A Combinatorial Heuristic Approach for Solving Real-Size Machinery Location and Road Design Problems in Forestry Planning

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The location and operation of harvest machinery, along with the design and construction of access roads, are important problems faced by forestry planners, making up about 55% of total production costs. One of the main challenges consists of finding a design that will minimize the cost of installation and operation of harvest machinery, road construction, and timber transport, while complying with the technical restrictions that apply to the operation of harvesting equipment and road construction. We can model the network design problem as a mixed-integer linear programming problem. This model is fed with cartographic information, provided by a geographic information system (GIS), along with technical and economic parameters determined by the planner. We developed a specialized heuristic for the problem to obtain solutions that enable harvesting economically profitable volumes at a low cost. This methodology was programmed into a computer system known as PLANEX and is being applied in nine forestry companies that report important benefits from its use.

Subject classifications: natural resources; forestry; discrete location; heuristics.
Area of review: OR Practice.

1. Introduction

The location of harvesting machinery and the construction of access roads are important decisions that must be taken as part of the forest-harvesting processes. The location of machinery, tractors, and towers determines the volume of timber that can be extracted, as well as part of the costs of harvesting. The construction of road networks connecting the machinery with existing roads to remove the timber constitutes an important part of overall costs. The planner looks for a location for the machinery and a road network design that will minimize total harvest costs. This problem, which we shall call machinery location and road design (MLRD), has traditionally been analyzed manually using maps. This is a slow, difficult process that only allows for the evaluation of a very limited range of possibilities.

Forestry companies have installed geographic information systems (GIS) that represent physical locations through databases that visually display the information. A GIS digitally represents topographical information, existing roads, and standing timber, among other data. The introduction of GIS has allowed the development of new systems capable of backing decision-making processes regarding spatial problems, including MLRD.

Current literature deals with this problem. Twito et al. (1987) describe PLANS, while Clement (1990) and Gordon et al. (1995) describe PLANZ. The former was developed by the U.S. Forest Service and the latter by its counterpart in New Zealand. These systems interact with a GIS and basically function as simulators. They take a machine with a given location, then calculate the area the equipment is capable of harvesting and the route that connects it to the harvesting area exit, using an existing road network. These systems evaluate the location of machinery but they do not provide the best equipment assignment or the best road network, nor can they generate additional alternatives to those proposed by the planner.

This paper offers a new approach toward dealing with the MLRD problem, based on a heuristic resolution of the underlying mixed-integer programming problem. The Industrial Engineering Department of the University of Chile, in cooperation with Oregon State University and a group of forestry firms, coordinated by Fundación Chile, implemented this methodology in a computer system known as PLANEX, which is currently operating in nine forestry firms. The planner can incorporate several parameters, including the technical characteristics of the machinery's operation, road construction, transport and harvest costs, exit points, and economic variables that restrict the harvest. The planner has the option to designate segments of roads and barriers that the paths cannot cross. The topographical information, timber volumes, and the existing road networks originate from the GIS. The methodology of analysis consists of dividing the terrain under study into small 10×10 meter cells, calculating and storing the topographical information, the volumes of timber available, and other relevant data for each cell.

We model MLRD using mixed-integer linear programming, thus obtaining a large-sized network design model, which uses the information stored in the cells. By applying a heuristic approach, especially designed for the model, we have obtained approximate solutions that allow for harvesting economically profitable volumes at a low cost. Forestry firms report significant savings from using this methodology, which have been evaluated to be around 15% to 20% of operating cost, as detailed in §6.

According to the information available, this approach is the most advanced of its kind and is the only one that provides an automatic solution to the problem. Furthermore, the approach is of interest in that it shows how optimization techniques can be empowered by GIS technology to improve companies' efficiency.

The following section describes in detail the MLRD problem. Sections 3 and 4 show the modeling methodology, alongside a mathematical optimization model that represents the problem we seek to solve. We then describe the heuristic developed to solve the problem. The results and benefits the companies have obtained through the application of this methodology are shown in §6. Finally, we present the conclusions in §7.

2. Description of the Problem

The harvesting operation involves a series of steps. First, standing trees are felled in a planned sequence. Once the trees have been felled, they are transported to a storage center (log landing) where they are loaded onto a truck and transported to processing centers. We are concerned here with hauling the trees to the storage centers. In flat areas, ground equipment such as crawler tractors or rubber-tired skidders are preferred. Logs are either pulled to the tractor by cable or the tractor moves to the logs and grabs them with a grapple. The logs are then dragged (skidded) to roadside and piled at the log landing. On steep slopes or fragile terrain, where tractors cannot operate, towers with aerial cables are used. The towers and log landings are located on high points. A traction cable (mainline) pulls a carriage with attached logs along a suspended cableway (skyline) up to the landing where the logs are released for temporary storage. The logs are subsequently loaded onto trucks, taking them to their destination. Occasionally the towers are located at lower positions and logs are dragged downhill.

Aerial cable operations, which have higher installation and operation costs than tractors, generally cover larger areas and thus require a smaller road network. The design of the road network that allows access to machinery takes into account distances, slopes, soil removal, and the minimum turn radius required by the transporting vehicles. Therefore, the main decisions to be taken by the planner are: (i) tower locations; (ii) areas harvested with skidders or with towers, taking into account the technical conditions required by the machinery; and (iii) the road network that provides the harvest machinery with access, enabling the wood to be transported from the field.

The goal is to locate the harvest machinery and design the road network at the lowest possible cost, while reducing the environmental impact upon the harvested terrain. Production costs include machinery installation and operating costs, the cost of road construction, and the cost of moving timber outside the harvesting area. Parts of the harvesting area may be left unharvested if the production costs per cubic meter exceed the preestablished ceiling.

