Visualizations of a *n*-ary Fragmentation Process with Neighborhood Interaction

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Abstract. It is defined a multiple fragmentation model in 2 dimensions. To represent the material defects, a random distribution of point flaws is considered. The number of generated fragments and breaking criterion also are random variables. The *n*-ary fragmentation process is produced by a breaking criterion that depends on the distribution of point flaws and neighboring fragments. The total mass is conserved. To stop each fragment iterative fracture a random stop is evaluated. The visualizations of the model present complex patterns of fracture that resemble real systems.

Keywords: Visualizations, Multiple Fragmentation Model
PACS: 62.20.Mk, 07.05.Tp

1. INTRODUCTION AND DEFINITION OF THE MODEL

The processes of fragmentation are complex phenomena that appear in different scale in nature and technology. Examples of fragmentation can be found in very large scale (astronomy), medium scale (mining processes) and microscopic scale (nuclear fission). In refs. [1,2,3] was given a long enumeration of some natural fragment size distributions of high energetic instantaneous breaking of brittle objects. This experimental evidence predicts a power-law behavior for small fragment masses:

$$F(s) = P(X(s) \leq s) \approx \alpha s^{-\beta} \quad \alpha, \beta > 0 \tag{1}$$

where s is the fragment's area or volume. The exponent β varies in the range [1.44,3.54], see refs. [1,2,3]. A mean-field type approximation to describe the fragmentation process can be formulated by means of the rate equations, see ref. [4]:

$$\frac{\partial c(x,t)}{\partial t} = -a(x)c(x,t) + \int_x^\infty a(y)c(y,t) f(x|y)dy \tag{2}$$

where: $c(x,t)$ is the concentration of fragments of mass less than x at time t, $a(x)$ is the rate at which fragments of mass x break into smaller ones and $f(x|y)$ is the conditional probability that a fragment of mass x was produced from a fragment of mass $y \geq x$. Using scaling and homogeneity assumptions some exact results were obtained, see ref. [4]. Several one and two dimensional fragmentation models have been analytically studied by the application of the rate equations. See for instance refs. [5,6,7].

The collision phenomena of solid but brittle discs with similar size were numerically studied in ref. [8]. To define the fracture process a molecular network of faults is considered. Using large-scale simulations and molecular dynamics techniques, the velocity, size and position distributions of the generated fragments were obtained. By

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means of a dynamical model of granular solids, a critical point between damage and fragmentation behavior was determined, see ref. [9]. Large-scale simulations and molecular dynamics techniques have also been applied to study fracture and fragmentation models in materials that present a previous distribution of cracks and/or flaws, see ref. [10].

Simple models for two-dimensional binary and *n*-ary fragmentation were numerically studied in refs. [11,12,13]. Its main assumptions are: material modeled as the discrete and continuous unit square; existence of q random point flaws that interact with the maximal net forces to produce the fracture; every piece fragmentation stops at each time step with probability p . A fragment size distribution with power-law behavior was determined for a wide range of the parameters.

In ref. [14], the explosive fragmentation process of two different two dimensional systems was studied: A disordered array of elastic beams and a Lennard-Jones liquid. For both models, the cumulative fragment size distribution has the form:

$$F(m) \propto m^{-\alpha} e^{-\left(\frac{m}{M_0}\right)} \tag{3}$$

in the long time limit, where M_0 is the typical fragment mass. This distribution was generalized for three dimensional experiments: Gypsum disks dynamic brittle fragmentation and elastic materials with cracks submitted to scalar strain fields, see ref. [15]:

$$F(s) \propto s^{\frac{(2D-1)}{D}} f_1\left(-\left(\tfrac{2}{\lambda}\right)^D s\right) + f_2\left(-s_0^{-1}\left(\lambda + s^{\frac{1}{D}}\right)^D\right) \tag{4}$$

where $F(s)$ is the number of fragments of size s , D is the Euclidean dimension of the system, λ is the penetration depth and f_1, f_2 can be approximated by exponential functions.

In ref. [16] were studied analytically and numerically two dimensional models that intent to describe the multiple fragmentation of flat brittle objects which suffer a strong impact on one of its sides. By analytical predictions and computational simulations it was obtained a power-law behavior for the fragment area distribution that exhibits two regimes, one of them proportional to $F(s) \propto s^{-\frac{1}{2}}$.

In this work a new model for *n*-ary fragmentation that generalizes refs. [11,12,13] to the case of fracture forces defined by neighborhood interaction is presented. The assumptions of the new model are:

1) The initial fragment is a continuous square or cube of linear size 1. To model the material imperfections are considered q random point flaws that remains fixed during the fragmentation process and interact with the distribution of forces (see refs. [8,9,10]). The parameter q was chosen constant.

