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TECHNOLOGY PROGRESS, CREDIT MARKET FRICTIONS AND LABOR MARKET FRICTIONS

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PROGRESO TECNOLÓGICO, FRICCIONES EN EL MERCADO DEL CRÉDITO Y FRICCIONES EN EL MERCADO LABORAL

Este trabajo modela la interacción entre progreso tecnológico (PT), fricciones en el mercado laboral y fricciones en el mercado del crédito, con el objetivo de estudiar cómo la respuesta del mercado laboral frente al aumento del PT se encuentra mediada por el grado de desarrollo del mercado del crédito. La hipótesis principal apunta al hecho que economías con fricciones en el mercado del crédito responden de manera diferente al PT que aquellas economías sin fricciones en dicho mercado, lo que determinaría finalmente el efecto sobre el mercado laboral. Acemoglu (2001) plantea que un mecanismo de este estilo podría servir como explicación alternativa al milagro del empleo en EEUU respecto a Europa durante la década del 80. Con este propósito se desarrolla un modelo teórico de búsqueda para modelar las imperfecciones en ambos mercados, siguiendo el esquema propuesto por Pissarides (1998), aplicado en etapas sucesivas como en Wasmer y Weil (2004). En el modelo, un empresario sigue tres etapas antes de iniciar la producción de su idea de negocio: i) *Recauda los fondos* necesarios desde el mercado del crédito que le permita iniciar su empresa; ii) *Busca un trabajador* en el mercado laboral para iniciar la producción de la firma y iii) *Produce el bien*, el que luego se transa en un mercado de bienes competitivos. Debido a las fricciones, las dos primeras etapas toman tiempo y necesitan recursos. Cuando se inicia la producción, la firma adquiere la tecnología más avanzada disponible cuya productividad es la más alta de la economía. Esta productividad se mantiene constante hasta que el empresario decide adoptar una nueva tecnología con mayor productividad, lo que implica la destrucción de la relación laboral, y también en este modelo, la destrucción de la relación de préstamo. Los principales hallazgos muestran que a medida que crece el PT el valor del trabajo para un empresario disminuye al igual que su vida útil, no obstante, existen dos posibles efectos sobre la congestión de equilibrio del mercado laboral y sobre el desempleo. El sentido de estos efectos depende del costo total involucrado en el proceso de abrir una vacante, el que a su vez se relaciona con las fricciones del mercado del crédito. Un aumento del PT en economías donde el costo total asociado a abrir vacantes es menor que la pérdida del valor del trabajo asociado al PT, incentiva al empresario a destruir las actuales relaciones de trabajo y préstamo, disminuyendo la congestión del mercado laboral. De esta manera, en equilibrio, un aumento del PT induciría mayor reasignación laboral y mayor desempleo. No obstante, economías donde abrir una vacante es un proceso costoso mayor a la pérdida del valor de un trabajo asociada al aumento en el PT, los empresarios prefieren mantener la firma en producción hasta que sea rentable y evitan la transición al mercado del crédito. En equilibrio, la congestión del mercado laboral aumenta y existe una mayor reasignación laboral, sin embargo, el efecto sobre el desempleo depende si el efecto positivo del PT en la congestión de mercado laboral supera, o no, a una fracción del efecto negativo que el PT tiene sobre la vida óptima del trabajo. Respecto a la eficiencia del mercado del crédito, entendida como la habilidad de unir a prestamistas con prestatarios, una economía con una alta probabilidad de encuentro entre estos agentes, presenta menores tiempos de vida de las relaciones de trabajo y una mayor congestión de equilibrio del mercado laboral. Lo primero, dado que encontrar financiamiento se hace más probable, mientras que lo segundo ocurre ya que en equilibrio se ofrecerán más vacantes en busca de trabajadores. No obstante, el efecto sobre el desempleo depende de cómo se balancean estas fuerzas.

TECHNOLOGY PROGRESS, CREDIT MARKET FRICTIONS AND LABOR MARKET FRICTIONS

This paper focus in the interaction between technological progress, labor market frictions and credit market frictions, in order to study at which degree the response of the labor market to an increase of technological progress is mediated by the degree of development of the credit market. The main hypothesis points to the fact that economies with and without frictions in the credit market respond differently to technological progress, which ultimately determine the effect on the labor market. Acemoglu (2001) suggests that a mechanism of this kind could serve as an alternative explanation to the U.S. employment miracle compared to Europe during the 80s. Following Pissarides (2000, Ch 3), a theoretical search model is developed to shape imperfections in both markets, applied in successive stages as in Wasmer and Weil (2004). In the model, an entrepreneur follows three stages before to produce her business idea: i) Fund raising from the credit market that allows her to start the business; ii) Search for a worker in the labor market to start production of the firm and iii) the production of good and selling it in a competitive market of goods. Due the friction, the first two stages are processes that take time and resources. When production begins, the firm acquires the most advanced technology available at the date of creation whose productivity is the highest in the economy. This productivity remains constant until the entrepreneur decides to adopt a new technology with higher productivity, which involves the destruction of the labor relationship, and as an assumption in this model, the destruction of the lending relationship. The main findings show that as the technological progress increases, the value of job for an entrepreneur decreases as well as its optimal life time. However, there are two possible effects on equilibrium labor market tightness and unemployment. The sense of these effects depends on the total cost engaged in the process of opening a vacancy, which in turn is related to credit market frictions. Thus, an increase in technological progress in economies where the total cost of open a vacancy is less than the loss of value of a job induced by the technological progress, encourages entrepreneur to destroy the current labor and lending relationships, reducing the labor market tightness. Thus, in equilibrium, an increase of technological progress induce greater labor reallocation and higher unemployment. On the other hand, an economy where open a vacancy is a expensive process, greater to the loss of value of a job associated with the increase in technological progress, entrepreneurs prefer to keep the firm in production and avoid the transition to the credit market. In equilibrium, the labor market tightness increases and there is greater job reallocation, however, the effect on unemployment depends on whether the positive effect of technological progress on the labor market tightness exceeds, or not, at a fraction of the negative effect that technological progress has on the optimal life time of a job. Regarding the efficiency of the credit market, defined as the ability to match lenders to borrowers, an economy with a high matching probability between these agents, has lower optimal life time of labor relationships and greater equilibrium labor market tightness. The former, since finding financing becomes more likely, while the latter occurs since in equilibrium more vacancies will be offered in search for workers. However, the effect on unemployment depends on how these forces are balanced.

A mis padres: a ti Mamá, a ti Papá

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Chapter 1

Introduction

I return to the question addressed by Mortensen and Pissarides (1998): how does technological progress affect the equilibrium unemployment?. The model presented here is a preliminary attempt to show that a more complete answer of this question requires a number of elements, and one of such element is the credit market. In particular, imagine an economy A and a second economy B both with same performance in the labor market in the steady state, but one with better credit market than the other. The hypothesis developed in this paper is that the difference in the performance of the credit markets has a role in the type of response of these two economies, in a context where technological progress is crucial in the economy.

This thesis studies how the interplay between an imperfect credit market and an imperfect labor market can influence the dynamics of job creation and job destruction in an economic environment governed by technological progress. The economic literature provides a consistent dynamic theory of equilibrium unemployment, job vacancies and wage formation (Pissarides 2000, Ch. 2; Aghion and Howitt, 1994; Mortensen and Pissarides 1998), but the dynamic interaction between technology progress, labor market and credit market has received less attention.

In the model proposed in this thesis, an entrepreneur who has a business idea must seek funds in the credit market in order to finance the search for a worker in the labor market. Market frictions imply that both credit and labor markets search are costly processes that take time to be achieved successfully. The analysis of the search frictions in both of these markets is based on the matching framework proposed by Pissarides (1985) completed with a Nash bargaining processes between entrepreneurs and financiers, as well as between entrepreneurs and workers.

After meeting a worker, the firm produces a single homogeneous good and receives a continuous flow of market-sales revenue, from which the wage rate is paid to the worker and the credit repayment to the financier. This first building block is similar to the framework proposed by Wasmer and Weil (2004). We extend it in order to allow for a continuous process of technological progress governing the dynamics of the economy, modeled as in Mortensen Pissarides (1998), but focused only on the case where existing jobs can not be easily updated to exploit a new technology. More precisely, there are different technological vintages in the economy, each one offering a different productivity level. When a job is created, the firm adopts the leading edge technology, which is kept fixed during the whole market life of the job-entrepreneur match so that its productivity remains constant over time. The technology frontier characterizes the state of the art technology that is available to all firms at a given point in time. Producing firms face rising wages in this framework since the outside option of a worker is endogenously increasing in the rate of technological progress. Hence, a constant productivity growth of the technology frontier renders all existing technologies gradually obsolete. This imply that jobs are destroyed when the worker-firm match is no longer profitable, as in Mortensen Pissarides (1998) but with an important difference here: the structure of the credit market determines the job destruction process. This is a key contribution of this thesis.

An entrepreneur having a producing firm has to decide whether (i) it continues to operate with the technology adopted at the date of creation or (ii) she chooses to destroy the job. In the basic model proposed here, the latter option implies that all relationships, between the entrepreneur and the financier as well as between the firm and the worker must be destroyed, which brings the entrepreneur back to credit market in order to begin the sequential process of search exposed above. The technology adoption decision made by a firm entails the determination of the life span of a job that maximizes its value.

This thesis focus on a world where the only option for entrepreneurs to keep their company in production is the destruction of a job to update to the the cutting-edge technology and thus to get a greater productivity. In this context our main findings can be summarized as follows. First, related to the efficiency of the credit market: an economy with low frictions and therefore a high matching probability between borrowers and lenders, has a lower optimal life span of a job since is more easily for an entrepreneur find funds for his project. This causes the entrepreneur has incentives to adopt the technology at the frontier, destroying the current job and loan relationship. In the opposite sense although with a similar mechanism, the labor market tightness rises since in equilibrium there will be more firms offering vacancies to find a worker who use the technology recently acquired. Second, related to the technological progress: an increase in the rate of technological progress reduces the equilibrium value of a job and diminishes their optimal life, but has two possible effects on the equilibrium labor market tightness depending on the total cost of posting a vacancy. In this model, the total cost of posting a vacancy is related to the efficiency of the credit market. The higher the efficiency of credit market, the lower the total cost of posting a vacancy. Accordingly, the equilibrium labor market tightness decreases if the reduction of a value of a job due the technological progress is greater than the total cost of posting a vacancy. Otherwise, the equilibrium labor market tightness increases. Third, regarding the implications for unemployment: this paper proposes that only when credit market conditions are efficient, i.e., the match between borrowers and lenders is done with relative efficiency and the reduced value of a job due to technological progress is greater than the total cost of opening vacancies, an increase in the rate of technological progress would produce more labor reallocation and an increase of unemployment in equilibrium. However, if the match between entrepreneurs and financiers is costly and takes time, to the point where the reduced value of a job due to technological progress is less than the total cost of opening vacancies, entrepreneurs have incentives to maintain their labor relationship with the worker and prevent the passage of fundraising. Thus, the effect on unemployment depends on whether the positive effect of technological progress on the labor market tightness exceed, or not, at a fraction of the negative effect that technological progress has on the optimal life of a job.

