Review of combinatorial problems induced by spatial forest harvesting planning☆

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Abstract

In the last two decades forest planning and management has become very much focused on spatially oriented decisions, due to the introduction of spatial details, such as road building, and environmental concerns, like wildlife protection, biodiversity, reducing erosion and improving water quality. This has led to modeling transformations and extensions in planning forest activities, as mixed integer decision variables are necessary to account for spatial restrictions. These new models are much harder to solve due to the added combinatorial complexity. In this paper, we discuss issues of modeling spatial criteria, as well as algorithms that have been developed to solve them, both heuristic and exact. We present these results as they have evolved in a state-of-the-art structure.

Keywords: Spatial forestry; Environment; Combinatorial algorithms

1. Introduction

Mathematical models have been used successfully in forest decision making for several decades. Decisions supported by models go from long range strategic ones to schedule harvesting and planting cycles with time horizons of a century or more to daily operational decisions in harvesting and transportation [17]. At the tactical level, decisions involve planning during which period, each unit will be harvested in order to satisfy expected future demands, necessitating decisions on road building for access to harvested units. This adds spatial characteristics to the planning process and requires linear programming (LP) models with 0–1 variables in order to address road building decisions.

The more recent concern with environmental conditions, such as protecting wildlife, reducing erosion, improving water quality and preserving scenic beauty, has added explicit spatial dimensions to planning. A well known spatial requirement demands that harvested areas be no larger than a certain size, typically 30–80 ha. Conceptually, one could end up with harvesting patterns akin to a chessboard, where black cells are to be harvested and neighboring cells are not. Regulating spatial impacts of harvest activity promotes greater forest health and long term sustainability. It achieves this by minimizing environmental impacts as well as providing desirable habitat to certain species.

☆ Partial funding for the first author was provided by the National Science Foundation (Geography and Regional Science Program and the Decisions, Risk, and Management Science Program) under grant BC-0114362. Partial funding for this research was also received from Nucleo Milenio Complex Engineering Systems and Fondecyt Projects: 1020784 and 1040520.

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In this paper, we analyze spatial requirements, both how they are modeled and solution algorithms. Part of the challenge is how to model spatial conditions sufficiently using simple and complex constraints. The second challenge addressed in this paper is solving these resultant spatial optimization problems as they typically are extensions of well known combinatorial problems that have proven to be intractable.

2. Forest planning

Traditional forest planning LP models have been in heavy use since these early 70s [14,24]. These models were developed to support long range decisions for large forest areas in terms of harvesting and planting. Time horizons are typically two rotations, which can be up to 180 years for some tree species. Decisions are centered around basic units or stands, representing reasonably homogeneous forest areas in terms of tree species, age, and site quality. Typically, stands are 10–500 ha in size.

Given a decision variable associated with the management of a stand through planning horizon T, it is possible to structure a basic harvest planning model. Representative harvest actions could be to clearcut, partially treat or selectively thin a stand. This is essentially the focus of strategic models [7,14]. In contrast to the strategic orientation, tactical level planning typically has a shorter temporal horizon and focuses on the areas or trees to be harvested. As such, spatial relationships and conditions become critical. We begin with the a simple model in this domain.

Notation

- x_i^t number of hectares of stand i to be harvested in period t
- H^t total volume harvested in period t
- c_i^t net return of harvesting one hectare of stand i in period t
- v_i^t volume per hectare obtained in stand i if harvested in period t
- a_i total area of stand i
- β^L lower productivity bound
- β^H upper productivity bound

Model 1

$$\text{Max} \quad \sum_t \sum_i c_i^t \cdot x_i^t$$

$$\text{s.t.} \quad \sum_i v_i^t \cdot x_i^t = H^t \quad t = 1, \ldots, T \tag{1}$$

$$\sum_i x_i^t \leqslant a_i \quad \forall i \tag{2}$$

$$\beta^L \cdot H^{t-1} \leqslant H^t \leqslant \beta^H \cdot H^{t-1} \quad t = 2, \ldots, T \tag{3}$$

$$x_i^t \geqslant 0 \quad \forall i, t$$

$$H^t \geqslant 0 \quad \forall t.$$

The objective of Model 1 is to maximize net present revenue. Constraints (1) track total volume harvested in period t. Constraints (2) limit hectares harvested to the total area of each stand. Constraints (3) impose a productivity condition on volume harvested in each planning period to be at most some percentage below or above the total harvest in the preceding period. As an example, a common deviation is 15% from the harvest in the previous period. Finally, all decision variables are continuous.

