INEQUALITY, INSTITUTIONS AND GROWTH*

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Abstract

The paper explores the relationship between the distribution of wealth and income, the institutional environment and the growth rate of an economy. A formal model is developed in which the members of society determine, through a political process, the extent of property rights protection provided by its institutions. We show that the level of protection is a function of the decisive agent's property share, and that for both a political process requiring a 'strict consensus' and for one consisting in 'majority voting', improvements in the distribution of property result in more secure property rights. The model also predicts that, since increased property rights protection reduces the adverse effects of socio-political instability on savings and investment, improvements in the distribution of wealth and income also lead to higher growth rates.

Resumen

Este artículo explora la relación existente entre la distribución del ingreso y la riqueza, la institucionalidad y la tasa de crecimiento de la economía. Se desarrolla un modelo formal en el que los miembros de la sociedad determinan, a través de un proceso político, el grado de protección de los derechos de propiedad provisto por sus instituciones. Mostramos que el grado de protección es una función de la participación en la propiedad que tiene el votante decisivo y que para procesos políticos se requieren “consenso estricto” o para aquellos consistentes con la “regla de la mayoría”, mejorías en la distribución de la propiedad conduce a tener derechos de propiedad más seguros. El modelo también predice que, dado que el aumento en la protección de los derechos de propiedad reduce los efectos adversos de la inestabilidad sociopolítica en ahorro e inversión, las mejorías en la distribución del ingreso y la riqueza también conducen a tasas de crecimiento mayores.


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1. Introduction

Early economic theories postulated that inequality might be necessary to generate the savings needed for growth in poor nations. According to Kaldor (1956), inequality is favorable for savings because the rich capitalists save a larger share of their income than the poor workers.\(^1\) However recent empirical research have found a negative and significant association between inequality and growth.\(^2\) Persson and Tabellini (1994) find a significant negative correlation between initial inequality and long-run growth using both a historical panel data set and post war cross sections. Alesina and Rodrik (1994) obtain the same results with cross country growth regressions for different data sets on inequality. Clarke (1995) and Perotti (1996) confirm that the negative correlation is robust across different inequality measures and to many specifications of the growth regressions. Deininger and Squire (1996a) confirm that the inverse relationship between unequal distribution of assets and growth holds for a new data set on inequality even when regional differences are taken into account.\(^3\)

This paper concentrates in two of the links that have been suggested to explain the observed negative correlation between inequality and growth.\(^4\) The main focus of this paper is the link relating inequality with growth through the quality of institutions. North and Thomas (1973) and North (1990), from a historical perspective, and Keefer and Knack (1997), Knack and Keefer (1995) and Barro (1996a), from cross-country evidence, indicate that institutions that protect property rights matter for economic growth. Moreover, Keefer and Knack (1995) contend that the quality of institutions worsens with inequality. The paper also deals with the socio-political violence and instability explanation. According to Barro (1991) and Alesina, Ozler, Roubini and Swagel (1996) socio-political violence and instability curtail growth, and Alesina and Perotti (1996) and Perotti (1996) argue that political instability increases with income inequality.

Several empirical studies give support to both the institutions and the socio-political instability hypotheses. Knack and Keefer (1995) and Keefer and Knack (1995) use institutional indicators (quality of bureaucracy, corruption in government, rule of law, expropriation risk, repudiation of contracts by governments, nationalization potential, contract enforceability) compiled by two private investment risk services (International Country Risk Guide (ICRG) and

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\(^1\) Schmidt-Hebbel and Serven (1996) use alternative inequality and saving measures on cross-section and panel data and provide no support for the notion that income inequality has any positive effect on aggregate saving.

\(^2\) For an extensive survey of the literature see Benabou (1996).

\(^3\) Note that this discussion concerns the correlation from inequality to growth, for a recent discussion on the Kuznets's hypothesis type of effects of growth on inequality see Bruno, Squire and Ravallion (1996).

\(^4\) See Murphy, Shleifer and Vishny (1989) for the market size link, Persson and Tabellini (1994) and Alesina and Rodrik (1994) for the redistribution link and Aghion and Bolton (1997) for the imperfect capital markets explanation.
Business Environmental Risk Intelligence (BERI)) in their cross-country growth regressions and find that "institutions that protect property rights are crucial to economic growth and investment" (Keefer and Knack (1995), p. 233). Barro (1996a) finds that the rule of law measure, which he interprets as "an indicator of the security of the property rights" (Barro (1996b), p. 146), has a positive and significant effect on growth. Barro (1996a) also finds that "the political instability variables are, however, not significantly related to growth when the rule of law index is also included in the regressions" (p. 25). Keefer and Knack (1995) evaluate the effect of inequality on the security of property rights and find that inequality reduces the security of property rights. Their result is robust across different measures of inequality and property rights.

There is also evidence concerning the socio-political instability mechanism. Using indicators of social unrest as political assassinations, mass demonstrations, political strikes and coups, Barro (1991) and Alesina, Ozler, Roubini and Swagel (1996) find that socio-political instability has a negative effect on growth. Barro (1991) uses a single growth equation estimation of the effect of political instability on growth while Alesina, Ozler, Roubini and Swagel (1996) use a simultaneous equation approach to avoid the problems of joint endogeneity. Alesina and Perotti (1996), Perotti (1994) and Perotti (1996) also find that there is a significant positive relationship between inequality and socio-political instability.

In this paper we concentrate on the institutions mechanism. We develop a formal model that explains the empirical findings discussed above. The model is composed of two building blocks. The first one is the formalization of the Keefer and Knack (1995) argument linking inequality to the quality of the institutions that protect the property rights. The second part links the institutions to the growth rate of an economy.

The informal argument provided by Keefer and Knack (1995) is that "inequality makes it more difficult for consensus to be reached about policies in general, and policies protecting property rights, in particular" (p. 3). We formalize the consensus requirement by assuming a strict consensus requirement where all agents must agree with the adopted institutional arrangement. This requirement, which is consistent with the possibility of sabotage and/or veto by minorities, leads to what the trade negotiations literature calls the convoy effect in which the pace of reform is dictated by the agent most reluctant to reform. The agent most reluctant to reform will be the agent with the smallest wealth endowment and his willingness to reform will be an increasing function of his share of wealth. This will result in the observed link between equality, measured as the poorest agent's share of wealth and the quality of institutions.

