

Optimizing maintenance service contracts under imperfect maintenance and a finite time horizon

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When a company decides to outsource a service, the most important reasons for doing so usually are to focus on core business, to be able to access high-quality services at lower costs, or to benefit from risk sharing. However, service contracts typically follow a structure whereby both owner and contractor attempt to maximize expected profits in a noncoordinated way. Previous research has considered supply chain coordination by means of contracts but is based on unrealistic assumptions such as perfect maintenance and infinite time-span contracts. In this work, these limitations are overcome by defining the supply chain through a preventive maintenance strategy that maximizes the total expected profit for both parties in a finite time-span contract. This paper presents a model to establish such conditions when maintenance is imperfect, and the contract duration is fixed through a number of preventive maintenance actions along a significant part of the asset life cycle under consideration. This formulation leads to a win-win coordination under a set of restrictions that can be evaluated a priori. The proposed contract conditions motivate stakeholders to continually improve their maintenance services to reach channel coordination in which both parties obtain higher rewards. Copyright © 2012 John Wiley & Sons, Ltd.

Keywords: maintenance; service contracts; imperfect maintenance; finite time horizon

1. Introduction

The introduction of standards such as PAS-55 [1] and ISO 14001 [2] and the increasing concern on sustainably managing of life cycle costs has intensified the use of asset management techniques to estimate resources from system design to operation and disposal [3, 4]. One way to achieve this is to balance in-house resources and to outsource business functions such as maintenance.

Before the 1970s, most equipment maintenance was performed with in-house resources. Nevertheless, because the systems have been growing in complexity, it is more competitive to supply system service using specialized external agents and equipment [5]. In the past decade, maintenance outsourcing has significantly increased in relevance. Outsourcing has become a business key to reach a competitive advantage because products and services can be offered by outside suppliers in a more efficient and effective way [6]. There has also been a paradigm shift in asset management, in which maintenance has evolved from a cost-generating activity to a value-adding function; currently, outsourcing is viewed not only as a way to ensure cost objectives but also as a way to access better quality of service and improve the product delivery capability [7]. Outsourcing also involves risk transfer. The cost of this transfer may be estimated as the difference between outsourcing a task and performing it in-house [8]. Through maintenance externalization, a set of advantages is obtained for the manufacturer, namely (i) best maintenance practices due to expertise of the providers and use of the latest maintenance technology, (ii) risk mitigation of high costs by setting for-purpose service contracts, (iii) reduction of capital investments, and (iv) ability of in-house managers to spend more time in the strategic aspects of the business. On the contrary, some disadvantages are (i) cost of contracting scarce services, making it possible for the contractor to increase monopolistic behavior, (ii) a potentially risky dependency, such as control of machine availability transferred to a contractor,

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(iii) loss of corporative know-how, and (iv) the need to supervise the achievement of contract goals (and related conflicts in case of nonperformance) and to manage external resources [9]. This last issue is critical because the employees reporting relationship may cause conflicts between the contractor and the manufacturer. Employees technically report to the contractor, although they are often under direct control of the manufacturer [7]. Possible litigation problems may also arise in the service sourcing relationship, such as an accident involving contractors [10]. The optimal risk profile involves a practical offsetting of operational risks and litigation risks [8].

A potential by-product of outsourcing is back sourcing. It refers to the internalization process after the outsourcing has failed. Whitten and Leidner [11] show that although the choice to outsource has been exhaustively considered by researchers, the decision to back source has not received equal attention. According to them, product quality, service quality, relationship quality, and switching costs are variables related to the decision to implement back sourcing. Likewise, internal strategic guidelines of organizations also have an effect on the decision to back source. Wong [12] suggests that service sourcing strategies can be influenced by power and politics at top level management because managers have different experiences, backgrounds, philosophies, and knowledge, which may impact the decision-making process.

The existence of ill-defined contracts often results in a difference between the service level delivered by contractors and the performance expected by manufacturers. This gap may become an important factor to consider when choosing between in-house or outsourcing [12]. Tseng *et al.* [13] point out that services provided by contractors should be explicit in maintenance contracts terms to avoid unilateral decisions by contractors or manufacturers. Tseng *et al.* state that this specification creates a certain rigidity of contractual terms, and factors such as scheduling of maintenance activities or flexibility for adopting new technologies have an impact on maintenance outsourcing coordination. In the current increasingly competitive industry scenario, effective channel coordination has become crucial, attracting the interest of numerous empirical and theoretical studies [14]. This situation emphasizes the need for designing performance-based contracts to achieve a win–win coordination for manufacturers and contractors at the same time, namely, a channel coordination.

Desired channel coordination is relevant not only to for-profit companies but also to service-oriented organizations. Nonprofit organizations have some characteristics that differentiate them from profit-centered companies, including the following: (i) nonprofit organizations do not have owners, (ii) these companies cannot distribute profits, and (iii) many of these organizations have tax privileges [15]. The focus of nonprofit organizations is on achieving a high service level. An example may be found in the defense industry, where equipment availability is critical to provide deterrence power to the country.

