Geostatistical modelling of rock type domains with spatially varying proportions: application to a porphyry copper deposit

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Synopsis

Plurigaussian simulation allows constructing lithofacies or rock type models that reproduce the contacts between facies in accordance with the geologist's interpretation. Its implementation requires inferring the local facies proportions, but the uncertainty in the true proportions is not accounted for. The simpler model with constant facies proportions may not yield realistic results, due to the possibility of obtaining facies at locations where it is geologically unlikely to find them.

This article presents a variation of the plurigaussian model, in which the facies proportions are represented by random fields. The realizations can be made conditional to soft geological information to account for local changes in the facies proportions. The model is illustrated via a case study of a porphyry copper deposit where four Gaussian random fields are simulated conditionally to drill hole data and to constraints on the probability of finding a given facies at specific locations (control points) in the deposit. Then the first two fields are truncated using the random thresholds defined by the last two, generating a three-facies model. The proposed random proportion model proves to be simple to use and to account for spatial variations of the geological characteristics and for the uncertainty in the facies proportions.

Keywords: categorical variable; lithofacies; truncated plurigaussian simulation; regionalized proportions

Introduction

Uncertainty quantification is a key aspect when assessing and classifying ore resources and reserves in mining applications, or when considering oil inventory in petroleum engineering. The uncertainty stems from a lack of knowledge about the geological genesis of the deposition and an inability to construct realistic genetic models of the mineral deposit or petroleum reservoir. Geostatistical simulation allows accounting for this uncertainty, by generating multiple plausible realizations of the distribution of petrophysical properties. However, practical implementation of conditional simulation in mining engineering has been focused on reproducing the variability of these properties within fixed geological domains; the uncertainty in the extent of these domains is often most consequential and should be accounted for when the geological model is not certain.

Geostatistical simulation of categorical variables representing geological domains, henceforth called facies or lithofacies, can be performed mainly in two ways. One is to consider object-based models such as the Boolean or dead leaves models.[1–5] The main problem of this approach is the difficult conditioning when many hard data must be honoured. Solutions are often iterative and may take a long time to converge to satisfy all the conditions in a single realization.

A second approach corresponds to pixel-based simulation methods. For instance, sequential indicator simulation[6] and transition probabilities[7] have been proposed to characterize the spatial extension of lithofacies. They provide a flexible framework, but often realizations lack realism from a geological standpoint. Gaussian-based models are an alternative to the indicator-based approach. Among these models, truncated Gaussian simulation is suited to cases when there is a sedimentary sequence of strata.[8–10] Plurigaussian simulation is more flexible and allows more complex transitions between lithofacies, based on a truncation rule for a multivariate Gaussian distribution.[11–12]

Plurigaussian simulation has found wide acceptance for modelling petroleum reservoirs. Mining applications have also been developed in the past few years.[13–16]

One important limitation of most categorical simulation methods is that handling spatial changes in the facies proportions becomes cumbersome. It often requires using non-stationary models that incorporate locally varying proportions, but parameter inference (in particular, variogram

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analysis) is not clearly laid. This paper proposes a variation of the plurigaussian model in order to combine three important aspects: (1) spatially varying facies proportions, (2) accounting for uncertainty in these proportions and (3) stationarity of the model. The idea is to model the facies proportions by random fields instead of deterministic fields and to incorporate geological knowledge by introducing information on the probability of finding a particular facies at given locations. This information is converted into conditioning data that ensure the reproduction of the spatial variations (trends) in the facies proportions.

In the following, the existing approaches to plurigaussian simulation are discussed, then the model with randomized facies proportions is introduced through a case study, and its realizations are compared with those obtained via conventional models.

Traditional plurigaussian models

Constant proportion model

Plurigaussian simulation aims at constructing realizations of categorical variables that represent lithofacies by truncating two or more Gaussian random fields. The simplest approach consists of assuming that the facies proportions are constant in space. The model is therefore defined by a truncation rule, a set of threshold values and the correlogram models of the Gaussian random fields to truncate.[5,11] One advantage is the ability to control and reproduce the contacts between facies as interpreted by the geologist. Another property of this model is its stationarity, which facilitates variogram analysis.

