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RESEARCH ARTICLE

Monopolistic competition in electricity networks with resistance losses

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Abstract We consider a pool type electricity market in which generators bid prices in a sealed bid form and are dispatched by an independent system operator (ISO). In our model, demand is inelastic and the ISO allocates production to minimize the system costs while considering the transmission constraints. In a departure from received literature, the model incorporates explicit description of the network details. The analysis shows that losses along transmission lines render the market imperfectly competitive. Indeed, competition among generators is qualitatively similar to the interaction among firms in a monopolistic competition setting. A lower bound for market prices is derived and it is shown that the costumers' cost of oligopolistic pricing is strictly positive. At a methodological level, we generalize standard oligopoly theory tools.

Keywords Electricity networks · Transmission losses · Spot markets · Monopolistic competition · Oligopoly theory

JEL Classification C61 · D43 · L94

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1 Introduction

A particularly common design in recently liberalized electricity markets organizes wholesale trading through an integrated market. In those markets, wholesale generators and costumers participate in a sequence of auctions. A central authority, hereafter referred to as independent system operator (ISO), coordinates the market participants with the purpose of optimizing the system operations. Examples of this organization can be found increasingly in the US, as well as in Colombia, Spain, and the UK.[1]

A key aspect differentiates wholesale electricity markets from virtually any other market, namely, electricity transfers take place on electricity grids, and electricity grids have a complex physics. Indeed, electricity transfers must respect transmission and generation constraints, Kirchhoff's laws, and unexpected network failures. The importance of network constraints on market outcomes has been recognized by several authors.[2] Indeed, network constraints may allow some generators to exercise market power even in the presence of several potential competitors. However, existing analytical frameworks for understanding the consequences of realistic network constraints on pricing strategies are limited in several aspects (we review the received literature below).

The present paper models a bid based integrated power market that functions on an arbitrary electricity network. In our model, generators submit prices in a sealed bid format and prices cannot be higher than the exogenously set price cap. The ISO uses the bids to represent the system costs and allocates production among generation units in order to minimize the cost of serving total demand. The model includes explicit description of the network details and their consequences on feasible allocations. We focus on the noncooperative outcome of the game among generators.

It is first shown that the interaction among generators can be understood in terms of standard economics. In fact, as a consequence of transmission losses along transmission lines, the game among generators is qualitatively similar to the interaction among firms facing Hicksian demands in a monopolistic competition setting. So, while the game analyzed becomes a first price auction in the no network case, the existence of transmission losses makes the game a competition in differentiated products. A simple two-generator example illustrates how the magnitude of transmission losses determines the intensity of market competition, which ranges from homogenous product Bertrand competition when there are no losses to purely monopolistic pricing when losses are sufficiently large.

While useful, the monopolistic competition analogy does not permit us to immediately derive estimates of equilibrium outcomes. The problem lies in the intrinsic non-differentiability of each generator's maximization problem (whose origin is precisely the presence of network constraints); we therefore carry out a quantitative stability analysis of the dispatch program to derive estimates of demand functions' slopes. Then, a lower bound for the markup that each generator sets is derived in terms of the network fundamentals. This bound shows that no matter the fine details of the

[1] See Wilson (2002) for additional details.

[2] For example, Borenstein et al. (2000) emphasize that "transmission constraints will be at the heart of market power issues in a restructured electricity market."

network, whenever a generator expects to be dispatched at some positive quantity, it exercises market power and raises its bid above its marginal cost. As a consequence, consumers' cost of oligopolistic pricing is strictly positive.

Though the presence of transmission losses is a key driver of our results, in practice they may be quite modest.[3] Nevertheless, their importance on market prices may be significant and indeed may be comparable to the impact of congestion on prices.[4] As a consequence, the introduction of transmission losses not only facilitates the analysis by "smoothing out" the model, but understanding its impact on pricing strategies is an interesting question by itself.

Prices above marginal costs echo the conclusions of previous works. However, our argument relies on a feature—namely the product differentiation pushed by the physics of power flows—that has not been formally captured by previous models and applies to a large set of electricity networks.

We explore a few examples to assess the gap between actual equilibrium prices and the lower bound. These examples show that the bound may bind, may provide a good approximation, or may be vaguely informative about equilibrium prices.

The methodology employed to analyze the model constitutes an additional contribution of this work. We borrow some tools from optimization theory, and study different stability concepts for minimization problems. Our approach contrasts with the standard framework for strategic competition on networks, which relies on properties for (convex and/or differentiable) variational inequalities and mixed complementarity problems. We view our methodology as a natural generalization of some techniques employed in oligopoly theory. New applications may arise in oligopoly models where demand functions are derived from complex optimization problems, as in Hotelling competition in several dimensions, or consumer problems with constrained decision sets.

Related literature This paper pertains to two strands of the economics literature on electricity markets. The first one analyzes the impact of generators' strategies in N-firm power pools on market outcomes. Green and Newbery (1992), in a pioneering paper, model the British pool by adapting the supply function equilibrium model introduced by Klemperer and Meyer (1989). Generalizations and extensions of Green and Newbery's model are given by Green (1999), and Newbery (1998). On the other hand, von der Fehr and Harbord (1993) do not smooth the bids and model the UK electricity pool as a first-price multi-unit auction. Fabra et al. (2006) analyze different auction formats for bid based pools. All of these models ignore the complexity of power transfers and analyze competition among generators not considering the transmission network as a potential source of market power.

A second set of works explicitly models the effects of network constraints on market outcomes. Borenstein et al. (2000) show that the competitive impact of small transmission lines may be important, while Joskow and Tirole (2000) emphasize the role of

[3] Although losses can amount to 3–8%, about half or more occur at distribution voltages that are not relevant to competition among generators.

[4] CAISO's studies illustrate this point. See, for example, Table 1 in page 12 of the August 2004 analysis of market-based price differentials at http://www.caiso.com/17ea/17eacf356fab0.pdf. We thank a referee for bringing this reference to our attention.

the structure of transmission rights. These authors model competition *à la* Cournot on small two-node networks. While these models are particularly tractable, the generality of the insights obtained is rather limited. Additionally, in these works market power is the consequence of binding transmission capacities. In contrast, in our model market power arises as a result of the intrinsic physics of power transfers.[5]

Since our model can be seen as a Hotelling-type model of monopolistic competition, this paper also relates to spatial competition models (e.g. Hotelling 1929; D'aspremont et al. 1979; Hobbs 1986; Anderson et al. 1989; Mulligan and Fik 1994). We contribute to this literature by adding a network to the analysis and deriving the differentiation among firms from network fundamentals. The methodology introduced to analyze the (non-differentiable) game among generators could also be exploited in other Hotelling-type models.

