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Resumen

En este trabajo estudiamos un modelo de financiamento en redes, aplicado a comunidades, en las cuales no existe colateral ni acceso al crédito, y el financiamiento es alcanzado mediante la confianza al interior de la comunidad. Modelamos la comunidad como una red no dirigida, en la que los miembros de la comunidad están conectados en la red mediante links y la confianza es la representación de los beneficios futuros de interacciones mutuas, modelada como capacidades de links. Cada miembro de la comunidad tiene una dotación exógena de recursos. Un único miembro tiene acceso a una oportunidad de proyecto, la que puede ser financiada por la comunidad mediante transferencias de recursos y un esquema de repagos. Caracterizamos el nivel de financiamiento que la comunidad puede alcanzar, y los factores que lo gobernan, como un problema de *flujo máximo de largo restringido*. La probabilidad de éxito del proyecto, la tasa de recuperación y la probabilidad de interacción tienen un efecto monótono en el nivel de financiamiento. Sin embargo, el efecto de la tasa de repago y del spread de intermediación, dependen de los tamaños relativos de la confianza de la comunidad y los recursos disponibles en ésta. La distribución *ex-ante* de riqueza es examinada en términos de mejoras estocásticas y su efecto en la capacidad de financiamiento de la comunidad, encontrándose una correlación negativa con *mean-preserving spreads* de la distribución de la riqueza. También estudiamos la implementación de una institución formal de castigo, la que impone un costo a los miembros que cometen default en sus repagos estipulados. Presentamos una parametrización bajo la cual la institución puede ser exitosamente auto implementada por la comunidad. Finalmente, derivamos implicancias relacionadas con la estructura de red y el bienestar social, y desarrollamos dos aplicaciones. La primera tiene relación con descalces entre oferta y demanda en mercados donde la demanda es desconocida pero ya fue comprometida, y el costo de externalizar la demanda es menor que el costo de no satisfacerla. La segunda aplicación tiene relación con la venta negociada de un bien homogéneo, producido por los miembros de la comunidad, a un agente externo a la comunidad. Los miembros de la comunidad difieren en su poder de negociación y enfrentan la decisión de vender en el mercado spot o vender a través de un miembro designado de la comunidad, quien negocia con el agente externo.

Summary

We study a model of network financing, applied to communities, in which no actual collateral nor access to credit are available, and financing is achieved by means of trust within the community. We model the community as an undirected network, in which community members are linked within it and trust is a representation of future benefits from mutual interactions, modeled as the links capacities. Each community member has a exogenous resource endowment. One single member has access to a project opportunity which can be financed by the community through resource transfers, upon agreeing to a repayment schedule. We characterize the financing level that the community can achieve, and the factors that govern it, as a *length constrained maximum flow* problem. The project's success probability, the repayment's recovery rate and the member's interaction probability have monotone effects on the financing level. However, the effect of repayment rate and intermediation spread depends on the relative sizes of community trust and resources. *Ex-ante* wealth distribution is examined in terms of stochastic improvements and its effects on the community's financing ability, leading to a negative correlation with *mean-preserving spreads* of the wealth distribution. We also study the implementation of a formal punishing institution, which imposes a cost to members of the community that default on their agreed upon repayments. We give a parameterization under which the institution can be successfully implemented by the community itself. Finally, we derive further implications related to network structure and social welfare, and develop two applications. The first one is related to supply-demand mismatches in markets where demand is unknown but already compromised, and outsourcing demand is less costly than not meeting it. The second application is for the negotiated sell of a homogeneous good, produced by community members, to an outside third party. Community members differ in their bargaining power, and face the choice of selling in the spot market or selling through a designated member who negotiates with the outside third party.

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1 Introduction

This work studies the capacity of a community, in the absence of enforceable contracts, to undertake a project of variable size owned by a single agent. The main focus here is the extent to which resources to invest in the project can be gathered from community members. In particular, we focus on the role that community members' interactions play with regard to the amount of resources credible transferred to the owner of the project. We also analyze the effects that shocks to the wealth distribution and the existence of formal institutions have on the community's capacity to credible transfer funds to the owner of the project.

We show that the maximal amount of resources that can be transferred to the owner of the project is determined by a restricted flow problem. This is known in the network literature as the *length constrained maximum flow problem.* In addition, *ex-ante* wealth distribution is correlated to the amount of resources transferred in the sense that stochastic improvements of the wealth distribution favor the transfer of more resources towards the project's owner. We also show that under certain parameterizations community members are willing to exert private effort in the creation of a formal institution able to punish community members that fail to obey the agreement on which transfers of resources were based on.

Our model borrows heavily from Karlan *et al* [2009]. Mainly, the model considers a community represented by an indirected network where each member is represented by a node and their connections by links. Each link has a given capacity that indicates the strength of the link in terms of how much linked members enjoy mutual interactions (by means of favors, future enterprises, kinship, etc). There is an interaction probability for the links to break, meaning that mutual interactions are no longer enjoyable. We call the sum of the strengths across all members of the community the the level of community trust. A community member chosen randomly comes up with a project with constant returns to scale and a stochastic technology. With a given probability, the project succeeds in which case it yields a positive return and with the complementarity probability the project fails in which case it yields nothing. The scale of the project is determined by the amount of resources that the owner of it is able to raise from the community members. Each member has a resource constraint, which accounts for his wealth or productive capacity. We assume that the investment is contractible, but the project return is not. Agents can transfer resources and expect a repayment afterwards. Despite the fact that there are no formal institutions, we assume that in the case of default, community members are able to recover at least a share of the amount they ought to receive from the owner of the project. We call this the recovery rate.

This could be due to community policing, burglary or threat of violence against the defaulter.

We show that improvements in the recovery rate and probability of success weakly increase the amount of resources transferred, whereas an increase in the interaction probability decreases that. The effects of changes in the repayment rate and the intermediation spread are ambiguous, and depend on the relative sizes of trust and resources within the community. If trust is high and resources are comparatively scarce, an improvement in the repayment rate is beneficial in the sense of the amount of resources transferred and an improvement in the intermediation spread is detrimental. If on contrary, trust is low and resources are comparatively abundant, the converses are true.

The intuition for the impacts of the recovery rate and the interaction probability is that both directly affect trust among community members, as well as the extent to which members are willing to participate. For larger recovery rates (and lower interaction probabilities), the portion of resources that are at stake is lower (either because more resources can be recovered in case of default or because defaults are less likely), so that trust can hold a larger amount of resources. In addition, a larger recovery rate (lower interaction probability) means a better expected return rate, which allows distant members to participate as well, extending the participation level among members. Hence, more resources can be transferred due to trust, and there are more resources available due to the expansion in participation. The probability of success has the same impact in the expected return rate, thus allowing for more resources to be available.

With respect to repayment rate and intermediation spread, a trade-off between trust and participation arises. A larger repayment rate (lower intermediation spread) means a better expected return rate, therefore participation is benefited from it, nevertheless, it also means that less resources can be transferred per link, due to a lower trust. This occur because the larger the repayment rate (lower intermediation spread) is, the larger the repayment itself is, which goes against trust. Therefore, which effect dominates depends directly on the relative sizes of trust and resources within the community.

A classic example of the benefits of trust in communities relates to the *Jewish diamond merchants*, that emerged as the dominant party in the diamond trading market, dominance that has lasted many centuries. Given the diamond's characteristics, it is hard to enforce court rules in its trading, and also, there is a liquidity issue because of the high value of diamonds, which makes it preferable for intermediaries to pay with credit rather than instantaneously. In this adverse context for trading, it is argued that the main reason for the Jewish's success as diamond merchants is their trust based community (which extends far beyond geographic borders), which allows them to trade diamonds in an efficient manner. Nowadays, these dominance can be particularly seen in the city of New York, where about $85 - 90\%$ of *The New York Diamond Dealers Club* members are Jewish.

“Magic Cheese”

In July 2006, one of the largest Ponzi schemes in Chile was revealed. In November 2004, a company by the name of *Fermex Chile S.A* was formed, and its business was the production of lactic ferments for export, to be used in the cosmetic industry in France. But the company did not produce the ferments itself, instead they sold packs (with the inputs for the ferments), for 250,000 pesos each (about USD 427 of the time) to “investors” and then promised to repurchase the fermented products (in form of tiny wheels of cheese) for double the initial amount (to be made in 8 payments, every 15 days each).

Initially, Fermex Chile S.A did repurchase the products, thus encouraging investors the reinvest and to bring new investors as well (new investors could only entry by recommendation of another active investor). Eventually, after more than a year operating, the size of the scheme was large enough, and no fermented cheese was repurchased any longer. It has been estimated that around 5,000 people fell into the scheme, for a total amount of 5,000 million pesos (about USD 11.7 million, and USD 2.341 on average per person). Only a 54% of the total amount (about 2,700 million pesos, USD 6.32 million) could be retreived to the affected people (proportional to their investment). The mayority of affected people belong to *Coltauco*, a 18,000 habitants town with aboriginal ancestors. The same scheme had already been done in Peru, for a total amount of USD 50 million.

1.1 Literature Review

The subject of social networks has been widely developed in the recent past years. It focuses on communities, and its interactions, that find it beneficial to cooperate inside the community. The main theories find their foundation in the field of sociology, mainly regarding to the concept of social capital. Coleman [1988] introduced this concept in an attempt to study the connection between social capital and human capital, arguing that a strong measure of social capital showed evidence of value in the process of creating human capital. The idea of communities cooperating to achieve a better status is not new. Besley, Coate and Loury [1993] developed their model for *rotating savings and credit associations* (ROSCAS), which enables members of a community, who individually desire the same durable consumption good, to save collectively and thus acquire the good sooner (in expectation), as opposed to the alternative of saving independently.

In the same line with social capital, there is a whole literature about *microfinance institutions* (MFI), which exploit the idea that social capital can be considered as a collateral, for loan access, for people or communities with otherwise no capital. It is argued that in the absence of formal collateral, formal loan institutions (such as banks) are subjected to imperfect information that is costly to overcome (few screening possibilities, monitoring costs and few enforcing possibilities). In this sense, social capital has proved to help alleviate the issue. The main work has been done with respect to peer selection and joint liability arrangements between MFI's and communities. Essentially, borrowers are asked to form groups and are made jointly liable for the each group's member repayment. This exploits the fact that community members have better knowledge about each other's type (effort, project's quality, repayment willingness, etc.) and that sanctions can be applied within the community, whereas the outside institutions lack this ability. Ghatak and Guinnane [1999] show that joint liability does indeed achieve better screening, induce peer monitoring and enforce repayment. This enables poor communities, to some extent, to gain access to credit, by means of reducing the adverse selection and moral hazard that loan institutions face (and are unable to overcome by traditional methods, e.g. collateral and other financial sanctions). Regarding to group formation mechanisms, Armendariz and Gollier [2000] consider a random matching mechanism (because there is no public information about borrower's types) among risky and safe borrowers. They show that under some conditions, joint liability contracts (in their paper is regarded as joint responsibility) can serve as a risk pooling mechanism, benefiting efficiency (since good borrowers, the safe ones, are not excluded due to high interest rates). Ghatak [1999] considers the types to be known for every member of the community, but not for outsiders (i.e. not for the banks or lending institutions). He shows that every borrower prefers to be matched with a safer borrower, and that under a population balance assumption, all borrowing groups are conformed by borrowers with the same risk (project's success probability). He also shows that joint liability, the average repayment rate among borrowing groups is higher than under individual liability. Aggregate expected surplus is also higher under joint liability than under individual liability.

