Detection and identification of ore grindability in a semiautogenous grinding circuit model using wavelet transform variances of measured variables

G.D. Gonzalez a,b,⁎, D. Miranda a, A. Casali b, G. Vallebuona b

a Electrical Engineering Department, University of Chile, Av. Tupper 2007, Santiago, Chile
b Mining Engineering Department, University of Chile, Av. Tupper 2069, Santiago, Chile

A B S T R A C T

A reduced dimension dynamic model subject to random disturbances for a semiautogenous grinding (SAG) circuit is developed that is able to handle changes in the characteristics of the new feed ore. This dynamic model, which is adjusted using data from a large mineral processing plant, has been developed with the objective of being useful for design and testing of fault identification and detection (FDI) systems, soft-sensors, automatic control systems, etc. The reduced dimension is a requirement in order to be able to obtain in a reasonable time statistical results to evaluate such systems. In this paper this SAG circuit dynamic model is used to test a method for detecting changes and identifying the grindability of the ore being processed by the SAG circuit. The method used — based on the variance of the continuous wavelet transform of measured circuit variables — incorporates improvements of a previous method. Results show that a step change of new feed ore is detected in about 30 to 80 min depending on the grindability change. This result may be considered to be adequate when taking into account that the response time for the mill hold-up to attain a new equilibrium value after the ensuing transient is of about 3 h. Identification of grindability in stationary operation gives near 100% of correct classification under the analysed conditions. The sensitivity of the FDI method to changes in circuit characteristics is also assessed and acceptable results are obtained.

1. Introduction

The principal aim of this work is to develop a method for on-line detection and identification of grindability of the ore being processed by a SAG mill. To test this procedure a SAG circuit model is developed which is able to handle changes in the characteristics of the fresh feed ore. The purpose of the model of the SAG mill circuit developed here is that it should behave like a real circuit from a qualitative point of view, and that from a quantitative point of view it should be within what might be expected of such circuits. Therefore the model must reproduce the main features of actual SAG mills in general, so that it may be used for designing and testing identification and detection systems for faults and operating conditions, automatic control systems, test soft-sensor designs, etc. The model developed here is of reduced dimension (i.e., five) in order to be able to obtain in reasonable time statistical results to evaluate such systems. The resulting systems may then be expected to be successfully tuned to a particular SAG mill or SAG circuit by adjusting their parameters, since the principal characteristics of these mills have been implicitly taken into account due to the involvement of the mill model in their design.

Such special dynamic model is not intended for mill design nor layout designs.

The quality of automatic control performance achievable in a closed circuit grinding operation is determined to a large extent by the ability to measure circuit variables and identify disturbances. Ore grindability is often one of the most important disturbances in grinding operations but, unfortunately, the possibility of measuring grindability on-line has seemed too remote, so the possibility of taking opportune corrective actions has been thwarted.

An estimation or measurement of ore grindability is also required for an optimal operation of mining facilities. A modern approach to grinding operations in the mining industry considers mining and mineral processing operations in a holistic manner, as in the Mine to Mill methods (JKTech, 2004). Mill throughput depends on ore grindability, usually characterized off-line for some ore samples (Morrell, 2004). Another possibility is to estimate the ore grindability from lithological characterization by image analysis and on-line measurements (Casali et al., 2001). However the mill is usually fed with ores from different mine sectors and also the ore is mixed in ore pass, stock piles, and so on. To overcome this problem it is better to identify on-line the grindability of the ore in the mill, so it can be correlated with the different ore types fed to the plant and with the mill throughput. A supervisory control system may be then designed to take the SAG mill circuit to optimal operating points by acting on
the set points (e.g., of hold-up) of the lower level SAG mill circuit control systems.