A second alternative that gives similar results is also explored. Instead of a strict consensus requirement, we assume that the quality of the institutions is decided in a majority voting political process over the single issue of property

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protection. The minimum consensus needed for an institutional reform would be the support of the majority which implies that the decisive agent would now be the median voter.

The second building block of the model corresponds to the effect of the quality of the institutional environment on the economy's growth rate. We focus on the ability of the institutional environment, understood as the rules of the game or the set of constraints that structure political, economical and social interaction, to minimize the adverse effect of socio-political instability on investment and growth. The formal rules (constitution, laws, property rights, etc.) and the mechanisms to enforce them (strong law enforcement organizations, independent judiciary power, check and balance mechanisms, well-defined administrative procedures, transparent decision making process, etc.) can constrain the actions of the agents that generate the socio-political instability as they attempt to obtain a larger share of output. Thus, an adequate institutional environment that assures the security of property and contractual rights can isolate economic activity from the negative effects of socio-political instability.

In the socio-political instability and violence models the institutional framework, which is taken as given, is inefficient in protecting property rights. The climate of instability results from the disadvantaged groups attempts to obtain a larger share of the output that is not properly protected. This creates a tragedy of the commons situation in which all the agents end up overconsuming. They are not willing to sacrifice present consumption because they fear that they will not be able to keep for themselves all the benefits of their investments. Improvements in the institutional framework's capacity to protect property rights reduce these disincentives to invest and, therefore, have a positive effect on the growth rate of the economy. In our model we formalize this argument by extending the model that Bardhan and Dayton-Johnson (1996) developed to analyze the effect of asset inequality on the conservation of a common-pool resource. Instead of assuming that there is common access to the whole stock of capital, we allow the members of the economy to assign exclusive property rights to a part of the economy's stock of capital.

Summing up and combining both building blocks, the society determines the degree of property rights protection by choosing how much of the initial

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7 These attempts can sometimes be violent. The non-violent mechanisms through which they can attempt to do so are: "nationalization; bursts of inflationary finance to sustain the incomes of the government bureaucracies and the military; the squeezing of the agricultural sectors in favor of politically powerful urban classes through exchange rate policies, price controls and monopolistic marketing boards; legislation and other measures that alter the bargaining power of labor; the allocation of highly desirable government and civil service jobs and university admissions to favored ethnic and tribal groups; and large scale bureaucratic corruption tolerated and condoned by the government." (Benhabib and Rustichini (1996), p. 126).
stock of capital will enjoy common access. Since the decision requires everybody's consent, the decisive preference is that of the poorest agent and the adopted degree of protection is a function of the share of property that belongs to him. Greater equality, measured or correlated with the poorest class' share of wealth, implies that they would be willing to protect more of the economy's assets and therefore, that the economy will build institutions that provide greater property rights security. And, as the agents consume more from the unprotected common-access stock of capital, total savings and investment increase as the society adopts a higher degree of protection and reduces the size of the unprotected stock of capital. This is what drives the equality-institutions-growth link in our model.

In addition to the Bardhan and Dayton-Johnson (1996) model, there are two papers that are closely related to the model presented here since they model property rights protection endogenously. The first one, Tornell (1997), permits the players of a dynamic model of the tragedy of the commons to incur in a one time loss to establish a system in which all the economy’s assets are privately owned. By focusing on the switch from common property to fully privatized property, Tornell does not analyze different degrees of protection. Tornell and Velasco (1992) permit the players to divert resources to lower productivity technologies that enjoy private access. Their model does consider a continuum of different degrees of protection by determining how much of the poor countries’ capital is protected in safe-low interest bank accounts in rich countries. But as in the case of Tornell (1997), they do not consider the link between wealth distribution and the security of the property rights.

The remaining of this paper is divided into four sections. The following section, Section 2, presents the model. The main results are discussed in Section 3. Section 4 presents possible extensions of the model. And in the fifth section we conclude.

2. The Model

The model consists in a T-periods dynamic game in which N players (or groups with identical members), that only differ in their expropriatory capacity, enjoy common access to the economy’s stock of unprotected capital. Each agent’s level of consumption from this unprotected stock of capital is limited by their appropriatory capacity and the consumption decisions of the other players. Through a political process, the players assign and protect property rights over part of the economy’s capital according to their relative appropriatory capacities.

In the basic version of the model the outcome of the game is determined in the first period. The players determine depth of the institutional reform is also decided in the beginning of the game and how much they will attempt to consume from the stock of capital that remains unprotected.
2.1. Assumptions

2.1.1. Preferences and Consumption

Each of N agents, \( i = 1, \ldots, N \), maximizes lifetime utility,

\[
U_i = U(C_{t,1}, \ldots, C_{t,i}, \ldots, C_{t,T}),
\]

where \( C_{t,i} \) is agent \( i \)'s consumption in period \( t, t = 1, \ldots, T \). Utility is derived from the consumption of a single good which can be instantaneously converted into capital. In each period agents can consume from the stock of unprotected (or commonly owned) capital \( (K_i^c) \) and from their individually owned or private stock of capital \( (K_{t,i}^p) \).

This simple version of the model uses an additive utility function with no discounting,

\[
U_i = \sum_{t=1}^{T} (C_{t,i}) = \sum_{t=1}^{T} (C_{t,i}^c + C_{t,i}^p),
\]

where \( C_{t,i}^c \) is agent \( i \)'s period \( t \) consumption from \( K_i^c \) and \( C_{t,i}^p \) is agent \( i \)'s period \( t \) consumption from his \( K_{t,i}^p \).

Consumption from the private stock of capital is constrained by:

\[
0 \leq C_{t,i}^p \leq K_{t,i}^p - q_{t,i},
\]

where \( q_{t,i} \) corresponds to a tax charged to cover the costs of establishing the required institutions to protect the property rights (i.e. judiciary system, law enforcers, etc.).

