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A time-hierarchical microeconomic model of activities

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Abstract The microeconomic approach to explain consumers' behavior regarding the choice of activities, consumption of goods and use of time is extended in this paper by explicitly including the temporal dimension in the choice-making process. Recognizing that some activities, such as a job and education, involve a long-term commitment and that other activities, such as leisure and shopping, are conducted and modified in the short term, we make these differences explicit in a microeconomic framework. Thus, a hierarchical temporal structure defines the time window or frequency of adjusting the variables of activities (duration, location and consumption of goods) and the magnitude of the resources (time and money) spent. We specify and analyze a stylized microeconomic model with two time scales, the macro and micro level, concluding that preference observations at the micro level, such as transport mode choice, are strongly conditioned by the prevailing choices at the macro scale. This result has strong implications for the current theory of the value and allocation of time, as well as on the location of activities, as illustrated by numerical example.

Keywords Value of time · Hierarchical decisions · Temporal scale · Long-term and short-term activities

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Introduction

In the last decades, several microeconomic models have been proposed to explain people's behavior, including the consumption of discrete and continuous goods, the choice of feasible activities within the urban system and the use of time. Analogous to the proposal of Lancaster (1966) in the theory of consumption, Becker (1965) proposed a basic theory of time use assuming that the utility function of households and individuals is dependent on a set of nonworking activities, which is produced by combining consumption goods and allocated time. The trade-off between working and leisure time causes the author to conclude that the value of time equals the consumer's wage rate. De Serpa (1971) added working time and a set of technological relationships between goods consumption and duration of each activity as arguments to the utility function, with consumption of any good requiring a minimum amount of time. This model contributed by differentiating time values allocated to each activity. Evans (1972) fundamentally modified this approach by assuming that time spent on activities is the only argument of the utility function, arguing that the role of goods is limited to making activities feasible. This argument questions the definition of activities and what yields utility (goods or time) and introduces a new set of constraints on the time allocated to different activities. By explicitly modeling the choice of transport mode as a component of the activities, Train and McFadden (1978) developed the specification of the indirect utility function for the mode choice; the parameters of this utility function yield time values in this case derived from the consumer's choice between the alternative time and cost options offered by different transport modes. In addition, Small (1982) presented a static microeconomic model that considers the scheduling of the individual's activities, including duration and starting time, finding an expression to compute the value of leisure time depending, among other things, on both the marginal contribution of traffic congestion and the starting time of activities; the model was applied to characterize work trips. In the same line, Winston (1987) developed a model of continuous scheduling of activities, showing that the value of time is a dynamic measurement for each individual that can vary according to each individual's environment. Juster (1990) showed empirically as well as descriptively the dependence between time allocation of activities and durables (goods that last between periods).

Later, Jara-Díaz and Guevara (2003) and Jara-Díaz and Guerra (2003) integrated the choice of activities and transport mode in the calculus of the time value. In both approaches, individuals choose their working hours and time spent on all other activities to attain a static equilibrium.

In sum, since the work of De Serpa, researchers have concluded that for a given individual, the value of time differs for each activity performed. More precisely, an individual's time value is composed of an individual's specific component that values the general scarcity of time, given a time budget, and another component that depends on whether the activity performed is work (generates income) or leisure (consumes income and time). Subsequently, empirical evidence has supported this argument. For example, Jara-Díaz et al. (2008) and Munizaga et al. (2008) developed an econometric approach to calculate and estimate the value of working time and the value of leisure time based on a utility maximization model, including goods and time allocated to activities. The econometric model was applied to data from three different locations: Santiago (Chile), Karlsruhe (Germany) and Thurgau (Switzerland). Their analysis shows that the average value of leisure time is less than the average wage rates in Santiago, unlike in the European cities. The average value of working time in Karlsruhe is positive, unlike in Santiago and Thurgau. Olguín (2008) used the 2001 origin-destination survey of Santiago to report different time values: by gender, age and residential location. Another interesting result of this study is that the value of leisure time of residents in the high-income area is 3–4 times the value of leisure time in other areas. Additionally, the overall reported value of working time was negative; however, unexpectedly, it was positive for individuals under 25 years and for women between 25 and 64 years. Greeven (2006) also obtained different time values associated with differentials in households' expenditures on long-term commitments, such as housing, domestic services, medical services, education, communications and consumer durables.

