



Endogenous residual claimancy by vertical hierarchies[☆]



Salvatore Piccolo^{a,*}, Aldo Gonzalez^b, Riccardo Martina^c

^a *Università Cattolica del Sacro Cuore (Milano) and CSEF, Italy*

^b *University of Chile, Chile*

^c *Università Federico II di Napoli and CSEF, Italy*

HIGHLIGHTS

- We study a model of vertical hierarchies where the allocation of residual claimancy is endogenous.
- Residual claimancy is affected by production externalities across hierarchies.
- Principals may prefer to retain a share of the surplus from production when dealing with inefficient types.

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ABSTRACT

In this note we study a model of vertical hierarchies where the allocation of residual claimancy is endogenous and is determined jointly with production and contractual decisions. We show that the (equilibrium) allocation of residual claimancy may be affected by production externalities across hierarchies in a non-trivial manner. Specifically, although revenue-sharing contracts foster agents' (non-contractible) surplus enhancing effort, we show that principals dealing with exclusive and privately informed agents might still prefer to retain a share of the surplus from production when dealing with inefficient (high-cost) types. This is because reducing the surplus share of those types reduces the information rent given up to efficient (low-cost) types by means of a 'generalized competing contracts' effect.

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1. Introduction

We study a vertical hierarchy model where the allocation of residual claimancy is endogenous and jointly determined with production and contractual decisions. The objective is to derive basic insights on the interaction between market forces and organization design under asymmetric information, so as to contribute to the existing literature on vertical contracting and optimal delegation.

Consider two uninformed principals, each dealing with an exclusive agent.¹ Agents are privately informed about their production costs, produce a verifiable output in the principal's behalf and

exert a surplus-enhancing effort which is non-verifiable in court. Production generates externalities across the two principal–agent pairs, which can be either positive or negative. Agents' types can be correlated. Principals offer direct revelation mechanisms specifying type-dependent surplus-sharing rules in addition to output decisions and monetary transfers. Contracts are secret and hence have no strategic value.

Two effects shape the equilibrium allocation of residual claimancy. On the one hand, by sharing the surplus from production with her agent, a principal is able to increase the agent's non-contractible effort, which makes production more appealing: a *surplus-enhancing* effect.² On the other hand, when agents' costs are correlated, rewarding an agent with a share of the firm's surplus generates an informational externality that affects the efficient types' rent. This effect emerges only if there are production externalities across the hierarchies. In particular, when residual claimancy is endogenous, the incentive of efficient types to manipulate their costs depends not only on the cost-saving rent that this strategy secures, but also on its effect on the firm's expected

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* Correspondence to: Dipartimento di Economia e Finanza, Via Necchi 5, 20123 Milano, Italy. Tel.: +39 3343961549.

E-mail address: salvaticcolo@gmail.com (S. Piccolo).

¹ For example, exclusive deals are largely enforced in the video-rental market. Blockbuster has its own downstream retailers (distributors): each of these outlets has an exclusivity right within a given geographic market where it competes with retailers distributing alternative brands.

² This effect is standard in moral hazard models with risk neutral players showing that revenue-sharing (or sell out) contracts are desirable insofar as they provide the right incentives to exert (non-contractible) effort into a project.

surplus through the competitive channel. Essentially, when a share of the firm's surplus is allocated to the agent directly through the contract offer, the agent's incentive to overstate his cost must weight the impact that this lie produces on the principal's beliefs about the competing agent, which affects the surplus that the principal expects to share with the agent, and hence the monetary incentives she will offer: a *competing-contracts* effect (Gal-Or, 1999; Martimort, 1996).

We show that efficient types are always made full residual claimants of the firm revenues. But, if costs are positively (negatively) correlated and outputs are strategic complements (substitutes), principals may benefit from retaining a share of these revenues when dealing with inefficient types. Essentially, sharing revenues with a high-cost agent increases the mimicking incentives of the low-cost type. By contrast, if costs are positively (negatively) correlated and outputs are strategic substitutes (complements) full residual claimancy is granted to the agents regardless of their types.