As an optimization problem, Model 1 is relatively simple to solve using commercial LP software. However, Model 1 lacks formal spatial relationships in structured conditions. Spatial relationships were introduced starting in the 1970s with the work of [28,29,15]. One important case of spatial characterization at this level corresponds to roads needed to access areas to be harvested. Road building necessarily implies decisions best represented by 0–1 variables. In order to include such decision making in the context of Model 1, extension is necessary. In particular, we need to be concerned with where harvesting is taking place and its final destination as well as the corresponding potential transportation network. A simple model reflecting the roading extension is presented below.

Additional notation

$$Z_{ij}^t = \begin{cases} 1 & \text{if road from node } i \text{ to } j \text{ is built in period } t \\ 0 & \text{otherwise} \end{cases}$$

f_{ij}^t timber flow from node i to j in period t
h_i^t timber flow produced in node i in period t (node i corresponds to an accumulation point for timber harvested in unit i)
d_{ij}^t cost of building road from i to j in period t
e_{ij}^t unit cost for timber flow on road from i to j in period t
D_i^t Demand at node i, period t
u_{ij} Capacity of road (i, j)

Model 2

$$\text{Max} \quad \sum_t \left(\sum_i c_i^t \cdot x_i^t - \sum_{i,j} d_{ij}^t \cdot Z_{ij}^t - \sum_{i,j} e_{ij}^t \cdot f_{ij}^t \right)$$

$$\text{s.t.} \quad h_i^t = v_i^t \cdot x_i^t \quad \forall t, i \tag{4}$$

$$Z_{ij}^t = \begin{cases} h_i^t & \text{for all } i \text{ productions nodes, } t \\ 0 & \text{for all } i \text{ road intersection nodes, } t \\ -D_i^t & \text{for all } i \text{ demand nodes, } t \end{cases} \tag{5}$$

$$f_{ij}^t \leqslant u_{ij} \sum_{\sigma \leqslant (t-1)} Z_{ij}^\sigma \quad \forall i, j \tag{6}$$

$$Z_{ij}^t \in [0, 1] \quad \forall i, j, t$$

$$f_{ij}^t \geqslant 0$$

$$h_i^t \geqslant 0$$

$$x_i^t \geqslant 0.$$

The objective of Model 3 includes revenue associated with harvesting as well as costs corresponding to road building and timber traffic flow. Constraints (4) define timber collected at stand i in period t, if there is any. Constraints (5) define flow conservation at network nodes representing timber producing, intersection and demand nodes. Finally, flow up to capacity is allowed in Constraints (6) on arcs that have been built in the current or previous planning periods.

Model 3, in contrast to Model 1, can be a difficult problem to be solve, particularly if the road network is dense. For this problem algorithmic enhancements have significantly improved solution approaches. These include strengthening of the formulation and Lagrangean relaxation of the formulation [1]. Further, heuristic solution development has been both active and successful for this model [17].

3. Environmental issues

In addition to roading concerns, the basic harvest scheduling model has been utilized to address environmental issues. Such issues include protecting wildlife, reducing erosion, and preserving scenic beauty [17]. The spatial form of harvesting plays an important role in how environmental issues may be modeled. A major restriction that is typically

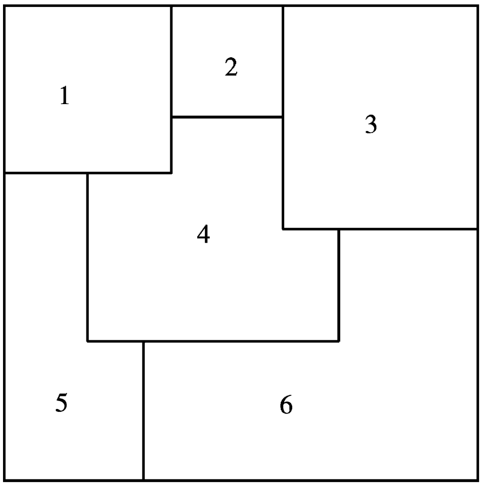

Fig. 1. 6 harvest block example.

imposed relates to not excessively harvesting a particular area. This is realized by limiting contiguous harvest areas to at most 30 to 80 hectares [20]. We will use a commonly defined maximum harvest size of 49 hectares in this exposition.