In contrast, this model is ill suited for characterizing phenomena that show clear trends in the facies proportions, for which the probability of occurrence of a facies cannot be considered constant in space. In such situation, a solution is to define a model with regionalized (spatially varying and deterministic) facies proportions.

Regionalized proportion model

One approach consists of considering a variation of the facies proportions with depth and of defining vertical proportion curves. Such curves depend on the choice of a reference surface and can account for stratigraphic correlation, considering proportional deformation, truncation of the facies through erosion, and superposition of a sedimentary facies over an eroded surface.[17] A new coordinate system can be defined, such that the reference surface represents the zero level. The inference of the parameters and the simulation are performed in this new system, and then the coordinates are back-transformed to the original stratigraphic space. The concept of vertical proportion curves has been applied to the simulation of sedimentary and fluvio-deltaic deposits.[8,18]

The plurigaussian model can also be generalized to account for lateral changes in the facies proportions. In this case, the proportions are calculated at specific locations from the empirical proportions observed on the surrounding data, then they are interpolated to the whole space, e.g. by inverse distance weighting or by kriging, leading to a three-dimensional map representing the local facies proportions.[19–21]

Drawbacks of the traditional models

The model with constant proportions is simple and straightforward to use. However, it is usually not realistic since, in general, some facies are more likely to prevail in some regions than in others. For instance, in the case of ore deposits, facies representing an oxide mineralization are rarely found in depth, hence the assumption of a constant probability of occurrence (constant proportion) is questionable.

On the other hand, although it is meant to create realistic images of deposits with a complex geology, the model with regionalized proportions leads to several difficulties.

(1) One should distinguish between two concepts: (i), the observed lithofacies proportions and (ii), the probabilities of finding these facies at given locations. The ergodicity property states that the spatial domains on which the proportions are calculated should be large with respect to the ranges of the random fields under study, in order to identify them with theoretical probabilities.[22] Now, this condition is clearly not fulfilled in the model: the proportions are calculated locally, on small domains centred on the locations of interest, therefore they may not match the probabilities of occurrence.

(2) Calculation of local proportions requires many data to be reliable. This situation is not always met in practice.

(3) Facies proportions need to be defined at every spatial location where the simulation has to be performed, even if no specific information is available at that location and therefore the true proportions are unknown. Consequently, the model is likely to understate the uncertainty in the less recognized parts of the deposit, by not accounting for the uncertainty in the facies proportions.[23–24]

(4) The interpolation procedure used to create the proportion maps is arbitrary and may produce artefacts in these maps (hence, unrealistic properties in the realizations): due to the smoothing effect of the interpolator, extreme proportions do not occur in undersampled areas. To illustrate this statement, an example is shown next.

(5) Even if the underlying Gaussian random fields are stationary, the facies indicators obtained by truncating such random fields are not stationary, which makes the calculation and fitting of their simple and cross variograms problematic. When dealing with a single vertical proportion curve, the experimental indicator variograms can be calculated for each horizontal level, then these local variograms are averaged and fitted.[12] The implementation for the general case of regionalized proportions with vertical and lateral variations is cumbersome, since it depends on the sampling configuration and often leads to variogram models that are difficult to interpret.

The next section presents a model that considers regionalized facies proportions but remains stationary. The idea is similar to that used with Poisson point processes: a non-stationary process with a regionalized density can be converted into a stationary process (called Cox process or doubly-stochastic Poisson process) by randomizing this density.[5] By analogy, regionalized facies proportions can be modelled by random fields instead of deterministic fields. For simplicity, the presentation is made through a case study to a mining data set.