Recent literature has pointed out the benefits of cooperation among communities without formal markets, with respect to the subjects of risk sharing (assurance) and borrowing. Bloch, Genicot and Ray [2008] and Ambrus, Mobius and Szeidl [2010] develop bilateral insurance models concerning to idiosyncratic endowment shocks. While Bloch *et al* focus mainly on the informational role of links inside the community, Ambrus *et al* focus on the collateral role of links, to ensure that risk is shared effectively.

Karlan, Mobius, Rosenblat and Szeidl [2009] develop a model for borrowing among members of a community. In their model, they also consider interactions to become collateral for the borrowing to be possible. The role of connections is that of allowing a member to borrow from other members, to whom he may not be directly connected, and intermediary connections serve as a guarantee for the borrowing to be returned. They characterize the borrowing capacity by means of maximal flows. Karlan *et al*'s paper will be a keystone for this work. While their focus is onto the borrowing between two members of the community, our focus is on the ability of the whole community to benefit from a project opportunity. Therefore, opposed to Karlan *et al*, where in equilibrium no transfers between members are expected, in this model the transfers occur in two ways, first as a resources gathering process for the project, and then as repayments. Hence, the amount of resources available in the community is a key feature.

The rest of this work is as follows. In the next section we present the model. In section

3, we analyze first the resources transfer schemes with and without intermediation spread. In section 4, we investigate the effects of wealth distribution, and in section 5 we analyze the implementation of a punishing institution. Section 6 discusses some implications of the model. Lastly, section 7 contains the concluding remarks.

2 Model

The model consists of a community with social interactions, represented by a network, in which a single agent obtains a contractible investment opportunity or project. The idea is that the rest of the community participate in the project. For this to take place, community members must benefit from transferring resources.

Formally, the community is represented by a network $G = (W, E)$, with W being the set of members in the community (nodes of the network) and E is the of links among members in W. In this context, interactions are allowed to be of different magnitudes. These are modeled by a capacity function $c : W \times W \to \mathbb{R}$,[1] such that if $(u, v) \in E$ (i.e. agents u and v are connected), then $c(u, v) = c(v, u) > 0$; and if $(u, v) \notin E$, then $c(u, v) = c(v, u) = 0$. In addition, each agent's link goes bad with a small (and exogenous) probability $\varepsilon > 0$, which is independent across links and agents.[2] We refer to this as the *interaction probability.* If a link has gone bad, neither of the two agents could enjoy mutual interactions. The link could also go bad for endogenous reasons that we will explain shortly.

Another feature of the model is that each agent $u \in W$ has a finite resource capacity $x(u) \geq 0$ (it can also be thought of as a production capacity).

The project is successful with probability $p \in (0, 1)$ and upon success it returns r per dollar invested, and upon failure it returns nothing. Furthermore, we will assume that the project has no liquidation value. The return r and the probability of success p are such that $p(1 + r) \geq 1$. That is, the per dollar net return is positive and constant. Hence, it is optimal to invest as much as possible. The outcome of the project is observable but not contractible.

The timing is as follows:

1. One agent only, say agent $t \in W$, draws an investment opportunity.

2. A repayment structure is stipulated and resources are transfered directly or indirectly[3]

[1] Along this work, the terms *capacity* and *trust* will be used interchangeably.

[2] Links go bad unilaterally.

[3] Indirect transfers are resources transferred from agents not directly connected to t, hence the resources follow a path through intermediary agents up to t.

along the network towards agent t.

3. The project is undertaken and the outcome is observed.

4. Each agent receives a private signal about the state of every own link.

5. Agents involved in the repayment structure decide whether or not to repay.

6. Afterward, linked agents play a pair-wise independent simultaneous cooperation game. When the link has not gone bad the payoff matrix is given by

	C	D
C	$c(u,v), c(u,v)$	$0, c(u,v)/2$
D	$c(u,v)/2, 0$	$-1, -1$

while when the link has gone bad, this is given by

	C	D
C	$-1, -1$	$0, 0$
D	$0, 0$	$0, 0$

We assume that the repayment action is publicly observed and can be distinguished from the project's outcome. Furthermore, upon default, each agent is able to recover a fraction $\tau \in [0, \bar{\tau}]$ ($\bar{\tau} \leq 1$) of the stipulated repayment.[4] This may represent the existence of some regulatory framework in the community, or the recovery capacity by own means (an agent may be able to steal back from the defaulter). It will be referred to as the *recovery rate*.

[4] We will assume that $p(1+r)\bar{\tau} < 1$. This is a sufficient condition to ensure that agents are worse-off upon default than when not transferring.

3 Analysis

3.1 Informal Grand Contract

The size of the project that agent t can undertake would depend on how much resources the community is able to transfer to him. Naturally, this requires an informal contract (due to the lack of formal markets in the community), that allows members of the community to decide the amount transferred and the intermediaries (if needed). Hence, the informal contract stipulates the repayment structure of transfered resources.

Since there is complete and perfect information, it is feasible to consider repayments that are contingent on the outcome of the project, as well as on the actions taken by any other agent. Hence, we will consider a sequential repayment scheme of the following form: after success, agent t promises to pay a return $\rho \leq r$ [5] per unit invested; while after failure, he pays nothing. Given that the project succeeds, if any agent at any repayment stage defaults, then the continuation repayment scheme, for the agents that transferred (directly or indirectly) resources to the defaulter, is τ of the repayment scheme at the previous stage. This follows because the agents being directly defaulted, can retrieve a fraction τ of the agreed upon repayment scheme.

3.2 Punishments

Let z be an arbitrary repayment amount that u has to make to v. One might think that if u fails to repay, v may punish him playing D in the cooperation game, leaving u with a maximum payoff of 0 (which is a kind of *minmax* strategy). The reason to include the probability ε for the link to go bad is to rationalize this punishing mechanism.

Agents u and v encounter themselves in a two stages game, in which first u has to decide whether or not to make the repayment z and afterwards, they play the cooperation game simultaneously. At the beginning of the game both agents already privately know whether

[5] It is clear that for transfers to exist, ρ must satisfy $1/p \leq 1 + \rho$.

the link has gone bad or not. Consider first u's strategy. If he knows that the link has gone bad, he will never make any repayment, since he will get 0 at the cooperation game anyway. If he does not know whether the link has gone bad or not, he believes it is still active with probability $(1-\varepsilon)$, therefore, if he makes the repayment he gets an expected continuation payoff of $(1-\varepsilon)c(u,v)$. If he does not make the repayment, he gets $(1-\tau)z$ (plus at least 0 in the continuation game). This poses a threshold of $\bar{z} = (1-\varepsilon)c(u,v)/(1-\tau)$ to the repayment size z, because if $z > \bar{z}$, u is always better off not making the repayment, which in turn leaves v worse off than not transferring at all. The threshold $\bar{z}$ will be referred to as the *repayment rationality*. This allows us to focus only on repayments $z \leq \bar{z}$, as stated in the following lemma:

Lemma 3.1 *Every resource transfer must satisfy the repayment rationality.*

For the equilibrium of this two stages game, consider the following strategies:

$$v \to \begin{cases} \text{play } C & \text{if received repayment and link is still active} \\ \text{play } D & \text{otherwise} \end{cases}$$

$$u \to \begin{cases} \text{make repayment } z \text{ and play } C & \text{if link is still active and repayment rationality holds} \\ \text{default and play } D & \text{otherwise} \end{cases}$$

Given the strategies and the repayment rationality, v's updated beliefs establish that after a repayment, u's link is active with probability 1; whereas that after a default, u's link has gone bad with probability 1. Therefore, if v observes a repayment, she correctly infers that u's link is still active and thus plays C, which makes optimal for her to play C in case that her link is still active, and to play D otherwise. And if she observes a default, she correctly infers that u's link has gone bad and thus it is optimal for her to play D.

On the other hand, if u's link is active and he decides not to make the repayment, he gets at most the size of the repayment after what is recovered by v, $(1-\tau)z$. If he decides to make the repayment, given v's strategy, it is optimal for him to play C with a payoff of $(1-\varepsilon)c(u,v)$, which given that $z \leq \bar{z}$, is larger than z. Hence, it is optimal for u to make the repayment if his link is still active. If his link has gone bad, the strategy is trivially optimal.

3.3 Two agents example

Suppose there are only two agents in the community, u and t. Given a repayment rate ρ, what amount of resources can be invested in the project? Agent t's resources can always be fully invested, so they are left out of the analysis and the focus is on the resources that u can transfer to t. Let f be the flow of resources that u transfers to t. First thing to be noticed is that f cannot exceed u's resource constraint, i.e. $f \leq x(u)$. Second, f must satisfy the *repayment rationality* condition (for if not, u would prefer not to transfer any resources at all), i.e. $f(1+\rho) \leq (1-\varepsilon)c(u,t)/(1-\tau)$. Third, u has to at least break-even (in expectation), i.e. $p[(1-\varepsilon)f(1+\rho)+\varepsilon\tau f(1+\rho)] - f \geq 0 \Leftrightarrow p[(1-\varepsilon)+\varepsilon\tau](1+\rho) \geq 1$.

Thus, there are two conditions about the size of the flow that u can transfer to t, and one condition about the repayment rate ρ. From the first two conditions, the relevant one is the strongest one, so it can be written as $f \leq min\left\{x(u), \frac{(1-\varepsilon)c(u,t)}{(1-\tau)(1+\rho)}\right\}$. This is equivalent to the max flow $s \to t$ in the modified network $\tilde{G} = (W \cup \{s\}, E \cup (s,u))$, with capacities $\tilde{c}(s,u) = x(u)$ and $\tilde{c}(u,t) = \frac{(1-\varepsilon)c(u,t)}{(1-\tau)(1+\rho)}$, subject to $p[(1-\varepsilon)+\varepsilon\tau](1+\rho) \geq 1$. The inclusion of node s serves two purposes, first, it allows the resource constraint to be included as a flow constraint, and second, it will allow for multi-sources flow, as will become clear shortly.

