J. Finan. Intermediation 21 (2012) 203–216

Contents lists available at SciVerse ScienceDirect

J. Finan. Intermediation

journal homepage: www.elsevier.com/locate/jfi

Optimal and efficient takeover contests with toeholds

Gino Loyola *

Department of Management Control, University of Chile, Diagonal Paraguay 257, Of. 1303, 8330015 Santiago, Chile

ARTICLE INFO

Article history:
Received 17 January 2011
Available online 14 October 2011

Keywords:
Takeovers
Toeholds
Corporate control
Optimal auctions

ABSTRACT

Target firms often face a takeover threat from raiders with prior stakes in its ownership (toeholds). Previous literature has shown that, when takeovers are modeled as standard auctions, toeholds induce more aggressive bids from raiders, which has two important consequences for the selling process: (i) the board of directors is no longer indifferent about the sale procedure used to get the highest price, and (ii) the target may not be assigned to the highest-value raider. This paper characterizes how the price-maximizing procedure should be in the presence of asymmetric toeholds. Our central result is that the optimal rule needs to be implemented by a discriminatory mechanism quite different from conventional auction formats. By imposing an extra-charge against high-toehold bidders, the optimal mechanism is able to extract more surplus from raiders who bid more aggressively. As a result, nonbidding shareholders benefit unambiguously from the toehold asymmetry. Furthermore, as this bias restores the symmetry in bidders' expected payoffs, the proposed mechanism also allows to allocate efficiently the target among them.

1. Introduction

Target firms often face a takeover threat from raiders with prior stakes in its ownership (toeholds). For example, Bradley et al. (1988) find that 34% of the bidders in their sample of 236 successful tender offers own toeholds, while Betton and Eckbo (2000) establish that 53% of initial bidders in their sample of over 1300 tender offers (including failed ones) have prior stakes in the target company. More recently, Betton et al. (2009) document that, although toeholds have steadily declined since the early 1980s, they are the norm in hostile takeovers, as more than 50% of this class of takeovers in their sample present bidders with previous participation in the target ownership.

* Fax: +56 (2) 6 35 16 79.
E-mail address: gloyola@fen.uchile.cl

1042-9573/$ - see front matter © 2011 Elsevier Inc. All rights reserved.
doi:10.1016/j.jfi.2011.08.002

The presence of toeholds rises interesting questions regarding the selling procedure that the board of directors – or any special committee on behalf of nonbidding shareholders – should use to extract the highest price from the potential buyers. These concerns arise mainly because, when takeovers are modeled as standard auctions, the presence of toeholds introduces additional incentives on raiders to bid more aggressively (Singh, 1998; Bulow et al., 1999).

This more aggressive bidding behavior has two sources. First, unlike conventional auctions, in a takeover contest with toeholds bidders can get a payoff not only when they win, but also when they lose the contest. In fact, since a toehold bidder owns a proportion of the target, losing transforms him into a *seller.* This implies that, conditional on losing, a toehold induces a *more* aggressive bidding behavior. Second, conditional on winning, a toehold also leads raiders to offer higher bids. This is because a prior stake in the target means lower costs of overbidding – by comparison with outside bidders –, as the amount of shares to be bought is smaller.

Toeholds strengthen, therefore, the traditional incentive to increase bids present in any auction, but in this case with the intention to possibly sell at a higher price. Previous literature (see Subsection 1.1) has concluded that, in the context of takeover battles, this more aggressive bidding behavior has two important implications for the selling process.

The first consequence is a break-down of the equivalence of standard auctions in terms of the target sale price they can attain, even when raiders possess symmetric stakes.[1] In such circumstances, non bidding shareholders – by means of the board of directors – should therefore pay special attention to the mechanism used to sell a company. The second implication of more aggressive bids is that the target firm may not be assigned to the highest-value raider. A well-known result is that, under asymmetric ownership structures, conventional auction formats cannot rule out ex post inefficient allocations of the target, as the toehold size of potential buyers can play a decisive role in the outcome of the bidding process.

From this, the current paper deals with the issue of how to run a takeover contest in the presence of toeholds from the nonbidding shareholders' perspective. Consequently, we analyze how the maximizing target price mechanism should be and how it could be implemented. In sharp contrast with the existing literature, our work is, to the best of our knowledge, pioneering in that it adopts a *normative* approach rather than a positive one. Thus, instead of taking a particular auction format as given for exogenous reasons, it characterizes how the optimal selling procedure should be. To this end, we construct a model based on the mechanism design approach introduced by Myerson (1981), assuming that each potential buyer derives gains from a particular synergy associated to run the firm. Two main features of our model are the possibility of asymmetry among bidders' toeholds and the existence of a bidder without toeholds (outside bidder).

In this setting, our central result points out that the optimal selling rule needs to be implemented by a procedure quite different from the traditional auction formats frequently used to model a takeover bidding process. In particular, we prove that this implementation is possible through a discriminatory second-price auction with a scheme of asymmetric payments that imposes an extra-charge against raiders with high toeholds.

This discriminatory pricing policy has the following rationale. By imposing a bias against high-toehold bidders, the optimal mechanism extracts more surplus from the stronger players in the game. In the context of takeovers, these advantaged players correspond to raiders who bid more aggressively due to their larger stake in the target. As a result, this non-conventional procedure exhibits two main properties: one relevant from a revenue perspective, and the other with important implications from a social efficiency viewpoint.

The first property of the optimal mechanism is that it pays the seller to adopt its discriminatory pricing rule, as we show that the expected selling price is *strictly* increasing not only in a common toehold (the symmetric case), but also in the degree of asymmetry in these stakes (the asymmetric case). It is worthy to stress that the last property contrasts strongly with the characteristics exhibited by traditional auction formats under the same value and ownership structures studied in the present work. Indeed, whereas we show that at the optimal procedure nonbidding shareholders benefit

[1] This is a classic result in auction theory: the so-called revenue equivalence principle (Myerson, 1981; Riley and Samuelson, 1981).

unambiguously from the asymmetry in toeholds, previous literature has however found a non-monotonic relation between the target sale price and this degree of asymmetry when conventional auctions are conducted (see for instance, Ettinger, 2009; Loyola, 2008a).

The second property of the proposed mechanism, and also contrary to standard auctions, is that its discriminatory pricing scheme induces an *efficient* allocation rule in the following sense: the optimal procedure ensures that, conditional on selling at all, the target company is assigned to the highest-value raider. This occurs as long as bidders' value realizations are sufficiently high and thus the reserve price involved in the optimal mechanism is not binding. Related to this point we have to acknowledge that, although the efficiency property is desirable and constitutes an important result itself, it is not a property pursued as the ex ante focus of our framework, but only the consequence from a price-maximizing objective.