After introducing both the relevance of maintenance outsourcing, its specific drivers and border conditions, and the need for contracts to attain channel coordination, the remaining sections of this paper are structured as follows. Section 2 shows the problem formulation noting the implications of imperfect maintenance and finite-horizon service contracts. The model formulation is explained in Section 3. Section 4 presents the coordination mechanisms for profit centered manufacturers. Section 5 describes the case of nonprofit centered manufacturers. Finally, Section 6 provides the conclusions of the work.

2. Problem formulation

Coordination in the supply chain (i.e., channel coordination) plays a relevant role in outsourcing. In the current dynamic environment, coordination of the parties is essential for services in the chain. Kumar [16] suggests that two types of coordination are necessary in supply chain management: horizontal coordination (between the players who belong to the related industry) and vertical coordination (across industry and companies). Although the need for coordination is becoming increasingly evident, efforts to create infrastructures to implement such coordination are still in the early stages. Kumar states that supply chains can create systems that integrate instant visibility and entire dynamic supply chains on an as-needed basis. Those chains are more likely to reach competitive advantages over those that do not adopt such systems [16].

There are several methods to achieve cooperation between manufacturer and contractor. A common practice is to use a *work package contract* that specifies a maintenance strategy and a cost structure acceptable to the contractor. This kind of contract falls into the category of *labor plus parts*, where there are no incentives for the contractor to improve performance [14] because the more services are required, the more the contractor earns. For the contractor, the usual focus is to keep customer loyalty by showing the capacity to outperform competitors [17].

Another issue to take into account when negotiating contracts is the system level at which the contract acts on a system. The contract may include the maintenance of (usually) a single component of a complex system, and it may also be an umbrella agreement or full-service contract covering the whole system. An example of the first case is presented by Tarakci *et al.* [14]. The same authors study a manufacturing system with multiple processes in which each component is maintained independently [18].

Considering the need for reaching effective coordination of the supply chain, Tarakci *et al.* [14] study incentives to maximize the total profit of the service chain. Contracts that aim to achieve a win–win coordination maximize the profits of

the players. According to Tarakci *et al.* [14], these contracts lead the contractor to improve the performance of maintenance operations. They demonstrate that this kind of contract can be an effective tool in achieving the desired overall coordination. Nevertheless, they consider both perfect maintenance for preventive actions and infinite horizon contracts. These two limitations could be improved in order to achieve a full realistic implementation of the model in the operational reality.

The inclusion of imperfect maintenance contributes to a realistic modeling of system failure rates. Changes in failure patterns strongly influence maintenance and replacement decisions [19]. Perfect maintenance considers that every maintenance action returns the system to its ‘as good as new’ condition. Nonetheless, Malik [20] points out that working systems under wear-out failures are not expected to be restored to a new condition and proposes the inclusion of a maintenance improvement factor for imperfect repairs. Furthermore, Nakagawa [21] suggests that failure rate functions on imperfect maintenance cases could be adjusted using a probability approach; thus, the action is perfect (‘as good as new’) with probability $(1-\alpha)$ and minimal (‘as bad as old’) with probability α . Zhang and Jardine [22] argue that enhancements by overhauls tend to be magnified by Nakagawa’s model, and there is a possibility that the failure rate could be bounded; consequently, the appropriateness of the model could be restrained. Zhang and Jardine present an optional approach in which the system failure rate function is in a dynamic modification between overhaul periods because this rate is considered between ‘as bad as old’ and ‘as good as previous overhaul period’ using a fixed degree. Zhang and Jardine’s approach is used in the model formulation of this paper. Because imperfect maintenance sets the system failure rate between a new condition and a previous to failure condition [23], the introduction of this realistic assumption is fundamental to model applicability.

An important issue that should be considered during the coordination process is the time horizon of the contract. This condition holds true not only because of the amortization of investments by the provider but also because the assets under consideration suffer, in general, an aging process that increases the need to perform maintenance and overhaul actions. In this regard, Lugtigheid *et al.* [4] focus on finite-horizon service contracts. They note the lack of literature for finite-horizon contracts and present several methods that consider a repair/replacement model for critical components. In our case, the focus is not on the component level but on the system level. Furthermore, Nakagawa and Mizutani [24] propose finite-interval versions for classic replacement models, such as models of periodic replacement with minimal repair, block replacement, and simple replacement. Regarding the aging process, it is often an effect of imperfect maintenance practices that can be modeled using different approaches, many of which are described in references such as [25–27]. Nakagawa and Mizutani also consider imperfect maintenance models but do not split costs into in-house and outsourcing costs. In this article, we focus on the well-known method described by Zhang and Jardine [22], but the application of the concepts to other approaches like virtual age models [28] is straightforward.

3. Model formulation

Let us consider equipment whose maintenance the manufacturer wishes to subcontract. According to the manufacturer’s own needs and considering the service supply chain benefits, the manufacturer intends to offer a contract that (i) maximizes the sum of expected profits for the parties along the duration of the contract and (ii) minimizes maintenance costs subject to a service level constraint. The first situation may appear when both parties are profit-centered (i.e., a mine site and a haul-truck maintenance contractor). In the second case, the manufacturer is committed to obtain a given service level and intends to minimize the maintenance costs while the contractor is profit-centered (i.e., a hospital and the critical equipment maintenance contractor).