Consumption from the common stock of capital will depend not only on the size of the stock, but also on the consumption decisions of other agents:

\[
0 \leq C_{t,i}^c \leq C_{t,i}^{\max} = \begin{cases} 
K_i^c - \sum_{i \neq h} (C_{t,i}^c) & \text{if } \sum_{i \neq h} (C_{t,i}^c) \leq (1 - \beta_h)K_i^c \\
\beta_h K_i^c & \text{if } \sum_{i \neq h} (C_{t,i}^c) > (1 - \beta_h)K_i^c
\end{cases}
\]

where \( \beta_h \) is agent \( h \)'s relative capacity of appropriation \( (0, 1) \) and \( \sum_{i=1}^{N} \beta_i = 1 \).

The above constraint implies that if an agent did not face competition to consume from the common stock of capital, he would have the capacity of consuming the whole stock. However, if other agents are also attempting to consume from the common stock of capital then it must be shared according to each agent's relative capacity of appropriation. The capacity of appropriation parameter reflects the agents' relative power to capture a larger share of the
common stock of capital either through direct expropriation and/or by manipulating the political system.\textsuperscript{8}

Agents are ordered according to their appropriatory capacity,

\[ 0 < \beta_1 \leq \beta_2 \leq ... \beta_N. \]

2.1.2. Capital Accumulation and Property Rights Protection

The economy starts period 1 with an aggregate endowment of capital \( K_1 \), arbitrarily set equal to 1, which expands following:

\[
K_t = (1 + g_t) \left( K_{t-1} - \sum_{i=1}^{N} (C_{t-1,i} + q_{t,i}) \right),
\]

where the exogenous growth rate \( g_t \in [0, N - 1] \). The upper limit on \( g_t \) is imposed to eliminate the possibility of an outcome in which property rights would not be required. It is also assumed that capital's growth rate is constant,\textsuperscript{9} \( g_t = g \) for \( t = 1, ..., T \).

Initially there are no property rights and all agents share common access to the aggregate stock of capital. However, as mentioned above, there can be an institutional reform to provide property rights protection. If the reform occurs let \( r, r \in (1, ..., T) \), denote the period in which the reform occurs. With the reform, each agent will be granted exclusive rights over his portion of the protected capital stock \( (K^p_{r,t}) \) plus common access to the stock of capital that remains unprotected \( (K^c_t) \).

The depth of the reform is measured by the degree of property rights protection, \( \phi, \phi \in (0, 1) \), which corresponds to the share of the total stock of capital for which there is exclusive access at the moment of the reform. Under the initial common access regime \( \phi = 0 \) while with an institutional framework that provides full property rights protection \( \phi = 1 \).

The stock of protected capital is distributed among the agents according to their relative appropriatory capacities.

\[ K^p_{r,t} = \beta_i K^p_r = \beta_i (\phi K_r). \]

The capital accumulation equations for \( t > r \) are:

\[ K^c_t = (1 + g) \left( K^c_{t-1} - \sum_{i=1}^{N} (C^c_{t-1,i}) \right) \text{ and } K^p_{r,t} = (1 + g)(K^p_{r,t-1} - (C^p_{t-1,i} + q_{t,i})). \]

\textsuperscript{8} In another section we extend the model to include absolute capacity of appropriation constraints.

\textsuperscript{9} This assumption simplifies the algebraic presentation without altering the results of the model.
The depth of the institutional reform and its date is determined through a political process. The main one used in this version of the paper is a mechanism requiring the consensus of all agents for both decisions. We also examine a majority voting alternative and find that the main results of the paper hold.

As mentioned above, the institutional reform is not free. There is a one-time aggregate cost, \( Q_r \), in period \( r \) of establishing the property rights protecting institutions. This cost is a function of the degree of property rights protection given by the institutional reform and also of the size of the aggregate stock of capital at the moment of the reform, \( Q_r = Q(K_r, \phi) \). We assume that the marginal cost of protection is positive and non-decreasing (the cost of protecting property increases with the size of the property protected \( \left( \frac{\partial Q}{\partial K_r}, \frac{\partial Q}{\partial \phi} > 0 \right) \) and \( \left( \frac{\partial^2 Q}{\partial K_r^2}, \frac{\partial^2 Q}{\partial \phi^2} \geq 0 \right) \). In this version of the model we use the following specification to simplify the algebra: \( Q(K_r, \phi) = N_q(K_r)^2 = N_q(\phi K_r)^2 \) where \( q \geq \frac{(1+g)^{T-1}-1}{2N(1+g)^{T-1}} \).  

Taxes are charged only in the period in which the reform occurs and the tax is the same for every agent. Thus, \( q_{t,i} = \frac{Q_r}{N} \) for \( t = r \) and \( q_{t,i} = 0 \) for \( t \neq r \).

2.2. Solution

The solution of the model is reached in three stages. In the first one, the subgame-perfect Nash equilibrium is obtained through backwards induction taking as given that there is an institutional reform in period 1 (\( r = 1 \)) which sets the degree of property rights protection at \( \phi = \phi^* \). In the second stage, it is shown that the institutional reform must occur in the first period, as was assumed in the first stage. The adopted degree of property rights protection, \( \phi^* \), is obtained in the third stage.

2.2.1. Stage I

In this stage each agent chooses the consumption strategy that maximizes his lifetime utility (equation 1) subject to the consumption constraints (equations 2 and 3) and the capital accumulation equations (4) taking as given that \( r = 1 \) and \( \phi = \phi^* \). Because agents share common access to the unprotected stock of capital, their consumption decisions will depend on the actions of the other agents. An agent's (agent h's) consumption strategy will be designed for two possible scenarios concerning the actions of other agents. In the defection scenario at least one of the other agents attempts to consume as much as possible

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10 The value of \( q \) is restricted to make the maximum possible adopted level of protection less than or equal to unity, the maximum feasible value. This assumption does not affect the results of the model, it only makes their presentation clearer.
from the stock of unprotected capital while in the cooperation scenario all other agents constrain their consumption attempts\textsuperscript{11} \( \left( \sum_{i \neq h} C_{i,t}^c < K_t^c \right) \).

The whole stock of unprotected capital will be consumed in the defection scenario independent of agent h's decision. An agent's consumption decision involves no trade-off: if one chooses not to consume a unit from the common stock of capital then the unit will not be reinvested to increase the future consumption possibilities, it will be consumed by the agent that is attempting to maximize his consumption \( \left( \frac{\partial \left( \sum_{i \neq h} C_{i,t}^c \right)}{\partial C_{i,h}^c} = -1 \right) \).