3.4 General Characterization

The idea exposed in the previous example can be easily extended to any network $G = (W, E)$, but first we introduce the concepts of flows and maximal flows.

Definition 3.2 *A flow $f : E \to \mathbb{R}$, relative to capacities c, a source $s \in W$ and a sink $t \in W$, is a function that satisfies*

(i) Capacity constraints: $f(u,v) \leq c(u,v)$ for all $(u,v) \in E$,

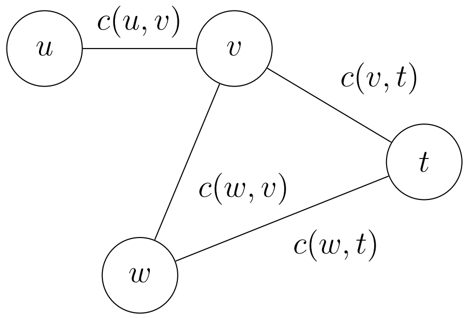
(ii) Antisymmetry constraints: $f(u,v) = -f(v,u)$ for all $(u,v) \in E$,

(iii) Flow conservation: $\sum_{w \in W} f(u,w) = 0$ for all $u \in W \setminus \{s,t\}$.

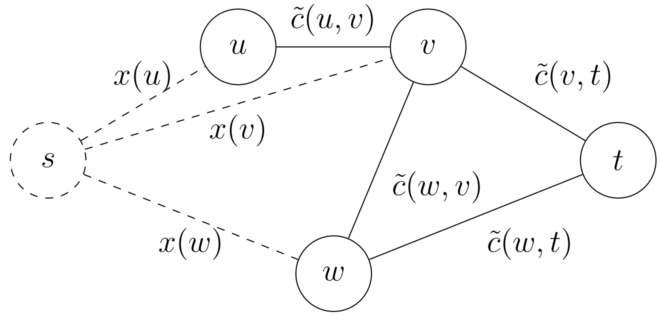
It will be referred to as a $s \to t$ flow. The value of the flow is the amount leaving the source s, i.e. $|f| = \sum_{w \in W} f(s,w)$. The maximal flow is therefore given by $\arg\max_f\{|f|\}$.

Additionally, if the flow is constrained to a certain path length l, it will be referred to as a $s \to t|_l$ flow.

In order to apply the two agents example's idea onto any network, it is necessary to transform it into the network $\tilde{G} = (W \cup \{s\}, E \cup \{(s,u)\}_{u \in W})$, with $\tilde{c}(s,u) = x(u)\ \forall u \in W$ and $\tilde{c}(u,v) = \frac{(1-\varepsilon)c(u,v)}{(1-\tau)(1+\rho)}\ \forall (u,v) \in E$, as is shown in figure 3.1. Given that agent t has the investment opportunity, the max flow $s \to t$ represents the maximal amount of resources that can be pledged to t, under a repayment rate ρ, so that the repayment scheme is followed through. However, this flow concept alone does not guarantee that every agent involved in it breaks even, since repayments are susceptible of being diluted along the network, in case that some links go bad.



(a) Network G.



(b) Network $\tilde{G}$.

Figure 3.1: 4 agents network.

To check whether a particular agent u is at least breaking even on expectation, it is necessary to account for every (u, t) path that the $u \to t$ flow uses. In the two agents example was already shown that, conditional on the project being successful, the expected repayment rate that any agent receives, due to a length 1 investment path, is $[(1 - \varepsilon) + \varepsilon\tau](1 + \rho)$. This shows that there is a factor $[(1 - \varepsilon) + \varepsilon\tau]$ related to the contingency of default at distance 1. This factor updates the expected repayment flow at every link from a particular path. Therefore, and using an inductive argument, a flow that goes through a path of length l, has an expected repayment rate given by $p[(1 - \varepsilon) + \varepsilon\tau]^l(1 + \rho)$. In conclusion, there is a maximal length for the path that any investment flow can take, and is given by $l(\rho) = \max\{l' \in \mathbb{N} \mid p[(1 - \varepsilon) + \varepsilon\tau]^{l'}(1 + \rho) \geq 1\}$.[6,7]

Theorem 3.3 *The maximal amount of resources that can be transfered to t, given a repayment rate ρ, is the length constrained max flow $s \to t|_{l(\rho)+1}$ in network $\tilde{G}$.*

For Theorem 3.3's proof, see Theorem 3.8's proof in Appendix, with $\gamma = 1$ and $\rho = r$.

The *Length Constrained Maximum Flow Problem* has been documented in the network field, for various purposes (Kodialam and Lakshman [2002]).

3.4.1 Comparative Statics

So far, there are mainly two aspects that determine the maximal amount of resources that can be invested (besides the actual resources): the trust among members, represented by $\tilde{c}$; and the participation reach of the community, represented by $l(\rho)$. As well, the main parameters affecting these aspects are the recovery rate τ, the repayment rate ρ, the interaction probability ε and the success probability p. The idea is to determine the effect of the parameters on the two main aspects, and thus, the net effect on the maximal amount of invested resources.

[6] The maximum is well defined since $[(1 - \varepsilon) + \varepsilon\tau]$ is strictly less than 1.

[7] Since the original network is being transformed by adding the node s and the respective additional links, there is an extra link that every path has to follow, which does not have any moral risk, namely, the (s, u) link for all $u \in W$. Thus, the allowed maximal length is effectively $l(\rho) + 1$.

First thing to notice is that an increase in the trust capacities $\tilde{c}$ is beneficial in terms of maximal flow, since more resources can be transferred among the participation reach (as long as there are resources left over). On the other hand, an increase in the participation reach $l(\rho)$ is also beneficial since it expands the community resources that may be transferred.

The results showed in the following proposition are straightforward:

Proposition 3.4

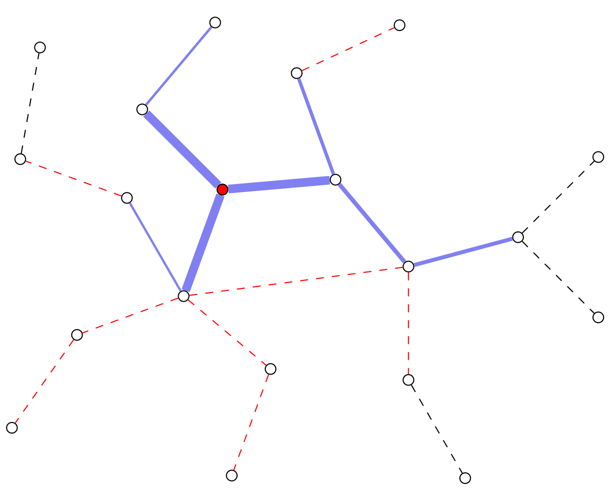
(i) An increase in the recovery rate τ shifts $\tilde{c}$ upwards and $l(\rho)$ weakly upwards. It follows that it weakly enhances the community's financing capacity.

(ii) A decrease in the interaction probability ε also shifts $\tilde{c}$ upwards and $l(\rho)$ weakly upwards. It follows that it also weakly enhances the community's financing capacity.

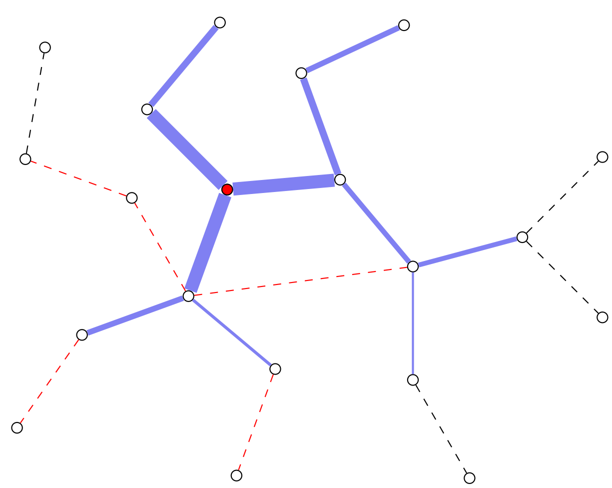
(iii) An increase in the success probability p shifts $l(\rho)$ weakly upwards. It follows that it weakly enhances the community's financing capacity.

The impact of ρ is somewhat less direct. For one, an increase in ρ shifts $\tilde{c}$ downwards, but on the other hand, it weakly shifts $l(\rho)$ upwards. This represents a trade-off between participation reach and per path flow. Which effect dominates will depend on the community's characteristics.

Proposition 3.5 *If trust is high (high $\tilde{c}$) relative to resources, an increase in ρ enhances the community's financing capacity, since the gain from expanding the resources available more than offsets the loss in trust. Analogously, if trust is low (low $\tilde{c}$) relative to resources, an increase in ρ lowers the community's financing capacity, since the trust effect dominates.*

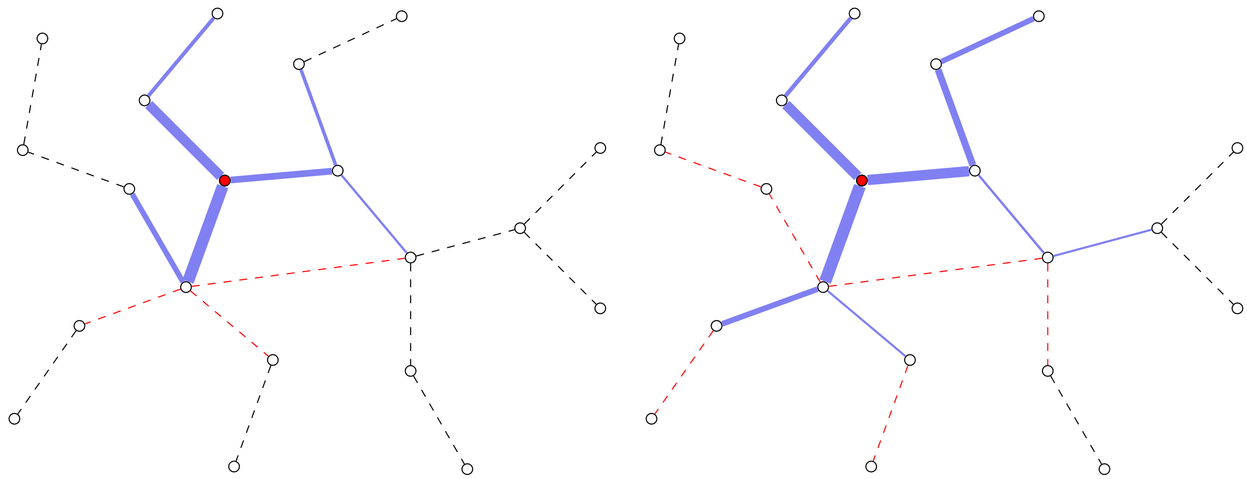


(a) Low τ.