The causes of the differences in time allocations to different activities were empirically studied by Levinson (1999), concluding that demography, socioeconomics, season and scheduling affect the distribution of time to various activities. His analysis also showed that the time allocated to an activity is positively correlated to the trip duration (travel time and activity duration). He also concluded that time allocated to activities, such as household activities or shopping, is correlated with an individual's gender, age, location and residential density, income, seasonality, marital status, age of children, and frequency of activities. Bullard and Feigenbaum (2007) proposed a general equilibrium model for a life cycle, in which a household's utility includes consumption and leisure in each period. The calibration shows significant changes in consumption over a life cycle. The model identifies the income, asset holdings and hours worked, in different temporal cohorts, for individuals older than 25 years. They estimated that the average age of maximum consumption of nondurable goods is close to 45 years old. In contrast, in terms of hours spent at work, on average, the peak is reached between the age of 50–55 years; after this period hours spent at work are likely to decline. Similarly, Guang-Zhen and Yew-Kwang (2009) studied life cycles using a microeconomic model to show age-dependent dynamics of the value of time. In their model, long-term choices are represented by an a priori structure of risks or investments assumed by an individual. In addition, they analyzed the changes in the value of time after the retirement of an individual and the influence of mortality and interest rates on the time valuation.

Another area with studies on time allocation to activities is the activity-based modeling, especially The Multiple Discrete-Continuous Extreme Value (MDCEV) models, where problems of multiple choice options are analyzed, for example. Here an individual decides to participate in multiple types of maintenance and leisure activities within a certain time period. For example, Bhat (2005, 2008) the estimation of time allocations and decisions of participation in various activities are modelled by through optimization conditions for a particular functional form of the utility function, subject to single budget constraint. In a recent work, Castro et al. (2012) incorporate multiple constraints, such as time and income obtaining an econometric expression for the time value as a resource.

The static microeconomic theoretical models of time allocation (static refers to a single and representative temporal cohort) assume that the activity/consumption choices are optimized simultaneously (fixed consumption/activities are assumed as exogenous parameters). Additionally, both decisions and exogenous elements of the model generate differences in the value of time as resource of each individual (for example, residential or work location, age, mode, transport cost, travel time, and in some cases the time allocated to work, education level, etc.). These exogenous decisions can be incorporated in the parameters of both, the utility function (change in taste) and the budget constraints [for example exogenous income or wage rate that depends on previous decisions, expenses and time commitments in budget constraints defined in Jara-Díaz et al. (2008)].

In addition, the value of time as resource can be different across individuals due to the variety of parameters associated with technological constraints (see De Serpa 1971; Evans

1972; Jara-Díaz 2003, 2007), for example, in the case that each activity has an individuals' specific minimum time allocation and, similarly, each consumption good has a lower exogenous bound, as proposed by Jara-Díaz et al. (2008).

However, this theoretical literature cannot explain explicitly the empirical findings that show differences in the value of time between individuals with similar resources but with differences such as age, housing type or location, gender, marital status, car ownership or number and age of children (see for example, Jara-Díaz et al. 2013).

In this paper, we explore the implications of relaxing the assumption of a simultaneous choice-making process, introducing time scales in a dynamic (inter-temporal) choice of activities in the context of an otherwise similar utilitarian approach as that commonly used in the literature. This dynamic assumption can explain what in static models is represented by some exogenous parameters, represented in our approach as decision variables in previous periods. A basic argument is that activities performed by an individual can be clearly differentiated by duration, i.e., the time window during which the choice lasts, and we assume this difference explains a structural dependency among activities and time values. We also propose that time windows are defined by the proportion of available resources (time and income) consumed or produced by the activity, in contrast to an arbitrary taxonomy. For example, we can easily observe that work and study involve time windows of years of commitment to a given choice and a major allocation of daily time and monetary resources, whereas other activities that involve less daily time, such as shopping or leisure activities, can be modified within a day or week and involve a comparatively small amount of time and income.