Fault detection and identification (FDI) methods will be used here to detect and identify the grindability of the ore, considering that different grindabilities produce different operating conditions which, although not considered fault situations, give rise to different characteristics of measured variables, some of which may serve to estimate the grindability of the ore in the mill. The variance of the continuous wavelet transform (Daubechies, 1992; Gonzalez, 2003, 2006) of the measured plant variables has been used in FDI methods. The method developed in Gonzalez et al. (2003, 2006) has been applied here to the identification of a grindability index, but also incorporating mean values directly in order to better discriminate between operating conditions determined by the grindability indexes.

2. The SAG circuit and its model for two kinds of ore

2.1. The SAG circuit model

Following the general objective stated above a SAG mill circuit dynamic model has been adjusted using data collected from a large mineral processing plant in Chile, in order to produce results approaching those which would be obtained for actual mills. In order to test the developed method, the SAG mill circuit dynamic model has to be able to handle changes in the feed ore characteristics.

The SAG circuit consists of a SAG mill and a screen as shown in the schematic diagram of Fig. 1. The dynamic model for the SAG mill is based on the constant ore model given in Amestica et al. (1993) and Amestica et al. (1996). In this case the ore sizes to be considered have been reduced to the five intervals shown in Table 1. This is required in order to have reasonably fast runs of this dynamic model, since several runs are necessary in order to generate statistical data.

An improvement with respect to the previous model (Amestica et al., 1996) is that the model has been expanded so that it is able now to keep track of two different kinds of ore. The reason for this is that if the characteristics of the feed ore change from a type A to type B set of characteristics, the ore in the mill is initially of type A and it takes time before it is ground out to the extent that all of it exits through the mill. Then, if grindabilities of the incoming ore may be represented by mixing a hard ore (e.g., of type A) and a soft ore (type B), this enlarged model may deal with feed ores whose grindabilities change with time.

Thus, if grindabilities of the incoming ore may be represented by mixing a hard ore (e.g., of type A) and a soft ore (type B), this enlarged model may deal with feed ores whose grindabilities change with time. In particular, let the type of feed ore undergo a step change from type A to type B. Then, if grindabilities of the incoming ore may be represented by mixing a hard ore (e.g., of type A) and a soft ore (type B), this enlarged model may deal with feed ores whose grindabilities change with time. In particular, let the type of feed ore undergo a step change from type A to type B. Then, if grindabilities of the incoming ore may be represented by mixing a hard ore (e.g., of type A) and a soft ore (type B), this enlarged model may deal with feed ores whose grindabilities change with time.
the general purpose of this model stated in the Introduction, it has been considered sufficient to account only for grinding from size $i$ to size $i+1$.

The mill feed flows for each size $j$ and ore type are $\mu_f \text{f}_{\text{A}j}$ and $(1-\mu)$ $\text{f}_{\text{B}j}$, and the discharge flows for each size $j$ are

$$F_{\text{sdA},j} = \frac{(1-c_E)K_{\text{mA}} \lambda_j}{\sqrt{H_s}}, \quad F_{\text{sdB},j} = \frac{(1-c_E)K_{\text{mB}} \lambda_j}{\sqrt{H_s}}, \quad j = 3, 4, 5 \quad (5)$$

where $c_E$ is the mill grate rejection coefficient and $K$ adjusts the total flow presented to the grate (Amestica et al., 1993, 1996).

The return flows of ore types A and B rejected by the screen are $\text{fs}_{\text{A}j}$ and $\text{fs}_{\text{B}j}$ (Fig. 2) for each size $j$. The water hold-up $H_w$ is given by

$$\frac{dH_w}{dt} = F_w - F_{\text{w}} \text{d}, \quad F_{\text{w}} = \left( \lambda_0 + \lambda_1 \right) H_w \quad (6)$$

where $F_{\text{w}} \text{d}$ is the water discharge flow, and $\lambda_0$ and $\lambda_1$ are model fitting parameters (Amestica et al., 1993, 1996).

Shared variables in these equations for ores type A and B are the total ore feed flow $F_{\text{f}}$, the power draft $P$ and the total ore hold-up $H_t$.