1.1. Related literature[2]

Our work has connections with, at least, two strands of the auction literature applied to the market for corporate control. First, it relates to auction-based models of takeover bidding processes in which raiders have prior stakes in the target. This literature has studied such class of contests using mainly two valuation environments: the independent private values (IPV) and the common value (CV) settings. The first environment is applicable to *trade* buyers, who derive idiosyncratic gains from taking over a company. Alternatively, the CV framework is applicable to *financial* buyers, who derive gains from restructuring strategies that their rival bidders would also implement after taking over the firm.

In both valuation environments, the auction theory approach has established that more aggressive bids induced by toeholds have two important results for the selling process.[3] The first result is that this higher bidding aggressiveness breaks the so-called revenue equivalence principle (Myerson, 1981; Riley and Samuelson, 1981). In the context of takeovers, this finding implies that the equivalence in terms of target's average sale price between standard auction formats no longer holds, as several papers have shown.

Under the IPV setting, the primary reference is Singh (1998), who analyzes a game in which a toehold bidder and an outside bidder compete to gain control of a company. In that framework, he shows the superiority of the second-price auction over the first-price auction. The major insight stemming from his model is what he calls the *owner's curse*. According to this phenomenon, the higher aggressiveness of the toeholder is so that in the second-price auction he is (rationally) willing to bid more than his valuation.[4] Since such an overbidding behavior is absent in the first-price auction due to the traditional trade-off present in this mechanism, the non-equivalence between both standard auctions emerges.[5] Ettinger (2008) confirms this result in a contest in which buyers have *symmetric* stakes in the seller's surplus, finding the same sale price dominance of the second-price auction over the first-price format.

Under the CV setting, the main reference is perhaps the work of Bulow et al. (1999), who study a takeover contest between two toehold bidders. They compare the sealed-bid first-price and the ascending-price (equivalent to the second-price one) auctions in both the symmetric and asymmetric cases. The analysis shows that with symmetric toeholds, the ascending auction performs better than the first-price auction in terms of the expected selling price per share. In contrast, when examining the asymmetric case, the model delivers opposite results whenever toeholds are very asymmetric and sufficiently small.

[2] This subsection reviews applications of auction theory to takeover with toeholds. However, there is also an extensive auction literature on takeover contests *without* toeholds. Some classic works in this line are, among others, Fishman (1988), Grossman and Hart (1988), and Harris and Raviv (1988). For a survey of more recent auction theory applications to the market for corporate control, see Dasgupta and Hansen (2007) and the references therein.

[3] Other applications of auction theory to takeover with toeholds examine the role of initial stakes on deterring rival bidders (Dewatripont, 1993; Ravid and Spiegel, 1999), and on mitigating the free-rider problem of Grossman and Hart (1980) (Shleifer and Vishny, 1986; Chowdhry and Jegadeesh, 1994).

[4] This overbidding phenomenon is encompassed by the more general model of Burkart (1995), which analyzes the case of two (potentially) toehold bidders and two-sided asymmetric information.

[5] This is the trade-off that bidders face between payoff from and chances of winning when forming their bids in a first-price auction.

The second result coming from more aggressive bids due to the presence of toeholds is concerned with efficiency. Prior research has demonstrated that when takeovers with initial stakes are modeled as conventional auction formats, the winner of the process may be the raider who does not value the target the most. This can occur because bidders with higher toeholds can take advantage of his larger aggressiveness, and also discourage more attractive bids from rivals with even higher target valuations. A finding of this nature has been established under both the IPV framework (Burkart, 1995; Ettinger, 2009) and the CV setting (Bulow et al., 1999).

Finally, the current paper is also related to the auction literature on takeover bids with asymmetrically informed raiders. More specifically, our proposed mechanism is in line with the established superiority of discriminatory procedures over traditional auction formats. Povel and Singh (2006), for instance, analyze takeover contests under a general value setting that allows both private and common value environments. They characterize the optimal selling procedure that a target company should design when it faces two outside bidders (*without* toeholds) who are asymmetrically informed. As we do, Povel and Singh also conclude about the optimality of imposing a bias against the strongest bidder (the better-informed one in their model), but by means of a two-stage procedure. Similarly, Dasgupta and Tsui (2003) examine in an interdependent value setting the properties of the "matching auction", a sequential procedure where the first mover is also the strong bidder. In their model, the strong player can be either the larger-toehold bidder or the better-informed one. Dasgupta and Tsui also find that the matching auction allows the target's seller to obtain a higher expected transaction price than with conventional auctions. However, and in contrast to our model, they obtain this result only when asymmetry is sufficiently large.

This paper proceeds as follows. Section 2 sets up a model of takeover contests in the presence of toeholds. Under this framework, Section 3 characterizes the optimal selling mechanism, and establishes its main properties. Finally, Section 4 concludes and stresses some policy implications. Most of the proofs are collected in the Appendix.

2. The model

The nonbidding shareholders of a target company (the *seller*), represented by the board of directors or a special committee, face a takeover threat from two possible risk-neutral buyers (the *bidders*). Before announcing the takeover, bidder i can have a common knowledge participation in the ownership of the target (a *toehold*), denoted by ϕ_i. Hence, the term $(1-\phi_1-\phi_2)$ corresponds to the effective participation of the seller in the selling surplus. We assume an ownership structure that allows both asymmetry in toeholds and the presence of a bidder without a prior stake so that $\phi_1 \geqslant \phi_2 \geqslant 0$.[6]

We will also refer to the players as follows: a bidder with toehold as a *bidding shareholder* (or *toehold bidder*), a bidder without toehold as an *outside bidder* (or *non-toehold bidder*) and the seller as *non bidding shareholders*.

The value that the initial shareholders (bidding and nonbidding ones) assign to the target company is denoted by t_0, which is common knowledge and is here normalized to zero. Given the ownership structure described above, we interpret t_0 as the common value that all shareholders assign to the firm when they own it *partially*. In other words, t_0 represents how much all shareholders value the firm under the current management, i.e., either before the takeover takes place or when this process is finally unsuccessful.

In contrast, we denote t_i as the private value that bidder i assigns to the target when he owns it *fully*. In consequence t_i can be interpreted as a private synergy that bidder i can exploit when he wins the contest and obtains absolute control of the company. It is also called the value "to run the firm" .[7] More formally, we model the value t_i as bidder i's private information, but for which it is common

[6] Our structure allows the presence of a non-toeholder, given its predominance in actual takeovers (see Bradley et al., 1988; Betton and Eckbo, 2000; Betton et al., 2009).