Hence, this thesis contributes to the study of the dynamic interaction between labor and credit frictional markets in a context where technological progress continuously affects firm's decisions, extending the framework of Mortensen and Pissarides (1998).

Why consider financial markets and technology progress?

Acemoglu (2001) tells a story of European unemployment and U.S miracle based on the credit markets context. During the 1960s Europe and U.S. had similar unemployment levels and they were both in a steady state situation. Technological changes during 70s and 80s shifted the growth sectors of the economy and new entrepreneurs and firms assumed a more important role. Acemoglu, argues that this process occurred more quickly in the U.S than in Europe thanks to a more fluid credit market, with institution like venture capitalist channeling funds to rapidly growing high-tech sectors of the economy.

Following Rajan and Zingales (1998), Acemoglu (2001) classifies productive sectors according to their credit dependence in the U.S in low, medium and highly credit-dependent industries. Acemoglu (2001) plots the share of employment in a number of European countries across these categories of credit-dependent relative to the share of the same categories in the U.S. and finds that except for Netherlands and UK, two countries that commonly thought to have the best credit markets, the European economies appear to have substantially less employment in the most credit-dependent industries. This evidence is suggestive that the most credit-dependent industries have relatively high employment in economies with better and more fluid credit markets relative to another with problem to allocate funds "in the hands of people with right skills" (Ibid.). The most interesting thing is that these high credit-dependent industries (electrical goods and office, and dataprocessing machine industries) grow faster with technological progress (Acemoglu, 2001), and there may be technological reasons why some industries depend more on external finance than others (Rajan and Zingales, 1998).

In a more recent paper Ilyna and Samaniego (2011) show that industries that grow faster in more financially developed economies display greater R&D intensity indicating that well-functioning financial markets tend to beneficiate industries where growth is driven by R&D as in the case of technology industries. Their major finding is that financial development stimulates economic growth, and henec employment rates, by relieving financing constraint on industries that grow by performing R&D. This evidence shows a relationship between industrial sectors that rely more on R&D activities that are governed by technological progress and financial development of credit market. This suggests that to evaluate how the number of jobs are affected by the technological progress, is important to consider the degree of financial development of an economy and its ability to connect investors and borrowers.

On the other hand, models where firm's financial structure is irrelevant to investment suppose that external funds provide a perfect substitute for internal capital. In general, with perfect capital markets this assumption is equivalent to say that a firm's investment decision, as adopting a new technology, is independent of its financial condition. These models may apply to mature companies, but for other firms, financial factors seem to matter and external capital is not a perfect substitute for internal funds. As Rajan and Zangales (1998, p.565) argue, very young firms are more dependent on external finance than older firms, and this is more pronounced in high-tech industries. Fazzari, Hubbard and Petersen (1988), show that financial factors affect the investment, emphasizing that the link between financing constraint and investment varies by type of firm. The investment of firms that exhaust all their internal funds, which opportunity cost is lower than external funds, should be more sensitive to fluctuations in their cash flow than firms that pay high dividends. In this line, Benhabib and Spiegel (2000) show that indicators of financial development are correlated with both total factor productivity growth and investment and suggest that financial backwardness may hinder the ability of agents to invest.

Finally, the current financial crisis of the recent years began with an unexpected shock on credit markets which led to the worst recession since the Great Depression. One of the main consequences of this recession has been the persistent decline in employment rates, which reached record levels in Europe during the 2012 and 2013. These evolutions suggest that new insights should be grasped from the analysis of the relationship between unemployment, credit market functioning and technological progress. This thesis aims at developing such an analysis by proposing a model featuring within the same framework credit and labor-market frictions as well as a continuous process of technological progress.

The rest of the thesis is organized as follows: Chapter 2 discusses the related literature dealing with the interplay between the labor and the credit markets, as well as, the closely related literature of technological progress and unemployment. Chapter 3 presents the setting of the model and the main analytical results. Finally, Chapter 4 briefly concludes.

Chapter 2

Related Literature

This thesis addresses the relationship between the labor and credit markets in the context of continuing technological progress. Particularly, it asks how these frictional markets interact and influence the dynamics of job decision-making and technology adoption, in an environment of productivity growth. This question has been partially addressed in the literature.

Pissarides (2000, Ch. 2) proposes a model to study long-run effects of growth on employment and argue that higher productivity growth increases the rate of return of vacancy creation; rises the rate of job creation, and diminishes the rate of unemployment. However, the effect of growth studied by Pissarides (2000, Ch. 2) is related to the notion of disembodied technological progress. According to this idea, new and existing jobs benefit from higher levels of productivitiy, no matter if they renew or replace their current capital stock. Although useful for the analysis, this perspective stands as a result of partial equilibrium and only takes into account the effect of productivity growth on job creation, without considering the potential impact of technological progress on job destruction.

Aghion and Howitt (1994) focus on how the rate of economic growth affects unemployment in the long-run. They emphasize the re-allocative aspect of growth. In this framework, unemployment is directly affected by job destruction rates, and indirectly affected by the incentive of firms to post vacancies, thus increasing the likelihood of an unemployed worker finding a job. One of the main contributions of these authors is the analysis of two competing effects of growth on unemployment: First, the capitalization effect, as in Pissarides (2000, Ch. 2). An increase in growth productivity raises the return from creating a firm and encourages the entry of more firms thereby promoting job creation. Secondly, the creative destruction effect. An increase in growth can reduce the life of a job therefore increasing job separation, discouraging vacancy posting, and diminishing the rate at which unemployed workers may match with a job. Mortensen and Pissarides (1998) study how technological progress affects the number of jobs at equilibrium. In a frictional model of labor market, they show that higher productivity growth induces higher unemployment rates when renovation cost is above a critical value. The opposite effect occurs when this cost is below the threshold. Other interesting result derived from this model is that an increase in technological progress, decrease the equilibrium value of a job, as well as, the equilibrium value of the labor market tightness. These findings suggest that credit market frictions can act as an accelerator of this effect given its intermediation character.

On the other hand, many researchers focus on credit market imperfections and their role on macroeconomic performance. While the ability of financial sector to amplify small shocks into large output fluctuations is well studied in the literature (Kiyotaki and Moore, 1997; Bernanke, Gertler and Gilchrist,1999); there is less knowledge about the impact of credit channel on the labor market outcomes.

Acemoglu (2001) develops a thesis that credit market frictions may play an important role in the increase of European unemployment. The main argument of the author begins with the assumption that the U.S. credit market is more developed and flexible in providing loans to new firms than the European credit market. Then, he follows with the idea that "these credit markets respond to new opportunities is an important contributor to the different employment performance in these economies". Taking two steady-state economies as starting point, -one with a better credit market than the other and both with low unemployment rates-, Acemoglu (2001) examines how these economies respond to the arrival of new technologies. In the economy with worse credit market, technological change has an adverse effect on unemployment; in the absence of an efficient credit market, those entrepreneurs who need liquidity to start a new business will not be able to access the necessary funds.

Wasmer and Weil (2004) assess the influence of credit markets imperfections on the labor market in a general equilibrium framework, with endogenous search frictions. Their results suggest that credit frictions act as an amplifier device of macroeconomic volatility through a financial accelerator, which magnitude is proportional to the credit gap - measured by the deviation of output to the perfect credit market level-. However, this model does not incorporate the notion of technical progress.

This thesis is closely related to the model of technical progress used by Mortensen and Pissarides (1998), but it does not include the possibility of updating a job. This idea can be interpreted as more radical technological progress which brings structural changes that make obsolete the job relationship which must necessarily be destroyed and replaced by new one. In addition, this thesis models technological progress with two imperfect markets *à la* Wasmer and Weil (2004).

Chapter 3

The Model

There are three types of agents in the economy: entrepreneurs that have an idea but need capital to produce, financiers [1] that have resources but no chance to produce, and workers, that offer their labor inelastically, but have no capital and no ideas. Time is continuous denoted by t, there are three stages in the economy, and the events are as follows. First, an entrepreneur needs to find a banker in order to finance recruiting costs in the labor market, and to adopt the latest technology available at date of creation. The search process in the credit market takes time and personal effort. With certain probability, an entrepreneur meets a suitable financier, negotiate a debt contract and together continue to operate as a firm. Second, once the firm is created, the banker-entrepreneur match goes to the labor market and looks for a worker to produce a single homogeneous consumption good. Each firm needs only one worker to operate. Labor market imperfections impose that the search process for a worker takes time and has a cost per unit of time which is financed by bank. With some probability, a firm finds a worker, a job is created and starts producing with the cutting-edge technology. Third, once a firm is producing, it receives a benefit flow for selling their output in a perfectly competitive product market, pays a wage to the worker, and a repayment flow to the banker. The agents of this economy are risk neutral and r is the per-period discount rate. By assumption, search costs in the credit and labor markets, as well as, unemployment income grow steadily with the productivity level performed at the technology frontier. To ensure the existence of a steady state with balanced growth in this economy, all costs in the model grow with productivity at the frontier, where the common growth factor is $p(t) = e^{gt}$ and g the exogenous rate of growth of productivity at the frontier such,

$$g = \frac{\frac{dp(t)}{dt}}{p(t)}$$

[1] In what follows I may refer as banker or financier interchangeably.

The technological progress building block closely follows Mortensen and Pissarides (1998), and is extended to include the interplay between frictions in the labor and credit markets, as in Wasmer and Weil (2004).