Let the total hold-up of the mill, including water hold-up $H_w$, be

$$H_{\text{nt}} = H_s + H_w. \quad (7)$$

Ore grindability is represented by a grindability index $f$ in the range [0.94, 1.06], where $f<0.94$ corresponds to hard ore, $f=1.06$ to soft ore, and $f=1$ to normal ore.

The particle size distribution of the feed ore is represented by four different vectors $f$ corresponding to fine ($f^1$), normal ($f^2$), coarse ($f^3$) and very coarse ($f^4$) size distributions.

This mill dynamic model is of the state/state-output class, resulting from the dynamic mass and water balances. The main components of the state vector are the ore masses of types A and B at the different sizes. Hence, given an initial state and the model inputs, a solution may be obtained. But, since the model in non-linear, an analytical solution is not practical, and simulation has been used.

2.2. Measured variables, disturbances and manipulated variables

The variables assumed to be measured are: $F_w = $water flow added to the mill input, $F_{\text{f}} = $fresh solids feed flow, $F_{\text{sd}} = $feed particle size distribution, $H_{\text{nt}} = $total hold-up of the mill (solids+water), $F_{\text{w}} = $return ore flow — consisting mainly of pebbles — rejected by the screen, and $P = $mill power draft.

The disturbances considered are: $v_{\text{w}} = $white noise zero mean random disturbance affecting measurement of sensor $k$, $w_j = $white noise zero mean random disturbance added to Eqs. (2) and (3) giving ore size $j$ in the mill, $w_{\text{w}} = $white noise zero mean random disturbance added to Eq. (6) giving the mass $H_s$ of water in the mill, $p_k = $random disturbance affecting the feed particle size distribution. These random disturbances — which are stochastic processes (Papoulis and Pillai, 2002) — have been added to account for uncertainties in model and variables. In addition, the following disturbances are considered: changes in the grindability index $f$, changes in the feed ore particle size distribution, and changes in the SAG circuit characteristics (represented by changes of the circuit model parameters).

The manipulated variables of the simulator are the feed flow $F_{\text{f}}$, and the feed flow added to the mill, $F_{\text{sd}}$. Usually there is a controller that fixes the ratio of ore and water flow, in this case according to $F_{\text{f}} = (F_{\text{f}} + F_{\text{sd}})$, so the only manipulated variable in this case is the feed ore flow $F_{\text{f}}$.

The screen performance is represented by rejection coefficients $c_{ij}$ affecting the mill discharge flows of sizes $j=3, 4$ and 5 entering the screen. Let the solids flow of size $j$ (see Table 1) passing the screen be $F_{\text{spj}}$ (Fig. 1). Then $F_{\text{spj}} = (1-c_{ij})F_{\text{sdj}}$, where $c_{ij}$ is the screen rejection fraction for ore flow $F_{\text{sdj}}$ of size $i$ ($i=3, 4, 5$). Sizes 1 and 2 are rejected by the mill grate, so $F_{\text{spj}}=0$ and $F_{\text{spj}}=0$. Then the rejected ore flows in Eqs. (2) and (3) for each size and ore type are $F_{\text{fA}ij} = c_{ij}F_{\text{sdA}j}$ and $F_{\text{fB}ij} = c_{ij}F_{\text{sdB}j}$.

The rejected ore flows $F_{\text{fA}j}$ and $F_{\text{fB}j}$ are conveyed back to the mill input where they join the new ore feed flows $F_{\text{fA}}$ and $F_{\text{fB}}$. The mesh opening size of the screen is in this case 12.7 mm (1/2"), so that the ore size 3 is rejected by the screen ($c_{ij}=1$), while ore size 5 passes the screen ($c_{ij}=0$) and a fraction $c_{ij}$ of ore size 4 is rejected. The rejection coefficient $c_{ij}$ has been used as a model adjusting parameter. The rejected flow for ore type A is then $F_{\text{fA}A} = F_{\text{fA}A} + F_{\text{fA}j},$ where $F_{\text{fA}A} = F_{\text{fA}A}$ and $F_{\text{fA}j} = c_{ij}F_{\text{sdA}j}$ and similarly for ore type B. It has been assumed that water rejection by the sieve is not significant.