[7] Alternatively, since we have normalized $t_0 = 0$, t_i can be interpreted as an incremental cash flow generated by the new control and management under bidder i (see Singh, 1998).

knowledge that it is independently and identically drawn according to c.d.f. F with support $[\underline{t}, \bar{t}]$, and $\underline{t} > 0$.[8] Moreover, we denote its density as f and define its hazard rate as $H(t_i) = f(t_i)/(1 - F(t_i))$.[9]

Implicit in the previous interpretation of t_i is the assumption that the takeovers modeled in the present paper are not partial. That is, all shareholders must sell their stakes to the winning contestant (and he must buy it) according to the price stated by the contest's rules. Related to this assumption is that selling decisions of nonbidding shareholders are assumed to be perfectly coordinated, either by the board of directors or a special committee. This implies that, in contrast to Grossman and Hart (1980), all nonbidding shareholders accept the highest offer and do not free-ride individually on t_i to get a higher sale price.

3. The optimal mechanism

As discussed in the Introduction, under the ownership structure described in the previous section, the equivalence among standard auctions in terms of selling price breaks down. From this, what is the price-maximizing selling procedure when toeholds are present should be a relevant question from the nonbidding shareholders' perspective. Furthermore, how different this optimal mechanism looks from traditional auction formats should also be an important matter from the same point of view.

Accordingly, in this section we address these issues and characterize the selling procedure that allows nonbidding shareholders to maximize the target price when facing a takeover threat from raiders with initial ownership stakes. To do that, we follow very closely the *mechanism design* framework proposed by Myerson (1981). In the context of our model, this mechanism consists of two elements: (i) an allocation rule that assigns the target firm to one of the bidders (or even to none of them), and (ii) a scheme of payments among bidders and shareholders.

The analysis hinges on the *revelation principle*, according to which it suffices to focus on direct revelation mechanisms. That is, mechanisms in that the unique piece of information that the seller asks each buyer to report is his valuation of the target (his "type"). This strategy to solve the problem works as it is always possible, under the assumptions of the model, to find a non-direct procedure that replicates the allocation rule of the optimal mechanism. As a consequence, our analysis consists of two stages. First, we characterize an optimal (but theoretical) mechanism based on information, in general, non-observed by the seller as bidders' valuations. Second, we look for a more realistic procedure that *implements* the optimal mechanism based on information indeed observed by the seller like, for example, buyers' bids (in an auction) or buyers' offers (in a bargaining process).

3.1. Optimal allocation rule

We denote the vector of value realizations of all bidders by t, i.e., $t = (t_1, t_2)$, with support $T = [\underline{t}, \bar{t}] \times [\underline{t}, \bar{t}]$.[10] Let us define $p_i(t)$ as the probability with which the optimal mechanism allocates the target company to bidder i, conditional on the vector of reported value realizations t (i.e., the *allocation rule*). Similarly, let us define $x_i(t)$ as the expected transfer from bidder i to shareholders for all shares, conditional on the same vector (i.e., the *payment scheme*). Let $Q_i(\widehat{t_i})$ be bidder i's conditional probability of winning when his report on the target's value is $\widehat{t_i}$ and his rivals tell the truth, i.e.,

$$Q_i(\widehat{t_i}) \equiv \int_{\underline{t}}^{\bar{t}} p_i(\widehat{t_i}, t_j) f(t_j) dt_j$$

for $i, j = 1, 2$, $i \neq j$. If bidder i reports $\widehat{t_i}$ when he values the target at t_i, his expected payoff is given by[11]

[8] The assumption $\underline{t} > t_0 = 0$ implies that it would always be ex-post efficient to sell the target company to one of the bidders. As we will see, however, the optimal mechanism does not necessarily satisfy this property.

[9] As it is standard in auction theory, we concentrate on the regular case, that is, increasing hazard rates.

[10] The characterization of the optimal mechanism can be easily extended to the case of more than two bidders. For the three-bidder case (two asymmetric toeholders and one outside bidder) the characterization can be obtained from the author upon request.

[11] For the sake of presentation, we have omitted the arguments of p_i and x_i, but it should be clear that $p_i = p_i(\widehat{t_i}, t_j)$ and $x_i = x_i(\widehat{t_i}, t_j)$, for all i.

$$U_i(\widehat{t}_i, t_i) \equiv \int_{\underline{t}}^{\bar{t}} [t_i p_i - (1 - \phi_i)x_i + \phi_i x_j] f(t_j) dt_j$$

for all t_i, $\widehat{t}_i \in [\underline{t}, \bar{t}]$ and for $i, j = 1, 2$, $i \neq j$. Note that, after integrating, this expected payoff can be seen as the sum of three components. The first one corresponds to the value to bidder i of controlling completely the target, t_i, weighted by the probability of winning the takeover contest, Q_i. The second term represents the expected payment from bidder i to selling shareholders (both nonbidding and bidding ones). This payment is a *net* transfer because it does not consider bidder i's stake in the target, as the term $(1 - \phi_i)$ points out. As is established below, notice that this payment may be positive even when bidder i *loses* the takeover contest (see Proposition 1). The last component corresponds to the expected payment received by bidder i from bidder j, proportional to his toehold in the target.[12]

We define bidder i's *truthtelling* payoff, that is, when he reports his true value of the target to the seller ($\widehat{t}_i = t_i$), as $V_i(t_i) \equiv U_i(t_i, t_i)$. Similarly, the seller's expected revenue when all bidders reveal their true type is given by

$$U_0 \equiv \sum_{i=1}^{2} \int_T (1 - \phi_1 - \phi_2) x_i(t) f(t) dt. \tag{1}$$

The optimal mechanism then solves the following problem:

$$\max_{x_i \in \mathbb{R},\ p_i \in [0,1]} U_0 \tag{2}$$

$$s.t.$$

$$V_i(t_i) \geqslant 0 \quad \forall t_i \in [\underline{t}, \bar{t}], \quad i = 1, 2 \tag{3}$$

$$V_i(t_i) \geqslant U_i(\widehat{t}_i, t_i) \quad \forall t_i, \widehat{t}_i \in [\underline{t}, \bar{t}], \quad i = 1, 2 \tag{4}$$

$$\sum_{i=1}^{2} p_i(t) \leqslant 1 \quad \text{and} \quad p_i(t) \geqslant 0, \quad i = 1, 2, \quad \forall t \in T \tag{5}$$

where (2) is the seller's expected revenue, (3) is bidder i's individual rationality constraint, (4) represents bidder i's incentive compatibility constraint, and (5) corresponds to the feasibility constraints of the problem.