A mining case study

Presentation of the data and the deposit

The data used in this section correspond to a set of 2 376 diamond-drill hole samples from an exploration campaign in a breccia complex of a porphyry copper deposit, located in the Central Andes and owned by Division Andina of CODELCO-Chile. The available information consists of the rock type prevailing at each sample location. The rock type model can be used to constrain the simulation of copper grades at a later stage of the resource evaluation project. Three main rock types can be distinguished (Figure 1).[25–26]

- cascade granodiorite (code 1), located in the eastern and southern parts of the sampled area. It is one of the host rocks of the breccia complex, with ages ranging from 20.1 to 7.4 million years
- tourmaline breccia (code 2), located in the central part of the sampled area. It consists of granodiorite clasts surrounded by matrix cement dominated by tourmaline and sulphides (chalcopyrite, pyrite, molybdenite and minor bornite). Its age ranges from 5.2 to 5.1 million years. The rock emplacement is related to the main alteration-mineralization event of the breccia complex
- other breccias (code 3), which outcrop in the western and southern parts of the sampled area. This group comprises three sorts of rocks: castellana (rock flour breccia), monolith and paloma breccias. It is younger than the tourmaline breccia (or contemporaneous) since granodiorite and tourmaline breccia clasts can be found into these breccias. Their emplacement relocated and diluted the previous tourmaline breccia mineralization.

Every rock type is in contact with the two others. The transition between granodiorite and tourmaline breccia is gradational and erratic, whereas tourmaline breccia and castellana breccia (hence the other breccias' group) have a sharper and well-defined contact. All these contacts are steeply dipping and quite continuous in depth. Henceforth, the granodiorite, tourmaline breccia and other breccias facies will be denoted by F_1, F_2 and F_3, respectively.

Stationary model with constant proportions

One approach to describe the rock types is to use a stationary plurigaussian model in which the facies proportions are constant in space. Under these conditions, one only has to define the truncation rule, threshold values and correlograms of the Gaussian fields that will be truncated.

Truncation rule and threshold values

The truncation rule controls the transitions between the different facies. It has to be chosen in accordance with the geological evidence and interpretation of the deposit and with the statistical behaviour of the data. Here, two independent standard Gaussian random fields (denoted by Y_1 and Y_2) will be used, so the truncation rule amounts to defining a partition of the bi-Gaussian space.

A simplified geological evolution of the sampled area can be summarized as follows: 20.1 million years ago, granodiorite (F_1) occupied the whole sampled area. Then, 5.2 million years ago, the emplacement of tourmaline breccia (F_2) divided the area and decreased the proportion of granodiorite. Later or simultaneously, the other breccia group (F_3) emplacement took place, decreasing the proportion of granodiorite and tourmaline breccia and leading to the current rock type configuration. Such an evolution can be used to define the truncation rule (Figure 2).

The rock type prevailing at location $\mathbf{x}$ is defined as follows:

$$\begin{cases} \mathbf{x} \in F_1 \Leftrightarrow Y_1(\mathbf{x}) < u_1 \\ \mathbf{x} \in F_2 \Leftrightarrow Y_1(\mathbf{x}) \geq u_1 \text{ and } Y_2(\mathbf{x}) < u_2 \\ \mathbf{x} \in F_3 \Leftrightarrow Y_1(\mathbf{x}) \geq u_1 \text{ and } Y_2(\mathbf{x}) \geq u_2 \end{cases} \quad [1]$$

