# Computing with Multi-row Gomory Cuts

Daniel G. Espinoza\*

Universidad de Chile, Department of Industrial Engineering, Santiago RM, 837-0439, Chile daespino@dii.uchile.cl

Abstract. Cutting planes for mixed integer problems (MIP) are nowadays an integral part of all general purpose software to solve MIP. The most prominent, and computationally significant, class of general cutting planes are Gomory mixed integer cuts (GMI). However finding other classes of general cuts for MIP that work well in practice has been elusive. Recent advances on the understanding of valid inequalities derived from the infinite relaxation introduced by Gomory and Johnson for mixed integer problems, has opened a new possibility of finding such an extension. In this paper, we investigate the computational impact of using a subclass of minimal valid inequalities from the infinite relaxation, using different number of tableau rows simultaneously, based on a simple separation procedure. We test these ideas on a set of MIPs, including MIPLIB 3.0 and MIPLIB 2003, and show that they can improve MIP performance even when compared against commercial software performance.

#### 1 Introduction

The most successful approach to solve general MIP today is branch and cut, where general cutting planes are a crucial factor for the overall performance. After the great success in the 90's of using general purposes cutting planes such as GMI cuts [9,5], a great deal of research was devoted to extend those ideas to find other families of general cuts that consistently outperform GMI cuts. However, results have been mixed, and although there are several extensions that in theory are at least as good as GMI cuts, in practice they do not seem to offer much advantage. Most of the extensions have focused on deriving inequalities from the master cyclic group problem introduced by Gomory and Johnson [11], which look at a single constrained problem.

The theoretical importance of looking at multi-row relaxations has been proved in a number of works. For instance, Cook et al. [6], show an example with infinite Chvátal-Gomory rank (i.e. obtaining the convex hull of the integer points by adding inequalities derived from one row relaxations is impossible). Andersen et al. [3], prove that by looking at inequalities generated from two row relaxations, the convex hull of the Cook-Kannan-Schrijver example, can be

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obtained by adding a single cut. This situation is extended to higher dimensions in the work of Yanjun Li and Jean-Philippe P. Richard.

An interesting recent development has been the work of Andersen et al. [3], Cornuéjols and Borozan [7] and Gomory [10]; who have proposed to look at the so-called infinite relaxation problem, which was also introduced by Gomory and Johnson [11], and where several constraint are considered at the same time. The novelty of this relaxation is that it works on a continuous relaxation, and looks at an arbitrary number of tableau rows at the same time. Cornuéjols and Borozan [7] show that any minimal valid inequality for the relaxation can be related to maximal, convex, lattice-free polyhedrons; thus identifying relevant inequalities with simple geometrical entities.

To the best of our knowledge, no computational test of the impact of using cuts derived from this relaxation have been published. The main contribution of this paper is to show that they are also very valuable in practice, not only improving the root LP integrality GAP (GAP<sub>LP</sub>) closed at the root node, but also in speeding-up the overall branch and cut performance when compared with CPLEX [12] defaults.

The rest of the paper is organized as follows. Section 2 presents the definition and basic results related to the infinite relaxation. Section 3 presents the basic computational problems, tradeoffs, and main ideas that guided the implementation, and also some further ideas to speed-up cut-generation and possible alternative choices. Section 4 explain our experiments, settings, and results.

### 2 The Infinite Relaxation

Consider a general mixed integer program (MIP)

$$\min cx$$

$$s.t. Ax = b$$

$$x_i \in \mathbb{Z} \ \forall i \in I$$

$$x_i \geq 0 \ \forall i = 1, \dots, n,$$
(1)

where  $I \subseteq \{1,\ldots,n\}$ ,  $A \in \mathbb{Q}^{m \times n}$  is of full row rank,  $c \in \mathbb{Q}^n$ ,  $b \in \mathbb{Q}^m$ , and  $x \in \mathbb{Q}^n$ . Branch and cut algorithms start by solving

$$\min cx$$

$$s.t. Ax = b$$

$$x_i \ge 0 \ \forall i = 1, \dots, n,$$
(2)

the LP relaxation of (1), and obtain an optimal basic solution of the form

$$x_B = f + \sum_{j \in N} r^j x_j,\tag{3}$$

where B is the set of basic variables satisfying  $B \subseteq \{1, ..., n\}, |B| = m, N$  is the set of non-basic variables defined as  $N = \{1, ..., n\} \setminus B$ , and  $f, r^j \in \mathbb{Q}^m, \forall j \in N$ ,

 $f \geq 0$ . The basic solution is  $x^* = (x_B, x_N) = (f, 0)$ , and is an optimal solution to (1) if and only if  $x_i^* \in \mathbb{Z}, \forall i \in I' = I \cap B$ . If not, then one might try to find a valid inequality cutting off  $x^*$  from the feasible region of (2).

One possibility is to consider the following relaxation of (1):

$$z = f + \sum_{i \in N \cap I} (r^i - a^i) s_i + \sum_{i \in N \setminus I} r^i s_i,$$

$$z \in \mathbb{Q}^{I'},$$

$$s > 0,$$
(4)

where we drop all basic continuous variables, drop the non-negativity constraints on the basic integer variables, and where  $a^i \in \mathbb{Z}^{I'}, \forall i \in I \cap N, z = x_{I'} - \sum_{i \in I \cap N} a^i s_i$ , and then relax  $s_i$  to be continuous. This relaxation was considered in [3,10] for the case |I'| = 2.

Gomory and Johnson [11] suggested relaxing (4) to an infinite-dimensional space; following the notation in [7]; it can be described as:

$$x = f + \sum_{finite} rs_r$$

$$x \in \mathbb{Z}^q$$

$$s \ge 0$$
(5)

where  $s_r$  is defined for every  $r \in \mathbb{Q}^q$ , and  $\sum_{finite}$  means that  $|r:s_r>0| \in \mathbb{N}$ , i.e. s has finite support. This is called the *infinite relaxation* and is denoted by  $R_f$ , where the feasible solutions of  $R_f$  are vectors (x,s) with finite support satisfying (5). Note that any valid inequality for (5) yields a valid inequality for (1).

Borozan and Cornuéjols [7] studied minimal valid inequalities for (5), proving the following theorem:

Theorem 1 (Minimal Valid Inequalities for  $R_f$  [7]). If  $f \notin \mathbb{Z}^q$ , then any minimal valid inequality that cuts off (f,0):

- i. Is of the form  $\sum_{finite} \psi(r) s_r \geq 1$ .
- ii.  $\psi$  is positive, subadditive, homogeneous, convex and piecewise linear.
- iii. If  $B_{\psi} = \{x \in \mathbb{Q}^p : \psi(x f) \leq 1\}$ , then  $B_{\psi}$  is convex, with no integral point in its interior. Furthermore  $f \in B_{\psi}$ .
- iv. If  $\psi$  is finite, then  $\psi$  is a continuous nonegative homogeneous convex piecewise linear function with at most  $2^q$  pieces.
- v. If  $\psi$  is finite, then f is in the interior of  $B_{\psi}$  and  $B_{\psi}$  is a polyhedron of at most  $2^q$  facets, and each of its facets contains an integral point in its relative interior.

One of the consequences of Theorem 1 is that it allow us to identify minimal valid inequalities  $\psi$  with the set  $B_{\psi}$ , providing a simple geometrical interpretation for them. We use this interpretation to chose a sub-family of minimally valid inequalities for (5). It is worth mentioning that the results of Theorem 1 where simultaneously conjectured (and partially proved) by Gomory in [10].

## 3 Selecting a Subclass of Valid Inequalities, and Separating Them

Thanks to the results in [7], the problem of finding minimal valid inequalities for (5), can be reduced to the problem of looking at maximal lattice-free polyhedra in  $\mathbb{Q}^q$ , where the lattice is just  $\mathbb{Z}^q$ . Although the characterization of all maximal lattice-free convex sets in the plane is known [15], such a characterization is unknown for arbitrary dimensions.