This type of restriction has been introduced into planning in the following way. Divide the area to be harvested into blocks of no more than 49 hectares. Thus, if a stand has 240 hectares, it is divided into five blocks, as an example. Geographic information systems (GIS) have played an important role in the generation and manipulation of data for these problems [3]. A basic form of expressing harvest area limitations, called adjacency, states that if a block j is harvested in period t then no block adjacent to j can be harvested for γ periods, where γ is the green up period reflecting the time needed until trees in block j grow to a minimum height. Thus, in Fig. 1, if block 2 is harvested in period t, blocks 1, 3, and 4 (adjacent blocks) cannot be harvested before period $t+\gamma$.

A basic model to reflect adjacency, essentially proposed by [28], is now detailed.

Let

$$x_i^t = \begin{cases} 1 & \text{if block } i \text{ is harvested in period } t, \\ 0 & \text{otherwise.} \end{cases}$$

Model 3

$$\text{Max} \quad \sum_t \sum_i c_i^t \cdot x_i^t$$

$$\text{s.t.} \quad \sum_i v_i^t \cdot x_i^t = H^t \quad \forall t \tag{7}$$

$$\beta^L \cdot H^{t-1} \leqslant H^t \leqslant \beta^H \cdot H^{t-1} \quad \forall t \tag{8}$$

$$x_i^t + x_j^t \leqslant 1 \quad \text{If } i, j \text{ are adjacent} \quad \forall t \tag{9}$$

$$\sum_t x_i^t \leqslant 1 \quad \forall i \tag{10}$$

$$x_i^t \in [0, 1] \quad \forall i, t$$

$$H^t \geqslant 0.$$

The objective of Model 3 is to maximize net present revenue. Constraints (7) and (8) track total volume harvested in each period and establish acceptable temporal variation in timber production, respectively. Constraints (9) impose adjacency restrictions for one period green up. The one period green up situation is presented for simplicity, but is

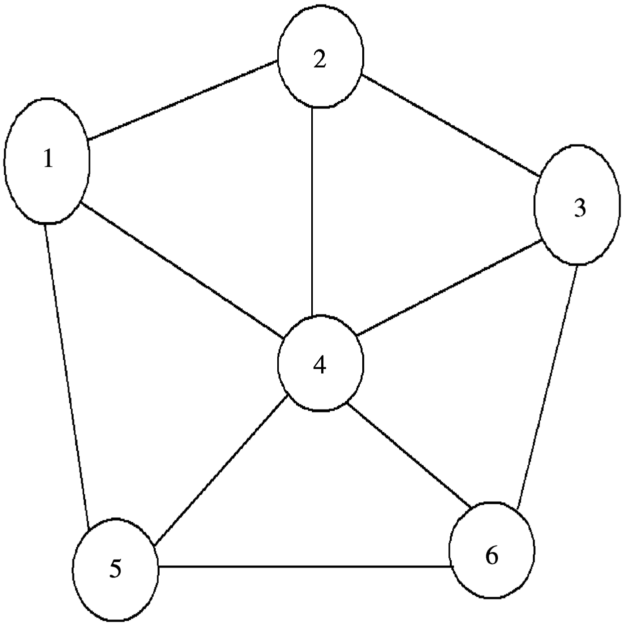

Fig. 2. Adjacency graph associated with 6 block example.

commonly imposed. Clearly with Constraints (9), one constraint is required for each pair of adjacent blocks. Constraints (10) establish that a block may only be harvested once in the planning horizon. Finally, integer and non-negativity requirements are also imposed.

In terms of problem structure, Model 3 may be considered a weak formulation. A primary issue is that there are too many constraints associated with (9). The continuous LP relaxation, as a result, contains many fractions in most cases [22]. However, it is possible to strengthen the basic formulation for Model 3.

If a graph of nodes and arcs is used to represent adjacency between harvest blocks, it is possible to view Model 3 in more standard mathematical terms. Fig. 2 shows adjacency relations suggested in Fig. 1. Each node represents a block and an arc exists when blocks i and j are deemed adjacent.

A stronger formulation, as suggested in [22], is to use cliques instead of pairwise adjacency constraints. Then, Constraints (9) can be replaced by

$$\sum_{i \in CL_r} x_i^t \leqslant 1 \quad \forall_r, t, \tag{11}$$

where CL_r is the set of blocks that are simultaneously adjacent to each other and r is an index to the set of cliques necessary to impose all conditions structured in Constraints (9). An example of a clique associated with Fig. 2 (and Fig. 1) is the following:

$$x_2^t + x_3^t + x_4^t \leqslant 1.$$

This is the clique imposing adjacency restrictions between nodes 2, 3 and 4 in Fig. 2.