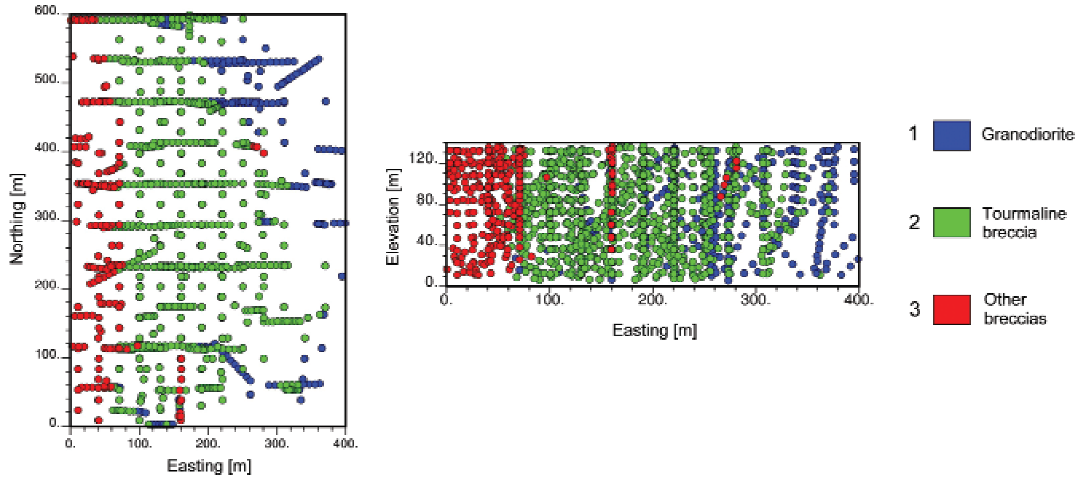

Figure 1—Plan view and cross-section showing lithofacies information

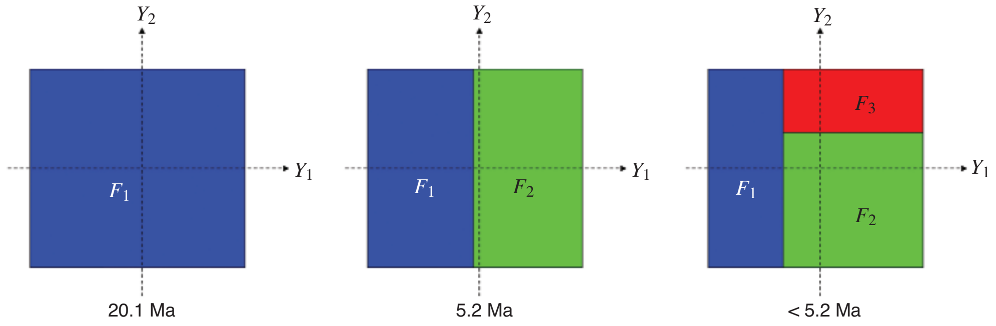

Figure 2—Definition of the facies partition according to the geological evolution of the deposit

where u_1 and u_2 are threshold values defined so that the model reproduces the proportion of each facies. In the case under study, 29% of the area of interest corresponds to granodiorite (F_1), 55% to tourmaline breccia (F_2) and 16% to the other breccias (F_3). These global proportions yield the following threshold values:

$$u_1 = -0.553 \text{ and } u_2 = 0.754. \qquad [2]$$

Variogram analysis

The correlograms ρ_1 and ρ_2 of the Gaussian random fields Y_1 and Y_2 are fitted according to their impact on the simple and cross-variograms of the facies indicators. The following nested cubic models lead to a satisfactory fitting:

$$\begin{cases} \rho_1 = 0.03cub\,(30m,\ 60m) + 0.02cub \\ (200m,\ 300m) + 0.95cub\,(800m.\infty) \\ \rho_2 = 0.6cub\,(200m,\ 500m) + \\ 0.4cub\,(200m,\ 1000m) \end{cases} \qquad [3]$$

In this equation, the distances in parentheses represent the ranges along the horizontal and vertical directions, respectively. The experimental indicator variograms and the theoretical curves deduced from the previous models are shown in Figure 3. Note that the cross-variogram between the indicators associated with granodiorite and other breccias is almost a pure nugget effect, which means that there is little spatial relationship between both facies in the area under study.

Stationary model with random proportions

Principles

To allow the facies proportions to vary in space, the idea is to replace both thresholds u_1 and u_2 in Equation [1] by two independent stationary Gaussian random fields U_1 and U_2, with means m_1 and m_2, unit variances and the same correlograms ρ_1 and ρ_2 as Y_1 and Y_2. Under these assumptions, Equation [1] becomes:

$$\begin{cases} \mathbf{x} \in F_1 \Leftrightarrow V_1(\mathbf{x}) < \dfrac{m_1}{\sqrt{2}} \\ \mathbf{x} \in F_2 \Leftrightarrow V_1(\mathbf{x}) \geq \dfrac{m_1}{\sqrt{2}} \text{ and } V_2(\mathbf{x}) < \dfrac{m_2}{\sqrt{2}} \\ \mathbf{x} \in F_3 \Leftrightarrow V_1(\mathbf{x}) \geq \dfrac{m_1}{\sqrt{2}} \text{ and } V_2(\mathbf{x}) \geq \dfrac{m_2}{\sqrt{2}} \end{cases} \qquad [4]$$

with $V_1(\mathbf{x}) = \dfrac{Y_1(\mathbf{x}) - U_1(\mathbf{x}) + m_1}{\sqrt{2}}$ and $V_2(\mathbf{x}) = \dfrac{Y_2(\mathbf{x}) - U_2(\mathbf{x}) + m_2}{\sqrt{2}}$.