For general dimension q, we can define the following full-dimensional maximal lattice-free bounded convex sets:

- 1. The simplex defined by the points  $\{0, \pm ke_i : i = 1, \dots, q\}$ .
- 2. The set  $B_a = \frac{1}{2} + \{x : a^{\delta}x \leq a^{\delta}\delta, \forall \delta \in \Delta\}$  where  $\Delta = \{\{-\frac{1}{2}, \frac{1}{2}\}^q\}, 0 < a^{\delta} \in \mathbb{Q}^q \text{ and } a_i^{\delta} \neq 0, \forall i = 1, \dots, q, \delta \in \Delta.$

These two classes of sets represent the two extremes in terms of number of facets; in the first family, each set has q + 1 facets, while in the second family, each set has  $2^q$  facets. Note also that each of their facets contains an integer point in their relative interior, thus they define minimal valid inequalities for (5).

For the case q=2, Cornuéjols and Margot [8] proved that all simplex-related sets (called triangle inequalities in [3]) are facet defining for  $R_f$ , but that not all  $B_a$  sets define facets of  $R_f$ . However, is easy to see that there exist an arbitrarily small perturbation  $\varepsilon$  of a, such that  $B_{a+\varepsilon}$  defines a facet of  $R_f$ . This observation, and the limited numerical precision of floating point representation, justify, from a practical point of view, overlooking the fact that some  $B_a$  do not define a facet of  $R_f$  for q=2. Although a similar result for arbitrary q is unknown, it seems reasonable to conjecture that related arguments should show the importance of the sets  $B_a$  in general.

This gives us a wide range of possible sets B to choose from. However, if we restrict ourselves to sets that are symmetric with respect to each coordinate axis, then, the only possible choice for B is the family  $B_a$ , where all  $a^{\delta} \equiv a$  for some  $a \in \mathbb{Q}^q_+$  (we assume that  $0 \notin \mathbb{Q}_+$ ). This restriction implies that the resulting cut should be invariant under multiplication of -1 to any constraint in (5).

From this point on, we focus on this kind of lattice-free sets. We assume that  $f \in (0,1)$ , and define  $f' = f - \frac{1}{2}e$ , where e is the vector of all ones. With this,  $\psi_a$  (the function related to  $B_a$ ) can be defined as follows:

$$\psi_a(r) = \begin{cases} 0 & \text{if } r = 0\\ 2 \max_{\delta \in \Delta} \left\{ \frac{\phi_{\delta}(a, r)}{a_o - \phi_{\delta}(a, f')} : \phi_{\delta}(a, r) > 0 \right\} & \text{if } r \neq 0 \end{cases}, \tag{6}$$

where  $\phi_{\delta}(a,b) = \sum (a_i \delta_i b_i : i = 1, \dots, q)$  and  $a_o = \frac{1}{2} a \cdot e$ .

Note that the amount of work to compute  $\psi_a(r)$  is exponential in q, however, one can speed up the process by using gray-code enumeration of  $\Delta$ . In our code we use Knuth's loopless gray binary generation (LGBG) algorithm [13] to speed-up the computation of  $\psi_a(r)$ , moreover, we compute  $\psi_a(r)$  for all required r at the same time. Additional speed gains can be achieved by noting that in LGBG, index i changes its value exactly  $2^{q-i}$  times during the algorithm, thus sorting each row in

decreasing order by number of non-zeros should decrease the amount of total work. Finally, another factor of two can be gained by maintaining a list of  $r: \phi_{\delta}(a, r) > 0$ .

Another problem is to choose appropriate vectors a. One possibility is to use branching pseudo-cost values (see [1,14] for details on pseudo-cost branching) to define the  $a_i$ . Instead, we use  $a_i = 1, \forall i = 1, \ldots, q$ , but select the fractional variables to consider using branching pseudo-cost information.

For integer non-basic variables we select  $a^i$  in (4), such that  $r^{i'} = r^i - a^i \in [-\frac{1}{2}, \frac{1}{2}]^q$ , in the hope of obtaining small coefficients for  $\psi_a(r^{i'})$ . Note, that such a choice may not be the best possible.

To improve numerical stability of the cuts, we choose from fractional variables that are away from the nearest integer by at least  $2^{-12}$ ; also, the ratio between the smallest and largest absolute value in the cut should not exceed  $2^{15}$ ; if the minimum non-zero absolute coefficient in the cut  $(|c|_{min})$  is above one, we divide the resulting cut by  $|c|_{min}$ ; we discard cuts whose violation is below  $2^{-10}$ ; finally, we add cuts only at the root node of the branch and cut run. The code is available at http://dii.uchile.cl/~daespino.

## 4 Computational Results

Our computing environment is a Linux 2.6.22 machine with 1Gb. of RAM, with a 3GHz. Intel Pentium 4 CPU with 1Mb of cache; all the code is written in C, and was compiled with GCC 4.2.0 with optimization flags -O3.

Our cutting scheme was embedded as a cut-callback in CPLEX 10.2, and is called after CPLEX has added its own cuts. In every call we add at most one cut, but the procedure may be called many times during the optimization process. Our procedure adds cuts only at the root node. We compare our results against CPLEX defaults, with pre-processing turned on; this include automatic generation of clique cuts, lifted cover cuts, implied bound cuts, lifted flow cover cuts, flow path cuts and Gomory fractional cuts.

Our test set of MIP instances contains all MIPLIB 3.0 [4], MIPLIB 2003 [2], and other problems from the literature. The full test set contained 173 problems, from where we discard all problems (29) where the GAP<sub>LP</sub> after solving with CPLEX 10.2 defaults was below 0.1%; then we discard all problems (34) CPLEX could solve to optimality within five seconds of CPU time; then we discarded all problems (15) where neither CPLEX nor our cutting procedure could improve the root LP bound<sup>1</sup>; finally, we discarded all problems (8) where our cutting procedure could not add any cut<sup>2</sup>. This reduced our test-set to 87 problems.

We tested six configurations, CPLEX defaults (C0), and the configurations T2N5, T5N5, T10N5, T10N1k and T15N1k<sup>3</sup>, where TxxNyy signifies the adding of up to yy cuts generated using up to xx tableau rows. The first four configurations where run with a time limit of one hour, while the last two configurations where run with a time limit of 20 minutes.

<sup>&</sup>lt;sup>1</sup> Table 1 contain the list of all such problems.

<sup>&</sup>lt;sup>2</sup> Table 2 contain the list of all such problems.

<sup>&</sup>lt;sup>3</sup> 1k stand for 1000, i.e. a thousand.

**Table 1.** Problems where neither CPLEX nor our procedure could improve the root LP value

berlin_5_8_0	CMS750_4	glass4	marketshare1	marketshare2
neos19	neos 818918	neos 823206	net12	noswot
p2m2p1m1p0n100	railway_8_1_0	rd-rplusc-21	usAbbrv.8.25_70	van

Table 2. Problems where our procedure could not add any cut to the problem

bg512142	dano3mip	dano3mip.pre	dg012142
harp2	mod011	momentum3	neos4

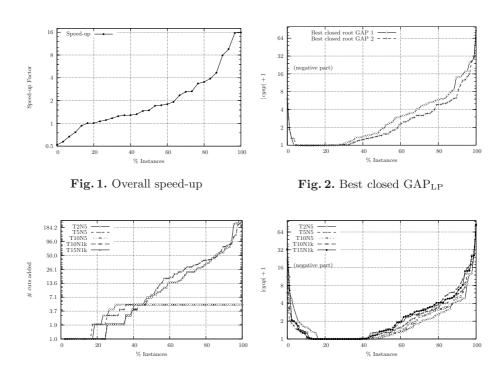


Fig. 3. Cuts added by configuration

Fig. 4. Closed  $GAP_{LP}$  by configuration

Tables 3, 4, 5 present our computational results over the reduced test-set. The first column indicates the problem name; the following six columns, give the root LP bound and the running time for the corresponding configuration; finally, the last column, has the optimal/best known solution for instance, and then the maximum of the closed GAP<sub>LP</sub>, defined as  $(Z_{LP} - Z_{CPX})/(Z_{IP} - Z_{CPX})^4$  over

<sup>&</sup>lt;sup>4</sup> where  $Z_{LP}$  is the root LP value for the configuration,  $Z_{IP}$  is the value of the optimal/best solution known for the problem, and  $Z_{CPX}$  is the value of the root LP obtained with CPLEX defaults.