Cliques provide enhanced problem structure [22]. Further, Constraints (11) are not only fewer than the number of required Constraints (9), but are much stronger as well. While pairwise constraints tend to lead to many fractional values, clique constraints are facet defining [25]. As a result, LP solutions have significantly fewer fractional variables and less branching is necessary. Results in [22] show that virtually no branching is required in the problems evaluated when cliques are imposed. This is in contrast to not being able to get an initial feasible solution when pairwise Constraints (9) were imposed.

Another approach to improving problem structure is to use column generation [2]. The sub-problems generate columns for the master problem by solving a maximum stable set problem to generate improved harvesting options with respect to adjacency. This approach has led to good results for realistic problem applications. Another alternative is dynamic programming, but application results have only been reported for smaller sized problems [11]. While exact formulations for the adjacency problem continue to be of interest, large planning applications often require heuristics solutions approaches. Good results have been obtained using metaheuristics such as tabu search and simulated annealing [26,21,5]. Typically in these approaches a neighboring solution involves changing the period of harvest of one block (including no harvest) or exchanging harvest period between two blocks.

4. Area restrictions

Adjacency relationships as structured in Model 3 reflect what has been termed by [20] the unit restriction model (URM). Implicit in the URM approach is that blocks are assumed to be of sufficient size such that no two adjacent blocks can be simultaneously harvested. When the typical maximum opening size is 49 ha, this means that blocks are assumed to be between 25 and 49 ha in size. However, it is not uncommon to find a forest area with blocks being comprised of basic cells with detailed spatial information. For example, basic cells of 5–20 ha are common when the above 49 ha maximum is stipulated.

This basically means that a somewhat different modeling approach may be required to model conditions when a maximum area of impact is being imposed. As such, a new approach has gained prominence in recent years. Given the basic cells, we can define the problem now in terms of deciding how to define blocks instead of carrying this out a priori. This new modeling approach has been called the area restriction model (ARM) [20]. Typically a block will have 3–8 cells, but this is dependent on cell size and the maximum area limit, α. It can be shown that by including the block building decisions into the process can significantly improve solution quality [23]. Unfortunately, this problem is substantially more difficult to solve, as the combinatorial complexity increases because of the options for forming blocks.

The problem can be described now as the need to form clusters of less than α ha such that all selected clusters for harvesting are not adjacent. Research in this area has principally focused on metaheuristic development to solve the ARM [12,16,4,8,26,5]. Recently, however, there has been increased interest in solving the ARM exactly. The ARM can be stated as follows: given basic cells, form clusters such that any feasible cluster is composed of contiguous cells with an area of at most α ha and that any two clusters are compatible in the sense that they are not adjacent to each other nor do they overlap.

If Φ_l is a feasible cluster of cells, then Ψ is defined as the set of all feasible clusters

$$\Psi = \{\Phi_l, \Phi_2, \ldots, \Phi_n\}.$$

As such, two clusters Φ_l and Φ_p are incompatible if they both contain cell k (they overlap) or if they are adjacent. For any cluster $S \in \Psi$, the set of incompatible clusters to cluster S is Ω_S.

Let a_k be the area of cell k, v_k^t be the volume of cell k in period t, and c_k^t be the net benefit for harvesting cell k in period t. For any cluster $S \in \Psi$, its area, volume and benefit, respectively, are

$$a_S = \sum_{k \in S} a_k,$$

$$v_S^t = \sum_{k \in S} v_k^t,$$

$$c_S^t = \sum_{k \in S} c_k^t.$$

Finally, consider a slightly revised decision variable associated with a cluster:

$$x_S^t = \begin{cases} 1 & \text{cluster } S \text{ is harvested in period } t, \\ 0 & \text{otherwise.} \end{cases}$$

