Modelling SAG milling power and specific energy consumption including the feed percentage of intermediate size particles

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ABSTRACT

Comminution, particularly milling, is on average the largest consumer of energy in mining. Actual comminution circuits consist in most of the cases in coarse crushing, SAG milling, pebble crushing and secondary ball milling. In these circuits the SAG mill is the largest energy consumer. In many engineering projects either a power equation and/or a specific energy equation are used for the designing of these mills but not always with acceptable results. In general these equations are used to predict power consumption as a function of mill size, level and density of the internal charge and % of critical speed. Almost none of these equations (with the notable exception of the Morrell’s model) consider explicitly the effect that the feed particle size has on the mill performance, particularly on the power and on the specific energy consumption of the mill. To address this fact new models are developed in this work able to predict power or specific energy consumption, including the usual design variables, but adding a variable that represents the feed size distribution. Operational data from 4 grinding circuits corresponding to 3 Chilean copper concentration plants are used and the %/C0./%/+1.00%/C0./152 +25 mm) is selected as independent variable. The results indicate that both models are able to estimate the required variables for all data sets. In the first model (power equation) the average error obtained was 3.7% and in the second model (specific energy) the average error was 6.8%. These models would be useful for performance optimisation of the test case mills and should be fitted (parameters values) again for other mills.

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1. Introduction

Comminution, especially milling, is the most important operation in mineral processing, both for its relevance to the downstream processes as well as for the high costs associated with the energy consumption. Comminution is generally the single largest consumer of energy on a mine site, accounting for an average of a 53% (CEEC the future, 2012). The average grade of ores is falling in most places in the mining industry, resulting in even higher costs, due to higher energy consumption per unit of metal production. Head grades are declining, energy costs are rising, and energy supply security is falling. The pressure for more energy efficient comminution is increasing as well as the negative impact on the license to operate. Comminution circuits are rarely designed (or even operated) to minimise energy consumption. The initial capital cost is a more common driver in their design. There is no doubt that many circuits can be made more energy-efficient by modifying their design or operating conditions (Valery and Jankovic, 2002). In this scenario, the optimisation of milling, especially SAG milling, the most widely used and energy consumer stage in the grinding circuit, acquires a very important role.

Usually, tumbling mills power equations have been derive from mechanics as the product of torque and rotational speed. Models for the prediction of the power drawn by ball, semi-autogenous and fully autogenous mills have been developed and cited in the technical literature (Turner, 1982; Austin, 1990; Moys, 1993; Morrell, 1996). However, none of these models considers the feed size distribution as a design variable. In many mine to mill type studies (Simkus and Dance, 1998; Kojovic, 2005; Michaux and Djordjevic, 2005; Morrell and Valery, 2001 and many others) has demonstrated the influence of the feed size distribution in the SAG mills throughput, power consumption and accordingly, specific energy. The feed size distribution together with the ore characteristics (hardness, lithology, alterations, etc.) are the most important factors affecting the SAG mills performance. Some operations have recognised an opportunity in the relationship between feed size and mill performance and manipulate feed size to obtain efficiency improvements (Morrell and Valery, 2001). The specific energy does not just depend on ore competence but also on factors such as feed size and ball load (Morrell, 2004). The ore competence is usually characterised by pilot plant testing, lab testing an even
plant testing and its nature is considered in SAG mills design and optimisation. In many cases the feed size distribution is even more influential than the ore characteristics (Simkus and Dance, 1998; Kojovic, 2005), but it is not considered for the design. A notable exception is the Morrell’s model (Morrell, 2008, 2009). In his approach the total specific energy of the entire AG/SAG circuit is firstly predicted using the SAG mill feed F80 and the ball mill cyclone overflow F80. Morrell’s model (Morrell, 2011) relates the feed, mill geometry and operating conditions to the specific energy of the circuit. However, the F80 is not enough to represent properly different size distributions, because different size distributions can have the same F80.

The feed size distribution is also the implicit result of the ore competence (affecting blasting and crushing performance). This distribution affects the SAG mill load and the load affects directly the mill power and the efficiency of the mill operation (Van Nierop and Moys, 2001). If a SAG mill has been designed with a conventional power or specific energy model which do not consider the projected feed size distribution it is likely that this mill will reach the expected power consumption but will not satisfy the design productive performance of the plant.