Let us provide some intuition on this program. First, individual rationality constraints ensure the participation of all bidders in the selling procedure no matter what their valuations for controlling completely the target company. Specifically, these constraints establish that each bidder's truthtelling payoff for any of his possible target values must be greater or equal than his outside utility. In our model, the outside utility even for a toehold bidder can be normalized to zero, as it is possible to show that the seller's optimal threat for the non-participating bidder is that the target remains under the current management and control (and thus, $t_0 = 0$). [13] Second, incentive compatibility constraints guarantee that the optimal mechanism is so that it will always be in all bidder's best interest to reveal their true target valuations to the seller. In more concrete terms, these constraints establish that each bidder's truthtelling payoff for any of his possible values must be greater or equal than the payoff coming from any value report different from the honest one. Lastly, feasibility constraints point out the properties that the optimal allocation rule represented by p_i (for all i) has to satisfy as it corresponds to a probability-based rule. Also, this last group of constraints allows for the possibility that the takeover eventually fails and the target remains under the current control and management.

Bulow and Roberts (1989) provide an alternative interpretation of the mechanism design problem, according to which a third-degree price discriminating monopolist (the seller) faces different markets (the bidders). Under this approach, the optimal mechanism design then requires the seller to be able

[12] In this model the outcomes of the selling process can be grouped in three possible events: bidder i or j wins the auction, or the takeover fails. Notice that the probabilities of these three events (p_i, p_j and $1 - p_i - p_j$) are implicitly considered in the second and third term of bidder's payoff, as x_i and x_j were defined as *expected* payments.

[13] This result is an application of the optimal threat rule established by Jehiel et al. (1996, 1999) to our model. See also Loyola (2007), Section 3, who formally derives this rule for a problem closer to that studied here.

to identify the marginal revenue of each bidder in order to extract surplus selectively from each of them. In particular, Bulow and Roberts define $c_i(t_i)$, bidder i's *marginal revenue*, as[14]

$$c_i(t_i) \equiv t_i - \frac{1}{H(t_i)}$$

for all i. Using this marginal revenue concept, and following Myerson (1981) (see more details in Appendix A), it can be shown that the optimal mechanism also solves a program equivalent to that above described. This alternative program is as follows:[15]

$$\max_{p_i, V_i(\underline{t})} \sum_{i=1}^{2} \left[-V_i(\underline{t}) + \int_T c_i(t_i) p_i(t) f(t) dt \right] \tag{6}$$

$s.t.$

$$V_i(\underline{t}) \geqslant 0, \quad \text{for all } i. \tag{7}$$

$$Q_i'(t_i) \geqslant 0 \quad \text{for all } t_i \in [\underline{t}, \bar{t}] \quad \text{and} \quad \text{for all } i. \tag{8}$$

$$\sum_{i=1}^{2} p_i(t) \leqslant 1 \quad \text{and} \quad p_i(t) \geqslant 0, \quad \text{for all } i \quad \text{and} \quad \text{for all } t \in T. \tag{9}$$

Thus, the mechanism design program can be simplified to a problem in which the optimal procedure is now characterized by two elements: (i) the allocation rule p_i, and (ii) the truthtelling payoff for the lowest-value bidder $V_i(\underline{t})$. As can be seen from (6), the seller's expected revenue can be expressed in terms of these two elements, and also in terms of marginal revenue functions $c_i(\cdot)$. In this alternative formulation, whereas (9) represents the original feasibility constraints of the problem, individual rationality and incentive compatibility constraints are substituted by sufficient conditions. In particular, as bidder i's utility is increasing in his value for the target – $V_i'(t_i) = Q_i(t_i) \geqslant 0$ for all t_i –, bidder i's individual rationality constraint for any of his possible values is ensured if this constraint holds true for the lowest-value bidder's, i.e., if (7) is satisfied. Furthermore, note that the probability of winning the takeover contest for bidder i is increasing in his target value, i.e. $Q_i'(t_i) \geqslant 0$. Hence, it will always be in bidder i's best interest to report his true valuation. Accordingly, it is possible to show that (8) suffices for the original incentive compatibility constraint (4) to hold.

After solving this program, the allocation rule and the lowest-value bidder's payoff at the optimal mechanism can be characterized. This is the content of the following lemma.

Lemma 1. *The optimal mechanism sets $V_i(\underline{t}) = 0$ and*

$$p_i(t) = \begin{cases} 1 & \textit{if } c_i(t_i) > \max\{0, c_j(t_j)\ \forall j \neq i\} \\ 0 & \textit{otherwise} \end{cases}$$

for all i, and for all $t \in T$.

This statement establishes that, in the presence of toeholds, the optimal allocation rule is *not* a discriminatory one. This conclusion follows from the fact that $c_i(\cdot) = c(\cdot)$ for all i, that is, all bidders exhibit the *same* marginal revenue function for the seller. This implies that even though bidders possess asymmetric toeholds, it is revenue maximizing for nonbidding shareholders to offer them the same chances of winning whenever they report the same value. This result is surprising because one would expect that, since a toehold induces a more aggressive bidding behavior, the seller should take it into account to design the optimal rule. Our interpretation is that, as opposed to cross-holdings (see Loyola, 2007), toeholds only impose links between bidders' *payments*, but *not* between bidders' *valuations*.

[14] Myerson (1981) calls this term bidder i's *virtual valuation*.

[15] Notice that this problem is identical to the optimization program in Myerson (1981), who does *not* consider the presence of toeholds.

Consequently, in the terminology of Bulow and Roberts (1989), the marginal revenue function (which depends only on valuations) ends up being the same for all bidders. This implies that the seller perceives all bidders as symmetric players, and hence, it is optimal to impose no bias and to attain a symmetric equilibrium.

However, as we will see in the next subsection, this optimal symmetric equilibrium requires the seller to introduce an asymmetry into the payment scheme.[16]

3.2. Implementation

Because all bidders provide the same marginal revenue, the implementation of the optimal allocation rule requires a scheme of payments that induces an efficient allocation. That is, an allocation which guarantees that the target firm be, conditional on selling at all, awarded to the bidder who values it the most. Since we have assumed that players are asymmetric in their toeholds, and thus in their expected payoff functions, the only way to attain an efficient allocation is to design a scheme of *personalized* payments. This implies that we must rule out any conventional auction, as it imposes symmetric payments on the players and thus results in an asymmetric and inefficient equilibrium. This fact is formalized in the next corollary.

Corollary 1. *An auction with a non-discriminatory pricing rule cannot implement the optimal selling mechanism.*

From the incentive compatible constraint, we show next that the optimal allocation rule can be implemented by a selling mechanism with an asymmetric scheme of transfers.