2.3. Adjusting the SAG circuit model to data from a real SAG circuit

The mill model was adjusted using the following nominal values for the SAG mill operation: mass of lumps $H_{\text{s1}} = 110$ (t), total mass of ore in the pulp $H_{\text{p}} = 46$ (t), total mass of ore $H_{\text{p}} = 156$ (t), mass of water $H_{\text{w}} = 20$ (t), mass of balls $H_b = 194$ (t), equivalent porosity $c_{eq} = 0.35$, lumps porosity $c_{l} = 0.35$, balls porosity $c_{b} = 0.35$, solids density $\rho_s = 2.7$ (t/m$^3$), ball fraction $f_b = 0.12$, slurry density $\rho_{sl} = 1.79$ (t/m$^3$) and solids concentration $C_7 = 0.7$ (Amestica et al., 1993, 1996). In addition, new ore feed flow $F_{\text{f}} = 1217$ (t/h), screen rejected ore flow $F_{\text{w}} = 134$ (t/h), and total mill ore feed flow $F_{\text{f}} = 1351$ (t/h). The mill diameter is $D=9.8$ (m), its equivalent (cylindrical) length is $L=4.6$ (m), and the mill internal volume is $V_m = 347$ (m$^3$). The size distributions for the fresh feed $f_i$, screen rejection $f_c$, circuit product $f_p$, mill discharge $f_m$, and feed $f_f$ streams are given by $f_i = [0.00 0.00 0.00 49.9 50.1]$, $f_c = [0.00 0.00 0.00 49.9 50.1]$, $f_p = [0.00 0.00 45.5 45.1]$, and $f_f = [29.7 13.5 49.1 7.6 0.1]$ for the sizes in Table 1.

2.4. Response of the SAG circuit model

The SAG circuit model was used to build the SAG circuit simulator using SIMULINK from MATLAB. The simulator includes an option of PID automatic control of the total mill hold-up $H_{\text{nt}}$, as well as a ratio control of the feed water flow $F_w$ in proportion to the solids feed flow $F_{\text{f}}$. Using the SAG circuit simulator the response of the simulated circuit under various operating conditions has been determined.

2.4.1. Evolution of hold-ups of different ores

With the hold-up control off, and the water ratio control set at 0.63, the grindability index of the feed ore undergoes a large abrupt change from $f_{\text{A}} = 0.94$ (hard ore) to $f_{\text{A}} = 1.06$ (soft ore), at $t=8$ (h), after the circuit had settled down following an initial transient. Fig. 3 shows how the

![Fig. 3](https://example.com/fig3.png)
Identification, on the other hand, refers here to estimating the grindability index which the ore will reach or has reached in stationary conditions, e.g., as in time interval [8, 16] in Fig. 4.

Important performance characteristics here are: (i) percentage of hits (correct identification) and of misses (miss-identification, including failure to identify), and (ii) insensitivity in the identification of a given parameter such as $\Gamma$ to changes in equipment characteristics — e.g., changes of mill grate opening, ball load, screen opening, mill parameters affecting grinding — and to changes in some variables such as feed ore particle size distribution.

3.2. Detection and identification methods

Fault Identification and Detection (FDI) methods (Chen and Patton, 1999) are suitable in general for identifying operating points or detecting changes in operating points, even though no fault may be present. FDI methods may broadly be classified in (a) Model Based FDI, i.e., those which require a model of the plant or unit (multi-model approach (Chen and Patton, 1999)), and (b) Feature Based FDI, i.e., those which do not. In this latter case the different operating conditions are found by directly analyzing features of the measured variables.

The Feature Based FDI method is used here. Examples of features extracted from measured variables are, mean values, variances, auto-correlation functions, cross-correlation functions, etc., either of the variables themselves or of transformations performed on the variables such as Fourier transform or series, and continuous or discrete wavelet transforms.

