Local neutrality of Corporate Tax systems

Autores:
Eugenio Figueroa B.
Pablo Gutiérrez C.
Ramón E. López

Santiago, Octubre de 2014
Local neutrality of Corporate Tax systems

Pablo Gutiérrez C. ¹  Ramón E. López¹,²  Eugenio Figueroa B. ¹

Abstract. This paper shows one important result, namely, that corporate tax systems that allow at least for two sources of investment tax deductions (e.g., accelerated arbitrary investment depreciation and deductibility of part of interest payments on the firm’s debt) can be, under certain plausible conditions, locally neutral. That is, they allow for the existence of at least one positive corporate tax rate that renders the user cost of capital equal to the undistorted (without taxes) level of this cost.

¹ Department of Economics, University of Chile
² AREC Department, University of Maryland at College Park
Local neutrality of Corporate Tax systems\textsuperscript{3}

1. Introduction

This paper shows one important result, namely, that corporate tax systems that allow at least for two sources of investment tax deductions (e.g., accelerated arbitrary investment depreciation and deductibility of part of interest payments on the firm` s debt) can be, under certain plausible conditions, \textit{locally} neutral. That is, they allow for the existence of at least one positive corporate tax rate that renders the user cost of capital equal to the undistorted (without taxes) level of this cost. To show this result we use a generalization of the simplest model of corporate taxes due to Hall and Jorgenson (1967) which allows us to derive such result with a minimum of algebraic clutter and with complete transparency. However, we do probe the validity of this result using a further extension of the Hall and Jorgensen model that in fact considers, in a reduced form manner, several of the additional complexities arising by allowing the firm` s internal sources, in conjunction with merely borrowing, to finance its investment.

Several methods have been considered to keep the corporate tax neutral, where it could raise government revenues without distorting capital investment choices. The best known methods devised by economists are the Imputed Income method (IIM) and the Cash Flow method (CFM), the first one being based on Samuelson (1964), and the second on Brown (1948).\textsuperscript{4} The IIM allows firms to deduct from taxes the true capital depreciation over time as

\textsuperscript{3} The authors thank Juan Pablo Torres-Martinez and Alfonso Montes for useful comments.

\textsuperscript{4} Also, Ruf (2012) shows that the taxable base attained from Samuelson` s method can be enhanced by forcing all pure profits to be part of the tax base.
well as the imputed interest costs of the new investment. The CFM permits full and instantaneous depreciation of investment at the time they are implemented without allowing any further deduction on interest payments.\textsuperscript{5}

All these methods are \textit{globally} neutral by permitting any corporate tax rate between 0 and 1 to have no effect on the user cost of capital. However, in reality very few countries use any of the globally neutral methods described in the literature (Klemm, 207). The main reason for the lack of applicability of these methods is their exceedingly difficult practical implementation (Devereux and Freeman, 1991). Instead, most tax codes allow for an assortment of capital deductions, but in a way that is far distant from the above methods.

The present paper introduces the concept of \textit{local} neutrality. We show that, depending on the structure of tax deductions and exemption allowances, in certain important cases, locally neutral positive tax levels exist. Moreover, we demonstrate that some of the most popular tax systems used in many countries can, under certain conditions, be rendered locally neutral. We show that the user cost of capital in many tax regimes existing around the world

\textsuperscript{5} More recently, Boadway and Bruce (1984) have shown that allowing for the true capital depreciation is not necessary for neutrality as long as the allowed depreciation rates mimic the true ones in terms of present value. Also, Devereux and Freeman (1991) propose an alternative method that consists of deducting the opportunity cost of the market value of the firm from the tax base. This is the so-called Allowance for Corporate Equity (ACE) method, which is based on the theoretical results shown by Boadway and Bruce (1984). Bond and Devereux (1995, 2003) generalize the theoretical results about the neutrality of the ACE method and Kleinbard et al. (2007) also analyzes it.
may be a non-monotonic function of the corporate tax rate. This, in turn, implies that in addition to the zero tax there may be other tax rates which allow for neutrality.