Proposition 1. *In the presence of toeholds, the optimal selling mechanism can be implemented by a modified second price auction with:*

(i) *A reserve price.*
(ii) *The following scheme of payments:*

$$x_i(t) = \begin{cases} z_i(t_j) + [\delta_i - 1] z_i(t_j) & \text{if } p_i(t) = 1 \\ \gamma_i z_j(t_i) & \text{if } p_i(t) = 0 \text{ and } p_j(t) = 1 \\ 0 & \text{otherwise} \end{cases}$$

for all i, j = 1, 2, $i \neq j$, and for all $t \in T$, where

$$\delta_i \equiv \frac{1 - \phi_j}{(1 - \phi_i - \phi_j)}, \quad \gamma_i \equiv \frac{\phi_i}{(1 - \phi_i - \phi_j)},$$

and $z_i(t_j) = \inf\{s_i : c_i(s_i) \geqslant 0 \text{ and } c_i(s_i) \geqslant c_j(t_j) \text{ for all } j \neq i\}$.

From this statement, one can derive the main properties of the optimal scheme of payments, which are set out in the following result.

Corollary 2. *The optimal scheme of payments has the following properties:*

(i) *It imposes a discriminatory policy with a winning extra-charge and a losing payment.*
(ii) *It generates a truthtelling and efficient mechanism.*
(iii) *It yields an average sale price increasing with common toeholds and asymmetry in these stakes.*

[16] The underlying rationale for this apparent contradiction between the allocation rule and the scheme of payments is the same as the one behind the break-down of the revenue equivalence principle. That is, when toeholds exist, revenues *do* depend on the entire payment scheme, not only on the transfers made by the lowest type bidder. As a result, it does not suffice to examine only the allocation rule to state the properties of the optimal mechanism. In fact, one needs to characterize the payment scheme fully as this is crucial in order to recognize the non-standard and discriminatory nature of the optimal selling procedure.

The intuition behind these properties is as follows. The first property emphasizes that, as compared to a situation without toeholds, the optimal scheme of payments includes two features: an extra-charge against the winner and a payment from the loser to selling shareholders, whenever these bidders have a positive initial stake in the target. Furthermore, it imposes a discriminatory policy with a bias against the largest-toehold bidder, as either the winning or losing payment eventually charged to him is higher.[17] Whereas the aim of this discriminatory policy is to take advantage of the more aggressive bidding behavior by toeholders, the rationale for a positive loser's payment is to capture the fact that losing transform a toehold bidder into a seller.

The second property follows from the fact that the bias involved in the scheme of payments induces *symmetric* objective functions for all bidders, which goes us back to the standard problem when there are no toeholds. In this context, it is well-known from the previous literature (e.g., Myerson, 1981) that the rules of a second-price auction with a reserve price guarantee that the proposed mechanism exhibits the following two properties. First, this mechanism is *truthtelling* as all possible raiders bid their true valuations. Second, it is also an *efficient* procedure in the sense that, as long as the reserve price is *not* binding, the target firm is sold to the raider who values it the most irrespective of his toehold size.[18] This result in terms of efficiency can be compared to Burkart (1995), who also studies a takeover contest with toeholds in an environment with IPV and a second price auction. Contrary to our finding, he shows that this takeover contest may be ex post *inefficient*, as there is a positive probability that the target be assigned to the lower-value bidder due to the overbidding behavior of toehold bidders. Of course, this difference in terms of efficiency lies in that whereas Burkart examines a conventional second price auction, we propose a modified version of this format auction.

Related to the last point, it is worthy to recognize that our mechanism design approach has *not* as an a priori objective to find an efficient allocation rule, but an optimal (i.e. price-maximizing) mechanism. Thus, although the efficiency of the characterized selling procedure constitutes a desirable and even a major result itself, it emerges as a consequence from the aim of nonbidding shareholders to maximize the expected target sale price.

Lastly, the third property of the optimal mechanism points up that the presence of toeholds benefits unambiguously the seller in both the symmetric and asymmetric cases, as winning and losing payments are increasing with these ownership stakes. As a result, in the symmetric case ($\phi_1 = \phi_2 = \phi$), it is possible to verify that the average target selling price increases with the common toehold ϕ. In addition, it is also possible to show that in the asymmetric case ($\phi_1 > \phi_2$), this price is also *strictly* increasing with $\varepsilon \equiv \phi_1 - \phi_2$, i.e., the degree of asymmetry in these stakes, which implies that it pays the seller to impose a discriminatory pricing policy. The last result contrasts, however, with the properties exhibited by the target selling price when conventional (non-discriminatory) auctions are run in an IPV setting.[19] For instance, Ettinger (2009) and Loyola (2008a) illustrate second-price auction cases in which a non-monotone relation between the expected price and the degree of toehold asymmetry emerges: the price is first increasing then decreasing in this degree of asymmetry. In light of that, it is then particularly striking from a practical viewpoint why conventional auctions without a discriminatory feature are so prevalent in actual takeovers, even when toeholds are present. This is certainly a puzzling result, which suggests that our approach should revise some of its assumptions in any future research aimed at accounting more fully for selling decisions in actual takeover processes.

We end this section with a comment on the applicability of the loser's payment included in the optimal mechanism. Payments from the loser to the seller in an optimal procedure are not so rare as one might expect. In fact, they are in line with similar results found in the literature devoted to characterizing optimal auctions when externalities exist. For instance, Goeree et al. (2005) show that the positive externalities present in fund-raising activities lead to discarding winner-pay auctions in favor of all-pay formats. In a result reminiscent of ours, they establish the optimality of an auction

[17] Moreover, this discriminatory policy gets exacerbated with the degree of asymmetry, as it is possible to show that the gaps of both winning extra-charges and losing payments are increasing with the difference in toeholds.

[18] It is worthy to note that the optimal mechanism cannot rule out the ex post *inefficient* outcome in which the reserve price is binding and, thereby, the target company remains under the current management. This may occur because, although we have assumed that $\underline{t} > t_0 = 0$, it is possible that $\max_{i=1,2} c_i(t_i) < t_0 = 0$ for realizations of $t_1$and t_2 sufficiently low (see Lemma 1).

[19] The model studied by Bulow et al. (1999) shows that in a CV setting more asymmetric toeholds tend to *lower* sale target prices.

with a reserve price and payments by the losers - a mix between participation fees and an all-pay auction run in a subsequent stage -, which depend on the degree of the externality. Moreover, Goeree et al. emphasize that some characteristics of this optimal mechanism are present in procedures used for raising funds in the real world. The loser-payment characteristic of our proposed mechanism in the takeover case is, therefore, not far from that exhibited by the optimal procedure in other contests with externalities.

4. Discussion and concluding remarks

Firms often face a takeover threat from raiders with prior ownership stakes, the so-called toeholds. Previous literature has shown that, when the takeover bidding process is modeled as a conventional (i.e. non-discriminatory) auction, toeholds induce a more aggressive bidding behavior from raiders holding these stakes. As a consequence, two principal results related to the selling process emerge, one relevant from a private perspective and the other important from a social viewpoint.