V_1 and V_2 are two independent stationary standard Gaussian random fields with correlograms ρ_1 and ρ_2 respectively. Accordingly, the prior model (Equation [4]) is not fundamentally different from the plurigaussian model with constant facies proportions (Equation [1]): it just amounts to replacing Y_1 and Y_2 by V_1 and V_2 and defining the means of the random fields U_1 and U_2 by (Equations [1], [2], [4]):

$$m_1 = u_1\sqrt{2} = -0.782 \text{ and} \\ m_2 = u_2\sqrt{2} = 1.066. \qquad [5]$$

At each location $\mathbf{x}$, one can define proportion random fields (P_1, P_2 and P_3) associated with the different facies:

$$\begin{cases} P_1(\mathbf{x}) = G[U_1(\mathbf{x})] \\ P_2(\mathbf{x}) = \{1 - G[U_1(\mathbf{x})]\} G[U_2(\mathbf{x})] \\ P_3(\mathbf{x}) = \{1 - G[U_1(\mathbf{x})]\}\ \{1 - G[U_2(\mathbf{x})]\} \end{cases} \qquad [6]$$

where $G(.)$ is the standard Gaussian cumulative distribution function.

Spatial variations in the facies proportions can be reproduced via the incorporation of conditioning information on the facies proportions P_1, P_2 and P_3 (or, equivalently, on the random fields U_1 and U_2). For example, the probability of finding granodiorite (F_1) in the western sector of the area of interest is deemed less than 5%. Such a condition (P_1 less than 0.05) implies a constraint on the threshold random field U_1 in this sector, namely that it is less than $G^{-1}(0.05)$ (Equation [6]).

Conditional simulation

Quantification of geological knowledge

The geological knowledge on the deposit is quantified by inequality constraints on the facies proportions at a set of 980 'control points' located on a regular grid with mesh size 40 m × 40 m × 20 m (Figure 4):

➤ in the central part of the deposit where the tourmaline breccia prevails, the geologist assumes that there is less than 5% chance of finding another rock type