Table 3. Results over reduced test set, part I  $\,$ 

Configuration	C0	T2N5	T5N5	T10N5	T10N1k	T15N1k	Results
	Root LP	Root LP	Root LP	Root LP	Root LP	Root LP	Best Sol.
Problem	Time (s)	Time (s)	Time (s)	Time (s)	Time (s)		CGap1/CGap2
	9770.47	9797.84	9854.74	9799.46	9799.46	9823.78	11503.4
alc1s1	3592.91	3593.45	3592.71	3594.60	1193.85	1194.55	4.86%/4.86%
	9804.58	9862.15	9869.86	9838.56	9838.56	9829.37	11503.4
A1C1S1		3600.33	3594.44	3584.48	9838.50	1196.85	
	3587.77						3.84%/3.84% 10938.8
A2C1S1	9468.86	9394.16	9394.16	9320.6	9320.6	9385.61	
	3590.22	3598.84	3598.90	3591.90	1195.77	1194.16	-5.08%/-5.08%
aflow30a	1075.14	1081.59	1074.58	1074.58	1074.58	1075.27	1158
	66.62	55.56	45.79	50.67	48.33	50.86	7.79%/7.79%
aflow40b	1059.61	1058.88	1059.61	1062.55	1062.55	1062.94	1168
	3585.42	3563.11	3597.93	3596.80	1193.03	1194.28	3.07%/2.71%
air04	55645.7	55664.6	55660.4	55663.3	55667	55664.9	56137
	42.33	75.30	48.47	64.48	154.15	405.37	4.33%/3.86%
air05	25957.8	25972	25970.5	25973.7	25978.6	25973.7	26374
anoo	31.48	83.59	79.81	66.11	500.72	295.92	5.00%/3.83%
B1C1S1	15496.3	15471.8	15463.4	15529	15529	15451.8	24881.7
BICIDI	3577.55	3586.90	3586.32	3579.90	1191.87	1192.43	0.35%/0.35%
B2C1S1	16012.5	16112.2	16165.8	15969.5	15969.5	16213	26282.5
D2C151	3587.55	3590.03	3588.81	3589.90	1190.73	1192.06	1.95%/1.49%
bc1	2.47	2.47	2.47	2.47	2.48	2.47	3.34
DCI	274.66	493.91	436.80	880.40	923.93	585.60	0.47%/0.26%
L =112=	873792	873792	873792	873792	873792	873792	878430
bell3a	7.09	7.17	7.53	7.22	7.54	7.28	0.00%/0.00%
	3.0605e+06		3.0605e+06	3.0605e+06	3.0605e+06	3.0605e+06	3.0678e + 06
biella1	3590.18	3591.99	3532.93	3593.41	1191.94	1195.15	-0.27%/-0.29%
	15.11	15.49	14.98	14.99	14.99	15.25	46.75
bienst1	431.12	850.79	331.77	475.22	463.64	503.64	1.20%/1.20%
	15.28	15.35	15.35	15.28	15.28	15.27	54.6
bienst2	3586.40	3599.01	3566.95	3591.94	1194.88	1195.43	0.18%/0.18%
	6086.37	6086.76	6087.58	6087.57	6088.08	6088.14	6217.86
blp-ar98	3563.80	3559.50	3563.07	3566.11	1165.68	1179.48	1.35%/0.92%
	3928.3	3928.38	3928.83	3928.46	3928.64	3928.81	4057.94
blp-ic97	3569.97	3377.47	3586.15	3400.25	1181.92	1176.14	0.41%/0.41%
	4376.19	4376.8	4378.04	4378.59	4381.97	4379.05	4531.39
blp-ic98	3560.93	3540.84	3595.65	3541.67	1142.70	1158.75	3.72%/1.55%
	2283.19	2283.67	2283.51	2284.5	2286.22	2287.46	2342.32
blp-ir98	541.68	606.95	725.03	467.55	922.14	973.49	7.22%/2.20%
	935.94	935.95	935.95	935.95	935.95	935.95	974
core2586-950	3423.24	3502.65	3587.97	3598.87	1411.34	1205.72	0.01%/0.01%
	1054.08	1054.08	1054.08	1054.08	1054.08	1054.08	1086
${\it core}4284 {\text -} 1064$	3598.72	3599.71	3599.47	3545.67	1196.53	1209.14	0.01%/0.00%
core4872-1529	1510.91	1510.91	1510.91	1510.91	1510.91	1510.91	1568
	3305.73	3598.09	3545.04	3599.30	1196.74	1196.19	0.00%/0.00%
danoint	62.73	62.73	62.73	62.73	62.73	62.73	65.67
	3589.38	3599.02	3598.22	3594.90	1198.09	1197.23	0.02%/0.02%
dc1c	1.7582e + 06				1.7575e + 06	•	1.8478e+06
	3591.28	3591.33	3591.63	3432.11	1192.99	1181.32	-0.84%/-0.84%
dc1l					1.7446e + 06		1.8517e + 06
	3588.70	3588.79	3588.84	3587.28	1194.80	1195.38	0.02%/0.02%
dolom1					6.5563e + 06		1.49e + 08
	3588.23	3586.81	3587.44	3585.06	1189.61	1190.51	0.00%/0.00%
ds	59.54	59.56	59.49	59.59	59.59	59.35	447.01
40	3584.19	3582.82	3585.46	3584.17	1194.23	1212.81	0.01%/0.01%
fast0507	172.15	172.17	172.18	172.18	172.18	172.18	174
iastuou/	2340.49	2862.75	1119.82	2467.56	1170.02	1250.19	1.95%/1.87%
0		2.5729e+07			2.5731e+07		2.5779e+07
gesa2-o	8.79	6.39	5.62	6.37	6.22	15.37	4.50%/4.50%
	560	560	560	560	560	560	1174
liu	3476.32	3467.47	3463.13	3452.70	1152.16	1153.31	0.00%/0.00%
			10		-52.10		/ 0.00/0