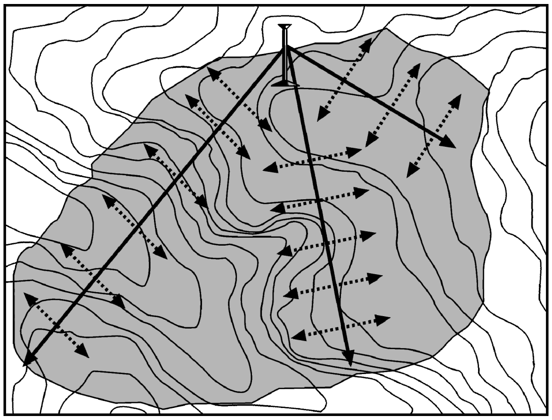
Harvesting and road network construction account for around 55% of total production costs, thus illustrating the importance of the problem under study. The costs of building the road network, prorated for volumes produced, usually range from \$3.50 to \$4.00/m^3, while the harvesting costs range from \$5.00 to \$9.00/m^3. Transport costs, which are approximately \$0.10/m^3 per kilometer, are less relevant in this case because only traffic inside the harvesting zone is considered. The costs quoted in this paper are representative of the Chilean forestry industry and resemble those of other countries with a dynamic forestry sector and similar topography, such as New Zealand, South Africa, and the northwestern United States.

The following provides a detailed description of the technical factors involved in installing and running harvesting machinery and designing access roads.

2.1. Harvesting with Aerial Equipment

Towers are mainly used to harvest areas with steep slopes, usually over 30%, where tractors are unable to operate. They are also used to harvest in fragile terrain to avoid the ground compaction caused by tractors. The tower consists of a structure about 10 meters high and two cables (skyline and mainline) of varying lengths, which are powered by an engine. The skyline cable is anchored to the lower part of the harvest area while the tower is usually located on

Figure 1. Aerial cable harvesting pattern.

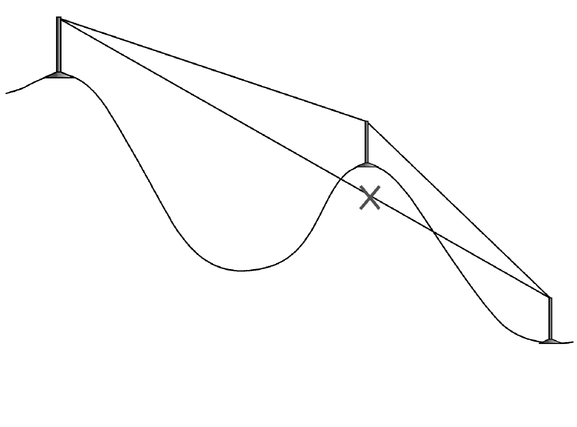


higher ground. The felled logs on the slope are attached to the mainline cable, which is used to pull the logs to the carriage and then pulls the carriage along the skyline to the landing. The mainline cable has a lateral reach of about 30 meters from the skyline. After all logs with lateral reach of the skyline have been yarded to the landing, the skyline cable is then anchored to a new point and the process is repeated. This process, shown in Figure 1, enables safe harvesting on very steep slopes in a circular pattern.

The tower is located in a position that is flat enough to provide for a landing area, where logs are unloaded, stored, and then loaded onto trucks that will move them out. These storage areas must be accessible via the road network.

The area covered by a tower corresponds to the area it can harvest. The cable yarding distance typically ranges from 300 to 1,000 meters and restricts the area covered by the tower. On the other hand, topographic conditions such as rivers or hills may affect the yarding distance in some directions, decreasing its capacity. An intermediate support—that is, a jack supported by trees—can allow the skyline to be maintained at a suitable height, increasing the load capacity of the skyline, accommodating topographic features such as hills, and thus increasing the yarding distance and the area covered by the tower; see Figure 2.

Figure 2. Use of intermediate supports helps to avoid a hill.



The most common operation carried out using towers is uphill yarding. It is also possible to use an auxiliary cable to operate in flat areas to pull the empty carriage back to the logs. Downhill harvesting is less frequent, due to increased potential for soil erosion as well as safety issues. The costs of harvesting using aerial equipment vary according to the type of operation, fluctuating between \$7.50 and \$9.50/m^3.

2.2. Harvesting Using Ground-Based Equipment

Tractors used to transport logs to the road edge are suitable for harvesting on flat and nonfragile areas. Topographical conditions limit use of this equipment. Tractors are inadequate for operating on fragile ground because they compact the soil, which brings with it additional costs to repair the soil damage to guarantee rapid growth of the next tree generation. Similar damage occurs when tractors operate on steep terrain. Mobility of loaded tractors is limited for moving uphill. Moving downhill on steep slopes is dangerous and can also lead to increased erosion and soil disturbance. Moreover, tractors may overturn if the lateral slope is too pronounced.

The operating costs of transport depend largely on the distance logs must be carried to reach the road. In practice, this limits the area covered by this type of equipment. The slopes encountered by equipment while operating also affect costs.

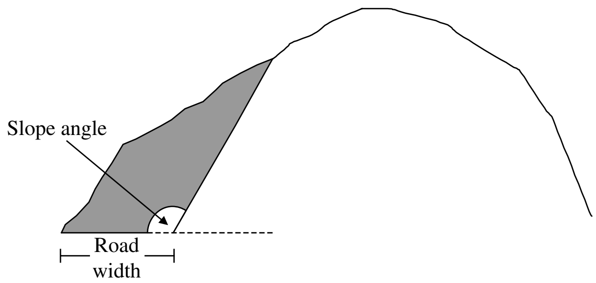
In addition to tractors and rubber-tired skidders, there are also other types of ground-based equipment, including forwarders (light tractors) and animals (oxen, mules, and horses). Ground-based equipment costs less to operate than aerial equipment on appropriate terrain, ranging from \$5.00 to \$7.50/m^3.