Hence, principals are more inclined to share revenues with efficient types rather than with inefficient ones. This result adds to the existing literature in three main respects. First, it extends the competing-contracts' effect introduced in Gal-Or (1999) and Martimort (1996).³ Second, one additional insights of our paper is that, once residual claimancy is endogenously determined, it can potentially play an important role in the welfare comparison between different organizational modes—e.g., common agency versus exclusive deals. Finally, it shows that production externalities may contribute in a non-obvious manner to determine the way contracting counterparts share the surplus generated by their relationship.⁴

The mechanism that our paper emphasizes is also different from those identified in earlier models with complete information or uncertainty with peak demand problems. Dana and Spier (2001) consider the use of revenue sharing in a supply chain with a perfectly competitive downstream market and stochastic demand. They demonstrate that a revenue-sharing contract can induce the downstream firms to choose supply-chain optimal actions, which is only one of the effects at play in our model. Mathewson and Winter (1985) and Desai (1997) also study franchise contracts when a retailer can exert costly effort to enhance revenue: they show that revenue sharing decreases the retailer's incentive to engage in such an effort. Differently from us, both these papers only focus on moral hazard, while we also consider adverse selection.

Finally, we also offer a contribution to the literature on input versus output monitoring and the choice of residual claimancy—e.g., Khalil and Lawaree (1995) and Maskin and Riley (1985).⁵ Both these models consider a single principal-agent set-up and are silent on the link between competition and residual claimancy. Cai and Cont (2004) also study how delegation contracts should be optimally designed to induce strategic advantages against a third party. However, they model the third party as a buyer, not as a competing hierarchy.

2. Set-up

There are two principals, P_1 and P_2 , and two exclusive agents, A_1 and A_2 . A_i ($i = 1, 2$) produces output q_i in P_i 's behalf. Firm i 's

³ Gal-Or (1999) considers positively correlated costs, while Martimort (1996) considers perfectly correlated types. In our model we allow also for negative correlation.

⁴ Notice that our results do not hinge on public contracts, as it is the case in Fershtman and Judd (1987), where vertical contracts have strategic effects insofar as firm owners can credibly commit to share revenues with their managers. In this sense, our analysis follows the approach taken in Katz (1991), who argues that observable contracts are not robust to secret renegotiation.

⁵ Hempelmann (2006) extends this approach to a single manufacturer-retailer relationship where the retailer is privately informed about his marginal cost of production.

surplus from production is $S^i(e_i, q_i, q_j)$. Players are risk neutral. P_i 's utility is

$$V^i(\cdot) = (1 - \alpha_i) S^i(e_i, q_i, q_j) - t_i, \quad i, j = 1, 2 \quad i \neq j,$$

where q_i is the output produced by A_i , t_i is the monetary transfer paid by P_i to A_i , e_i is a non-contractible surplus-enhancing effort exerted by A_i and $\alpha_i \in [0, 1]$ denotes the share of the surplus $S^i(\cdot)$ that P_i allocates to A_i —i.e., α_i measures the extent to which A_i is made residual claimant of firm- i 's surplus. A_i 's utility is

$$U^i(\cdot) = t_i - \theta_i q_i - \psi^i(e_i) + \alpha_i S^i(e_i, q_i, q_j), \quad i, j = 1, 2 \quad i \neq j,$$

where $\theta_i \in \Theta_i \equiv \Theta$ ($i = 1, 2$) denotes A_i 's marginal cost of production and is private information. The type-space is $\Theta \equiv \{\underline{\theta}, \bar{\theta}\}$, with $\bar{\theta} > \underline{\theta}$. A_i 's monetary effort cost is $\psi^i(e_i)$.

We use a version of the revelation principle to characterize the equilibrium of the game—see, e.g., Martimort (1996). P_i offers to A_i a direct revelation mechanism

$$C_i \equiv \{t_i(m_i), q_i(m_i), \alpha_i(m_i)\}_{m_i \in \Theta}$$

that maps A_i 's report m_i about his cost θ_i into a monetary transfer $t_i(m_i)$, an output $q_i(m_i)$ and a share of the surplus $\alpha_i(m_i)$.⁶ Contracts are secret: neither P_j nor A_j can observe C_i .

C_i is a simplified version of the Baron and Myerson (1982) mechanism, with the additional (linear) revenue-sharing component $\alpha_i(\theta_i)$. In our setting, though, contracts are incomplete: P_i cannot condition contract C_i neither on A_i 's effort e_i nor on A_j 's output q_j .⁷ For the sake of realism, we rule out the possibility of paying the agents as a non-linear function of realized profits and focus on the simplest case where the upstream principals offer revenue-sharing based on a percentage of realized revenue (surplus).