Such time-scale differences are a common feature in many physical and biological systems, which can provide relevant knowledge based on their dynamic structures and which have recently been applied to study social and economic systems (see Gunderson and Holling 2002). These dynamic systems are modeled assuming a hierarchy in the involved processes, with different time scales for each sub-system, which is the approach borrowed in this paper to model human activities. There are other features common to these systems, such as stochastic shocks and memory effects as well as conditions on the decision sequences, all implying a potentially strong complexity in the dynamic process. However, despite the relevance of these features to the individual's behavior, to keep this paper focused and simple, they are not included in our model; instead, we deliberatively focus on the effects of introducing the temporal scale in the decision process.

Our model considers a hierarchical structure in the activity decision-making process based on the time windows in which choices are made, representing both the speed of decision changes and the amount of resources consumed. For simplicity, we concentrate on two temporal scales, named the macro and micro scale, which are considered sufficient to understand the basic effects that can later be extended to a multi-scale structure. Using this approach, we seek to analyze the dynamics of an individual's behavior and, particularly, to study the effects that long-term choices may have on a set of short-term decisions and on the interpretation of the value of time derived from choices observed at the micro scale. An important assumption of this model is the timing or synchrony of decision making at the macro and micro levels. We assume that at the time of the beginning of each macro scale, called the adjusting point, all short-term and long-term decisions are adjustable. We also consider that individuals make decisions based on the current economic information in each given time window, i.e., myopic decisions, although the hierarchical approach can be extended using future expectations without fundamentally modifying our main conclusions.

With the microeconomic approach presented here, we indicate that the time value associated with micro-scale activities is naturally dependent on the set of choices made at the macro scale, including time and money expenditures in durables, as observed in the above-mentioned empirical studies. The macro-micro bidirectional dependency among activities is not explicit in previous microeconomic models, where all activities belong to the same hierarchy level. Thus, the time-hierarchical model provides a more comprehensive choice-making framework, where long and short-term choices, or all choices in life, become theoretically integrated in a dynamic approach. A practical conclusion for transport studies follows: the value of time estimated from the standard econometric calibration of transport mode choice models is bound to be different according to the individual's choice of durables (housing, car(s), education level, job) and the location of macro-scale activities, such as work and study; the implication is that there is a potential bias in the estimation of time values in the applied research. We also conclude on how to specify short-term utility functions based on the conditionality of durables and long-term assignments of time and money budgets.

The essential aim of the paper is to improve the understanding of individuals' behavior in the urban context, where individuals face a highly complex set of interconnected decisions. Considering this complexity, we question the assumptions of the standard optimization paradigm adopted from the microeconomic literature and introduce a fundamental structure of the decision process, the temporal structure. In this paper, we particularly concentrate on the effect of this approach on understanding time values. Our aim is to develop a better model of urban agents. We also derive and discuss the practical implications of our model for applications.

Microeconomic hierarchical model of consumer

The proposed model considers the hierarchy of activities depending on the frequency of consumption changes of both continuous and discrete goods over a specific time horizon. To analyze this hierarchy, the feasible range of activity choices is divided into two scales, called the macro scale (long-term activities) and micro scale (short-term activities). Macro scale choices are exemplified by work and study; after defining their basic contract conditions (salary or fees, duration and location), they remain essentially fixed during the macro time period. The micro scale includes decisions that can be adjusted frequently (e.g., daily), exemplified by leisure and shopping. An individual's choice of the location of activities is assumed to be made at the corresponding scale of activity; however, decisions regarding transport mode are made at the micro scale, irrespective of the activity to which the individual is traveling. Activities are defined as self-contained, meaning that activities do not share any action, goods or time, and they are interdependent, meaning that to be feasible, activities may need resources acquired through other activities. At each temporal scale, consumers make choices assuming that all exogenous variables, such as prices, are constant during the time window; hence, the longer the time window, the stronger the price prediction the consumer has to make.

The model also assumes that certain goods obtained at the macro scale are durable in nature, noting that in economic theory, such goods are defined as those goods (or services) that, once acquired, can be used several times; they are also known as reusable assets (Sullivan and Stevens 2003). In addition, the characteristics of these goods may affect the level of consumption at the micro scale. The set of decisions made by the agent in the short and long term is based on his/her information at the beginning of the respective time window. The model developed in this paper is deterministic and continuous; however, it can easily be extended to discrete choices, such as residential location of activities (Pérez and Martínez 2003) or transport mode choice (Train and McFadden 1978), as shown in the simulation example below. In the proposed model, we assume that decisions are based on an individual's current preferences only, ignoring—for simplicity—other effects, such as memory, learning in consumption, and the temporal dependence of decisions across different time windows and scales.

