Universidad de Chile
Facultad de Economía y Negocios
Escuela de Economía y Administración

Vessel Routing Problem Under Uncertainty of Demand

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Participantes
Felipe Andrés Vega Fink

Profesor Guía
Juan Pablo Torres Cepeda

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Abstract

In this paper it is introduced an optimization to solve the Vehicle Routing Problem (VRP) with uncertainty of demand. The focus is to minimize the transportation costs while satisfying all the given constraints of the problem. The demand uncertainty is solved by applying a distribution fitting to the historical demand data provided by a break-bulk sea shipping company; therefore this is a real world implementation of the VRP with uncertainty of demand. Various scenarios are generated, each with randomized demand from each port's distribution.

Keywords: Optimization, vehicle routing, demand uncertainty, distribution fitting

1. Introduction

Many transport optimization models have been developed to improve the performance in the shipping industry. Several models focus on reducing cost and transit time, for land, air or sea transportation (Okita et al. 2004).

Sea shipping has been standardized to optimize performance, where the biggest breakthrough was the introduction of the container. These reusable steel boxes provided easier transportation, optimizing space, standardizing gear for the load and discharge process and even to protect the goods in a safer way. Since the beginning of the containerization in early 1960s to 1990, the trade grew from 0.45 trillion dollars to 3.4 trillion dollars. It grew by a factor of 7. (Bernhofen et al., 2013)

However, not every cargo fits in a container, projects like a windmill, an oversized gear, or heavy machinery. Also there is cargo that is inefficient to consolidate in a container like metal, cement, concentrate, or grain (Bornozis, 2006). And other cargo preferred to avoid containerization because of their size. Most break bulk cargoes are highly valuable products. (Shipping Australia's Break Bulk Shipping Study). Break-Bulk carrier is a ship that has wide vaults to carry cargo, which can carry volumetric cargo that would not fit in a container.

In this paper, a real world problem is addressed to implement an optimization under uncertainty of demand. The current scenario of the optimization is a shipping company under uncertainty of demand (demand defined as the cargo demanded by a destination). The cargo transported is mainly break-bulk[1] or bulk, this means that the sea vessels provide a restriction of volume or weight depending on the properties of the cargo.

Therefore the aim of the optimization in this work is to solve the optimization with uncertainty of demand, space restrictions for the cargo and find the minimal amount of cargo to justify the cost of going to a port.

In this paper, we address the vehicle routing problem (VRP) with demand uncertainty and the transshipment of cargo. Various authors have studied VRP and similar problems. The basic problem is the Vehicle Routing Problem or VRP (Danzing & Ramser 1958), this approach minimizes transportation costs while satisfying the demand, but it has strong assumptions regarding the distribution of the uncertainty. Therefore Sungur et al. (2008) research comes in; they solved this problem, but with a robust solution that optimizes the worst-case value over all data uncertainty. Other studies of transportation problems used column generation to maximize driver productivity and minimize time and miles.

Break-Bulk shipping is a less studied area, the uncertainty of this service and a smaller percentage of participation compared to the Liner shipping (container) makes it an area less developed in optimization modeling. Meanwhile VRP with demand uncertainty has been studied but not applied to a shipping Break-Bulk transportation, therefore this papers aims to fill that gap. But also to go a bit further, to apply this model to a Shipping company from a developing country, as is Companias de Navegacion Interoceanicas (CCNI) from Chile.

This paper extends the work of Sungur et al. (2008), focusing on the implementation of their uncertainty of demand problem on the real world scenario of a shipping company of a developing county.

Also adding the uncertainty of demand solved with the distribution fitting of the historical data for a real world company in the sea shipping industry.

[1] Break-bulk is a volumetric cargo that does not fit in a simple container because of it`s dimensions.

The structure of this paper is organized as follows. Section 2 presents the relevant literature for this case. The problem is presented in Section 3, which shows the characteristics of the situation to optimize the model. Section 4 presents formulations of the vessel routing problem demand uncertainty. Experimental results are shown and analyzed in Section 5. And the paper concludes in section 6 with a discussion of the main findings and conclusions.