For tractability of analyses, we limit ourselves to the following conditions:

1. The system failure rate function follows a Weibull distribution with shape parameter β (integer) and

$$\beta > 1 \tag{1}$$

2. A preventive maintenance (PM) action restores the system to *almost as good as new* condition according to [22]:

$$\lambda_k(t) = \alpha\lambda_{k-1}(t - T) + (1 - \alpha)\lambda_{k-1}(t) \tag{2}$$

where t represents time, k corresponds to the index of the k -th preventive action, and α is the maintenance improvement factor,

$$0 \leq \alpha \leq 1$$

3. Corrective maintenance is *minimal*.
4. Direct (spare+labor) costs and length of PM are C_p (money units, mu) and T_p (time units, tu), respectively.
5. Direct costs and length of corrective maintenance are, respectively, C_r (mu) and T_r (tu).
6. The interval between PM is T (tu).

7. The contractor is free to select the age T at which PM will be performed.
8. The basic service fee is p (mu/tu).
9. The contractor sets a minimum expected profit π (mu/tu) to take part in the contractual relationship.
10. The net revenue of the manufacturer after production costs is R (mu/tu).
11. The contract duration is from the beginning of a system life cycle to the end of the n -th overhaul.

Before the first PM, the failure rate is

$$\lambda(t) = \lambda_0 \beta t^{\beta-1}, \quad t < T \tag{3}$$

The expected number of failures N after n overhauls is

$$N(nT) = \sum_{i=0}^n \binom{n}{i} \alpha^{n-i} (1-\alpha)^{i-1} N_0(iT) \tag{4}$$

where

$$N_0 = \int_0^{nT} \lambda(t) dt$$

for $n \geq \beta$, β integer is

$$N(nT) = \kappa \lambda_0 T^\beta$$

where κ depends on α and n . Some values are shown in Table I.

The expected interval availability during the contract is

$$A(nT) = \frac{nT - N(nT)T_r}{n(T + T_p)} \tag{5}$$

and the expected profit for the buyer is

$$\Pi_m(nT) = RA(nT) - p \tag{6}$$

The expected maintenance (direct) costs are

$$c_i(nT) = \frac{nC_p + N(nT)C_r}{n(T + T_p)} \tag{7}$$

which leads to the expected contractor profit

$$\Pi_c(nT) = p - c_i(nT) \tag{8}$$

Following the lead of Tarakci *et al.* [14], when Equations 6 and 8 are compared for a fixed fee p , it is clear that the manufacturer wishes to maximize availability (which, in our case, is equal to utilization), whereas the contractor wishes to minimize maintenance costs. It is necessary to propose a contract that achieves collaboration for both parties. With that in mind, the expected profit of the service chain is

$$\Pi(nT) = RA(nT) - c_i(nT) \tag{9}$$

To achieve channel coordination, it is necessary to maximize $\Pi(nT)$; this situation, however, will hardly ever be reached if both the manufacturer and the contractor try to maximize their own objective functions, as shown in the following Lemma.

Table I. κ vs n .	
β	κ
1	n
2	$n^2(1-\alpha) + n\alpha$
3	$n(n-1)(n-2)(1-\alpha)^2 + 3n(n-1)(1-\alpha) + n$

Lemma 1

Define

$$g_{\alpha}(T) = \kappa \lambda_0 \left((\beta - 1)T^{\beta} + \beta T_p T^{\beta-1} \right)$$

Then,

1. The optimal solution that maximizes the manufacturer's profit is T_m^* , which satisfies

$$g_{\alpha}(T_m^*) = n \frac{T_p}{T_r} \tag{10}$$

2. The optimal solution that maximizes the contractor's profit is T_c^* , which satisfies

$$g_{\alpha}(T_c^*) = n \frac{C_p}{C_r} \tag{11}$$

3. The optimal solution that maximizes the total profit is T^* , which satisfies

$$g_{\alpha}(T^*) = n \frac{RT_p + C_p}{RT_r + C_r} \tag{12}$$

This result is equivalent to the one developed by Tarakci *et al.* [14]; nevertheless, there are some significant differences between both results. Note that the definition of $g_{\alpha}(T)$ differs from the definition of $g(T)$ proposed by Tarakci *et al.* in a κ factor that depends on both n and α . This factor is important because it takes into account the fact that the contract has a finite time horizon and that overhauls do not leave the system in an *as good as new* condition.

With the following Lemmas, we discuss the dependence of the function $g_{\alpha}(T)$ on the model's constants and the effect of finite time horizon and imperfect maintenance hypothesis in setting optimal PM intervals.

Lemma 2

In *ceteris paribus* condition,

1. The optimal maintenance intervals for the manufacturer, the contractor, and the service chain decrease in scale and shape parameters of the process failure-rate function.
2. The optimal PM interval T_m^* for the manufacturer increases in the PM time T_p , but that of the contractor (T_c^*) decreases in PM time.