(b) High τ.

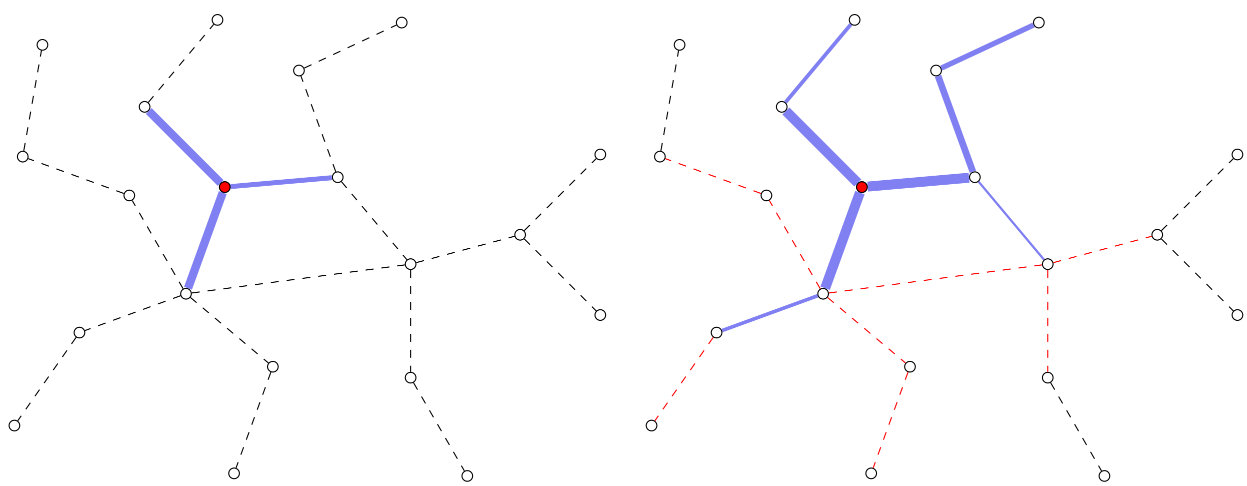
Figure 3.2: Increase of *recovery rate* τ.



(a) High ε. *(b)* Low ε.

Figure 3.3: Decrease of *interaction probability* ε.

Figures 3.2, 3.3 and 3.4[8] show the effects of increments in τ, $-\varepsilon$ and p respectively. They also demonstrate that changes in trust are continuous, while changes in reach are discrete. Therefore, figure 3.2 shows a change in trust only, whereas figure 3.3 shows a change in both trust and reach. Figure 3.4 shows a change in reach only.



(a) Low p. *(b)* High p.

Figure 3.4: Increase of *probability* p.

[8] In these figures, as well as in figures 3.5 and 3.6, the red node denotes agent t, the blue edges denote the actual max flow, the red dashed edges denote the extent of the participation reach (of course blue edges imply participation reach) and the black dashed edges represent social relations among agents. The thickness of the blue edges is proportional to the flow's magnitude.

3.5 General Characterization with *Intermediation Spread*

So far, the model considers that agents transfer other member's resources just for the sake of it.[9] A refinement to the model is to consider the repayment, in case of success, to be the return rate of the project, r, adjusted by an intermediation factor $\gamma \in (0,1)$, which might be considered as an *intermediation spread* (actually, $(1-\gamma)$ accounts for the spread).[10] In this way, agents do get a benefit from transferring other agent's resources.

This new approach of the model needs a refinement in network flows, which is the concept of *generalized flow*. This concept considers that each link has a *loss factor*, which makes the outgoing flow from a particular link less than the ingoing flow. In this fashion, and considering that the spread γ is the loss factor, the repayments are a generalized flow, which is being diluted, and that dilution accounts for the intermediation benefits. More formally,

Definition 3.6 *A generalized flow, with loss factor γ, is a flow as defined in Definition* 3.2*, with the antisymmetry constrained modified as follows:*

$$f(u,v) = -\gamma f(v,u) \textit{ for all } (u,v) \in E$$

The concepts of generalized maximal flow and generalized path length constrained flow are trivially extended from Definition 3.2.

To extend the previous characterization to this new setting, let network $\hat{G} = (W \cup \{s\}, E \cup \{(s,u)\}_{u \in W}, \gamma)$, with $\hat{c}(s,u) = +\infty \ \forall u \in W$ and $\hat{c}(u,v) = \frac{(1-\varepsilon)c(u,v)}{(1-\tau)(1+r)} \ \forall (u,v) \in E$, be the new transformation of the original network $G = (W,E)$. The generalized max flow $t \to s$[11] in $\hat{G}$ is the maximal amount of resources that can be transfered to t, considering only capacity constraints (in other words, considering only the "trust" conditions). In order to incorporate the resource constraints, it is necessary to approach the generalized max flow problem from its path formulation, adding the resource constraints explicitly:

Definition 3.7 (Generalized Max Flow - Path Formulation) *Let $\mathcal{P}$ be the set of (s,t) paths and let $F(P)$ be the flow associated to path $P \in \mathcal{P}$. The linear problem to be solved is:*

[9] Actually, an agent that transfers another agent's resources, gets a benefit in expectation due to the probability ε. But it still does not seem enough to account for transfers of other agent's resources.

[10] The *spread* is considered to be multiplicative, rather than additive, to keep in line with an extended notion of flow, the *generalized flow*, which is introduced in the next paragraph.

[11] These is the same as a $s \to t$ flow with loss factor $1/\gamma$ (which is actually a gain factor).

$$
\begin{array}{lll}
\underset{\{F(P)\}_{P\in\mathcal{P}}}{maximize} & \sum_{P} F(P) & \\
s.t & \sum_{P\,:\,e\in P} \gamma^{d(e|P)} F(P) \leq \dfrac{(1-\varepsilon)c(e)}{(1-\tau)(1+r)} = \hat{c}(e) & \forall e \in E \\
& \sum_{P\,:\,e\in P} F(P) \leq x(e) & \forall e \in \{(s,u)\}_{u\in W} \\
& F(P) \geq 0 & \forall P \in \mathcal{P}
\end{array}
$$

where $d(e|P)$ is the position of link e in path P (relative to agent t).

The first constraint guarantees that the repayment is incentive compatible and the second constraint guarantees resources feasibility. This is a linear programing problem, which can be solved with various solver applications.

Analogous to the previous characterization, each path that contains a positive flow has to be constrained to a maximal length, so that agents at least break-even. In this setting, the expected repayment rate for an agent at distance 1 from t, conditional on the project being successful, is $[(1-\varepsilon)+\varepsilon\tau](1+r)\gamma$, and thus, the relevant factor is $[(1-\varepsilon)+\varepsilon\tau]\gamma$. This implies that the maximal path length that any resources flow can go through is given by $l(\gamma) = \max\{l' \in \mathbb{N} \mid p\left([(1-\varepsilon)+\varepsilon\tau]\gamma\right)^{l'}(1+r) \geq 1\}$. In this way, it is sufficient to restrict the set $\mathcal{P}$ to those paths that not exceed the length $l(\gamma)+1$, i.e. $\mathcal{P}_{l(\gamma)+1}$.[12,13]

Theorem 3.8 *The maximal ammount of resources that can be transferred to agent t, given an intermediation spread $(1-\gamma)$, is the solution of the generalized max flow problem of Definition* 3.7*, with (s,t) paths $\mathcal{P}_{l(\gamma)+1}$, in network $\hat{G}$.*

See proof in Appendix.

The solutions to the problem in Definition 3.7 give the amount of resources that can be transferred along the network, but they allow for allocations that should not be observed at all. This occur since the formulation in Definition 3.7 does not take into account that agents prefer to first transfer own resources rather than other's resources. For the optimization problem to account for it, it is necessary to modify it in the following manner:

Definition 3.9 (Generalized Max Flow - Path Formulation) *Consider the maximization problem in Definition* 3.7*, and let the objective function be changed for:*

$$
\underset{\{F(P)\}_{P\in\mathcal{P}}}{maximize} \quad \sum_{P}[1-\delta d^{*}(P)]F(P)
$$

[12]Again, the actual allowed path length is $l(\gamma)+1$, because of the (s,u) links.

[13]For details about the constrained paths finding process, see the Appendix.

where $d^*(P)$ is the number of agents in path P that have own resources and δ is an arbitrarily small parameter.

The term $\delta d^*(P)$ adds a "cost" to the path in terms of how many agents with own resources that path has. Therefore, the maximization will allocate flows through the least costly paths whenever possibly, which in turn means that resources are transferred firstly from own resources. Since δ can be an arbitrarily small parameter, it can be fixed so that $\delta d^*(P) < 1$ for every path P,[14] which ensures that the gain of transferring resources always surpasses the "cost" of the path, thus preventing suboptimal solutions. In other words, paths that have the same number of agents with own resources, will be perfect substitutes (whenever the constraints allow it), whereas paths with different number of agents with own resources can be unequivocally ordered.

This formulation still misses a point, which is that agents, whenever possible, would prefer to transfer resources through shorter paths, thus minimizing the intermediation costs they bare. This cannot be achieved by formulating the problem to simply use the shortest paths possible, since two agents with different length paths should be equally allowed to transfer their own resources. Therefore, it has to be formulated so that each agent uses its own shortest paths whenever possible (which is different than shortest paths being used globally in the network), regardless of what others do. This detailed formulation is shown in the Appendix.[15]

3.5.1 Comparative Statics

As in the previous characterization, the main aspects that modulate the community's investment are the trust among members, $\hat{c}$, and the participation reach, $l(\gamma)$. The parameters τ, ε and p have the same effect as in Proposition 3.4. The parameter ρ is now changed for the two parameters γ and r.

If γ is increased, representing a lower intermediation spread, the terms $F(P)$ in the first set of constraints are directly affected and reduced by it. This is because the lower the intermediation is, the higher the repayment will be, which goes against the trust aspect (due to $\hat{c}$). On contrary, the increment in γ shifts the participation reach $l(\gamma)$ weakly upwards, resulting in a trade-off similar to the one with ρ.