We derive the necessary and sufficient conditions for local neutrality. Under certain conditions developed below, there are two tax rates which yield neutrality: a zero tax rate and just one positive tax rate. A government wanting to maximize tax revenues would of course choose the latter. However, any deviation from the optimal (locally neutral) tax rate, whether above or below it, causes efficiency losses. Thus, we have a razor’s edge solution with two hazards: a tax rate above (below) the optimal one may cause efficiency losses by distorting the user cost of capital upwardly (downwardly).

We consider here tax systems that permit arbitrary capital depreciation allowances in combination with tax allowances for the interest paid on the portion of the investment that is financed with borrowing (as opposed to the totality of the imputed interests of the capital invested). We denote this method as Mixed Method (MM). The importance of the MM is that variants of this method are used by most countries around the world (Mirrless et al. 2011).

2. Local neutrality: A formal definition

We first provide a formal definition of the concepts of global and local neutrality.

Definition 1. Define a tax system conformed by the corporate net income tax \( \tau \) and a subset \( \mu = \{ \mu_i \}_{i=1}^n \) of non-tax instruments (investment allowances, debt interest deductions, accelerated capital depreciation, etc.). The corporate tax \( \tau \) is **globally neutral** if and only if all the possible values of \( 0 \leq \tau \leq 1 \) are mapped by the following correspondence:
\[ \Omega(\mu) = \{ \tau \in (0,1]: c(\tau; \mu) = c^* \} = (0,1] \] (1)

where, \( c^* \) is the undistorted user cost or rental price of capital. The corporate tax \( \tau \) is locally neutral if and only if at least one non-zero \( \tau = \tau^* \) is mapped by the following correspondence:

\[ \Omega(\mu) = \{ \tau^* \in (0,1]: c(\tau; \mu) = c^* \} \neq \emptyset \] (2)

3. The asset price and the user cost of capital

Consider the following generalization of the capital arbitrage condition proposed originally by Hall and Jorgenson (1967) which results in the MM case:

\[ \frac{\partial}{\partial \tau} \left[ q(\tau) \right] = \int_{\tau}^{\infty} \left[ (1 - \tau)c(s)e^{-[\delta + \lambda(s-t)]}ds + \tau \chi q(\tau) \right] \] (3)

Where \( q(\tau) \) is the asset market price of capital at acquisition time \( \tau \), \( 0 \leq \tau \leq 1 \) is the corporate tax rate, \( c(s) \) is the user cost or rental price of capital at time \( s \) (equal to the cost of capital services), \( i \) is the interest rate faced by the firm after the tax allowances to be defined below, \( \delta \) is the true rate of capital depreciation which is assumed constant and exponential, and \( 0 \leq \chi \leq 1 \) is the proportion of the investment value that is allowed to be deducted from the tax base.\(^7\)

The after-tax interest rate \( i \) faced by the firm is different from the market interest rate when the tax code allows interest costs paid on the debt to be tax deductible. We assume that a maximum proportion \( \eta \) of the interest costs paid by the firm is allowed to be tax-deductible.

\(^6\) We first assume that the firm under consideration is a mature one, which implies that taxes on dividends are not relevant in the determination of the user cost of capital (Korinek and Stiglitz, 2008).

\(^7\) A common form of allowance for the investment value is the accelerated depreciation allowance in which case \( \chi \) is the present value of the (accelerated) depreciation tax allowance. The tax depreciation formula is of course different from the true depreciation.
and that the firm fully uses this benefit. We also assume that a proportion $\beta$ of the investment cost is financed through borrowing. For simplicity we consider $\beta$ as exogenous.$^8$ However, we relax this assumption later. Then the effective interest cost of capital to the firm considering this tax benefit is,

$$i(t) = (1 - \beta)r(t) + \beta(1 - \tau_\eta)r(t) = (1 - \eta \beta \tau)r(t)$$

(4)

The arbitrage condition (3) assumes perfect capital markets and that firms maximize profits by equating the marginal product of capital to the after tax rental or user cost of capital, $c(s)$. As Hall and Jorgenson show, the rental cost of capital can be calculated from the relationship between the asset price of new capital and the discounted present value of future services derived from this asset.