Lemma 2 is completely analogous to the one proved by Tarakci *et al.*, and it shows the same intuitive facts for our case; if there is a higher process deterioration rate, more frequent overhauls will be necessary from the point of view of all players. However, if an improvement in the PM time is made for the contractor, the effect on optimal times will be the opposite for the other parties, and channel coordination will be more difficult to reach.

On the other hand, in a first analysis of Equations (10), (11), and (12), it would appear that if n is increased, then the optimal intervals will be increased as well. This observation is not valid because g_{α} is a function that depends on n , so it is not straightforward how variations on n affect the value of optimal PM intervals.

The following Lemmas show an interesting relationship between $\kappa(\alpha, n)$ and n , which allow us to understand the dependence of the optimal PM intervals on contract's time horizon.

Lemma 3

Let

$$\kappa(\alpha, n) = \sum_{i=0}^n \binom{n}{i} \alpha^{n-i} (1-\alpha)^{i-1} i^{\beta}$$

$\alpha \in [0, 1]$, $\beta \geq 1$ and $n \in \mathbb{N}$, then

$$\kappa(\alpha, n) \geq n \tag{13}$$

Lemma 4

Let $n, i \in \mathbb{N}$, $n \geq i$, $\beta \in \mathbb{R}$, $\beta \geq 1$, then

$$\left(n \binom{n+1}{i} - (n+1) \binom{n}{i} \right) i^{\beta} \geq (n+1) \binom{n}{i-1} (i-1)^{\beta} \tag{14}$$

Lemma 5

Let $\alpha \in [0, 1]$, $\beta \geq 1$ and $n \in \mathbb{N}$, then

$$\frac{n}{\kappa(\alpha, n)} \geq \frac{n+1}{\kappa(\alpha, n+1)} \tag{15}$$

Tarakci *et al.* [14] showed that the optimal PM intervals, assuming an infinite time horizon and a renewal process, were given by the following:

$$g(T_{m_{rp}}^*) = \frac{T_p}{T_r} \tag{16}$$

$$g(T_{c_{rp}}^*) = \frac{C_p}{C_r} \tag{17}$$

$$g(T_{rp}^*) = \frac{RT_p + C_p}{RT_r + C_r} \tag{18}$$

According to its definitions, g_α and g are related as follows:

$$g_\alpha(T) = \kappa(\alpha, n)g(T) \tag{19}$$

Hence, we can rewrite optimality conditions of Lemma 1 in terms of $g(T)$:

$$g(T_{m_{n+1}}^*) = \frac{n+1}{\kappa(\alpha, n+1)} \frac{T_p}{T_r} \leq g(T_{m_n}^*) = \frac{n}{\kappa(\alpha, n)} \frac{T_p}{T_r} \leq \frac{T_p}{T_r} = g(T_{m_{rp}}^*) \tag{20}$$

$$g(T_{c_{n+1}}^*) = \frac{n+1}{\kappa(\alpha, n+1)} \frac{C_p}{C_r} \leq g(T_{c_n}^*) = \frac{n}{\kappa(\alpha, n)} \frac{C_p}{C_r} \leq \frac{C_p}{C_r} = g(T_{c_{rp}}^*) \tag{21}$$

$$g(T_{n+1}^*) = \frac{n+1}{\kappa(\alpha, n+1)} \frac{RT_p + C_p}{RT_r + C_r} \leq g(T_n^*) = \frac{n}{\kappa(\alpha, n)} \frac{RT_p + C_p}{RT_r + C_r} \leq \frac{RT_p + C_p}{RT_r + C_r} = g(T_{rp}^*) \tag{22}$$

Inequalities in Equations (20), (21), and (22) follow from Lemmas 3, 4, and 5. As $g(T)$ is an increasing function, it is straightforward that optimal PM intervals for finite horizon contracts are shorter than or equal to optimal PM intervals for an infinite horizon contract, in *ceteris paribus* condition. Even more, PM intervals will be shorter in contracts with more terms agreed; that is, only because the contract is longer, more PM are needed.

On the other hand, considering the $\kappa(\alpha, n)$ definition, a lower α allows an increase in κ . This result is very intuitive because a low improvement in failure rate after an overhauling creates incentives to do PM more often.

Moreover, it is easy to prove that

$$\lim_{\alpha \rightarrow 1} \kappa(\alpha, n) = \lim_{\alpha \rightarrow 1} \sum_{i=0}^n \binom{n}{i} \alpha^{n-i} (1-\alpha)^{i-1} i^\beta = n \tag{23}$$

Thus, looking at Equations (20), (21), and (22), we can conclude that if perfect overhauls are performed (renewal process), optimal conditions will not depend on n , and the decisions over PM intervals will not depend on contract time horizon.

Because all right-hand side expressions in optimal PM interval condition in Equations (20)–(22) are weighed by $n/\kappa(\alpha, n)$, which does not depend on T , Lemma 3 in the paper of Tarakci *et al.* [14] is applicable. It allows us to state an analogous Lemma.

Lemma 6

The relationships among the optimal PM intervals for the manufacturer, the contractor, and the service chain are given by (i) $T_c^* = T_m^* = T^*$ if $C_p/C_r = T_p/T_r$; (ii) $T_c^* > T^* > T_m^*$ if $C_p/C_r > T_p/T_r$; and (iii) $T_c^* < T^* < T_m^*$ if $C_p/C_r < T_p/T_r$.