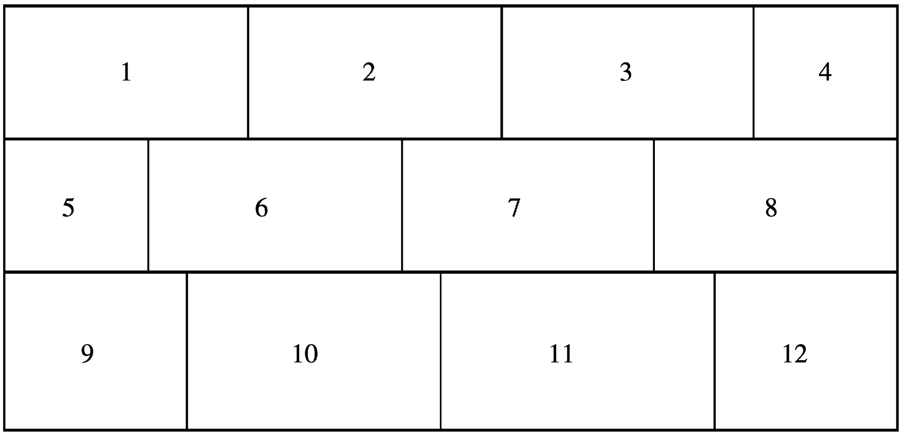

Fig. 3. Example of cell configuration.

With this notation, the ARM may be formulated as follows:
Model 4

$$\text{Max} \quad \sum_t \sum_S c_S^t \cdot x_S^t$$

$$\text{s.t.} \quad \sum_S v_S^t \cdot x_S^t = H^t \quad \forall t \tag{12}$$

$$\beta^L \cdot H^{t-1} \leqslant H^t \leqslant \beta^H \cdot H^{t-1} \quad \forall t \tag{13}$$

$$\sum_t x_S^t \leqslant 1 \quad \text{for each cell } k \tag{14}$$

$$x_S^t + x_l^t \leqslant 1 \quad \forall S, l \in \Omega_S \;\; \forall t \tag{15}$$

$$x_i^t \in [0, 1] \quad \forall \text{ clusters } S \in \Psi \;\; \forall t$$

$$H^t \geqslant 0.$$

Model 3 maximizes total net revenue. This is a straightforward extension of Model 3 to account for area restrictions rather than adjacency between blocks. Constraints (12) and (13) ensure harvest volume flow. Constraints (14) impose that a cluster is harvested at most once. Constraints (15) prevent incompatible clusters from simultaneously being selected, as at most either $x_S^t = 1$ and cluster S is chosen, or $x_l^t = 1$ and one of the clusters in Ω_S (incompatible to cluster S) is chosen. Finally, decision variables are defined as non-negative and integer in the case of x_S^t.

The structure of Model 4 is not particularly good in practice. The number of constraints (15) equals the number of clusters, which grow rapidly with the size of the problem. The formulation is also weak, in the sense that the LP relaxation is highly fractional. [18] presented a tighter formulation of Model 4. Consider the adjacency relations of Fig. 3. Now, Fig. 4 is the corresponding adjacency graph G where each cell corresponds a node i, and arc (i, j) exists if cells i and j are adjacent.

As suggested previously, two clusters, S and S', will be incompatible if they either share a cell i or have cells $i \in S$ and $j \in S'$ such that cells i, j are adjacent. In such a case arc (i, j) exists in the adjacency graph G. Let $SO(i, j)$ be the set of all clusters S such that $i \in S$, $j \in S$ or both i and $j \in S$. Thus, all clusters in $SO(i, j)$ are mutually incompatible. The model formulation used in [18] is based of restricting clusters related to each arc (i, j) in G.

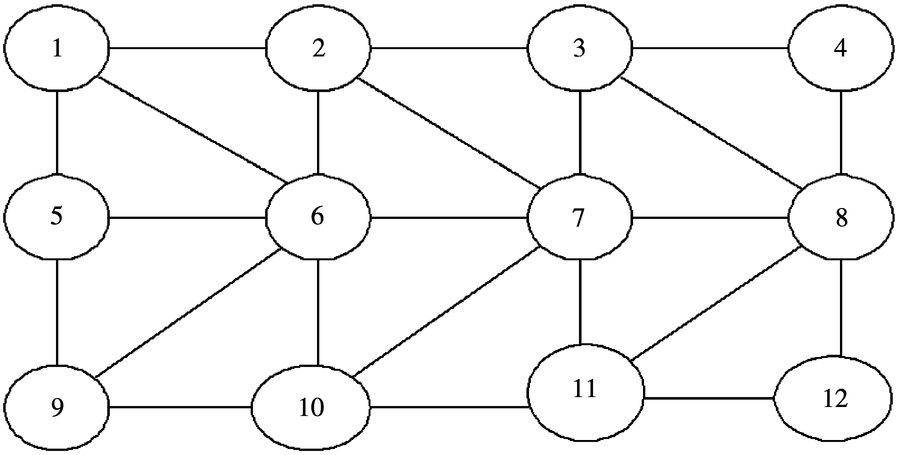

Fig. 4. The adjacency graph for the cell configuration in Fig. 3.