The main objective of this work is the development of mathematical models able to be used as predictive tools to optimise power consumption and/or specific energy in operating SAG mills (it is not intended for design, but could help with it). Even if the mill total load is a key variable and can be indirectly measured with bearing pressure measurements, it was decided to discard it as an input variable. The main reason is that this kind of model must be useful to relate independent input variables with the dependent variables SAG mill power and specific energy, to assure models with fixed parameters. The total load it is not an input, in the sense that cannot be directly modified at will, it is the result of changes in other really independent variables such as the ball charge, the feed rate, the feed size distribution, the mill velocity and the % solids. So if a model is developed to be used as a tool to decide operating conditions and also to explicitly include the feed size as a model input it is necessary to avoid variables too correlated between them so that any change could be identified to its effects, without the need to fit new model’s parameters every time. Since the total load is a function of the above mentioned independent variables (besides other mill characteristics as the discharge system for example), to make evident the effect of these variables they should not go together with the total load which is related to them, to avoid affecting the robustness of the models. For a given mill to have a combination of feed size, ball load, mill speed and % solids will represent the total load.

2. Operational data

Operational data from 4 grinding circuits corresponding to 3 Chilean copper concentration plants, all of them treating copper sulphide ores, are used in this work. A simplified grinding flow sheet, generic in the sense that represents all circuits considered in this work, is presented in Fig. 1. Also shown are the main SAG mill variables included in this study. All others were discarded due to its low availability or because the requirements of the models developed in this work.

The lever arm is the basis for the power models. The power is usually calculated as torque multiplied by the rotational speed and torque is equal to the force applied multiplied by the lever arm. Besides being a function of the geometric variables, such as mill diameter, mill length, grates openings, and pebble ports size; the power equation is also function of all the operational variables selected for this study. The applied force is a function of the mass inside the mill and it is composed of balls, ore and water. The ball charge is a function of the bulk fraction of the SAG mill volume (Jb) occupied by balls; the ore retained in the mill is the result of the volumetric filling which depends on the ore size distribution (specifically the % +6’ and the % −6’ +1”), on the rotational speed (N/Nc) and on the solid concentration by weight fraction inside the mill which is a function of the solid concentration by weight fraction in the SAG mill feed (Scw); and the water retained in the mill is related to the ore retained in the mill by the solid concentration by weight fraction inside the mill which is a function of the solid concentration by weight fraction in the SAG mill feed (Scw). The lever arm is a function of the rotational speed (N/Nc) and the power is a function of the rotational speed (N/Nc) as indicated before in the basic equation. Of course, ball charge plus ore retained inside the mill plus water retained inside the mill could be represented and indirectly measured by the total load. However it will not be done in this work.

As was mentioned before, the main objective of this work is the development of mathematical models able to be used as predictive tools to optimise power consumption and/or specific energy in operating SAG mills, assuming that the ore competence was already included in the basic design of the mills (for example using DW, in the specific energy equation (Morrell, 2004), but studying the inclusion of the feed size distribution (not just the F80) as a model variable, represented by an operation relevant size fraction.

**Fig. 1.** SABC A/B milling circuit. Scw = solid concentration by weight fraction in the SAG mill feed; Jb = balls charge (bulk fraction of the SAG mill volume); N/Nc = fraction of the SAG mill critical speed; Pc = SAG mill power consumption (kW); % −6’ +1” = % of the fresh feed in the size range −152 +25 mm.
In fact the later can be modelled as a function of the others. Additionally, as has been shown by Powell et al. (2001), ball loads high enough (over 12% as in the cases studied in this work) contribute to a significant portion of the power draw. Analogous considerations for not including it as an independent variable were taken with respect to the slurry pooling detrimental effect on loss of throughput (Powell et al., 2001) and accordingly on the specific energy as was studied by Powell and Valery (2006). The case study mills have all the same discharge system, resulting in a lower variation in the effect of the slurry pooling on power (Moys et al., 1996). As an important independent variable, the feed size distribution (not just the F80) was chosen as a model variable, represented by an operation relevant size fraction. The feed size distribution can be modified at will by mine to mill procedures, by primary crushing changes and by using the segregation in the stock pile. These models were developed to decide, for example, a reduction in the feed size to increase the ball charge at the same power as a way to increase the SAG mill throughput. As has been stated by Powell et al., 2001, for high mill loads, it is possible to increase mill throughput by changing the load conditions increasing the ball charge. They are not intended for design where the Morrell’s model (Morrell, 2011) is at present probably the best option, even if they could help with it including the feed size as a correcting factor. These models will be especially useful for performance optimisation of the test case mills. For other mills and/or minerals the model structure could be maintained, but the model parameters should be fitted again, determining new parameters values, likely different.