In the cooperation scenario an agent does face a trade-off when deciding how much to consume from the common stock of capital: if an agent decides not to consume one unit, the unit will be reinvested and future consumption possibilities will be expanded \( \left( \frac{\partial \left( \sum_{i \neq h} C_{i,t}^c \right)}{\partial C_{i,h}^c} = 0 \right) \).

**Period T** In period T the game and the agents’ lives end therefore there is no reason for them to postpone their consumption. This is true for both scenarios. All agents will attempt to consume as much as possible from both their private stocks of capital and from the unprotected stock of capital. Since this is true for all agents, equation 3 implies:

\[
C_{i,h}^p = K_{i,h}^p \quad \text{and} \quad C_{i,h}^c = \beta_h K_T^c \quad \text{for all} \ h = 1, \ldots N.
\]

**Period T-1** In period T-1 agents choose \( C_{i-1,h}^p \) and \( C_{i-1,h}^c \) to maximize their lifetime utility taking into account the period T consumption decisions. In this period, the optimal \( C_{i-1,h}^c \) does depend on the type of scenario faced by the agent making the choice while the decision of \( C_{i-1,h}^p \) does not. The agent's problem is

\[
\text{Maximize } U_h = \sum_{t=1}^{T} (C_{i,t})_h - \sum_{t=1}^{T-2} (C_{i,t,h})_p + C_{i-1,h}^p + C_{i-1,h}^c + C_{i-1,h}^c + C_{i-1,h}^c = \sum_{t=1}^{T-2} (C_{i,t})_h + (1 + g) (K_{i-1,h}^p - C_{i-1,h}^p) + C_{i-1,h}^c + \beta_h (1 + g) (K_{i-1,h}^c - \sum_{i=1}^{N} C_{i-1,i}^c)
\]

\text{\textsuperscript{11} If agents choose to constraint their consumption attempts they will constraint them to zero. This is due to the linearity of the functions that results in corner solutions where agents either choose to consume as much as possible or not to consume at all. However, the cooperation scenario would yield the same results if other agents consume a fixed amount of the common stock of capital (by fixed we mean independent of agent h's consumption choice, at least as long as } C_{i,h}^c \leq K_t^c - \sum_{i \neq h} C_{i,h}^c.\)
subject to

\[ 0 \leq C_{T-1,h}^p \leq K_{T-1,h}^p, \ 0 \leq C_{T-1,h}^c \leq C_{T-1,h}^{\max} \text{ and} \]

\[ \frac{\partial}{\partial C_{i,h}^c} \left( \sum_{i \neq h}^o C_{i,T}^c \right) = \begin{cases} 0 \text{ in the cooperative scenario} \\ -1 \text{ in the defection scenario.} \end{cases} \]

The first-order condition for \( C_{T-1,h}^p \) is:

\[ \frac{\partial U_h}{\partial C_{T-1,h}^p} = 1 - (1 + g). \]

Since \( g > 0 \), the above condition implies that in \( T-1 \) consumption from the private stock of capital will be postponed until period \( T \): \( C_{T-1,h}^p = 0 \) and \( C_{T-1,h}^p = (1 + g) K_{T-1,h}^p \).

The first-order condition for \( C_{T-1,h}^c \) in the defection scenario is:

\[ \frac{\partial U_h}{\partial C_{T-1,h}^c} = 1 > 0. \]

The above condition implies that in the defection scenario every agent's optimal choice is to attempt to consume as much as possible from the common stock of capital and therefore no unprotected capital will be left for the following period: \( C_{T-1,h}^c = \beta_h K_{T-1}^c \) and \( C_{T,h}^c = 0 \).

The defection scenario is an equilibrium outcome of the subgame for \( T > T-2 \). If there is at least one agent that is maximizing his consumption from the unprotected stock of capital then replicating him is every agents' best response.

The first-order condition for \( C_{T-1,h}^c \) in the cooperative scenario is:

\[ \frac{\partial U_h}{\partial C_{T-1,h}^c} = 1 - \beta_h (1 + g). \]

The above condition implies that in the cooperative scenario an agent's optimal decision depends on their relative capacity of appropriation. If \( \beta_h > \frac{1}{1+g} \) then agent \( h \) will choose not to consume from the unprotected stock of capital in period \( T-1 \). On the other hand, if \( \beta_h \leq \frac{1}{1+g} \) then agent \( h \) will attempt to consume as much as possible from the unprotected stock of capital in period \( T-1 \).
For the cooperative scenario to be an equilibrium outcome for this subgame, it must be the case that no agent chooses to maximize his consumption from the unprotected stock of capital if none of the other agents is doing so. Since $g < N-1$, there will always be at least one agent for which $\beta_h = \frac{1}{1+g}$.\footnote{\(\beta_1 < \frac{1}{1+g}\) always because $\beta_1 \leq \frac{1}{N}$. Note that this is true even in the case of perfect equality where $\beta_i = \frac{1}{N}$ for all $i$.} For that agent attempting to consume as much as possible from the stock of unprotected capital is a strictly dominating strategy. Therefore the cooperative scenario is not an equilibrium outcome of this subgame.

