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SYMPOSIUM

Walrasian prices in markets with tradable rights

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Abstract In this paper, we consider an exchange economy where there is an external restriction for the consumption of goods. This restriction is defined by both, a cap on consumption of certain commodities and the requirement of an amount of rights for the consumption of these commodities. The caps for consumption are imposed exogenously due to the negative effects that the consumption may produce. The consumption rights or licenses are distributed among the agents. This fact leads to the possibility of establishing license markets. These licenses do not participate in agents' preferences, however, the individual's budgetary constraint may be modified, leading to a reassignment of resources. Our aim is to show the existence of a Walrasian equilibrium price system linking tradable rights prices with commodity prices.

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1 Introduction

Tradable-license systems are the focus of current interest in market-based natural resources or environmental policies. For example, a system of licenses is interesting as it could provide a mean to achieve decentralized solutions to set restrictions on fishing for certain fish species or in order to organize a market of emission licenses or pollution rights. For general references, see Ellerman et al. (2008); Joskow et al. (1998) and Newell et al. (2005).

A license confers the agents holding it, the right to consume. In the examples above, the right is to capture a certain amount of a protected species of fish or to emit pollutants at a certain rate. However, it is not always desirable to allow such rights to be transferred on a one-to-one basis. In a market system, these licenses should be tradable, and the desirable rule governing exchange of licenses or rights should be based on a market-price system.

Models of "Cap System" with tradable licenses have been analyzed by several authors during the last 40 years, (see Baumol and Oates 1988; Ellerman et al. 2008 and Montero 2001 as general references). However, the literature on models focusing on pricing rights in a purely competitive basis is scarce. In this issue, (Burniaux and Martins 2011) analyze the consequences of imposing an unilateral carbon emission constraint defined exogenously in a general equilibrium model with two countries, and (Chichilnisky 2011) studies the existence of equilibrium prices in a sustainable market.

A precise formulation of an emission license model with a competitive basis appears in the seminal paper by Montgomery (1972). In a scenario where an exchange of such licenses between polluters at different locations is considered, Montgomery shows that a market equilibrium in emission licenses exists and that, with some restrictions on the initial allocation of licenses, the market equilibrium is efficient. Later, Boyd and Conley (1997) were the first to directly treat the efficiency problem in presence of externalities opposed to an indirect way through *Arrovian* commodities, arguing that essential non-convexities highlighted by Starrett (1972) are due to unboundedness of the negative effects of an externality, rather than the externality itself.

Conley and Smith (2005) extended the Boyd and Conley model to allow firms to benefit from public goods and be damaged by externalities, proving the existence of a competitive equilibrium and stating a first welfare theorem. Their main result could be viewed as a type of general equilibrium Coase theorem. More recently, the paper by Mandel (2009) focuses on the influence on the general equilibrium of an economy of the opening of a licenses market. Assuming there existed an equilibrium before the opening of allowances market, the paper describes the changes in the firms' behavior, which guarantee that an equilibrium can be reached in the enlarged economy.

The models considered by Boyd and Conley, Conley and Smith, or Mandel imply to re-consider the pollutants as crucial consumption goods as well as key input factors for production, which drive them to the necessity of re-defining the individual preferences and production sets in order to take into account these new factors in their formulations. The problem we see in this approach is that the equilibrium solution critically depends on the assumptions on the set of properties that define the preferences (and production sets) of new goods and thus, the result is specific for those assumptions. How to model changes in preferences (and production sets) in the presence of new goods in the market is certainly an open question, for which we do not have a satisfactory answer. Complementarily, Chipman and Guoqiang (2011) is considered the presence of tradable pollution right in the economy. However, as a crucial difference with our work, authors assume that agents' preferences depend explicitly on the pollution right.

In this paper, we consider a scenario in which limits to the consumption of certain commodities have been established exogenously and that the consumption of these commodities requires the availability of certain amount of rights or licenses for its consumption. The scenario may reflect a situation where, due to binding international agreements, limits to excessive consumption of certain raw materials or limits to the capture of protected species have been established in order to restrict the potential negative effects produced by their consumption. These negative effects may be, for instance, greenhouse effects, different types of environmental pollution, or the risk of extinction of a fish species.

Our aim is to set a simple model of an economy in order to show the existence of an equilibrium price system linking tradable license prices with commodity prices and to highlight the immediate consequences on equilibrium prices when limits to consumption are set.

For it, we consider an exchange economy with externalities (the individual's preference depends on private consumption goods chosen by this individual and on the entire consumption plan chosen by other agents in the economy). The enforcement of licenses for the consumption is exogenous; the amount of such licenses is defined by an exogenous mapping that associates pollutants with consumption plans. In our model, licenses do not participate directly in preferences. However, the requirement of licenses for the consumption of specific commodities leads to the existence of a licenses market, and consequently, licenses become tradable modifying the budgetary constraints of agents.