Variational inequalities have proven useful to study power markets. Wei and Smeers (1999), for example, model the long-term interaction among firms which build capacities and commit their outputs (Day et al. (2002) and Pang and Hobbs (2004) offer interesting generalizations). Hobbs et al. (2000) formulate the problem of finding a power market equilibrium as a mathematical program with equilibrium constraints. Ehrenmann and Neuhoff (2004) study a market design similar to the one studied in this paper and compare it to an unbundled system. These works address the problem of computation of equilibrium in pure strategies (which not always exist).[6] More closely related to the present work, Hu and Ralph (2007) study the problem of pure strategy existence in a model similar to ours.

The rest of the paper Section 2 gives a simple illustration of some of our results. Section 3 presents our general model, and introduces definitions, notation, and assumptions. Section 4 presents and exemplifies our results. Section 4 additionally discusses some extensions. Section 5 presents some final remarks. Supporting material and proofs are presented in the Appendix.

2 An example

Let us start by considering a symmetric two-node network. At each node $v = 1, 2$ there is an inelastic demand d and a generation firm with linear cost function $c(q_v) = cq_v$, with $c > 0$. Nodes are joined by a transmission line with resistance $r \geq 0$. This means that if the flow through the line from node 2 to node 1 is f, then the power loss is given by rf^2. We split the losses evenly among the nodes,[7] and consider the following balance constraints:

[5] More recently, Cho (2003) provides welfare theorems for a network market. In Cho's model, the physical environment in which agents interact is considerably more complex than Borenstein et al.'s. Yet Cho restricts participants' sophistication and studies the competitive equilibrium of the market. The present paper not only permits a more general network structure but also focuses on the more realistic case of strategic interaction.

[6] Some authors have also developed routines to solve for equilibrium in models with transmission losses. See Chen et al. (2006) and Bautista et al. (2007).

[7] There is no theoretical argument in favor of any particular way to split the losses. Here we split them evenly for the sake of tractability. Our general analysis can be applied to other conventions.

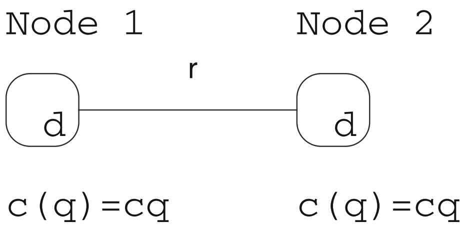

Fig. 1 Two-node model

$$d + \frac{r}{2} f^2 = q_1 + f,$$
$$d + \frac{r}{2} f^2 = q_2 - f.$$

These equalities may be interpreted as market-clearing conditions, in the sense that, at each node, total demand (local demand plus losses) has to equal total supply (local production plus effective flow). Firms compete bidding prices p_v representing constant costs per unit. Given the prices p_1, p_2, the ISO minimizes $p_1 q_1 + p_2 q_2$ while respecting the nodal balances (and the non-negativity constraints). Figure 1 illustrates this simple model.

This two-player game is symmetric. If $r = 0$, then there are no flow losses and we can think of the network as consisting of a single node. The game is just an homogenous product Bertrand game and generators bid prices that equal their marginal costs c. No market power is exercised.

Things are different when $r > 0$. In this case the marginal cost of power from generator 2 at node 1 is a strictly increasing function (and, of course, so is the marginal cost of power at node 2 from generator 1). Then, if generator 1 increases its bid a little, the quantity at which it is dispatched is just slightly smaller. Consequently, each generator faces a *continuous* non increasing demand function. Indeed, in the Appendix, it is shown that the quantities at which generators are dispatched are

$$Q_1(p_1, p_2) = d + \frac{1}{2r}\left(\frac{p_2 - p_1}{p_2 + p_1}\right)^2 + \frac{1}{r}\left(\frac{p_2 - p_1}{p_2 + p_1}\right),$$
$$Q_2(p_1, p_2) = d + \frac{1}{2r}\left(\frac{p_1 - p_2}{p_2 + p_1}\right)^2 + \frac{1}{r}\left(\frac{p_1 - p_2}{p_2 + p_1}\right).$$

Consequently, the game can be seen as a Bertrand competition model with differentiated products.

It should not be surprising that generators are able to exercise market power. In fact, it is not hard to see that

$$\frac{\partial Q_1(p_1, p_2)}{\partial p_1} = -\frac{1}{r}\frac{4p_1^2}{(p_1 + p_2)^3}$$

and therefore the necessary optimality condition for a symmetric equilibrium $\bar{p} = p_1 = p_2$ is $(\bar{p} - c)\frac{1}{r}\frac{4\bar{p}^2}{4\bar{p}^3} = d$. The only solution to this equation is $\bar{p} = c/(1 - 2dr)$

and indeed constitutes an equilibrium (see Appendix for details). Firms bid strictly above marginal costs, c, and there is no power flow along the transmission line. The very existence of transmission losses enables generators to set higher prices.[8]

In this model, market power has nothing to do with binding constraints. This contrasts with previous works where the source of network-driven market power is either generation or transmission constraints (e.g. von der Fehr and Harbord 1993; Borenstein et al. 2000). In this sense, market power is structural: regardless of the installed capacity and the details of the transmission network, prices will be above marginal costs.

3 The General Model

Now we present a general version of the simple two-node model introduced in the previous section. Consider an oriented connected graph (V, E), where V is the set of vertexes and E is the set of edges. Each vertex $v \in V$ is a network node. Each edge $e \in E$ represents a high-voltage transmission line. There is only one generator at each node $v \in G$, where $G \subseteq V$. Each node $v \in V$ has an inelastic demand $d_v \geq 0$.

Generators bid, in a sealed bid form, prices $p_v \in [0, p^*]$, where p^* is an exogenously defined price cap. After knowing the vector of bids $p = (p_v)_{v \in G}$, the ISO minimizes the overall cost $\sum_{v \in G} p_v q_v$ of serving the (inelastic) market demands by determining the quantities $q = (q_v)_v \in \mathrm{R}^G$ that the generators produce and managing the flows $f = (f_e)_e \in \mathrm{R}^E$ over transmission lines.

Not every tuple (f, q) is feasible, however. One of the main innovations of the present work is the explicit consideration of network constraints. These network constraints, which must be respected by the dispatch process, are detailed below.