Differentiating Equation (3) with respect to time, using (4), and assuming that $q$ is constant we have$^9$,

$$c = q[1 - \tau \chi] \frac{(r(1 - \eta \beta \tau) + \delta)}{(1 - \tau)}$$

(5)

Importantly, the user cost of capital in the MM system is affected by the tax rate $\tau$ in a non-linear fashion. Moreover, under certain conditions to be derived below it is possible that the tax affects the function $c$ in a non-monotonic way. We note that if the tax rate is zero (and hence

$^8$ This assumption has been widely used in the literature (see Boadway and Bruce (1984), Bond and Devereux (1995, 2002) and Bustos et al, (2004), among several others).

$^9$ This assumption is consistent with the fact that the asset price of capital is exogenous to the firm in a context of static expectations.
tax deductions are irrelevant) equation (5) yields the well-known arbitrage condition for the rental price of capital, which shows that the rental price of capital is equal to the cost of maintaining a unit of capital in production for one period, which in turn is equal to the foregone interest rate plus the depreciation,

$$c = q(r + \delta)$$  \hspace{1cm} (6)

3.1 Local neutrality under MM

Below we show the necessary and sufficient conditions under which the MM is locally neutral. When these conditions are satisfied, whether the actual tax is neutral or not, depends on the tax level itself.

Under the MM system the user cost of capital may under certain conditions be a U-shaped function of the corporate tax rate, so the corporate tax is not globally neutral, but could be locally neutral. In particular, if we differentiate the user cost of capital, expression (5), with respect to $\tau$ and evaluate the resulting expression at $\tau = 0$, we obtain,

$$\frac{\partial c(\tau=0)}{\partial \tau} = q [(r + \delta)(1 - (1 - \tau)) - \eta \beta r]$$ \hspace{1cm} (7)

The sign of expression (7) is ambiguous depending on the parameters of the tax system. Thus we have the following lemma,

**Lemma 1.** The user cost of capital is decreasing at the origin (when the tax rate is zero) if and only if,

$$\frac{1-\chi}{\eta \beta} < \frac{r}{r+\delta}$$ \hspace{1cm} (8)
**Proof**: If we differentiate \( c(\tau) \) with respect to \( \tau \), evaluate this at \( \tau = 0 \) and using the fact that

\[
\frac{1-\chi}{\eta \beta} < \frac{\tau}{r+\delta}, \quad \text{we have that} \quad \frac{\partial c(\tau)}{\partial \tau} \bigg|_{\tau=0} < 0.
\]

That is, if the strength of the tax benefits (represented by \( \eta \beta \) and \( \chi \)) is sufficiently large relative to the market rate of interest, the effect of introducing a small corporate tax on the user cost of capital is negative. We want to show that the corporate tax is locally neutral if the condition (8) holds.

As the tax rate increases, however, the effect of the tax on the user cost of capital may turn positive. This happens when the tax rate increases above \( \tau^c \), which is the rate that yields \( \frac{\partial c(\tau^c)}{\partial \tau} = 0 \). This point corresponds to the minimum level of the user cost given that the user cost function is convex in the tax rate. Thus, \( \tau^c \) is the critical tax rate at which the slope of the cost curve turns from negative to positive. The cost function is therefore increasing in \( \tau > \tau^c \). The following lemma provides an explicit value for \( \tau^c \).

**Lemma 2.** The critical corporate tax rate at which the user cost function reaches its minimum level is,

\[
\tau^c = 1 - \sqrt{1 - \frac{\eta \beta r - (r+\delta)(1-\chi)}{\eta \beta r \chi}}
\]  
(9)

**Proof.** See appendix.

From (9) it is clear that as long as condition (8) is satisfied then \( 0 < \tau^c < 1 \).