The restrictions of the model primarily affect the agents' consumption sets. Agents may not consume certain quantities of specific commodities even when these form part of their endowments. Secondly, it may affect the agents' budget sets, since in order to consume, they will need to have the required rights. If an agent does not have those licenses, she may buy them investing part of her income coming from her endowments, or on the contrary, if she has any licenses left over, she could sell them to get an additional income.

It is also assumed that the estimated negative effects, and consequently the licenses required for the consumption of specific commodities, could depend not only on the quantity of those commodities but also on the entire consumption plan selected by the consumer. Our objective here is to reflect the situation in which a consumption plan entailing high technology, may involve less adverse effects, and consequently require fewer consumption licenses than another less technological consumption plan.

This model assumes that each agent is endowed with a certain amount of each type of the required licenses for consumption and also assumes that licenses are perfectly divisible and tradable. The agent's choice of a specific consumption plan requires that she has the inherent license for that consumption.

Our approach differs from other previous works in several aspects. Firstly, we do not explicitly consider production. In our model, agents evaluate their utility considering all the consequences involved in their consumption plan. Thus, our model is a pure exchange market in which the consumption rights or licenses are traded at the same time as the commodities, that is, licenses must be required at the same time that contracts for raw materials are signed, no matter the raw materials purpose. Therefore, and more importantly, we do not require to measure the actual negative effects of consumption. Instead, we suppose the existence of an external mapping that evaluates the potential negative effects derived from each contract, by mapping every consumption plan (or contract) into a theoretical amount of licenses of each type. Secondly, we do not need to introduce any other *type of good* in agents preferences and neither in the production sector, which avoid us from justifying how preferences and/or production sets could be distorted by the introduction of these new goods in the market.

Due to the presence of externalities in consumption (as we setup the model in Sect. 2), we introduce the concept of *Nash–Walras equilibrium* as a competitive outcome in our framework. This concept coincides with the standard Walras notion if we were not to consider externalities.

In Sect. 3, we prove a Walras' Law for our equilibrium concept. The main result of this paper is Theorem 1 in Sect. 4, which establishes the existence of a Nash–Walras equilibrium under general conditions on the fundamentals of the economy. Finally, Sect. 5 is devoted to the conclusion remarks and further developments.

2 The model

Following the standard Arrow-Debreu model, let us consider an economy with $m \in \mathbb{N}$ consumers and $\ell \in \mathbb{N}$ different consumption goods; the consumption set of consumer $i \in I = \{1, 2, \ldots, m\}$ is denoted by $X_i \subseteq \mathbb{R}^\ell$, and each consumer i is endowed with consumption goods denoted by $\omega_i \in X_i$. We set $\omega = \sum_{i \in I} \omega_i$, $X = \prod_{i \in I} X_i$ and given $i \in I$, we define

$$X_{-i} = \prod_{j \in I \setminus \{i\}} X_j.$$

In order to incorporate externalities in consumption, preferences of an individual $i \in I$ will be represented by a utility function

$$u_i : X_{-i} \times X_i \to \mathbb{R}.$$

We assume that limits to the consumption of certain commodities have been established exogenously due to binding international agreements established, where consumption of these commodities requires the availability of certain licenses. After an exogenous *Cap-setting Process*, limits to consumption are given by the mapping

$$f : \mathbb{R}^{\ell}_{+} \rightarrow \mathbb{R}^{k}_{+},$$

which defines the amount of the each type of $k \in \mathbb{N}$ negative effects that could produce the consumption of the allocation $x \in \mathbb{R}^{\ell}_{+}$.

For $j \in K = \{1, \ldots, k\}$, the *Cap-setting Process* sets a limit $R_j \in \mathbb{R}_{++}$ on the total allocation of the economy; we set

$$R = (R_j) \in \mathbb{R}^{k}_{++}.$$

In our model, the Cap-setting process mentioned above implies that for each $j \in K$, any consumption plan $x_i \in X_i,\ i \in I$, should comply with

$$\sum_{i \in I} f_j(x_i) \leq R_j,$$

where f_j denotes the $j \in K$ component of f that defines the caps to consumption that have been exogenously established. Observe that $f_j(x_i)$ could be the amount of commodity j representing a certain raw material for which a cap has been established in order to restrict the potential negative effects that this consumption will produce. However, here, we are considering a more general setting; in this model, each one of the potential negative effects and, consequently each cap, is measured globally in the sense that it depends not only on the amount of a given commodity but on the global consumption plan of the individuals. Proceeding in this way, we have in mind, for example, that a more technological consumption plan may produce less negative effects than a technologically poorer alternative.