4. Coordination mechanisms for profit-centered manufacturers

4.1. The cost subsidization contract

The basis for establishing the cost subsidization contract is given by Tarakci *et al.* [14]. If $T_c^* > T^*$, the manufacturer agrees to subsidize the cost of the PM to make it more attractive for the contractor, who wishes to maximize profit. Let ΔC_p be that bonus, and then, the effective cost observed by the contractor of a PM is

$$C'_p = C_p - \Delta C_p \quad (24)$$

To obtain $T_c^{*'} = T^*$, we know that,

$$g_\alpha(T^*) = n \frac{RT_p + C_p}{RT_r + C_r} = n \frac{C'_p}{C_r} = g_\alpha(T_c^{*'}) \quad (25)$$

then,

$$C'_p = C_r \frac{RT_p + C_p}{RT_r + C_r} \quad (26)$$

and,

$$\Delta C_p = C_p - C'_p \quad (27)$$

The result showed by Equation (26) is remarkable because it is exactly the same as the one obtained for infinite time contracts with renewal process failure.

The expected profit for the contractor is now

$$\Pi_c(nT) = p - c_i(nT) + \frac{n\Delta C_p}{n(T + T_p)} = p - c_i(nT) + \frac{\Delta C_p}{T + T_p} \quad (28)$$

and for the manufacturer

$$\Pi_m(nT) = RA(nT) - p - \frac{n\Delta C_p}{n(T + T_p)} = RA(nT) - p - \frac{\Delta C_p}{T + T_p} \quad (29)$$

Lemma 7

Channel coordination can be achieved using cost subsidization contract with $p \in [p_1, p_2]$, where

$$p_1 = \pi + c_i(nT^*) - \frac{\Delta C_p}{T^* + T_p} \geq 0 \quad (30)$$

$$p_2 = RA(nT^*) + \pi - \Pi(nT_c^*) - \frac{\Delta C_p}{T^* + T_p} \geq p_1 \quad (31)$$

If $T_c^* < T^*$, the manufacturer agrees to subsidize the cost of the corrective maintenance to make it more attractive for the contractor who wishes to maximize profit. Let ΔC_r be that bonus, and then, the effective cost observed by the contractor of a PM is

$$C'_r = C_r - \Delta C_r \quad (32)$$

To obtain $T_c^{*'} = T^*$, we know that

$$g_\alpha(T^*) = n \frac{RT_p + C_p}{RT_r + C_r} = n \frac{C'_r}{C_r} = g_\alpha(T_c^{*'}) \quad (33)$$

then,

$$C'_r = C_p \frac{RT_r + C_r}{RT_p + C_p} \quad (34)$$

and,

$$\Delta C_r = C_r - C'_r \quad (35)$$

The expected profit for the contractor is now

$$\Pi_c(nT) = p - c_i(nT) + \frac{N(nT)\Delta C_r}{n(T + T_p)} \quad (36)$$

and for the manufacturer

$$\Pi_m(nT) = RA(nT) - p - \frac{N(nT)\Delta C_r}{n(T + T_p)} \quad (37)$$

Lemma 8

Channel coordination can be achieved using cost subsidization contract with $p \in [p_1, p_2]$, where

$$p_1 = \pi + c_i(nT^*) - \frac{N(nT^*)\Delta C_r}{n(T^* + T_p)} \geq 0 \quad (38)$$

$$p_2 = RA(nT^*) + \pi - \Pi(nT_c^*) - \frac{N(nT^*)\Delta C_r}{n(T^* + T_p)} \geq p_1 \quad (39)$$

4.2. The uptime target and bonus contract

The basis for establishing the uptime target and bonus contract is given by Tarakci *et al.* [14]. If the contractor achieves an uptime level above a target uptime τ , the manufacturer agrees to increase the contract attractiveness by a per-unit-time bonus B . Both contractor's profit and manufacturer's profit, respectively, become

$$\Pi'_c(nT) = p - c_i(nT) + B[A(nT) - \tau]^+ \quad (40)$$

where $[x]^+ = \max\{x, 0\}$,

$$\Pi'_m(nT) = RA(nT) - p - B[A(nT) - \tau]^+ \quad (41)$$

Thus, the manufacturer selects B , τ , and p to encourage the contractor to choose the interval T^* to maximize the profit $\Pi'_c(nT)$. In this case, the channel coordination is set by the following Lemma.