 ${\bf Table~4.}~{\rm Results~over~reduced~test~set,~part~II}$ 

Configuration	C0	T2N5	T5N5	T10N5	T10N1k	T15N1k	Results
Problem	Root LP	Root LP	Root LP	Root LP	Root LP	Root LP	Best Sol.
	Time (s)	Time (s)	Time (s)	Time (s)	Time (s)		
manna81	-13229.5	-13227.5	-13226.8	-13226.1	-13221.9	-13220.8	-13164
	3540.39	3548.56	3548.17	3524.98	1181.63	1180.20	13.22%/5.21%
mas74	10506.2	10555.8	10558.9	10535.5	10535.5	10538.2	11801.2
	2548.22	3203.12	2437.62	2299.57	1185.31	1180.43	4.07%/4.07%
mas76	38908	38925.2	38934.3	38911.1	38911.1	38911.8	40005.1
	241.62	189.35	175.21	325.39	319.60	210.48	2.40%/2.40%
misc07	1425	1425	1425	1425	1425	1425	2810
	23.53	38.43	28.10 -611.85	44.54	61.50	21.00	0.00%/0.00% -607.21
mkc1	-611.85	-611.85	39.64	-611.85 99.49	-611.85	-611.85	0.00%/0.00%
	19.25 -582.39	20.48 -582.39	-582.39	-582.39	89.70 -579.15	58.91 -581.79	-563.23
mkc	3547.53	3550.57	3562.28	3567.50	-579.15 1172.77	1181.56	16.91%/0.00%
-	96250.1	96250.1	96250.1	96250.1	96250.1	96250.1	10.9176/0.0076
momentum1	3592.25	3590.55	3594.12	3593.23	1193.81	1182.12	0.00%/0.00%
	10702.1	10702.1	10705.4	10702.6	10702.6	10705.2	12314.2
momentum2	3591.55	738.17	3288.60	3593.40	1196.20	749.59	0.20%/0.20%
			1.9702e+07				1.98e+07
msc98-ip	3583.89	3549.60	3593.76	3586.20	1196.60	1195.54	0.00%/0.00%
	-22066.1	-22067.3	-22066.2	-22063.3	-22061.5	-22067	-21718
mzzv11	548.37	471.03	819.70	541.25	491.16	551.90	1.32%/0.81%
	-20830.7	-20826.7	-20789	-20787.2	-20787.2	-20822.6	-20540
mzzv42z	130.53	140.58	155.44	157.69	161.51		14.96%/14.96%
	-1187.33	-1185.79	-1185.77	-1185.12	-1184.66	-1185.34	-1135
neos10	23.15	22.85	21.30	29.77	26.81	61.57	5.10%/4.23%
	6	6	6	6	6	6	9
neos11	446.38	405.21	192.80	216.16	231.75	203.14	0.00%/0.00%
10	9.51	9.51	9.51	9.51	9.51	9.51	13
neos12	644.55	971.43	1111.82	883.34	1099.59	656.93	-0.01%/-0.01%
10	-112.97	-112.75	-107.98	-109.42	-112.4	-112.63	-95.47
neos13	3596.18	3596.89	3696.19	3596.15	1190.17	1197.57	28.51%/28.51%
neos14	66464.1	66387.1	66381.6	66439.7	66439.7	66536.2	74333.3
Heos14	1555.23	1181.56	1426.66	1037.64	1007.33	1193.10	0.92%/-0.31%
neos15	70411.4	70172.5	70409	70418.4	70418.4	70415.8	80835
neos13	3591.99	3597.23	3595.22	3588.92	1197.49	1198.19	0.07%/0.07%
neos17	0.03	0.03	0.03	0.02	0.02	0.02	0.15
ncosii	3057.75	315.74	776.98	514.16	517.12	1192.86	0.13%/0.13%
neos18	13	13	13	13	13	13	16
	325.23	3575.88	2547.07	3572.41	1183.89	1051.34	0.00%/0.00%
neos20	-470.8	-470.8	-470.8	-470.8	-470.8	-470.8	-434
	186.19	61.84	84.35	134.95	124.60	124.43	0.00%/0.00%
neos21	2.72	2.74	2.73	2.73	2.74	2.75	7
	91.88	107.15	89.11	108.62	107.98	114.33	0.71%/0.45%
neos22	777536	777786	777676	777702	777702	777821	779715
	66.65	52.88	49.44	36.72	38.65		13.09%/11.51%
neos23	63.81	63.81	63.81	63.82	64.16	63.82	137
	653.86	1958.24	2709.85	813.83	1195.49	525.27	0.48%/0.02%
neos2	-4069.79	-4056.09	-4039.14	-4070.69	-4028.29	-3927.78	454.86
neos3	14.81	15.08	13.36	16.30	19.03	52.29	3.14%/0.68%
	-5664.36	-5630.32	-5655.1	-5643.04	-5643.04	-5731.26	368.84
	54.28	53.25	54.06	69.07	63.54	80.71	0.56%/0.56%
neos5	13.33	13.32	13.33	13.33	13.33	13.33	15
	3197.48	3579.64	3580.21	3367.37	1193.82	1194.55	0.00%/0.00%
neos7	692631	693168	693268	692631	692631	692532	721934
	137.84	59.85	59.72	51.88	51.01	1091.68	2.17%/2.17%
neos9	793.25	792.25	793.5	791.75	793.14	791.75	798
	3587.56	3586.40	3585.06	3586.00	1191.12	1190.05	5.26%/5.26%
nsrand-ipx	50181.8	50183.1	50184.8	50184.2 3609.45	50187.8 1185.31	50186.2	51200
	3588.37	3575.79	3583.57	ou09.45	1185.31	1198.76	0.58%/0.30%