On the other hand, we assume that for each $j \in K$, there is a type of *license* and that each individual $i \in I$ is endowed with an amount of each of them. Formally, each agent $i \in I$ is endowed with a vector

$$r_i = (r_i^j) \in \mathbb{R}^{k}_{+}$$

in such a way that

$$\sum_{i \in I} r_i^j = R_j, \quad j \in K.$$

If agent $i \in I$ decides to consume $x \in X_i$ then she must have an *amount* $f(x) \in \mathbb{R}^{k}_{+}$ of each consumption right (license). One key assumption in our model is that consumption rights can be traded in the market and that they do not participate in the individual's preferences. The fact that licenses can be traded in the market implies that any individual may exchange them with consequences on the size of her budgetary set; similar to prices of consumption goods, prices for licenses will be determined endogenously as part of the equilibrium.

Thus, the difference

$$r_i - f(x) \in \mathbb{R}^k$$

defines the amount of licenses that individual $i \in I$ may sell in the market (those for which the corresponding component is positive) and those she needs to buy since his initial endowment of the corresponding license is not enough to support the consumption of x (negative components).

If the price for licenses is $s \in \mathbb{R}^k_+$, then the consumption of x, as already mentioned, implies that the total wealth she can obtain (or pay if negative) from trading them in the market is:

$$s \cdot [r_i - f(x)] \in \mathbb{R}.$$

In the following, Δ denotes the simplex in $\mathbb{R}^{\ell+k}$ and for $n \in \mathbb{N}_+$ and $x,\ y \in \mathbb{R}^n$, we say that $x \leq_n y$ iff $x_i \leq y_i$, for each $i = 1, 2, \ldots, n$, $x <_n y$ iff $x \leq_n y$ and $x \neq y$ and, $x <<_n y$ iff $x_i < y_i$, for each $i = 1, 2, \ldots, n$. Finally, 0_n is zero in $\mathbb{R}^n$.

Definition 1 For $(p, s) \in \Delta$, the budgetary set for individual $i \in I$ at prices (p, s) is defined by

$$B_i(p, s) = \{\xi_i \in X_i \mid p \cdot \xi_i \leq p \cdot \omega_i + s \cdot [r_i - f(\xi_i)]\}.$$

Definition 2 An economy with consumption rights and externalities is defined as

$$\mathcal{E}_R = (X_i, (u_i), (\omega_i), (r_i), f)_{i \in I}.$$

The corresponding economy without consumption rights ("exchange economy with externalities") is denoted by

$$\mathcal{E} = (X_i, (u_i), (\omega_i))_{i \in I}.$$

In order to define the equilibrium notion for economy $\mathcal{E}_R$, we consider feasibility in both consumption goods and consumption of licenses.

Definition 3 We say that $x = (x_i) \in X$ is a feasible allocation for the economy $\mathcal{E}_R$ if

$$\sum_{i \in I} x_i \leq_\ell \omega \in \mathbb{R}^\ell_+$$

and

$$\sum_{i \in I} f(x_i) \leq_k R \in \mathbb{R}^k_+.$$

The set of feasible allocation for the economy $\mathcal{E}_R$ is denoted by $\mathcal{F}_R$.

Remark 1 Observe that the endowments $(\omega_i) \in X$ need not to be a feasible allocation for the economy $\mathcal{E}_R$. This occurs if for some j

$$\sum_{i \in I} f_j(\omega_i) > R_j.$$

More generally, for $j \in K$ suppose that f_j is a convex function and $f_j(0_\ell) = 0$; if $x = (x_1, x_2, \ldots, x_m) \in X$ allocates the total endowment, that is, $\sum_{i \in I} x_i = \omega$, then we have that

$$f_j(\omega/m) \leq \frac{1}{m} \sum_{i \in I} f_j(x_i) \leq \frac{1}{m} R_j.$$

Consequently, if $R_j < m f_j(\omega/m)$, the cap is effective. That is, it is not possible to allocate the total endowment of the economy.

Finally, the definition below is a natural extension of the competitive equilibrium notion we have for an exchange economy.[1]

Definition 4 We say that $((p^*, s^*), (x_i^*)) \in \Delta \times \mathbb{R}_+^{m\ell}$ is a Nash–Walras equilibrium for the economy $\mathcal{E}_R$ if

(a) $x^* = (x_i^*) \in \mathcal{F}_R$
(b) for each $i \in I$, $x_i^* \in B_i(p^*, s^*)$, and x_i^* maximizes $u_i(x_{-i}^*, \cdot)$ on $B_i(p^*, s^*)$.

3 Walras' Law and some direct consequences

We begin this section with the following straightforward lemmata, which will be useful to show the Walras' Law in our context (Proposition 1).