Once a firm finds a worker, the job is created at the technology frontier, and maintains this technology until the job destruction. The job destruction process can occur by either exogenous or endogenous reasons. Exogenously a separation schock arrives according to a Poisson process. Endogenously, for obsolescence of job, a match eventually becomes non profitable. As we shall discuss below, wages and repayment flows to financiers grow with to productivity at the frontier. This implies that there is a point in time where the productivity of a job -constant overtime- will be lower than the costs of keeping production active in the product market. This means that the production relationship between firm and worker must be destroyed, either to upgrade the technology or to leave the market. The timing of events requires that, once a job is destroyed, the entrepreneurs search again for funding in the credit market.

3.1 Matching

The existence of labor markets frictions implies that workers and entrepreneurs do not meet easily, and must devote efforts in order to correctly match. This process is modeled following Pissarides (1985), who uses a matching technology that allows to form worker-entrepreneurs pairs able to produce. A matching function is a constant return to scale technology $m(u, v)$ that produces a flow of matches between firms and workers from two inputs: unemployed workers [2]u, and job vacancies v. The degree of congestion of labor market is measured by the ratio $\frac{v}{u} = \theta$, usually referred as labor market tightness in the literature. The arrival rate at which an entrepreneur finds a suitable worker is defined by.

$$\frac{m(u,v)}{v} = m(\theta^{-1}, 1) \equiv q(\theta) \qquad q'(\theta) < 0 \tag{3.1}$$

The higher the labor market tightness, i.e., the higher the number of available vacancies relative to the the numbers of unemployed workers, the lower the probability that an entrepreneur fills a vacancy.

On the credit market side, to begin the production process an entrepreneur must find a banker to finance the recruiting cost. This search implies that some economic resources will be spent. Following Wasmer and Weil (2004), this model assumes that the entrepreneur has no funds

[2] A worker can be employed or unemployed. I assume that only unemployed workers search for job, i.e, no exists on-the-job search.

or capital to go to the labor market and to find a worker. Therefore, an entrepreneur must borrow money from a financier. Since there are frictions in the credit market, an entrepreneur who wants to begin a new proyect must put some efforts to encounter a suitable financier. This process takes time due to the frictions, that come from the same sources as those present in the labor market. Therefore, I use the same labor-market framework to model this stage. Let $h(B, E)$ be the constant return to scale credit market matching function, homogeneous of degree one, that uses two inputs: B, the number of bankers searching for ideas to finance, and E, the numbers of entrepreneurs looking for funds to start their projects. The credit market tihgtness is defined then as $\frac{E}{B} = \phi$. The arrival rate at which an entrepreneur finds a banker is then,

$$\frac{h(B,E)}{E} = h(\phi^{-1}, 1) \equiv z(\phi) \qquad z'(\phi) < 0 \tag{3.2}$$

The higher the credit market tightness, i.e., the higher the number of entrepreneurs raising funds relative to the the numbers of available financiers, the lower the probability that an entrepreneur finds funds to produce his idea.

3.2 Value Functions

The asset values of an entrepreneur in the credit, labor and goods markets, are denoted by E_i with $i = c$, l, g respectively. The Bellman equations are given by the following expressions,

$$rE_c\left(t\right) = -dp\left(t\right) + z(\phi)\left[E_l\left(t\right) - E_c\left(t\right)\right] + \dot{E}_c\left(t\right) \tag{3.3}$$

$$rE_l\left(t\right) = q\left(\theta\right)\left[E_g\left(t,\, t\right) - E_l\left(t\right)\right] + \dot{E}_l\left(t\right) \tag{3.4}$$

$$rE_g\left(\tau,\, t\right) = \max\left\{p\left(\tau\right)y - w\left(\tau,\, t\right) - \rho + \lambda\left[E_c\left(t\right) - E_g\left(\tau,\, t\right)\right] + \dot{E}_g\left(\tau, t\right), 0\right\} \tag{3.5}$$

The date of creation of a job determines its technology vintage. Following Mortensen and Pissarides (1998; 1999) and Pissarides (2000, Ch. 3) the model distinguishes the date of creation of a job, τ, from the current time, t. An entrepreneur who is looking for funds to produce his idea, incurs in a flow cost $dp(t)$. With arrival rate of $z(\phi)$ the entrepreneur has a succesfull match with a suitable financier, and then, is able to search for a worker in the labor market stage. This search process is succesfully funded by the bank, at rate $q(\theta)$. When a match with a worker is performed, the job is created with the cutting-edge technology available at the date of creation $E_g\left(t, t\right)$. In the product market stage, a firm created in a date $\tau < t$, has an output of $p(\tau)$, pays a wage rate of $w(\tau, t)$, and pays a flow of $\rho(\tau, t)$ to the bank. As we shall see, this repayment flow is negotiated between banks and entrepreneurs according to a Nash-bargaining process. An exogenous shock can destroy the job with arrival rate of λ, in which case, the entrepreneur goes back to the credit market stage. The choice of max between profits net of costs and zero in the expression (3.5) reflects an endogenous exit decision.

Symetrically, the asset value of a bank in the credit, labor and goods markets, are denoted by B_i with $i = c,\ l,\ g$ respectively. The Bellman equations that describe their evolution are:

$$rB_c\left(t\right) = -kp\left(t\right) + \phi z\left(\phi\right)\left[B_l\left(t\right) - B_c\left(t\right)\right] + \dot{B}_c\left(t\right) \tag{3.6}$$

$$rB_l\left(t\right) = -\gamma p\left(t\right) + q\left(\theta\right)\left[B_g\left(t,\ t\right) - B_l\left(t\right)\right] + \dot{B}_l\left(t\right) \tag{3.7}$$

$$rB_g(\tau,\ t) = \rho + \lambda\left[B_c\left(t\right) - B_g(\tau,\ t)\right] + \dot{B}_g\left(\tau, t\right) \tag{3.8}$$

When a bank is looking for projects to fund, it incurs in a search cost of $kp(t)$ per period. This search process ends when the financier meets a suitable entrepreneur with an interesting project, which occurs at rate $\phi z(\phi)$. Once a match entrepreneur-bank is performed, the labor market stage begins, and this pair is named as a firm. The bank finances the recruiting cost of the firm $\gamma p(t)$, which process succeeds at arrival rate $q(\theta)$. Then, in the product market stage, the firm is in production and the bank receive a repayment flow of ρ until the relationship between worker and firm is destroyed, in which case the banker seeks again an idea to finance.

The third type of agents present in the economy, workers, can be either employed or unemployed. In this model, there is no on-the-job-search and the value of a worker at each state, are defined by the following Bellman equations,

$$rU\left(t\right) = bp(t) + \theta q\left(\theta\right)\left[W\left(t,\ t\right) - U\left(t\right)\right] + \dot{U}\left(t\right) \tag{3.9}$$

$$rW\left(\tau, t\right) = \max\left\{w\left(\tau, t\right) + \lambda\left[U\left(t\right) - W\left(\tau, t\right)\right] + \dot{W}\left(\tau, t\right), 0\right\} \tag{3.10}$$

An unemployed worker receives non-labor benefits $bp(t)$, which can be interpreted as social security or as any type of income derived from non-labor activity. Note that when a new job is created, the value of the match for the worker is embodied with the cutting-edge technology denoted by $W(t,t)$ in the right-hand side of equation (3.9). When a worker finds an entrepreneur and job worker match begins, he bargains a wage that is a function of the technology vintage at the date of creation and varies with the rate of technological change.

Once the job is destroyed, the worker moves to the unemployment pool and engage in an active search for a job again.

In order to obtain finite present values, the model requires the following assumptions: *(i)* $r + \lambda - g > 0$ and *(ii)* $r + q(\theta) - g > 0$. Moreover, to ensure positive gains we assume that *(iii)* $y - b - \gamma > 0$. This assumptions are maintained throughout this paper. Also the model assumes, that every financier and entrepreneur can entry in the credit market by simply incurring in the corresponding search cost. This implies, that in equilibrium all profits be exhausted to the zero level by the free entry of agents. This free entry condition implies that in equilibrium $E_c(t) = 0 = B_c(t)$. From equation (3.6) and (3.3), and by free-entry condition in the credit market, we have:

$$B_l(t) = \frac{kp(t)}{\phi z(\phi)} \tag{3.11}$$

$$E_l(t) = \frac{dp(t)}{z(\phi)} \tag{3.12}$$

These are the values for the bank and the entrepreneur to participate in the labor market stage. For each agent, equations (3.11) and (3.12) show that the value of participating in the labor maket stage must be equal to the cost asociated to find a counterpart. These values rise with the productivity at the frontier, as previously mentioned, to ensure a balanced growth path. Since $\phi z(\phi)$ and $z(\phi)$ are the instantaneous arrival rates, according to Poisson law their inverse is the length of duration associated to the search process. When ϕ increases, i.e. there are more entrepreneurs relative to bankers, the equilibrium value of a matched bank declines and the equilibrium value of a matched entrepreneur increases because the former have to search relatively less than the latter.

The fact that an entrepreneur must go to the credit market before heading to labor market, implies that no free entry condition exists in the labor market. New vacancies enter the market according to equation (3.12) for all t. This implies that

Proposition 1 *The Job Creation condition is given by,*

$$E_g(t, t) = \frac{(r + q(\theta) - g)}{q(\theta)} \frac{dp(t)}{z(\phi)} \tag{3.13}$$

Proof. *Sustituting equation (*3.12*) into (3.4).*

For an entrepreneur, the value of a job match, expressed in terms of productivity, is equal to the expected discounted cost to search for a financier. Labor and credit markets tightnesses at time t satisfy equation (3.13), which means that the creation of new jobs depends on the characteristics of both, the credit and labor market. Recalling that $q'(\theta) < 0$ and $z'(\phi) < 0$, a rise in θ or ϕ increases the value of new jobs, so that in a $(\theta, E_g(t,\, t))$ space, the job-creation condition is upward sloping.

3.3 Bargaining Process

A firm is created before the job creation. This means that an entrepreneur meets a banker before knowing the worker, and hence, the negotiation of the repayment flow is made before of the negotiation of wage contract. Since the agents in this economy are rational and correctly anticipate the effects of their actions in the future, we solve the model backward.