Table 5. Results over reduced test set, part III

Problem								
Problem	Configuration		T2N5	T5N5	T10N5	T10N1k	T15N1k	Results
Solitary	Problem							
msrand-lays         3579.66         3583.67         3579.68         3571.66         1193.02         1201.12         1.58%/0.30%           mg08         204.28         204.31         204.37         204.37         120.37         120.37           mw04         16380.3         16771.3         16781.9         16781.3         16792.6         16787.9         1680.0           op11217         -20         -19         -10         -19         -19         -19         -19         -19 <td></td> <td>Time (s)</td> <td>Time (s)</td> <td>Time (s)</td> <td>Time (s)</td> <td>Time (s)</td> <td>Time (s)</td> <td>CGap1/CGap2</td>		Time (s)	CGap1/CGap2					
mug08         204.28         204.31         204.33         204.33         204.33         204.33         204.37         21.03.12         0.95%/0.5%         21.           nw04         16380.3         16771.3         16781.3         16781.3         16792.6         16787.9         16860.         0.92%/0.54%         nw04         158.00         46.19         51.81         52.10         112.66         907.47         85.59%/83.29%         98.25%/93.20%         19.19         10         19         30         20         20         20 <td></td> <td>50181.8</td> <td>50183.1</td> <td>50184.8</td> <td>50184.2</td> <td>50187.8</td> <td>50186.2</td> <td>51200</td>		50181.8	50183.1	50184.8	50184.2	50187.8	50186.2	51200
nuglos         47.44         63.14         73.89         90.37         126.99         185.04         9.2%/0.54%           nw04         16380.3         16713.3         16771.9         16781.3         16792.6         16787.9         16865           opt1217         -20         -19         55.90%/25.00%         25.00%/25.00%         25.00%/25.00%         20         0         0         0         0         0         0.00%/00%         0	nsrand_ipx	3579.66	3583.67	3579.68	3571.66	1193.02	1203.12	0.58%/0.30%
163803   16771.3   16791.9   16781.3   16792.6   16787.9   16866   16792.6   16787.9   16866   16792.6   16787.9   16866   16792.6   16787.9   16866   16792.6   16787.9   16866   16792.6   16787.9   16866   16792.6   16787.9   16866   16792.6   16787.9   16866   16792.6	00	204.28	204.31	204.33	204.33	204.37	204.37	214
nw04         58.00         46.19         51.81         52.10         112.66         907.47         85.59%/83.25%           opt1217         3553.84         3542.62         3541.18         3530.90         1193.49         1190.32         25.00%/25.00%           pk1         0	nuguo	47.44	63.14	73.89	90.37	126.99	185.04	0.92%/0.54%
opt1217         58.00         46.19         51.81         52.10         112.66         997.47         85.978/88,29%           opt1217         3553.84         3542.62         3544.18         3530.90         1193.49         1190.35         25.00%/25.00%           pk1         153.16         143.84         112.66         162.38         168.66         195.87         0.00%/0.00%           protfold         359.98         3599.40         399.94         3598.46         1199.45         1197.11         16.53%/11.63%           qap10         335.29         3535.5         333.45         333.51         333.52         333.51         133.84         12.26         11.63         346.42         23.97         11.63         347.02         440.23         34.47         23.94         293.04         293.04         1923.04         293.04         132.88         348.70         23.94         132.88         34.87         23.94         132.88         34.87         23.94         132.88         34.87         23.94         132.88         34.87         23.94         132.88         34.87         23.94         132.88         34.87         23.94         132.88         34.87         23.94         132.88         34.83         34.94         34.94	0.4	16380.3	16771.3	16779.9	16781.3	16792.6	16787.9	16862
opt1217         3553.84         3542.62         3544.18         3530.90         1193.49         1190.35         25.00%/25.00%           pkl         0	nw04	58.00	46.19	51.81	52.10	112.66	907.47	85.59%/83.25%
plat	1017	-20	-19	-19	-19	-19	-19	-16
poth         153.16         143.84         112.56         162.38         168.46         195.87         0.00%/0.00%           protfold         -41.09         -39.92         -41.09         -39.42         -39.78         -5.5%/11.63%           apple         333.55         339.94         3599.40         3598.46         1199.45         1199.71         15.5%/11.63%           apple         333.55         333.5         333.48         333.51         333.55         507.83         0.34%/0.23%           qu         -923.04         -926.83         -929.04         -7.66         7.46         68.85         0.00%/0.00%           rail507         3570.21         3320.93         3564.49         3559.79         1160.16         125.59         1.82%/1.40%           ran14x18-1         3362.27         3363.59         3363.68         3363.22         3363.59         3585.63         359.10         1195.36         1195.84         0.38%/0.38%           roll3000         12243.9         12257.5         12257.8         12259.4         12260.1         125.96/2.41%           roll40000         4985.46         985.53         398.53         988.46         985.46         985.56         10177.50           seymour         407.1	opt1211	3553.84	3542.62	3544.18	3530.90	1193.49	1190.35	25.00%/25.00%
protfold	1.1	0	0	0	0	0	0	11
proteidd         3599.88         3599.09         3599.09         3598.46         1199.45         1199.71         16.53%/11.63%           qap10         333.5         951.54         339.66         450.92         486.25         507.83         .344%/0.23%           qiu         -923.04         -926.83         -996.83         -992.04         -923.04         -126.07         -132.88         17.73         -172.18         172.18         172.18         172.18         172.18         172.18         172.18         172.18         172.18         172.18         172.18         172.18         182.14         <	pkI	153.16	143.84	112.56	162.38	168.46	195.87	0.00%/0.00%
Sample		-41.09	-39.92	-41.09	-41.09	-39.42	-39.78	-31
qap10         333.5         333.5         333.48         333.51         333.52         333.51         344/0.23%           qiu         -923.04         -926.83         -926.83         -926.83         -923.04	protfold		3599.40	3599.40	3598.46	1199.45	1199.71	16.53%/11.63%
qap10         395.29         951.54         399.66         450.92         486.25         507.83         0.34%/0.23%           qiu         77.25         92.21         95.20         77.66         74.46         68.85         0.00%/0.00%           rail507         172.15         172.17         172.17         172.17         172.17         172.18         173.37         122.37/14         182.28         182.28         182.28         363.22         363.29         363.93         363.68         363.22         363.22         361.99         373.33         363.03         385.63         385.63         385.63         385.63         385.63         385.64         985.66         1975.54         192.99         191.90         191.90         191.90         191.90         191.90         191.90         191.90         191.90         191.90         191.90         191.90         191.90		333.5	333.5	333.48	333.51	333.52		340
qiu         -923.04         -926.83         -926.83         -923.04         -923.04         -923.04         -122.8°           rail507         372.15         172.17         172.17         172.17         172.17         172.17         172.18<	qap10							0.34%/0.23%
qui         77.25         92.21         95.20         77.66         74.46         68.85         0.00%/0.00%           rail507         3570.21         3320.39         3564.49         3559.79         1160.16         1255.95         1.82%/1.46%           ran14x18.1         3588.84         358.75         3363.59         3363.68         3363.22         3363.22         3363.29         3363.68         3363.22         3363.22         3363.99         373.35           roll3000         12243.9         12257.5         12257.8         12259.4         12259.4         12260.1         1289           rout         985.46         985.53         3595.53         3595.53         3595.53         395.46         985.56         1077.5           seymourt         405.86         405.9         405.92         405.93         405.97         405.93         410.7           seymourt         407.17         407.2         407.63         407.63         408.2         408.17         222%/133           seymourt         407.17         407.2         407.63         407.63         408.2         408.17         225%/133           seymourt         353.68.81         358.51.81         3599.40         3599.24         1191.94         191								
rail507	qiu							
ran1507								174
ran14x18_1 3362.27 3363.59 3363.68 3363.22 3363.22 3361.99 3733 3588.84 3587.58 3587.68 3589.10 1195.36 1195.40 0.38%/0.38% roll3000 12243.9 12257.5 12257.8 12259.4 12259.4 12259.1 12260.1 12899 rout 928.46 3592.55 3595.23 3595.23 1194.49 1197.01 2.52%/2.41% rout 928.46 982.25 985.51 982.60 825.46 985.26 1197.0.09 seymour1 405.86 405.9 405.92 405.93 405.97 405.93 410.77 2180.19 2450.5 2407.13 2436.9 1200.26 1203.71 2257/1.33% seymour 3682.73 1505.80 3590.40 1305.40 1200.26 1203.71 2257/1.33% seymour 3682.73 1505.80 3590.40 1305.40 1195.45 1191.93 6.50%/2.95% siena1 10163e+07 1.0163e+07 1	rail507							
ranl4x18-1 ranl4x18-1 roll3000 12243-9 12257-5 12257-8 12259-4								3735
roll3000         12243.9         12257.5         12257.8         12259.4         12259.4         12260.1         12890           rout         3592.85         3596.57         3595.23         3592.53         1194.49         1197.01         2.52%/2.41%           rout         298.64         985.53         985.46         985.56         1077.5           zeymour1         405.86         405.9         405.92         405.93         405.97         405.93         410.70           zeymour1         407.17         407.2         407.63         407.63         408.2         408.17         422           seymour         407.17         407.2         407.63         407.63         408.2         408.17         422           seymour         3582.73         3590.80         3590.46         3590.24         1195.45         1191.93         6.50%/2.95%           sienal         1.0163e-07	ran14x18_1							
roll 3592.85 3596.57 3595.23 3592.53 1194.49 1197.01 2.52%/2.41% rout 985.46 985.53 985.53 985.46 985.6 985.56 1077.5t 219.81 299.25 195.17 126.08 122.64 81.27 0.11%/0.09% seymour1 405.86 405.9 405.92 405.93 405.97 405.93 410.76 2186.19 2455.05 2762.13 2433.69 1200.26 1203.71 2.22%/1.33% seymour 3582.73 3590.80 3590.46 3590.24 1195.45 1191.93 6.50%/2.95% sienal 3592.54 3593.03 3592.21 3593.66 1197.19 1199.08 0.00%/0.00% 3592.40 3593.66 1395.25 1199.39 6.50%/2.95% 3593.60 3590.46 3590.24 1195.45 1191.93 6.50%/2.95% 3593.60 3590.46 3590.24 1195.45 1191.93 6.50%/2.95% 3593.60 3590.46 3590.24 1195.45 1191.93 6.50%/2.95% 3593.60 3590.46 3590.24 1195.45 1191.93 6.50%/2.95% 3593.60 3590.46 3590.24 1195.45 1191.93 6.50%/2.95% 3593.60 3590.46 3590.24 1195.45 1191.90 80.00%/0.00% 3590.46 3590.24 1195.45 1191.93 80.00%/0.00% 3590.46 3590.24 1195.45 1191.93 80.00%/0.00% 3590.46 3590.24 1195.45 1191.93 80.00%/0.00% 3590.46 3590.24 1195.45 1191.93 80.00%/0.00% 3590.46 3590.24 1195.45 1191.93 80.00%/0.00% 3590.46 3590.24 1195.45 1191.93 80.00%/0.00% 3590.46 3590.24 1195.45 1191.93 80.5391.60 80.5391.60 8								
rout	roll3000							