From these results we can demonstrate the following proposition:
**Proposition 1.** If a corporate tax system is MM and condition (8) holds then the corporate tax is locally neutral with two neutral tax rates, zero and \( \tau^* = \frac{\eta \beta r - (r + \delta)(1 - \chi)}{\eta \beta r \chi} \).

**Proof.** Using definition 1 we find a positive tax rate \( \tau \) that makes the after tax user cost of capital equal to the undistorted rental price of capital; that is,

\[
c = q[1 - \tau^* \chi] \frac{(r(1 - \eta \beta \tau^*) + \delta)}{(1 - \tau^*)} = q(r + \delta)
\]  

Equation (10) can be explicitly solved for \( \tau^* \) yielding,

\[
\tau^* = \frac{\eta \beta r - (r + \delta)(1 - \chi)}{\eta \beta r \chi} \tag{11}
\]

If condition (8) is satisfied then the neutral tax \( \tau^* \) in (11) is positive.

Moreover, using (9) and (11) it follows that,

\[
\tau^c = 1 - \sqrt{1 - \tau^*}.
\]

Which of course implies \( \tau^* > \tau^c \).

This means that, as shown in Figure 1, the optimal tax rate is on the upward side of the (U-shaped) cost function.
If condition (8) is satisfied then the neutral tax \( \tau^* \) in (11) is positive. Then from Definition 1 it follows that if condition (8) is satisfied there exist a unique positive corporate tax (\( \tau^* \)) that is locally neutral.

We have the following corollary to Proposition 1,

**Corollary 1.1.** If a corporate tax system is MM and condition (8) holds then the level of capital investment is higher (lower) than its neutral level if \( 0 < \tau < \tau^* \) (\( \tau > \tau^* \)) and is equal to its neutral level if \( \tau = \tau^* \). Moreover, investment is increasing (decreasing) in the tax rate within the \( 0 - \tau^c \) (\( \tau^c - \tau^* \)) interval.

**Proof.** Follows directly from Proposition 1.

3.2 Locally Neutral Corporate Tax and the Neutrality Loci

Local neutrality as defined in Definition 1 can be used to determine the combinations of \( \chi \) and \( \beta \) that make a particular (positive) tax rate, \( \bar{\tau} \), optimal for given values of \( r \) and \( \delta \). Consider an
arbitrary tax, \(0 < \bar{\tau} < 1\). Then define all the combinations of \(\chi\) and \(\eta\) satisfying condition (8) that solve the neutrality equation,

\[
c_k(\bar{\tau}; \chi, \eta \beta) = r + \delta
\]

Using (5) and (12) we obtain,

\[
\chi = \frac{(r+\delta) - \eta \beta r}{(r+\delta) - \eta \beta r}
\]

Equation (13) allows us to analytically determine the combinations of \(\chi\) and \(\eta \beta\) under which \(\tau\) is globally or locally neutral. In particular, when the existing parameters of the tax system allow \(\chi\) as defined in (13) to be independent of \(\bar{\tau}\), the system is globally neutral. Such is the case, for example, when \(\eta=0\), which renders \(\chi=1\). In general, if \(\eta \beta > 0\) then the level of \(\chi\) that solves (12) is a function of \(\bar{\tau}\). From (13) it follows that the slope of the loci is negative (\(\partial \chi / \partial \eta \beta < 0\)).

