)^2(n-3)}{16\alpha n}v + Cv^{1-n}$, as usual, we chose $C = 0$. With this equilibrium, we can calculate the utilities of the agents. For the player, given the signal, we have:

$$\mathbb{E}[U|v] = \frac{(1+\alpha)^2}{16\alpha n}3v^n$$

And the expected utility before the auction:

$$\mathbb{E}[U] = 3\frac{(1+\alpha)^2}{16\alpha n(n+1)}$$

For the player, given the winner of the auction:

$$\mathbb{E}[V|v_{win}] = \frac{(1+\alpha)^2}{8\alpha}v_{win}$$

Before the auction:

$$\mathbb{E}[V] = \frac{(1+\alpha)^2}{8\alpha}\frac{n}{n+1}$$

Finally, the owner of the player gets:

$$\mathbb{E}[b_win] = \frac{(1+\alpha)^2(n-3)}{16\alpha(n+1)}$$

Case 2: $\alpha < \frac{1}{3}$

With this α, the player will play $u(b) = (1-\alpha)v(b) + b$, so the utility of a team is:

$$\mathbb{E}[U|v] = \{\frac{1}{4\alpha v}[(1+\alpha)v - (1-\alpha)v(b)]^2 - b\}[b^{-1}(b)]^{n-1}$$

To find an equilibrium, we have to maximize and then force $b(v)$ to be the arg max. The FOC is:

$$0 = \{-2(1-\alpha)v'(b)\frac{1}{4\alpha v}[(1+\alpha)v-(1-\alpha)v(b)]-1\}[b^{-1(b)}]^{n-1}+\{\frac{1}{4\alpha v}[(1+\alpha)v-(1-\alpha)v(b)]^2-b\}[b^{-1}(b)]^{n-}$$

If we are in an equilibrium, $b(.)$ solves this, thus:

$$0 = \{-(1-\alpha)\frac{1}{b'(v)} - 1\}v + \{\alpha v - b\}\frac{n-1}{b'(v)}$$

And we have the following differential equation:

$$b'(v) + \frac{n-1}{v}b(v) = [n\alpha - 1]$$

The solution is $b = \frac{n\alpha-1}{n}v + Cv^{1-n}$, again we chose $C = 0$, and with this we calculate the utilities of the agents. Before doing that, it is important to notice that for $\alpha < \frac{1}{n}$ the strategy is decreasing on v, so we have to assume that $\alpha \geq \frac{1}{n}$. Now, for the team, the expected utility after the auction is:

$$\mathbb{E}[U|v] = \frac{v^n}{n}$$

And the expected utility

$$\mathbb{E}[U] = \frac{1}{n(n+1)}$$

For the player, given the winning bid:

$$\mathbb{E}[V|v_{win}] = (1-\alpha)v_{win}$$

And before the auction:

$$\mathbb{E}[V] = \frac{1-\alpha}{n+1}$$

Finally, for player's owner, the utility is

$$\mathbb{E}[b_{win}] = \frac{n\alpha - 1}{n+1}$$

Now we have solved the two cases, the question that emerges is, how much information is acquire in both cases? An to answer that we have to decide who is the agent that buys the information. In this schema could be the player, that gives more accurate information about his talent, the owner, who can give tapes or test results of the player to the team, or the team itself, who can investigate or pay someone to have better information about its valuation.

7.4.3 Everyone pays

For the *all pay auction*, the player will have the same strategy used previously in the section *Standard Auction*. For him it does not matter if the loser pays or not, the only important thing is that the bid made by the winner have to be paid. As the previous section, we will have to possible strategies for the player, depending on the information available at the moment of biding.

Now, in general, the expected utility of a team given his signal, and the player strategy, equals to:

Using the results of the previous section, the expected utility of a team given the signal equals to:

$$\mathbb{E}[U|v] = \frac{1}{4\alpha v}[(1+\alpha)v - [u(b)-b]]^2[b^{-1}(b)]^{n-1} - b$$

Where $u(b)$ is the strategy used by the player. And the conditions that $b(.)$ must suffice are:

$$\frac{(n-1)[b^{-1}(b)]^{n-2}}{4\alpha v b'(b^{-1}(b))}[(1+\alpha)v-[u(b)-b]]^2 - \frac{[u'(b)-1]}{2\alpha v}[(1+\alpha)v-[u(b)-b]][b^{-1}(b)]^{n-1} - 1 = 0$$

In an equilibrium, it will turn into:

$$\frac{(n-1)v^{n-3}}{4\alpha b'(v)}[(1+\alpha)v - [u(b)-b]]^2 - \frac{[u'(b)-1]}{2\alpha}[(1+\alpha)v - [u(b)-b]]v^{n-2} - 1 = 0$$

Case 1: $\alpha \leq \frac{1}{3}$

In this case the strategy of the player is $u(b) = \frac{1+\alpha}{2}v(b) + b$. In the equilibrium, the beliefs are correct, and the FOC now is :

$$\frac{(n-1)}{16\alpha b'(v)}(1+\alpha)^2 v^{n-1} - \frac{1}{8\alpha b'(v)}(1+\alpha)^2 v^{n-1} - 1 = 0$$

Solving the differential equation;

$$b(v) = \frac{(n-3)}{16\alpha n}(1+\alpha)^2 v^n$$

In this case, the utilities of the agents are as follows. For the player, given the signal, we have:

$$\mathbb{E}[U|v] = \frac{3}{16\alpha n}(1+\alpha)^2 v^n$$

And the expected utility before the auction:

$$\mathbb{E}[U] = \frac{3}{16\alpha n(n+1)}(1+\alpha)^2$$

For the player, given the winner of the auction:

$$\mathbb{E}[V|v_{win}] = \frac{(1+\alpha)^2}{8\alpha} v_{win}$$

Before the auction:

$$\mathbb{E}[V] = \frac{(1+\alpha)^2}{8\alpha}\frac{n}{(n+1)}$$

Finally, the owner of the player gets:

$$\mathbb{E}[\sum b_w in] = n\mathbb{E}[\frac{(n-3)}{16\alpha n}(1+\alpha)^2 v^n] = \frac{(n-3)(1+\alpha)^2}{16\alpha(n+1)}$$

Case 2 $\alpha < \frac{1}{3}$

In this case, $u(b) = (1-\alpha)v(b) + b$, and the FOC for a team is:

$$\frac{(n-1)}{4\alpha b'(v)}[2\alpha]^2 v^{n-1} - \frac{1-\alpha}{2\alpha b'(v)}[2\alpha v]v^{n-2} - 1 = 0$$

$$\frac{(n-1)}{b'(v)}\alpha v^{n-1} - \frac{1-\alpha}{b'(v)}v^{n-1} - 1 = 0$$

The solution to this differential equation is:

$$b(v) = (\alpha - \frac{1}{n})v^n$$

And the utilities of the players are as follows. For the team, the expected utility after the auction is:

$$\mathbb{E}[U|v] = \frac{v^n}{n}$$

And the expected utility

$$\mathbb{E}[U] = \frac{1}{n(n+1)}$$

For the player, given the winning bid:

$$\mathbb{E}[V|v_{win}] = (1-\alpha)v_{win}$$

And before the auction:

$$\mathbb{E}[V] = \frac{1-\alpha}{n+1}$$

Finally, for player's owner, the utility is

$$\mathbb{E}[\sum b_j] = n\mathbb{E}(b) = n(\alpha - \frac{1}{n})\frac{1}{n+1} = \frac{n\alpha - 1}{n+1}$$