rout 219.81 292.25 195.17 126.08 122.64 81.27 0.11%/0.09% seymour1 405.86 405.9 405.92 405.93 405.97 405.93 410.77   2186.19 2455.05 2762.13 2433.69 1200.26 1203.71 2.22%/1.33% seymour 407.17 407.2 407.63 407.63 408.2 408.17 422%/1.33% seymour 3582.73 3590.80 3590.46 3590.24 1195.45 1191.93 6.50%/2.95% sienal 1.0163e+07 1.0163e+								
seymour1         405.86         405.9         405.92         405.93         405.97         405.93         410.76           2186.19         2455.05         2762.13         2433.69         1200.26         1203.71         2.22%/1.33%           seymour         407.17         407.2         407.63         407.63         408.2         408.17         2.22%/1.33%           sienal         1.0163e+07         1.0664e+08         2.0164         2.2217e         2.217e         2.217e	rout							
seymour         2186.19         2455.05         2762.13         2433.69         1200.26         1203.71         2.22%/1.33%           seymour         407.17         407.2         407.63         407.63         408.2         408.17         422           sienal         1.0163e+07         1.026         1.026         1.026         1.026         1.026         1.026         1.026         1.026         1.026								
seymour         407.17         407.2         407.63         407.63         408.2         408.17         423           3582.73         3590.80         3590.46         3590.24         1195.45         1191.93         6.50%/2.95%           sienal         1.0163e+07         3.06%(0.00%)         3.06%(0.00%)         3.02336e         1.0264e         1.221e+08	seymour1							
seymour         3582.73         3590.80         3590.46         3590.24         1195.45         1191.93         6.50%/2.95%           sienal         1.0163e+07         6.64e+08         6.5391e+08         6.221e+08         4.2218e+08         4.2223e+08         4.2219e+08         4.2460.799           sp98ic         355.49.89         3556.06         3562.27         3558.49         1184.27         1175.80         2.57%/2.16%           spe3bic         44430e+08								
sienal         1.0163e+07         2.026         3.566.86         1.017.85         1.267.79         0.366/0.287         3.568.96         1.017.85         1.267.79         0.366/0.028         4.3219e+08         4.32219e+08         4.32219e+08         4.32219e+08	seymour							
seenal         3592.54         3593.03         3592.21         3593.56         1197.19         1199.08         0.00%/0.00%           sp97ar         6.5388e+08         6.5391e+08         6.64e+08           sp97ic         3591.66         3568.81         3588.03         3568.96         1170.85         1267.79         0.36%/0.32%           sp97ic         3533.65         3554.51         3551.67         3524.83         1136.89         1190.07         1.48%/0.79%           sp98ar         5.2499e+08         5.2504e+08         5.2509e+08         5.2511e+08         5.2509e+08         5.25								
sp97ar         6.5388e+08         6.5391e+08         6.5391e+08         6.5391e+08         6.5391e+08         6.5391e+08         6.5391e+08         6.5391e+08         6.5391e+08         6.64e+08         3591.66         3568.81         3588.03         3568.96         1170.85         1267.79         0.36%/0.32%         229         229         0.36%/0.32%         3591.67         3542.83         1136.89         1190.07         1.48%/0.79%         4.3e+08         3553.65         3554.51         3551.67         3542.83         1136.89         1190.07         1.48%/0.79%         4.49%/0.79%         3549.89         3556.06         3562.27         3558.49         1184.27         1175.80         2.57%/2.16%         3549.89         3556.06         3562.27         3558.49         1184.27         1175.80         2.57%/2.16%         352.86         3563.69         3580.06         1190.08         1174.74         2.53%/2.16%         352.86         3563.69         3580.06         1190.08         1174.74         2.53%/2.16%         352.16%         352.16%         362.27         22         22         22         22         22         22         22         22         22         22         22         22         22         22         22         22         22         22         22	siena1							
spy7ar         3591.66         3568.81         3588.03         3568.96         1170.85         1267.79         0.36%/0.32%           sp97ic         4.2211e+08         4.2217e+08         4.2218e+08         4.223e+08         4.2219e+08         4.3e+08           3553.65         3554.51         3551.67         3542.83         1136.89         1190.07         1.48%/0.79%           sp98ar         5.2499e+08         5.2504e+08         5.2508e+08         5.2509e+08         5.2511e+08         5.2509e+08         5.3e+08           3549.89         3556.06         3562.27         3558.49         1184.27         1175.80         2.57%/2.16%           sp98ic         4.4430e+08         4.4445e+08         4.4445e+08         4.4448e+08         4.4444e+08         4.5141e+08           stein45         22         22         22         22         22         22         22         33           swath1         41.33         37.34         86.34         124.20         309.76         1195.45         4.85%/4.07%           swath2         195.24         104.00         527.60         40.76         195.96         458.98         4.32%/2.47%           swath3         343.09         344.09         344.07         344.13 <t< td=""><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></t<>								
sp97ic         4.2211e+08         4.2217e+08         4.2217e+08         4.2218e+08         4.2213e+08         4.2219e+08         4.3e+08           3553.65         3554.51         3551.67         3542.83         1136.89         1190.07         1.48%/0.79%           sp98ar         5.2499e+08         5.2504e+08         5.2509e+08         5.251e+08         5.2509e+08         4.4448e+08         4.4448e+08         4.4448e+08         4.4448e+08         4.4448e+08         4.4448e+08         4.4418e+08         4.4418e+08         4.4418e+08         4.4418e+08 <t< td=""><td>sp97ar</td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></t<>	sp97ar							
sp97ic         3553.65         3554.51         3551.67         3542.83         1136.89         1190.07         1.48%/0.79%           sp98ar         5.2499e+08         5.2504e+08         5.2508e+08         5.2509e+08         5.2511e+08         5.2509e+08         5.3e+09           sp98ic         3554.89         3556.06         3562.27         3558.49         1184.27         1175.80         2.57%/2.16%           sp98ic         3550.63         3550.63         3563.69         3580.06         1190.08         1174.74         2.53%/2.16%           stein45         22         22         22         22         22         22         22         22         22         30           swath1         338.68         339.99         339.36         340.32         340.54         340.64         379.07           swath2         343.09         343.89         344.07         344.13         344.27         344.91         385.2           swath3         343.09         344.09         344.16         344.13         344.27         345.07         397.76           swath3         373.88         374.41         375.53         375.21         379.4         376.08         467.43           swath         366.58								
sp98ar         5.2499e+08         5.2504e+08         5.2508e+08         5.2509e+08         5.2511e+08         5.2509e+08         5.3e+08           3549.89         3556.06         3562.27         3558.49         1184.27         1175.80         2.57%/2.16%           sp98ic         4.4430e+08         4.4445e+08         4.4445e+08         4.4445e+08         4.4445e+08         4.4444e+08         4.5141e+08           3550.63         3550.63         3552.86         363.69         3580.06         1190.08         1174.74         2.53%/2.16%           stein45         22         23         34         340.00         340.00         340.03         340.32         340.54         340.64         379.07         380.55         340.77         344.17         344.13 <th< td=""><td>sp97ic</td><td></td><td></td><td></td><td></td><td></td><td></td><td>·</td></th<>	sp97ic							·
sp98ar         3549.89         3556.06         3562.27         3558.49         1184.27         1175.80         2.57%/2.16%           sp98ic         4.4430e+08         4.4445e+08         4.4445e+08         4.4445e+08         4.4444e+08         4.4444e+08         4.5141e+08           3550.63         3552.86         3563.69         3580.06         1190.08         1174.74         2.53%/2.16%           stein45         22         23         340.00         370.00         390.36         340.32         340.54         340.64         379.07         385.3         345.07         344.								
sp98ic         3549.89         3550.06         3562.27         3558.49         1184.27         117.80         2.57%/2.16%           sp98ic         4.4430e+08         4.4443e+08         4.4448e+08         4.4444e+08         4.4444e+08         4.5141e+08           stein45         22         22         22         22         22         22         22         22         22         22         22         22         22         36.21         36.00%/0.00%         36.11         0.00%/0.00%         36.31         36.32         35.56         36.11         0.00%/0.00%         37.00%         38.68         339.99         339.36         340.32         340.54         340.64         379.07         37.74         86.34         124.20         309.76         1195.45         4.85%/4.07%         344.13         344.27         344.91         385.24         385.24         383.30         344.07         344.13         344.27         344.91         385.24         385.24         384.90         344.09         344.16         344.13         344.29         345.07         397.76         397.76         397.76         397.76         397.76         385.35         385.35         385.35         385.35         385.35         385.35         385.35         385.35         <	sp98ar							·
spysic         3550.63         3552.86         3563.69         3580.06         1190.08         1174.74         2.53%/2.16%           stein45         22         22         22         22         22         22         22         22         22         338.68         339.99         339.36         340.32         340.54         340.64         379.07           swath1         41.33         37.34         86.34         124.20         309.76         1195.45         4.85%/4.07%           swath2         343.09         343.89         344.07         344.13         344.27         344.91         385.2           swath3         343.09         344.09         344.16         344.13         344.29         345.07         397.7c           swath3         783.86         535.92         789.25         186.25         393.72         866.58         3.62%/1.96%           swath         373.88         374.41         375.53         375.21         379.4         376.08         467.41           swath         3494.23         3533.15         3513.14         3535.65         1141.32         1199.99         5.90%/1.77%           timtab1         443780         438838         485907         446072         446072<								