Nodal balances At each node, available power must satisfy nodal demand. There are power flow losses in the transmission lines. A well known way to approximate losses is as a quadratic functions. Indeed, if the flow over $e \in E$ is f_e, the loss is given by $r_e f_e^2$, where $r_e \geq 0$ is the line resistance. Assuming that losses are split between the nodes associated to each line, the nodal power balances are[9]

$$\sum_{e \in K_v} \frac{r_e}{2} f_e^2 + d_v = q_v + \sum_{e \in K_v} f_e sgn(e, v), \quad v \in G \tag{3.1}$$

$$\sum_{e \in K_v} \frac{r_e}{2} f_e^2 + d_v = \sum_{e \in K_v} f_e sgn(e, v), \quad v \in V \backslash G, \tag{3.2}$$

where K_v is the set of transmission lines connecting node v and $sgn(e, v)$ is equal to 1 or -1 depending on the orientation of the graph and whenever $e = (v, w)$, $sgn(e, v) = -sgn(e, w)$. We also denote $K = \cup_{v \in G} K_v$. The left hand side of (3.1) is

[8] In fact, in this two-node model transmission losses play a role similar to that of transportation costs in a Hotelling model.

[9] Under our working assumption (A2), these equalities could also be written as inequalities that turn out to bind. This is not always the case; see, for example, Chao and Peck (1996).

half the sum of all the losses related to node v plus nodal demand d_v. The right hand side of (3.1) is the production of generator v plus the sum of effective flows.
Generation constraints Each generator has a nonempty production set

$$q_v \in [0, \bar{q}_v], \tag{3.3}$$

where $\bar{q}_v \geq 0$.
Transmission constraints Each transmission line $e \in E$ has a maximum safe capacity: $\underline{f}_e \leq f_e \leq \overline{f}_e$, where $\underline{f}_e \leq 0 \leq \overline{f}_e$. More generally, we consider the constraint

$$f \in F, \tag{3.4}$$

where $F \subset \mathrm{R}^E$ is a convex compact set. This formulation is general enough as to include Kirchhoff's voltage law constraints and other network constraints.[10]

We denote by Ω the set of pairs $(f, q) \in \mathrm{R}^E \times \mathrm{R}^G$ satisfying (3.1)–(3.4). We assume that Ω is nonempty.

After knowing the bids $p = (p_v)_{v \in G}$, the ISO runs the following *dispatch program*

$$\min \left\{ \sum_{v \in G} p_v q_v \mid (f, q) \in \Omega \right\}. \tag{3.5}$$

In words, the ISO optimizes the system operations as if the auction prices were generators' marginal costs. Of course, in practice this needs not be so for generators could game the dispatch process to maximize their profits.

We define $OPT(p) = \min\{\sum_{v \in G} p_v q_v \mid (f, q) \in \Omega\}$ and the set

$$Q(p) = \{q \in \mathrm{R}^G \mid \exists f \in \mathrm{R}^E, (f, q) \text{ is a solution of (3.5)}\}$$

of optimally generated quantities $q = (q_v)$. This set is nonempty and compact.

Generator v's payoff depends on the quantity at which it is dispatched and the clearing price at its node. When bids are $p = (p_v)_{v \in G}$ and the dispatch is $q \in Q(p)$, the payoff for generator v is

$$u_v(p_v, q_v) = p_v q_v - c_v(q_v), \tag{3.6}$$

where c_v is a real-valued cost function.

We are interested in the noncooperative outcome of this game. To be more precise, an equilibrium is a tuple of distributions $(F_v)_{v \in G}$ with supports contained in $[0, p^*]$, such that for some selection $\hat{q}(p) \in Q(p)$, $p \in [0, p^*]^{|G|}$, given the dispatch rule $\hat{q}$, no generator can improve its expected payoff by unilaterally modifying its prescribed distribution F_v.

In the sequel, we consider the following realistic assumption.

[10] Kirchhoff's voltage law: change in voltage over any loop is equal to zero. See Schweppe et al. (1988).

A1
(a) *Existence of transmission losses:* For all $e \in E$, $r_e > 0$;
(b) *Existence of demand:* For some v, $d_v > 0$.

For $v \in G$ and a vector of flows $f_{K_v} \in \mathrm{R}^{K_v}$ representing flows along transmission lines connecting node v, consider

$$T_v(f_{K_v}) := \sum_{e \in K_v} \frac{r_e}{2} f_e^2 + d_v - \sum_{e \in K_v} f_e sgn(e, v).$$

We make the following assumption on the network.

A2 For all $f \in F$ and all $v \in G$, $T_v(f_{K_v}) \geq 0$.

This condition restricts the structure of the electricity network; however it should not be deemed as particularly demanding. For example, it holds if at each node $v \in G$, $d_v = 0$, and v is connected by only one transmission line to the electricity network.

We also make the following assumption on firms' costs.

A3 For all v, c_v is convex and continuously differentiable and its derivative at 0, $c_v'(0) = \lim_{y \to 0} \frac{c_v(y) - c_v(0)}{y}$, is strictly positive.

Assumption (A3) implies that each firm is willing to produce a positive quantity only if the per unit price is strictly positive.

4 Analysis

4.1 Preliminary results: product differentiation and equilibrium existence

We begin by proving the following important and simple lemma.

Lemma 1 *If for all v, $p_v > 0$, then $Q(p)$ is a singleton. Moreover $Q(\cdot)$ is continuous on* $]0, p^*]^{|G|}$.

This lemma formally states that in our model competition is in differentiated products. At first glance, this property may not seem intuitive. Indeed, if the dispatch process were unconstrained, the whole market demand would be allocated to the generator bidding the lowest price. In this case, as usual in homogenous Bertrand games, a discontinuity unambiguously appears when two or more generators bid the same price. The key assumption allowing us to prove this continuity result is the existence of transmission losses. The intuition for this result has already been presented in Sect. 2.

It is natural to interpret the dispatch program as a minimum expenditure problem and the tuple $(Q_v)_{v \in G} \colon [0, p^*]^{|G|} \to \mathrm{R}_+^{|G|}$ as Hicksian demands. Moreover, by employing necessary optimality conditions, it is not hard to show that the demand system $(Q_v)_{v \in G}$ satisfies a law of demand.

Since our game is qualitatively similar to a Bertrand game with differentiated products, the existence of equilibrium can be analyzed using standard tools. Moreover, the equilibrium can be taken symmetric if the network and cost functions have a symmetric structure.

Proposition 2 *There exist distributions* $(F_v)_{v\in V}$ *that form an equilibrium.*

Note that in the standard first price auction, payoff functions are discontinuous and therefore existence cannot be ensured by employing straightforward fixed point arguments. While our game becomes a first price auction in the no network case, the existence of network constraints greatly simplifies the existence proof. Ensuring existence in pure strategies is hard because generators' payoffs are non-convex; we get back to this point in Sect. 4.4.

Because there are many complex network constraints, it is not possible to establish the kind of complementarity exhibited by the functions $(Q_v)_{v\in G}$. Furthermore, the demand functions (Q_v) are not differentiable even in simple networks. We therefore need to introduce new tools to understand more fully the interaction among generators.