Notation

Relevant sets and indices:

- Δ*:* Duration of a micro-time window; 1 day by default. All activities, macro or micro, and their consumption goods and durations are defined in a common "time unit" Δ, irrespective of the temporal scale. The 1-day time window is not arbitrary because it is the biological cycle in which activities are scheduled. One week is also considered a plausible time window, although it is also organized as a set of days with different activities.
- $V\Delta$: Duration of a macro-time window, e.g., 1–5 years. V is the number of micro-time windows in the macro window.
- J: Sets of time scales. In the two-scales model $J = \{j_1 = m, j_2 = M\}$, with m denoting micro and M denoting macro scales. In addition, there are sets of feasible activities and consumption goods, denoted respectively Ω^j and Λ^j, associated with scale j. $\Omega^M \cap \Omega^m = \emptyset; \Lambda^M \cap \Lambda^m = \emptyset$.
- *i, k*: Indices for activities and consumption goods, respectively.
- *n,v*: Time window indices denoting the vth micro-time window in the nth macro-time window; $v = 1,\ldots,V; n = 1,\ldots,N$. Note that a double index for time is needed to differentiate between macro- and micro-time windows.

Parameters:

- $r_i^{n,v}$: Income ($r_i^{n,v} \geq 0$) or cost ($r_i^{n,v} < 0$), per time unit obtained in the *micro* activity *i* in the micro-time window (*n,v*); $\forall i \in \Omega^m$, vector $r^{n,v} \equiv (r_i^{n,v}, \forall i \in \Omega^m)$
- $R_i^{n,v}$: Income ($R_i^{n,v} \geq 0$) or cost ($R_i^{n,v} < 0$) per time unit obtained in *macro* activity *i* in time period (*n,v*); $\forall i \in \Omega^{\mathrm{M}}$, vector $R^{n,v} \equiv \left(R_i^{n,v}, \forall i \in \Omega^M\right)$
- $p_k^{n,v}$, $P_k^{n,v}$: Unitary price of good *k,* micro and macro level, respectively, in *(n,v);* $k \in \Lambda^m U \Lambda^{\mathrm{M}}$, vectors $p^{n,v} \equiv \left(p_k^{n,v}, \forall k \in \Lambda^m\right); P^{n,v} \equiv \left(P_k^{n,v}, \forall k \in \Lambda^{\mathrm{M}}\right)$
- $I^{n,v}$: Exogenous income obtained in (*n,v*) from real estate rents, capital investments in previous periods, bank loans, inheritance, etc.
- $C^{n,v}$: Adjustment costs associated with transactions of durable goods.

Decision variables

The classical decision variables are the allocation of time and the consumption of goods for each activity. The new variables in this model are the time and wealth surpluses, which are transferred from the macro scale to the micro scale; these variables are called dynamic variables.

Decision variables in the direct utility specification:

- $X_k^n, x_k^{n,v}$: Consumption of good k (macro and micro, respectively)—per unit of time Δ—$\forall k \in \Lambda^{\mathrm{M}} U \Lambda^{\mathrm{m}}$, vectors $x^{n,v} \equiv \left(x_k^{n,v}, \forall k \in \Lambda^m\right), X^n \equiv \left(X_k^n, \forall k \in \Lambda^{\mathrm{M}}\right)$.
- $T_i^n, t_i^{n,v}$: Time allocated to activity i (macro or micro, respectively)—per unit of time Δ—$i \in \Omega^M U \Omega^m$, vectors $t^{n,v} \equiv \left(t_i^{n,v}, \forall i \in \Omega^m\right),\ \ T^n \equiv \left(T_i^n, \forall i \in \Omega^M\right)$.