Figure 2 shows three loci of local neutrality for three different values of \(\bar{\tau}\), \(\tau^0 < \bar{\tau} < \tau^2\); for each value of \(\bar{\tau}\), the combinations of \(\chi\) and \(\eta \beta\) that make \(\bar{\tau}\) locally neutral describe the corresponding locus of local neutrality. Each one of the three loci depicted in the figure satisfies the condition in Equation (8) and \(0 < \eta \beta < 1\). These loci are all above the grid area of the figure which represents the set of combinations of the values of \(\chi\) and \(\eta \beta\) that do not satisfy condition (8), in which case local neutrality is infeasible and only a zero tax level is neutral.
Figure 2: Three loci for which the corporate income tax is locally neutral

An important implication illustrated in Figure 2 is that any tax rate may be made optimal by adjusting just one of the parameters characterizing the tax system ($\eta$ or $\chi$) as long as the new parameter combination satisfies condition (8).

3.3 Local neutrality and the maximum tax-free rate of return to capital

Globally neutral tax systems allow a maximum rate of return to capital ($MRR$) equal to the opportunity cost of capital to be tax free. That is the taxable base of these systems is equal to the excess profits above the $MRR = r$. In this section we show that locally neutral tax systems also allow a tax exempt $MRR$. However, unlike the case of globally neutral systems, in the locally neutral systems the $MRR$ is not fixed but rather dependent on the tax rate itself. We show that the $MRR$ for locally neutral systems could even be higher than the MRR of globally neutral systems.

Proposition 2. The taxable base of globally or locally neutral tax systems is equal to the excess of the actual profit rate and the MRR, where
If $\chi = 1$ and debt interests are not tax deductible ($\eta = 0$) then $MRR = r$, the system is globally neutral. If $0 < \chi < 1$, debt interest rates are deductible ($\eta \beta > 0$) and condition (8) is satisfied then the system is locally neutral.

**Proof.** See appendix

**Corollary 2.1** The MRR of a locally neutral system is greater or equal to the opportunity cost of capital ($r$) if and only if the following condition is met,

$$\frac{1 - \chi}{\eta \beta (1 - \tau \chi)} \leq \frac{r}{r + \delta}$$

**Proof.** Follows directly from (14).

4. **A generalization**

The model used so far is extremely simple designed to show the main results with as little complexities as possible. In this section we show that the key qualitative results presented above are, in fact, robust to a generalization of the model that recognizes the existence of more complex sources of financing that may include the firm’s own funds in addition to borrowing. The main consequence of this generalization is that $\beta$ may be endogenous. In addition, if the firm uses internal sources of finance we must explicitly consider two things. First, the role of other taxes affecting shareholders, including personal income taxes and capital gain taxes. Second, the degree of integration of the tax system ($s$), which corresponds to the proportion of the taxes paid by the firm that is credited to the personal taxes paid by
shareholders. With respect to this last point, we allow for partial \(0 < s < 1\) as well as total \(s = 1\) integration.

Define the function, \(\theta = \frac{(1-\tau+\tau r)[1-m(y^t_i)]}{(1-\tau)(1-z)}\), representing the opportunity cost of a retained dollar in terms of foregone dividends, which is a slight generalization of the formula originally derived by King (1974), by allowing for varying degrees of tax integration; where \(m(y^t_i)\) is the personal marginal tax that the shareholders pay for the dividends as a function of their total income, \(y^t_i\), and \(z\) is the tax rate applicable to capital gains.

In this context, the discount factor of the firm will be equal to a weighted average of the after-tax cost of capital to the firm \((r(1-\tau\eta))\) plus the opportunity cost to shareholders of foregoing dividends \((\theta r)\), using endogenous weights \(\beta\) and \(1 - \beta\), respectively.

\[
i(\tau) = [\beta r(1 - \tau\eta) + (1 - \beta)\theta r].
\]  

(15)

By replacing (15) in (3) and differentiating with respect to time, we obtain the following expression for the cost of capital:

\[
c(\tau) = \frac{(1-\tau r)[\beta r(1-\tau\eta)+(1-\beta)\theta r+\delta]}{(1-\tau)}
\]  

(16)

The firm chooses \(\beta\) to minimize (16). Thus, if \(\theta > 1\) then the cost of a dollar to shareholders in the firm is higher than outside the firm, in which case the firm will distribute all its profits and hence will finance their new investments entirely via borrowing; that is, \(\beta = 1\). By contrast, if \(\theta < 1\) then the cost of a dollar to shareholders inside the firm is less
or equal than outside the firm, implying that firms will have incentives to finance at least part of the new investments via retained profits.\footnote{This case is more frequently encountered in many tax codes. For example, in the US after the tax reform of 2003, by equalizing \( m \) and \( z \) made \( \theta = 1 \) given that \( s = 0 \) (Chetty and Saez, 2005).} In this case we have that \( 0 \leq \beta < 1 \).