4.2 Main results: the exercise and cost of market power

The main purpose of this subsection is to provide a lower bound for the markup set by each generator in equilibrium and estimate the consumer cost of oligopolistic pricing. In order to derive those results, the following result is key; see the Appendix for a proof.

Proposition 3 *Take a profile* $p \in]0, p^*]^{|G|}$ *and fix a generator* $w \in G$. *Consider* $\hat{p}_w \in]0, p^*]$ *such that* $p_w - \hat{p}_w$ *is small enough. Then,*

$$|Q_w(p) - Q_w(\hat{p}_w, p_{-w})| \leq |p_w - \hat{p}_w| \frac{\left(|K_w|(1 + \max_{e\in K_w} r_e \max\{\overline{f}_e, |\underline{f}_e|\}\right)^2}{\sigma},$$

where $\sigma = \frac{1}{2}\min_{v\in G}\{p_v(\min_{e\in K_v} r_e)\}$.

Proposition 3 shows that the demand system depends Lipschitz continuously on bids.[11] The result provides an estimate of the slope of the demand function faced by each generator and therefore it will be useful to study the trade-off each generator solves when bidding. To see intuitively why differentiability of the demand system cannot obtain, note that in the presence of several constraints, the solution of an optimization problem will typically be defined by parts. So, the solution exhibits several kinks and therefore will not be differentiable in general.

So far we have not analyzed whether equilibrium prices will be inefficiently high. We recall that the standard way to answer that question in Bertrand games with differentiated products is to derive first order conditions for each player's maximization problem (consult Vives 1999). This approach cannot be followed in our general model for demand functions are not differentiable. However, the Lipschitz property stated above allows us to give an estimation of equilibrium markups.

[11] Milgrom and Segal (2002) study differentiability properties of the value of a parameterized family of optimization programs. In a departure from those envelope results, we are interested in regularity results for optimization problems in terms of the *set* of optimal solutions.

Proposition 4 *Define the payoff that w obtains at equilibrium by playing the pure strategy price p_w as*

$$\alpha_w(p_w) = \mathrm{E}_{p_{-w}}[u_w(p_w, Q_w(p_w, p_{-w}))] = \int u_w(p_w, Q_w(p_w, p_{-w}))\mathrm{d}F_{-w}(p_{-w}),$$

where $\mathrm{E}_{p_{-w}}$ is the expectation with respect to the product distribution $\prod_{v \neq w} F_v$ and u_w is defined in (3.6). Take p_w maximizing α_w over $[0, p^*]$.[12] *Then, either $p_w = p^*$ or*

$$\mathrm{E}_{p_{-w}}|p_w - c'_w(Q_w(p))| \geq \frac{\mathrm{E}_{p_{-w}}[Q_w(p_w, p_{-w})]}{\bar{\eta}_w} \tag{4.1}$$

where

$$\bar{\eta}_w = 2\frac{|K_w|^2\big(1 + \max\{r_e \max\{\overline{f}_e, |\underline{f}_e|\} \mid e \in K_w\}\big)^2}{\min_{v \in G}\{c'_v(0)(\min_{e \in K_v} r_e)\}}.$$

If generator w has a linear cost function, $c_w(q) = c_w q$, then

$$p_w - c_w \geq \frac{\mathrm{E}_{p_{-w}}[Q_w(p_w, p_{-w})]}{\bar{\eta}_w}. \tag{4.2}$$

To prove this result we establish an optimality-like condition for generator w's maximization problem. Such optimality-like condition can be derived since Proposition 3 provides an estimate for the slope of the demand function.

A consequence of the result above is that each generator raises its price above marginal costs whenever it is dispatched with positive (but eventually small) probability. That can happen, for example, because the generator is a big producer or because the network isolates it with some demand (see also Example 6). On the other hand, if the generator bids prices close to its marginal costs, then it expects to be dispatched at a low quantity. More generally, the impact of any network constraint on market prices can be estimated by its effect on quantities normalized by a slope term; see Sect. 4.3.

We can interpret $\bar{\eta}_w$ as a measure of the slope of the demand function faced by w. Roughly speaking, the greater $\bar{\eta}_w$, the more elastic the demand function that the generator faces and the more aggressive its behavior.

The market outcome may or may not be efficient. For example, in the only equilibrium of the symmetric generator model analyzed in Sect. 2, production is efficient. However, the consumers' cost of market power is strictly positive, as stated in the following result.

Corollary 5 *Under the assumptions of Proposition 4, suppose additionally that for all v, $c_v(q_v) = c_v q_v$. Denote $c = (c_v)_{v \in G}$. Then, for any equilibrium distribution F*

[12] Note that, in general, the equilibrium may be in mixed strategies. In any mixed strategy equilibrium, generator w chooses prices that maximize $\alpha_w(p_w)$ with probability 1.

and any price p in the support of F there exists $\kappa(p) \geq 0$, such that

$$OPT(p) \geq OPT(c) + \kappa(p).$$

Moreover, defining $\psi(p) = (\psi_v(p_v))_{v \in G}$ by $\psi_v(p_v) = \frac{\mathrm{E}_{p_{-v}}[Q_v(p)]}{\bar{\eta}_v}$,

$$\kappa(p) = OPT(\psi(p)),$$

and it is strictly positive provided all players expect to be dispatched at positive quantities given p.

4.3 Examples

In this subsection we illustrate our results. We start by showing the impact of capacity constraints on market prices. This reasoning could be used to carry out a variety of comparative statics exercises (for example, an increase in transmission losses).

Example 6 (Exploiting generation constraints) Any feasible pair (f, q) satisfies

$$\sum_{v \in G} q_v \geq \sum_{v \in V} d_v + \sum_{e \in E} r_e f_e^2 \geq \sum_{v \in V} d_v. \tag{4.3}$$

Therefore, for all w

$$q_w \geq \sum_{v \in V} d_v - \sum_{v \in G, v \neq w} \bar{q}_v.$$

Assuming costs are linear,

$$p_w - c_w \geq \frac{\sum_{v \in V} d_v - \sum_{v \in G, v \neq w} \bar{q}_v}{\bar{\eta}_w}.$$

This expression shows how total demand impacts markets' pricing. In particular, it shows that if total network demand $\sum_{v \in V} d_v$ increases sufficiently, then the distribution of market prices will get closer and closer to the price cap and the bound will eventually bind.[13]

The following example complements the analysis of Sect. 2.