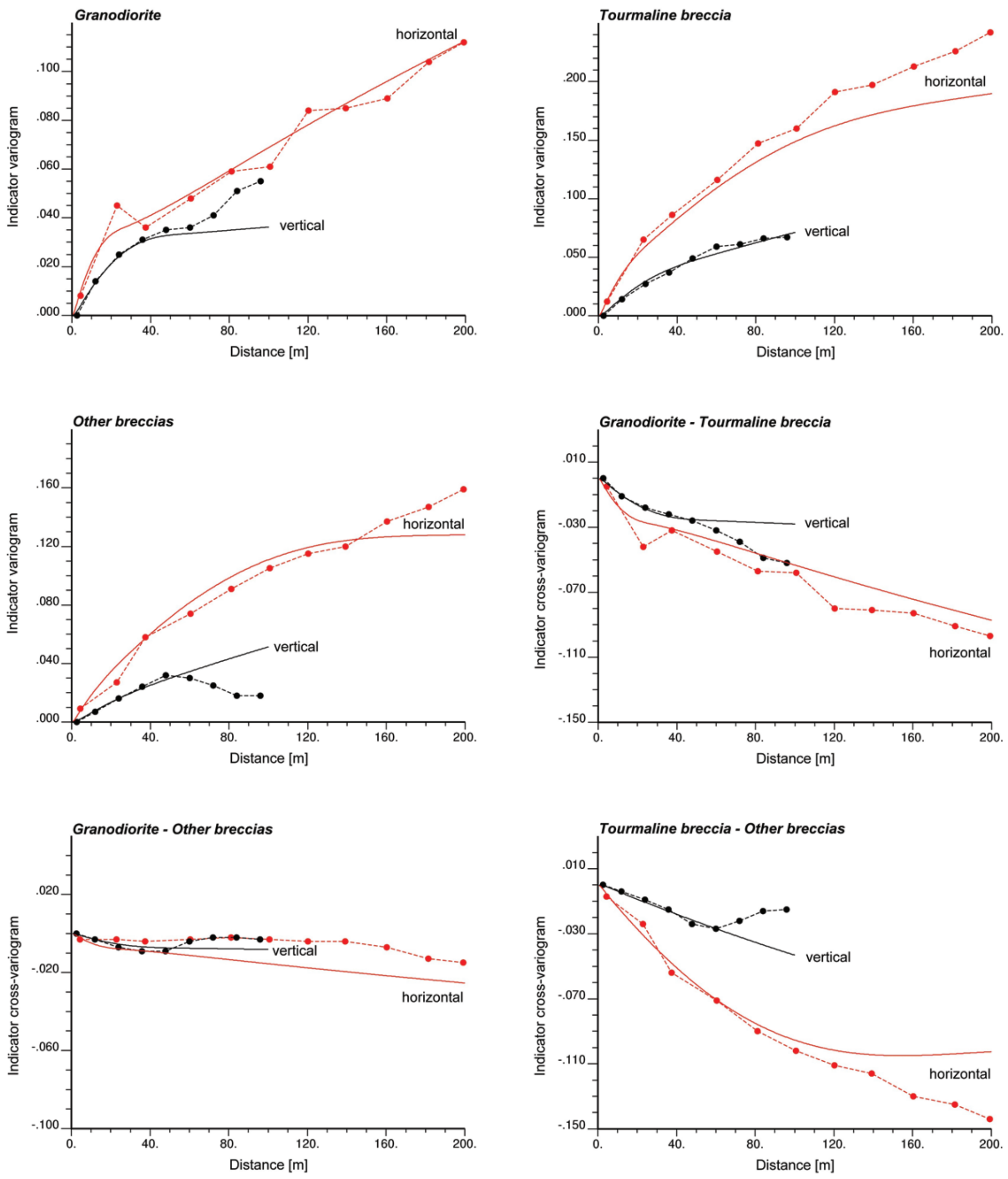

Figure 3—Simple and cross-variograms of the facies indicators (dash lines: sample variograms, continuous lines: variogram models)

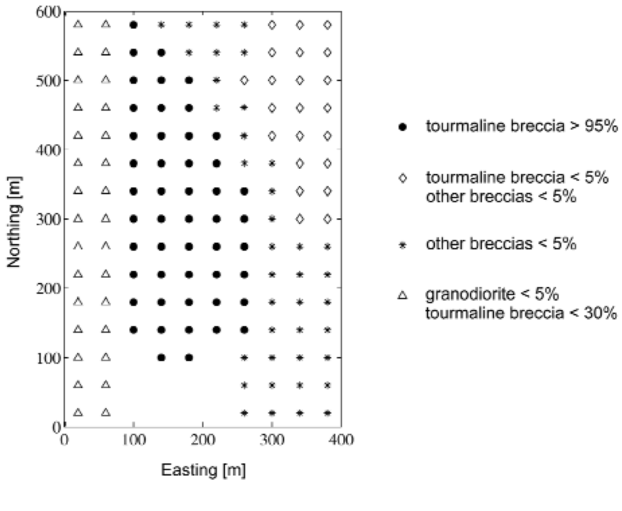

Figure 4—Mapping of interval constraints according to geological knowledge

- ➤ in the east and northeast parts, there is less than 5% chance of finding tourmaline breccia and less than 5% chance of finding other breccias
- ➤ in the southeast part of the deposit and in the contact zone between tourmaline breccia and granodiorite, there is less than 5% chance of finding other breccias
- ➤ in the western part, there is less than 5% chance of finding granodiorite and less than 30% chance of finding tourmaline breccia.

All these constraints are assumed to remain constant with depth, because of the geological continuity along the vertical direction.

Steps for conditional simulation

In practice, the simulation of facies (rock type domains) can proceed according to the following steps.