The timing is as follows:

1. Agents observe costs.
2. Principals offer contracts.
3. Agents report types, exert effort and produce.
4. Payments materialize.

The equilibrium concept is PBE. Since contracts are private, we assume that agents have *passive beliefs*: regardless of the contract offered by his own principal, an agent always believes that the other principal offers the equilibrium contract.⁸

Technical assumptions:

A1 the vector of costs $\theta = (\theta_1, \theta_2)$ is drawn from a joint cdf such that:

$$\begin{aligned} - \Pr(\underline{\theta}, \underline{\theta}) &= v^2 + \rho, \\ - \Pr(\bar{\theta}, \bar{\theta}) &= (1 - v)^2 + \rho, \\ - \Pr(\underline{\theta}, \bar{\theta}) &= \Pr(\bar{\theta}, \underline{\theta}) = v(1 - v) - \rho. \end{aligned}$$

The marginal distribution is: $\Pr(\theta) = v$ and $\Pr(\bar{\theta}) = 1 - v$, ρ is the correlation index between θ_1 and θ_2 : $\rho > 0$ ($<$) means positive (negative) correlation between types.⁹

⁶ We ruled out the case of relative performance evaluations which would require: (i) C_i to be contingent on A_j 's cost parameter θ_j ; (ii) lottery contracts that are difficult to enforce in practice—see, e.g., Bertolotti and Poletti (1996) on the issue of first-best implementation with correlated types and competing hierarchies.

⁷ Mechanisms based on relative performance evaluation are generally viewed as pro-competitive practices by antitrust authorities and thus banned.

⁸ This assumption captures the idea that since principals are independent and act simultaneously, a principal cannot signal to his agent information that he does not possess about the other principal's contract—i.e., the *no signal what you do not know* requirement.

⁹ The case of positive correlation captures those instances where firms' production technologies are affected by aggregate factors such as public expenditures in R&D, changes in the fiscal pressure, etc. By contrast, the case of negative correlation can be useful to capture those situations, such as R&D races, where the technological success of a firm—e.g., a patent that reduce the firm's marginal costs—excludes that of its rivals, which remains with the same cost structure.

A2 $S^i(\cdot)$ and $\psi^i(\cdot)$ are symmetric and quadratic:

$$S(e_i, q_i, q_j) = \kappa + e_i q_i + \beta q_i - q_i^2 + \delta q_i q_j$$

$$i, j = 1, 2 \quad i \neq j, \quad (1)$$

$$\psi(e_i) = e_i^2 / 2\phi \quad i = 1, 2, \quad (2)$$

with $\beta > \bar{\theta}$ and $\phi > 0$.

The parameter δ measures the magnitude of strategic complementarity ($\delta > 0$) or substitutability ($\delta < 0$) between outputs. A positive δ implies that principals' reaction functions are upward sloping and a negative δ implies that principals' reaction functions are downward sloping.

A3 Non-negative probabilities:

$$\Pr(\underline{\theta}, \bar{\theta}) = \Pr(\bar{\theta}, \underline{\theta}) \geq 0 \Leftrightarrow v(1-v) \geq \rho \quad \text{if } \rho \geq 0,$$

$$\min\{\Pr(\bar{\theta}, \bar{\theta}), \Pr(\underline{\theta}, \underline{\theta})\} \geq 0 \Leftrightarrow \min\{(1-v), v\} \geq \sqrt{|\rho|} \quad \text{if } \rho < 0.$$

Finally:

A4 $\Delta\theta \equiv \bar{\theta} - \underline{\theta}$ small and

$$0 < \phi < \min\left\{2 - \frac{\delta\rho}{v(1-v)^2}, 2 - \delta\right\}.$$

$\Delta\theta$ small allows us to derive the key result by using Taylor approximations, without affecting its main insights; ϕ small allows us to deal with concave maximization problems.

3. Complete information

Suppose that each principal observes her own agent's type but not that of the rival.