The wage contract negotiated between an entrepreneur and a worker takes as given the repayment flow to financier, previously negotiated. Entrepreneurs and banks, on the other hand, take into account that their negotiation will influence the wage bargaining in the labor market.

3.3.1 Endogenous wage

Worker and employer negotiate the surplus of their relationship according a rule that maximizes the Nash product at every t,

$$w(\tau, t) = argmax[E_g\,(\tau,\, t) - E_c(t)]^{1-\alpha}[W(\tau, t) - U(t)]^{\alpha}$$

with $\alpha \in [0, 1]$ the worker's share parameter representing the barganing power of the worker in the negotiation. Using the first-order condition, and the free-entry condition $E_c(t) = 0$, I show that worker and entrepreneur share the surplus according to the following rule,

$$\alpha E_g\,(\tau,\, t) = (1-\alpha)[W(\tau, t) - U(t)] \tag{3.14}$$

Proposition 2 *The wage at time t on a job with technological vintage τ, is an increasing function of: i) The total cost that the entrepreneur incurs during the full lenght of the search devoted to find a financier, $\frac{d}{z(\phi)}$; ii) the credit and labor market tightnesses, ϕ and θ; and a decreasing function of the repayment flow ρ.*

$$w(\tau, t) = \alpha[p(\tau) - \rho] + (1-\alpha)p(t)\left[b + \theta\frac{\alpha}{1-\alpha}\left(r + q(\theta) - g\right)\frac{d}{z(\phi)}\right] \tag{3.15}$$

Proof. *See Appendix A.1.*

The larger is the total search cost in the credit market, the larger is the wage paid by the entrepreneur to the worker. Since the efforts made by the entrepreneur to find a banker willing to finance his idea involve a number of sizeable costs, the entrepreneur is willing to pay a higher wage in order to keep his production relationship. An alternative interpretation is the following: an increase in the credit market tightness, i.e, a large number of entrepreneurs looking for funds relative a searching financier, can be seen as lack of liquidity in the credit market. This lack of liquidity, lowers the matching probability between a banker and entrepreneur accordingly to $p'(\phi) < 0$, increases the search time for rising funds, and delays the start-up of a firm. In response to this, the wage raises as a way to mantain the production of the firm and to avoid a job destruction and subsequent transition to the credit market. The expression obtained in proposition 2 confirms some previous results in the literature: first, as in Mortensen and Pissarides (1998), the wage is increasing in θ. In fact, in absence of frictions in the credit market, equation (3.15) is analogous to their result; Second, the wage rate is a weighted average of firm's output net of the repayment flow to the bank (henceforth the operating profits) and the flow value of the outside option of the worker. The latter grows at the rate of technological progress, since the outside option of workers improves during the employment because all vacancies offer a higher wage (new jobs are created at the frontier productivity). Third, as in Wasmer and Weil (2004), a marginal unit increase in the repayment flow from the entrepreneur to the banker, leads to a decrease in wage of α.

Rewriting equation (3.15),

$$w(\tau, t) = \alpha[p(\tau) - \rho] + (1-\alpha)p(t)\Psi(\theta, \phi) \tag{3.16}$$

where $\Psi(\theta, \phi)$ is the reservation wage, which reads as follows

$$\Psi(\theta, \phi) = b + \theta \frac{\alpha}{1-\alpha}\left(r + q(\theta) - g\right)\frac{d}{z(\phi)} \tag{3.17}$$

3.3.2 Endogenous repayment flow

The loan contract between a banker and a entrepreneur is a result of a bargaining process. The bank finances the labor search costs γ and the entrepreneur repays an amount of ρ as long as the firm produces. This is a formulation similar to Wasmer and Weil (2004): entrepreneurs and banks are known before the employer knows the worker, and are able to anticipate their negotiation will affect the wage bagaining. According to a Nash-Bargaining rule, bank and entrepreneur share the surplus of their relationship as follows,

$$\rho = argmax\left[B_l(t) - B_c(t)\right]^{\eta}\left[E_l(t) - E_c(t)\right]^{1-\eta}$$

Where $\eta \in (0,1)$ is the bargaining power of the banker. The first order condition for optimal sharing of the surplus between entrepreneurs and bankers, and the free-entry conditions imply,

$$(1-\eta)(1-\alpha) B_l(t) = \eta E_l(t) \tag{3.18}$$

The term $(1-\alpha)$ in the left-hand side of equation (3.18) comes from the fact that, in this negotiation, entrepreneurs take into account the effect on the bargaining on their future negotiation between entrepreneur with a worker. The optimal bargaining repayment flow is given by,

Proposition 3 *The bargained repayment flow from the entrepreneur to financier is a function of productivity at the date of creation and the productivity of the cuting-edge technology:*

$$\rho = \eta_\alpha \left[p(\tau) y - w(\tau, t)\right] + (1-\eta_\alpha)\left[\frac{\gamma p(t)}{q(\theta)}(r+\lambda-g)\right] \tag{3.19}$$

with $\eta_\alpha = \frac{\eta}{1-\alpha(1-\eta)} > \eta$.

Proof. *See Appendix A.2*

The equilibrium repayment flow is a weighted average of the firm's output net of the wage given the technology at the date of creation, and the loan made by the bank. The weight, $\eta_\alpha = \frac{\eta}{1-\alpha(1-\eta)} > \eta$, depends not only on the bargaining powers of the bank and the entrepreneur, but also, on the bargaining power of the worker in the wage bargaining. If workers have all the bargaining power in the wage negotiation, the repayment flow is $p(t) - w(\tau, t)$. This implies that bankers experiment an increase in that would we call the "effective" bargaining power, i.e. η_α, in the negotiation of the loan contract, and this effective bargaining power increase with the worker's bargaining power in the wage negotiation. Finally, as in the endogenous wage case, equation (3.19) rises with productivity of newly created jobs. Since jobs produce output with technology at the creation date τ, there is a point in time when the labor product is lower than its costs, and, consequently the job-entrepreneur match ends. Substituting, equations (3.11) and (3.12) in expression (3.18), we obtain the equilibrium credit-market tightness,

$$\phi^* = \frac{(1-\eta)(1-\alpha)}{\eta}\frac{k}{d} \tag{3.20}$$

This result is similar to Wasmer and Weil (2004), and Petrosky and Wasmer (2013). The equilibrium credit-market tightness is constant since both sides of the credit market have free-entry conditions. Note that to ensure the existence of a steady state with balanced growth, costs k and d, should grow at the same rate. In particular, the common growth factor is $p(t) = e^{gt}$.

3.4 Optimal life of a Job

In this model there is no job updating, in the sense that the employer cannot upgrade to the technology frontier with the current labor relationship. In other words, the employer knows from the moment it finds a worker (and starts the production) when technology should be changed. In this framework, the technology adoption decision is modeled as the choice of firms regarding of the optimal life time of the job. When a job is created, firm's owners are able to anticipate the profitable duration of a job. Beyond this moment, the job becomes unprofitable, since wages and repayment flows grow with productivity of new jobs at the frontier. At this point, an entrepreneur who wants to continue his business, chooses to destroy the job and start a new one, by moving to the credit market. By combining equation (3.16) and (3.19), and substituting into (3.5), we obtain a value of an existing job created at τ,

$$(r+\lambda)\,E_g(\tau,t) = (1-\alpha)\,(1-\eta)\left\{p\,(\tau)\,y - p\,(t)\left(\Psi\,(\theta,\phi) + \frac{\gamma}{q\,(\theta)}\,(r+\lambda-g)\right) + \dot{E}_g\,(\tau,t)\right\} \tag{3.21}$$

The firm chooses the life of the job that maximizes its value after creation. The optimal life chosen by an entrepreneur at the time when a job is created is dynamically consistent, since it is the same optimal life at subsequent times and coincides when the value of a job is zero (Pissarides 2000, Chapter 3). Let T be the optimal life of a job, integrating equation (3.21) from the current time, t, to the date of job destruction, $\tau + T$, then the optimal value of the job at any time t satisfies

$$E_g(\tau,t) = \max_T\left\{\int_t^{\tau+T}(1-\alpha)\,(1-\eta)\left[p\,(\tau)\,y - p\,(s)\left(\Psi\,(\theta,\phi) + \frac{\gamma}{q\,(\theta)}\,(r+\lambda-g)\right)\right]e^{-(r+\lambda)(s-t)}ds\right\} \tag{3.22}$$

Substituting the balanced-growth condition whereby a new job value is proportional to productivity at the technological frontier, i.e. $E_g\,(t,\,t) = p\,(t)\,E_g$ when $\tau = t$, and normalizing $p(0) = 1$, the following proposition follows,

Proposition 4 *The maximal value of a job for an entrepreneur, is an increasing function of the bargaining power of the entrepreneur in both negotiations, and a decreasing function* *of the credit market thightness ϕ, the labor market thightness θ, and, the loan accorded with the bankγ.*

$$E_g(0,0) = \max_T \left\{ \int_0^T (1-\alpha)(1-\eta) \left[y - e^{gs} \left(\Psi(\theta,\phi) + \frac{\gamma}{q(\theta)} (r+\lambda-g) \right) \right] e^{-(r+\lambda)s} ds \right\} \tag{3.23}$$

Equation (3.23) represents the maximal value of a job. At the same time, it stems from the optimal decision regarding the life-span of a job, given the posibility of obsolescence. As such eq.(3.23) can be interpreted as a job destruction condition. This implies that the credit-market equilibrium outcome influences the job destruction, through search frictions, captured by the reservation wage in Lemma 2 eq.(3.17) , as well as, through the negotiating process captured by $(1-\eta)$ that may change by institutional or policy arrangements.

The larger the credit market tightness, the smaller the maximal value of an existing job. A tighter credit market rises the worker's reservation wage. The underlying mechanism is as follow: In presence of more entrepreneurs than bankers, it is more costly to the employers to find a suitable financier, which means that is more valuable for the employer the current labor relationship, resulting in a higher wage.