2.3. Road Design

Chilean forests, like many of the world's forests, are mainly located in mountainous areas with large topographic variation, making the design of access roads for harvesting equipment quite complex. Location of roads must consider the following technical conditions: (i) maximum slope, which may vary depending on whether the truck is going uphill or downhill; (ii) minimum turn radius necessary for trucks to turn, which rules out very tight curves and tight switchbacks; and (iii) earthwork when the road crosses hillsides. To avoid cut slope failure, the earth cannot be removed vertically and must maintain an angle that guarantees the stability of the hill. The road width and the slope angle, as shown in Figure 3, determine the volume of soil to be removed, which has a significant impact on road construction costs. In very steep areas, the excavated soil must be transported to a disposal site because placement of the excavated soil on the lower side of the road would be unstable.

Taking technical conditions into consideration, the objective is to build a road network that provides access to harvest machinery at the lowest possible cost. The analysis

Figure 3. Earthwork.



includes the construction cost and the cost of transporting the wood from the landing area to the exit from the harvesting area. When existing roads are used no extra construction costs are incurred, although improvement costs may apply—for instance, improving a dirt road and upgrading the surface to gravel. An optimum road design not only means significant savings for forestry firms, but also constitutes a positive environmental impact.

3. Modeling Methodology

Our approach consists of discretizing (rasterizing) the terrain under study by dividing it into 10×10 meter square cells. We associate spatial coordinates (x-y-z) with each cell. The coordinates (x-y) refer to the center of the cell, while the z coordinate refers to its height, as we explain in the next section. The planner identifies exit cells to which the timber is transported. The physical and economic aspects of the problem are defined as attributes of the cells and the relationships between them. We conceptualize the problem as a network where the nodes represent the cells and the arcs are the road segments or harvesting options joining a machine location cell with a harvested cell. From this perspective, the problem aims to optimize a network design.

The volume of wood to be harvested in the cells is obtained from a GIS. We also need to calculate cell height and slope. This information is essential to establish the feasibility of building a road segment or calculating the number of cells available for harvesting by equipment located in a base cell.

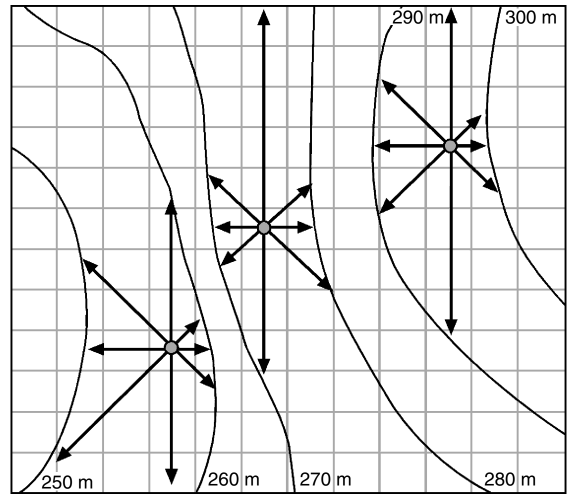
Moreover, to incorporate the minimum turn radius for trucks, we relate various road vertices to each cell to identify the cell from which we access a road segment, and eliminate turns that are too tight for a truck to negotiate.

The financial limits of the operation are incorporated as a maximum cost per cubic meter for each cell. This includes operating costs and prorated fixed costs, roads, and landing areas. The economic restrictions differ depending on whether we use ground or aerial-based equipment.

3.1. Digital Terrain Model

To calculate the height of each cell, we use the contours that represent the altitude of the terrain. First, we calculate

Figure 4. Approximate height of each cell with nearest contour line.



the distances between the cell and the nearest contour in eight different directions, as shown in Figure 4. The height of the cell in question is calculated as the average of these eight height measurements, weighted inversely to the distance to the cell.

To calculate the slope of the cell, we used its height, as well as that of the neighboring eight cells. Then, we calculate plane S, which is the best suited to these nine points, using least squares. The method determines the plane that minimizes the sum of the square difference between the height of the nine cells and the height the plane predicts for each cell. The slope of the cell is determined by $\hat{n}$, normal vector to the plane S, as shown in Figure 5.

3.2. Feasible Road Networks

Each cell can connect to its neighboring cell via an undirected arc that represents a road segment. The gradient of the road and the slope of the cells, given that soil removal

Figure 5. Approximate tangent plane calculated by least squares.

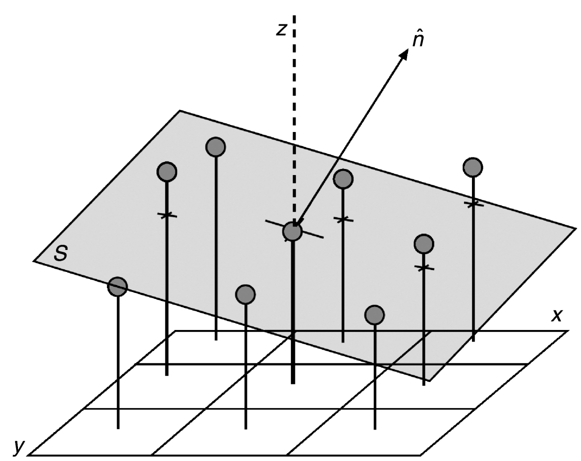
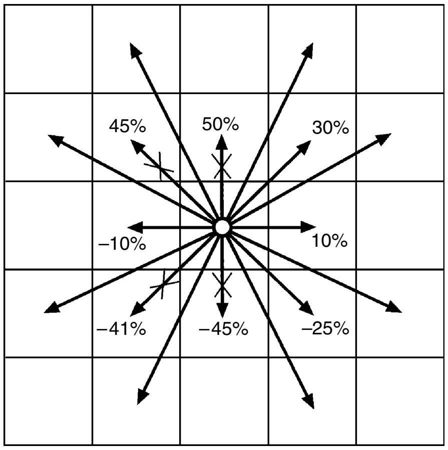


Figure 6. Generating a feasible network when restricted by a maximum 40% gradient.



may be excessive, determine the feasibility of constructing that arc.