Lemma 1. *There exists a unique symmetric PBE where agents are full residual claimants of the firms' surplus and are left with no rents—i.e., $\alpha^*(\theta_i) = 1 \forall \theta_i \in \Theta$, $i, j = 1, 2$, and:*

$$t^*(\theta_i) = \theta_i q^*(\theta_i) - \max_{e_i \geq 0} \left\{ \sum_{\theta_j \in \Theta} \Pr(\theta_j | \theta_i) S(e_i, q^*(\theta_i), q^*(\theta_j)) - \psi(e_i) \right\} \quad \forall \theta_i \in \Theta, \quad i, j = 1, 2 \quad i \neq j,$$

where $q^*(\cdot) : \Theta \rightarrow \mathfrak{R}_{++}$ solves:

$$\beta - (2 - \phi) q^*(\theta_i) + \delta \mathbb{E}_{\theta_j} [q^*(\theta_j) | \theta_i] = \theta_i$$

$$\forall \theta_i \in \Theta, \quad i, j = 1, 2 \quad i \neq j.$$

When there are no rents to be grabbed, principals cannot lose by making agents full residual claimants of the firms' surplus. This maximizes their incentive to exert effort, and thus profits that are extracted via the fixed transfer.

4. Asymmetric information

Under asymmetric information principals learn their agents' types through costly contracting: they must give up an information rent in order to screen types.

A_i 's expected utility (in a truthful equilibrium) is:

$$U_i(\theta_i) \equiv t_i(\theta_i) - \theta_i q_i(\theta_i) + \max_{e_i \geq 0} \left\{ \alpha_i(\theta_i) \sum_{\theta_j \in \Theta} \Pr(\theta_j | \theta_i) \times S(e_i, q_i(\theta_i), q_j(\theta_j)) - \psi(e_i) \right\}.$$

C_i is acceptable by A_i if and only if:

$$U_i(\theta_i) \geq 0 \quad \forall \theta_i \in \Theta.$$

Moreover, A_i truthfully reports his type if the following Bayesian incentive compatibility constraints hold:

$$U_i(\theta_i) \geq t_i(m_i) - \theta_i q_i(m_i) + \max_{e_i \geq 0} \left\{ \alpha_i(m_i) \sum_{\theta_j \in \Theta} \Pr(\theta_j | \theta_i) \times S(e_i, q_i(m_i), q_j(\theta_j)) - \psi(e_i) \right\} \quad \forall m_i \neq \theta_i.$$

Denote by $q^e(\cdot) : \Theta \rightarrow \mathfrak{R}_{++}$ the symmetric output function in a separating equilibrium. As standard, assume that efficient types mimic inefficient ones.¹⁰ Hence, only the participation constraint of the high-cost types and the incentive constraint of the low-cost types matter:

$$U_i(\bar{\theta}) \geq 0, \quad (3)$$

$$U_i(\underline{\theta}) \geq U_i(\bar{\theta}) + \Delta\theta q_i(\bar{\theta}) + \delta \alpha_i(\bar{\theta}) q_i(\bar{\theta}) \times \left[\underbrace{\sum_{\theta_j \in \Theta} \Pr(\theta_j | \underline{\theta}) q^e(\theta_j) - \sum_{\theta_j \in \Theta} \Pr(\theta_j | \bar{\theta}) q^e(\theta_j)}_{= \frac{\rho(q^e(\underline{\theta}) - q^e(\bar{\theta}))}{v(1-v)}} \right]. \quad (4)$$

P_i 's maximization problem is:

$$\mathcal{P} : \max_{\theta_i \in \Theta} \sum_{\theta_j \in \Theta} \Pr(\theta_j | \theta_i) [S(e_i(\theta_i), q_i(\theta_i), q^e(\theta_j)) - \theta_i q_i(\theta_i) - \psi(e_i(\theta_i)) - U_i(\theta_i)],$$

subject to

(3)–(4),

$$\alpha_i(\theta_i) \in [0, 1] \quad \forall \theta_i \in \Theta,$$

$$e_i(\theta_i) = \psi'^{-1}(\alpha_i(\theta_i) q_i(\theta_i)) \quad \forall \theta_i \in \Theta.$$