**Period $t > 1$** The equilibrium outcome of the subgame of $\tau > 1$ can be generalized to:

\[
C_{t,h}^c = \begin{cases} 
0 & \text{and} \ C_{t,h}^v = K_{t,h}^v \text{ for } h = 1, \ldots, N \text{ and } t = \tau, \ldots, T-1, \\
C_{t,h}^c = \frac{C_{t,h}^c}{C_{t,h}^c} = \max C_{t,h}^c = \beta_h K_i^c \text{ for } h = 1, \ldots, N \text{ and } t = \tau + 1, \ldots, T.
\]

**Period 1** In period 1 the problem faced by the agents is the same as in the case of $t = 2, \ldots, T-1$ with the only difference that the tax to finance the institutional reform is charged in this period. This difference implies that $K_{2,h}^p = (1+g) \left( K_{1,h}^p - C_{1,h}^p - q_{1,h} \right)$ and that $0 \leq C_{1,h}^p \leq K_{1,h}^p - q_{1,h}$. In the maximization problem this difference implies that equation (5) becomes:

\[
C_{1,h}^p, C_{1,h}^c \quad \text{Max } U_h = \sum_{t=1}^{T} (C_{t,h}) = C_{1,h}^p + C_{1,h}^c + \sum_{t=2}^{T} (C_{t,h}) = C_{1,h}^p + C_{1,h}^c + C_{T,h}^p + C_{2,h}^c
\]

\[
= C_{1,h}^p + C_{1,h}^c + (1+g)^{T-1} \left( K_{1,h}^p - C_{1,h}^p - q_{1,h} \right) + \beta_h (1+g) \left( K_i^c - \sum_{i=1}^{N} (C_{i,i}) \right)
\]

and the constraint (6) becomes:

\[
0 \leq C_{1,h}^p \leq K_{1,h}^p - q_{1,h}, 0 \leq C_{1,h}^c \leq C_{T-1,h}^c.
\]

The other constraint (7) is not affected by the tax:

\[
\frac{\partial}{\partial C_{t,h}^c} \left( \sum_{i \neq h} C_{t,i}^c \right) = \begin{cases} 
0 & \text{in the cooperative scenario} \\
-1 & \text{or} \text{ the defection scenario.}
\end{cases}
\]

As can be seen above, the only difference between the problem for period 1 and the problem for any other period is the inclusion of a lump-sum tax that doesn't affect the optimality conditions. Therefore the optimal period 1 consumption choices are:
$C^p_{i,h} = 0$ and $C^c_{i,h} = C^\text{max}_{i,h} = \beta_h K^c_i$ for $h = 1, \ldots, N$.

Given $r = 1$ and $\phi = \phi^*$, the solution to the game is:

$$
C^p_{i,h} = 0 \quad \text{and} \quad C^p_{i,h} = K^p_{i,h} \quad \text{for} \quad h = 1, \ldots, N \quad \text{and} \quad t = 1, \ldots, T - 1
$$

(8)

$$
C^c_{i,h} = 0 \quad \text{and} \quad C^c_{i,h} = \beta_h K^c_i \quad \text{for} \quad h = 1, \ldots, N \quad \text{and} \quad t = 2, \ldots, T
$$

$$
K^p_{i,h} = (1 + g)^{T-1} (\beta_h \phi^* - q(\phi^*)^2) \quad \text{and} \quad K^c_i = 0 \quad \text{for} \quad h = 1, \ldots, N \quad \text{and} \quad t = 2, \ldots, T - 1
$$

(9)

$$
K^p_{i,h} = \beta_h \phi^* \quad \text{and} \quad K^c_i = (1 - \phi^*) \quad \text{for} \quad h = 1, \ldots, N \quad \text{and}
$$

$$
U_h = (1 + g)^{T-1} (\beta_h \phi^* - q(\phi^*)^2) + \beta_h (1 - \phi^*) \quad \text{for} \quad h = 1, \ldots, N.
$$

If reform occurs in period 1, each agent will consume in period 1 and in period $T$ only. In period 1 each agent will attempt to consume as much unprotected capital as possible and therefore all of it will be depleted during that period. On the other hand, they will postpone their consumption from their protected private capital until period $T$.

2.2.2. Stage II

If there is an institutional reform to assign and protect property rights over the economy's stock of capital, it must be the case that the reform occurs in period 1. Otherwise, if the reform would be implemented in a period later than period 1, there would be no capital left to protect and therefore there would be no gain from reforming.

Notice that heterogeneity (non identical $\beta$'s) has not played a role in the results obtained until now. The results are the same for the case of identical capacities of appropriation.

2.2.3. Stage III

In this stage the equilibrium outcome of the game and the optimal timing of reform are used together with the political consensus requirement to obtain the adopted level of property rights protection, $\phi^*$.\(^{13}\) We model the notion that wide-ranging reforms, as the establishment of institutions capable of protecting property rights, require a certain degree of political consensus. In this version of the paper we use a strict consensus requirement by which all agents must agree with the institutional reform.\(^{14}\) This alternative allows the possibility of sabo-

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\(^{13}\) Since $\phi^*$ can equal zero, the possibility of no reform is not being excluded.

\(^{14}\) In the following section a majority voting process is considered.
tage or veto by minorities where institutional reform can be blocked by the agents that oppose it. This consensus requirement leads to a "convoy effect" in which the pace of reform is dictated by the agent most reluctant to reform. In terms of the institutional reform being considered here, this implies that the adopted degree of property rights protection will be the one preferred by the agent desiring the lowest degree of protection:

$$\phi^* = \text{Min} (\phi_1, \ldots, \phi_N),$$

where $\phi_h$ is agent $h$'s preferred degree of protection.

$\phi_h$ maximizes agent $h$'s lifetime utility (equation 9):

$$\phi_h \text{ Max } U_h = (1 + g)^{T-1} (\beta_h \phi - q (\phi)^2) + \beta_h (1 - \phi)$$

The first-order condition yields that:

$$\phi_h = \frac{\beta_h ((1 + g)^{T-1} - 1)}{2q (1 + g)^{T-1}}.$$  

Since $g > 0$, $\phi_h$ is an increasing function of $\beta_h$ and hence $\phi_1 \leq \phi_2 \leq \ldots \leq \phi_N$. The adopted degree of property rights protection will be the one preferred by the poorest agent: $\phi^* = \phi_1$.

3. Results

3.1. Degree of Property Rights Protection

The solution of the model yields several interesting results concerning the degree of property rights protection:

First, equation (8) indicates that there is a problem of overconsumption which can be ameliorated with property rights protection. Only protected capital will be reinvested, any capital left unprotected will be completely consumed in period 1.

Second, equation (10) indicates that there will be a positive degree of property rights protection established in period 1. This is due to the fact that $\beta_1 > 0$ and $g > 0$. $\beta_1 > 0$ implies that the decisive agent will get a positive share of the protected property while $g > 0$ implies that he will support some degree of property rights protection since this will expand his future consumption possibilities as protected capital is reinvested.