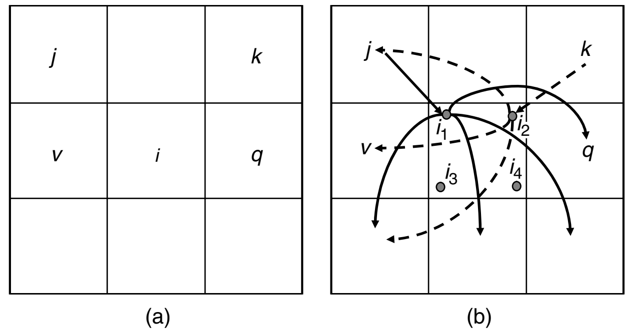
Restrictions on maximum gradients are directly incorporated into this model. For an arc that joins the $\{i, j\}$ cells the gradient is equal to $(h_j - h_i)/d_{ij}$, where d_{ij} is the distance between the cells on the horizontal plane and (h_i, h_j) represents the height of the respective cells.

The $\{i, j\}$ arc is included in the feasible road network if the absolute value of the gradient is less than the maximum gradient specified by the planner. In some instances slope conditions are difficult, and few of these eight defined directions allow feasible connections for the cell. In these cases, additional directions are generated connecting to cells one layer away. Figure 6 shows this calculation by displaying 16 possible directions for connecting one cell. We define this network as the set of cells V and the set of undirected arcs E^V. The road network is undirected because the construction of the segment enables timber to flow both ways.

The flow in this network presents some difficulties for the incorporation of the minimum turn radii required by the transport vehicles. As shown in Figure 7(a), if we get to cell i from cell j, the arc toward cell q is feasible in terms of the required turn radii, but this is not the case with the arc toward cell v because it would produce too tight a curve. On the other hand, if we get to cell i from cell k, the direction toward cell v is feasible, although this is not the case when moving toward cell q. The example shows that the flow feasibility in the $\{i, v\}$ and $\{i, q\}$ arcs depends on the direction from which the flow reaches cell i, which cannot be established using this representation.

To solve this problem, we lay out a new extended network, which will allow us to naturally incorporate this restriction. We associate a number of vertices to each cell, one for each direction from which the cell can be reached, as shown in Figure 7(b) for cell i. If we get to cell i from cell j, arcs from all vertices of j $(j_1, j_2, \ldots)$ are associated with vertex i_1. The arcs pointing to the cells that can

Figure 7. The original cell configuration (a) leads to a feasible road network (b) when taking minimum turn radii into account.



Note. Several nodes are associated with each cell.

be accessed from vertex i_1 are shown as an unbroken line, including the arc that points to cell q, although not to cell v. If we reach cell i from cell k, we get to vertex i_2, from which we can reach the cells that point to the arcs shown as dotted lines, including cell v, but not cell q.

We use this network to determine the timber flows along the roads, so we therefore require a directed representation. We characterize this network by the node set N, with a maximum of 16 vertices associated with each cell because of its feasible directions, and E^N, the set of directed arcs.

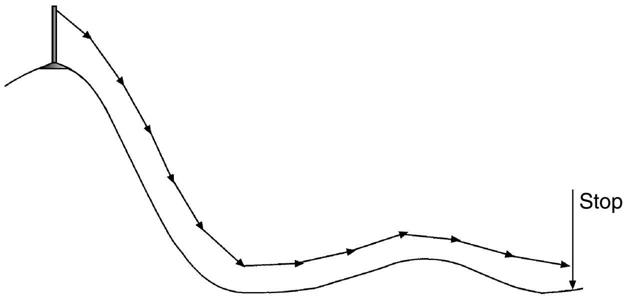
The planner may alter the potential road network using road barrier or priority roads options. Barriers are lines that the timber flow cannot cross and they therefore eliminate arcs from the potential network. This is a useful option for modeling rivers or other barriers that roads cannot cross. Priority roads are segments to be incorporated without needing to verify their technical feasibility, adding arcs to the potential network. This is a useful option for adding road segments that the planner considers both suitable and technically feasible.

The cost of constructing segment $\{i, j\}$ depends on the type of road surface (soil or gravel), the length of the arc, the width of the road, and earthwork. The cost of the road surface is proportional to the length of the arc and width of the road, while the cost of the earthwork is proportional to the volumes of the excavation. The cost of transport per flow unit is proportional to the distance and may differ depending on whether it is uphill or downhill. Calculating the earthwork of a segment of the road is somewhat intricate. It is based on a topographical analysis of the cells that make up the segment, using the tangent planes as explained in detail in Epstein et al. (2000).

3.3. Scope of Harvesting Equipment

The planner provides all the potential locations for the towers, expressed as grid cells. To calculate the area covered by a tower, we determine the maximum reach by the equipment in multiple directions. The scope of the tower is the polygon defined by the maximum radial coverage; see Figure 1. There are several criteria for determining where a

Figure 8. End of line determined by maximum skyline cable length.

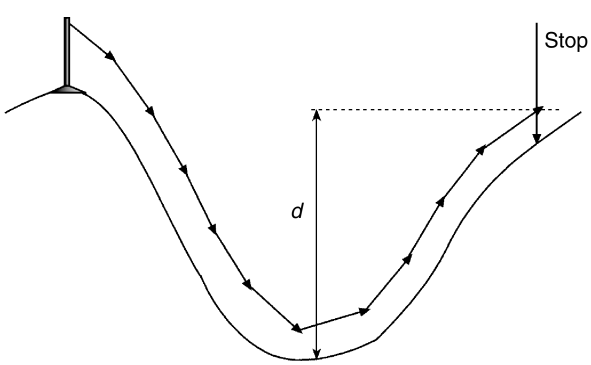


line stops: First, when the line reaches the maximum harvesting distance defined by the length of the skyline, as shown in Figure 8. Second, when the difference between the height of the line and the lowest point is greater than a given value, as shown in Figure 9. Third, when the cable has been laid out too flat, preventing the carriage from moving. This typically can happen in areas with small slopes, where tractors are more suitable.