Propositions 3 and 4 below show that this generalization leaves the qualitative conclusions of the previous analysis remain intact.

**Proposition 3.** If \( \theta \geq 1 \) and the firm chooses \( \beta \) to minimize the cost of capital, we have:

\[
c(t) = \frac{(r(1 - \eta) + \delta)}{(1 - \tau)}(1 - \chi \tau)
\]

So, if \( \lim_{t \to 1} c(\tau) \to \infty \) and \( \chi \geq \frac{(1 - \eta)r + \delta}{r + \delta} \), then the tax system is locally neutral.

**Proof:** See appendix.

**Proposition 4.**

If \( \theta < 1 \) and the firm chooses \( \beta \) to minimize the cost of capital, we have:

\[
c(t) = \frac{(\beta r(1 - \eta \tau) + (1 - \beta) \theta r + \delta)}{(1 - \tau)}(1 - \chi \tau)
\]

So, if \( \lim_{t \to 1} c(\tau) \to \infty \), then there is locally neutrality.

**Proof:** See appendix.

According to Proposition 4 (\( \theta < 1 \)) the system can be locally neutral without having any tax allowance because of the existence of other taxes which makes this simply a second best condition. However, the Corollary 4.1 below shows that if the tax system do allow for more than one tax allowances then the corporate tax rate that makes the system locally neutral (\( \tau^{ts} \)) is higher than the corporate tax rate that makes the tax system locally neutral in the case of no
tax allowances ($\tau_1$); that is, when more than one tax allowances exist we have a corporate tax rate that is greater than the second best rate.

**Corollary 4.1.** Assume that $\theta < 1$ and $\lim_{t \to 1} c(t|\beta, \eta, \chi) \to \infty$. Then we have that

$$c(t|\beta_2, \eta, \chi) > c(t|\beta_1, \eta = 0, \chi = 0),$$

where $\beta_1$ and $\beta_2$ is the proportion of capital for which the firm uses debt as a source of finance without and with tax allowances, respectively, and $\tau^{ts} > \tau^{sb}$.

**Proof.** See appendix.

Figure 2 below illustrate this result.

![Figure 2: Cost of capital and local neutrality when $\theta \leq 1$](image)

Figure 2: Cost of capital and local neutrality when $\theta \leq 1$
5. Conclusion

This paper has introduced the concept of local neutrality of tax systems. We have identified the conditions under which local neutrality of the corporate tax is feasible, showing that these conditions are likely to be satisfied in the context of tax codes used by many countries. The practical importance of the analysis in this paper is to demonstrate that, with minor modifications that do not fundamentally alter the basic structure of the most popular existing tax codes, it is possible to attain local neutrality of the corporate tax system. That is, by adjusting the parameters of the tax system so that condition (8) is satisfied, local neutrality attains. The remaining issue is to identify the precise level of the corporate tax rate that is effectively optimal. We have derived an analytical expression that may help guiding the identification of such optimal tax rate. We also show the existence of a double peril in setting the tax rate under local neutrality; inefficiency may occur if the positive tax is set either above or below the optimal rate.