Lemma 9

Channel coordination can be achieved using uptime target and bonus contract with $\tau \in [\tau_1, \tau_2]$ and $p \in [p_1, p_2]$, where

$$\tau_1 = \frac{\Pi(nT^*) - \pi}{R} \quad (42)$$

$$\tau_2 = A(nT^*) - \frac{c_i(nT^*) - c_i(nT_c^*)}{R} \quad (43)$$

$$p_1 = R\tau + \pi - \Pi(nT^*) \quad (44)$$

$$p_2 = R\tau + \pi - \Pi(nT_c^*) \quad (45)$$

4.3. Case study

Let us consider a customized version of the case described by Tarakci *et al.* [14]. Parameter values are shown in Table II. We are interested in a study of the optimal interval T for contractor, manufacturer, and service chain, respectively. Figure 1 displays such values in terms of $g_\alpha(T)$, whereas Figure 2 presents a study of κ in terms of β and n . In summary, Table III exhibits results for this initial case. Figures 3 and 4 show a study of availability in terms of T and a study of the expected profits, respectively. Then, the optimal duration of the contract is $n(T^* + T_p) = 5(9.56 + 1) = 52.80$.

Table II. Initial parameters.	
Parameter	Value
λ_0	0.001
β	3
T_p	1
T_r	0.30
C_p	8
C_r	0.40
R	15
p	2.50
π	2.50
α	0.90
n	5

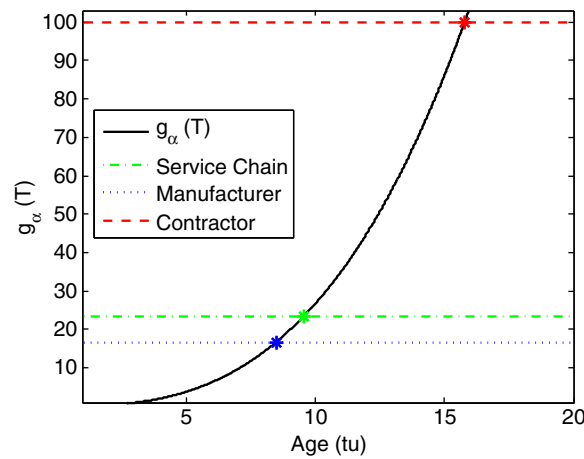


Figure 1. Study of $g_\alpha(T)$ for $n = 5$, where * indicates each optimal value.

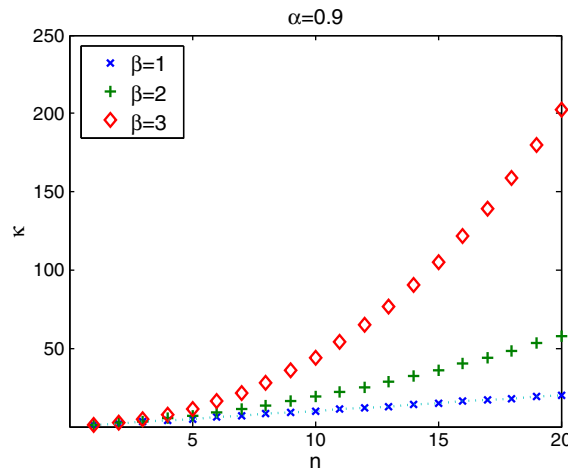


Figure 2. Study of κ .

Table III. No incentives; results.					
	T	A	Π	Π_m	Π_c
T_m^*	8.48	0.850		10.21	
T^*	9.56	0.848	11.88		
T_c^*	15.79	0.777			1.67

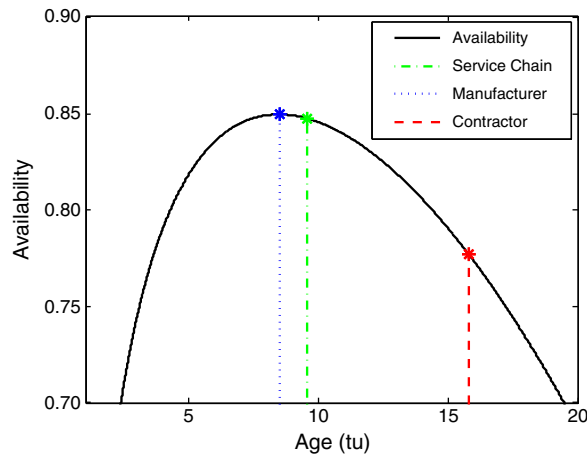


Figure 3. Study of A , where * indicates each optimal value.

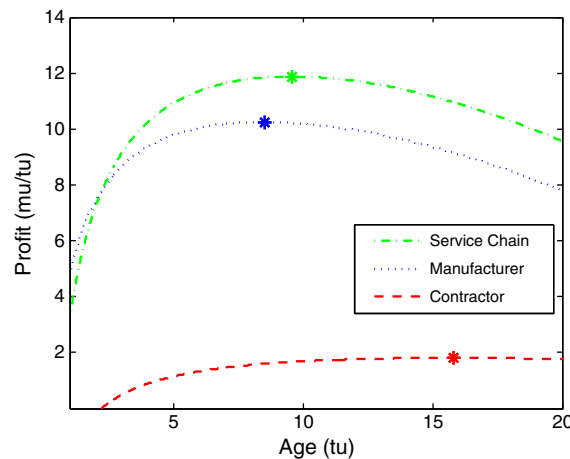


Figure 4. Study of the expected profits.

When the optimal PM interval (T^*) is set, then the fee p to be adopted by the manufacturer for satisfying the contractor's minimum expected profit, i.e. the equilibrium service fee, is: 3.33. If Π_c is below π , as this case with $p = 2.50$, then the contractor does not respond well to that profit. Hence, it is necessary to seek ways to create an incentive for the contractor.