In the present SAG circuit case the feature used is the continuous wavelet transform (CWT) variance of measured variables (Gonzalez et al., 2003, 2006). In this case this method is applied to measured variables of the SAG circuit of Fig. 1 (see Section 2.2). An overall view of such method shall be given here. Details may be found in the cited references.

The operations in this method comprise two stages: (i) feature extraction and (ii) feature processing.

3.2.1. Feature extraction

Let $y_i(t)$ be a measurement performed on the SAG circuit, e.g., $F_s(t)$, when the ore grindability index is $\Gamma(t)$, and let $Z_i(t)$ be the feature extracted from a given measurement $y_i(t)$, or a combination of the features extracted from a set of measurements (e.g., $F_s(t)$, $F(t)$, $F_d(t)$), when $\Gamma(t)$ is known. Feature $Z_i(t)$ shall be called characteristic template for grindability index $\Gamma$, for a given set of measurements, and are found off-line using experimental and historical records obtained from the plant. Feature $S_n(t)$, obtained on-line from measurement $y_j(t)$, or combining features extracted from a set of such measurements, when the grindability index $\Gamma$ is unknown, shall be called sample template. It will be used to estimate the unknown grindability index $\Gamma(t)$ under the current operating conditions by comparison with the set of characteristic templates.

When the variables exhibit a stochastic nature, as in this case (e.g., see Fig. 7) feature extraction from measured variables involves expected values. This is in general difficult, and sometimes impossible, because statistical parameters and functions (e.g., joint probability densities) are required which are generally unavailable from a practical point of view. However, these expected values may be estimated by means of time averages of a single outcome of the stochastic process (Gonzalez et al., 2003, 2006).

3.2.2. Feature processing

The sample template $S_i(t)$, and the characteristic templates $Z_i(t)$ ($i=1,...,m$) are now processed in order to distinguish, with acceptable error, which is the prevailing condition of the plant being analysed, by finding to which characteristic template the sample template is closest to, according to a given criterion. Methods that may be used to this end are, hypothesis testing, concatenation of functions, principal
consider the measured variables for a given random nature of the variables in the SAG circuit, the proper way to find the CWT of measured variables (e.g., in the SAG circuit case, \( F_{\gamma}(t) \), \( P(t) \), \( F_{\omega}(t) \)). The CWT of \( y(t, \Gamma) \) is:

\[
W(a, b, \Gamma) = \frac{1}{\sqrt{a}} \int_{-\infty}^{\infty} y(t, \Gamma) \Psi \left( \frac{t-b}{a} \right) dt.
\]  

(8)

It turns out that if \( y(t, \Gamma) \) may be considered a wide sense stationary random process — at least within a time frame — then the expected value of Eq. (8) is zero and its variance only depends on scale \( a \). Due to random nature of the variables in the SAG circuit, the proper way to consider the measured variables for a given \( \Gamma \) is as realizations — or outcomes — of a stochastic process \( y(t, \Gamma, \xi) \) where \( \xi \) accounts for the randomness and selects one of all the possible measurements that could have been obtained because of the random nature of the variables (Papoulis and Pillai, 2002). This implies that the CWT is random and has an expected value and a variance \( \mathbb{E}[W^2(a, b, \Gamma, \xi)] = V(a, \Gamma) \), where \( W(a, b, \Gamma, \xi) \) is the CWT of measured variable \( y(t, \Gamma, \xi) \) when the grindability index is \( \Gamma \). It turns out that (Gonzalez et al., 2003, 2006) these CWT variances \( V \) are only functions of the real variable \( a \) and of the parameter \( \Gamma \) and may be used as characteristic templates corresponding to operating condition determined by \( \Gamma \). But \( V=V(a, \Gamma) \) does not account for the mean of \( y(t, \Gamma, \xi) \) (Gonzalez et al., 2006), which may be important in the identification of \( \Gamma \). Then let the characteristic template for the \( j \)th measured variable — e.g., \( F_{\gamma}(t), P(t), F_{\omega}(t) \) — for operating condition \( \Gamma \), include such mean, i.e.,

\[
Z_j(a, \Gamma) = E \left[ W_j^2(a, b, \Gamma_i, \xi) \right] + E[y_j(\Gamma_i)].
\]  