Dynamic variables (macro-micro transference):

- $\boldsymbol{S^{n,v}}$: Surplus of wealth per time unit, which is transferred from the macro scale in time period (n,v) to the micro-time window v; ($\boldsymbol{S^{n,v}} \in \boldsymbol{R}$) (with $\boldsymbol{S^{n,v}}$ a debt (negative) or a saving (positive).
- $\boldsymbol{\tau^n}$: Time saved from the macro window n for all micro-scale activities.

Note that the model parameters, such as prices, and variables have a double time superscript (*n,v*), which is important to denote the macro-scale window and the specific micro-scale window within the macro scale, assuming that the micro-scale superscript v is reset at the beginning of each macro-time window n.

Minimum or maximum time constraints

Within each time window, following Jara-Díaz (2003), we assume that the decision of time assigned to a certain activity depends on the level of goods consumption. At the macro scale, the decision depends on the consumption goods at that scale only; however, at the micro scale, the decision depends on the consumption choices at the macro as well as the micro scale. This assumption reflects that some micro-scale activities require inputs decided in the long term (infrastructure, durables). For example, a car is a durable good required to be chosen as a transport mode; housing is another durable good whose type (floor space, land lot size and building quality) affects micro-scale choices, such as leisure time.

Let us define the following technological constraints associated with the macro scale, per unit of time Δ:

$g_i^n\left(T_i^n, X^n\right) \geq 0, \forall i \in \Omega^M$: The times and goods allocated to each macro activity within a time window are mutually dependent; for example, the time allocated to work is limited by the consumption of durables, such as car ownership, residential and job location. We particularly consider the two different forms proposed by De Serpa (1971) and extended by Jara-Díaz (2003).

$\underline{D}_i^n(X^n) \leq T_i^n \leq \bar{D}_i^n(X^n), \forall i \in \Omega^M$: The maximum/minimum feasible time for the realization of an activity is constrained by the quantity of consumption goods purchased in the same macro-scale time window n. Note that the macro technological constraints are fixed for the entire time window n.

With regard to the micro scale, we identify the following types of constraints, $g_i^{n,v}(t_i^{n,v}, x^{n,v}; X^n) \geq 0, \forall i \in \Omega^m$; these constraints relate the time assigned to a short-term activity at the micro scale with both micro and macro consumption decisions. As described for the macro scale, this constraint can be as follows:

$$\underline{d}_i^{n,v}(x^{n,v}, X^n) \leq t_i^{n,v} \leq \bar{d}_i^{n,v}(x^{n,v}, X^n)$$

That is, in both scales, i.e. micro and macro decisions, the time allocated to any activity may be jointly bounded by the resources obtained from the activity's macro and micro scales or by endogenous and exogenous constraints. For instance, the technology and physical equipment at home generate different bounds on the time allocation for home activities. The specific features of the job and the residential locations generate more or less options to assign time to leisure as well as to the availability of transport modes. Note that v is an indicator of the deterioration of the durable goods X, and therefore, such technological constraints at the micro level vary over time.

Optimization model

In this section we propose a two-level microeconomic model of consumers' behavior defined in the same time window (n,v), for macro or long-term and micro- or short-term activities.

Long-term decisions are made at the beginning of each macro-time window (n), which is micro-time window $(n,1)$, along with all micro decisions associated with this micro time window. In subsequent micro-time windows, the consumer adjusts his/her short-term decisions conditional on macro choices (durables). The set of optimal choices made at the macro level in $(n,1)$ is determined by the following:

$$\max_{X,T} U_M^{n,v}(X^n, T^n)$$

Subject to

$$F_M^{n,v} \equiv \left\{ \begin{array}{c} R^{n,v}T^n - P^{n,v}X^n - C^{n,v} + I^{n,v} - S^{n,v} = 0 \\ \sum_{i\in\Omega^M} T_i^n = \Delta - \tau^n \\ \underline{D}^n(X^n) \le T^n \le \bar{D}^n(X^n), \end{array} \right\} \tag{1}$$

In this case $v = 1$; however, we use a general notation because $F_M^{n,v}$ in (1) applies also at any $v > 1$.