The mills used in the grinding circuits considered in this study, named GC1 to GC4, are shown in Table 1:

The data collected during periods of time of: a year (GC1), four months (GC2 and GC3) and two weeks (GC4); was analysed and filtered to remove outliers, periods where the sensors were out of order, maintenance campaigns, periods where the throughput was affected by problems corresponding to other unit operations of each concentration plant (flotation, solid/liquid separation, etc.) or by problems inherent to the mining operation and so on. In Table 2 it is shown the variation range of the variables for each SAG circuit. Fresh feed size distribution was measured with Split-Online® (GC1), WipFrag® (GC2 and GC3) and sampling, screening and off line size analysis in the lab (GC4).

To analyse the experimental data and in order to show the effect of the fresh feed size, %−6′+1′ (−152 +25 mm) was chosen. This particular size range was used because in many plants treating copper ores it is well known its detrimental effect on the SAG mill throughput. It is also possible to use the −1′ fraction that shows a positive effect on the SAG mill throughput (the power consumption decreases, but the specific energy decreases more) because the mineral goes faster to the mill discharge, leaving a greater part of the grinding task to the secondary ball mills. Besides, finer feeds tend to perform better than coarser feeds (Morrell and Valery, 2001). In the test case mills, to use the −1′ fraction it is almost redundant due to the fact that the +6′ fraction is small (3% average and exceptionally as high as 8%) and it is maintained relatively constant (see Fig. 2), at least in the studied plants, by the operations personnel. For this reason, the other two fractions are strongly correlated between them (see Fig. 2) so just one should be used. To change one of them will produce a proportional change in the other.

In Figs. 3 and 4 the data is presented in order to show the effect of the fresh feed size, particularly the %−6′+1′ (−152 +25 mm), on the SAG mill power consumption (Pc) in kW and on the SAG mill specific energy consumption (Ecs) in kW h/t, obtained by dividing the power consumed (kW) by the fresh feed rate (t/h).

It is important to notice that the behaviour shown in Fig. 4 is affected by the dependence of the power consumption on the mill sizes. This fact will be addressed later on in this paper.

3. Modelling

3.1. Specific energy model

As can be seen in Fig. 3, the effect of the independent variable (%−6′+1′) on the mill specific energy is similar for all mills analysed, so a generic common behaviour (with a common slope) is assumed, but the position of the trend lines is different for all of them. The different mill sizes are implicitly considered in the specific energy because they are related with the treatment capacity in each mill.

From Fig. 2, a linear behaviour should be considered as follows,

\[
Ecs = a(\% − 6′ + 1′) + c
\]  

(1)

Usual minimisation software (mean square of residuals) was used to determine the best estimator for a common slope (a), obtaining a value of 0.063. The position (c) of the trend lines should also be explained and for doing so the other involved and measured variables were considered in the form of a multivariable linear equation. The parameters of Eq. (1) were adjusted with the experimental data, obtaining a correlation of 99.6% for the “c” equation, resulting as follows,

\[
C = −45N/Nc + 75.3Jb + 18.3 \frac{1}{Scw}
\]  

(2)

Using the common slope value and replacing Eq. (2) in Eq. (1), the specific energy model is as follows,

\[
Ecs = 0.063(\% − 6′ + 1′) + 75.3Jb − 45N/Nc + 18.3 \frac{1}{Scw}
\]  

(3)

where Ecs is the specific energy in kW h/t, %−6′+1′ is the % of the fresh feed in the size range −6′+1′ (−152 +25 mm), Jb is the bulk fraction of the SAG mill volume occupied by the balls charge. N/Nc is the fraction of the SAG mill critical speed. Scw is the solid concentration by weight fraction in the SAG mill feed.

3.2. Power model

As can be seen in Fig. 4, the effect of the independent variable (%−6′+1′) on the mill power consumption is similar for all mills analysed, but the position of the trend lines is different for each one. The obvious reason is the effect of the mill size which is quite strongly correlated between them (see Fig. 2) so just one should be used. To change one of them will produce a proportional change in the other.