(9)

For a single measurement \( y_j(t, \Gamma_i) \) made on the plant — following Gonzalez et al. (2006), let the sample template be defined as

\[
S_{mj}(a, \Gamma_i) = \frac{1}{T} \int_{0}^{T} W_j^2(a, b, \Gamma_i) dB + \frac{1}{T} \int_{0}^{T} y_j(t, \Gamma_i) dt.
\]  

(10)

It may be shown that this sample template is an unbiased estimator of the characteristic template \( Z_j(a, \Gamma) \) given by Eq. (9) (Gonzalez et al., 2006).

In the case of the SAG circuit, three basic operating conditions corresponding to three grindability indexes \( \Gamma \) have been chosen: (i) for hard ore with \( \Gamma_1 = 0.94 \), (ii) for medium ore with \( \Gamma_2 = 1.00 \), and (iii) for soft ore with \( \Gamma_3 = 1.06 \). Then, for each of the following measured variables indexed by \( j \): (a) feed ore flow \( F_{\gamma}(it)(j=1) \), (b) power draft \( P \), \( (j=2) \), and (c) rejected flow \( F_{\omega} \), \( (j=3) \), there are three characteristic templates \( Z_j(a, \Gamma_i) \) for \( F_{\gamma} \) corresponding to a given measurement when the ore in the mill is hard with coarse particle size.

3.3. Application to SAG circuit grindability index estimation

3.3.1. Feature extraction and templates

The feature used in the method originally developed by Gonzalez et al. (2003, 2006), is the variance of the continuous wavelet transform (CWT) of measured variables (e.g., in the SAG circuit case, \( F_{\gamma}(t) \), \( P(t) \), \( F_{\omega}(t) \)). The CWT of \( y(t, \Gamma) \) is

\[
\Gamma(a, \Gamma) = \frac{1}{\sqrt{a}} \int_{-\infty}^{\infty} y(t, \Gamma) \Psi \left( \frac{t-b}{a} \right) dt.
\]  

Following the good results reported by Gonzalez et al. (2006), the three characteristic templates for \( \Gamma \) as well as the three sample templates for the unknown \( \Gamma \) have been concatenated, i.e., its scales axis have been juxtaposed, as shown in Fig. 6. Let this concatenated characteristic template be \( Z(a, \Gamma) \) and the concatenated sample template be \( S_m(a, \Gamma) \) corresponding to the measured variables \( F_{\gamma}, P, F_{\omega} \).

Further template processing includes (i) discretisation of scales \( a \), (ii) dimension reduction using PCA (Mardia, 1979), (iii) projection of the reduced dimension templates into the Fisher space (Mardia, 1979), and decision on the Fisher space as to which characteristic template the sample template is closest to (Gonzalez et al., 2003, 2006).

4. Results

Results have been obtained by programming both the SAG circuit model and the grindability detection and identification method using MATLAB and Simulink.
4.1. Generation of templates

For each known value of the three grindability indexes \( \Gamma_i \) (i.e., 0.94, 1.00, and 1.06), 100 outcomes \( S_m(\Gamma_i) \), \( r = 1 \) to 100, of sample template variances were generated for each of the five values of \( J_0 \) in the set \{0.11, 0.12, 0.13, 0.14, and 0.15\}. These \( S_m(\Gamma_i) \) were determined using measurements of length \( T = 20 \text{ (h)} \) of the measured variables \( F_{sf}, P \) and \( F_{sr} \). Then characteristic template \( Z(\Gamma_i) \) has been determined by averaging these \( Q = 500 \) sample templates for a given \( \Gamma_i \) (Gonzalez et al., 2006). The result is three characteristic templates which are represented by three points in the Euclidean vector space 9F14. The projection of these templates onto the reduced PCA of dimension 8 is then projected to Fisher space of dimension 2 giving the characteristic templates \( Z_1(\Gamma_1), Z_2(\Gamma_2), \) and \( Z_3(\Gamma_3) \) (Gonzalez et al., 2006).