Lemma 1 *Suppose that $f : \mathbb{R}_+^\ell \to \mathbb{R}_+^k$ is continuous and that for $i \in I$ and for any $x_{-i} \in X_{-i}$, $u_i(x_{-i}, \cdot) : X_i \to \mathbb{R}$ is locally non-satiated.*[2] *Given $((p^*, s^*), (x_i^*))$ a Nash–Walras equilibrium of $\mathcal{E}_R$, if for $x_i \in X_i$ holds that $u_i(x^*) \leq u_i(x_{-i}^*, x_i)$, then*

$$p^* \cdot x_i \geq p^* \cdot \omega_i + s^* \cdot [r_i - f(x_i)].$$

Proof Suppose that $p^* \cdot x_i < p^* \cdot \omega_i + s^* \cdot [r_i - f(x_i)]$. Since f is continuous, there exist $\epsilon > 0$ such that

$$p^* \cdot x_i' < p^* \cdot \omega_i + s^* \cdot \left[r_i - f(x_i')\right]$$

[1] In the following, for $x = (x_i) \in X$, we adopt the notation $u_i(x) = u_i(x_{-i}, x_i)$.

[2] That is, for any $x_{-i} \in X_{-i}$, $\epsilon > 0$ and $x_i \in X_i$, there exists $x_i' \in B(x_i, \epsilon) \cap X_i$ such that $u_i(x_{-i}, x_i) < u_i(x_{-i}, x_i')$, where $B(x_i, \epsilon)$ is the open ball with center x_i and radius ϵ.

for all $x_i' \in B(x_i, \epsilon)$. Therefore, by local non-satiation, there is a point $z \in B(x_i, \epsilon)$ such that $u_i(x_{-i}^*, x_i) < u_i(x_{-i}^*, z)$ and then $u_i(x^*) < u_i(x_{-i}^*, z)$, which contradicts that (x_i^*) is the equilibrium allocation at prices (p^*, s^*). $\square$

A direct consequence of Lemma 1 is the following proposition.

Proposition 1 Walras' Law *Under the conditions of Lemma 1, if* $((p^*, s^*), (x_i^*))$ *is a Nash–Walras equilibrium of* $\mathcal{E}_R$ *then*

$$p^* \cdot \left[\sum_{i \in I} x_i^* - \omega\right] = 0, \quad s^* \cdot \left[\sum_{i \in I} f(x_i^*) - R\right] = 0.$$

Proof From Lemma 1, for each $i \in I$, $p^* \cdot x_i^* = p^* \cdot \omega_i + s^* \cdot \left[r_i - f(x_i^*)\right]$, which leads us to conclude

$$p^* \cdot \left[\sum_{i \in I} x_i^* - \omega\right] + s^* \cdot \left[\sum_{i \in I} f(x_i^*) - R\right] = 0. \tag{1}$$

Since $\sum_{i \in I} x_i^* \leq_\ell \omega$, $\sum_{i \in I} f(x_i^*) \leq_k R$ and $(p^*, s^*) \in \mathbb{R}_+^{\ell+k}$ follows that $p^* \cdot \left[\sum_{i \in I} x_i^* - \omega\right] \leq 0$ and $s^* \cdot \left[\sum_{i \in I} f(x_i^*) - R\right] \leq 0$, which along with (1) implies the desired result. $\square$

Remark 2 For a Nash–Walras equilibrium $((p^*, s^*), (x_i^*))$, the fact that the requirement of licenses may effectively restrict the consumption of a good $k \in \{1, 2, \ldots, \ell\}$ corresponds to $\sum_{i \in I} x_{ik}^* - \omega_{ik} < 0$; under this situation, the Walras' Law implies that $p_k^* = 0$. Note that this fact does not depend on the distribution of licenses among individuals but only depends on the aggregate amount of licenses. In this situation, as we will see in the next example, the amount of licenses assigned to each individual could have consequences on their welfare in the equilibrium, allowing further analysis regarding public policy through the assignment of licenses among agents.

Suppose that the amount of licenses effectively restricts the consumption of a good $k \in \{1, 2, \ldots, \ell\}$ and that for some consumer i the good k is desirable, that is, for any positive λ, $u(x_i^* + \lambda e_k) > u(x_i^*)$, where e_k is the kth vector of the canonic basis of $\mathbb{R}^\ell$. From these assumptions, it immediately follows that $p_k^* = 0$ and, from the budgetary constrain, we have for some $j \in K$, $s_j^* > 0$ and $f_j(x_i^* + \lambda e_k) > f_j(x_i^*)$. Consequently, an effective cap on a commodity implies that the equilibrium price of that commodity is zero and that the price of the corresponding license becomes the relevant price.

On the contrary, note that when the level of licenses is high enough, the price of the license becomes zero at the equilibrium and the economy becomes equivalent to a classical exchange market with externalities $\mathcal{E}$.