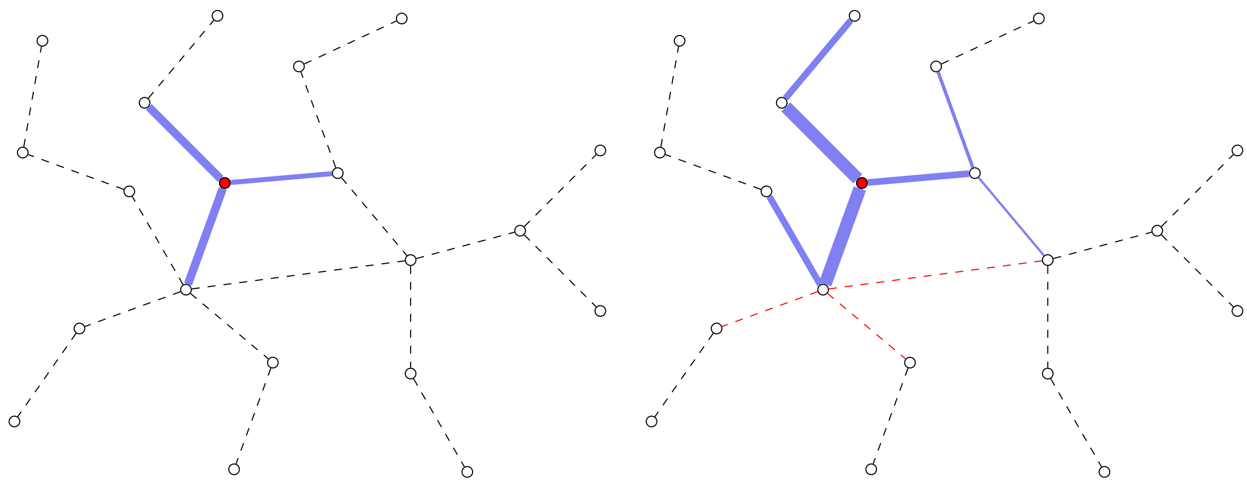
The effect of an increase in r is similar to the one of γ, but it directly affects $\hat{c}$ rather than indirectly.

Proposition 3.10 *If trust is high (high $\hat{c}$) relative to resources, an increase in γ or r enhances the community's financing capacity, since the gain from expanding the resources*

[14]It is sufficient to chose $\delta < 1/(|W| - 1)$.

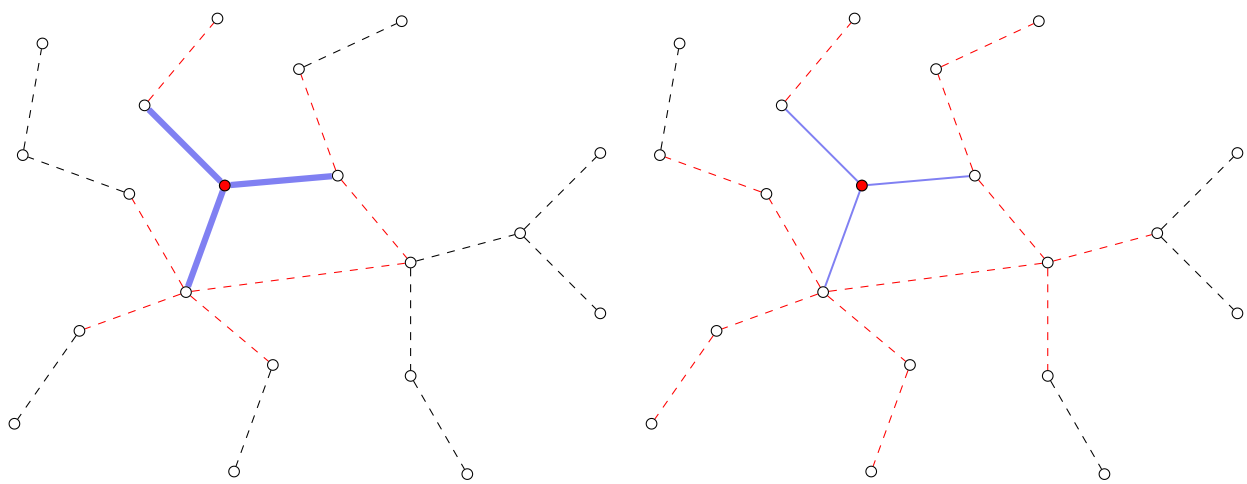
[15]Note that this consideration is relevant only for the actual resources flows, and not for the magnitude of the max flow solution.

available more than offsets the loss in trust. Analogously, if trust is low (low $\hat{c}$) relative to resources, an increase in γ or r lowers the community's financing capacity, since the trust effect dominates.



(a) High $(1-\gamma)$. *(b)* Low $(1-\gamma)$.

Figure 3.5: Decrease of *intermedation spread* $(1-\gamma)$ at high trust.



(a) Low r. *(b)* High r.

Figure 3.6: Increase of *interest rate* r at low trust.

Figures 3.5 and 3.6 show the effects of increments in γ and r respectively. They represent the two different scenarios: firstly, figure 3.5 is an increment in γ when trust is high relative to resources, and thus the increment in reach is the predominant effect; secondly, figure 3.6 is an increment in r when trust is low relative to resources, and thus the increment in reach is not useful since the extra resources cannot be transferred (due to low trust) and this is the effect that dominates.

4 Wealth Distribution Analysis

To examine the role that wealth or resources distribution play, we will assume that the resource capacities $x(u)$ are randomly drawn from an i.i.d distribution F over the network's agents, with support $[0, b]$. Conditional on t being the agent with the project, what is the expected amount of resources that can be effectively invested?

Given the capacity constraints c, each $t \to s$ generalized max flow in $\hat{G}$,[16] with (s, t) paths $\mathcal{P}_{l(\gamma)+1}$, measures the level of "trust" inside the community, towards agent t. Let $\mathcal{T}^{st}(c) \subset \mathbb{R}^{|W|}$ be the set of maximal flows, characterized as vectors $T \in \mathcal{T}^{st}(c)$, whose u-*th* component corresponds to the flow between s and u for that particular max flow T. In other words, for a particular $T \in \mathcal{T}^{st}(c)$, the u-*th* component, T_u, corresponds to the maximal amount of own resources that agent u is able to transfer due to "trust" only. Therefore, consider a fixed trust vector $T \in \mathcal{T}^{st}(c)$, then the expected amount of resources that the community can effectively invest through t, is given by a truncated expectation in the following manner:

$$\mathbb{E}[I(t)]_F = \mathbb{E}\left[\sum_u h_u(x_u)\right]_F$$

where

$$h_u(x_u) = \begin{cases} x_u & \text{if } x_u \leq T_u \\ T_u & \text{if } x_u > T_u \end{cases}$$

It can be re-written as follows:

[16]The value of the flow is unique, but there may be numerous flows achieving the same maximal value.

$$
\begin{aligned}
\mathbb{E}[I(t)]_F &= \int_{[0,b]^{|W|}} \sum_u h_u(x_u)dF(x) \\
&= \sum_u \int_0^b h_u(x_u)dF(x_u) \\
&= \sum_u \int_0^{\min\{T_u,b\}} x_u dF(x_u) + \sum_u T_u \int_{\min\{T_u,b\}}^b dF(x_u) \\
&= \sum_u \left\{ \min\{T_u, b\}F(\min\{T_u, b\}) - \int_0^{\min\{T_u,b\}} F(x_u)dx_u \right\} \\
&\qquad + \sum_u T_u[1 - F(\min\{T_u, b\})] \\
&= \sum_u \min\{T_u, b\} - \sum_u \int_0^{\min\{T_u,b\}} F(x_u)dx_u
\end{aligned}
$$

where $F(x)$ and $F(x_u)$ represent the joint distribution and the marginal (one dimensional) distribution respectively. The second equality is due to the additive separability of the expectation and the fourth equality is due to the integration by parts of the left-hand side integrals.

In it self, the previous expression is not of much use, but it becomes useful to asses the effect of changes in the distribution F. Consider that the resource capacities distribution changes from F to G (over the same support), the change then in the amount of resources that can be effectively invested through t changes as follows:

$$
\Delta\mathbb{E} = \mathbb{E}[I(t)]_F - \mathbb{E}[I(t)]_G = \sum_u \int_0^{\min\{T_u,b\}} [G(x_u) - F(x_u)]dx_u
$$

It is clear that a sufficient condition to determine the sign of the previous expression is the concept of *second order stochastic dominance (SOSD)*.[17] In fact, if $F \geq_{SOSD} G$, then $\int_0^t F(t)dt \leq \int_0^t G(t)dt \; \forall t \in [0, b]$, which directly implies that $\Delta\mathbb{E} \geq 0$.

The preceding argument is valid for all $T \in \mathcal{T}^{st}(c)$, hence, it is globally valid, and we can state the following proposition:

Proposition 4.1 *Let F and G be two distributions over $[0, b]$ such that $F \geq_{SOSD} G$, conditional on $t \in W$ being the agent with the project, $\Delta E = \mathbb{E}[I(t)]_F - \mathbb{E}[I(t)]_G$ is non-negative.*

A nice feature of Proposition 4.1 is that it allows for comparisons among distributions related by the concept of *mean-preserving spread*. A mean-preserving spread is the same as

[17] Obviously, as *first order stochastic dominance (FOSD)* implies (SOSD), (FOSD) is also a sufficient condition.

(SOSD), restricted to distributions with the same mean. This is of some interest because the notion of mean-preserving spread is related to the notion of resources inequality, in this case, the *ex-ante* distribution of resources in the community. Thus, the implication of the previous result is that an improvement in the *ex-ante* resource distribution (but remaining its expected value), leads to a higher level of investment capacity for the community.

Corollary 4.2 *Let F and G be two distributions over $[0, b]$ such that F is an improvement over G in terms of resources inequality (i.e. G is a mean-preserving spread of F), then $\Delta E = \mathbb{E}[I(t)]_F - \mathbb{E}[I(t)]_G$ is non-negative.*

5 Punishing Institution

The role that institutions play in a society can be quite decisive in its political and economical evolution, explaining a great deal of differences between societies performances, and ultimately, society's emergence. In terms of performance, economic institutions are directly responsible for it, and shape the fashion in which resources are seized. However, political institutions do also play an important role, since it is the political institutions the ones that shape which economic institutions will emerge and last.