2. Literature Review

A wide range of literature concerning vehicle routing optimization and intermodal freight transportation is currently available. Escudero et al. (2012) solved the drayage problem with transit time uncertainty. They applied two different methods, the first was a Two-Phase heuristic algorithm, where all possible combinations of tasks are analyzed and then combined tasks are inserted into routes. And the second methods was a Genetic Algorithm, which is a stochastic metaheurisic algorithm based on the evolutionary theory. To improve performance they introduced the concepts of penalty costs of certain actions and improvement factor, which demands a certain minimum of improvement to change the current combination. The comparison of the two possible methods ended with the first one as the best fitted for the task, because of the speed to solve and the flexibility of adaption. Katayama & Yurimoto (7th International Symposium on Logistics) focused on the Load Planning Problem for Less-than-Truckload. The method to solve the problem was a Lagrangian Relaxation (LR), the results of the experimentation suggested that the (LR) can perform a good job of identifying a lower bound of the problem, but it is lacking an adaptation to the real world. Ileri et al. (2006) focused on minimizing the cost of daily drayage operations. The column-generation with Tree Orders was used to solve the problem and to find cost-effective schedules. Sun et al. (2014) extends and refines the work done in Ileri et al. (2006). The focus was on fast solving an optimization of daily dray operations across intermodal freight network in the face of constant changing data. This problem focused on maximizing driver's productivity meanwhile minimizing miles and time. And continue with the column-generation considering traffic congestion, integration with commercial transportation system and address imbalance of empty containers that get accumulated in certain regions.

In Arnold et al. (2003) was modeled an intermodal transportation system. The model focused on optimally locating rail or road terminals for freight transport. To solve the model a heuristic approach was used. The paper aims to analyze the impact of variations in the supply of transport of the rail and road freight transport in the Iberian Peninsula. The heuristic method was used due to the requirement of time limit to provide a solution.

Powell & Koskosidis (1992) applies a tree constraint to solve a shipment routing sub problem that is extracted from real world considerations. A family of algorithms is investigated to find a solution to the routing sub problem. The used algorithms are a Hierarchical solution, Gradient-Based local search and Primal Dual Methods. The Primal Dual Methods analyzed where the Sub gradient Optimization (SGO), Multiplier Adjustment Algorithm and Dual Ascent Algorithm. The SGO algorithm showed the best execution time and quality of the upper and lower bounds. Therefore the SGO is the best suited seems best suited to provide a good solution in relatively short time.

Varelas et al. (2013) is a paper that presents a toolkit that Danaos Corporation developed to optimize ship routing. The toolkit, named ORISMAS, solves the problem of least-cost voyage versus faster voyage. This is achieved through the integration of financial data, hydrodynamic models, weather conditions, and marketing forecasts. Due to ORISMAS in 2011 the revenues where increased in $1.3 million from time saving and $3.2 million from fuel savings. This is considering 30 vessels that where operated with ORISMAS.

Christiansen et al. (2003) made a literature review of the current status of ship routing and scheduling up to the year 2003. Christiansen et al. (2003) provides relevant research to in terms of liner and tramp. Liner services are defined as a bus line, because it operates with a published itinerary that the ship must stick to it. The Liner shipping must take decisions at different instances, such as route and schedule design, fleet size and mix (combinations of cargoes to maximize revenues), fleet deployment and cargo booking (choose which cargoes are accepted or rejected for the voyages).

A tramp services follows the available cargoes, it works like a taxi. Compared with industrial shipping (such as oil) this area has been less researched. Principally this is due to the large number of small operations in the tramp business. Tramp ships follow the available cargo to transport and sometimes they reach agreements that specify quantities, destinations, transit time and a payment per ton. The optimization is done as maximization instead of industrial shipping considering both costs and revenues. The optimization is done with a LP-Relaxed solution approach with a sub problem of shortest path. Other was solved with formulated as a set packing problem with an algorithm for generating all possible schedules a priori. Fagerholt (2003) developed a heuristic hybrid search algorithm to solve the ship scheduling problems. Also the paper comments the differences of sea transport with other types. Ships pay port fees; draft is a function of weight of the load that affects the possibility to dock in ports. Plus the ports operate in international trade, therefore crossing multiple jurisdictions. And the ships can be diverted at sea.