2) At each step all the fragments will be broken in n fragments unless they satisfy the stop condition. The neighboring fragments apply a force f in the piece boundary, defined by

$$f \propto l_b s \tag{5}$$

   where l_b, s are the boundary length and area of the neighboring fragment.

   The largest force will be chosen to break the piece. The fracture lines are defined by the largest force and the n nearer point flaws. Moreover, every fracture line generates an additional random point flaw inside the fragment to simulate the effect of the fracture process.

3) The total mass (area) is conserved: The sum of the new fragments area is the same of the original fragment.

4) There are two situations in which the fragmentation process of a piece stops:

a) If the fragment area is smaller than the minimal fragment size or cutoff: m_{fs}.

b) With a constant probability p.

The random stop applies for fragments of area less or equal to the critical area a_c, introduced to represent the fact that larger fragments have more probability to be broken than the smaller ones. The parameter a_c will be chosen as a random variable in [0,1].

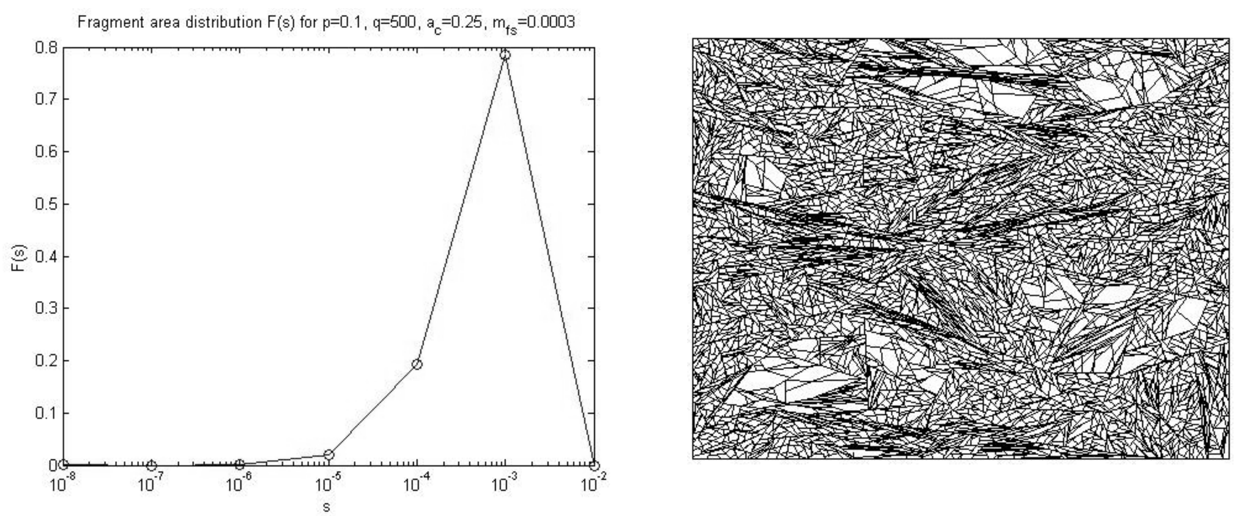
In the next section are presented the numerical results of the model defined in 1) – 4). The objective of this study was to determine the visualizations and the fragment size distribution.

2. VISUALIZATIONS AND NUMERICAL RESULTS

The numerical study was performed by medium scale simulations. The methodology for the simulations was the following:

- The parameters p, q, a_c, m_{fs} were chosen: $p = 0.08, 0.09, 0.1; q = 300, 500; a_c = 0.25; m_{fs} = 0.0001, 0.0003, 0.0005$.
- The results were averaged over several independent random initial conditions, defined by the initial forces and the random distribution of point flaws.
- The process of fragmentation evolves according to the rules 1) - 4) defined in the previous section.

It was determined that the fragment area distribution $F(s)$ follows approximately a log-normal distribution in the logarithmic scale, for the values of the parameters studied.



(a) (b)

FIGURE 1. Fragment area distribution $F(s)$ (a) and visualization (b) for $p = 0.1, q = 500, a_c = 0.25, m_{fs} = 0.0003$

The visualizations of the model present patterns of fracture that simulate real fragmentation processes of brittle materials like, for instance, glass. The log-normal distribution can be explained due to:

- The similarity in the fragments typical length
- The low frequency of the larger fragment

CONCLUSIONS

A new model for multiple fragmentation was studied numerically. Its main characteristic is that the forces of fracture are defined by the fragments of the neighborhood. By medium-scale simulations it was determined approximately a log-normal distribution in the logarithmic scale for the fragment area distribution. This result can be explained due to the similarity in the fragments typical length and the low frequency of the larger fragment. The visualizations of the model resemble real fragmentation processes of brittle materials.

ACKNOWLEDGMENTS

This work was supported by CONICYT Chile, grants FONDECYT 1050808 and 1070269 and UTFSM DGIP grant 240721.

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