Similarly, the set of optimal short-term choices in any micro-time window (n,v) is the solution of the following:

$$\max_{x,t} U_m^{n,v}(x^{n,v}, t^{n,v})$$

Subject to

$$F_m^{n,v} \equiv \left\{ \begin{array}{c} r^{n,v}t^{n,v} - p^{n,v}x^{n,v} + S^{n,v} = 0 \\ \sum_{i\in\Omega^m} t_i^{n,v} = \tau^n \\ \underline{d}^{n,v}(x^{n,v}, X^n) \le t^{n,v} \le \bar{d}^{n,v}(x^{n,v}, X^n) \end{array} \right\} \tag{2}$$

Following De Serpa (1971) and Jara-Díaz (2003), problems (1) and (2) assume that utility depends on goods and time, and the feasible set is defined by income and time budgets, in addition to technological constraints. These problems are interdependent of the set of savings in money ($S^{n,v}$) and time (τ^n) for the macro-level problem (1), which defines budget constraints for the micro-level problem (2). Additionally, macro decisions imply consumption of durables that modify the set of feasible micro-scale options. The macro income constraint contains a fixed adjustment cost associated with the transaction of durables, denoted as $C^{n,v}$. Note that even though adjustment costs only affect the variation of durables consumption, the technological relation between goods and time causes these costs to have an indirect effect on the time allocation of long-term activities. For the micro-scale problem (2), we assume that income is exhausted in every period v, i.e., there are no intertemporal money savings. This assumption simplifies the consumer behavior model to focus on the hierarchical structure of the decision process. However, an extension introducing intertemporal savings to this model would yield a more dynamic model.

The microeconomic problems (1) and (2) define two different dynamics in the decision-making process.

a. Vertical dynamic (hierarchical): this dynamic is analyzed through time (τ^n), monetary ($S^{n,v}$) budget transferences between the different decision scales, and the effects of durables in micro-scale choices.
b. Horizontal dynamic (temporal): this dynamic is explained by the temporal variation of the parameters associated with the economy over time, together with the variation in the perception of the durables. An example is the deterioration or changes in specific features of the durables, such as residential location externalities.

Hierarchical choice

In this section, we show the process of decision-making based on a hierarchical structure starting with macro-scale choice. In every time window $(n,1)$, the individual adjusts her/his long-term decisions; we call this the adjusting point. At this point, the individual is assumed to solve both problems, macro and micro, simultaneously, using the parameter α that determines the weight placed on macro or long-run utility versus micro or short-run utility. Note that the simultaneous macro-micro choice allows the influence of the micro scale on the macro scale because at the adjusting point all choices are made simultaneously; thus, at this point in time, short-term choices directly affect long-term choices and vice versa.

Thus, at every adjusting point $(n,1)$ the individual solves the following:

$$\max_{X,T,x,t}\left(\alpha U_M^{n,1}(X^n,T^n)+(1-\alpha)U_m^{n,1}\left(x^{n,1},t^{n,1}\right)\right)$$

Subject to:

$$F_M^{n,1}\cup F_m^{n,1}\equiv\left\{\begin{array}{c}R^{n,1}T^n-P^{n,1}X^n-C^{n,1}+I^{n,1}+r^{n,1}t^{n,1}-p^{n,1}x^{n,1}=0\\ \sum\limits_{i\in\Omega^M}T_i^n+\sum\limits_{i\in\Omega^m}t_i^{n,1}=\Delta\\ \underline{D}^n(X^n)\le T^n\le\bar{D}^n(X^n),\\ \underline{d}^{n,1}\left(x^{n,1},X^n\right)\le t^{n,1}\le\bar{d}^{n,1}\left(x^{n,1},X^n\right)\end{array}\right\} \tag{3}$$

The set $F_M^{n,1}\cup F_m^{n,1}$ is the simultaneous macro-micro feasible set at $(n,1)$. In problem (3) micro choices affect macro decisions. For example, the residential location is a macro choice made not only based on the environmental amenities but also with regards to accessibility to micro-choice activities (e.g., shopping, social activities, interaction with other agents). In the set $F_M^{n,1}\cup F_m^{n,1}$, the macro and micro income and time constraints are added, and therefore, the transferences of budget ($S^{n,v}$) as well as time (τ^n) make no sense at the adjusting point because the hierarchical structure is lost; analytically, we have $R^{n,1}T^n-P^{n,1}X^n-C^{n,1}+I^{n,1}+r^{n,1}t^{n,1}-p^{n,1}x^{n,1}=0$ and $\sum_{i\in\Omega^M}T_i^n+\sum_{i\in\Omega^m}t_i^{n,1}=\Delta$. We also make two other simplifying assumptions: first, we consider that the individual makes myopic decisions based on the current economic information in each given time window; secondly, we assume that the total or hierarchical utility is separable in the macro and micro utilities, using the alpha parameter as the weighting factor. These assumptions may be relaxed to model other contexts associated with decision making (for example, perfect foresight dynamic or non-separable utilities) without affecting our main conclusions regarding the macro-micro dependencies.