Dumas et al. (1990) takes a generalization of the Vehicle Routing Problem (VRP) that is the pick up and delivery problem with time windows (PDPTW). The VRP focuses on providing a design of routes that present minimum cost for a set of vehicles that service a known demand. The PDPTW constructs an optimal route to satisfy different requests, such as pickup and deliver under capacity, time windows and precedence constraints. The algorithm used is a column generation algorithm scheme with a constrained shortest path as sub problem. The findings are that the time windows and the distribution of pickup demand are the most significant parameters, both having higher influence on the running time of the algorithm. If the nodes in the problem are fewer than ten, then the shortest path is an efficient way to generate feasible routes.

Sungur et al. (2008) addresses the problem of uncertainty of demand and considering this, use the robust approach. The robust solution provides a good solution for all possible data uncertainty. A normal VRP solution just will find an optimal value, which due to the uncertainty could not be a good solution. The robust solution is an attractive option to formulate the problem, since it does not require distribution assumptions on the uncertainty. This model will be adapted in this work to solve the current problem of a break-bulk shipping company.

3. Problem Description

As the break-bulk transportation was relegated to a secondary concern, few models attempted to develop models of optimization for this sort of transportation and fewer for a company that is located in a developing country. As any transportation problem there is the trade off between a least-cost voyage and a faster voyage (cost savings against time savings). The least-cost voyage is affected by a variety of expenses. The Bunker cost (fuel cost) is one of the most important in this industry; it is a key component for the whole sea shipping industry. The cost is determined by the amount of bunker consumed in the route and by the price of the IFO140 and MDO, which are determined by the trading markets. The company is a price taker on the fuel market and does not considering derivatives to reduce the price variation. Every sea vessel has it's own fuel consumption of IFO140 and MDO, and as CCNI's fleet is heterogeneous, the bunker cost for every voyage depends on the distance and the Vessel assigned.

Another cost that has influence in the results is the hire of the ships (rental cost). The rental of every ship (as CCNI does not own any Vessel that can transport break-bulk Cargo) has a daily tariff, this tariff or hire depends on a wide range of vessel's characteristics, some of them are the age of the ship, consumption, type of ship, cargo capacity, gear capacity etc. Therefore every ship has it's own hire. CCNI focuses on having a multipurpose vessel. A multipurpose vessel is able to transport container, bulk and break-bulk Cargo. Other vessel that is used by CCNI is the Single Decker Bulk Carrier, which is a ship that just has vaults, and cannot carry container. But the single decker bulk carrier is less used than the multipurpose vessel. Another cost that plays an important role is the port cost, every time a ship arrives to a port, it has to pay certain fees, such as usage of pilot services, light dues, dockage cost, etc. All this costs and more are summarized as port costs, which are highly correlated with the type of vessel, it's size and the amount of days the ship remains at the port. And the faster voyage receives the benefit of a lower hire costs for the fewer time of rental.

The company does more than just move the cargo from one place to another; it also incurs in the costs related to the cargo. These costs are variable, depending on the cargo type, amount and specification. The list of costs is long, but some of them are lashing labor, lashing materials, loading, discharge, pre-loading, post-discharge, stuffing, stripping, Freight-Forwarder, Agency Costs, usage of certain gear and others. The costs related to the cargo tend to vary, because of the demand uncertainty that this service has. The cargo in this services differs from the containerized services in the heterogeneousness of it's cargo, some pieces may differ in length, some are heavy machinery that has certain requirements in terms of lashing or loading requirements, some type of cargo might never been loaded/discharged in certain ports and the company has no knowledge of the costs that are going to be incurred before the cargo is shipped.

In contrast to the Liner Shipping, the break-bulk shipping has no time windows as the time of loading and discharge of the non-containerized cargo is variable and the company is unable to estimate the length of the stay in the port, therefore it would create congestion in the port. But that is a variable element that extends further than the current model to be solved.

The demand of break-bulk transportation is unstable and variable, due to fluctuations in production, economic cycles, competitors, etc. Those are characteristics of the spot market as is the break-bulk. Contracts are hard to get and the price competition generates a price war. For this reason, the demand for the service has to be estimated to solve the optimization problem. An analysis of distribution of the demands has to be done in order to archive an optimum solution. The demand can be divided in North Bound (NB) and South Bound (SB).