(1) Co-simulate the Gaussian random fields Y_1, Y_2, U_1 and U_2 at the sample locations and at the control

points, conditionally to the available data (drill hole data and interval constraints defined at the control points). This step can be performed by an iterative technique known as Gibbs sampler. At each iteration, one selects a point at random among the drill hole locations and control points and updates the Gaussian random fields at the selected point, conditionally to the values taken by the Gaussian fields at the other points.[27–29] In the present case, the Gibbs sampler is stopped after 1 000 000 iterations, so that the values of the Gaussian fields Y_1, Y_2, U_1 and U_2 at each conditioning point (drill hole sample or control point) are updated 300 times on average.

(2) Simulate the Gaussian fields Y_1, Y_2, U_1 and U_2 at the locations where the facies realizations are required. Since these fields are independent, each field is conditioned only to its values at the sample locations and at the control points obtained in the previous step. The simulation can be made with any Gaussian simulation method. In the case study, the turning bands algorithm is used.[22]

(3) Apply the truncation rule to obtain a rock type model (Equation [4]).

Results and discussion

Figure 5 displays nine conditional realizations of the facies, three of them corresponding to a random proportion model, three to a constant proportion model (Equation [1]), and the last three to a regionalized proportion model where local facies proportions are calculated in 100 × 100 m^2 grid cells and interpolated to the whole space by squared inverse distance weighting. The results call for several comments.

First, the constant proportion model is not always realistic. For instance, breccias can appear in the east and southeast parts of the deposit: although few drill holes are available in these sectors, the occurrence of breccias is highly improbable from a geological point of view. The regionalized proportion model improves this situation by imposing local facies proportions. However, this model has the following drawbacks:

- The interpolation of the proportions may be inaccurate and lead to misleading interpretations. For instance, the proportion of tourmaline breccia seems to be overestimated in the southeast part of the deposit (Figure 5)
- The interpolation smoothes out the proportions and creates artefacts in the proportion maps, which affects