We also observe in (3.23) that the larger the loan made by financier to entrepreneur, the smaller the maximal value of a job. Moreover, If the labor market tightness rises, the reservation wage increases (see proposition 2), and also the expected time spent in searching a worker

The first order condition of equation (3.23), conduces to

$$e^{gT^*} \left(\Psi(\theta,\phi) + \frac{\gamma}{q(\theta)} (r+\lambda-g) \right) = y \tag{3.24}$$

The optimal life of a job is chosen such that the cost at the end of the horizon is equal to the stationary match product. An equilibrium solution is characterized by credit - and labor-market tightnesses, as well as the optimal life span of a job (ϕ^*, θ^*, T^*) that verify equations (3.20), (3.13) and (3.23). Using the balanced-growth equation, the job creation condition (3.13) can be recast as,

$$E_g = \frac{r+q(\theta)-g}{q(\theta)} \frac{d}{z(\phi)} \tag{3.25}$$

Given θ^*, $\phi^*$and the optimal job age at destruction T^*, the Equations (3.25) and (3.23) with (3.24) imply that,

Proposition 5 *The equilibrium value of a job is given by the crossing of job creation and job destruction conditions, and is written as,*

$$\frac{r+q\left(\theta^{*}\right)-g}{q\left(\theta^{*}\right)} \frac{d}{z\left(\phi^{*}\right)}=E_{g}^{*}(0,0)=(1-\alpha)(1-\eta) y \int_{0}^{T^{*}}\left[1-e^{g\left(s-T^{*}\right)}\right] e^{-(r+\lambda) s} d s \tag{3.26}$$

Proof. *The left-hand side is the job creation condition (**3.25**) evaluated at the equilibrium values of θ^* and ϕ^*. This continuous expression has a positive slope in the (E_g, θ) space since $q'(\theta) < 0$. The right-hand side is the job destruction condition (3.23) after assessing the first order condition (3.24). Note also that equation (3.23) is decreasing in θ by proposition 4 and is decreasing monotonically in (E_g, θ) space. Hence an equilibrium exist and is unique.*

In the right-hand side of equation (3.26) the term $(1-\alpha)(1-\eta)$ represent that the equilibrium value of a job for the entrepreneur is a function of their bargaining power in the labor market and their bargaining power in the credit market. The fact that the entrepreneur must negotiate a repayment flow with a banker, implies that higher bargaining power of the financier reduces the value of a job for an entrepreneur. This expression is similar to Mortensen and Pissarides (1998) with the exception of the presence $(1-\eta)$, that tell us that the presence of credit market, not only reduce the value of a job due the frictions but also by the negotiation process in the rising funds stage. Therefore, in the absence of this credit market frictions, the value of a job is higher $(\eta = 0)$ and the expression corresponds with the finding of Mortensen and Pissarides (1998).

The couple (η, ϕ) can be seen as the *credit market pressure* faced by the entrepreneur. An increase in the credit market pressure, as we call, is an increase in any of this two parameter: the bargaining power of financier η, and/or the credit marker tightness ϕ, present in the optimal life time, that reduce the equilibrium value of a job by equation (3.23). The mechanism by which this ocurr is the reduction of the optimal life of the job and the structure of the bargaining power within the credit market.

To compute the equilibrium replace ϕ^* defined by virtue of equation (3.20) in (3.26). Note that ϕ^* is a function of a set of parameters fixed in this model: the bargaining power of financier, η; the bargaining power of worker, α; the search cost of investments opportunities for the bank k, and the cost of fundraising for the entrepreneur, d. So we can take it as constant in the model, as follows,

$$\frac{r+q\left(\theta^{*}\right)-g}{q\left(\theta^{*}\right)} \frac{d}{z\left(\frac{(1-\eta)(1-\alpha)}{\eta} \frac{k}{d}\right)}=(1-\alpha)(1-\eta) y \int_{0}^{T^{*}}\left[1-e^{g\left(s-T^{*}\right)}\right] e^{-(r+\lambda) s} d s \tag{3.27}$$

This is an equation that links the equilibrium labor market tightness θ^*, and the optimal life of a job T^*, by mean of a set of parameters. Some remarks can be made from this expression:

Proposition 6.a *The surplus value of a match decreases with the rate of tehcnological progress, g, for every value of market tightness θ. More formally,*

$$\frac{\partial E_g^*}{\partial g} = -\int_0^T \left[se^{gs} \left(b + \theta \frac{\alpha}{1-\alpha} \frac{d}{z(\phi)} \left(r + q(\theta) - g - \frac{1}{s} \right) + \frac{\gamma}{q(\theta)} \left(r + \lambda - g - \frac{1}{s} \right) \right) \right] \times (1-\alpha)(1-\eta) e^{-(r+\lambda)s} ds < 0$$

Proof . From equation (3.23) and using the envelope theorem, we have that $\frac{\partial E_g^*}{\partial g} = \frac{\partial E_g}{\partial g} \mid_{(g,\ T^*(g))}$

***Proposition* 6.b** *The equilibrium labor market tightness θ^*increases with the rate of technological progress, g, if $\frac{\partial E(\theta^*,g;\Omega)}{\partial g} < \frac{1}{q(\theta^*)} \frac{d}{z(\phi^*)}$, and decreases otherwise. This can be shown by*

$$\frac{\partial \theta}{\partial g} = \left(\frac{\overbrace{q(\theta^*)^2}^{(+)}}{\underbrace{-\underbrace{\frac{\partial E(\theta^*,g;\Omega)}{\partial \theta^*}}_{(-)} \underbrace{q(\theta^*)^2}_{(+)} - \underbrace{q'(\theta^*)}_{(-)} \underbrace{\frac{(r-g)}{q(\theta^*)^2} \frac{d}{z(\phi^*)}}_{(+)}}_{(+)}} \right) \left(\underbrace{\frac{\partial E(\theta^*,g;\Omega)}{\partial g}}_{(-)} + \underbrace{\frac{1}{q(\theta^*)} \frac{d}{z(\phi^*)}}_{(+)} \right) \tag{3.28}$$

Proof. The rate at which entrepreneur and worker meet decreases with the labor market tightness as show eq. (3.1), and also the value of a job (see proposition 4). For more details, see Apendix A.3.

The sense of the movement of the labor market tightness respect to the rate of the technological progress depends on the search cost for fundraising, and therefore, in the level of effciency of the credit market. If these costs are sufficiently small, namely $\frac{\partial E(\theta^*,g;\Omega)}{\partial g} > \frac{1}{q(\theta^*)} \frac{d}{z(\phi^*)}$, there exists a negative relationship between technological progress and the equilibrium labor market tightness. In the other hand, if these costs are sufficiently large, namely $\frac{\partial E(\theta^*,g;\Omega)}{\partial g} < \frac{1}{q(\theta^*)} \frac{d}{z(\phi^*)}$, the relationship between the rate of technological progress and the equilibrium labor market tightness is positive.

***Proposition* 6.c** *The optimal life of a job also decreases in the rate of technological progress.*

$$\frac{\partial T^*}{\partial g}=\frac{\left(\frac{\frac{1}{q(\theta^*)}\frac{d}{z(\phi^*)}\frac{\partial E(\theta^*,g;\Omega)}{\partial\theta^*}q(\theta^*)^2-\frac{\partial E(\theta^*,g;\Omega)}{\partial g}q'(\theta^*)(r-g)\frac{d}{z(\phi^*)}}{-\frac{\partial E(\theta^*,g;\Omega)}{\partial\theta^*}q(\theta^*)^2-q'(\theta^*)(r-g)\frac{d}{z(\phi^*)}}\right)-(1-\alpha)(1-\eta)ye^{gT^*}\int_0^{T^*}(T^*-s)e^{-s(r+\lambda-g)}ds}{(1-\alpha)(1-\eta)ge^{gT^*}y\int_0^{T^*}e^{-s(r+\lambda-g)}ds}<0$$

Proof. See Appendix A.4

Proposition 6 shows us some interesting things about the relationship between technology progress and labor and credit markets. The higher the technological progress, the lower the maximal value of a job for an entrepreneur (prop. 6.a). The underlying mechanism is that the cost involved in the job, namely, the wage rate paid to the worker and the repayment flow paid to the bank, increase in the productivity at the frontier which, in turn, increases in the exogenous rate of technological progress. This raise in the job costs reduces the value of a job, a result present in Mortensen and Pissarides (1998).

An increase in the rate of technological progress, however, produces two different types of effect on equilibrium labor market tightness, depending on the total costs that the entrepreneur faces before posting a vacancy. Note that these costs are related with the level of efficiency of credit market modeled by $z(\phi)$. As credit market becomes more efficient in matching borrowers and financiers, the matching probability $z(\phi)$ increases and therefore costs are reduced[3]. Thus, in an economy with low frictions in the credit market (high level of $z(\phi^*)$), the higher rate of technological progress, the lower the equilibrium labor market tightness. In fact, in a scenario with no credit market frictions (i.e. $z(\phi^*)$ approaches infinity) the result becomes Pissarides (2000, Ch. 3). On the other hand, an economy with high frictions in the credit market, and hence high costs of posting a vacancy, higher rates of technological progress increase the equilibrium labor market tightness. The economic explanation behind this fact is related to two effects of technology progress present in the model. First, the job destruction effect or the loss of value of a job due to technological progress, represented by $\frac{\partial E(\theta^*,g;\Omega)}{\partial g}$ in the second term in brackets in eq. 29 (see proposition 6.a.), that consider the decision of entrepreneurs of destroy the job due the increasing costs induced by the rate of technological progress. A second effect, novel in this model, is the direct effect of g on job creation,

[3]Focus in the second term in brackets in eq. (3.28). This expression is formed by the sum of the terms $\frac{\partial E_g^*}{\partial g}<0$ and $\frac{1}{q(\theta^*)}\frac{d}{z(\phi^*)}>0$. Note that the latter is simply another form of the search cost (or hiring cost) faced by entrepreneur, c, in the model of Pissarides (2000, Ch. 1 p.10) considering, in this case, as a function of the credit market, $c(\phi)$. Indeed, we can name $c(\phi^*)=\frac{d}{z(\phi^*)}$, and then we would return to Pissarides (2000, Ch. 1), $\frac{c(\phi^*)}{q(\theta^*)}$.

represented by $\frac{1}{q(\theta^*)}\frac{d}{z(\phi^*)}$ in eq. (29). If the job destruction effect dominate the job creation effect of g, $\left|\frac{\partial E(\theta^*,g;\Omega)}{\partial g}\right| > \frac{1}{q(\theta^*)}\frac{d}{z(\phi^*)}$, i.e. the loss of value of a job due the technological progress is greater than the total costs of posting a vacancy, entrepreneurs prefer to destroy the job since the costs of creating jobs are relatively lower. This immediately increases the pool of unemployed workers and reduced the tightness of the labor market. Then the entrepreneurs decide to begin a new search for funds, once they get these funds, they post a vacancy, find a worker, and start the production. If on the other hand, the job creation effect dominates the effect of job destruction $\left|\frac{\partial E(\theta^*,g;\Omega)}{\partial g}\right| < \frac{1}{q(\theta^*)}\frac{d}{z(\phi^*)}$, implies that it is so expensive the process of posting vacancies that the entrepreneur decides to avoid the step of having to raise funds to post a vacancy, which would reduce the pool of unemployed workers relative to vacancies offered on the labor market, and therefore increases the labor market tightness. On the other hand, an increase in technological progress unambiguously reduces the optimal lifetime of a job. So, in summary, an increase in the rate of technological progress imply: i) a reduction of value of a job, ii) a decrease (increase) of the equilibrium labor market tightness in economies with high costs of posting vacancies, which are related with an efficient (inefficient) credit market, and iii) lowered the optimal lifetime of a job.