<table>
<thead>
<tr>
<th>Table 1</th>
<th>Summary of grinding mills.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Circuit</td>
<td>GC1</td>
</tr>
<tr>
<td>SAG mills</td>
<td>1</td>
</tr>
<tr>
<td>Size</td>
<td>38′ × 20′ (11.6 × 6.1 m)</td>
</tr>
<tr>
<td>Installed power (kW)</td>
<td>19,388</td>
</tr>
<tr>
<td>Ball mills</td>
<td>2</td>
</tr>
<tr>
<td>Circuit P_{80} (μm)</td>
<td>150</td>
</tr>
</tbody>
</table>
be important to normalise this behaviour with respect to the mill size. As it was discussed in the introduction, all the usual power equations presented in the specialised literature include the term $D^{2.5}L$, with $D$ and $L$ being the internal diameter and length of a mill. For this reason, the dependent variable $P_c$ (Fig. 4) was divided by $D^{2.5}L$, obtaining the results for the normalised power presented in Fig. 5.

From Fig. 5, a linear behaviour should be considered as follows,

$$\frac{P_c}{D^{2.5}L} = m(\% - 6'' + 1'') + n$$  \hspace{1cm} (4)

In Fig. 5, the trend lines are much closer than those no normalised shown in Fig. 4, but also the slopes are quite similar. Usual minimisation software (mean square of residuals) was used to determine the best estimator for a common slope ($m$), obtaining a value of 0.0348. Of course the position ($n$) of the trend lines should be also explained and for doing so the other involved and measured variables were considered in the form of a multivariable linear equation. The parameters of Eq. (4) were adjusted with the experimental data, obtaining a correlation of 99.8% for the 

$$n = -6.7N/Nc + 26.4Jb + 4 \frac{1}{Scw}$$  \hspace{1cm} (5)

Using the common slope value and replacing Eq. (5) in Eq. (4), the power consumption model is developed as follows,

$$P_c = D^{2.5}L \left( 0.0348(\% - 6'' + 1'') + 26.4Jb - 6.7N/Nc + 4 \frac{1}{Scw} \right)$$  \hspace{1cm} (6)

where $P_c$ is the power consumption in kW, $D$ is the internal diameter of the mill in m, $L$ is the internal diameter of the mill in m, $\% - 6'' + 1''$ is the % of the fresh feed in the size range $-6'' + 1'' (-152 + 25$ mm), $Jb$ is the bulk fraction of the SAG mill volume occupied by the balls charge, $N/Nc$ is the fraction of the SAG mill critical speed, $Scw$ is the solid concentration by weight fraction in the SAG mill feed.

4. Results and discussion

The quality of the models presented in the previous chapter is tested in terms of their goodness of fit and their predictive capability. For the goodness of fit the relation between modelled and experimental data is shown comparing with the identity line. For the predictive quality, some data not used for the model parameter


<table>
<thead>
<tr>
<th>Circuit</th>
<th>GC1</th>
<th>GC2</th>
<th>GC3</th>
<th>GC4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Power consumption (MW)</td>
<td>16.8–19.6</td>
<td>10.1–13.9</td>
<td>4.6–5.9</td>
<td>11.0–13.1</td>
</tr>
<tr>
<td>Fresh feed rate (t/h)</td>
<td>1891–2135</td>
<td>1406–1844</td>
<td>779–1139</td>
<td>2513–2857</td>
</tr>
<tr>
<td>Specific energy (kW h/t)</td>
<td>8.0–10.4</td>
<td>5.5–9.0</td>
<td>4.2–7.4</td>
<td>3.3–5.1</td>
</tr>
<tr>
<td>Ball charge, bulk (%)</td>
<td>19</td>
<td>13–13.5</td>
<td>12–13.5</td>
<td>13–14</td>
</tr>
<tr>
<td>Speed, % of critical speed</td>
<td>70–71</td>
<td>74</td>
<td>78</td>
<td>67–78</td>
</tr>
<tr>
<td>Solid concentration by weight (%)</td>
<td>74–78</td>
<td>67</td>
<td>65</td>
<td>73</td>
</tr>
</tbody>
</table>

Table 2

Summary of operational data ranges in each SAG mill.

Fig. 2. Fresh feed size distribution, in the SAG mill of grinding circuit GC1. Size ranges: $+6'' (+152$ mm), $-6'' +1'' (-152 +25$ mm) and $-1'' (-25$ mm).

Fig. 3. SAG mill specific energy consumption ($Ecs$) in kW h/t, in 4 grinding circuits (GC1 to GC4) at different % of the fresh feed in the size range $-6'' +1'' (-152 +25$ mm).