The first result is the fact that, even with symmetric toeholds, conventional auctions are no longer equivalent in terms of the selling target price they attain. Thus, finding out what is the optimal selling procedure should be a relevant concern for nonbidding shareholders. The second result is that conventional auction formats can induce ex post inefficient allocations, in the sense that these mechanisms cannot guarantee that the target will always end in the hands of the highest-value raider irrespective of his toehold size.

To address the first question, we have characterized how a target firm should be sold when bidders possess prior stakes in its ownership, and the objective of nonbidding shareholders is to maximize the expected selling price. We formally establish that this optimal mechanism corresponds to a non-conventional auction with a scheme of asymmetric payments that imposes a bias against toeholders. The rationale of such a discriminatory pricing policy is the fact that a conventional mechanism is unable to induce a symmetric – and thus also an efficient – allocation rule, as it preserves the initial advantage of toehold bidders. In contrast, a scheme of asymmetric winning extra-charges and losing payments allows both to take advantage of the higher aggressiveness of toeholders and go back to a symmetric environment.

Interestingly, the optimal mechanism, by restoring the symmetry in bidder's expected payoffs, has the additional and desirable property of inducing an efficient allocation rule. Although not defined as an a priori objective of our framework, this efficiency property constitutes a major result itself, and deserves an additional regulatory policy-oriented comment. As above mentioned, previous literature has established that asymmetry in toeholds can have a dramatic effect on the efficiency of the takeover process (see Burkart, 1995; Ettinger, 2009, for IPV settings; and Bulow et al., 1999 for a CV environment). This has questioned one of the main properties generally attributed to auctions in takeovers: the fact that the target resources are assigned to the buyer that ensures their highest-value use. This efficiency concern has been argued – besides market transparency and competition-reducing considerations – as a further rationale for tighter disclosure rules, as they impose a lower threshold in terms of toehold size prior to submitting a tender offer (see for instance Burkart, 1995, p. 1510). On the contrary, our normative approach suggests that a discriminatory procedure restores this efficiency property, and that its application may thus weaken, at least, this specific line of reasoning behind a more demanding disclosure rule.

We have also demonstrated that nonbidding shareholders benefit from the discriminatory mechanism, as the target average sale price is strictly increasing both in the common toehold and in the degree of asymmetry in these stakes. The latter finding is in sharp contrast with the properties of conventional auction formats in takeover battles, which then leads to opposite policy implications. For instance, Bulow et al. (1999) show that in general the asymmetry in toeholds lowers prices in common-value ascending auctions. As a result, they recommend the "level the playing field" practice, according to which it may be revenue increasing to sell toeholds very cheaply to the buyer with the smaller stake in the target. Our normative approach suggests, on the contrary, that the seller should follow strategies with the aim of preserving this asymmetry. Accordingly, the board of directors should block or discourage the entrance of new shareholders suspected of becoming competitors against the incumbent toeholder in a future takeover battle.

Our model suggests that benefits per share from buying toeholds, even at zero cost, may be completely neutralized by a mechanism that is optimal from nonbidding shareholders' point of view. This would lead to that the optimal toehold may be zero (or lower than it could be thought at first glance) if bidders can anticipate that such a mechanism will indeed be implemented. The same result regarding a low optimal toehold has already been attained by other works, which however have based their conclusions on agency costs (e.g., Goldman and Qian, 2005; Betton et al., 2009).[20] A natural extension of our model is, therefore, to exploring the rationale for toeholds as an endogenous decision in a context that includes elements such as private benefits of control, management resistance to hostile takeovers, and favoritism for a 'white knight' who preservers these benefits after the takeover.

We recognize that the optimal procedure here characterized yet differs from real-world takeover processes in several aspects, especially in those features concerning the presence of an active private process prior to publicly announcing offers (Boone and Mulherin, 2007; Hansen, 2001). For instance, in that process it is usual the coexistence of auctions and bargaining procedures. By contrast, we present a unique implementation of the theoretical revenue-maximizing procedure based on an auction format (a modified SPA). Hence, it would be interesting to explore if an alternative negotiation-based selling procedure can also implement the optimal mechanism, or although suboptimal, if it can yield an average target price fairly close to the maximal one. Further, it would be worthy to incorporate other elements to the model that could provide us with conditions under which an auction-based or a bargaining procedure is better from a nonbidding shareholders' perspective.

Other characteristics of actual takeover processes – documented by Hansen (2001) – not mapped by our optimal mechanism are: (i) the use of two bidding stages (preliminary 'declarations of interest' and subsequent binding bids), and (ii) the revelation of proprietary information from the seller to a reduced group of bidders selected according to their declarations of interest. In contrast, our optimal procedure is a direct selling mechanism in which: (i) only one 'message' (one bid) about each bidder's target value is required by the seller, (ii) bidders do not need seller's information different from the contest's rules to form their valuations and thus, their bids, and (iii) the seller does not exclude a priori any potential buyer as she designs a selling procedure that guarantees the participation of all bidders in the process through individual rationality constraints.

As all these differences between real-world takeovers and our optimal procedure involve an important challenge for our approach, the framework here developed should then be seen as a baseline model from which additional elements may be incorporated to provide a better representation of actual takeover processes. Essentially, we require a model that makes endogenous the seller's decision about the eventual number of buyers (and thus also about auctions vs. negotiations) based on the effects that interdependence of valuations and information among bidders can have on expected revenues. As a starting point, the IPV setting should therefore be abandoned in favor of a framework that includes both a common value element and some statistical dependence among bidders' signals. This can be especially pertinent if we consider that recent contributions from auction literature, both theoretical and empirical, have shown that a non-monotone relation between revenues (or bids) and number of bidders can emerge in more general valuation and information structures.[21]

Acknowledgement

This paper is based on Chapter 2 of my Ph.D. dissertation at Universidad Carlos III de Madrid. I am especially indebted to M. Angeles de Frutos for her comments and guidance. I would also like to thank the comments by Claudio Bonilla, Antonio Cabrales, Freddy Coronado, Rafael Crespí, Luciano de Castro, Juan-José Ganuza, Ángel Hernando, Vijay Krishna, Diego Moreno, and the seminar participants at the AEFIN-Palma de Mallorca 2007, ESEM-Milan 2008, EFMA-Milan 2009, CEA Workshop 2009, AFFI-Saint

[20] Mathews (2007) also offers an explanation to the optimal acquisition of these ownership stakes, but based on optimal surplus extraction from subsequent raiders by the target firm and an alliance partner.