In the early American history, a vast set of examples can be found regarding different models of institutions and its outcome. In South America, mainly extractive institutions were established, allowing the Spaniard colonizers to gather huge amounts of gold and silver and retrieving it to Spain. The previously prevalent hierarchical structure of indigenous settlements helped in doing so, since the Spaniards needed only to take the position of the leader to coerce the rest of the population into their favor. This outcome is very distant to what happened in North America. English settlers tried to force the indigenous population to work for them, without any success. Soon after failure, the elite of English settlers in North America tried to impose a structure of forced labor among English settlers, restricting power to the elite only. This institutions also failed, since settlers were left with little or no incentives. Later, they realized that proper incentives had to be given, so political and economical power was distributed more evenly among settlers by means of more inclusive institutions, giving way to early images of democracy. The different outcomes and institutions that prevailed explain in some way the differences in performance that North and South America have exhibited up until today. Institutions have to appeal to the preferences of the people in charge of implementing them, so if the power is concentrated in few people (as in the case of South America in its early history, or in many African countries), it is more likely for extractive institutions to emerge, pleasing those few in the power. But, in societies where the power can not be sustained by just a few (as in early North America), extractive institutions are more likely to fail, giving way to inclusive institutions.

In the context of our model, let us think about the existence of an institution, which imposes a punishment to agents that do not behave accordingly with the stipulated informal contract. Such an institution can be thought of as a sort of fine, or jail, any of this possibilities reflecting a cost to the ones that deviate. The institution is costly and has to be founded by the community, in terms of personal provisions to it, e.g. time spent working for the institution, or effort as well.

Let κ be the cost, in terms of personal provisions, of the institution, and let S be the size of the punishment (i.e. the cost that represents the institution to the ones that deviate). The institution is only achieved when the total amount of personal provisions exceed the cost of it, i.e. when $\sum_u h_u \geq \kappa$, where $h_u \in H_u = [0, H]$ is the provision of agent $u \in W$. The provision is costly as well, with cost function $\psi : H_u \to \mathbb{R}$.

In the presence of such an institution, the incentives that agents face are shifted in the following manner: for any level z of repayment that u has to make to v, for it to be rational, it has to satisfy $z \leq [(1-\varepsilon)c(u,v) + S]/(1-\tau)$. Therefore, the institution improves the trust among agents in the community, and this in turn, improves (potentially) the amount of resources that the community can invest.

Let I be the total amount of resources that the community invests while no institution exist, and let I' be the same amount when the institution does exist. The gross surplus of the institution is given by $p(1+r)(I'-I) \geq 0$, therefore, it will be efficient to implement the institution whenever this surplus surpasses the cost of provision, κ. Now, the cost of provision, κ, depends on the cost structure ψ of the agents. For instance, if ψ is a linear function, then it is efficient to implement the institution whenever $p(1+r)(I'-I) \geq \psi(\kappa)$; whereas if ψ is a convex function, then it is efficient to implement the institution whenever $p(1+r)(I'-I) \geq \sum_u \psi(\kappa/|W|)$. However, the efficient provision of the institution might not be achievable. If ψ is linear, and the sets H_u are unbounded or sufficiently large, then it is trivial to find an optimal allocation among agents (just take h_u so that $\psi(h_u)$ equals the change of utility of each agent when the institution is provided), but if the sets H_u are not large enough, it might not be possible anymore (because some might not be able to provide as much as they should due to the change of utility). When ψ is convex, then the substitution of provision among agents is not perfect anymore, making it more difficult to achieve efficient provision.

5.1 Provision Game

For exposition simplicity, we will focus on the limiting case $\varepsilon \to 0$,[18] so that there are no repayment defaults on equilibrium (see Appendix for the regular case). Each agent has to simultaneously choose a provision level $h_u \in H_u$, and payoffs are given by

$$\pi_u(h_u, h_{-u}) = \begin{cases} U_u(S) - \psi(h_u) & \text{if } \sum_u h_u \geq \kappa \\ U_u(0) & \text{if } \sim \end{cases}$$

where

[18] Refer to the Appendix for more details.

$$U_u(\cdot) = \sum_{P \in \mathcal{P}: u \in P, (s,u) \notin P} p\gamma^{d|_P}(1+r)\mathbb{E}[F(P,\cdot)](1-\gamma) + \sum_{P \in \mathcal{P}: (s,u) \in P} p\gamma^{d|_P}(1+r)\mathbb{E}[F(P,\cdot)]$$

and the expectation is taken over a uniform distribution among all the solutions of Definition 3.9. The first term represents the intermediation profits of agent u, while the second term represents the profits from own resources.

Each agent's maximal provision, given that the institution is implemented, is given by

$$\bar{h}_u = \psi^{-1}(U_u(S) - U_u(0)) \tag{5.1}$$

Proposition 5.1 *(i) A pure equilibrium of the Provision Game always exist, and moreover, (ii)*

- *If* $\max\{\bar{h}_u\} \geq \kappa$*, then in every equilibrium the institution is implemented (multiple equilibria strategies).*
- *If* $\max\{\bar{h}_u\} < \kappa$ *and* $\sum \bar{h}_u \geq \kappa$*, then there are two kinds of equilibria, one where the institution is implemented, and the other where is not implemented.*
- *If* $\max\{\bar{h}_u\} < \kappa$ *and* $\sum \bar{h}_u < \kappa$*, then in equilibrium provisions are all zero, and the institution is not implemented.*

(iii) The set of equilibria that implement the institution is denoted by $\{h_u^*\}_{u \in W}$ *such that* $\sum_u h_u^* = \kappa$ *and* $h_u^* \in [0, \bar{h}_u]$ *for all* $u \in W$*.*

Proof. (i) Take the empty allocation (i.e. $h_u = 0$ for all u), if it is an equilibrium, existence is proven. If it is not an equilibrium, it means that a single agent, v, is willing to provide $h_v = \kappa$, which is an equilibrium. (ii) If $\max\{\bar{h}_u\} \geq \kappa$, by (i) it is an equilibrium, and since an agent alone can implement it, it follows that in every equilibrium the institution is implemented. Else, if $\max\{\bar{h}_u\} < \kappa$ and $\sum \bar{h}_u \geq \kappa$, then no agent alone can implement the institution, thus there is an equilibrium without implementation, but there is enough aggregated provision for the institution to be implemented. Finally, if $\max\{\bar{h}_u\} < \kappa$ and $\sum \bar{h}_u < \kappa$, no agent alone can implement the institution and there is no sufficient aggregate provision to implement it either. Therefore, the equilibrium is one with zero provision. (iii) Take any allocation (if exists) $\{h_u^*\}$ such that $h_u^* \leq \bar{h}_u$ for all u and $\sum_u h_u^* = \kappa$, since every agent is providing less than his maximal provision, no one is willing to lower his provision since it would prevent the institution from being implemented; in turn, no one has incentives to increase his provision, since it makes no improvement at all. □

5.2 Comparative Statics

As Proposition 5.1 states, there is a maximum cost for the institution to be implemented, given by $\bar{\kappa} = \sum_u \bar{h}_u$. This maximal value is dependent on the regulation level τ. Notice that each $\bar{h}_u$ is determined by the difference $U_u(S) - U_u(0)$. In turn, each $U_u(\cdot)$ depends on τ via the capacities $\hat{c}$, which determine the maximum flows. It is clear that $U_u(\cdot)$ is weakly increasing in τ, for the capacities $\hat{c}$ are increasing in τ.

To see whether an improvement in the regulation level τ eases the maximum cost, we have to see how both $U_u(0)$ and $U_u(S)$ change with τ. Thus, we have to look at

$$\begin{aligned}
\Delta U_u &= U_u(S) - U_u(0) \\
&= \sum_{P \in \mathcal{P}: u \in P, (s,u) \notin P} p\gamma^{d|_P}(1+r)\left\{\mathbb{E}[F(P,S)] - \mathbb{E}[F(P,0)]\right\}(1-\gamma) + \\
&\quad \sum_{P \in \mathcal{P}: (s,u) \in P} p\gamma^{d|_P}(1+r)\left\{\mathbb{E}[F(P,S)] - \mathbb{E}[F(P,0)]\right\}
\end{aligned}$$

If the institution is already implemented, and the only constrains active in the maximization problem in Definition 3.9 are the capacity constraints (for every maximal flow) (condition (2)), then a marginal increment in the regulation τ shifts the maximum cost level upwards. Denote by τ_1 (if it exists), the maximum regulation level for which no resource constraint is active, given the institution being implemented. Then it follows that an increment in the capacities $\hat{c}$ due to the institution, is larger when τ is larger, and thus, the benefits from the institution, represented by 5.1, are larger since the only constraints active are the "trust" constraints, so there are actually more resources available for every agent that is already transferring resources.

Formally, consider M_i and m_i, for $i = 0, S$, the best and worse maximal flow for a particular agent, on a particular path. Then $\mathbb{E}[F(P,i)] = \frac{M_i + m_i}{2}$, and thus $\mathbb{E}[F(P,S)] - \mathbb{E}[F(P,0)] = \frac{M_S - M_0}{2} + \frac{m_S - m_0}{2}$. Notice that each upper limit M_i is a capacity constraint along the path, i.e. $\frac{c(e)+i}{(1-\tau)(1+r)}$ for some e along the path, or the agent's resource constraint (but given condition (2), it will be a capacity constraint). Moreover, it will be the same path's link for $i = 0, S$, since capacities grow proportionally due to the institution. The lower limits m_i, under condition (2), are either 0 or a capacity constraint along the path (again, it will be the same path's link along the path), for if it is not a capacity constraint and not zero, it would mean that at some point there is a resource constraint active. Thus, $M_S - M_0 = S/((1-\tau)(1+r))$ and $m_S - m_0 = 0$ or $m_S - m_0 = S/((1-\tau)(1+r))$, which are clearly increasing in τ.

However, if for some agent v his resource constraint becomes active when the institution is implemented (for some maximal flow), it means that the extra resources that he transfers in some of the maximal flows, given the implementation of the institution, are being truncated.

Therefore, if τ is marginally incremented, then $U_v(0)$ is fully incremented, whereas $U_v(S)$ has some paths that are truncated, and thus $\bar{h}_v$ is decreasing, and it is not clear whether $\bar{\kappa}$ is incremented or not.

Analogous to condition (2), if the institution is implemented and all resources can be transferred (condition (3)), then a marginal increase in τ is only detrimental in terms of the institution provision. The reason for this is quite simple, if the institution allows all resources to be transferred, then an increment in τ has no impact on the profits (under institution), whereas it potentially has a positive impact in absence of the institution, therefore, the terms $\bar{h}_u$ are only weakly decreasing. The level of τ at which it starts to be detrimental (if it exists), τ_2, corresponds to the level of regulation that just allows the whole community resources to be invested, given the institution being implemented.

Thus, a marginal increment in the regulation τ is only beneficial, for the purposes of the institution, in the interval $[0, \tau_1]$ (if τ_1 exists), whereas it is only detrimental in the interval $[\tau_2, \bar{\tau}]$ (if τ_2 exists). The impact among the interval $[\tau_1, \tau_2]$ is undetermined.