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stein45         24.73         26.15         26.43         30.23         35.56         36.11         0.00%/0.00%           swath1         338.68         339.99         339.36         340.32         340.54         340.64         379.07           swath2         343.09         343.89         344.07         344.13         344.27         344.91         385.2           swath2         195.24         104.00         527.60         40.76         195.96         458.98         4.32%/2.47%           swath3         343.09         344.09         344.16         344.13         344.29         345.07         397.77           swath3         783.86         535.92         789.25         186.25         393.72         866.58         3.62%/1.96%           swath         373.88         374.41         375.53         375.21         379.4         376.08         467.41           stmtab1         443780         438838         485907         446072         446072         462240         764772           timtab2         563628         560505         566589         558124         558124         575232         1.1111e+0e           timtab2         3593.70         359.85         3592.81         3583.98								
swath1         338.68         339.99         339.36         340.32         340.54         340.64         379.07           swath1         41.33         37.34         86.34         124.20         309.76         1195.45         4.85%/4.07%           swath2         343.09         343.89         344.07         344.13         344.27         344.91         385.2           swath3         343.09         344.09         527.60         40.76         195.96         458.98         4.32%/2.47%           swath3         783.86         535.92         789.25         186.25         393.72         866.58         3.62%/1.96%           swath         373.88         374.41         375.53         375.21         379.4         376.08         467.41           swath         3494.23         3533.15         3513.14         3535.65         1141.32         1199.99         5.90%/1.77%           timtab1         443780         438838         485907         446072         446072         462240         76477           timtab2         3583.77         3593.85         3592.81         3581.24         558124         558124         575232         1.1111e+0           tr12-30         129675         129675	stein45							30
swath1         41.33         37.34         86.34         124.20         309.76         1195.45         4.85%/4.07%           swath2         343.09         343.89         344.07         344.13         344.27         344.91         385.3           swath3         195.24         104.00         527.60         40.76         195.96         458.98         4.32%/2.47%           swath3         343.09         344.09         344.16         344.13         344.29         345.07         397.76           swath3         378.86         535.92         789.25         186.25         393.72         866.58         3.62%/1.96%           swath         373.88         374.41         375.53         375.21         379.4         376.08         467.41           timtab1         443780         43838         485907         446072         446072         462240         76477.77           timtab2         358.77         3593.85         359.34         3570.40         1196.11         1193.77         31.12%/13.12%           timtab2         3583.77         3593.85         3592.81         3583.98         1194.40         1194.57         2.12%/0.54%           tr12-30         3593.70         3596.10         3586.60								
41.33         37.34         86.34         124.20         309.76         1195.45         4.85%/4.07%           swath2         343.09         343.89         344.07         344.13         344.27         344.91         3855.2           swath3         195.24         104.00         527.60         40.76         195.96         458.98         4.32%/2.47%           swath3         343.09         344.09         344.16         344.13         344.29         345.07         397.76           swath         373.88         374.41         375.53         375.21         379.4         376.08         467.41           timtab1         443780         438838         485907         446072         446072         462240         764777           timtab1         3567.05         3587.46         3589.34         3570.40         1196.11         1193.77         13.12%/	swath1							
$\begin{array}{c} \text{swath2} \\ \text{swath3} \\ \text{swath3} \\ \end{array} \begin{array}{c} 195.24 \\ \text{symath3} \\ \end{array} \begin{array}{c} 343.09 \\ 344.09 \\ 344.09 \\ \end{array} \begin{array}{c} 344.16 \\ 344.16 \\ \end{array} \begin{array}{c} 344.13 \\ 344.29 \\ 345.07 \\ \end{array} \begin{array}{c} 345.07 \\ 397.76 \\ 397.76 \\ \end{array} \\ \end{array} \\ \begin{array}{c} 378.86 \\ 535.92 \\ \end{array} \begin{array}{c} 789.25 \\ 789.25 \\ \end{array} \begin{array}{c} 186.25 \\ 393.72 \\ \end{array} \begin{array}{c} 366.58 \\ 36.65 \\ \end{array} \begin{array}{c} 36.65 \\ 3.62\%/1.96\% \\ \end{array} \\ \\ \text{swath} \\ \begin{array}{c} 373.88 \\ 374.41 \\ 375.53 \\ 375.21 \\ \end{array} \begin{array}{c} 375.21 \\ 379.4 \\ 376.08 \\ \end{array} \begin{array}{c} 366.08 \\ 3.62\%/1.96\% \\ \end{array} \\ \\ \text{swath} \\ \begin{array}{c} 443780 \\ 3583.15 \\ 3533.15 \\ 3513.14 \\ 3535.65 \\ \end{array} \begin{array}{c} 1141.32 \\ 1199.99 \\ 5.90\%/1.77\% \\ \end{array} \\ \begin{array}{c} 462240 \\ 764775 \\ 36777 \\ 3567.05 \\ 3587.46 \\ 3589.34 \\ 3589.34 \\ 3570.40 \\ \end{array} \begin{array}{c} 1196.11 \\ 1193.77 \\ 13.12\%/13.12\% \\ \end{array} \\ \begin{array}{c} 11111400 \\ 1193.77 \\ 13.12\%/13.12\% \\ \end{array} \\ \begin{array}{c} 129675 \\ 129675 \\ 3593.70 \\ 3593.70 \\ 3596.10 \\ 3586.60 \\ 3544.45 \\ \end{array} \begin{array}{c} 1198.44 \\ 1197.57 \\ 12984 \\ 1197.57 \\ 23.72\%/9.38\% \\ \end{array} \\ \begin{array}{c} 129675 \\ 3582.16 \\ 3589.28 \\ 3589.58 \\ 3581.38 \\ \end{array} \begin{array}{c} 1194.40 \\ 1194.41 \\ 1195.11 \\ 10.02\%/0.00\% \\ \end{array} \\ \begin{array}{c} 129742 \\ 29140e+07 \\ 2.9142e+07 \\ 2.9142e+07 \\ 2.9142e+07 \\ 2.9142e+07 \\ 2.9141e+07 \\ \end{array} \begin{array}{c} 4328\% \\ 2.914767 \\ 2.9141e+07 \\ 3.01e+07 \\ 3.01e+07 \\ \end{array} $								
$\begin{array}{c} 195.24 & 104.00 & 527.60 & 40.76 & 195.96 & 458.98 & 4.32\%/2.47\% \\ swath3 & 343.09 & 344.09 & 344.16 & 344.13 & 344.29 & 345.07 & 397.76 \\ 783.86 & 535.92 & 789.25 & 186.25 & 393.72 & 866.58 & 3.62\%/1.96\% \\ swath & 373.88 & 374.41 & 375.53 & 375.21 & 379.4 & 376.08 & 467.41 \\ 3494.23 & 3533.15 & 3513.14 & 3535.65 & 1141.32 & 1199.99 & 5.90\%/1.77\% \\ timtab1 & 443780 & 438838 & 485907 & 446072 & 446072 & 462240 & 764777 \\ 3567.05 & 3587.46 & 3589.34 & 3570.40 & 1196.11 & 1193.77 & 13.12\%/13.12\% \\ timtab2 & 563628 & 560505 & 566589 & 558124 & 558124 & 575232 & 1.1111e+06 \\ 3583.77 & 3593.85 & 3592.81 & 3583.98 & 1194.40 & 1194.57 & 2.12\%/0.54\% \\ tr12-30 & 129675 & 129675 & 129728 & 129762 & 129894 & 130596 \\ 3593.70 & 3596.10 & 3586.60 & 3544.45 & 1198.44 & 1197.57 & 23.72\%/9.38\% \\ trento1 & 5.1830e+06 & 5.1830e+06 & 5.1830e+06 & 5.1830e+06 & 5.1830e+06 & 5.830e+06 \\ 3582.16 & 3590.92 & 3590.58 & 3581.38 & 1194.41 & 1195.11 & 0.02\%/0.00\% \\ 1MMTS & 2.9137e+07 & 2.9137e+07 & 2.9140e+07 & 2.9142e+07 & 2.9142e+07 & 2.9141e+07 & 3.01e+07 \\ \hline \end{array}$	swath?	343.09	343.89		344.13			385.2
$\begin{array}{c} \text{swath3} \\ \text{swath} \\ \\ \text{swath} \\ \\ \text{swath} \\ \\ \begin{array}{c} 373.88 \\ 374.41 \\ 375.53 \\ 375.21 \\ 379.4 \\ 376.08 \\ 375.21 \\ 379.4 \\ 376.08 \\ 376.08 \\ 376.08 \\ 376.08 \\ 376.08 \\ 376.08 \\ 377.77 \\ 379.4 \\ 376.08 \\ 467.41 \\ 376.08 \\ 379.41 \\ 376.08 \\ 379.41 \\ 379.4 \\ 379.08 \\ 379.4 \\ 379.08 \\ 379.4 \\ 379.08 \\ 379.4 \\ 379.08 \\ 379.4 \\ 379.17 \\ 379.4 \\ 379.17 \\ 379.4 \\ 379.17 \\ 379.4 \\ 379.17 \\ 379.1$	SWAUIIZ	195.24	104.00		40.76		458.98	
$\begin{array}{c} 83.86 \\ 535.92 \\ \hline \\ 805.92 \\ \hline \\ 805$	ewath3	343.09	344.09	344.16	344.13	344.29	345.07	397.76
$\begin{array}{c} \text{swath} \\ \text{3494.23} \\ \text{3533.15} \\ \text{3533.15} \\ \text{3513.14} \\ \text{3535.65} \\ \text{1141.32} \\ \text{1199.99} \\ \text{5.90\%/1.77\%} \\ \text{timtab1} \\ \text{3567.05} \\ \text{3587.46} \\ \text{3589.34} \\ \text{3570.40} \\ \text{1196.11} \\ \text{1193.77} \\ \text{13.12\%/13.12\%} \\ \text{3583.77} \\ \text{3593.85} \\ \text{3592.81} \\ \text{3583.87} \\ \text{3593.85} \\ \text{3592.81} \\ \text{3583.98} \\ \text{1194.40} \\ \text{1194.57} \\ \text{129675} \\ \text{129728} \\ \text{129762} \\ \text{129762} \\ \text{129762} \\ \text{129894} \\ \text{130599} \\ \text{130599} \\ \text{15.1830e+06} \\ \text{5.1830e+06} \\ 5.1830$	Swating	783.86	535.92	789.25	186.25	393.72	866.58	3.62%/1.96%
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	annth.	373.88	374.41	375.53	375.21	379.4	376.08	467.41
$\begin{array}{c} \text{timtab1} \\ \text{3567.05} \\ \text{3587.46} \\ \text{3589.74} \\ \text{3589.34} \\ \text{3570.40} \\ \text{1196.11} \\ \text{1196.11} \\ \text{1193.77} \\ \text{13.12\%/13.12\%} \\ \text{13.12\%/13.12\%} \\ \text{129675} \\ \text{129675} \\ \text{129675} \\ \text{129675} \\ \text{129675} \\ \text{129670} \\ \text{3583.00} \\ \text{3586.60} \\ \text{3584.45} \\ \text{1198.44} \\ \text{1197.57} \\ \text{23.72\%/9.38\%} \\ \text{129462} \\ \text{129762} \\ \text{129762} \\ \text{129762} \\ \text{129762} \\ \text{129762} \\ \text{129762} \\ \text{129894} \\ \text{1305.99} \\ \text{1305.99} \\ \text{3593.70} \\ \text{3596.10} \\ \text{3586.60} \\ \text{3544.45} \\ \text{1198.44} \\ \text{1197.57} \\ \text{23.72\%/9.38\%} \\ \text{3581.66} \\ \text{3580.90} \\ \text{3582.16} \\ \text{3590.92} \\ \text{3590.58} \\ \text{3581.38} \\ \text{1194.41} \\ \text{1195.11} \\ \text{10.02\%/0.00\%} \\ \text{30.16+07} \\ 3$	swatn	3494.23	3533.15	3513.14	3535.65	1141.32	1199.99	5.90%/1.77%
$ \begin{array}{c} 3567.05 \\ 3587.46 \\ 3589.34 \\ 3570.40 \\ 1196.11 \\ 1193.77 \\ 13.12\% / 13.12\% \\ 158.124 \\ 575232 \\ 1.1111e+06 \\ 3583.77 \\ 3593.85 \\ 3592.81 \\ 3583.98 \\ 1194.40 \\ 1194.57 \\ 129762 \\ 129762 \\ 129894 \\ 130596 \\ 130596 \\ 13959.70 \\ 3593.70 \\ 3596.10 \\ 3586.60 \\ 3544.45 \\ 1198.44 \\ 1197.57 \\ 23.72\% / 9.38\% \\ 1194.41 \\ 1197.57 \\ 23.72\% / 9.38\% \\ 1194.41 \\ 1195.11 \\ 0.02\% / 0.00\% \\ 110059 \\ 120059$	Airest o la 1	443780	438838	485907	446072	446072	462240	764772
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	пппарт	3567.05	3587.46	3589.34	3570.40	1196.11	1193.77	13.12%/13.12%
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	4:41-0	563628	560505	566589	558124	558124	575232	1.1111e+06
$ \begin{array}{c} tr12\text{-}30 \\ \hline \\ & 129675 \\ \hline \\ & 3593.70 \\ \hline \\ & 3593.70 \\ \hline \\ & 3596.10 \\ \hline \\ & 3586.60 \\ \hline \\ & 3544.45 \\ \hline \\ & 1198.44 \\ \hline \\ & 1197.57 \\ \hline \\ & 23.72\%/9.38\% \\ \hline \\ \\ trento1 \\ \hline \\ \\ & 5.1830e+06 \\ \hline \\ & 5.2874e+06 \\ \hline \\ & 3582.16 \\ \hline \\ & 3590.92 \\ \hline \\ & 3590.58 \\ \hline \\ & 3581.38 \\ \hline \\ & 1194.41 \\ \hline \\ & 1195.11 \\ \hline \\ & 0.02\%/0.00\% \\ \hline \\ & 11000000000000000000000000000000000$	timtab2	3583.77		3592.81		1194.40		2.12%/0.54%
$ \frac{3593.70}{5.830e+06} \frac{3596.10}{5.1830e+06} \frac{3580.50}{5.1830e+06} \frac{3544.45}{5.1830e+06} \frac{1198.44}{5.1830e+06} \frac{1197.57}{5.1830e+06} \frac{23.72\%/3.88\%}{5.2874e+06} $		129675	129675		129762	129762	129894	130596
$ \begin{array}{c} \text{trento1} \\  \\ & 5.1830 \text{e} + 06 \\ \hline \\ & 3582.16 \\ \hline \\ & 3590.92 \\ \hline \\ & 3590.92 \\ \hline \\ & 3590.58 \\ \hline \\ & 3581.38 \\ \hline \\ & 1194.41 \\ \hline \\ & 1195.11 \\ \hline \\ & 0.02\%/0.00\% \\ \hline \\ & 110MTS \\ \hline \\ \\ & 110MTS \\ \hline \\ & 110MTS \\ \hline \\ \\ & 110MTS \\ \hline$	tr12-30							23.72%/9.38%
trentol 3582.16 3590.92 3590.58 3581.38 1194.41 1195.11 0.02%/0.00%  1IMTS 2.9137e+07 2.9137e+07 2.9140e+07 2.9142e+07 2.9142e+07 2.9141e+07 3.01e+07								5.2874e+06
2.9137e+07 2.9137e+07 2.9140e+07 2.9142e+07 2.9142e+07 2.9141e+07 3.01e+07	trentol							
2001.01 0.41/M/0.41/M	UMTS							
		5554.04	5551.09	3333.94	5510.55	1134.00	1130.10	0.41/0/0.41/0