[21] For theoretical approaches, see Bulow and Klemperer (2002), Hendricks et al. (2003), Pinkse and Tan (2005) and Loyola (2008b). For literature that provides empirical evidence in favor of this class of relation in several markets organized as auctions, see Hong and Shum (2002), Hendricks et al. (2003), Athias and Nunez (2009) and DeBrock and Smith (1983).

Malo 2010, and BALAS-Santiago 2011. The Editor (Viral V. Acharya) and two anonymous referees provided very helpful comments and suggestions. The usual disclaimer applies.

Appendix A. The optimal mechanism problem

The optimal mechanism solves problem described by Eqs. (2)–(5). From Myerson (1981), standard substitutions and computations lead to state the equivalence between the incentive compatibility constraints and the following two conditions:

(i) $\frac{\partial V_i(t_i)}{\partial t_i} = Q_i(t_i)$

(ii) $\frac{\partial Q_i(t_i)}{\partial t_i} \geqslant 0$

These conditions allow to replace (4) by (ii) and

$$V_i(t_i) = V_i(\underline{t}) + \int_{\underline{t}}^{t_i} Q_i(s_i)ds_i. \tag{10}$$

Similarly, (3) is guaranteed to hold if $V_i(\underline{t}) \geqslant 0$ for all i. Hence, straightforward computations allow us to rewrite the seller's expected payoff and simplify the maximization problem as presented in Section 3.

Appendix B. Proofs

Proof of Lemma 1. From (6), it is in the seller's interest to make $V_i(\underline{t}) = 0$ for all i because $V_i(\underline{t}) > 0$ is suboptimal and setting $V_i(\underline{t}) < 0$ violates the individual rationality constraint. Moreover, $H'(t_i) > 0$ implies that $c_i'(t_i) > 0$ and thereby $\partial p_i(t)/\partial\, t_i \geqslant 0$, so that $Q_i'(t_i) \geqslant 0$ is satisfied for all i. Finally, since $t_0 = 0$, the optimal allocation rule is found by comparing for a given $t = (t_1, t_2)$ the terms $c_i(t_i)$, whenever they are positive. The solution sets then $p_i(t) = 1$ iff $c_i(t_i) > \max\{0, c_j(t_j)\ \forall j \neq i\}$. □

Proof of Proposition 1. On the one side, for any value t_j consider

$$z_i(t_j) = \inf\{s_i : c_i(s_i) \geqslant 0 \text{ and } c_i(s_i) \geqslant c_j(t_j) \text{ for all } j \neq i\}$$

for all i, i.e., the infimum of all winning values for i against t_j. Then, in equilibrium

$$p_i(s_i, t_j) = \begin{cases} 1 & \text{if } s_i > z_i(t_j) \\ 0 & \text{if } s_i < z_i(t_j) \end{cases} \tag{11}$$

and

$$\int_{\underline{t}}^{t_i} p_i(s_i, t_j)ds_i = \begin{cases} t_i - z_i(t_j) & \text{if } t_i \geqslant z_i(t_j) \\ 0 & \text{if } t_i < z_i(t_j) \end{cases} \tag{12}$$

for all i.

On the other side, substitute $Q_i(s_i)$ into (10), and change the order of integration. After rearranging, the truthtelling payoff of the bidder with the lowest signal can be written as

$$V_i(\underline{t}) = \int_{\underline{t}}^{\bar{t}} \left\{ t_i p_i(t) - [1-\phi_i]x_i(t) + \phi_i x_j(t) - \int_{\underline{t}}^{t_i} p_i(s_i, t_j)ds_i \right\} f(t_j)dt_j \tag{13}$$

for all i and $t_i \in [\underline{t}, \bar{t}]$. Since it is optimal $V_i(\underline{t}) = 0$ for all i, then sufficient conditions for (13) to hold are:

$$t_i p_i(t) - [1-\phi_i]x_i(t) + \phi_i x_j(t) = \int_{\underline{t}}^{t_i} p_i(s_i, t_j)ds_i$$

for all i and for all state t. If we fix a particular state $t = (t_i, t_j)$, two cases are possible: (i) a winning bidder exists, or (ii) the target company is not awarded to any bidder. Applying (11) and (12), the solution of this system of equations for these both cases yields the desired scheme of asymmetric payments. □

Proof of Corollary 2.

(i) First, from Myerson (1981), note that the term $z_i(t_j) > 0$ defined in Proposition 1 corresponds to the payment from bidder i when he wins the takeover contest under the optimal mechanism *without* toeholds. This implies that when the winner is a bidder with a toehold ($\phi_i > 0$ and thus, $\delta_i > 1$), his payment has an *extra-charge* as compared to the payment he would make in case of having no toeholds. This extra-charge is given by $[\delta_i - 1]z_i(t_j)$. Second, since $z_j(t_i) > 0$, when the loser is a bidder with an initial stake ($\phi_i > 0$ and thus, $\gamma_i > 0$), his payment is *positive*, i.e. $\gamma_i z_j(t_i) > 0$. Third, from $\phi_1 > \phi_2$, it follows that $\delta_1 > \delta_2$ and $\gamma_1 > \gamma_2$. Thus, it is clear that the scheme of transfers proposed imposes a discriminatory policy with a bias against the bidder with the largest initial stake.

(ii) After substituting the optimal scheme of payments of Proposition 1 into bidder i's truthtelling payoff, it simplifies to

$$V_i^*(t_i) = \begin{cases} t_i - z_i(t_j) & \text{if } p_i(t) = 1 \\ 0 & \text{otherwise} \end{cases}.$$

The scheme of transfers induces, therefore, symmetric objective functions for all bidders, as in the standard problem when there are no toeholds. In this case, as Myerson (1981) has shown, the rules of a second-price auction with reserve price guarantee that the proposed mechanism is truthtelling. In addition, this auction format ensures that, as long as the reserve price is not binding, the procedure is also efficient in the sense that the object for sale is assigned to the highest-value bidder.

