

# Groundwater Management Using Model Reduction via Empirical Orthogonal Functions

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**Abstract:** This work presents a novel approach for solving groundwater management problems with reduced computational effort. We replace a groundwater flow model governed by a partial differential equation with a simple model governed by an ordinary differential equation. Model reduction is achieved with empirical orthogonal functions, i.e., principal components. Replacement of the full-scale model by a reduced model allows implementation of the embedding approach for optimal groundwater management. Comparing the results obtained with the full-scale simulation model, preliminary analyses show that the reduced model is able to reproduce head variations in the flow domain with good accuracy and, to a certain degree, the sensitivities of head with respect to pumping. A key advantage of the reduced model is that it is simple and easy to solve, and in many instances captures the dominating characteristics of the original model. In view of the many sources of uncertainty influencing groundwater simulation, the accuracy provided by a reduced model may be sufficient for planning purposes. As with other examples of model reduction presented in recent research efforts, the methodology shows promise in presenting general trends, but does not eliminate the need for the original model when more detailed analyses are needed.

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## Introduction

Numerical simulation of groundwater flow and transport constitutes the primary tool for evaluating the impact of human intervention on aquifer systems. In particular, approaches that use simulation and optimization techniques such as aquifer yield maximization (Feyen and Gorelick 2004; McPhee and Yeh 2004), conjunctive use of surface and groundwater (Emch and Yeh 1998; Philbrick and Kitanidis 1998; Barlow et al. 2003), contaminant plume containment (Shafike et al. 1992; Ritzel et al. 1994), and aquifer remediation and seawater intrusion mitigation (Duckstein et al. 1994; Reed and Minsker 2004; Hilton and Culver 2005) have been applied in screening the most efficient solutions to management problems. For a comprehensive review of simulation-optimization approaches to groundwater management, see Wagner (1995). As documented in the literature, simulation-optimization approaches require the inclusion of the groundwater (simulation) model, in some fashion, within the constraint set of the optimization (management) model. The influence-coefficient method (Becker and Yeh 1972; Maddock 1972; Ahlfeld and

Heidari 1994) achieves this goal by combining a first-order Taylor series approximation and the sensitivity of groundwater head to pumping/recharge. On the other hand, the embedding approach includes the finite difference or finite element approximations of the groundwater governing equations directly in the constraint set of the optimization model (Yeh 1975; Gorelick 1983; Gharbi and Peralta 1994; Feyen and Gorelick 2004). As a result, this approach is practical in cases where the groundwater model can be represented by a small to moderate number of computational nodes, in the order of tens of thousands. In the last decade, with the steady increase in computing power and data storage capacity, groundwater models have experienced a tendency toward large dimensionality. In many cases, the model may involve hundreds of thousands of nodes.

Additionally, there exist numerous situations where the special characteristics of the management problem in question require repeated runs of the simulation-optimization model. This is particularly true in instances where various decision makers and stakeholders are expected to require fast model outputs in contexts of real-time conflict resolution (Thiessen and Loucks 1992; Cai et al. 2004). Hence, in many instances, computationally expensive simulation models, while providing a more accurate and detailed description of the system under consideration, may be too slow to execute. The appropriate level of model complexity, given data availability and intended model application, has been the subject of intense research and reflection ever since simulation models became ubiquitous research and decision-making tools. Hydrologic simulation models are always too complex relative to the information contained in observed data. Additionally, observation errors combined with model structure uncertainty make it impossible to identify a unique model to represent the natural system. Consequently, it is plausible to seek a simple model to capture, in a consistent way, the behavior of a natural system. Another plausible approach is to seek a useful model for a par-

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