The cost of harvesting a cubic meter of a cell depends on the type of tower and its location. The anchoring of the equipment and the construction of the landing area determine the cost of installing a tower.

The procedure for determining the area covered by ground-based equipment is different. Ground-based equipment is limited by prescribed tangential and perpendicular gradients. We begin at the base cell where the equipment is located and begin to inch forward, taking into account the tangential and perpendicular slopes until we are further from the cell base than the maximum distance established by the planner. All cells visited should be accessible to the equipment in the base cell. However, the planner could decide that a zone should not be harvested, for instance, to protect a special site, limiting the area covered by the equipment. The base cell is acceptable if the volume harvested is greater than the minimum established by the operations policies. We evaluate all the cells as if they were potential base cells.

Figure 9. End of line determined by a height difference of value d between the end of line and its lowest point.



The cost of harvesting a cubic meter from a cell is a function of the ground-based equipment, the distance between the harvested cell and the base cell, and the intermediate gradients in the distance traveled by the equipment. There is also an installation cost for each landing area, which depends on the type of ground-based equipment selected.

4. Mathematical Model

The following mixed-integer linear programming model represents MLRD as an uncapacitated network design problem. This problem is NP-hard, containing the Steiner Tree Problem as a special case. We now present one formulation for the problem under study.

Sets and Parameters

V Set of cells.

N Set of road vertices.

E^V Set of undirected road arcs on set of cells V. They represent the potential road segments that can be built.

E^N Set of directed road arcs on the set of road vertices N. They represent the pairs of vertices between which timber may be transported. This representation takes into account the turn radii, given that set N implicitly considers the cell, which provides access to the segment.

$N^{V(i)} \subset N$ Set of road vertices related to cell $i \in V$.

$c(n) \in V$ Cell related to road vertex $n \in N$.

$X \subset V$ Set of harvesting area exit cells. The timber exits the area under study on arrival at these points. One of these is a dummy exit, where timber that is not profitable for harvesting or is technically unfeasible is sent.

Q Set of machine types.

$V^q \subset V$ Set of cells where q-type machinery can be located.

$G_i^q \subset V$ Set of cells that can be harvested from cell i with q-type machine.

$H_j^q \subset V^q$ Set of cells where q-type machinery can be located so that cell j is accessible.

v_i Timber volume in cell i.

$D = \sum_{i \in V} v_i$ Total demand volume in area under study.

p_e Cost of constructing road arc $e \in E^V$.

a_i^q Cost of installing q-type machinery in cell i.

b_{ij} Cost of transport per volume of timber in arc$(i, j) \in E^N$.

g_{ij}^q Cost per volume of timber harvested in cell j from cell i using machine q.

Variables

x_e One if road segment $e \in E^V$ is constructed, zero if not.

y_i^q One if q-type machinery is located in cell $i \in V$, zero if not.

w_{ij}^q Volume of timber harvested in cell j from cell i using q-type machines.

f_{ij} Timber flow in arc$(i, j) \in E^N$.

h_{in} Timber flow from cell $i \in V$ toward the road vertex $n \in N$ in the same cell.

t_n Timber flow from the road vertex $n \in N$ to the related exit cell. The variable exists only when $c(n)$ belongs to X.

Objective Function

The objective is to minimize costs of road construction, machinery installation, harvesting, and transportation:

$$\min \sum_{e\in E^V} p_e x_e + \sum_{q\in Q}\sum_{i\in V^q} a_i^q y_i^q + \sum_{q\in Q}\sum_{i\in V^q}\sum_{j\in G_i^q} g_{ij}^q w_{ij}^q + \sum_{(i,j)\in E^N} b_{ij} f_{ij}.$$

Restrictions

(1) Flow balance in each road vertex:

$$\sum_{k\in N:\,(n,k)\in E^N} f_{nk} - \sum_{k\in N:\,(k,n)\in E^N} f_{kn} - h_{c(n),n} + t_n = 0 \quad \text{for all } i\in V,\ n\in N^{V(i)}.$$

(2) All the timber must be harvested or be accounted as unreachable or economically unfeasible for harvesting:

$$\sum_{i\in X}\sum_{n\in N^{V(i)}} t_n = D.$$

(3) The timber that is transported from one cell to an associated road vertex must equal the timber harvested from that cell:

$$\sum_{n\in N^{V(i)}} h_{in} - \sum_{q\in Q}\sum_{j\in G_i^q} w_{ij}^q = 0 \quad \text{for all } i\in V.$$

(4) To harvest from cell i using q-type machinery, the machine must be installed:

$$\sum_{j\in G_i^q} w_{ij}^q \leqslant \left(\sum_{j\in G_i^q} v_j\right) y_i^q \quad \text{for all } q\in Q,\ i\in V^q.$$

(5) Timber volume restricts the harvest:

$$\sum_{q\in Q}\sum_{i\in H_j^q} w_{ij}^q \leqslant v_j \quad \text{for all } j\in V.$$

(6) To transport timber along a road, the latter must be built:

$$f_{ij} \leqslant D\cdot x_e \quad \text{for all } (i,j)\in E^N,\ e=\{c(i),c(j)\}.$$

(7) Nonnegativity and integrality of the variables:

$$w, f, h, t, r \geqslant 0,$$

$$x, y \in \{0,1\}.$$

The model allows us to harvest timber up to a maximum cost. With this in mind, we use a dummy exit to which timber that is unprofitable for harvesting or technically unreachable is sent. All cells are connected to this dummy exit with a single transport cost equal to the maximum harvest cost set by the planner.