Both (3) and (4) bind. Hence, A_i 's rent is:

$$U_i(\underline{\theta}) = \underbrace{\Delta\theta q_i(\bar{\theta})}_{\text{Standard rent}} + \underbrace{\frac{\alpha_i(\bar{\theta}) q_i(\bar{\theta}) \delta \rho (q^e(\underline{\theta}) - q^e(\bar{\theta}))}{v(1-v)}}_{\text{Competing contracts}}. \quad (5)$$

The first term of (5) captures the rent that an efficient type enjoys in a single principal–agent relationship ($\delta = 0$): low-cost types overstate their type to negotiate higher transfers. The second term is a generalized version of the competing-contracts' effect highlighted by Gal-Or (1999) and Martimort (1996), which depends on the nature of downstream externalities (δ) and on the degree of correlation between types (ρ). In the standard case where efficient types produce more than inefficient ones – i.e., $q^e(\underline{\theta}) > q^e(\bar{\theta})$ – this effect mitigates A_i 's incentive to overstate his type if and only if $\delta\rho < 0$.

\mathcal{P} rewrites as:

$$\max \left\{ \sum_{\theta_j \in \Theta} \Pr(\theta_j | \theta_i) \sum_{\theta_j \in \Theta} \Pr(\theta_j | \theta_i) [S(e_i(\theta_i), q_i(\theta_i), q^e(\theta_j)) - \theta_i q_i(\theta_i) - \psi(e_i(\theta_i))] - v q_i(\bar{\theta}) \left[\Delta\theta + \frac{\alpha_i(\bar{\theta}) \delta \rho (q^e(\underline{\theta}) - q^e(\bar{\theta}))}{v(1-v)} \right] \right\}.$$

¹⁰ This conjecture will be verified ex-post.

s.t. $\alpha_i(\theta_i) \in [0, 1]$,
 $e_i(\theta_i) = \psi'^{-1}(\alpha_i(\theta_i) q_i(\theta_i)) \quad \forall \theta_i \in \Theta$.

Differentiating w.r.t. outputs:

$$\begin{aligned} & \beta + 2\alpha_i(\underline{\theta}) q_i(\underline{\theta}) \phi - 2q_i(\underline{\theta}) \\ & + \delta \mathbb{E}_{\theta_j} [q^e(\theta_j) | \underline{\theta}] - \alpha_i(\underline{\theta})^2 q_i(\underline{\theta}) \phi = \underline{\theta}, \\ & \beta + 2\alpha_i(\bar{\theta}) q_i(\bar{\theta}) \phi - 2q_i(\bar{\theta}) + \delta \mathbb{E}_{\theta_j} [q^e(\theta_j) | \bar{\theta}] - \alpha_i(\bar{\theta})^2 q_i(\bar{\theta}) \phi \\ & = \bar{\theta} + \underbrace{\frac{\nu}{1-\nu} \left[\Delta\theta + \frac{\alpha_i(\bar{\theta}) \delta \rho (q^e(\underline{\theta}) - q^e(\bar{\theta}))}{\nu(1-\nu)} \right]}_{\text{Distortion}}. \end{aligned}$$

Low-cost types' output equalizes (expected) marginal revenues to marginal costs. High-cost types are forced to produce a downward distorted output for rent extraction reasons. This distortion increases in $\alpha_i(\bar{\theta})$ iff $\delta \rho > 0$.

Differentiating w.r.t. $\alpha_i(\underline{\theta})$ and $\alpha_i(\bar{\theta})$:

$$\begin{aligned} \nu(1 - \alpha_i(\underline{\theta})) q_i(\underline{\theta}) \phi - \lambda_i(\underline{\theta}) + \mu_i(\underline{\theta}) &= 0, \\ (1 - \nu)(1 - \alpha_i(\bar{\theta})) q_i(\bar{\theta}) \phi - \frac{\delta \rho (q^e(\underline{\theta}) - q^e(\bar{\theta}))}{1 - \nu} \\ + \lambda_i(\bar{\theta}) - \mu_i(\bar{\theta}) &= 0, \end{aligned}$$

with complementary slackness:

$$\begin{aligned} \lambda_i(\theta_i) \alpha_i(\theta_i) &= 0, \quad \lambda_i(\theta_i) \geq 0 \quad \forall \theta_i \in \Theta, \\ \mu_i(\theta_i) (1 - \alpha_i(\theta_i)) &= 0 \quad \mu_i(\theta_i) \geq 0 \quad \forall \theta_i \in \Theta, \end{aligned}$$

where λ (θ_i) and μ (θ_i) are the multipliers associated with $\alpha_i(\theta_i) \geq 0$ and $\alpha_i(\theta_i) \leq 1$.