4.2. Identification results

Fig. 8 shows squared distances from the sample template projected into the Fisher space for \( \Gamma = 0.94 \) to 1.00 characteristic templates projected in that space when \( \Gamma \) changes from 1.00 (normal) to 0.94 (hard). The smallest distance is seen to correctly identify the unknown \( \Gamma \).

4.2.1. Identification of basic grindability indexes

In this case identification was done for cases when the actual grindability index had the supposedly unknown values of 0.94, 1.00 and 1.06. One hundred repetitions of the experiment were performed — using data collected from the SAG circuit model of Section 2 for each one of the three grindability values. The hits and misses then determine the percentages shown in Table 2, which contains identification results which include the effect of using both the mean value and template concatenation. Table 2 also has the results when only one of the three measured variables, \( F_{sr} \), is used for determining characteristic templates and sample templates. In each of such cases the same procedure for obtaining the Fisher space as in the concatenated case is followed, but considering the CWT variances for only one measured variable, and the corresponding covariance matrix. It may be seen that when concatenation is used there is 100% correct identification of \( \Gamma \), while if only one measurement such as \( F_{sr} \) is employed correct identification is at most 67.2 % (for \( \Gamma = 1.06 \)), and miss-identification attains a maximum of 34.6% when \( \Gamma = 1.00 \) is identified as \( \Gamma = 1.06 \). Results using only single measurements \( F_{sr} \) and \( P \) are similar to those using only \( F_{sr} \).

The benefit of adding the mean value of the measured variables to the variances and sample variances may be seen in Table 3. It may be seen that only when using both CWT and mean value together with concatenation 100% hits is achieved. The CWT by itself only gives a maximum of 61.8% correct identification (when \( \Gamma = 1.06 \)) without concatenation, which increases to 68% with concatenation. The identification result for both CWT and mean is only 67.2% (when \( \Gamma = 1.06 \)) without concatenation, but increases to 100% when concatenation is used. Similar results are obtained for feed flow \( F_{sf} \) and power draft \( P \).

4.2.2. Identification of grindabilities within a continuous range

Good results were achieved in the identification of grindability within a range using concatenation when the circuit was operating with ore having a grindability index \( \Gamma \) in the continuous range [0.94, 1.06]. In this case a linear interpolation procedure was used around the cases for \( \Gamma = 0.96, 1.00, \) and 1.06. Pairs of actual and identified \( \Gamma \), i.e. (actual \( \Gamma \), identified \( \Gamma \), are (0.940, 0.940), (0.970, 0.970), (0.990, 0.997), (1.000, 1.000), (1.010, 1.003), (1.030, 1.030), and (1.060, 1.059).

4.2.3. Sensitivity in the identification of \( \Gamma \) with respect to deviation from nominal conditions

In an ideal case the identification method should be insensitive to changes of circuit parameters when they change from the set of nominal parameters with which the templates were determined. However, it may be expected that there will exist some sensitivity, but it should be relatively small. In order to test such sensitivity, several tests were run where a set of selected parameters whose values have been randomly chosen in each test. These parameters are \( \epsilon_{sf} \) and \( K \) in Eq. (5), and \( \lambda_3 \) in Eq. (6) and are randomly generated from Gaussian distributions with the following means and relative standard deviations (SD/mean %): for \( \epsilon_{sf} \), 2.5% and 15.0%; for \( K \), 960 and 75%; and for \( \lambda_3 \) 34 and 15.0%. One hundred tests were made to obtain \( S_{mr}(\Gamma) \) when the ore in the circuit had either grindability index 0.94, 1.00, or 1.06. The results obtained for these random variations for normal particle size show the method is reasonably insensitive. In fact, correct identification percentages of grindability index \( \Gamma \) for normal particle size using concatenation, were found to be between 99.0% and 98.8%, while mistaken identification varied from 0.0% to 1.2%. The results for intermediate and coarse feed size are similar.