A typical large problem would have 1,000 hectares with 75,000 timber cells. Without considering the turn-radii constraints and depending on the geographical conditions, we could have about 400,000 potential road segments and 100,000 road vertices for such a problem. Under normal considerations, the number of potential locations could be 300 for the towers and 5,000 for the tractors. The flow-balance constraints, Type (1), add one restriction for each road vertex, while the capacity constraints, Type (6), add one restriction for each road segment. The continuous-flow variable f, and the binary road design variable x, account for the majority of the decision variables. In a case like this, the model will end with approximately 0.6 million constraints and 1.7 million variables. While the LP relaxation of this size is solvable with current commercial codes, the integer problem is difficult to solve. At the end of §5 we report our results on test instances that confirm this observation. If we add the turn-radii constraints (see §3.2), the problem becomes considerably larger, making it even more difficult to obtain good solutions in reasonable time using an exact approach.

Diaz et al. (2005) solve problem instances up to 500 hectares using CPLEX 8.1 in a reasonable time. Problem-specific solution approaches have also been proposed. In Vera et al. (2003), a Lagrangian relaxation approach decomposes the problem into two basic components, and both subproblems are strengthened to obtain better solutions. The first component is a plant location problem, where the machines are the plants and timber cells act as customers if they can be reached from a machine. The second component is a fixed-charge network flow problem, involving road building and timber-flow variables, where the capacities are much larger than actual flows. In this case, the problem is known to be difficult to solve; see Balakrishnan et al. (1989) and its references.

A second approach by Epstein and Olivares (2001) uses a flow decomposition formulation, where timber flows are identified by origin, in a dual-ascent algorithm. This approach leads to better results than the first one, and both are somewhat superior to the straight use of a commercial code, but neither is able to solve more than medium-size problems.

From these experiences, we determined the need to develop a heuristic approach to solve real-size instances, which is detailed in the next section.

5. Model Solution

We have developed an algorithm similar to the "minimum cost routes" heuristic proposed by Takahashi and

Matsuyama (1980) to identify minimum Steiner Trees with a guaranteed error factor of two. While their algorithm selects at each step, among all the pairs of leaf nodes, the arcs on the shortest path of the nearest pair of such nodes, our method calculates road-building shortest paths between machine locations and exit cells, rather than every pair of timber cells. Thus, their theoretical bound error does not apply to our problem because of the flow timber costs and the existence of directed arcs.

In each iteration, our method installs equipment in a feasible location according to minimum-cost criteria per cubic meter of timber harvested and transported out of the harvesting area. The cover radius of the equipment determines the volume to be harvested. The costs include equipment installation and operation, along with the costs of building additional roads to connect the equipment to the road network and the costs involved in transporting timber out of the harvesting area. A minimum-cost route algorithm, taking into account the construction and transport costs, determines the optimum route that would connect the equipment to the exit. The heuristic selects the equipment that carries the lowest cost per volume of wood evacuated. It also updates the existing roads; adding the new segments to connect the selected equipment. The harvested cells are no longer eligible. The algorithm repeats this process as long as there is timber to be harvested and the average minimum cost does not exceed a value previously defined by the planner. The heuristic may leave part of the wood unharvested due to technical obstacles or for economic reasons. Figure 10 presents the algorithm in a schematic format, with some simplifications to allow for better understanding.

In the dynamics of the planning process, one or two forest engineers carry out the analysis, usually in one session lasting a couple of hours. The planners will test a series of scenarios with different parameter sets, such as harvesting cost or road slopes. In addition, they test alternative harvesting policies, such as the level of use of cable logging in fragile areas. Therefore, we need an algorithm that runs in a few minutes to be able to use the optimization methodology in a real planning context where several tests are performed on each instance.

When we incorporate the turn-radii constraint to our typical 1,000 hectare problem with 75,000 timber cells and 400,000 potential road segments, we obtain the much larger extended network with 1.5 million road vertices and 5 million potential road segments, as explained in §3.2 and Figure 7. Therefore, the practical efficiency of the algorithm depends on how Step 1 is implemented. For the algorithm to run in a few minutes, the calculations in Step 1 must take advantage of information obtained in previous iterations, especially regarding the coverage of the equipment and the optimum routes connecting the potential locations to the harvesting area exit in the best possible way.

To calculate in an efficient way the volume harvested by a machine in a given location, called A in Step 1(ii), we make an initial coverage analysis, which calculates the cells that are accessible for each type of machine and for each potential location. This information, which is independent of the timber volume in each cell, is stored in arrays. When a machine is installed in Step 4, the volume from these cells is fixed to zero for future iterations, as it will not be available any longer. Then, the calculation of A reduces to going through the array of accessible cells and adding the timber volume.

Figure 10. Heuristic algorithm based on the shortest routes.

Step 0. We assign the volume of timber in each cell. Initially, the road network is made up of existing roads.

Step 1. For each type of machine and for each potential location we calculate:

(i) The cells that could be harvested.
(ii) The volume of timber in these cells, which we refer to as A.
(iii) The cost of installing machinery in the location, which we refer to as B.
(iv) The cost of harvesting timber in the cells, which we refer to as C.
(v) The cheapest road connecting the machinery to an exit, ensuring the road complies with slope and earthwork restrictions. Moreover, the timber flow must comply with the minimum turn radii. The cost of each road arc is considered to be equal to the cost of arc construction, if building is required, plus the harvested timber flow cost. To comply with the turn radii, we use the extended network.
(vi) The cost associated with building road segments on the previously established route, which we refer to as D.
(vii) The cost of transporting timber from the machine to the harvesting area exit, which we refer to as E.

Step 2. For each alternative evaluated in Step 1, we calculate the average cost of the operation, which is equal to $(B+C+D+E)/A$.

Step 3. If the average minimum cost is less than or equal to the maximum cost approved by the planner, go to Step 4. If not, END.

Step 4. The machinery is installed in the location that guarantees the minimum average cost. We assign zero value to the harvested cells, which means that future machines will not be able to harvest the cells again. We also build the road segment on the optimum route, enabling the segments to be used in successive iterations without paying the cost of construction. We update the laid-out network used to calculate timber flows. Go to Step 1.