(iii) First, let U_0^* be the seller's expected revenue under the optimal mechanism, and hence, define $\rho_0^* \equiv U_0^*/(1 - \phi_1 - \phi_2)$, the average sale price under the same procedure. From (1) and Proposition 1, it follows directly that ρ_0^* is increasing with both the winning extra-charge and the losing payment. Second, consider the symmetric toeholds structure (i.e. $\phi_1 = \phi_2 = \phi > 0$). In this case, both the winner's extra-charge and the loser's payment are increasing in the common toehold, as it is easy to check that $\partial\delta_i/\partial\phi > 0$ and $\partial\gamma_i/\partial\phi > 0$ for all i. Simple application of the chain rule from these two facts implies that, at the optimal mechanism, the average sale price is increasing with the size of common toeholds. Lastly, consider the asymmetric toeholds case (i.e. $\phi_1 > \phi_2 > 0$). Let us define $\varepsilon \equiv \phi_1 - \phi_2$, the degree of asymmetry, so that the parameters of the winning extra-charge and the losing payment can be rewritten as

$$\delta_1 = \frac{1 - \phi_2}{1 - 2\phi_2 - \varepsilon}, \quad \delta_2 = \frac{1 - \phi_2 - \varepsilon}{1 - 2\phi_2 - \varepsilon},$$
$$\gamma_1 = \frac{\phi_2 + \varepsilon}{1 - 2\phi_2 - \varepsilon}, \quad \gamma_2 = \frac{\phi_2}{1 - 2\phi_2 - \varepsilon}.$$

Hence, one can easily verify that for a fixed ϕ_2, it holds that $\partial\delta_i/\partial\varepsilon > 0$ and $\partial\gamma_i/\partial\varepsilon > 0$ for all i. Again, the application of the chain rule allows us to establish that, at the optimal mechanism, the average sale price is also increasing with the degree of asymmetry in toeholds. □

References

Athias, L., Nunez, A., 2009. The more the merrier? Number of bidders, information dispersion, renegotiation and winner's curse in toll road concessions. IDHEAP/SPAN, University of Lausanne and LET, University of Lyon.

Bradley, M., Desai, A., Kim, E.H., 1988. Synergistic gains from corporate acquisitions and their division between the stockholders of target and acquiring firms. J. Finan. Econ. 21, 3–40.

Betton, S., Eckbo, B.E., 2000. Toeholds, bid jumps, and expected payoff in takeovers. Rev. Finan. Stud. 13, 841–882.

Betton, S., Eckbo, B.E., Thorburn, K.S., 2009. Merger negotiations and the toehold puzzle. J. Finan. Econ. 91, 158–178.

Boone, A., Mulherin, H., 2007. How are firms sold? J. Finan. 62, 847–875.

Bulow, J., Huang, M., Klemperer, P., 1999. Toeholds and takeovers. J. Polit. Economy 107, 427–454.
Bulow, J., Klemperer, P., 2002. Prices and the winner's curse. RAND J. Econ. 33, 1–21.
Bulow, J.I., Roberts, J., 1989. The simple economics of optimal auctions. J. Polit. Economy 97, 1060–1090.
Burkart, M., 1995. Initial shareholdings and overbidding in takeover contests. J. Finan. 50, 1491–1515.
Chowdhry, B., Jegadeesh, N., 1994. Pre-tender offer share acquisition strategy in takeovers. J. Finan. Quant. Anal. 29, 117–130.
Dasgupta, S., Hansen, R.G., 2007. Auctions in corporate finance. In: Eckbo, B.E. (Ed.), Handbook of Corporate Finance: Empirical Corporate Finance, Handbooks in Finance Series, vol. 1. North-Holland/Elsevier, pp. 87–143 (Chapter 3).
Dasgupta, S., Tsui, K., 2003. A 'matching auction' for targets with heterogeneous bidders. J. Finan. Intermediation 12, 331–364.
DeBrock, L., Smith, J., 1983. Joint bidding, information pooling, and the performance of petroleum lease auctions. Bell J. Econ. 14, 395–404.
Dewatripont, M., 1993. The leading shareholder strategy, takeover contests and stock price dynamics. Europ. Econ. Rev. 37, 983–1004.
Ettinger, D., 2008. Auctions and shareholdings. Ann. Econ. Statist. 90, 233–257.
Ettinger, D., 2009. Takeover contests, toeholds and deterrence. Scand. J. Econ. 111, 103–124.
Fishman, M.J., 1988. A theory of preemptive takeover bidding. RAND J. Econ. 19, 88–101.
Goeree, J.K., Maasland, E., Onderstal, S., Turner, J.L., 2005. How (not) to raise money. J. Polit. Economy 113, 897–926.
Goldman, E., Qian, J., 2005. Optimal toeholds in takeover contests. J. Finan. Econ. 77, 321–346.
Grossman, S.J., Hart, O., 1980. Takeover bids, the free-rider problem, and the theory of the corporation. Bell J. Econ. 11, 42–64.
Grossman, S.J., Hart, O., 1988. One share-one vote and the market for corporate control. J. Finan. Econ. 20, 175–202.
Hansen, R.G., 2001. Auctions of companies. Econ. Inquiry 39, 30–43.
Harris, M., Raviv, A., 1988. Corporate control contests and capital structure. J. Finan. Econ. 20, 55–86.
Hendricks, K., Pinkse, J., Porter, R.H., 2003. Empirical implications of equilibrium bidding in first price common value auctions. Rev. Econ. Stud. 70, 115–145.
Hong, H., Shum, M., 2002. Increasing competition and the winner's curse: evidence from procurement. Rev. Econ. Stud. 69, 871–898.
Jehiel, P., Moldovanu, B., Stachetti, E., 1996. How (not) to sell nuclear weapons. Amer. Econ. Rev. 86, 814–829.
Jehiel, P., Moldovanu, B., Stachetti, E., 1999. Multidimensional mechanism design for auctions with externalities. J. Econ. Theory 85, 258–293.
Loyola, G., 2007. How to sell to buyers with crossholdings. Working Paper Economic Series No. 07-50, Carlos III University, Madrid, Spain.
Loyola, G., 2008a. Optimal takeover contests with toeholds. Working Paper Economic Series No. 08-32, Carlos III University, Madrid, Spain.
Loyola, G., 2008b. On bidding markets: the role of competition. Working Paper Economic Series No. 08-33, Carlos III University, Madrid, Spain.
Mathews, R.D., 2007. Optimal equity stakes and corporate control. Rev. Finan. Stud. 20, 1059–1086.
Myerson, R., 1981. Optimal auction design. Math. Operations Res. 6, 58–73.
Pinkse, J., Tan, G., 2005. The affiliation effect in first-price auctions. Econometrica 73, 263–277.
Povel, P., Singh, R., 2006. Takeover contests with asymmetric bidders. Rev. Finan. Stud. 19, 1399–1431.
Ravid, A.S., Spiegel, M., 1999. Toehold strategies, takeover laws and rival bidders. J. Banking Finance 23, 1219–1242.
Riley, J.G., Samuelson, W.F., 1981. Optimal auctions. Amer. Econ. Rev. 71, 381–392.
Shleifer, A., Vishny, R.W., 1986. Large shareholders and corporate control. J. Polit. Economy 94, 461–488.
Singh, R., 1998. Takeover bidding with toeholds: the case of the owner's curse. Rev. Finan. Stud. 11, 679–704.