Model 5

$$\text{Max} \quad \sum_t \sum_S c_S^t \cdot x_S^t$$

$$\text{s.t.} \quad \sum_S v_S^t \cdot x_S^t = H^t \quad \forall t \tag{16}$$

$$\beta^L \cdot H^{t-1} \leqslant H^t \leqslant \beta^H \cdot H^{t-1} \quad \forall t \tag{17}$$

$$\sum_{t,S/k\in S} x_S^t \leqslant 1 \quad \text{for all cells } k \tag{18}$$

$$\sum_{S\in SO(i,j)} x_S^t \leqslant 1 \quad \forall t \text{ and each arc } (i,j) \in G \tag{19}$$

$$x_i^t \in [0, 1] \quad \text{for all cluster } S \in \Psi \text{ and } \forall t$$

$$H^t \geqslant 0.$$

Model 5 maximizes total net revenue, as have the previous models. Constraints (16) and (17) ensure harvest volume flow. Constraints (18) impose that a cell be harvested at most once. Constraints (19) restrict incompatibilities from occurring, with respect to a cell or adjacency. Note that Model 5 has a relatively small number of constraints compared to Model 4, as the adjacency restrictions require only one constraint for each arc in the adjacency graph G. Further, constraints (19) dominate constraints (14). Finally, decision variables are defined as non-negative and integer in the case of x_S^t.

While Model 5 is a much more compact and stronger formulation, it can be used to solve only small planning problems in practice, as discussed below.

As with the URM, deriving clique constraints tends to produce a very strong formulation, as maximal cliques in G clearly dominate arc Constraints (15) Such facet defining constraints have proven to be beneficial [25]. However, the number of maximal cliques turns out to be prohibitive in practice. To address this, [10] developed a constraint projection approach, where a graph of clusters GC is defined. In this graph, each feasible cluster u represents a node and an arc (u, u') will join nodes u, u' if clusters u, u' are incompatible.

To illustrate this, Fig. 5 shows a small fraction of a graph GC. Consider the following clusters from Fig. 3:

Cluster C_1 has cells (1,2,6).

Cluster C_2 has cells(6,7).

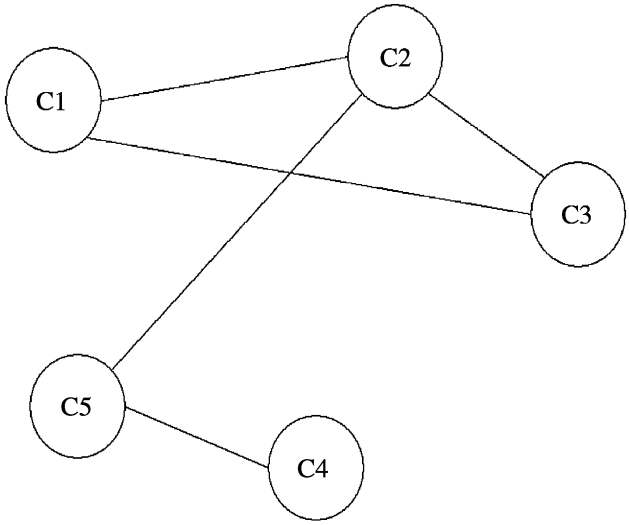

Fig. 5. Incompatibility graph in GC.

Cluster C_3 has cell (5,9).
Cluster C_4 has cell (12).
Cluster C_5 has cells (8,11).
Then, the representation in Fig. 5 of this small subset of clusters indicates cluster incompatibilities through arcs.

Consider now the clique S of nodes $(2, 3, 7)$ in Fig. 4. Then, it can be proven that all feasible clusters in GC that contain any node in the clique s are incompatible. Let the set of these clusters be SC_1. Then, any two clusters in the maximal clique SC_1 in G have a corresponding arc in the cluster graph GC. For example, in Fig. 5, clusters C_1, C_2, C_3 are incompatible.

This projected network structure gives a slightly different model form.