The efficient calculation of road building and transportation costs is more complicated. First, we connect all exit nodes to an artificial node. These connection arcs have no road-building or transportation cost. We calculate the shortest path from the artificial node to the potential machine locations, based on the road-building cost, using an implementation of Dijkstra's algorithm. The complexity of this operation (see Ahuja et al. 1993) is equivalent to that of calculating one shortest path. We obtain as a result the tree of shortest paths, which is the base to calculate road-building and transportation costs for the potential machine locations.

The road-building cost, called D in Step 1(vi), is obtained directly from this tree. The transportation cost, called E in Step 1(vii) is calculated by multiplying the timber volume, the distance in the shortest path tree to the

Table 1. Problem description.

	Nodes				Arcs				
Problem	Exit	Timber	Roads	Machine	Roads	Machine—road	Machine—timber	Total nodes	Total arcs
S (2 ha)	1	25	8	9	13	9	112	43	134
SM (10 ha)	1	1,016	14	10	17	10	1,917	1,041	1,944
M (40 ha)	1	3,635	106	63	154	63	16,081	3,805	16,298
Lnr (210 ha)	1	3,843	264	238	364	238	22,961	4,346	23,563
L (210 ha)	1	21,001	264	238	364	238	74,243	21,504	74,845

artificial node, and a unit transportation cost defined by the planner.

When a machine is installed in Step 4, the arcs of the associated route are considered as incorporated and their road-building cost is fixed to zero for future iterations, as they will be available at no cost. In addition, the labels of the visited nodes are recalculated, which allows the new tree of shortest paths to be determined using Dijkstra's algorithm and the previous solution as a starting point. Because modifications are minor in each iteration, using the previous solution increases efficiency considerably. The determination of the optimal tree of shortest paths is approximate, as the cost of transportation is not included. This simplification leads to a much faster algorithm. Considering transportation costs in the determination of this tree would add significant complexity, requiring the calculation of a shortest path from each potential location, because different volumes of timber from cells would lead to different transportation costs. This simplification does not significantly distort the problem, as the road-building costs are an order of magnitude larger than the transportation costs to the artificial node. Obviously, when evaluating and comparing the alternatives, the transportation cost on the road network is added.

To validate this heuristic solution approach, we carried out tests for a set of five small to medium-sized problems. Table 1 shows the characteristics of each problem, in terms of physical parameters and dimensions of the models. Real instances are significantly larger than these tests. We solved these problems using the MIP formulation described in §4 on a PC with a 2.4 GHz Pentium 4 processor running CPLEX 8.0 with the default settings and a processing time limit of three hours. While problems S and SM were solved to optimality in less than a minute, CPLEX found solutions with relatively large optimality gaps for the three larger instances after the three hours. Also, the solutions found by CPLEX were similar or inferior to the ones obtained by the heuristic. We note that the heuristic method needed less than a minute to solve any of these instances, including the largest one. To evaluate the quality of these solutions, we calculated a dual bound to the problems using the approach described in Epstein and Olivares (2001). As a result, we see that for the medium-size problem M the heuristic solution has a guaranteed error of 4.7%. The guaranteed errors for the larger problems Lnr and L are 3.7% and 3.5%, respectively. Table 2 shows these results.

6. Application and Results

We have applied this methodology in the PLANEX system that runs in Windows on a PC. PLANEX aims to solve problems involving up to 1,000 hectares in about two minutes on a 2.4 GHz Pentium 4 computer. The Chilean forestry firms Bosques Arauco, Forestal Bío-Bío, Forestal CELCO, Forestal Copihue, Forestal Millalemu, Forestal Mininco, Forestal Monteaguila (Shell group), and Forestal Valdivia have been using this system since the mid-1990s. It is also used by Cartón de Colombia, the Colombian branch of the Irish firm Smurfit.

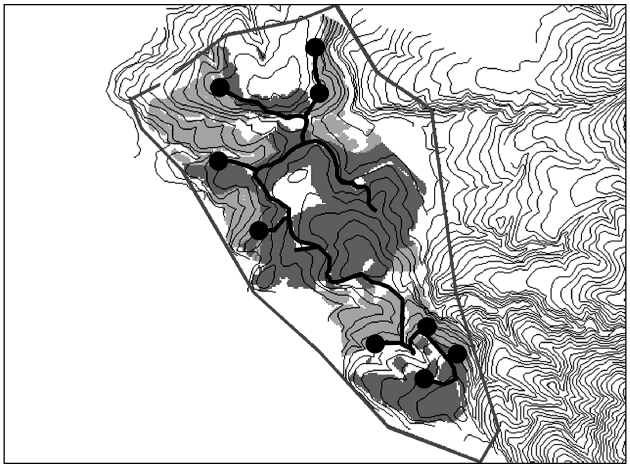
The user interface displays the information graphically and includes elements such as: (i) tower location; (ii) area harvested by aerial equipment; (iii) area harvested by ground equipment; (iv) unharvested areas; (v) new roads constructed; (vi) old roads used; and (vii) old roads not used. Figure 11 shows an example problem solution as displayed by the system.

The system also provides a written report of the solution, which includes the following information: (i) tower locations, using georeferenced coordinates; (ii) average harvest costs for the different equipment types, area, and volume harvested; (iii) volume of timber not to be harvested due to

Table 2. Numerical results for the comparison of CPLEX and the heuristic solutions.

Problem	CPLEX solution (B&B gap)	Dual bound	Heuristic solution	Comparison CPLEX vs. heuristic (%)	% gap (%) Heuristic vs. dual	% gap (%) CPLEX vs. dual
S	4,723 (0%)	4,723	4,723	0.00	0.00	0.00
SM	28,824 (0%)	28,824	28,832	−0.03	0.03	0.00
M	73,211 (29%)	69,935	73,246	−0.05	4.73	4.68
Lnr	646,935 (19%)	610,802	633,277	2.16	3.68	5.93
L	647,515 (18%)	610,703	632,349	2.40	3.54	6.03

Figure 11. Graphic solution.