Example 1 In order to define economy $\mathcal{E}_R$, suppose $m = 2$, $\ell = 2$ and that individual's preferences are given by $u_1(x_1, x_2) = u_2(x_1, x_2) = x_1^\alpha x_2^{1-\alpha}$, with $0 < \alpha < 1$. Endowments of goods are $(\omega_{i2}, \omega_{i2}) \in \mathbb{R}_+^2$, $i = 1, 2$; set $\omega_j = \omega_{1j} + \omega_{2j} > 0$, $j = 1, 2$. Additionally, suppose that $K = 1$, $f(x_1, x_2) = bx_2$ (with $b > 0$) and the endowment for licenses is $r_i \geq 0,\ i = 1, 2$. Set $R = r_1 + r_2 > 0$. The economy $\mathcal{E}$ is defined by u_i and $(\omega_{i1}, \omega_{i2}),\ i = 1, 2$, as before. As follows, assume that good one is the *numerary* and prices for good two and licenses are denoted by p and s, respectively. From the monotonicity of the involving functions, the consumer's problem for agent $i = 1, 2$ is

$$\max_{x_{i1}, x_{i2}} x_{i1}^{\alpha} x_{i2}^{1-\alpha} \text{ s.t. } x_{i1} + p x_{i2} = \omega_{i1} + p\omega_{i2} + s\left[r_i - b x_{i2}\right], \quad x_{i1},\ x_{i2} \geq 0,$$

whose unique solution is

$$x_{i1}(p, s) = \alpha\left[\omega_{i1} + p\omega_{i2} + s r_i\right], \quad x_{i2}(p, s) = (1-\alpha)\left[\frac{\omega_{i1} + p\omega_{i2} + s r_i}{p + bs}\right], \quad i = 1, 2.$$

The equilibrium conditions for goods one and two are, respectively,

$$x_{11}(p, s) + x_{21}(p, s) = \omega_1 \quad \Leftrightarrow \quad \alpha\left[\omega_1 + p\omega_2 + sR\right] = \omega_1 \tag{2}$$

$$x_{12}(p, s) + x_{22}(p, s) \leq \omega_2 \quad \Leftrightarrow \quad (1-\alpha)\left[\frac{\omega_1 + p\omega_2 + sR}{p + bs}\right] \leq \omega_2. \tag{3}$$

Combining (2), (3) and the budget constraint, for any $s \geq 0$

$$s\left[R - b\omega_2\right] \leq 0. \tag{4}$$

For the case $R > b\omega_2$, the unique equilibrium price is

$$s^c = 0, \quad p^c = \left(\frac{1-\alpha}{\alpha}\right)\frac{\omega_1}{\omega_2},$$

which coincides with the equilibrium price for the economy $\mathcal{E}$. For the case $R = b\omega_2$, there are infinite equilibrium prices $(p, s) \in \mathbb{R}^2_+$, parameterized by the relation $p + sb = p^c$.

For the case $R < b\omega_2$, from (4), we have that $s \geq 0$. However, note that $s = 0$ is not an admissible solution, since in such a case, the aggregated equilibrium demand for consumption good two would be equal to those obtained for economy $\mathcal{E}$ (i.e., ω_2), which is not a feasible allocation from the side of the licenses. Consequently, we may assume $s > 0$ and then, in order to preserve feasibility from the side of the licenses, (3) holds that

$$b(1-\alpha)\left[\frac{\omega_1 + p\omega_2 + sR}{p + bs}\right] \leq R.$$

If we denote by R' the consumption effectively employed by agents, we have

$$b(1-\alpha)\omega_1 + p[b(1-\alpha)\omega_2 - R'] + s[b(1-\alpha)R - bR'] = 0, \tag{5}$$

from which, along with (2) we conclude that

$$p\left[\omega_2 - \frac{R'}{b}\right] + s[R - R'] = 0.$$

Since $R' \leq R < b\omega_2$, in order to obtain positive equilibrium prices, we must impose $R' = R$, which lead us to conclude that the equilibrium price for good two is $p^* = 0$, and from (2), the equilibrium price for licenses should be

$$s^* = \frac{(1-\alpha)\omega_1}{\alpha R}.$$

Regarding good one, the equilibrium allocation is

$$x_{i1}^r = \alpha\left[\omega_{i1} + \frac{(1-\alpha)\omega_1}{\alpha R} r_i\right] = \alpha\omega_{i1} + (1-\alpha)\omega_1 \frac{r_i}{R}, \quad i = 1, 2, \tag{6}$$

which, for individual $i = 1, 2$, would be greater than those obtained in the exchange economy without consumption rights, provided that

$$\frac{r_i}{R} > \frac{\omega_{i2}}{\omega_2}.$$

Regarding good two, given $\delta = R - b\omega_2 > 0$, the aggregated demand at the equilibrium is given by

$$(1-\alpha)\left[\frac{\omega_1 + sR}{bs}\right] = \omega_2 - \frac{\delta}{b} < \omega_2. \tag{7}$$

Note that $R < b\omega_2$ implies that for some $i = 1, 2$, $r_i < b\omega_{i2}$. Thus, the initial endowment of goods and licenses does not necessarily belong to the budgetary set for this individual, at any price. This fact is relevant in our model, since it implies that we cannot use standard arguments to prove the existence of equilibrium in our setting by considering an extended economy where consumption rights (licenses) appear as new commodities in the market, even though they do not directly participate in agent's preferences.