3.4.1 Efficiency of credit market

As credit market become more efficient, which can be interpreted as an increase in the matching probability $z(\phi)$, the optimal life time of a job decreases and the equilibrium labor market tightness rises by virtue of equation (3.26). This mean that an economy with a more efficient credit market, the value of a job for entrepreneurs is higher than an economy with looser credit market performance. This notion introduce the following corollary,

Corollary 1: *An increase in the matching probability in the credit market, $z(\phi^*)$, reduces the optimal lifetime of a job, T^*, and increase the equilibrium labor market tightness, θ^**

Proof. *From (3.26) we can obtain* $\frac{\partial T^*}{\partial z(\phi^*)} = \frac{-\frac{d}{z(\phi^*)}\frac{r+q(\theta^*)-g}{q(\theta^*)}}{(1-\alpha)(1-\eta)ge^{gT^*}y\int_0^{T^*}e^{-s(r+\lambda-g)}ds} < 0$ *and* $\frac{\partial \theta^*}{\partial z(\phi^*)} = -\frac{(r+q(\theta^*)-g)q(\theta^*)}{q'(\theta^*)(r-g)} > 0$. *For more details see Appendix A.5.*

The fact that the optimal lifetime of a job is reduced with a more efficient credit market, is explained since those entrepreneurs in process of adopting the technology with the higher productivity of the economy, will find more quickly a financier willing to invest in their business. This would give incentives to break the current employment and lending relationship, and engage into the search for a new lending and job relationship. In the same vein, the higher the matching probability in the credit market, the higher the equilibrium labor market tightness, because there will be more companies offering vacancies which imposing externalities each other in the search process for a worker.

3.4.2 Implications for unemployment

Implications for unemployment and job flows can be derived by the balance between job creation and job destruction in steady state. Note that the job creation at time t is $JC(t) = \theta^* q(\theta^*) u(t)$, and job destruction is given by, $JD(t) = e^{-\lambda T^*} JC + \lambda [1 - u(t)]$. This expression includes the notion that the job destruction is equal to the flow of jobs that attain the age of obsolescence and the flow of jobs that receiving an exogenous shock of destrucion. In steady-state, the unemployment rate equates job creation and job destruction flows over time, as follows,

$$u^* = \frac{\lambda}{\lambda + \left(1 - e^{-\lambda T^*}\right) \theta^* q(\theta^*)} \tag{3.29}$$

Equation (3.29) has the same form of Mortensen and Pissarides (1998, p. 742), but with the difference that the optimal life of a job in this model is a function of credit market. As we seen, as credit market becomes more efficient (modeled as an increase in $z(\phi)$) the optimal life time of a job is reduced and the equilibrium labor market tightness rises (Corollary 1). In a general equilibrium framework, however, the effect on unemployment depends on whether the positive effect of matching probability on the labor market tightness exceed, or not, at a fraction of the negative effect that matching probability has on the optimal life of a job. More formally,

Corollary 2: *An increase in the matching probability in the credit market, $z(\phi^*)$, can reduce the equilibrium unemployment rate, u^*, if ,*

$$\left| \frac{\partial T^*}{\partial z(\phi^*)} \lambda \left(\frac{e^{-\lambda T^*}}{1 - e^{-\lambda T^*}} \right) \left(\frac{\theta^* q(\theta^*)}{q(\theta^*) + \theta^* q'(\theta^*)} \right) \right| < \frac{\partial \theta^*}{\partial z(\phi^*)}$$

Proof. *It follows from Corollary 1 and eqn. (3.29). For details see Appendix A.6*

Corollary 2 suggests the following: an increase in the efficiency of the credit market generates two forces affecting u that moving in the opposite direction. If the force generated by the positive movement of θ^* (relative to an increase in $z(\phi^*)$) is greater than a proportion of the negative movement of T^* (compared with the same increase in $z(\phi^*)$), then the equilibrium unemployment decrease. The mechanism by which unemployment decreases, is that while jobs are being destroyed, in equilibrium, sufficiently vacancies are opening for the creation of another new. On the other hand, if the inequality is reversed in the corollary, increase the equilibrium unemployment, because they are destroying more jobs than those being reallocated.

With regard to technological progress: If the cost of posting a vacancy is lower than the reduction of value of a job due the obsolescence induced by technological progress, an increase in the technological progress rate reduces the equilibrium labor market tightness and the optimal life of a job (proposition 6), and consequently the equilibrium unemployment raises. This part of the result is in line with Mortensen and Pissarides (1998) to establish that more rapid technological progress under these assumptions induces more labor reallocation and so higher unemployment. However, in the opposite scenario, if the cost of posting a vacancy is higher than the reduction of value of a job due to obsolescence, the effect of an increase in technological progress rate reduces the optimal life time of a job, but raises the equilibrium market tightness. In this context the implications on unemployment are ambiguous, since θ^*decrease but T^*increase. The key thing is that to respond, in a model which focuses in creative destruction effect, how does technological progress affect the equilibrium number of jobs? the presence of frictions in the credit market plays an important role to understand implications on the labor market.

Corollary 3: *The movement of the unemployment rate of the economy by an increase of the rate of technological progress depends on the search cost in the credit market,*

- *If search cost in credit market is lower than the value reduction of a job due the technology progress ($\frac{\partial\theta}{\partial g} < 0$ see prop. 6b), an increase in this rate unambiguously increases the unemployment rate of the economy.*

Proof: *by simple inspection of eq. (3.29) using proposition 6.b ($\frac{\partial\theta}{\partial g} < 0$) and 6.c ($\frac{\partial T}{\partial g} < 0$)*

- *If search cost in credit market is higher than the value reduction of a job due the technology progress ($\frac{\partial\theta}{\partial g} > 0$ see prop. 6b), an increase in this rate can reduce the equilibrium unemployment rate if*

$$\frac{\partial T^*}{\partial g}\lambda\left(\frac{e^{-\lambda T^*}}{1-e^{-\lambda T^*}}\right)\left(\frac{\theta^* q\left(\theta^*\right)}{q\left(\theta^*\right)+\theta^* q'\left(\theta^*\right)}\right) < \frac{\partial\theta^*}{\partial g}$$

Proof: *See Appendix A.7*

Importantly however, that while the equilibrium rate of unemployment decreases, productivity in this economy tends to stagnate, as entrepreneurs prefer to continue with the technology at the date of creation, which remains constant and whose gap to productivity at the cutting-edge technology, increases continuously.

Chapter 4

Conclusion

The model developed in this thesis shows the interaction between the technological progress in presence of labor and credit market frictions. I try to understand how technological progress affect the equilibrium number of jobs in an economy with a more efficient credit market in comparison to one that does not have one.

The following are the major findings: i) first, the fact that the entrepreneur must negotiate a repayment flow with a banker, implies that a higher bargaining power of the financier reduces the value of a job for an entrepreneur. The mechanism is not only related due the frictions, but also by the negotiation process in the fund raising stage. In effect, in the absence of this credit market friction, the value of a job is higher and the expression corresponds with Mortensen and Pissarides (1998). The bargaining power of financier and the credit market tightness can be seen as a credit market pressure faced by the entrepreneur. An increase in these credit market pressures reduces the equilibrium value of a job. ii) Second, an economy with low frictions in the credit market has a lower optimal life span of a job and a higher equilibrium labor market tightness. This fact is motivated since is more easily for an entrepreneur find funds for his project, setting up in this way, incentives to adopt the technology with the highest productivity. This process requires the entrepreneur to search for a worker and therefore post a vacancy, increasing the tightness between firms searching for a worker. iii) Third, an increase in the rate of technological progress reduces the equilibrium value of a job and diminishes their optimal life, but have two possible effects on the equilibrium labor market tightness. The direction of these effects depends on the total cost of posting a vacancy which, in turn, is related to the efficiency of the credit market. A negative effect takes place if the reduction of the value of a job due the technological progress is greater than the total cost of posting a vacancy; and a positive effect otherwise. The intuition behind this fact is that, with an increasing technological progress, a better developed credit market creates incentives for entrepreneurs to destroy labor relationship and return to get financing with more ease relative to an inefficient credit market. iv) Fourth, regarding the consequences for unemployment, an economy where the reduction in the value of a job induced by technology progress is greater than the total cost of posting a vacancies, an increase in the rate of technological progress produce more labor reallocation and a higher unemployment. However, in the opposite scenario, where the total cost of posting a vacancies is greater than the the reduction in the value of a job induced by technology progress, exists a threshold where an increase in the rate of technological progress can reduce the equilibrium unemployment.

In sum, this thesis propose that to respond how does technological progress affect the equilibrium number of jobs? the presence of frictions in the credit market play an important role. This model is an exploratory attempt to do that.

However, a number of questions remain open, and can be treated as an extension of this basic model. I mention three: first, further research should be devoted to the analysis of updating the job relationship as in Mortensen Pissarides (1998) case of renovation. Second, since the credit market is fricctional the match entrepreneur-financier may be as a valuable asset, and consequently, job destruction does not necessarily mean the destruction of bank-entrepreneur match. And third, to effectively examine the dynamics of technology adoption, allow new entrepreneurs to adopting technology into a variety of technologies available in the economy.