Model 6

$$\text{Max} \quad \sum_t \sum_S c_S^t \cdot x_S^t$$

$$\text{s.t.} \quad \sum_S v_t^S \cdot x_S^t = H^t \quad \forall t \tag{20}$$

$$\beta^L \cdot H^{t-1} \leqslant H^t \leqslant \beta^H \cdot H^{t-1} \quad \forall t \tag{21}$$

$$\sum_t \sum_{S/k \in S} x_S^t \leqslant 1 \quad \text{for each cell } k \tag{22}$$

$$\sum_{S \in SC_j} x_S^t \leqslant 1 \quad \forall t \text{ and each maximal clique} \in G \tag{23}$$

$$x_S^t \in [0, 1] \quad \text{for all cluster } S \in \Psi \text{ and } \forall t$$

$$H^t \geqslant 0.$$

Model 6 maximizes total net revenue, as have the previous models. Constraints (20) and (21) track and bound harvest volume flow, respectively. Constraints (22) ensure that a cell be harvested at most once. Constraints (23) impose that for any maximal clique S in the adjacency graph G, at most one feasible cluster in the set SC_j be harvested in a given period. Finally, decision variables are defined as non-negative and integer in the case of x_S^t.

It should be noted that Constraints (23) can be lifted to be facet forming as shown in [10]. Lifting these constraints, however, is complex and was not convenient to implement. Model 6 is, nevertheless, quite strong. While Constraints (19) define incompatible clusters along an arc in Model 5, Constraints (23) define constraints incompatible along a clique, thus, Constraints (23) are much stronger, and usually contain hundreds of nonzero components. Also, since the adjacency graph G is not very dense, all cliques in G can be defined explicitly. Since cells usually have 5–20 ha, clusters will typically have 3 to no more than 8 cells, and they can also be explicitly defined. Thus, a preprocessor program can identify Constraints (23).

Models 5 and 6 have been empirically evaluated in [10]. Results showed that Model 6 performed far better than Model 5. Model 6 was capable of solving medium sized problems, whereas Model 5 could not. However, as the number of periods grows, Model 6 does encounter limitations. For example, for a 16×16 grid problem with 5 periods a gap of 0.22% was obtained after a CPU time of 4 h, but the problem could not be solved for 7 periods in 8 h CPU time using a CPLEX 7.1 version on a Pentium III personal computer with a 700 mHz processor and 1 GB of RAM. An explanation for the degradation of results as the number of periods increases lies in the rigidity of the constraints that limit timber harvest variations from one period to the next as hard constraints. Current research is focusing on how to improve results for multiple periods as well as increasing capabilities for solving larger sized problems.

It is possible to compare structural relationships between the basic continuous Model 1 and Models 3–6, which handle spatial harvest restrictions. Model 2, which considers road building, is of a different spatial nature, so will not be included in this comparison.

Define the following:

$$x = [x_1^1, \dots, x_S^1, x_1^{\mathrm{T}}, \dots, x_S^{\mathrm{T}}]^t,$$

$$H = [H_1, \dots, H_T]^t,$$

$$c = [c_1^1, \dots, c_S^1, \dots, c_S^{\mathrm{T}}]^t,$$

$$a = [a_1, \dots, a_S]^t,$$

$$\mathbf{V} = \begin{pmatrix} v_1^1 & \dots & v_S^1 & & & \\ & & & \ddots & & \\ & & & v_1^{\mathrm{T}} & \dots & v_S^{\mathrm{T}} \end{pmatrix}, \quad \mathbf{B} = \begin{pmatrix} \beta^L & -1 & & & \\ -\beta^H & 1 & & & \\ & & \ddots & & \\ & & & \beta^L & -1 \\ & & & -\beta^H & 1 \end{pmatrix},$$

$$\mathbf{U} = \begin{pmatrix} 1 & 0 & \dots & 0 & & 1 & 0 & \dots & 0 \\ & & & & \ddots & & & \dots & 0 \\ 0 & \dots & 0 & 1 & & 0 & \dots & 0 & 1 \end{pmatrix}, \quad \mathbf{E} = \begin{pmatrix} E & & \\ & \ddots & \\ & & E \end{pmatrix}.$$

With this notation, Table 1 provides a comparison of Models 1, 3, 4, 5 and 6. The first column indicates the Model in question. The second column defines the formulation of each model using the variables, vectors and matrices defined above. Note that Models 4–6 are the same as Model 3, but vary with respect to the E matrices. The third column defines decision variables x_i^t. The next column defines the volume parameters, and is followed by the specification of the E matrices (expressed as incompatibilities for harvesting). Note that the differences in Models 4–6 lie in how clusters incompatibilities are defined. The last two columns characterize and give references to each model, respectively. With this, it is possible to identify similarities and differences in each of the presented models.

5. Additional spatial problems

Considering other environmental needs, in particular wildlife behavioral patterns, other spatial conditions have been used, leading to more complex formulations. Only heuristic approaches have been proposed so far to handle these problems.