Finally, from (6), the presence of consumption rights in the market implies a redistribution of good one between agents that otherwise may not be reached as a competitive outcome in the economy $\mathcal{E}$, unless a redistribution of endowments is carried out. However, from (7), we also have that the presence of consumption rights (licenses) may effectively restrict the consumption of goods, implying an *excess of supply* that may not be assigned to any individual. Thus, consumption rights may not necessarily be interpreted as a tax mechanism whose role is to reach a certain point in the contract curve of the economy $\mathcal{E}$. Indeed, Karp and Zhang (2011) show the advantage of quotas over emissions taxes in a model with asymmetric information. On the other hand, from Eq. 6, we have a redistribution effect as a result the introduction of consumption rights.

4 Existence of equilibrium

For the existence of equilibrium in our model, we will consider standard hypotheses on the fundamentals of the economy. The strongest condition we are assuming for the existence of equilibrium result is **SS** (*a survival* condition).

Assumption C. For each $i \in I$, $X_i \subseteq \mathbb{R}^{\ell}_{+}$ is convex, closed and $0_\ell,\ \omega_i \in X_i$.
Assumption SS. For each $i \in I$, $\omega_i \in \mathbb{R}^{\ell}_{++}$ and $r_i \in \mathbb{R}^{k}_{++}$.
Assumption R. For each $j \in K$, $f_j : \mathbb{R}^{\ell}_{+} \to \mathbb{R}_{+}$ is convex, continuous and $f_j(0_\ell) = 0$ (i.e., $f(0_\ell) = 0_k$).
Assumption U. For each $i \in I, u_i : X \to \mathbb{R}$ is continuous and for each $x_{-i} \in X_{-i}, u_i(x_{-i}, \cdot) : X_i \to \mathbb{R}$ is locally non-satiated and quasi-concave.

In order to facilitate the demonstration of our main result, we introduce the auxiliary economy $\mathcal{E}^M_R$, which differs from $\mathcal{E}_R$ only in the consumption sets that now, for individual $i \in I$ is defined by[3]

$$X_i^M = X_i \cap cl B\,(0_\ell, M\|\omega\|),$$

with $M > 1$ a given constant [4] We set $X^M = \prod_{i \in I} X_i^M$ and for $i \in I$, define

$$X_{-i}^M = \prod_{j \in I \setminus \{i\}} X_j^M.$$

Lemma 2 *Under Assumptions* **C**, **SS**, *and* **R**, *for* $i \in I$, *the correspondence*

$$B_i^M : \Delta \to X_i^M \mid B_i^M(p,s) = \left\{\xi_i \in X_i^M \mid p \cdot \xi_i \leq p \cdot \omega_i + s \cdot [r_i - f(\xi_i)]\right\}$$

is continuous.

Proof From Assumption **C**, it follows directly that for each $i \in I$, B_i^M is a closed correspondence. Since X_i^M is compact, it is upper semi-continuous.

Now, in order to show the lower semi-continuity of B_i^M at any point $(p_0, s_0) \in \Delta$, let G be any open set such that $B_i^M(p_0, s_0) \bigcap G \neq \emptyset$ and let ξ belonging to this set. Observe that by Assumption **SS**, we have that

$$0 < p_0 \cdot \omega_i + s_0 \cdot [r_i - f(0_\ell)],$$

[3] The closure of $A \subseteq \mathbb{R}^n$ is denoted by $cl A$ and the Euclidean norm of $x \in \mathbb{R}^n$ by $\|x\|$.

[4] Note that from feasibility condition for consumption bundles, any relevant consumption plan x_i for an individual $i \in I$ should comply with $0_\ell \leq x_i \leq_\ell \omega$ and therefore $\|x_i\| \leq \|\omega\|$.

and therefore, from the convexity of f, we conclude that for all $\lambda \in [0, 1)$

$$p_0 \cdot \lambda\xi < p_0 \cdot \omega_i + s_0 \cdot [r_i - f(\lambda\xi)].$$

Let be $\lambda_0 < 1$ such that $\lambda_0\xi \in G$. Since f is continuous, there exists $\epsilon > 0$ such that $\| (p, s) - (p_0, s_0) \| < \epsilon$ implies that

$$p \cdot \lambda_0\xi < p \cdot \omega_i + s \cdot [r_i - f(\lambda_0\xi)],$$

from which we deduce that $B_i^M(p, s) \bigcap G \neq \emptyset$ for all $(p, s) \in \Delta$ such that $\| (p, s) - (p_0, s_0) \| < \epsilon$. This last assertion finally leads us to conclude that B_i^M is a continuous correspondence as required. □

Theorem 1 Existence of Equilibrium *Under Assumptions* **C**, **SS**, **R**, *and* **U**, *there exist a Nash–Walras equilibrium for economy* $\mathcal{E}_R$.