Because of $T_c^* > T^*$, it becomes necessary to increase the frequency of the PM. Considering the previous example, now, the aim is to find a bonus ΔC_p , which sets the optimal point for the contractor with the one of the service chain. As shown in Table III, this point is located at $T^* = 9.56$. By evaluating Equation (27), it is possible to calculate the bonus, and then,

$$\Delta C_p = 6.12$$

Figure 5 shows a study of the profits for both parties (Equations (29) and (28) as a function of T). Note that even if the bonus is obtained, the contractor still does not respond because the expected profit is below π . By using Equations (30) and (31), the limits of the service fee are $p_1 = 2.75$ and $p_2 = 3.67$. Following the lead of the work of Tarakci *et al.* [14], the extra profit from channel coordination is the difference between p_2 and p_1 , which in this case is 0.92. This can be

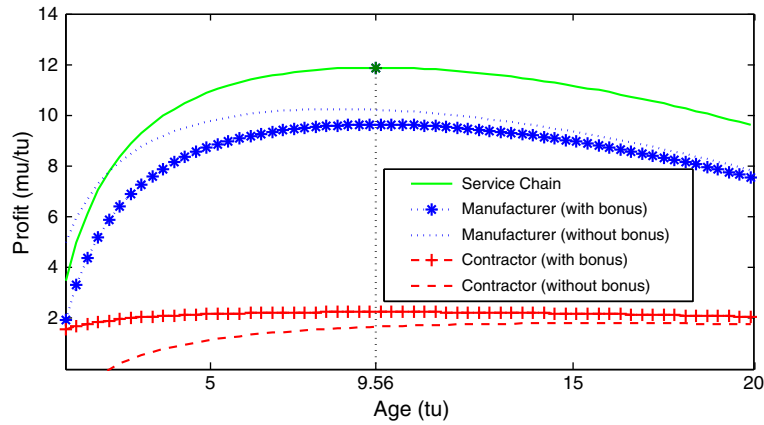


Figure 5. Expected profits in terms of T with bonus for preventive actions.

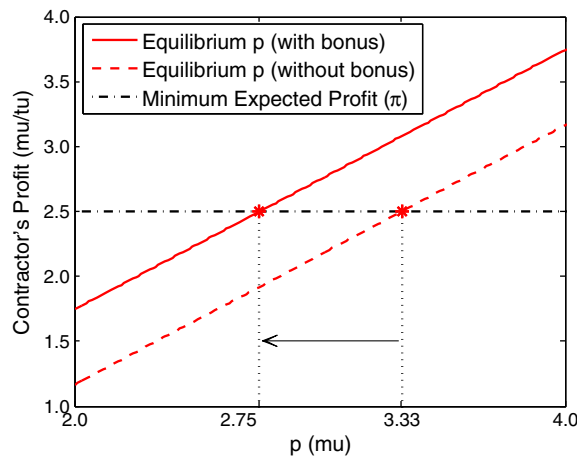


Figure 6. Decreasing of equilibrium service fee.

distributed between the contractor and the manufacturer by choosing p ($p \in [p_1, p_2]$). For example, if p is set at 3.30, then the contractor's profit is 3.05 (greater than the contractor's minimum expected profit), and the manufacturer's profit is 8.84. Under these conditions, the contractor receives an incentive for entering the coordination process. The sensitivity of this calculation is shown in Figure 6, where the equilibrium service fee decreases because of the incentive given by cost subsidization.

5. Nonprofit centered manufacturers

In several organizations (military, public services, etc.), the main interest is to provide a contracted service level (i.e., availability or deterrence power) at a minimum direct cost. In general, a reference value is set (from benchmarks with similar foreign organizations or just imposed) because of budget or capacity constraints. Let us consider the case where A_r is the reference availability. Note that the manufacturer is interested in achieving the contracted service level to obtain an achievable profit for his or her own benefit and for the contractor's as well.

5.1. Bonus to preventive actions

Let $T_{m_2}^*$ be the interval that achieves

$$A(T_{m_2}^*) = A_r$$

A_r should be feasible, so a necessary condition is

$$A_r \leq \max(A) \tag{46}$$

Note that the manufacturer only has to give the bonus to the contractor if $T_{m_2}^* \leq T_c^*$. Otherwise, the contractor is already working on the interval; therefore, the manufacturer does not have to provide the incentive. Depending on the cost ratios, it is necessary to evaluate the potentially feasible solutions according to roots of

$$\kappa \lambda_0 T_r T^\beta + (A_r - 1)nT + A_r n T_p = 0 \quad (47)$$

If the left-hand side of the aforementioned equation is considered, it is efficiently reasonable to choose the largest T that achieves A_r . To provide an incentive for the contractor to set $T = T_{m_2}^*$, a C_p'' , it must be perceived such that,

$$n \frac{C_p''}{C_r} = g_\alpha(T_{m_2}^*) \quad (48)$$

which allows to set the incentive at

$$\Delta C_p = C_p - C_p'' \quad (49)$$

5.2. Case study

Let us consider the case previously analyzed. The manufacturer has set the reference availability at $A_r = 0.83$. If Equation (47) is evaluated, then two potential solutions appear