Appendix

Proof of Lemma 2:

We have:

\[ c(\tau) = \frac{[r(1 - \beta \eta \tau) + \delta]}{(1 - \tau)}(1 - \chi \tau) \]

We want to find the minimum of \( c(\tau) \), so:
Solving (A1'), we have:

\[
\begin{align*}
\tau_1 &= \frac{-2\beta \eta r \chi + \sqrt{4\beta^2 \eta^2 r^2 \chi^2 + 4\beta \eta r \chi (r + \delta)(1 - \chi) - \beta \eta r}}{2\beta \eta r} \\
\tau_2 &= \frac{-2\beta \eta r \chi - \sqrt{4\beta^2 \eta^2 r^2 \chi^2 + 4\beta \eta r \chi (r + \delta)(1 - \chi) - \beta \eta r}}{2\beta \eta r}
\end{align*}
\]

But \( \tau_2 > 1 \) and this is inconsistent with a corporate tax rate, so:

\[
\tau^c = \frac{2\beta \eta r \chi - \sqrt{4\beta^2 \eta^2 r^2 \chi^2 + 4\beta \eta r \chi (r + \delta)(1 - \chi) - \beta \eta r}}{2\beta \eta r \chi} = 1 - \sqrt{\frac{1 - \beta \eta r - (r + \delta)(1 - \chi)}{\beta \eta r \chi}}
\]

Also, since \( c(\tau) \) is strictly convex it follows that \( \tau^c \) corresponds to a minimum.

**Proof of Proposition 2**

Consider the profit maximization of a firm affected by a profit tax, \( \tau \),

\[
\text{Max}_K \{(1 - \tau)F(K) - [(1 - \tau \chi)((1 - \eta \beta \tau)r + \delta)]qK} \quad \text{(A2)}
\]

Without loss of generality, by using appropriate units, we can assume that \( q = 1 \). We rewrite (A2) as,

\[
\max_K \{F(K) - (r + \delta)K - \tau [F(K) - \beta \eta r K - (r + \delta)\chi K + \beta \eta r \chi K] \} \quad \text{(A3)}
\]
The expression in square brackets in (A3) is the tax base or taxable profit. If the tax base is less than or equal to zero then the firm pays no corporate tax. Thus, if

\[ F(K) - \{ (\beta \eta r + ((r + \delta) \chi - \beta \eta r \chi \tau) K \} \leq 0 \]  
\hspace{1cm} (A4)

Then the firm is tax exempted. Inequality (A4) can be rewritten as,

\[ F(K) - \delta K - \{ (\beta \eta r + (r + \delta) \chi - \delta - \beta \eta r \chi \tau) K \} \leq 0, \]  
\hspace{1cm} (A5)

Define the profit rate as \( \pi / K \equiv [F(K) - \delta K] / K \). Thus, from (A4) it follows that the tax base is less or equal to zero if,

\[ MRR = \beta \eta r (1 - \tau \chi) + (r + \delta) \chi - \delta \]  
\hspace{1cm} (A6)

If \( \chi = 1 \) and \( \beta = 0 \) then \( MRR = r \).

The optimal tax that maximizes tax revenues without distorting the user cost of capital is the one that makes \( MRR = r \). That is, using (A6) we obtain that the optimal tax, \( \tau^* \), is defined by,

\[ \beta \eta r (1 - \chi \tau^*) + (r + \delta) \chi - \delta = r \]  
\hspace{1cm} (A7)

Solving (A7) we obtain that the optimal tax is

\[ \tau^* = \frac{\beta \eta r - (r + \delta)(1 - \chi)}{\beta \eta r \chi}. \]  
\hspace{1cm} (A8)

When \( \tau = \tau^* \) the system is neutral. So, if \( \tau < \tau^* \) the system allows for the firms to avoid paying taxes at rates above the opportunity cost of capital and if \( \tau > \tau^* \) then the tax-free rate of return is below the opportunity cost of capital and therefore distorts investments.