4.3. Detection results

Fig. 7 shows the evolution of the measured variables when feed ore changes, at \( t = 10 \text{ (h)} \), from normal \( \Gamma = 1.00 \) to hard \( \Gamma = 0.94 \). The feed ore flow \( F_{sf} \) is seen to decrease due to the action of the hold-up control

![Image](303x74 to 552x267)

<table>
<thead>
<tr>
<th>Table 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Correct identification (hits) percentage of ( \Gamma ) using CWT, mean values, and both, for only ( F_{sr} ) and for the concatenated case</td>
</tr>
<tr>
<td>Using</td>
</tr>
<tr>
<td>Actual ( \Gamma )</td>
</tr>
<tr>
<td>Fsr only</td>
</tr>
<tr>
<td>Mean only</td>
</tr>
<tr>
<td>CWT and Mean</td>
</tr>
</tbody>
</table>

![Image](713x381 to 890x542)

Fig. 7. Measured variables when a step change in the grindability index \( \Gamma \) from 1.00 to 0.94 happens at \( t = 10 \text{ (h)} \).
system in order to keep the total hold-up close to its fixed set point. Fig. 8 shows in the Fisher space the corresponding squared distances from the sample template and the three characteristic templates for hard, medium and soft ore. Mean detection delay \( t_{D} \) is determined here (see Section 3.1) by averaging several test results for detection times \( t_{D} \). The detection delay may be handled by means of a forgetting factor \( \lambda \) (Ljung, 1987; Gonzalez et al., 2006) which ponders past data in an exponentially decreasing form. The greater \( \lambda \), the greater the mean time between false detections — which is a good feature — but also the greater the detection time — which is undesirable. A compromise between rapid detection and low probability of false detection was reached by using a forgetting factor \( \lambda = 0.96 \). Mean detection times \( t_{D} \) for all possible step changes \( \Gamma \) between the three basic grindability indexes are: from 1.00 to 1.06, 62 (min); from 1.06 to 1.00, 33 (min); from 1.00 to 0.94, 36 (min); from 0.94 to 1.00, 58 (min); from 0.94 to 1.06, 78 (min); and from 1.06 to 0.94, 42 (min).

5. Conclusions

The behaviour of the dynamic model of reduced dimension developed here has shown to be good from a qualitative point of view, since its response is similar to that of actual SAG mill circuit. Concatenation of the characteristic templates, as well as of the sample templates, has resulted in important improvements in the identification of the grindability index, as compared with the cases in which templates for only a single measurement are used.

The detection and identification method considering the mean value of the measured variables, in addition to the variance of their continuous wavelet transform, has produced a considerable improvement in the identification of the ore grindability in all cases. For example, even with the help of concatenation, the correct identification percentages of normal ore type are 35% when only CWT variances are used, and 20% if only mean values of the measured variables are used. However, if both are taken into account this percentage increases to 100%.

The method was found to be reasonably insensitive to variations of some selected parameters, as expected due to the design.

Detection of a step change of grindability of the incoming ore has been found to take from somewhat more than 30 min to a maximum of 78 min for an extreme ore change.

The estimation of the grindability index may serve for a supervisory control (or an experienced operator) acting mainly on the fresh ore feed set point, to drive the circuit to desired operating points. Such action may be improved using measurement or estimation of other variables, e.g., feed size, ball loading.

It should be clear that the method developed here may be used not only for detecting and identifying other operating conditions in a SAG circuit, but for fault detection and identification in other plants of the minerals industry as well as in other industries.

Acknowledgements

Research leading to this paper has received funding from Project FONDECYT 1020741, and from the Departments of Mining and of Electrical Engineering of the University of Chile.

References