Example 7 Consider the electricity market described in Sect. 2 but now additionally suppose that the transmission line has limited capacity $f \in [-\bar{f}, \bar{f}]$, where $\bar{f} > 0$. For simplicity, assume that $d = 1$. In the no line capacity case, we already proved that the only symmetric equilibrium is given by $\bar{p} = \min\{c/(1 - 2dr), p^*\}$, provided

[13] This (formal) result is not obvious. It would be possible that the network isolates some generator and the demand increase could take place far away from that generator. The generator still will bid close to the price cap because the network is connected and, at some demand level, the ISO will need its production.

$1 - 2dr > 0$. Suppose that $\bar{f} > 0$ is large enough so that even when the line capacity is $\bar{f}$, the symmetric equilibrium is still an equilibrium. A sufficient condition for this is that the capacity is large enough so that the system is feasible even if one of the generators does not produce. This can be equivalently expressed as $r\bar{f} = 1 - \sqrt{1-2r}$. This condition is by no means necessary (see Borenstein et al. 2000). Moreover, in practice the line capacity will hardly be so large. As will be seen below, the RHS of the bound is decreasing in $\bar{f}$ so we are actually studying the performance of the bound in a worst-case scenario.

We can also use Proposition 4 to bound prices. The bound can be equivalently written as

$$\frac{\bar{p} - c}{c} \geq \frac{r}{2(1 + r\bar{f})^2} = \frac{r}{2(2 - \sqrt{1-2r})^2}.$$

The following table compares the actual markup to the lower bound.

r	$\frac{\bar{p}-c}{c} = \frac{2r}{1-2r}$	$\frac{r}{2(2-\sqrt{1-2r})^2}$	$\frac{\frac{r}{2(2-\sqrt{1-2r})^2}}{(\bar{p}-c)/c}$
0.01	0.02	0.005	0.24
0.05	0.11	0.022	0.20
0.1	0.25	0.041	0.16
0.2	0.67	0.067	0.1

We also note that

$$\lim_{r \to 0+} \frac{\frac{r}{2(2-\sqrt{1-2r})^2}}{(\bar{p} - c)/c} = \frac{1}{4}$$

so for $r \approx 0$ the bound accounts for 25% of the ratio $(\bar{p} - c)/c$.

The following example illustrates the role of binding transmission constraints.

Example 8 Consider a two node model as that in Sect. 2, but now assume that demand in node 1 is 0, while demand in node 2 is $d_2 = 1$. We also assume that costs are linear and heterogenous, given by $c_1 q$ and $c_2 q$ respectively with $c_1 < c_2$. The transmission line has capacity $\bar{f} > 0$. The market is so that firm 2 bids its marginal cost $p_2 = c_2$.[14] Given p_1, the dispatch program is

$$\min\{p_1 q_1 + c_2 q_2 \mid q_1 = \frac{r}{2} f^2 + f,\ q_2 = d + \frac{r}{2} f^2 - f,\ f \in [0, \bar{f}],\ q_i \geq 0, i = 1, 2\}$$

In this game, firm 1 is efficient because $c_1 < c_2$; however firm 1 is strategic so aims to maximize profits. We will analyze the pricing behavior of firm 1 when $\bar{f}$ is close to 0 so that the line is congested.

[14] We could assume that in that node there are several firms that behave competitively.

The optimal flow takes the form

$$f(p_1) = \min\left\{\bar{f}, \frac{1}{r}\frac{c_2 - p_1}{c_2 + p_1}\right\}$$

The line will be congested when the price p_1 is small enough so that $f(p_1) = \bar{f}$. Define $\bar{p}_1$ as the only solution to $\frac{1}{r}\frac{c_2-p_1}{c_2+p_1} = \bar{f}$ and note that given that $\bar{f}$ is small enough, $\bar{p}_1 = c_2\frac{1-r\bar{f}}{1+r\bar{f}} < c_2$ is in fact the optimal equilibrium price. Indeed, as shown in the Appendix, this will indeed be firm 1's optimal strategy provided

$$2c_2(r\bar{f})^3 \leq \left(c_2\frac{1-r\bar{f}}{1+\bar{f}} - c_1\right)(1+r\bar{f})^2$$

Therefore, for all such $r\bar{f}$,

$$\frac{\bar{p}_1 - c_1}{c_1} = \frac{c_2/c_1 - 1 - (c_2/c_1 + 1)r\bar{f}}{1 + r\bar{f}}.$$

Let $M(r\bar{f})$ be the RHS of this equality.

Proposition 4 offers the following estimate

$$\frac{\bar{p}_1 - c_1}{c_1} \geq r\bar{f}\frac{1 + \frac{r\bar{f}}{2}}{2(1 + r\bar{f})^2}.$$

Let $m(r\bar{f})$ be the RHS of this equality. We set $c_1 = 1$ and compute $m(r\bar{f})/M(r\bar{f})$ for different values of $r\bar{f}$ and $c_2 > 1$.

$r\bar{f} \diagdown c_2$	2	3	5
0.01	0.005	0.002	0.001
0.05	0.028	0.013	0.006
0.1	0.068	0.029	0.014
0.2	0.229	0.076	0.032

When $r\bar{f}$ is close to 0 (say, $r\bar{f} = 0.01$), firm 1 is facing a demand which is almost inelastic and therefore the equilibrium is so that firm 1 "undercuts" firm 2 by setting a price slightly below c_2. The bound, on the other hand, uses an estimate of the demand slope which is not precise. The bound therefore gets worse the more room firm 1 has to undercut firm 2 (the more different c_2 and c_1 are).

4.4 Extensions

The working paper version of the present work contains a number of extensions worth mentioning. First, demand is allowed to be random and the demand probability distribution is common knowledge. Second, firms are allowed to bid supply functions which must belong to an exogenously determined set (for example, firms bid convex nondecreasing cost functions). Third, each generator is paid its bid plus a capacity premium (that is, the nodal prices are the shadow values of electricity). It can be shown that to deal with the existence problem, none of our working assumptions is required. The key to proving that result is to introduce a topology on the sets of supply functions that ensures continuity of solutions and Lagrange multipliers for minimization problems. In this paper we have preferred to simplify the model in order to emphasize the economic and methodological substance of our contribution. Consult Escobar and Jofré (2008) for the more general results. In our view, the present framework could be used not only to analyze other phenomena in power markets (for example, collusion or contracts), but also to study a wider class of network markets.

We have assumed that each electricity plant bids independently of the others. Of course, in practice each plant is owned by a company that controls several generating units and, in principle, can manage these units to maximize the sum of profits. After careful inspection of our results, the monopolistic competition analysis can be extended to that more general context without significant difficulty.