Proof of Proposition 3

18
First we show that if $\theta \geq 1$ then $\beta = 1$.

The firm’s problem is:

$$\min_{\beta} C(\tau | \eta, \chi) = \frac{(1 - \tau \chi)(\beta(1 - \eta \tau)r + (1 - \beta)(\theta r) + \delta)}{(1 - \tau)}$$

$$1 \geq \beta \geq 0$$

If $\beta \neq 1$, then

$$\frac{(1 - \tau \chi)(\beta(1 - \eta \tau)r + (1 - \beta)(\theta r) + \delta)}{(1 - \tau)} > \frac{(1 - \tau \chi)(1 - \eta \tau)r + \delta}{(1 - \tau)}$$

So, $\beta = 1$ yields a global minimum.

We know that $c(\tau)$ is a continuous function, then using lemma 1 and $\lim_{\tau \to 1} c(\tau) \to \infty$, we have that $\tau^* \in (0, 1)$

$$c(\tau^* | \beta, \eta, \chi) = r + \delta$$

Proof of Proposition 4

Using continuity of $c(\tau | \beta, \eta, \chi)$, $\lim_{\tau \to 1} c(\tau | \beta, \eta, \chi) \to \infty$ and the fact that $C(0, \beta_1, 0, 0) < r + \delta$, then we have $\tau^* \in (0, 1)$ $c(\tau^* | \beta, \eta, \chi) = r + \delta$.

Proof of corollary 4.1:

First we show that if $\theta < 1$, and $\beta_2 < \beta_1$ then $(1 - \eta \tau) > \theta r$ and if $\beta_2 < \beta_1$ then $(1 - \eta \tau) < \theta r$.

If the firm does not have tax deductions, then it solves the following problem:

$$\min_{\beta_1} c(\beta_1 | \tau, \eta, \chi) = \frac{(\beta_1 r + (1 - \beta_1)(\theta r) + \delta)}{(1 - \tau)}$$

If the firm wants to minimize the cost of capital after tax deduction, then

$$\min_{\beta_2} c(\beta_2 | \tau, \eta, \chi) = \frac{(1 - \tau \chi)(\beta_2(1 - \eta \tau) + (1 - \beta_2)(\theta r) + \delta)}{(1 - \tau)}$$
So, if $\beta_2 < \beta_1$, then

$$\beta_1(1-\eta r) + (1-\beta_1)\theta r > \beta_2(1-\eta r) + (1-\beta_2)\theta r$$

$$(\beta_1 - \beta_2)(1-\eta r) > (\beta_1 - \beta_2)\theta r$$

$$(1-\eta r) > \theta r$$

If $\beta_2 > \beta_1$

$$\beta_1(1-\eta r) + (1-\beta_1)\theta r > \beta_2(1-\eta r) + (1-\beta_2)\theta r$$

$$(\beta_1 - \beta_2)(1-\eta r) > (\beta_1 - \beta_2)\theta r$$

$$(1-\eta r) < \theta r$$

Also, we show that if $0 < 1$ and $0 \leq \beta_i \leq 1$ for $i = \{1, 2\}$ then,

$$c(\tau|\beta_2, \eta, \chi) < c(\tau|\beta_1, 0, 0)$$

Suppose that,

$$c(\tau, \beta_2, \eta, \chi) > c(\tau, \beta_1, 0, 0)$$

Then

$$\frac{(1-\chi r)(\beta_2(1-\eta r) + (1-\beta_2)(\theta r) + \delta)}{(1-\tau)} > \frac{(\beta_1 r + (1-\beta_1)(\theta r) + \delta)}{(1-\tau)}$$

$$\beta_2 > \beta_1 \left( \frac{(1-\theta)}{(1-\chi r)(1-\eta r) - \theta r (1-\tau)} \right) + \frac{\tau \chi \theta}{(1-\chi r)(1-\eta r) - \theta r (1-\tau)} \quad (A9)$$

But if $\beta_2 > \beta_1$ then $(1-\eta r) < \theta r$; hence from (A9) we have that $\beta_2 < 0$, but this is a contradiction.

Then using Proposition (4) it follows that $\tau^{ts} > \tau^{sb}$.
References