Fig. 4. SAG mill power consumption ($P_c$) in kW, in 4 grinding circuits (GC1 to GC4) at different % of the fresh feed in the size range $-6'' +1'' (-152 +25$ mm).
The predictive capability is tested using the model to estimate values that were not used for the parameter estimation procedure. This testing was done with data in the medium as well as in the high specific energy levels and not in the low specific energy ranges where there is not enough data to do so. The predictive results shown in Fig. 6, show accordance between predicted and measured values with respect to the general trend, but the variation is high due to the reasons already explained (the direct effect of the feed flow rate).

The goodness of fit of the power model (Eq. (6)) to represent the experimental data, for the 4 SAG mills studied, can be seen in Fig. 7. Some points not used for the model parameters fitting are also included as predicted values.

As can be observed in Fig. 7, the power model fit well the measured data and over a wide range of power values. As can be observed from the predicted values, the prediction is also good enough, in the medium as well as in the high power levels. Unfortunately, there is not enough experimental data in the low power range to do the same, but overall the prediction seems to be quite good.

After analysing the results of this work it can be observed that the specific energy model is not good enough due to the variability, intrinsic to the feed rate variability, as was discussed previously. The use of experimentally determined specific energy indices such as SPI or Axb or DW, or pilot testing when it is possible, is still a better way (but not perfect) to characterise the ore competence and to estimate the specific energy for SAG milling. However, if the evident effect of the particle size distribution of the feed is not considered, the SAG mill design would be at least in risk.

On the other hand, the power model developed in this work presents a quite good estimation capability and has the possibility to be used for many applications, but not for designing as was previously indicated. As an example, it has the potential to be used for grinding optimisation associated with mine to mill type of studies or for the implementation of pre crushing or for the expansion of a grinding plant in terms of t/h treated. In the Chilean copper concentrators the tendency has been to increase the balls charge ($J_b$) in the SAG mills as a mean to increase productivity (in most cases $J_b$ of 12–19% are actually used). Of course the mill power is a limit for doing so. The power model presented in this paper (Eq. (6)) allows to study, for a given operating SAG mill ($D, L, Pc, Nc$ and % solids known) the reduction of the intermediate feed size fraction ($\leq 50 \mu m$) that will allow a certain increase in the balls...
charge, keeping the power consumption fixed. For example, for an 36’ × 17’ SAG Mill, with a power consumption of 11.7 MW, operating at 73% solids, 13.5% balls charge and at 76% of the critical speed, with 50% of the feed in the size class –6” +1”; it could be possible to increment in 2% the balls charge (to 15.5%), reducing the % –6” +1” to 34.8%, with no changes in the power mill consumption. Accordingly, this will generate the possibility of increasing the SAG mill throughput, without affecting the product particle size going to the secondary ball mills.

For SAG mill design, this model could help by generating a correction factor over the power (kW) calculated in the usual way by multiplying the specific energy in kW h/t (determined from experimental testing) by the projected capacity in t/h. The power calculated in the usual way is a basic value which does not consider the variability of the feed size distribution, and could be corrected for the feed size distribution expected in the plant, using the model presented in this paper that does include this variable.

5. Conclusions

Two new SAG milling models are developed in this work, able to predict power or specific energy consumption, including the usual design variables such as mill size; balls charge level, solids concentration in the SAG mill feed and % of critical speed, but adding a variable that represents the feed size distribution. The –6” +1” (–152 +25 mm) size class was used because in many plants treating copper ores it is well known its detrimental effect on the SAG mill grinding capacity. It is also possible to use the –1” fraction that shows a positive effect on the same variable, but these two fractions are strongly correlated between them (see Fig. 2), since the +6” fraction is small and it is maintained relatively constant by the operations personnel.

Operational data from 4 grinding circuits corresponding to 3 Chilean copper concentration plants are used to both: modelling and testing their predictive capabilities. The results indicate that both models are able to estimate the required variables for all data sets. In the first model (power equation) the average error obtained was 3.7% and in the second model (specific energy) was 6.8%. The power model developed in this work (Eq. (6)) presents a quite good estimation capability and has the potential to be used for many applications such as operating SAG mill optimisation associated with mine to mill type of studies. Care should be taken in the sense that since the case study mills are all of the same kind (high aspect mills, grate discharge, with high ball loads) and all of them treating sulphide copper ores, the models parameters should be fitted again for other cases (different mills or different ores). The model structure can be the same, but the parameters values will change.

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