Note. The darker zones represent areas harvested by ground equipment, while the lighter shades represent areas harvested by aerial equipment. The thick lines represent the roads constructed, while the dark spots represent tower locations.

high economic cost and difficult access; (iv) cost of road construction, including new roads and the improvement of existing roads; (v) cost of transporting timber; and (vi) total volume harvested and average total costs.

After identifying the first heuristic solution, the planner aims to improve it, either by altering the current solution or evaluating a completely new alternative. The options for carrying out this search are modifying the technical and economic parameters of the plan doing a sensitivity analysis, or adding guides to the solution, such as forcing the location of some of the towers, proposing new road segments, or introducing new barriers that roads cannot cross. In the case of the latter, the planner provides a partial solution and the system automatically completes it. The planner can then optimize the problem once more and compare both solutions in terms of operations and costs. This ability to generate, save, and evaluate different alternatives in an objective and quick fashion has proved to be a key to the system's success.

Manual planning tends to use the existing road network, without adequately evaluating the high costs of repairing them and making them useful. There is also a tendency to maximize the area harvested by ground equipment because the variable harvest costs are lower than those of aerial equipment, but this practice does not take into account the increased investment in roads. Furthermore, manual planning does not rigorously consider the technical restrictions regarding road gradients and equipment coverage. Manual planning is unable to suitably balance the marginal costs of harvest and road building. The planner must follow a single line of preestablished logic to solve the problem. In practice, it is too cumbersome to generate different alternatives and to evaluate them quantitatively to make an optimal selection.

Using this methodology has shown the following advantages: (i) tendency to use fewer roads, with economic and environmental benefits; (ii) the solutions effectively lead to lower total costs; and (iii) solutions comply absolutely with technical restrictions, such as road gradients and the area covered by the equipment. The system allows the planner to look for new ways to approach the problem, enhancing the planner's perspective when searching for better solutions. Furthermore, the system provides complete statistics for each solution, simplifying the process of comparing various alternatives. In contrast to manual planning, which requires several days to construct and evaluate one alternative, the computerized system allows the planner to evaluate multiple alternatives in a few hours. Moreover, the planner can concentrate more on the most difficult instances.

The impact of PLANEX on the planning procedures of forestry firms can be seen, as an example, in the Chilean company Forestal Bío-Bío, owned by UBS Resource Investment Incorporated. Forestal Bío-Bío has a plantation area of approximately 50,000 hectares and harvests around 500,000 cubic meters of wood per year, equivalent to harvesting about 1,100 hectares. The company first implemented the system in the 1995/96 season to program the harvesting of just 896 hectares. In the following 1996/97 season, Bío-Bío planned 9,450 hectares using PLANEX, which rose to 10,350 hectares in the 1997/98 season, and to 12,021 hectares in the 1998/99 season. Essentially, this involved planning the whole tactical harvesting program for five years.

Figure 12 shows the decrease in harvesting and road costs per cubic meter during this period. In addition, the chart shows an increase in the proportion of use of aerial equipment compared to ground-based machinery. Figure 13 presents the increase of harvested hectares per road kilometer and the decrease of road cost per harvested cubic meter. This indicates that the road network was used much more efficiently. Compared to manual planning, PLANEX reduced the number of roads from 10% to 60%, depending on the instance. For example, we compare the manual and

Figure 12. Harvest and road costs and proportion of aerial and ground-based equipment upon incorporating PLANEX.

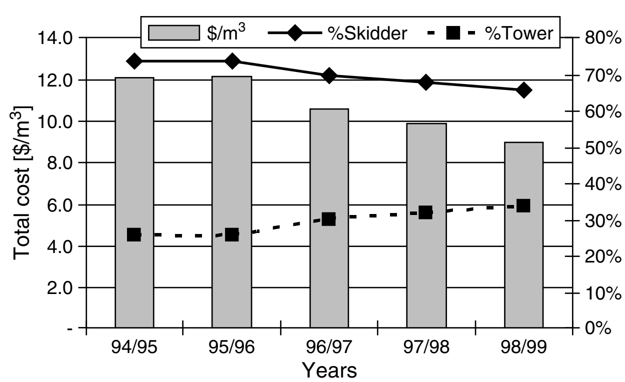
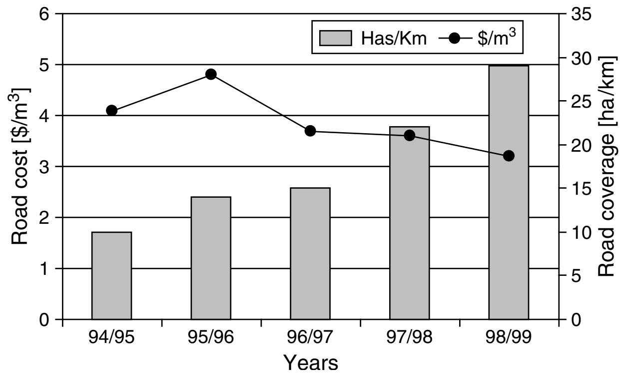


Figure 13. Road costs and road coverage upon incorporating PLANEX.



the system solutions on the "Cosmito" lot. The computerized system reduced road costs by $90,000 and raised harvest costs by $30,000, resulting in net savings of $60,000. In summary, for this period, harvest and road costs fell by $2.1 per cubic meter, or 17%, the equivalent of almost $1,000,000 per year. The other companies report similar results.

7. Conclusions

To prepare a forest area for harvesting, it is necessary to find a location for the machinery and build access roads to it. This problem is central to forestry firm logistics due to the significant costs involved and the environmental impacts associated, especially when they operate in mountainous zones.

This paper presents a heuristic approach that supports these decisions and has been applied in nine forest firms. The firms reported significant benefits from using the system to solve real problems.

According to the information available, this is the most advanced application of its type, permitting automatic solutions in short time periods. This paper shows an application that successfully combines operations research techniques with geographic information systems, with high impact in forest operations planning.

Acknowledgments

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