4. Formulation of the Problem

4.1 VRP formulation

The formulation of the problem is the following:

The ship has a volume capacity of *V cbm*[2], a weight capacity of *W tons*, and a container capacity of T Teu[3], to consider the limitations of the vessels. Let P be the set of ports. Whether or not a cargo is shipped from port *i* to *j* depends on x_{kij} and it takes value 1 if the cargo *k* is transported from to port *i* to port *j*. c_{kij} is the variable that represents the variable costs of the cargo *k* that is originated from port "i" has as destination port "j". v_k is the volume of cargo *k* and is exclusive to the cargo that goes SB. w_k is the weight of the cargo *k* that only takes a value other than cero if the cargo goes NB. Also t_k refers to the amount of teus, t_k is particular to the cargoes *k*'s that are containerized.

D_{ij} is the variable that denotes the number of days that takes a voyage between ports *i* and *j*.

Y_{ij} is a variable that determines the if the vessel sails from port i to port j. Takes value 1 if the vessel sails from port *i* to port *j* and cero other case.

$\varphi_i$is the port cost of port *i* and μ_i is the variable that shows how many times the vessel is to be docked at the port i, therefore μ_i can only take integer values.

B is the bunker price designated for the voyage and H is the hire (price per day) for the vessel.

[2] Cbm: cubic meter
[3] TEU: Twenty-foot equivalent unit

θ_i is a binary variable that takes value 1 if cargo is to be shipped to port i. And β_i is a binary variable that takes value 1 if cargo is to be shipped from port i.

Z_i^V, Z_i^W and Z_i^T respectively denote the total volume, weight and teus of the cargoes when the vessel sails from port i.

And d_i^v, d_i^w and, d_i^t are the demands of port i for volumetric cargo, normal cargo (weight cargo) and containerized cargo respectively.

$$Min \sum_{i\in P}\sum_{j\in P}\sum_{k\in K} c_{kij}x_{kij} + \sum_{i\in P}\sum_{j\in P} B + H \; * D_{ij} * Y_{ij} + \sum_{j\in P}\varphi_i * \mu_i \tag{1}$$

$$s.t. \sum_{i\in P}\sum_{j\in P} x_{kij} \leq 1 \quad for\ k \in K \tag{2}$$

$$x_{kij}, \theta_i, \beta_i \in \;\; 0{,}1 \tag{3}$$

$$v_k; w_k > 0 \tag{4}$$

$$\mu_i \geq \frac{\theta_i + \beta_i}{2} \;\; ; \; \forall i \; \in P \tag{5}$$

$$\sum_{j\in P} Y_{ij} \geq \beta_i \;\; ; \; \forall i \; \in P \tag{6}$$

$$\sum_{i\in P} Y_{ij} \geq \theta_j \;\; ; \; \forall j \in P \tag{7}$$

$$\sum_{j\in P}\sum_{k\in K} x_{ijk} * v_k - \sum_{j\in P}\sum_{k\in K} x_{jik} * v_k + \sum_{j\in P} Y_{ji} * Z_j^V \; = Z_i^V for\ i \in P \setminus i \neq j \tag{8}$$

$$\sum_{j \in P} \sum_{k \in K} x_{0jk} * v_k = Z_0^V \qquad (9)$$

$$\sum_{j \in P} \sum_{k \in K} x_{ijk} * w_k - \sum_{j \in P} \sum_{k \in K} x_{jik} * w_k + \sum_{j \in P} Y_{ji} * Z_j^W = Z_i^W \; for \; i \in P \setminus i \neq j \qquad (10)$$

$$\sum_{j \in P} \sum_{k \in K} x_{0jk} * w_k = Z_0^w \qquad (11)$$

$$\sum_{j \in P} \sum_{k \in K} x_{ijk} * t_k - \sum_{j \in P} \sum_{k \in K} x_{jik} * t_k + \sum_{j \in P} Y_{ji} * Z_j^T = Z_i^T \; for \; i \in P \setminus i \neq j \qquad (12)$$

$$\sum_{j \in P} \sum_{k \in K} x_{0jk} * t_k = Z_0^T \qquad (13)$$

$$Z_i^v \leq V \qquad \text{for } i \in P \qquad (14)$$

$$Z_i^w \leq W \qquad \text{for } i \in P \qquad (15)$$

$$Z_i^T \leq T \qquad \text{for } i \in P \qquad (16)$$

$$\sum_{i \in P} \sum_{k \in K} x_{ijk} * V_k \leq d_j^v \qquad for \; j \in P \setminus i \neq j \qquad (17)$$

$$\sum_{i \in P} \sum_{k \in K} x_{ijk} * W_k \leq d_j^w \qquad for \; j \in P \setminus i \neq j \qquad (18)$$

$$\sum_{i\in P}\sum_{k\in K} x_{ijk} * t_k \leq d_j^t \quad for\ j \in P \setminus i \neq j$$