In some markets, transmission constraints are taken into account only after the spot market is cleared. In others, firms' bids include startup costs as well as schedules of marginal costs. Those extensions are difficult because a key component of our analysis has been to see the dispatch process as a consumer theory problem; the consumer theory analogy breaks down, for example, when transmission constraints are considered only after the spot market is cleared. Some of our qualitative insights may apply; it would be interesting to study how our results would extend to those more realistic situations.

It would also be interesting to explore how robust the results are to alternative market designs. It is possible, for example, to use alternative pricing rules. Even with alternative pricing rules, our estimates of demand's slopes still apply. So, provided those pricing rules are sufficiently "smooth" as functions of the bids, we conjecture that our method of proof should go through and prices should be inefficiently high.

For numerical purposes, one would like to find sufficient conditions guaranteeing pure strategy existence for the game among generators. (We only show that a mixed strategy equilibrium exists.) However, the optimization problem characterizing the pure strategy best-replies is hardly a concave program. Consequently, techniques based on Kakutani's theorem (as our approach) do not seem suitable for approaching the pure strategy equilibrium existence problem. We are also considering such a broad class of electricity networks that it does not seem possible to state a general pure strategy equilibrium existence theorem. Our game does not possess a supermodular structure either. The question of pure strategy existence is an interesting and demanding research avenue.[15]

5 Concluding remarks

This paper shows that resistance losses in transmission networks matter. While the complexity of power transfers has been fully taken into account in an arbitrarily

[15] Hu and Ralph (2007) offer some results and examples.

large network, the economics of competition in electricity networks is surprisingly simple. Indeed, it has been argued that the model can be interpreted as a game among monopolistic competitors facing Hicksian demands. In particular, the physics of power flows renders the market naturally imperfectly competitive. Moreover, each generator exploits its monopolistic position by setting prices strictly above marginal costs. The bound for market prices shows that the impact of any other network constraint can be measured by its effect on quantities adjusted by a slope term.

We have explored a few examples to assess the tightness of the bound. In Example 6 we show conditions under which our lower bound binds. In Example 7, we have revisited our main example and showed that the bound can explain around 20 % of the ratio $(p-c)/c$. Example 8 shows that when transmission constraints are considered and the line is small, our lower bound does poorly when firms' costs are heterogenous enough. We hope these three examples clarify how our bound relates to equilibrium prices.

Our model ignores and simplifies several aspects of actual markets, of course. We, however, hope that the insights offered by this work are deemed as useful by economists who evaluate and design actual markets.

6 Appendix

This Appendix consists of three parts. In the first one, we go over the details of our leading example. In the second one, we detail the calculations for the equilibrium in Example 8. Finally, we provide proofs of the main results.

6.1 The example

The ISO solves the following optimization problem

$$\min\left\{p_1q_1+p_2q_2 \mid q_1+f=d+\frac{r}{2}f^2,\ q_2-f=d+\frac{r}{2}f^2,\ q_i\geq 0, i=1,2\right\}.$$

Substituting for q_i from the balance constraints and relaxing the constraints $q_i\geq 0$, we obtain

$$\min\left\{p_1\left(\frac{r}{2}f^2-f\right)+p_2\left(\frac{r}{2}f^2+f\right) \mid f\in \mathrm{R}\right\}.$$

The only solution to this problem is given by

$$f(p_1,p_2)=\frac{1}{r}\frac{p_1-p_2}{p_1+p_2}.$$

Therefore, the optimal quantities are given by

$$Q_1(p)=\max\left\{0, d+\frac{r}{2}f(p)^2-f(p)\right\},\ Q_2(p)=\max\left\{0, d+\frac{r}{2}f(p)^2+f(p)\right\}.$$

Given $p_2 \geq c$, there exists $\bar{p}_1 > p_2$ such that for all $p_1 < \bar{p}_1$, $Q_1(p_1, p_2) > 0$. On this set,

$$\frac{\partial Q_1(p_1, p_2)}{\partial p_1} = \frac{1}{r}\left(\frac{p_2 - p_1}{p_2 + p_1} + 1\right)\frac{-2p_1}{(p_1 + p_2)^2} = -\frac{1}{r}\frac{4p_1^2}{(p_1 + p_2)^3}.$$

Now, define firm 1's profit function as $u_1(p_1, p_2) = (p_1 - c)Q_1(p_1, p_2)$ and note that it is enough to find firm 1's best reply on the set $[c, \bar{p}_1]$. Let us look for a symmetric equilibrium $p_1 = p_2 = \bar{p}$. The necessary condition for an interior equilibrium $\bar{p} < p^*$, $\partial u_1(\bar{p}, \bar{p})/\partial p_1 = 0$, has as only solution $\bar{p} = c/(1 - 2rd)$. If $c/(1 - 2dr) > p^*$, then no interior equilibrium exists and the only solution to the (symmetric) optimality conditions is $\bar{p} = p^*$. To show that we indeed found an equilibrium, it is enough to show that $u_1(p_1, p_2)$ is concave in $p_1 \in [c, \bar{p}_1]$ when $p_2 = \bar{p}$. But note that

$$\begin{aligned}
\frac{\partial^2 u_1(p_1, p_2)}{\partial p_1^2} &= 2\frac{\partial Q_1(p_1, p_2)}{\partial p_1} + (p_1 - c)\frac{\partial^2 Q_1(p_1, p_2)}{\partial p_1^2} \\
&= -\frac{2}{r}\frac{4p_1^2}{(p_1 + p_2)^3} + (p_1 - c)\frac{-4}{r}\frac{2p_1p_2 - p_1^2}{(p_1 + p_2)^4} \\
&= -\frac{4}{r}\left(\frac{2p_1^2}{(p_1 + p_2)^3} + (p_1 - c)\frac{2p_1p_2 - p_1^2}{(p_1 + p_2)^4}\right) \\
&= -\frac{4p_1}{r}\left(\frac{p_1^2 + 4p_1p_2 - 2cp_2 + cp_1}{(p_1 + p_2)^4}\right).
\end{aligned}$$

This second derivative is negative provided $p_1^2 + 4p_1p_2 - 2cp_2 + cp_1 \geq 0$. Given that $p_1 \geq c$ and the polynomial is increasing when $p_1 > 0$, this condition is equivalent to $2c + 2p_2 \geq 0$. The following result summarizes the discussion.

Proposition 9 *Suppose that* $1 - 2dr > 0$. *The only symmetric equilibrium for the two-node model introduced in Sect. 2 is given by* $\bar{p} = \min\{\frac{c}{1-2dr}, p^*\}$.