Third, equation (10) also indicates that the security provided by the institutions established in period 1 is a positive function of the growth rate of reinvested capital. This result is evident, since the benefit of stronger institutions is that it increases the size of capital reinvested and allowed to grow for future consumption: the larger the rate at which reinvested capital grows, the greater the benefits of reform.
Fourth, equation (10) also shows that the degree of protection is inversely correlated to the cost of establishing the institutions. Not surprisingly, the extent of the reform decreases with the cost of reform.

Fifth, equation (10) yields that the level of property rights protection given by the established institutions is positively correlated with the share of the property allocated to agent 1, \( \beta_1 \). The consensus requirement makes agent 1 the decisive agent. How much of the capital he wants protected (and actually ends up being protected) will depend on how much of it will he get.

If equality is measured by (or correlated to) the share of the property assigned to the poorest agent then this fifth implication of the model is consistent with Keefer and Knack’s (1995) finding that inequality reduces the security of property rights. An obvious question that arises is whether measuring equality by \( \beta_1 \) is an appropriate measure.\(^{15}\) From a theoretical point of view there is no correct measure and there is no single one universally accepted. But fortunately, from an empirical point of view, it seems that the choice of the precise measure is not crucial. The main measures used in the literature are very highly correlated. Clarke (1995) reports that the simple correlations between the most common measures of income distribution (the coefficient of variation, Theil’s index, the Gini coefficient and the share of total income earned by the poorest 40 percent of the population to the share of income earned by the richest 20 percent of the population) are in the (0.87,0.99) range.

### 3.2. Growth

In this economy there is only one good that can instantaneously be converted into capital, therefore the growth rate of capital is also the growth rate of the stock of output, \( G \), the growth rate of the economy from period 1 until the end of the game, is a function of the adopted degree of property rights protection:

\[
G = \frac{(K_T - K_1)}{K_1} = \frac{K_T}{K_1} - 1 = \sum_{i=1}^{N} K_{T,i}^p - 1 = (1 + g)^{T-1}(\phi - Nq(\phi^2)) - 1.
\]

The first and second derivatives of \( G \) with respect to \( \phi \) indicate that if \( \phi \in (0, \phi^*_G = \frac{1}{2 N q}) \), \( G \) increases with \( \phi \). Since the maximum degree of property rights protection achievable in the economy is \( \frac{(1 + g)^{T-1}}{2 N q (1 + g)^{T-1}} \) when \( \beta_1 = \frac{1}{N} \) then it must

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\(^{15}\) Notice that \( \beta_1 \) is not only the poorest agent’s endowment of protected property, but it is also the share of the unprotected stock of capital that the poor agent will be able to consume. Therefore \( \beta_1 \) also corresponds to the poorest agent’s share in wealth and, since in this model there is no other source of income, it also corresponds to his share in total income.
be the case that $\phi^* < \phi^*_G$. This implies that expansions in the size of the protected stock of capital will result in higher growth rates as more capital is saved and reinvested. This result, which holds even in the case of perfect equality ($\beta_i = \frac{1}{N}$ for all $i$), is due to the fact that when agents choose $\phi$ to maximize their utility they take into account not only that higher $\phi$ can result in higher savings and more output available for consumption in period $T$, but also that higher $\phi$ implies less present consumption. This implication of the model is consistent with the cross-country finding that growth expands as the institutional capacity to protect property rights improves (Knack and Keefer (1995) and Barro (1996a)).

A related result is that the economy's growth rate is an increasing function of $\beta_1$. As stated above, the larger the share of the property allocated to agent 1, the larger the degree of property rights protection that he will support and that society will establish. And the larger the degree of property rights protection, the larger the growth rate. The combination of both results implies that a more equal distribution of wealth and income leads to higher savings, investment and growth rates. As mentioned in the introduction, there is abundant evidence consistent with this prediction of the model (see Clarke (1995), Perotti (1996), etc.).

3.3. Socio-Political Instability

We interpret socio-political instability as resulting from the attempts of the agents to obtain a larger share of the unprotected assets. Specifically, socio-political instability is viewed as a necessary by-product of an agent's appropriation of part of the common capital. Agents or groups must "compete" for those additional units of consumption from the unprotected capital by showing and weighting their relative strength. These demonstrations of power might be violent in some occasions (political assassination, military coups, etc.), non-violent in others (labor strikes, mass demonstrations, nationalizations, etc.), but in most cases will generate a climate of instability.

Given the above interpretation, socio-political instability will be proportional to the level of consumption from the unprotected stock of capital and, since in this version of the model all of the unprotected capital is consumed in the first period, instability will arise only at the beginning of the game and will be proportional to $(1 - \phi^*)$.\footnote{Richer results are obtained in an extension of the model: there is instability in the $T$ periods and instability is affected by variables other than only $\phi^*$.} The model yields several results concerning instability:

The first implication is derived from the result concerning the effect of inequality on the degree of property rights protection. As discussed above, in-
equality reduces the degree of property rights protection which implies that instability generates more socio-political instability. This implication of the model is consistent with Alesina and Perotti (1996), Perotti (1994) and Perotti’s (1996) finding that there is a significant positive correlation between inequality and socio-political instability, measured as an index that combines several indicators of social unrest using the method of principal components.

The second implication is derived from the result concerning the effect of property rights protection on growth. That higher degrees of protection have a positive effect on the growth rate implies that instability has a negative effect on growth. As discussed in the introduction, this implication of the model is consistent with the empirical findings of Barro (1991), Alesina, Ozler, Roubini and Swagel (1996).

The third implication of the model is that instability should have no effect on growth after controlling for the degree of property rights protection since the effect of instability on growth is determined by the size of property that remains unprotected. This implication is consistent with Barro’s (1996a) finding that “the political variables are, however, not significantly related to growth when the rule of law index is also included in the regressions” (p. 25).