Solving problem (3), the individual obtains the following demands for durable goods and long-term time allocated to activities, conditional on the parameters of the optimization problem:

$$X_k^{n*}\left(\varepsilon_M^{n,1},\varepsilon_m^{n,1},\Delta\right),\forall k\in\Lambda^M,T_i^{n*}\left(\varepsilon_M^{n,1},\varepsilon_m^{n,1},\Delta\right),\forall i\in\Omega^M, \tag{4}$$

where $\varepsilon_M^{n,v}\equiv\left(R^{n,v},P^{n,v},I_C^{n,v},\theta_M^{n,v},\alpha\right)$, $\varepsilon_m^{n,v}\equiv\left(r^{n,v},p^{n,v},\theta_m^{n,v}\right), I_C^{n,v}=I^{n,v}-C^{n,v}$ and $\theta_M^{n,v}$ are the parameters of technological constraints in (*n, v*).

We note that the set of long-term choices defined in (4) at (*n,1*) remains fixed for all (*n,v*) and macro-scale decisions depend on the macro utility weight parameter α. However, note that the utility obtained from this set changes along the macro-time window because exogenous parameters of the economy (prices, income, technological constraints), as well as the individual's perceptions, change. In addition, the deterioration of durables generates variations in the utility perceived by the agents in each micro-time period.

Long-term commitments (durable goods, fixed-time activities) define savings, e.g., money transfer (cost / revenue committed) and time transfer for short-term decisions, as follows:

$$S^{n,v}=R^{n,v}T^{n*}-P^{n,v}X^{n*}+I_C^{n,v} \tag{5}$$

$$\tau^n=\Delta-\sum_{i\in\Omega^M}T_i^{n*}(\cdot) \tag{6}$$

when $v>1$, then $I_C^{n,v}=I^{n,v}$ because the adjustment costs are zero and therefore not considered in the computation of $S^{n,v}$. Note that while τ^n is constant for all time windows *v* in (*n,v*), the money saving for every *v* also changes in every time window *v* following variations in the exogenous parameters of the economy. Formally,

$$S^{n,v}(\varepsilon_M^{n,v}|\varepsilon_M^{n,1},\varepsilon_M^{n,1},\Delta),\ \tau^{\mathrm{n}}(\varepsilon_M^{n,1},\varepsilon_M^{n,1},\Delta) \tag{7}$$

An example, τ^n is the individual's daily time (Δ assumed 24 h) available after assigning time to work or education, and $S^{n,v}$ is his/her remaining income from labor and investments interest, which is allocated for leisure and shopping after expenditure on macro scales decisions: durable goods (e.g. car), housing expenses (taxes, rent, public services, mortgages, among others), loan payments, fixed costs of education, etc.

On the other hand, we have the following decision process at the micro level. After solving the macro optimization problem (3) at the adjusting point $(n,1)$, the individual makes short-term decisions in every time window (n,v) for $v\neq 1$. This process is modeled as solving micro problem (2), assuming that the macro decisions are fixed, such that the solution to this problem generates the following optimal choice set:

$$x_k^{n,v}\left(\varepsilon_m^{n,v},S^{n,v},\tau^n,X^n\right),\quad \forall k\in\Lambda^m,\qquad t_i^{n,v}\left(\varepsilon_m^{n,v},S^{n,v},\tau^n,X^n\right),\quad \forall i\in\Omega^m \tag{8}$$

From specification (8), it is clear that micro activities change along the duration of the macro-time window due to changes in the economy, and decisions, such as job location, residential location, time allocated to work and consumption of durables, define the dynamics of short-term decisions. In addition, changes in income, real estate rents and other long-term monetary assets also generate differences in short-term decisions. Additionally, in (8) the parameter α implicitly affects the micro-scale consumption and time through the macro consumption X^n.