Proposition 1. *There exists a unique symmetric PBE where $\alpha^e(\underline{\theta}) = 1$,*

- $\alpha^e(\bar{\theta}) = 1 \Leftrightarrow \delta \rho \leq 0$,
- $\alpha^e(\bar{\theta}) \in (0, 1) \Leftrightarrow \delta \rho > 0$, with

$$\alpha^e(\bar{\theta}) \approx 1 - \frac{\nu \delta \rho (2 - \phi - \delta) \Delta\theta}{\phi (\beta - \underline{\theta}) (1 - \nu) (\nu (\nu - 1)^2 (2 - \phi) - \delta \rho)}.$$

Two forces shape the equilibrium residual claimancy. Since the effort equalizes the marginal benefit $\alpha_i q_i$ to the marginal cost $\psi'(e_i)$, a higher α_i promotes effort and increases the surplus that P_i shares with A_i . But, the allocation of residual claimancy also affects the competing-contracts' effect. To understand why, two cases must be considered.

1. ($\delta \rho < 0$) Consider first $\delta < 0$ and $\rho > 0$. Because types are positively correlated, A_i anticipates that if he overstates his cost, P_i will believe that A_j is more likely to be inefficient and that hierarchy- i 's expected surplus is high due to strategic substitutability. But this belief will induce P_i to reduce A_i 's monetary transfer.

Next, assume that $\delta > 0$ and $\rho < 0$. Because types are negatively correlated, A_i anticipates that if he overstates his cost, P_i will believe that A_j is less likely to be inefficient and that hierarchy- i 's expected surplus is high due to strategic complementarity. Again, this belief induces P_i to reduce A_i 's monetary transfer.

In these cases, the competing-contracts' and the effort-enhancing effects point in the same direction: it is optimal to award full residual claimancy to all types.

2. ($\delta \rho > 0$) Consider first $\delta < 0$ and $\rho < 0$. Because types are negatively correlated, A_i anticipates that if he overstates his cost, P_i believes that A_j is less likely to be inefficient and that hierarchy- i 's expected surplus is low due to strategic substitutability. This will induce P_i to increase A_i 's monetary transfer to compensate him for the reduction of surplus induced by a tougher competitor.

Next, assume that $\delta > 0$ and $\rho > 0$. Because types are positively correlated, A_i anticipates that if he overstates his cost, P_i believes that A_j is more likely to be inefficient and that hierarchy- i 's expected surplus is low due to strategic complementarity. This induces P_i to increase A_i 's monetary transfer to compensate him for the reduction of surplus due A_i 's low (expected) output.

In both cases there is a trade-off between the effort-enhancing effect and the competing-contracts' effects: principals retain a fraction of the firms' surplus when dealing with inefficient types.^{11, 12}

5. Conclusion

We developed a model of supply chains where the division of surplus between contracting counterparts is affected by production externalities in a substantial manner. Principals are more inclined to share revenues with efficient rather than with inefficient types.

Appendix

Proof of Lemma 1. The derivative of P_i 's objective w.r.t. $\alpha_i(\theta_i)$ is $q_i(\theta_i) (1 - \alpha_i(\theta_i)) \phi \geq 0$. Hence, $\alpha^*(\theta_i) = 1 \quad \forall \theta_i$. The rest of the proof is standard and thus omitted. ■

Proof of Proposition 1. Clearly, $\alpha^e(\underline{\theta}) = 1$. Hence, $q^e(\underline{\theta})$, $q^e(\bar{\theta})$ and $\alpha^e(\bar{\theta})$ solve:

$$\beta - (2 - \phi) q^e(\underline{\theta}) + \delta \mathbb{E}_{\theta} [q^e(\theta) | \underline{\theta}] = \underline{\theta}, \tag{A.1}$$