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Appendix A

A.1.: Proof of Proposition 2

$$w(\tau, t) = argmax[E_g(\tau, t) - E_c(t)]^{1-\alpha}[W(\tau, t) - U(t)]^{\alpha}$$

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$$\alpha[E_g(\tau, t) - E_c(t)] = (1-\alpha)[W(\tau, t) - U(t)]$$

But $E_c(t) = 0$, then the nash bargaining condition is given by,

$$\alpha E_g(\tau, t) = (1-\alpha)[W(\tau, t) - U(t)] \tag{A.1}$$

Rearranging terms in equation (3.10) we have,

$$(r+\lambda)W(\tau, t) = w(\tau, t) + \lambda U(t) + \dot{W}(\tau, t) \qquad / - (r+\lambda)U(t)$$

$$W(\tau, t) - U(t) = \frac{w(\tau, t) - rU(t) + \dot{W}(\tau, t)}{r+\lambda} \tag{A.2}$$

From eq. (A.2) and equation (3.5) in equation (A.1)

$$\alpha[p(\tau) - w(\tau, t) - \rho + \dot{E}_g(\tau, t)] = (1-\alpha)[w(\tau, t) - rU(t) + \dot{W}(\tau, t)]$$

Rearranging terms,

$$w(\tau, t) = \alpha[p(\tau) - \rho + \dot{E}_g(\tau, t)] + (1-\alpha)rU(t) - (1-\alpha)\dot{W}(\tau, t) \tag{A.3}$$

From the Nash bargaining condition (A.1) with $\tau = t$ we have,

$$W(t, t) - U(t) = \frac{\alpha}{1-\alpha}E_g(t, t) \tag{A.4}$$

Equation (A.4) in (3.9),

$$rU(t) = p(t)b + \theta q(\theta)\frac{\alpha}{1-\alpha}E_g(t,\, t) + \dot{U}\,(t) \tag{A.5}$$

We must obtain an expression for $E_g(t,\, t)$. From the free entry condition (3.12) and equation (3.4), $E_g(t,\, t)$ is given by,

$$\frac{(r + q\,(\theta) - g)}{q\,(\theta)}\frac{dp(t)}{z(\phi)} = E_g\,(t,\, t)$$

replace this result in equation (A.5)

$$rU(t) = p(t)\left[b + \theta\frac{\alpha}{1-\alpha}\left((r + q(\theta) - g)\,\frac{d}{z(\phi)}\right)\right] + \dot{U}\,(t)$$

At this point, we have a final expression for $rU(t)$. Put this into equation (A.3),

$w(\tau, t) = \alpha[p(\tau) - \rho + \dot{E}_g\,(\tau, t)] + (1-\alpha)\left[p(t)\left[b + \theta\frac{\alpha}{1-\alpha}\left((r + q(\theta) - g)\,\frac{d}{z(\phi)}\right)\right] + \dot{U}\,(t)\right] - (1-\alpha)\dot{W}\,(\tau, t)$

Since the sharing rule in (A.1) also holds for the capital gains terms $\alpha\dot{E}_g\,(\tau, t) = (1-\alpha)[\dot{W}\,(\tau, t) - \dot{U}\,(t)$, the final expression for the wage is given by

$$w(\tau, t) = \alpha[p(\tau) - \rho] + (1-\alpha)p(t)\left[b + \theta\frac{\alpha}{1-\alpha}\left((r + q(\theta) - g)\,\frac{d}{z(\phi)}\right)\right] \tag{A.6}$$

Appendix A

A.2: Proof of Proposition 3

$$\rho = argmax \left[B_l\left(t\right) - B_c\left(t\right)\right]^{\eta} \left[E_l\left(t\right) - E_c\left(t\right)\right]^{1-\eta}$$

By proposition 1$\frac{\partial w}{\partial \rho} = -\alpha$, First order condition then is,

$$\left(1-\eta\right)\left(1-\alpha\right)\left[B_l\left(t\right) - B_c\left(t\right)\right] = \eta\left[E_l\left(t\right) - E_c\left(t\right)\right] \tag{A.1}$$

Free entry conditions on the credit market for banks and entrepreneurs, imply that equation (A.1) is,

$$\left(1-\eta\right)\left(1-\alpha\right)B_l\left(t\right) = \eta E_l\left(t\right) \tag{A.2}$$

From value functions, we have:

$$E_l(t) = \frac{q(\theta)}{r+q(\theta)-g}\left[\frac{p(t)-w-\rho}{r+\lambda-g}\right] \tag{A.3}$$

$$B_l(t) = \frac{q(\theta)}{r+q(\theta)-g}\frac{\rho}{r+\lambda-g} - \frac{\gamma p(t)}{r+q(\theta)-g} \tag{A.4}$$

Substituting equations (A.3) and (A.4) into (A.2), we obtain,

$$\eta\left[\frac{q(\theta)}{r+q(\theta)-g}\frac{p(t)-w-\rho}{r+\lambda-g}\right] = (1-\eta)(1-\alpha)\left[\frac{q(\theta)}{r+q(\theta)-g}\frac{\rho}{r+\lambda-g} - \frac{\gamma p(t)}{r+q(\theta)-g}\right]$$

$$\eta\left[p(t)-w-\rho\right] = (1-\eta)(1-\alpha)\left[\rho - \frac{\gamma}{q(\theta)}p(t)\left(r+\lambda-g\right)\right]$$

$$\eta\left[p(t)-w\right] + (1-\eta)(1-\alpha)\left[\frac{\gamma}{q(\theta)}p(t)\left(r+\lambda-g\right)\right] = \rho\left[1-\alpha\left(1-\eta\right)\right]$$

Rearranging terms

$$\rho = \frac{\eta}{1-\alpha(1-\eta)}\left[p\left(t\right)-w\right] + \frac{\left(1-\eta\right)\left(1-\alpha\right)}{1-\alpha(1-\eta)}\left[\frac{\gamma p\left(t\right)}{q\left(\theta\right)}\left(r+\lambda-g\right)\right] \tag{A.5}$$

if $\eta_{\alpha} = \frac{\eta}{1-\alpha(1-\eta)}$, this implies that

$$\rho = \eta_{\alpha}\left[p\left(t\right)-w\right] + \left(1-\eta_{\alpha}\right)\left[\frac{\gamma p\left(t\right)}{q\left(\theta\right)}\left(r+\lambda-g\right)\right] \tag{A.6}$$

Appendix A

A.3: Proof of Proposition 6.b

To Obtain $\frac{\partial\theta}{\partial g}$, we implicitly derive the equilibrium equation (3.27), as follows,

$$\underbrace{\frac{r+q\left(\theta^{*}\right)-g}{q\left(\theta^{*}\right)}\frac{d}{z\left(\frac{(1-\eta)(1-\alpha)}{\eta}\frac{k}{d}\right)}}_{\Theta(\theta^{*},g;\Omega)}=\underbrace{(1-\alpha)\left(1-\eta\right)y\int_{0}^{T^{*}}\left[1-e^{g(s-T^{*})}\right]e^{-(r+\lambda)s}ds}_{E(\theta^{*},g;\Omega)}$$

$$\frac{\partial\theta}{\partial g}:\quad\frac{\partial\Theta\left(\theta^{*},g;\Omega\right)}{\partial g}+\frac{\partial\Theta\left(\theta^{*},g;\Omega\right)}{\partial\theta^{*}}\frac{\partial\theta}{\partial g}=\underbrace{\frac{\partial E\left(\theta^{*},g;\Omega\right)}{\partial g}}_{(-)}+\underbrace{\frac{\partial E\left(\theta^{*},g;\Omega\right)}{\partial\theta^{*}}}_{(-)}\frac{\partial\theta}{\partial g}$$

Note that $\frac{\partial E(\theta^{*},g;\Omega)}{\partial g}<0$ and $\frac{\partial E(\theta^{*},g;\Omega)}{\partial\theta^{*}}<0$ by virtue of proposition 6.1 and proposition 4 respectively. Also we know for the labor matching function eq. (3.1) that $q'\left(\theta^{*}\right)<0$, so,

$$\frac{\partial\theta}{\partial g}:\quad-\frac{1}{q\left(\theta^{*}\right)}\frac{d}{z\left(\phi^{*}\right)}-\frac{\overbrace{q'\left(\theta^{*}\right)}^{(-)}\left(r-g\right)}{q\left(\theta^{*}\right)^{2}}\frac{d}{z\left(\phi^{*}\right)}\frac{\partial\theta}{\partial g}=\underbrace{\frac{\partial E\left(\theta^{*},g;\Omega\right)}{\partial g}}_{(-)}+\underbrace{\frac{\partial E\left(\theta^{*},g;\Omega\right)}{\partial\theta^{*}}}_{(-)}\frac{\partial\theta}{\partial g}$$

Rearranging terms and solving for $\frac{\partial\theta}{\partial g}$, we have

$$\frac{\partial\theta}{\partial g}=\left(\frac{\overbrace{q\left(\theta^{*}\right)^{2}}^{(+)}}{\underbrace{-\underbrace{\frac{\partial E\left(\theta^{*},g;\Omega\right)}{\partial\theta^{*}}}_{(-)}\underbrace{q\left(\theta^{*}\right)^{2}}_{(+)}-\underbrace{q'\left(\theta^{*}\right)}_{(-)}\underbrace{(r-g)\frac{d}{z\left(\phi^{*}\right)}}_{(+)}}_{(+)}}\right)\left(\underbrace{\frac{\partial E\left(\theta^{*},g;\Omega\right)}{\partial g}}_{(-)}+\underbrace{\frac{1}{q\left(\theta^{*}\right)}\frac{d}{z\left(\phi^{*}\right)}}_{(+)}\right)$$

Appendix A

A.4: Proof of Proposition 6.c

Following the same approach that the previous proof, we have to implicitly derive the equilibrium equation,

$$\underbrace{\frac{r+q\left(\theta^{*}\right)-g}{q\left(\theta^{*}\right)}\frac{d}{z\left(\frac{(1-\eta)(1-\alpha)}{\eta}\frac{k}{d}\right)}}_{\Theta(g;\Omega)}=\underbrace{(1-\alpha)\left(1-\eta\right)y\int_{0}^{T^{*}}\left[1-e^{g(s-T^{*})}\right]e^{-(r+\lambda)s}ds}_{E(T^{*},g;\Omega)}$$

$$\frac{\partial T^{*}}{\partial g}:\quad \frac{\partial\Theta\left(\theta^{*},g;\Omega\right)}{\partial g}+\frac{\partial\Theta\left(\theta^{*},g;\Omega\right)}{\partial\theta^{*}}\frac{\partial\theta}{\partial g}=\frac{\partial E\left(T^{*},g;\Omega\right)}{\partial g}+\underbrace{\frac{\partial E\left(T^{*},g;\Omega\right)}{\partial T^{*}}}_{Leibniz\ \ Rule}\frac{\partial T^{*}}{\partial g} \tag{A.1}$$