(19)

The constraint (1) represents the objective function to minimize, with considering all the costs related to the model. Constrain (2) restricts the vessel to be able to transport cargo k just one time. The constraint (5) imposes the ship to go to port i when cargoes that are going to be transported have port i as origin or destination. Constraints (6) and (7) impose that the vessel has to arrive to or sail from a certain port due to the supply of cargo. Constraints (8) - (13) are the cargo flows from the ports. Constraints (14)-(16) are the vessel's capacity restrictions. The constraints (17) - (19) are demand limitations where cargo transported to port i cannot be higher than it's demand in terms of volume and weight.

4.2 Demand Uncertainty

The optimization mentioned in section 4.1 represents a problem where the demand is certain and the company has knowledge of the demand of cargoes. But for this services most of the time, mainly earlier to the three weeks before the beginning of the voyage, demand is uncertain. Therefore values d_i^v , d_i^w and d_i^t are uncertain. To address that uncertainty an analysis of the distribution of the historical values was made. The weekly distribution of each port was analyzed and values where assigned, due to the considerable amount of observations that had cero cargo transported, a double distribution was used. In the first stage a binominal distribution was used, where 1 referred to a week when cargo is transported and cero otherwise. And this binominal helped address the first part of the distribution of the demand. Later to combine the binominal, the amount of cargo transported per week (volume, weight or teu) was fitted to distributions. Each port has a specific distribution for the two demands, a demand of break-bulk cargo and containerized cargo. This distribution assigns a random value of the total weekly demand for each port. In Table 1 the first row show the different ports, each port has a break-bulk distribution for its demand that is the second column. And the containerized cargo distribution is shown in the third column of the table. Also five new variables were added, the first pair are the random binomial values for the demand of each port. ω_i^B refers to the break-bulk cargo and ω_i^C to the containerized cargo. The other three variables are random numbers distributed fitted to the data of the port and cargo type φ_i^C , $\varphi_i^{B/V}$ and $\varphi_i^{B/W}$. The first is the random value of the total sum of containers to be demanded by port i. $\varphi_i^{B/V}$ and $\varphi_i^{B/W}$ are the random values of the total break-bulk cargo demanded by port *i* but separated by volume and weight.

Table 1 Respective distributions assigned to fit each port.

Ports	BB Distrib	TEU Distrib
1	INVGAUSS	InvGauss
2	LogNorm	Expon
3	LogNorm	-
4	InvGauss	Expon
5	InvGauss	InvGauss
6	InvGauss	Expon
7	InvGauss	-
8	Expon	InvGauss
9	Pareto2	InvGauss
10	Weibull	InvGauss
11	Pert	Weibull
12	Expon	Expon
13	Triang	InvGauss
14	Pert	Expon
15	-	Expon

In consequence constraints (2) and (8) - (18) are no longer valid due to the lack of individual cargo to be shipped. So a variation of that constraint is used. x_{kij} is replaced by τ_{ij}^B and τ_{ij}^C, those are binomial variables for break-bulk and containerized cargo respectively. τ_{ij}^B and τ_{ij}^C, takes value 1 if the total cargo from port i to port j is to be transported by the vessel. And v_{ij}, t_{ji} and w_{ji} are reformulated to refer to the total volume, teus or tons that are demanded from port *i* to *j*.