Table 1
Comparison of Models 1, 3–6

Model #	Formulation	Meaning of x_i^t	Meaning of v_i^t	Incompatibility encoded in matrix E	Characteristics	Reference
1	Max cx s.t. $Vx = H$ $Bx \leqslant 0$ $Ux \leqslant a$ $x \geqslant 0$ $H \geqslant 0$	Area of block i harvested in period t.	Volume per area obtained in block i if harvested in period t	Not applicable	Lacks formal spatial relationship Easy to solve	–
3	Max cx s.t. $Vx = H$ $Bx \leqslant 0$ $Ux \leqslant 1$ $Dx \leqslant 1$ $x \in \{0, 1\}$ $H \geqslant 0$	1 if block i is harvested in period t.	Volume obtained if block i is harvested in period t	Pairs of blocks adjacent In strengthened formulation cliques of blocks.	Defines harvesting blocks a priori, which can lead to lower quality solutions	Thompson et al. [28]
					Can solve strengthened formulation using cliques or through Column Generation.	Murray and Church [22] Barahona et al. [2]
4	Same as model 3, but with difference E matrix	1 if cluster i is harvested in period t.	Volume obtained if cluster i is harvested in period t	Pairs of clusters that are adjacent or overlap	Block construction is part of the decision. Weak formulation Very difficult to solve	Murray [20]
5	Same as model 3, but with different E matrix	1 if cluster i is harvested in period t.	Same as model 4	Set of clusters that intersect an edge	Block construction is part of the decision Better formulation Easier to solve than model 4, but still difficult	Martins et al. [18]
6	Same as model 3, but with different E matrix	1 if cluster i is harvested in period$t$.	Same as model 4	Set of clusters that intersect a clique	Block construction is part of the decision Stronger formulation Easy to solve for medium sized instances and small number of periods	Goycoolea et al. [10]

(1) *Animals corridors*: In some cases, animals have several habitats of patches of mature trees, to move safely from one habitat to another. SNAP2 was developed by [27] and has been used by the US Forest Service to address corridor issues, among other things. Heuristics, including solving shortest paths and Steiner problems, are integrated to obtain acceptable solutions.
(2) *Old growth patches*: Other animals species require large contiguous patches of old growth or mature trees to thrive. In these situations, while preserving adjacency constraints in the harvesting process, we also need to consider preserving clusters of cells of mature trees that are not harvested, so that in any given period there is a sufficient number of contiguous areas of old growth of at last a minimum given size area. Caro et al. [6] detail an ARM oriented problem that considers old growth, both in terms of a minimum areas required for each old growth patch as well as a lower bound on total old growth preserved. A tabu search approach was developed. Tests carried out showed that significant improvements, both in objective value as well as reduction in CPU time, can be obtained careful implementation of tabu concepts, such as neighborhood reduction, intensification, diversification, alternative tabu criteria.

6. Discussion and conclusions

The objective of this paper was to analyze the state of the art in spatial forest management models as well as indicate research challenges. We have mainly focused on forest management planning from the perspective of spatial restrictions imposed by environmental concerns. One challenge is to express these concerns into mathematical models. A much used modeling form is the adjacency property, which does not allow neighboring blocks to be harvested unless a minimum green up time has passed. For this problem, exact formulations have been proposed and have been found to be solvable in some circumstances. However, in practice heuristic approaches are typically used, like tabu search and simulated annealing.

It is interesting to note the similarities in harvest planning with facility layout, an area that continues to receive considerable attention in optimization research. The idea in facility layout design is to configure a building so that interacting functions are in close proximity to each other [9]. As such, there is a clear interest in the notion of adjacency (see [13]). However, in harvest scheduling the goal is to ensure that treated areas do not violate a stipulated maximum contiguous area (or are not adjacent), whereas in facility layout planning the intent is to promote efficiency in overall design. Nevertheless, there would be no doubt in cases where certain activities should not be juxtaposed, like a boiler room and a storage room containing combustible materials [19]. Interestingly, many of the same heuristic approaches (genetic algorithms, simulated annealing and tabu search) have been used to solve both harvest scheduling and facility layout problems.

As detailed in the paper, harvest scheduling problems become much harder combinatorially if we include in the model the decision on forming the cutting blocks starting with basic, smaller cells (Models 4–6). This problem has proven to thus far be intractable for large and multiple period problems using exact approaches. As a consequence only heuristic approaches have been relied upon. Given this, significant algorithmic challenges remain.

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