6 Implications

6.1 Network Structure and Welfare

As in Karlan *et al* [2009], their implications about network structure over welfare also apply almost directly in this framework. Propositions 1 and 2 in Karlan *et al*, regarding to monotonicity among connections and closure respectively, have their similar here.

Definition 6.1 *A network with capacities c_1 is said to be more strongly connected than network with capacities c_2 if no link has lower capacity under c_1 than under c_2, i.e. $c_1(u,v) \geq c_2(u,v)$ for all $(u,v) \in E$.*

Then, as in Karlan *et al*, the following proposition follows immediately:

Proposition 6.2 *If the network with capacities c_1 is more strongly connected than the network with capacities c_2, then for any agent t with the investment opportunity, trust and payoffs are higher under c_1.*

This is true because if $c_1(u,v) \geq c_2(u,v)$ for each $(u,v) \in E$, then every feasible flow under c_2 is also feasible under c_1, thus trust is at least as high under c_1 as under c_2. It follows that agents under c_1 are transferring more own resources and/or others resources, which in any case leads to higher payoffs for every agent.

Definition 6.3 *The network neighborhood of t has a higher closure than the neighborhood of t' if*

(i) t and t' have the same total number of (s,i) paths, $i = t, t'$,

(ii) For each n, $P^t(n) \geq P^{t'}(n)$.

where $P^t(n)$ denotes the share of paths t has with agents to whom he has at least n paths.

Again as in Karlan *et al*, there is a relation between neighborhood closure and agent's t welfare.

Proposition 6.4 *If trust, measured by $\hat{c}$, is low relative to resources, a neighborhood with* *higher closure leads to a higher payoff to t. In contrast, if trust is high relative to resources, a neighborhood with higher closure leads to a lower payoff to t.*

The reason for this is that if trust is low relative to resources, then having a higher closure means that t "boosts" trust by means of more connections, which yields a higher amount of resources transferred to him. On contrary, if trust is high relative to resources, then a higher closure means that t has less reach, so he is not accessing to resources that would be able to be transferred due to trust, which yields a lower amount of resources transferred to him.

The notion of *betweenness centrality*, first introduced by Freeman [1977], can also be applied to this framework, related to intermediation. However, it has to be modified in order to account for the resources capacities. We will restrict the analysis to networks that exhibit the tree topology.

Since the network is a tree, there are no cycles, therefore, for every pair of agents, there is only one path (i.e. the shortest path). Consider agents v and k in W, if v lies on the (k,t) path, the amount that v may intermediate from k to t is the minimum between k's resources, $x(k)$, and the residual (k,t) path's trust capacity. The residual (k,t) path's trust capacity is the remaining capacity after agents closer to t, in (k,t) path, transfer their own resources first, i.e. $\max\left\{\hat{c}(k,t) - \sum_{j\in(k,t)\setminus\{k\}} x(j), 0\right\}$. The (k,t) path's trust capacity, $\hat{c}(k,t)$, is the least trust capacity of the links that conform the path, adjusted by γ, i.e. $\min\{\gamma^{-d(e|P)}\hat{c}(e)|e\in(k,t)\}$.

Let us now define the *weighted betweenness centrality* for a generic network, from a path perspective.

Definition 6.5 *Let $\mathcal{P}^{sp}(k,t)$ denote the set of shortest paths between k and t. The weighted betweenness centrality, of node v, relative to node t, is defined by*

$$C_{WB}(v|t) = \sum_{k\in W\setminus\{v,t\}} \sum_{P\in\mathcal{P}^{sp}(k,t)} \frac{\mathbb{1}_{v\in P}}{|\mathcal{P}^{sp}(k,t)|} \cdot \min\left\{ x(k), \max\left\{ \hat{c}(k,t) - \sum_{j\in(k,t)\setminus\{k\}} x(j), 0 \right\}\right\}$$

The weighted betweenness centrality introduces a score for a certain agent v, to lie on the shortest paths between all other agents and agent t, weighted by the amount of resources he may transfer.

Proposition 6.6 *Consider network G is a tree. Let $u, v \in W\setminus\{t\}$ be two different agents. If $C_{WB}(u|t) \geq C_{WB}(v|t)$, then the expected amount of resources intermediated by u is larger (weakly) than v's expected amount of intermediated resources.*

Essentially, if G is a tree, it is possible to uniquely rank every agent in terms of the resources they may intermediate. This is captured by the centrality notion. If G is not a tree, an agent may belong to the second shortest path, or higher order paths, and it is not clear how this different orders rank among each other.

This idea can also be applied before agent t is revealed. This is done by adding the centrality relative to every agent, i.e $\sum_{w\in W\setminus\{v\}} C_{WB}(v|w)$. This will yield an ex-ante correlation with expected intermediation amounts.

6.2 Supply-Demand Mismatch Application

Consider a services market in which supply is fixed and demand is variable, and assume that the cost of stock shortage is larger than the cost of outsourcing the service to some competitor. This might be the case that the service is already sold, but the actual time at which the service ought to be given, is unknown to the service providing firm. This framework allows for some degree of cooperation among competitors, in the sense that the mismatches between supply and demand for a given firm are compensated with other firms mismatches, which diminishes the distortions.

Formally, this framework can be represented as a network (most likely a complete network among all firms), where the resource constraints are each firm's supply, and the capacity constraints are the trust among firms. Now, in some markets, particularly big markets, it is expectable to find contracts and legal agreements between firms (e.g. overbooked airlines and their alliances), but in rather smaller markets, it is possible to find that the supply-demand mismatches compensation is based on social trust and thus this max-flow theory applies. However, a trivial extension has to be done, since this is a multi-sink problem. Consider the network $\bar{G} = (W \cup \{s\} \cup \{\bar{t}\}, E \cup \{(s,u)\}_{u\in W} \cup \{(u,\bar{t})\}_{u\in W})$, with $\bar{c}(s,u) = x(u)$, $\bar{c}(u,v) = \tilde{c}(u,v)\ \forall (u,v) \in E$ and $\bar{c}(u,\bar{t}) = y(u)\ \forall u \in W$, where $y(u)$ is the demand received by firm u.

Proposition 6.7 *Given demands $\{y(u)\}_{u\in W}$, it follows by Theorem* 3.8*, extended in the previously explained manner, that the maximal amount of demand that can be actually met, by means of transfers between firms in order to alleviate the supply-demand mismatch, is the solution of the max flow problem of Definition* 3.7*, in network $\bar{G}$.*

The idea is that each firm u is at least able to respond to the minimum between its own supply and demand (considering the direct $s \to \bar{t}$ flow through u). Then, firms with excess of supply can also respond to other's demand, up to the trust constraints or the own resource (supply) constraints.

6.3 Negotiated Goods Sell

Consider a community in which every member produces the same good. Members among the community differ in their bargaining power to sell the goods to a third party (consider, for example, farmers that grow crops and sell them as productive input to a bigger industry). Thus, every agent may sell the goods directly, or through another community member, with a better bargaining position. Assume that there can only be one negotiation, and agents that do not sell through this negotiation, have to sell their goods in the spot market.

We can approach this situation from the point of view of the community welfare, by analyzing which agent can produce the most community welfare, were he to be appointed to negotiate. There is a clear trade-off between bargaining power and network location (accounting for trust and resources). This trade-off can be studied in the framework of our max-flow theory. Consider a normalized spot market price of 1, and that the bargaining power is the probability $\theta \in (0,1)$ of selling at a high price $\bar{R}$ (otherwise, the price is the spot price). This yields an expected price, for agent u, of $\mathbb{E}[R_u] = 1 + \theta_u(\bar{R} - 1)$. In terms of our model, the expected price is equivalent to the nominal expected return of the project, $p(1+r)$, and the resources represent the goods to be sold. Therefore, the welfare produced by agent u is given by:

$$\begin{aligned} WF_u &= I(u) \cdot \mathbb{E}[R_u] + \left(\sum_v x(v) - I(u) \right) \cdot 1 \\ &= I(u) \cdot \theta_u(\bar{R} - 1) + \sum_v x(v) \end{aligned}$$

where $I(u)$ is the amount of goods that u can gather to sell (i.e. the solution of Definition 3.7).

Since $\bar{R}$ and $\sum_v x(v)$ do not depend on the negotiating agent, the trade-off between bargaining power, θ_u, and the agent's capacity of gathering goods, $I(u)$, is clearly revealed. Then, the agent that maximizes the community's welfare is simply the agent that maximizes the product of goods gathered and bargaining power, as in WF_u.

7 Conclusions

In this work we present a simple model of cooperation among community members, to benefit from the realization of a project initially owned by a single member. We characterize the maximal amount of resources that the community members can transfer, directly or indirectly along the network, in order to undertake the project. We show that this maximal amount is given by the concept of *length constrained maximum flow*, which is the classical max flow concept, but restraining the length of every flow to a permissible maximum. This is interesting for projects such as investment opportunities, where the returns are scalable in the investment. It also applies for productive opportunities, in which the production requires an initial amount of working capital, hence, the maximum flow indicates the largest project size feasible to be undertaken. We also show that some parameters of the problem, such as the recovery rate, the interaction probability and the project's probability of success have a monotone impact on the community's financing ability. Specifically, improvements in these parameters result in a weakly enhancements of the community's financing ability. This is achieved through two main aspects of the community: the trust within it, and the participation reach that the owner of the project possesses. On contrast, parameters such as repayment rate or intermediation spread, have ambiguous impacts, depending on the structure of the underlying network. A trade-off appears between trust and participation reach. Therefore, in communities with relative high trust compared to resources, the participation reach enhancement is predominant, thus increasing the repayment rate or lowering the intermediation spread is beneficial. The converse is also true.

Another key feature of the model is that resources are finite and matter, they are a crucial constraint in the problem. We examine the effects in changes on the *ex-ante* wealth distribution. Assuming that each member's wealth or resources are independently drawn from a continuous random distribution, we show that better distributions, in terms of second order stochastic dominance, leave the community with better resources transfers possibilities. Perhaps what is more relevant, is that even the concept of *mean-preserving spread* (MPS) is sufficient to make predictions. In deed, what a MPS does, is to leave the expected amount of resources unchanged, but increases the dispersion of the distribution, thus making it more unequal. Hence, if a distribution is an MPS of a second distribution, the later allows the community to gather more resources than the former.

We also showed that it is possible to augment the resources gathering possibilities by implementing a punishing institution. The role of the institution is to increase the personal cost of defaulting, which immediately increases trust within the community. We propose a provision game, in which the community members have to exert an individual amount of effort, and the institution is implemented whenever a provision threshold is achieved. Hence, when the institution is not too costly, it can be implemented in a decentralized fashion by the community members, as a step towards self governance.