$$T_1 = 5.60 \text{ and } T_2 = 12.05$$

These potential solutions can be seen in Figure 7. If both solutions are assessed in Equations (48) and (49), two different bonuses are obtained. For the sake of generality, the following example is enunciated. If the service level is set at the contractor's availability, namely $A_r = 0.777$, the two potential solutions for T are $T_a = 3.64$ and $T_b = 15.79$. Potential bonuses are $\Delta C_{pa} = 7.87$ and $\Delta C_{pb} = 0$, respectively. In spite of two numerical solutions for achieving the same A_r , if the PM interval is set as equal to the contractor's interval, there is no need for any incentive. Expectedly under these circumstances, the higher solution for T is the most economically suitable.

Because of the highest T achieves an identical desired availability with the lowest cost for the manufacturer, the most efficient option is to estimate the bonus (ΔC_p) using $T_2 = 12.05$. Then,

$$\Delta C_p = 4.34$$

In the same way, if the manufacturer sets the feasible maximum availability like a target, namely when $A_r = 0.8497$, then $T_{m_2}^* = 8.48$ (the same as T_m^*). But in this case, C_p'' is 1.33, and the manufacturer must pay a bonus of 6.67 for each preventive action. This result is consistent with the bigger challenge of increasing the service level. Finally, Figure 8 shows the increase of the contractor's profit from a no incentive contract to a bonus for preventive actions contract.

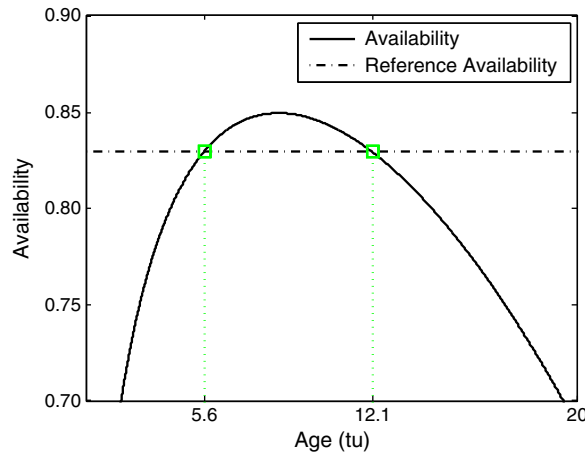


Figure 7. Potential solutions by setting $A_r = 0.83$.

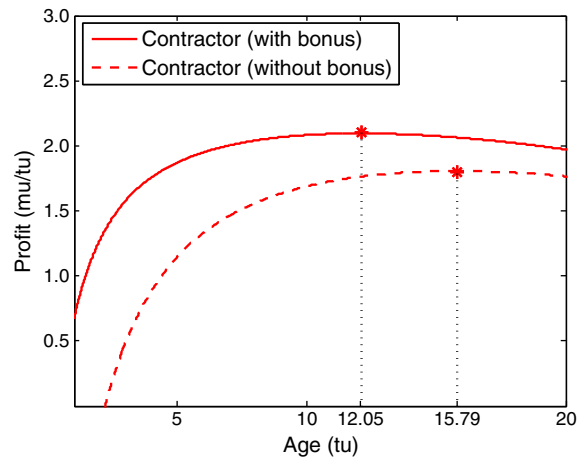


Figure 8. Profit for the contractor by setting the feasible maximum availability.

6. Conclusions

This paper introduces a model that defines contractual conditions to coordinate the supply chain. This is achieved by setting a PM strategy that maximizes the total expected profit for manufacturers, and contractors, who usually try to optimize profits separately. In particular, the method finds the optimal interval between PM for the contractor, manufacturer, and service chain, respectively. The study is an extension of previous works because it considers imperfect maintenance and finite-horizon service contracts and also shows how these affect stakeholder's decision making. Also, previous works have been expanded to include not only profit-centered manufacturers but nonprofit centered manufacturers as well. We have evaluated how to achieve the desired supply channel coordination to encourage both players to optimize their actions altogether and thus to achieve the increase of their expected profits.

For profit-centered manufacturers, we have found the optimal duration of a contract that reaches channel coordination. However, there are scenarios where the expected profit for the contractor is not enough to drive changes in the PM interval. One way to provide an incentive for the contractor is that the manufacturer pays a bonus for each PM performed by the contractor, provided that interventions allow achieving the supply chain optimization. We estimated such bonus.

For nonprofit centered manufacturers, we evaluated the optimal interval to achieve a reference availability delivered by the manufacturer. Again, we provide a bonus that motivates the contractor to achieve the desired availability, maximizing profits for itself and the entire supply chain.

Finally, we demonstrated that the model achieves win-win coordination of the supply chain. Both manufacturer and contractor are encouraged to continually improve their maintenance services, as profits increase relative to those obtained when no coordination occurs.

A direct expansion to the present formulation would consider how the replacement may affect the terms of the contract. The model can also be expanded to consider the maximization of the service chain discounted profit over an infinite time horizon.

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