$$s.t. \sum_{i\in P}\sum_{j\in P} \tau_{ij}^B \leq 1 \qquad (20)$$

$$\sum_{i\in P}\sum_{j\in P}\tau_{ij}^{C} \leq 1 \tag{21}$$

$$d_i^C = \varphi_i^C * \omega_i^C = \sum_{j\in P} t_{ji} \qquad for\ i \in P \tag{22}$$

$$d_i^{B/V} = \varphi_i^{B/V} * \omega_i^B = \sum_{j\in P} v_{ji} \qquad for\ i \in P \tag{23}$$

$$d_i^{B/W} = \varphi_i^{B/W} * \omega_i^B = \sum_{j\in P} w_{ji} \qquad for\ i \in P \tag{24}$$

$$\sum_{j\in P}\tau_{ij}^{B} * v_{ij} - \sum_{j\in P}\tau_{ji}^{B} * v_{ji} + \sum_{j\in P} Y_{ji} * Z_j^V = Z_i^V for\ i \in P \setminus i \neq j \tag{25}$$

$$\sum_{j\in P}\tau_{0j}^{B} * v_{0j} = Z_0^V \qquad \setminus j \neq 0 \tag{26}$$

$$\sum_{j\in P}\sum_{k\in K}\tau_{ij}^{B} * w_{ij} - \sum_{j\in P}\sum_{k\in K}\tau_{ji}^{B} * w_{ji} + \sum_{j\in P} Y_{ji} * Z_j^W = Z_i^W\ for\ i \in P \setminus i \neq j \tag{27}$$

$$\sum_{j\in P}\tau_{0j}^{B} * w_{0j} = Z_0^W \qquad \setminus j \neq 0 \tag{28}$$

$$\sum_{j\in P}\sum_{k\in K}\tau_{ij}^{C} * w_{ij} - \sum_{j\in P}\sum_{k\in K}\tau_{ji}^{C} * w_{ji} + \sum_{j\in P} Y_{ji} * Z_j^t = Z_i^t\ for\ i \in P \setminus i \neq j \tag{29}$$

$$\tau^{C}_{0j} * w_{0j} = Z^{t}_{0} \qquad \backslash\, j \neq 0$$
$$_{j \in P}$$

(30)

With this reformulation of the problem in 3.1, the uncertainty of demand is solved and the problem can work with an estimated demand that is based on the distribution of the historical demand. Now the problem can be solved even weeks in advance.

4.3 Income-Cost equilibrium

To improve the cost minimization, an analysis of the income and cost of the transported cargo for each port was done. This has to be done due to the lack of information about cost associated to the cargoes due to the fact that the demands of ports are uncertain. And to solve that, random numbers with certain distribution were created, but no cost linked to those demands. Therefore another tool has to be developed to consider the economically beneficial cargoes. The income per amount of cargo was brought up against the cost per amount of cargo. As the economies of scale are present in this problem, the higher the amount of cargo, the bigger was the growth of income in comparison to the growth of cost. Therefore equilibrium could be found to set a minimum cargo that covers at least all the costs that the company will have to incur to transport certain amount of cargo form a port.

$$\alpha^{C}_{i} \quad \begin{matrix} 1\ if\ d^{C}_{i} \geq R^{C}_{i} \\ 0\ if\ d^{C}_{i} < R^{C}_{i} \end{matrix} \qquad for\ i \in P$$

(31)

$$\alpha^{B}_{i} \quad \begin{matrix} 1\ if\ d^{B}_{i} \geq R^{B}_{i} \\ 0\ if\ d^{B}_{i} < R^{B}_{i} \end{matrix} \qquad for\ i \in P$$

(32)

$$\alpha^{C}_{i} \geq \frac{_{j}\ \tau^{C}_{ij}}{1000} \qquad for\ i \in P$$

(33)

$$\alpha^{B}_{i} \geq \frac{_{j}\ \tau^{B}_{ij}}{1000} \qquad for\ i \in P$$

(34)

R_i^C and R_i^B are the restrictions of container and break-bulk for port i, that were originated from the income-cost equilibrium. The new variables α_i^C and α_i^B from constraints (31) and (32) take value 1 if demand from port i is higher than the restriction. And constraints (33) and (34) restrict the cargo to be transported by the vessel to cero if the cargo demanded by port *i* is not superior to the restriction of port *i*. With the new properties of the estimated demand, the objective function is reformulated extracting the costs related to cargoes. The new objective function is

$$Min \sum_{i \in P} \sum_{j \in P} (B + H) * D_{ij} * Y_{ij} + \sum_{j \in P} \varphi_i * \mu_i \quad (35)$$

5 Experimental analyses

In this section, the performance of the model is evaluated and parameters to apply a comparison will be determined. To measure the effectiveness of the model that has been created in section 3, a set of scenarios were created. Each scenario has it's own randomized demand for each port, so each scenario is unique. The model will have to optimize every scenario in a different way, therefore it will be possible to create a comparison of the different results and evaluate the benefit of the model created. The results of the NB scenario are resumed in Table 2. The table synthetizes the relevant information from the scenario after the optimization took place. The first parameter is the sum of ports, which is the number of ports that the vessel will have to visit in the respective scenario. The cost is the monetary value of the scenario. Demand CNT is the demanded container cargo and the CNT transported represents the amount of TEU's that are transported considering the respective demand. Demand BB and BB Transported work in the same logic as the previews columns but applied to the break-bulk cargo.