6.2 Example 8

When the line capacity is not binding, the quantity at which 1 is dispatched is given by the expression

$$Q_1(p_1) = \frac{1}{r}\frac{c_2 - p_1}{c_2 + p_1} + \frac{1}{2r}\left(\frac{c_2 - p_1}{c_2 + p_1}\right)^2$$

so that the marginal utility is given by the expression

$$\begin{aligned}
&\frac{\mathrm{d}}{\mathrm{d}p_1}\{(p_1 - c_1)Q_1(p_1)\}|_{p_1=\bar{p}_1} \\
&\quad = \bar{f}\left(1 + \frac{r\bar{f}}{2}\right) - (1 + r\bar{f})\left(c_2\frac{1 - r\bar{f}}{1 + r\bar{f}} - c_1\right)\frac{1}{rc_2}\frac{(1 + r\bar{f})^2}{2(r\bar{f})^2}.
\end{aligned}$$

Bounding the first term on the RHS by $\bar{f}(1 + r\bar{f})$ shows that this derivative is negative under the condition in the main text. Therefore, at $p_1 = \bar{p}_1$ firm 1 is in the decreasing portion of the profit function and from concavity (which can be shown as we did in the previous subsection) it follows that firm will indeed find $p_1 = \bar{p}_1$ optimal.

6.3 Proofs of Sect. 4.1

Proof of Lemma 1 It is straightforward to show that the dispatch problem can be rewritten as

$$\begin{aligned} \bar{DP}(p) \quad & \min_{f_K \in \mathrm{R}^K,\, q \in \mathrm{R}^G} \sum_{v \in G} p_v q_v \\ & T_v(f_{K_v}) = q_v, \quad v \in G \\ & 0 \le q_v \le \bar{q}_v, \quad v \in G \\ & f_K \in P\Omega, \end{aligned}$$

where $P\Omega \subseteq \mathrm{R}^K$ is the set of vectors f_K such that there exists f_{-K} with $(f_K, f_{-K}) \in F$ (properly ordered). Consider the following *flow problem*

$$\min_{f_K \in PP\Omega} \sum_{v \in G} p_v T_v(f_{K_v}), \tag{6.1}$$

where

$$PP\Omega = \{f_K \in P\Omega \mid T_v(f_{K_v} \le \bar{q}_v,)\}$$

is a convex set. Since $p_v > 0$, if (f_K, q) is a solution of $\bar{DP}$, then f_K is a solution to (6.1), $T_v(f_{K_v}) = q_v$, $v \in G$, and the optimal values of both problems coincide. But $r_e > 0$ and $p_v > 0$, so the flow problem (6.1) is a strictly convex program. We deduce the existence of a unique f_K solving (6.1) which in turn implies the desired uniqueness result. The continuity of $(Q_v)_{v \in G}$ on $]0, p^*]^{|G|}$ follows as a consequence of Berge's maximum theorem. □

Proof of Proposition 2 Restrict each generator v to bid on the set $A_v := [c'_v(0), p^*]$, and consider *any* selection $q(p) \in Q(p)$, $p \in [0, p^*]^{|G|}$. From (A3), $(q_v)_{v \in G}$ is a continuous function on $\prod_{v \in G} A_v$. Therefore, the existence of equilibrium distributions $(F_v)_{v \in G}$ follows as a consequence of Glicksberg's theorem (see Fudenberg and Tirole 1991). □

6.4 Proofs of Sect. 4.2

Consider the following abstract optimization problem

$$\min_{x \in X} h(x), \tag{6.2}$$

where $X \subseteq \mathrm{R}^N$ and $h: \mathrm{R}^N \to \mathrm{R} \cup \{-\infty, +\infty\}$, and we define $A(h) \subset X$ as its set of optimal solutions. We already argued that A is typically not differentiable. Under the second order growth condition, however, A is a Lipschitz correspondence. For the sake of simplicity, we assume that A is a function; see Bonnans and Shapiro (2000) for more general results.

We say that the abstract optimization problem (6.2) satisfies the *second order growth condition* if there exists a neighborhood N of $A(h)$ and $\sigma > 0$ such that

$$h(x) \geq h(A(h)) + \sigma|x - A(h)|^2, \quad \forall x \in X \cap V.$$

Lemma 10 *Suppose that the abstract optimization problem* (6.2) *satisfies the second order growth condition and* $\hat{h} - h$ *is a Lipschitz function with constant* κ *on* $X \cap N$. *Then*

$$|A(h) - A(\hat{h})| \leq \frac{\kappa}{\sigma}.$$

Proof As h satisfies the second order growth condition,

$$\begin{aligned}\sigma|A(h) - A(\hat{h})|^2 &\leq h(A(\hat{h})) - h(A(h)) \\ &\leq \big(h - \hat{h}\big)(A(\hat{h})) - \big(h - \hat{h}\big)(A(h)) \\ &\leq \kappa|A(\hat{h}) - A(h)|,\end{aligned}$$

where the last inequality follows since $h - \hat{h}$ is a Lipschitz function. □

We now return to the analysis of solutions to the dispatch program. We define $f(p) \in \mathrm{R}^K$ as the solution to the flow problem (6.1). From the proof of Lemma 1, $Q_v(p) = T_v(f_{K_v}(p))$. To prove Proposition 3, we need the following second order growth condition.

Lemma 11 *Suppose that* $p_v > 0$ *for all* $v \in G$. *Then,*

$$\sum_{v \in G} p_v T_v(f^*_{K_v}) \geq OPT(p) + \sigma|f^*_K - f(p)|^2$$

holds for all $f^*_K \in PP\Omega$, *where* $\sigma = \frac{1}{2}\min_{v \in G}\{p_v(\min_{e \in K_v} r_e)\}$.

Proof Without loss of generality, suppose that $OPT(p) = 0$. It is not difficult to deduce that T_v is strongly convex on R^{K_v}: For all $\gamma \in]0, 1[$ and $f_{K_v}, f^*_{K_v} \in \mathrm{R}^{K_v}$

$$T_v(\gamma f_{K_v} + (1-\gamma)f^*_{K_v}) + \sum_{e \in Kv} \frac{r_e}{2}\gamma(1-\gamma)|f_e - f^*_e|^2 \leq \gamma T_v(f_{K_v})$$
$$+(1-\gamma)T_v(f^*_{K_v}). \tag{6.3}$$

□

Since $p_v > 0$

$$p_v\left(T_v(\gamma f_{K_v} + (1-\gamma)f^*_{K_v}) + \sum_{e\in K_v}\frac{r_e}{2}\gamma(1-\gamma)|f_e - f^*_e|^2\right)$$
$$\le p_v(\gamma T_v(f_{K_v}) + (1-\gamma)T_v(f^*_{K_v}))$$
$$= \gamma p_v T_v(f_{K_v}) + (1-\gamma)p_v T_v(f^*_{K_v}).$$