3.4. Welfare

This section shows how changes in the distribution of wealth affect social welfare. To avoid tautological results, as would be the case if we used a Rawls “egalitarian” type of social welfare function, an utilitarian social welfare function, \( W = \sum_{i=1}^{N} U_i \), is assumed. Substituting (9) yields,

\[
W = \sum_{i=1}^{N} U_i = (1 + g)^T (1 + \phi - q(\phi)^2) + \beta_1 (1 - \phi) = (1 + g)^T (\phi - Nq(\phi)^2) + (1 - \phi).
\]

The first and second derivatives of \( W \) with respect to \( \phi \) imply that social welfare is maximized with \( \phi_W^* = \frac{(1 + g)^T - 1}{2Nq(1 + g)^T - 1} \) and that it is an increasing function of the degree of property rights protection if \( \phi \in (0, \phi_W^*) \). Since the adopted degree of property rights protection in the economy, \( \phi^* \), belongs to \( (0, \phi_W^*) \) then it must be the case that expansions in the size of the protected stock of capital will result in higher welfare.

Since \( \phi^* \) is an increasing function of \( \beta_1 \), there are two interesting results concerning the effect of equality on welfare. First, improvements in the distribution of wealth result in higher levels of welfare. Second, since \( \phi^* = \phi_W^* \) when \( \beta_1 = \frac{1}{N} \), welfare is maximized with perfect equality.
4. Extensions

This section extends the model by allowing the adopted degree of property rights protection result from a majority voting political process instead of using the strict consensus requirement and also by restricting the agents' absolute capacities of appropriation.

4.1. Majority Voting

An alternative to the strict consensus requirement used in the previous section is to represent the political process as one of majority voting over the single issue of property protection. The adopted policy in this political process will be that for which no majority of voters can be formed to alter it. In this model, since the agents' preferences over the adopted policy are single peaked, Black's (1948) result holds and therefore the adopted policy is determined by the median voter's, agent m's, preferred policy. This result is consistent with the notion that successful reforms require the middle class' support. In the case without absolute capacity restrictions equation (10) implies that the resulting degree of property rights protection would be:

$$\phi^* = \phi_m = \begin{cases} \frac{\beta_m ((1 + g)^{T-1} - 1)}{2q(1 + g)^{T-1}} & \text{if } \beta_m \in (0, \frac{1}{N}) \\ \text{or} \\ \min \left( \frac{\beta_m ((1 + g)^{T-1} - 1)}{2q(1 + g)^{T-1}}, 1 \right) & \text{if } \beta_m > \frac{1}{N} \end{cases}$$

As discussed above, the $\beta$ in this model correspond to the agents' shares of income and wealth. Since in the real world actual income and wealth distributions are skewed to the right, the median agent's endowment is smaller than the economy's mean endowment $(\frac{1}{N})$. If equality is measured or correlated to the the median agent's share or if the shares of the poor agents (those for which their $\beta < \frac{1}{N}$) move in the same direction, then all the results concerning the effects of inequality obtained for the strict consensus requirement hold for the majority voting mechanism. On the other hand if changes in the distribution of the relative capacities of appropriation involve transfers between agents 1 and agent m then the implications of the model do depend on the specific measure of equality and on the assumed political process. However from an empirical and a theoretical point of view, this last case is not very relevant.17

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17 Cross-country data indicate that the income shares of the middle and the poorest quintiles are positively correlated. And from a theoretical point of view, the density functions commonly used to represent actual income distributions (for example the log-normal) have the characteristic that the income shares of all of the poorest 50 percent of the population are inversely correlated to the dispersion of the function.
4.2. Absolute Capacity of Appropriation Restrictions

The model can also be extended to include the possibility of poor agents facing absolute capacities of appropriation restrictions which imply that an agent or group of agents do not have the capacity (knowledge, education, capital, influence) to consume the whole stock of common capital even if they are the only agents attempting to do so. The constraint described by (3) is modified to incorporate these absolute capacity constraints:

\[
0 \leq C_{t,h}^c \leq C_{t,h}^{\text{max}} = \begin{cases} 
\min \left( \alpha_h K^c_i, K^c_i - \sum_{i \neq h} (C_{t,i}^c) \right) & \text{if } \sum_{i \neq h} (C_{t,i}^c) \leq (1 - \beta_h) K^c_i \\
\beta_h K^c_i & \text{or } \sum_{i \neq h} (C_{t,i}^c) > (1 - \beta_h) K^c_i
\end{cases}
\]

where \( \alpha_h \) is agent \( h \)'s absolute capacity of appropriation (\( \alpha_i \geq 0 \) and \( \sum_{i=1}^N \alpha_i = A \)). The relative capacity constraints are now \( \beta_i = \frac{\alpha_i}{A} \) (\( \beta_i \geq 0 \) and \( \sum_{i=1}^N \beta_i = 1 \)).

Agents are still ordered according to their relative and absolute capacities of appropriation. It is assumed that capacity constraints are binding only for poor agents (\( \beta_i \leq \frac{1}{N} \) for \( i \in \{1, \ldots, p\} \)) and that \( \sum_{i=1}^p \alpha_i < 1 \). These assumptions are made to ensure that the game can result in a new type of equilibrium (semi-cooperative equilibrium) in which only some agents attempt in every period to consume as much as possible from the unprotected stock of capital while the rest post-pone their consumption until the last period of the game. It is also assumed that the median voter is a poor agent.\(^{18}\) Hence,

\[
0 \leq \alpha_1 \leq \ldots \alpha_m \leq \ldots \leq \alpha_p < 1 \leq \alpha_{p+1} \leq \ldots \leq \alpha_N
\]

The solution to this version of the model can be reached following the same steps as in the basic model with the only difference that now there exists another possible scenario. In the basic model there were only two possible scenarios: a cooperative one in which all agents decided to post-pone their consumption until the end of the game (cooperate) and a defection one in which all agents attempted to maximize their consumption from the common stock of capital in period 1 (defect). It was shown above that \( g < N - 1 \) implied that the defection scenario was the only equilibrium of the game. This result was due to the fact that agents were not capacity constrained therefore if only one agent preferred to defect he would consume all of the common stock of capital in period 1 and this forced the defection of all other agents. Absolute capacity

\(^{18}\) This assumption is not necessary for the results, it is just consistent with real world income and wealth distributions.
constraints raises the possibility of a semi-cooperative equilibrium in which some agents defect but, since they are not able to consume all of the common capital, the rest of the agents still might choose to cooperate.