And the last column refers to the total distance that the vessel will have to sail that is measured in nautical miles. The different scenarios provide a wide variety of information after the constraints have been met. For instance in scenario 8 NB the break-bulk demand was 222.5 wt, but it does not exceed the equilibrium point stated for the ports, therefore none of the break-bulk cargo should be loaded. Having a voyage with just 50 TEU's to be shipped, turns out to be unproductive. For this reason, the voyage in scenario 8 NB should be a blank sailing (no voyage is made). Other important aspect is the automatic reduction of ports, if the cargo is not sufficient to satisfy the port equilibrium constraints, then the port is skipped. As is the case of scenario 1 NB, where 5 ports are in the vessel's itinerary in comparison with the 6 of scenario 2 NB or the 4 ports of scenario 4 NB. The omission of a port provides a variety of benefits such as the avoidance of the port cost. Also bunker cost and hire cost due to the deviation in time and fuel consumption.

Table 3 shows the same parameters Table 2 but applied to the scenarios of the SB voyages. The results can be used and analyzed as in Table 2. For instance, the scenario 4 NB has the lowest cost of the bunch of scenarios. But also has the lower amount of ports in the itinerary. That can compensate the fewer cargo, therefore lower income that it will receive in comparison to the like of scenario 6 where the cargo is almost the double with 3276.9 cbm.

Table 2 Result of NB scenarios

Scenario NB	Sum of Ports	Costs	Demand CNT	CNT Transported	Demand BB	BB Transported	Distance
1	5	$494.276	5	0	2377,9	2106,9	7221
2	6	$574.393	84	72	1114,7	1114,7	8334
3	6	$561.443	50	48	5975,1	5975,1	8080
4	4	$534.620	36	32	36,8	0	7779
5	6	$561.443	133	130	1651,2	1554,9	8080
6	6	$561.443	83	58	3732,8	3635,5	8080
7	5	$494.276	64	55	375,8	260,3	7221
8	5	$494.276	64	50	222,5	0,0	7221
9	5	$537.312	0	0	868,2	868,2	7814
10	6	$561.443	146	143	0	0	8080

Table 3 Results of SB scenarios

Scenario SB	Sum of Ports	Costs	Demand CNT	CNT Transported	Demand BB	BB Transported	Distance
1	8	$647.686	129	120	928	928	8910
2	8	$587.481	211	196	1319	1319	8004
3	7	$656.020	109	94	2038	2038	9388
4	6	$541.704	35	19	1972	1674	7916
5	8	$623.357	38	36	2452	2452	8828
6	8	$635.346	124	117	3277	3277	8942
7	9	$659.337	240	239	1900	1868	8929
8	6	$624.432	183	162	0	0	9057
9	6	$541.704	77	41	3642	3642	7916
10	9	$659.337	278	279	2545	2545	8929

6 Conclusions

In this study, a problem of a vehicle routing problem with uncertainty of demand was proposed. This work has shown that it is feasible to implement a VRP with the distribution of historical demand to solve the uncertainty in a real world scenario. The simple VRP was derived to a transshipment problem, with constraints of capacity for volume, weight and quantity of containers. Also was implemented an equilibrium point of income versus cost, to provide the minimum amount of cargo that brings a positive result to the operation. The work is applicable to a real-world scenario that can provide relevant information such as when to implement a blank sailing, which ports to omit despite that it has a demand of products, or even to end the voyage in a port earlier than it should. With the measures the deficiency created by the lack of cargo can be reduced. This study proofs that the VRP with the distribution of demand and equilibrium point is feasible in the real world scenario. Moreover it can be applied to the sea transportation of break-bulk and containers a like. This tool can be of assistance to the decision takers and may bring better results to the companies.

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