Proof For $i \in I$ define the function

$$u_i^* : \Delta \times X^M \times X_i^M \to \mathbb{R} \mid u_i^*((p, s), x, z) = u_i(x_{-i}, z),$$

and the correspondence

$$\mathbf{B}_i^M : \Delta \times X^M \to X_i^M \mid \mathbf{B}_i^M((p, s), x) = B_i^M(p, s).$$

Note that under Assumption **U**, the demand correspondence of the auxiliary economy $\mathcal{E}_R^M$, D_i^M defined by

$$D_i^M : \Delta \times X^M \to X_i^M \, | D_i^M((p, s), x) = \{\xi_i \in \mathbf{B}_i^M((p, s), x) \mid u_i(x_{-i}, \xi_i) \geq u_i(x_{-i}, z),$$
$$\forall z \in \mathbf{B}_i^M((p, s), x)\},$$

is compact and convex valued and from Lemma 2 and the Maximum Theorem (Berge 1997), it is upper semi-continuous.

Following the standard approach, for the *additional agent (the market)*, we define the function

$$u_0^* : \Delta \times X^M \times \Delta \to \mathbb{R} \, |u_0^*((p, s), x, (p', s'))$$
$$= p' \cdot \left(\sum_{i \in I} x_i - \omega \right) + s' \cdot \left(\sum_{i \in I} f(x_i) - R \right),$$

and the constant correspondence

$$B_0^M : \Delta \times X^M \times \Delta \to \Delta \mid B_0^M((p, s), x, (p', s')) = \Delta.$$

The demand of the *market* is defined by the correspondence,

$$
\begin{aligned}
&D_0^M : \Delta \times X^M \to \Delta \mid D_0^M((p,s),x) \\
&= \left\{ (p',s') \in \Delta \mid (p'-\tilde{p}) \cdot \left(\sum_{i\in I} x_i - \omega \right) + (s'-\tilde{s}) \cdot \left(\sum_{i\in I} f(x_i) - R \right) \right. \\
&\qquad \left. \geq 0, \forall (\tilde{p},\tilde{s}) \in \Delta \right\},
\end{aligned}
$$

which is convex and compact valued and, again by the Maximum Theorem (Berge 1997), it is upper semi-continuous.

Thus, if we define

$$D^M : \Delta \times X^M \to \Delta \times X^M \mid D^M = \prod_{i=0}^{m} D_i^M,$$

follows ifmmediately that D^M is compact and convex valued and upper semi-continuous and since $\Delta \times X^M$ is convex and compact, from Kakutani's Fixed Point Theorem we conclude that there exist $((p^*,s^*),(x_i^*)) \in \Delta \times X^M$ such that

$$((p^*,s^*),(x_i^*)) \in D^M(((p^*,s^*),(x_i^*))),$$

that is,

(i) for each $i \in I$, $x_i^* \in D_i^M((p^*,s^*),(x_i^*))$,
(ii) $(p^*,s^*) \in D_0^M((p^*,s^*),(x_i^*))$.

From condition (i), x_i^* maximizes $u_i(x_{-i}^*,\cdot)$ on the budget set $\mathbf{B}_i^M((p^*,s^*),x^*)$. On the other hand, since $x_i^* \in \mathbf{B}_i^M((p^*,s^*),x^*)$, $i \in I$, we have that

$$p^* \cdot x_i^* \leq p^* \cdot \omega_i + s^* \cdot \left[r_i - f(x_i^*) \right],$$

which, by summing in all the agents, it leads us to conclude

$$p^* \cdot \left(\sum_{i\in I} x_i^* - \omega \right) + s^* \cdot \left(\sum_{i\in I} f(x_i^*) - R \right) \leq 0. \tag{8}$$

From condition (ii) and inequality (8), holds that for each $(p,s) \in \Delta$

$$p \cdot \left(\sum_{i\in I} x_i^* - \omega \right) + s \cdot \left(\sum_{i\in I} f(x_i^*) - R \right) \leq 0.$$

Taking $s = 0_k$ and letting p be each vector the canonic basis of $\mathbb{R}^\ell$, the last inequality implies that

$$\sum_{i \in I} x_i^* - \omega \leq_\ell 0_\ell.$$

In the same way, taking $p = 0_\ell$ and letting s be each vector of the canonic basis of $\mathbb{R}^k$, we conclude that

$$\sum_{i \in I} f(x_i^*) - R \leq_k 0_k.$$

Thus, all the foregoing implies that $((p^*, s^*), (x_i^*)) \in \Delta \times X^M$ is an equilibrium for economy $\mathcal{E}_r^M$. In order to show that $((p^*, s^*), (x_i^*))$ is also an equilibrium for economy $\mathcal{E}_R$, let us suppose that for some $i \in I$ there exists $\tilde{x}_i \in X_i \setminus X_i^M$ such that