In the context of network structure and welfare, we replicate some of the results found in Karlan *et al.* We show that trust is monotone in the capacities, therefore welfare is weakly monotone in the capacities. We also show that neighborhood closure plays a similar role in this model. Provided that trust is low relative to the community's resources, if the project's owner has a higher neighborhood closure, then he attains a higher payoff. The converse also follows.

Finally, we derive two applications as extensions of the model. The first one is an application for markets, mainly services markets, where supply is fixed and demand is variable, but it has already been compromised, though the actual time when the service has to be provided is unknown. Assuming that firms interact with each other, we are able to characterize the maximal demand that can be actually met at a specific time. The characterization follows as an extension of the original model, and is also expressed in terms of a max flow problem. The second application is one that incorporates differentiation among community members in terms of their bargaining power to sell a homogeneous good to an outside buyer. We think of a community that produces a productive input and has the opportunity to appoint one member to negotiate with the outside buyer. This situation exhibits a trade-off between the goods that a community member can gather to sell and his bargaining power. We characterize this from a welfare point of view.

Possible extensions of the model are, among others, to allow for only partial observability of other member's actions, thus allowing for more complex strategies, and to incorporate intermediation as a strategic part of model, in line with the network's bargaining theory.

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9 Appendix

A Additive Spread

To consider an additive spread, the first set of constraints in Definition 3.7 has to be modified in the following manner:

$$\sum_{P\,:\,e\in P}(1+r-d(e|P)\gamma)F(P)\leq\frac{(1-\varepsilon)c(e)}{(1-\tau)}=\hat{c}(e)\ \forall e\in E$$

and the set of (s,t) paths, $\mathcal{P}$, has to be restricted to $\mathcal{P}_{l^a(\gamma)+1}$, with $l^a(\gamma)=\max\{l'\in\mathbb{N}|p([1-\varepsilon]+\varepsilon\tau](1+r-l'\gamma)\geq 1\}$.

The problem is no longer a generalized flow problem (although it can be formulated as a series of consecutive generalized max flow problems), but the solution can be characterized as a *weak flow.*

Definition 9.1 *A weak flow $h:\ E\to\mathbb{R}$, relative to capacities c, a source $s\in W$ and a sink $t\in W$, is a function that satisfies*

(i) Capacity constraints: $h(u,v)\leq c(u,v)$ for all $(u,v)\in E$,

(ii) Antisymmetry constraints: $h(u,v)=-h(v,u)$ for all $(u,v)\in E$,

(iii) Weak flow conservation: $\sum_{w\in W}f(u,w)\leq 0$ for all $u\in W\setminus\{s\}$.

The solution will be a $t\to s|_{l^a(\gamma)+1}$ weak flow, in network $\hat{G}$, which in every node leaves the amount of resources equivalent to the future intermediation profits (i.e. it is a flow in the space of resources and not in the space of repayments, since the focus is on the amount of resources that are transferred).

Under an additive spread, Proposition 6.6 can be restated as

Proposition 9.2 *Consider network G is a tree. Let $u,v\in W\setminus\{t\}$ be two different agents.*

If $C_{WB}(u|t) \geq C_{WB}(v|t)$, then the intermediation profits of u is larger (weakly) than v's intermediation profits.

Essentially, under additive spread, there is a direct association between intermediated resources and intermediation profits.

B Theorem 3.8 Proof

Proof. First, we show that any feasible $s \to t|_{l(\gamma)+1}$ flow in network $\hat{G}$ is a feasible resources transferring scheme. In fact, any agent transferring (directly or indirectly) resources to t is at least breaking even, since every flow follows a path of at most $l(\gamma)+1$ length, which by definition is the largest path length that leaves agents better off than breaking even. Also, since the transferring scheme takes the form of a flow, no pair of agents engage on a transfer that will exceed the repayment rationality, which is a necessary condition for transfers to exist as stated in Lemma 3.1. Notice that since the repayment scheme is contingent on any agent's action, agents that act as intermediaries do not incur in any risk, therefore they are willing to participate as such.

Second, we show that any transfers scheme larger than the max flow is not a feasible resources transferring scheme. In fact, if transfers are larger than the max flow, it means that at some link the capacity constraint is being violated. Let (u, v) be the link, and the net flow be from u to v (thus v has to repay u afterwards), then v prefers not to make the agreed upon repayment and defaults, leaving u worse off than not transferring. This violates Lemma 3.1.

□

C Max Flow Problem - Path Formulation

Definition 9.3 (Generalized Max Flow - Path Formulation) *Let $\mathcal{P}$ be the set of (s,t) paths and let $F(P)$ be the flow associated to path $P \in \mathcal{P}$. Let also $\mathcal{P}_u$ be the ordered set (in terms of path length, being the shortest path the first element) of (u,t) paths (avoiding s), and z_e and z_P be slack variables for links and paths respectively. The problem to be solved is:*

$$
\begin{aligned}
\underset{\substack{\{F(P)\}_{P\in\mathcal{P}},\\ \{z_e\}_{e\in E},\{z_P\}_{P\in\mathcal{P}}}}{\textit{maximize}} \quad & \sum_P [1-\delta d^*(P)]F(P) - \eta \sum_u \sum_{i=1}^{|\mathcal{P}_u|} z_{p_u^{i-1}} F(P_u^i) + \theta \sum_P z_P \\
s.t \quad & \sum_{P\,:\,e\in P} \gamma^{d(e|P)} F(P) + z_e = \hat{c}(e) && \forall e \in E \\
& \sum_{P\,:\,e\in P} F(P) \leq x(e) && \forall e \in \{(s,u)\}_{u\in W} \\
& z_P \leq z_e && \forall e \in P, \forall P \in \mathcal{P} \\
& z_e, z_P \geq 0 && \forall e \in E, P \in \mathcal{P} \\
& F(P) \geq 0 && \forall P \in \mathcal{P}
\end{aligned}
$$

where $d(e|P)$ is the position of link e in path P (relative to agent t), $\eta << \theta$ are sufficiently small parameters, P_u^i is agent u's i-th path, and $z_{P_u^0} = 0$ for all $u \in W$.

Again, the first constraint is for the repayment to be rational and also sets the value for the slack variables for each link. The second constraint is for the feasibility of resources. The third constraint, together with the term $\theta \sum_P z_P$,[19] obtains the slack variables for each path (the slack variable for each path is the lowest slack variable of the links that conform the path). The fourth constraint is to ensure that slack variables are positive and no unfeasible flow is obtained. The terms $\eta \sum_u \sum_{i=1}^{|\mathcal{P}_u|} z_{p_u^{i-1}} F(P_u^i)$ introduce a cost to the paths, proportional to the slack variable of the previous path (which is by definition shorter). Therefore, the shortest path available for every agent (i.e. the shortest paths that are not saturated) perfectly substitute for each other, regardless of the length, because their cost is zero (since the previous path is saturated, and hence, the slack variable is zero), and shorter paths are always less costly than larger paths (in fact, are not costly at all).

The drawback of this formulation is that it is no longer a linear programming problem, since the objective function contains variables that multiply each other, hence, its resolution is considerably harder.

D Punishing Institution

If ε is not limiting to zero, there are defaults in expectation, thus the expected repayment is reduced and there is also an expected defaulting value for intermediaries. The $U_u(\cdot)$ functions take the following form:

[19] It is worth to note that since θ is sufficiently small, it will not occur that z_P and z_e rise above an optimal allocation level, because this would lower the flows in $F(P)$, which in the objective function have a greater marginal return. And since $\eta << \theta$, the slack variables z_P will effectively rise up to $\min\{z_e\}$.

$$U_u(\cdot) = \sum_{P\in\mathcal{P}:u\in P,(s,u)\notin P} p\Gamma^{d|_P}(1+r)\mathbb{E}[F(P,\cdot)][(1-\gamma)+\gamma\varepsilon(1-\tau)] + \sum_{P\in\mathcal{P}:(s,u)\in P} p\Gamma^{d|_P}(1+r)\mathbb{E}[F(P,\cdot)]$$

where $\Gamma = [(1-\varepsilon)+\varepsilon\tau]\gamma$.

Proposition 5.1 is still valid with the new definition of $U_u(\cdot)$.

In concern with the comparative statics, now the intermediation profits (left-hand sum) are not only a fraction $(1-\gamma)$, but also there is the term $\gamma\varepsilon(1-\tau)$ which is the profit in case that the link has gone bad and default is the best response. The expected repayment is also reduced by the term $\Gamma < \gamma$, accounting for the possible defaults.

The difference between payoffs due to the institution are as follows:

$$\begin{aligned}
\Delta U_u &= U_u(S) - U_u(0) \\
&= \sum_{P\in\mathcal{P}:u\in P,(s,u)\notin P} p\Gamma^{d|_P}(1+r)\left\{\mathbb{E}[F(P,S)] - \mathbb{E}[F(P,0)]\right\}[(1-\gamma)+\gamma\varepsilon(1-\tau)] + \\
&\quad \sum_{P\in\mathcal{P}:(s,u)\in P} p\Gamma^{d|_P}(1+r)\left\{\mathbb{E}[F(P,S)] - \mathbb{E}[F(P,0)]\right\}
\end{aligned}$$

Γ is clearly increasing in τ, so the profits from own resources transferred, under condition (2), are still increasing. For the intermediation profits, there are to opposing terms, Γ is increasing in τ but $[(1-\gamma)+\gamma\varepsilon(1-\tau)]$ is decreasing, thus the product of both can be either increasing or decreasing. However, the variation of the product due to τ is modulated by ε, whereas the variation in $\mathbb{E}[F(P,S)] - \mathbb{E}[F(P,0)]$ is not. Therefore, whenever ε is small relative to $S/((1-\tau)(1+r)$, the profits from intermediation, under condition (2), are increasing in τ.

E Paths Finding Algorithm

In order to solve the problem stated in Definition 3.7, it is necessary to first obtain the set of (s,t) paths $\mathcal{P}_{l(\gamma)+1}$. This is achieved by the following recursive algorithm, which receives the network $G(W,E)$, an initial void path p (starting at node s), the node t, the maximal allowed path length $l(\gamma)+1$ and the length of the current p path, h.

Input: $G(W, E)$, p, t, $l(\gamma) + 1$, h
Output: Set of (s, t) paths $\mathcal{P}_{l(\gamma)}$

```
def FindPaths(G,p,t):
    if h<l(γ)+1 then
        Let x be the last node of path p;
        foreach edge xy : y ∈ E do
            if y ∉ p then
                if y == t then
                    P_{l(γ)+1} ← p − y;
                else
                    FindPaths(G,p − y,t,h + 1);
                end
            end
        end
    end
end
```