$$\begin{aligned} & \beta + 2\alpha^e(\bar{\theta}) q^e(\bar{\theta}) \phi - 2q^e(\bar{\theta}) + \delta \mathbb{E}_{\theta} [q^e(\theta) | \bar{\theta}] - \alpha^e(\bar{\theta})^2 q^e(\bar{\theta}) \phi \\ & = \bar{\theta} + \frac{\nu}{1 - \nu} \left[\Delta\theta + \frac{\alpha^e(\bar{\theta}) \delta \rho (q^e(\underline{\theta}) - q^e(\bar{\theta}))}{\nu(1 - \nu)} \right], \end{aligned} \tag{A.2}$$

$$(1 - \nu) (1 - \alpha^e(\bar{\theta})) q^e(\bar{\theta}) \phi - \frac{\rho \delta (q^e(\underline{\theta}) - q^e(\bar{\theta}))}{1 - \nu} = 0. \tag{A.3}$$

At $\Delta\theta = 0$:

$$\alpha^e(\bar{\theta})|_{\Delta\theta=0} = 1,$$

$$q^e(\underline{\theta})|_{\Delta\theta=0} = q^e(\bar{\theta})|_{\Delta\theta=0} = q^* = \frac{\beta - \underline{\theta}}{2 - \delta - \phi} > 0.$$

Linearizing (A.1)–(A.2):

$$-(2 - \phi) \frac{\partial q^e(\underline{\theta})}{\partial \Delta\theta} \Big|_{\Delta\theta=0} + \delta \mathbb{E}_{\theta} \left[\frac{\partial q^e(\theta)}{\partial \Delta\theta} \Big|_{\Delta\theta=0} \Big| \underline{\theta} \right] = 0, \tag{A.4}$$

$$\begin{aligned} & -2(1 - \phi) \frac{\partial q^e(\bar{\theta})}{\partial \Delta\theta} \Big|_{\Delta\theta=0} + \delta \mathbb{E}_{\theta} \left[\frac{\partial q^e(\theta)}{\partial \Delta\theta} \Big|_{\Delta\theta=0} \Big| \bar{\theta} \right] \\ & = 1 + \frac{\nu}{1 - \nu} + \frac{\rho \delta \left[\frac{\partial q^e(\underline{\theta})}{\partial \Delta\theta} \Big|_{\Delta\theta=0} - \frac{\partial q^e(\bar{\theta})}{\partial \Delta\theta} \Big|_{\Delta\theta=0} \right]}{(1 - \nu)^2}, \end{aligned} \tag{A.5}$$

$$\begin{aligned} & -(1 - \nu) \phi q^* \frac{\partial \alpha^e(\bar{\theta})}{\partial \Delta\theta} \Big|_{\Delta\theta=0} \\ & - \frac{\rho \delta \left[\frac{\partial q^e(\underline{\theta})}{\partial \Delta\theta} \Big|_{\Delta\theta=0} - \frac{\partial q^e(\bar{\theta})}{\partial \Delta\theta} \Big|_{\Delta\theta=0} \right]}{1 - \nu} = 0. \end{aligned} \tag{A.6}$$

¹¹ Notice that, since $\alpha^e(\bar{\theta})$ is decreasing in $\Delta\theta$, the first order effect of $\Delta\theta$ on $\alpha^e(\bar{\theta})$ is negative. Hence, more generally, principals may even decide not to give any share of the surplus to inefficient agents, who are then offered the Baron–Myerson outcome.

¹² Clearly, if $\delta = 0$ or $\rho = 0$, there is no competing-contracts' effect: agents are made full residual claimants of the firms' revenues.

Hence:

$$\frac{\partial \alpha^e(\bar{\theta})}{\partial \Delta \theta} \Big|_{\Delta \theta=0} = - \frac{v \delta \rho (2 - \phi - \delta)}{\phi (\beta - \underline{\theta}) (1 - v) (v (1 - v)^2 (2 - \phi) - \delta \rho)} \Rightarrow$$

$$\alpha^*(\bar{\theta}) \approx \max \left\{ 1, 1 + \Delta \theta \frac{\partial \alpha^e(\bar{\theta})}{\partial \Delta \theta} \Big|_{\Delta \theta=0} \right\}$$

$$= \max \left\{ 1, 1 - \frac{v \delta \rho (2 - \phi - \delta) \Delta \theta}{\phi (\beta - \underline{\theta}) (1 - v) (v (1 - v)^2 (2 - \phi) - \delta \rho)} \right\}.$$

Finally, A4 implies that high-cost types do not mimic. ■

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