By Leibniz Rule $\frac{\partial E(T^{*},g;\Omega)}{\partial T^{*}}=\int_{0}^{T^{*}}\frac{\partial E(s,T^{*},g;\Omega)}{\partial T^{*}}ds+\underbrace{E(T^{*},T^{*},g,\Omega)}_{0}1+\underbrace{E(T^{*},T^{*},g,\Omega)0}_{0}$ as follow,

$$\frac{\partial E\left(T^{*},g;\Omega\right)}{\partial T^{*}}=(1-\alpha)\left(1-\eta\right)ge^{gT^{*}}y\int_{0}^{T^{*}}e^{-s(r+\lambda-g)}ds$$

Replace this result into eq. **(A.1),**

$$\frac{\partial T^{*}}{\partial g}=\frac{-\frac{1}{q(\theta^{*})}\frac{d}{z(\phi^{*})}-\frac{q'(\theta^{*})(r-g)}{q(\theta^{*})^{2}}\frac{d}{z(\phi^{*})}\frac{\partial\theta}{\partial g}-(1-\alpha)\left(1-\eta\right)ye^{gT^{*}}\int_{0}^{T^{*}}\left(T^{*}-s\right)e^{-s(r+\lambda-g)}ds}{(1-\alpha)\left(1-\eta\right)ge^{gT^{*}}y\int_{0}^{T^{*}}e^{-s(r+\lambda-g)}ds}$$

Note that $\frac{\partial\theta}{\partial g}$is the result of Appendix A.3, so we can replace it to obtain its effect on T^{*}

$$\frac{\partial T^{*}}{\partial g}=\frac{\left(\frac{\frac{1}{q(\theta^{*})}\frac{d}{z(\phi^{*})}\frac{\partial E\left(\theta^{*},g;\Omega\right)}{\partial\theta^{*}}q(\theta^{*})^{2}-\frac{\partial E\left(\theta^{*},g;\Omega\right)}{\partial g}q'(\theta^{*})(r-g)\frac{d}{z(\phi^{*})}}{-\frac{\partial E(\theta^{*},g;\Omega)}{\partial\theta^{*}}q(\theta^{*})^{2}-q'(\theta^{*})(r-g)\frac{d}{z(\phi^{*})}}\right)-(1-\alpha)(1-\eta)ye^{gT^{*}}\int_{0}^{T^{*}}(T^{*}-s)e^{-s(r+\lambda-g)}ds}{(1-\alpha)(1-\eta)ge^{gT^{*}}y\int_{0}^{T^{*}}e^{-s(r+\lambda-g)}ds}<0$$

Appendix A

A.5.: Proof of Corollary 1

We have to implicitly derive the equilibrium equation,

$$\underbrace{\frac{r+q(\theta^*)-g}{q(\theta^*)}\frac{d}{z(\phi)}}_{\Theta(z(\phi);\Omega)} = \underbrace{(1-\alpha)(1-\eta)\,y\int_0^{T^*}\left[1-e^{g(s-T^*)}\right]e^{-(r+\lambda)s}ds}_{E(T^*,z(\phi);\Omega)}$$

$$\frac{\partial T^*}{\partial z(\phi)}: \quad \frac{\partial\Theta(z(\phi);\Omega)}{\partial z(\phi)} + \frac{\partial\Theta(z(\phi);\Omega)}{\partial T^*}\frac{\partial T^*}{\partial z(\phi)} = \frac{\partial E(T^*;\Omega)}{\partial z(\phi)} + \underbrace{\frac{\partial E(T^*;\Omega)}{\partial T^*}}_{Leibniz\ \ Rule}\frac{\partial T^*}{\partial z(\phi)} \tag{A.1}$$

By Leibniz Rule $\frac{\partial E(T^*,g;\Omega)}{\partial T^*} = \int_0^{T^*}\frac{\partial E(s,T^*,g;\Omega)}{\partial T^*}ds + \underbrace{E(T^*,T^*,g,\Omega)}_{0}1 + \underbrace{E(T^*,T^*,g,\Omega)0}_{0}$ as follow,

$$\frac{\partial E(T^*,g;\Omega)}{\partial T^*} = (1-\alpha)(1-\eta)\,ge^{gT^*}y\int_0^{T^*}e^{-s(r+\lambda-g)}ds$$

Replace this result into eq. (A.1)

$$\frac{\partial T^*}{\partial z(\phi)}: \quad -\frac{d}{z(\phi)^*}\frac{r+q(\theta^*)-g}{q(\theta^*)} = (1-\alpha)(1-\eta)\,ge^{gT^*}y\int_0^{T^*}e^{-s(r+\lambda-g)}ds\frac{\partial T^*}{\partial z(\phi)}$$

Rearranging terms, we obtain,

$$\frac{\partial T^*}{\partial z(\phi)} = \frac{-\frac{d}{z(\phi)^*}\frac{r+q(\theta^*)-g}{q(\theta^*)}}{(1-\alpha)(1-\eta)\,ge^{gT^*}y\int_0^{T^*}e^{-s(r+\lambda-g)}ds} < 0$$

The same procedure is followed for $\frac{\partial\theta^*}{\partial z(\phi)}$,

$$\frac{\partial\theta^*}{\partial z(\phi)}: \quad \frac{\partial\Theta(\cdot)}{\partial z(\phi)} + \frac{\partial\Theta(\cdot)}{\partial\theta^*}\frac{\partial\theta^*}{\partial z(\phi)} = \frac{\partial E(\cdot)}{\partial z(\phi)} + \frac{\partial E(\cdot)}{\partial\theta^*}\frac{\partial\theta^*}{\partial z(\phi)}$$

Solving and rearranging terms we have,

$$\frac{\partial\theta^*}{\partial z\left(\phi\right)} = -\left(r + q\left(\theta^*\right) - g\right)\frac{q\left(\theta^*\right)}{q'\left(\theta^*\right)\left(r - g\right)} > 0$$

Appendix A

A.6.: Proof of Corollary 2

Note that for $\frac{\partial u}{\partial z(\phi^*)}$ simply derive the equation (3.29) with respect to $z(\phi^*)$. Since u^* is a function of $T^*(\phi^*)$ and $\theta^*(\phi^*)$, we can rewrite as $u^*\left(T^*\left(\phi^*\right), \theta^*\left(\phi^*\right)\right)$. From Corollary 1, we know that $\frac{\partial T^*}{\partial z(\phi^*)} < 0$ and $\frac{\partial \theta^*}{\partial z(\phi^*)} > 0$. We just need to solve as follows,

$$\frac{\partial u^*}{\partial z(\phi^*)} = \underbrace{\frac{-\lambda}{\left[\lambda + \left(1 - e^{-\lambda T^*}\right)\theta^* q\left(\theta^*\right)\right]^2}}_{(-)} \left\{ \underbrace{\lambda e^{-\lambda T^*} \frac{\partial T^*}{\partial z\left(\phi^*\right)} \theta^* q\left(\theta^*\right)}_{(-)} + \underbrace{\left(1 - e^{-\lambda T^*}\right)}_{(+)} \underbrace{\frac{\partial \theta^*}{\partial z(\phi^*)}}_{(+)} \underbrace{\left(q\left(\theta^*\right) + q'\left(\theta^*\right)\theta^*\right)}_{(+)} \right\}$$

The latter expression is negative is,

$$\left| \frac{\partial T^*}{\partial z(\phi^*)} \lambda \left(\frac{e^{-\lambda T^*}}{1 - e^{-\lambda T^*}} \right) \left(\frac{\theta^* q\left(\theta^*\right)}{q\left(\theta^*\right) + \theta^* q'\left(\theta^*\right)} \right) \right| < \frac{\partial \theta^*}{\partial z(\phi^*)}$$

Appendix A

A.7.: Proof of Corollary 3

To proof this part of the corollary we take the results of proposition 6.b with $\frac{\partial \theta}{\partial g} > 0$ and proposition 6.c ($\frac{\partial T}{\partial g} < 0$). From equation (3.29) we can obtain $\frac{\partial u}{\partial g}$ as follow,

$$\frac{\partial u}{\partial g} = \frac{-\lambda^2 e^{-\lambda T^*} \frac{\partial T^*}{\partial g} \theta^* q\left(\theta^*\right) - \lambda \left(1 - e^{-\lambda T^*}\right) \left[\frac{\partial \theta^*}{\partial g}\left(q\left(\theta^*\right) + \theta^* q'\left(\theta^*\right)\right)\right]}{\left[\lambda + \left(1 - e^{-\lambda T^*}\right) \theta^* q\left(\theta^*\right)\right]^2}$$

Note that the first term of the numerator $-\lambda^2 e^{-\lambda T^*} \frac{\partial T^*}{\partial g} \theta^* q\left(\theta^*\right)$is positive by $\frac{\partial T}{\partial g} < 0$. Since we take in this part of the Corollary $\frac{\partial \theta}{\partial g} > 0$, the second term $-\lambda \left(1 - e^{-\lambda T^*}\right) \left[\frac{\partial \theta^*}{\partial g}\left(q\left(\theta^*\right) + \theta^* q'\left(\theta^*\right)\right)\right]$ is negative. Of course, the denominator is positive since it is a quadratic term. Rearranging terms, $\frac{\partial u}{\partial g} < 0$ is given by the expression in the numerator that meets the following condition,

$$\frac{\partial T^*}{\partial g} \lambda \left(\frac{e^{-\lambda T^*}}{1 - e^{-\lambda T^*}}\right) \left(\frac{\theta^* q\left(\theta^*\right)}{q\left(\theta^*\right) + \theta^* q'\left(\theta^*\right)}\right) < \frac{\partial \theta^*}{\partial g}$$