If \( \beta_{p+1} > \frac{1}{1+g} \) then the semi-cooperative outcome is, together with the defection outcome, an equilibrium of the game. This condition ensures that the rich agents, capable of consuming the complete stock of common capital, \( i \in (p+1, N) \), will not defect in the semi-cooperative scenario. In this equilibrium poor agents defect while the rest cooperate:

\[
\begin{align*}
C_{i,i}^p &= 0 \text{ and } C_{i,i}^c = K_{i,i}^c \text{ for } i = 1, \ldots, N \text{ and } t = 1, \ldots, T-1 \\
C_{i,i}^c &= \alpha_i K_i^c \text{ and } C_{i,i}^p = \beta_i K_T^c \text{ for } i = 1, \ldots, p \text{ and } t = 1, \ldots, T-1 \\
C_{i,i}^p &= 0 \text{ and } C_{i,i}^c = \beta_i K_T^c \text{ for } i = p+1, \ldots, N \text{ and } t = 1, \ldots, T-1
\end{align*}
\]

\[
K_{i,i}^p = (1+g)^{T-1}(\beta_i \phi^* - q(\phi^*)^2) \text{ and } K_{i,i}^c = (1+g)^{T-1}
\left(1 - \sum_{i=1}^{p} \alpha_i\right) K_{i-1}^c
\]

for \( i = 1, \ldots, N \) and \( t = 2, \ldots, T \).

\[
K_{i,i}^p = \beta_i \phi^* \text{ and } K_{i,i}^c = (1-\phi^*) \text{ for } i = 1, \ldots, N
\]

The degree of property rights protection is determined in the first period and corresponds to the one that maximizes agent \( m \)'s lifetime utility:\(^{19}\)

\[
\phi^* = \phi_m = \begin{cases} 0 & \text{if } \frac{(1+g)^{T-2}}{A} \sum_{i=1}^{p} \alpha_{i,T-1} + \sum_{t=1}^{T-1} \sum_{i=1}^{p} \alpha_{i,t} \leq T-1 \\
\frac{\beta_m}{2q} \left( \sum_{i=1}^{p} \alpha_{i,T-1} + \frac{A}{(1+g)^{T-2}} \left(1 - \sum_{t=1}^{T-1} \sum_{i=1}^{p} \alpha_{i,t}\right) \right) & \text{otherwise.}
\end{cases}
\]

Extending the model to include absolute capacity restrictions results in several new implications concerning socio-political instability:

First, it is not longer true that socio-political instability arises only in the first period of the game. In the semi-cooperative outcome of the game there is socio-political instability throughout the whole game as poor agents attempt to maximize their consumption from the common stock of capital in all \( T \) periods of the game.

\(^{19}\) \( U_m = (1+g)^{T-1}(\beta_m \phi - q \phi^*) + (1-\phi) \left( \sum_{t=1}^{T-1} \alpha_m \left(1+g\right)\left(1 - \sum_{i=1}^{p} \alpha_{i,t}\right)^{T-1} + \beta_m \left(1+g\right)\left(1 - \sum_{i=1}^{p} \alpha_{i,t}\right)^{T-1} \right) \)
Second, socio-political instability does no longer result in the consumption of the complete stock of unprotected capital. The size of the common stock of capital consumed when there is political instability is a function not only of the degree of property rights protection as in the case of the basic model but also of the absolute capacity constraints faced by the poor agents \(\sum_{i=1}^{p} \alpha_i\).

Third, the adopted degree of property rights protection is still an increasing function of the share of property belonging to the decisive agent but under the semi-cooperative outcome of the game it is also a function of the absolute capacity constraints faced by the poor agents: the larger the costs of instability (in terms of overconsumption of the unprotected stock of capital), the larger the incentives to protect it.

Fourth and last, the growth rate of the economy is no longer just a function of the degree of property rights protection and the exogenous rate g. In the semi-cooperative equilibrium socio-political instability (measured by or correlated to the absolute capacity constraints faced by the poor agents \(\sum_{i=1}^{p} \alpha_i\)) has a negative effect on growth. Unlike the defection outcome, this is true even after taking into account the adopted degree of property rights protection.

5. CONCLUDING REMARKS

The main contribution of this paper is that it formalized the explanation proposed by Keefer and Knack (1995) which links inequality to growth through the quality of institutions. This was done by combining two building blocks. The first one, which established the link between the distribution of wealth and the quality of institutions, was introduced by assuming a strict consensus requirement where all agents must agree with the adopted institutional arrangement. This requirement led to a result in which the depth of the reform to establish and protect property rights was dictated by the agent most reluctant to reform, in this case the one with the smallest wealth endowment. And since an agent's willingness to reform increases with the agent's share of wealth, the model yielded the observed link between inequality, measured as the poorest agent's share in wealth, and the quality of institutions.\(^{20}\)

The second block, the one linking the quality of institutions to the growth rate of the economy, results from considering that property rights protection minimizes the adverse effect of socio-political instability on investment. The climate of instability results from the attempts of the different agents to obtain a larger share of the stock of capital that is not properly protected. This creates a tragedy of the commons situation in which all the agents end up overconsuming.

\(^{20}\) A similar result was obtained when a majority voting political process was considered.
ing since they are not willing to sacrifice present consumption as they fear that they will not be able to keep for themselves all the benefits of their investments. As a result, only the protected stock of capital, that is the one for which there is exclusive access, will be saved and reinvested.

Combining both building blocks, the society determines the degree of property rights protection by choosing how much of the initial stock of capital will enjoy common access. Since the decision requires everybody's consent, the decisive preference is that of the poorest agent and the adopted degree of protection is a function of the share of property that belongs to him. Greater equality, measured or correlated with the poorest class' share of wealth, implies that they would be willing to protect more of the economy's assets and therefore, that the economy will build institutions that provide greater property rights security. And, as the agents consume more from the unprotected common-access stock of capital, total savings and investment increase as the society adopts a higher degree of protection and reduces the size of the unprotected stock of capital. This is what drives the equality-institutions-growth link in our model.

Another contribution of the formal framework developed in the paper is that it gives theoretical support to the following empirical findings of the literature on socio-political instability: inequality generates increased socio-political instability; socio-political instability has a negative effect on growth; and socio-political instability should have no effect on growth after controlling for the degree of property rights protection.

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