(a) $u_i(x_{-i}^*, \tilde{x}_i) > u_i(x_{-i}^*, x_i^*)$,
(b) $p^* \cdot \tilde{x}_i \leq p^* \cdot \omega_i + s^* \cdot \left[r_i - f(\tilde{x}_i)\right].$

Taking $\tilde{\lambda} \in]0, 1[$ close enough to one, Assumption **C** implies that $\tilde{\lambda}\tilde{x}_i \in X_i$ and from Assumption **U**, $u_i(x_{-i}^*, \tilde{\lambda}\tilde{x}_i) > u_i(x_{-i}^*, x_i^*)$. Moreover, condition (b) above directly implies

$$p^* \cdot (\tilde{\lambda}\tilde{x}_i) < p^* \cdot \omega_i + s^* \cdot \left[r_i - f(\tilde{x}_i)\right]. \tag{9}$$

Additionally, from Assumption **R** it is easy to check that $-f(\tilde{x}_i) \leq_k -f(\tilde{\lambda}\tilde{x}_i)$, and then, considering that $s^* \in \mathbb{R}_+^k$, inequality (9) finally implies

$$p^* \cdot (\tilde{\lambda}\tilde{x}_i) < p^* \cdot \omega_i + s^* \cdot \left[r_i - f(\tilde{\lambda}\tilde{x}_i)\right]. \tag{10}$$

For $\mu \in]0, 1[$ define

$$x_i^\mu = \mu x_i^* + (1 - \mu)\tilde{\lambda}\tilde{x}_i.$$

From (10) and Assumption **R**, holds that $p^* \cdot x_i^\mu < p^* \cdot \omega_i + s^* \cdot \left[r_i - f(x_i^\mu)\right]$, and from the quasi-concavity of $u_i(x_i^*, \cdot)$, $u_i(x_{-i}^*, x_i^\mu) \geq u_i(x_{-i}^*, x_i^*)$.

Note now that for $\mu \in]0, 1[$ close enough to one, x_i^μ belongs to X_i^M and therefore, from Assumption **U** we have that for some $\epsilon > 0$ there exists $\bar{x}_i \in X_i^M \cap B(x_i^\mu, \epsilon)$ such that

$$u_i(x_{-i}^*, \bar{x}_i) > u_i(x_{-i}^*, x_i^\mu) \geq u_i(x_{-i}^*, x_i^*).$$

Finally, choosing μ sufficiently close to 1, the continuity of f implies that

$$p^* \cdot \bar{x}_i \leq p^* \cdot \omega_i + s^* \cdot [r_i - f(\bar{x}_i)],$$

which contradicts the fact that x_i^* maximizes $u_i(x_{-i}^*, \cdot)$ on $\mathbf{B}_i^M((p^*, s^*), x^*)$. □

5 Conclusions

This paper deals with the problem of setting a price system for licenses in an economy in which consumption caps exist, and in order to consume, agents are required to have the corresponding license for consumption. This leads to the establishment of a market for rights or licenses.

Examples of this situation are the European Union Emissions Trading System established in 2005 to reduce greenhouse effects under the Kyoto Protocol (see Ellerman and Joskow 2008). Also, there are other cap-and-trade systems for emissions that have been implemented in the US. In these kind of systems, the price of licenses is set depending on the cost of controlling the negative effects.

Our model can be used not only on emission control systems but also to deal with any other license-based models where permits are required in advance. Such rights are for instance, aircraft landing licenses, or fishing licenses in a region where the amount of captures is regulated. It is also possible to consider such model to control road congestion by distributing total transit rights for specific links such that flow capacity ratios are limited on these links.

In our approach, agents evaluate their utility considering all the consequences involved in their consumption plan and the consumption plans of the other consumers. Licenses must be acquired at the same time as contracts for raw materials are signed. Thus, prices of licenses are linked to prices of commodities. Our model is based on the existence of an exogenous function that evaluates the potential negative effects derived from each contract. This mapping associates to every consumption plan (contract); a theoretical amount of licenses of each type and consequently to measure the actual negative effects of consumption is not required in our model. Our aim is to analyze the immediate consequences of setting a cap with a trade system of licenses in a simple model of general equilibrium.

We have shown that under standard conditions on the fundamentals of the economy, equilibrium exists. Our analysis points out that if the cap is effective for a raw material, the price of this commodity becomes irrelevant at the equilibrium and is the price of the corresponding license that matters. Given that we deduce that the effectiveness of the cap only depends on the total amount of licenses, the political welfare aspects derived from the distribution of these allowances among the agents become the relevant problem for the planner of the cap-and-trade system.

Finally, we would like to remark that in this paper, we are not considering the political welfare aspects derived from the distribution of the licenses among the agents. We shall focus on this problem in a future study, but it is worth remarking the existence of redistributive effects in the economy with caps for externalities and consumption rights.

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