Now, pick $f_K, f^*_K \in PP\Omega$ and $\gamma \in]0,1[$. Since $\sigma \le \frac{r_e}{2} p_v$,

$$p_v T_v(\gamma f_{K_v} + (1-\gamma)f^*_{K_v}) + \sigma \sum_{e\in K_v}\gamma(1-\gamma)|f_e - f^*_e|^2$$
$$\le \gamma p_v T_v(f_{K_v}) + (1-\gamma)p_v T_v(f^*_{K_v}).$$

By summing over $v \in G$, we deduce that

$$\sum_{v\in G} p_v T_v(\gamma f_{K_v} + (1-\gamma)f^*_{K_v}) + \sigma\gamma(1-\gamma)|f_K - f^*_K|^2$$
$$\le \gamma \sum_{v\in G} p_v T_v(f_{K_v}) + (1-\gamma)\sum_{v\in G} p_v T_v(f^*_{K_v}).$$

Finally, since $OPT(p) = 0$,

$$0 \le \sum_{v\in G} p_v T_v(\gamma f_{K_v} + (1-\gamma)f^*_{K_v})$$

and then

$$\sigma\gamma(1-\gamma)|f(p) - f^*_K|^2 \le (1-\gamma)\sum_{v\in G} p_v T_v(f^*_{K_v})$$

for all $f^*_K \in PP\Omega$. Since $1-\gamma > 0$, $\sigma\gamma|f(p) - f^*_K|^2 \le \sum_{v\in G} p_v T_v(f^*_{K_v})$. By taking $\gamma \to 1$ the claimed inequality follows. $\square$

Proof of Proposition 3 For simplicity, suppose that $\bar{f}_e \ge |\underline{f}_e|$ for all e. Define $\hat{p} = (\hat{p}_w, p_{-w})$, $h(f_K) = \sum_{v\in G} p_v(T_v(f_{K_v}))$, and $\hat{h}(f_K) = \hat{p}_w(T_w(f_{K_w})) + \sum_{v\in G\setminus\{w\}} p_v(T_v(f_{K_v}))$. As $h(f_K) - \hat{h}(f_K) = p_w T_w(f_{K_w}) - \hat{p}_w T_w(f_{K_w})$,

$$|(h(f_K) - \hat{h}(f_K)) - (h(f^*_K) - \hat{h}(f^*_K))|$$
$$\le |p_w - \hat{p}_w||T_w(f_{K_w}) - T_w(f^*_{K_w})|$$
$$\le |p_w - \hat{p}_w||K_w|(1 + \max_{e\in K_w} r_e\overline{f}_e)|f_K - f^*_K|.$$

By virtue of Lemmas 10 and 11,

$$|f_{K_w}(p) - f_{K_w}(\hat{p})| \leq |f_K(p) - f_K(\hat{p})| \leq |p_w - \hat{p}_w| \frac{|K_w|(1 + \max_{e \in K_w} r_e \overline{f}_e)}{\sigma}$$

Consequently,

$$|Q_w(p) - Q_w(\hat{p})| \leq |p_w - \hat{p}_w| \frac{\left(|K_w|(1 + \max_{e \in K_w} r_e \overline{f}_e)\right)^2}{\sigma}$$

which proves the result. □

Proof of Proposition 4 Let $p_w < p^*$ and let us prove (4.1). For each $\Delta p > 0$ small enough (such that $p_w + \Delta p < p^*$), $\alpha(p_w) \geq \alpha(p_w + \Delta p)$. Consequently,

$$\begin{aligned} &p_w \mathrm{E}_{p_{-w}}[Q(p_w, p_{-w})] - \mathrm{E}_{p_{-w}}[c_w(Q_w(p_w, p_{-w}))] \\ &\quad \geq (p_w + \Delta p)\mathrm{E}_{p_{-w}}[Q_w(p_w + \Delta p, p_{-w})] - \mathrm{E}_{p_{-w}}[c_w(Q_w(p_w + \Delta p, p_{-w}))]. \end{aligned}$$

By reordering,

$$\begin{aligned} &p_w \mathrm{E}_{p_{-w}}[Q_w(p_w, p_{-w}) - Q_w(p_w + \Delta p, p_{-w})] \geq \Delta p \mathrm{E}_{p_{-w}}[Q_w(p_w + \Delta p, p_{-w})] \\ &\quad + \mathrm{E}_{p_{-w}}[c_w(Q_w(p_w, p_{-w})) - c_w(Q_w(p_w + \Delta p, p_{-w}))]. \end{aligned} \tag{6.4}$$

Take now a sequence of positive reals $(\Delta p_n)_{n \in \mathrm{N}}$ converging to 0 and a define $L^{\Delta p_n}(p_{-w}, d) = c'_w(Q_w(p_w + \Delta p_n, p_{-w}))$. Then,

$$\begin{aligned} &L^{\Delta p_n}(p)\,(Q_w(p_w, p_{-w}) - Q_w(p_w + \Delta p_n, p_{-w})) \\ &\quad \leq c_w(Q_w(p_w, p_{-w})) - c_w(Q_w(p_w + \Delta p_n, p_{-w})). \end{aligned}$$

Together with (6.4) this implies that

$$\begin{aligned} &\mathrm{E}_{p_{-w}}\left[\left(p_w - L^{\Delta p_n}(p)\right) \frac{Q_w(p_w, p_{-w}) - Q_w(p_w + \Delta p_n, p_{-w})}{\Delta p_n}\right] \\ &\quad \geq \mathrm{E}_{p_{-w}}[Q_w(p_w + \Delta p_n, p_{-w})] \end{aligned}$$

From Proposition 3

$$\mathrm{E}_{p_{-w}}[|p_w - L^{\Delta p_n}(p)|\, \bar{\eta}_w\,] \geq \mathrm{E}_{p_{-w}}[Q_w(p_w + \Delta p_n, p_{-w}))]. \tag{6.5}$$

Since $(L^{\Delta p_n})_{n \in \mathrm{N}}$ is a sequence of bounded functions, without loss of generality, it pointwise converges to L. Since c'_w is continuous, by taking $n \to +\infty$, we deduce that $L(p) = c'_w(Q_w(p_w, p_{-w}))$. By virtue of (6.5), (4.1) follows from the Dominated Convergence Theorem. □

Proof of Corollary 5 From Proposition 4, with probability 1, $p_w \geq c_w + \psi_w(p_w)$. Thus for all q, $p \cdot q \geq c \cdot q + \psi(p) \cdot q$. By taking min on both sides, it follows that $OPT(p) \geq OPT(c) + OPT(\psi(p)) \geq OPT(c) + \kappa(p)$. Clearly $\kappa(p)$ is positive provided $\psi_v(p_v) > 0$ for all v. □

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