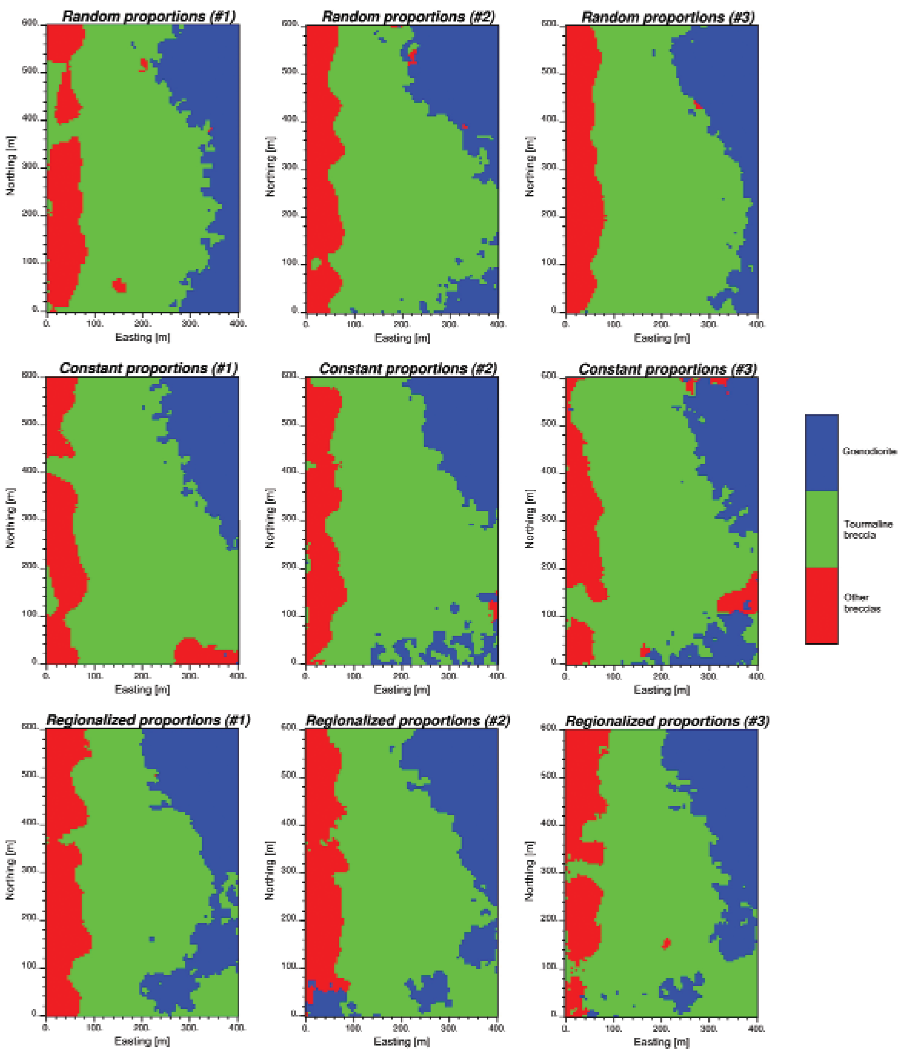

Figure 5—Rock type models obtained by plurigaussian simulations (representation of the bench of elevation 25 m)

the properties of the realizations. These artefacts are avoided in the random proportion model (Figure 6)

➤ Because the proportions are deterministic, the realizations do not account for the uncertainty in the facies that prevail in the regions where the geology is not well known.

The realizations corresponding to the random proportion model combine the advantages of the constant and regionalized proportion models: one avoids the occurrence of improbable facies in given sectors of the deposit, thanks to the constraints imposed at the control points, and still works in a stationary framework that facilitates parameter inference and variogram analysis. The reproduction of non-stationary patterns (spatial variations in the facies proportions) is deferred to the conditioning process.

Conclusions

The introduction of proportion random fields into the plurigaussian model combines two interesting aspects: (1), the prior model remains stationary, which is a very favourable situation for variogram analysis and (2), the facies proportions vary in space and can therefore account for changes in the geological characteristics of the deposit or reservoir. In addition to its simplicity and straightforwardness, the proposed approach allows integrating geological knowledge in the model (soft information) and considers the uncertainty in the facies proportions, especially in the sectors that are not well recognized, since it does not need to know the exact proportions at every location.

The random proportion model is quite general; the constant proportion model corresponds to the case when there is no geological constraint; while the regionalized proportion model is obtained by assigning hard constraints (exact values) to the facies proportions at each location to simulate. It is hoped that the presented case study will stimulate the application of plurigaussian simulations to mining and petroleum industry.

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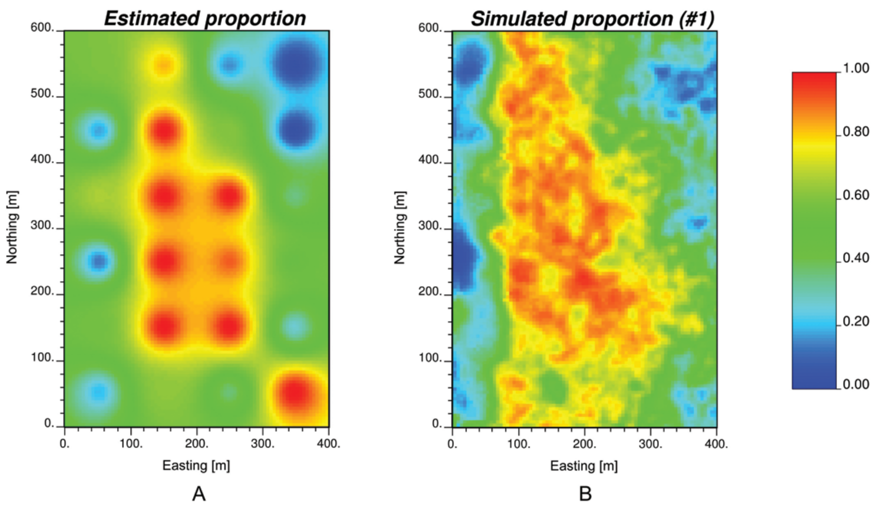

Figure 6—A, estimated and B, simulated tourmaline breccia proportion for the bench of interest

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