all configurations, and then the maximum of the same quantity over the  $\mathrm{T}^*\mathrm{N5}$  configurations.

Figure 1 summarizes the best speed-up factor over CPLEX defaults obtained over all instances (31) that finished to optimality on all six configurations, the geometric average speed-up was 31%. Also, looking at problems where at least one configuration had finished to optimality, CPLEX was faster by at least 5% in 10 cases, while in 16, 16, 14, 11, 9 cases configurations T2N5, T5N5, T10N5, T10N1k, T15N1k, where faster by at least 5% respectivelly. Figure 2 summarizes the best closed GAP<sub>LP</sub> over CPLEX default (1) and the best closed GAP<sub>LP</sub> over CPLEX when we limit the code to add up-to five cuts at the root node (2). The number of cases where each configuration had the best root LP value where 18, 19, 27, 22, 43, 38, for C0, T2N5, T5N5, T10N5, T10N1k and T15N1k respectively.

Figure 3 shows the number of cuts added for each configuration. Note that for the T\*N1k configurations, more than 80% of the instances required less than 40 cuts. On the other hand, it seems that the more tableau rows we use to generate cuts, the less number of total cuts our procedure needs. Figure 4 shows the closed GAP<sub>LP</sub> for each configuration; where negative values (i.e. the procedure performs worse than CPLEX) are displayed in the left part of the graph. Again from this figure it seems clear that there are advantages to considering more than one tableau row at the same time; in fact, the results for two tableau rows are consistently poorer than configurations that use more tableau rows in the cutting procedure.

### 5 Final Remarks and Conclusions

We have shown that even simple subclasses of inequalities derived from the infinite relaxation can have an important impact both on overall branch and cut performance, and on the  $GAP_{LP}$  closed at the root node. These results point towards both trying to identify important classes of inequalities for  $R_f$  for higher dimensions, and to find good computational implementation choices for them.

Although the implementation is numerically conservative, still, there are some numerical issues when cuts are used within the branch and bound tree. There are also instances where the cuts generated tend to be parallel to previous ones, causing again numerical issues.

There are many possibilities to explore, like adding more than one cut in every iteration, choosing different sets of tableau to work on, choosing different ground sets  $B_a$ , and testing the impact of inequalities derived form simplex-like ground sets.

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