Note that at the micro level, for all $i \in \Omega^m$, we have $t_i^{n,v}(\varepsilon_m^{n,v}, S^{n,v}, \tau^n, X^n)$ and $x_k^{n,v}(\varepsilon_m^{n,v}, S^{n,v}, \tau^n, X^n)$, which are increasing functions with decreasing rates $S^{n,v}$ and τ^n:

$$\frac{\partial t_i^{n,v}}{\partial S^{n,v}} \geq 0, \quad \frac{\partial^2 t_i^{n,v}}{\partial \left(S^{n,v}\right)^2} \leq 0, \quad \frac{\partial t_i^{n,v}}{\partial \tau^n} \geq 0, \quad \frac{\partial^2 t_i^{n,v}}{\partial \left(\tau^n\right)^2} \leq 0, \quad i \in \Omega^m$$

$$\frac{\partial x_k^{n,v}}{\partial S^{n,v}} \geq 0, \quad \frac{\partial^2 x_k^{n,v}}{\partial \left(S^{n,v}\right)^2} \leq 0, \quad \frac{\partial x_k^{n,v}}{\partial \tau^n} \geq 0, \quad \frac{\partial^2 x_k^{n,v}}{\partial \left(\tau^n\right)^2} \leq 0, \quad k \in \Lambda^m$$

which follows directly from observing that $S^{n,v}$ and τ^n are constraints in problem (2).

This two-level (macro-micro) hierarchical process can be extended to a multilevel process. At the adjusting point of each level, the individual optimizes choices of all lower levels, i.e., all levels with shorter time windows, and defines the initial time and money savings for these lower levels. Note that at the adjusting point, changes on macro-scale activities represent shocks in the micro-scale choice process.

Note that the formulation and calculation of $S^{n,v}$ as well as τ^n propose an extension to the definition of resources commitment (expense $E_c^{n,v}$ and time $T_c^{n,v}$) initially defined in Jara-Díaz and Guevara (2003) and Jara-Díaz (2007), later used in other econometric works. The committed expenses ($E_c^{n,v}$) and time ($T_c^{n,v}$) are obtained from a particular microeconomic model with exogenous technological constraints of the form $X_k^{n,v} \geq X_{k,min}^{n,v}$, $T_i^{n,v} \geq T_{i,min}^{n,v}$, $x_k^{n,v} \geq x_{k,min}^{n,v}$ and $t_i^{n,v} \geq t_{i,min}^{n,v}$. In particular, $E_c^{n,v}$ and $T_c^{n,v}$ are defined as the resources—in money and time—generated by the decisions that agents set to an exogenous minimum value. Compared to our model, such committed resources may be equal to $S^{n,v}$ and τ^n only in some cases, but from an analytical standpoint they are different. First, while $E_c^{n,v}$ and $T_c^{n,v}$ are based on very specific technological constraints that only consider exogenous minimum time and consumption for activities and goods, respectively, in our hierarchical model the values $S^{n,v}$ and τ^n are obtained endogenously from a more general formulation of macro scale decisions, including possible interactions between constraints on time, consumption and technology. In our model, τ^n can include activities whose time is not set to a minimum $\left(T_{i,min}^{n,v}\, or\, t_{i,min}^{n,v}\right)$. Additionally, even in activities where it is generally assumed that individuals allocate minimum time, such as home repairs, in our model the requirements associated with these activities vary continuously in time, therefore, $t_{i,min}^{n,v}$ also varies between micro time windows. Finally, given the simultaneity of the optimization consumer problem used to find $E_c^{n,v}$ and $T_c^{n,v}$ they naturally depend on the economy at the time (n, v). Conversely, according to Eqs. (5) and (6), the values of $S^{n,v}$ and τ^n depend on the parameters of the economy and technological constraints at the adjusting point (n, 1), and on some other parameters (e.g. prices) at every time window.

Conditional indirect utility functions

From problem (1) we now define the indirect macro utility function evaluated at any time window (n, v). This utility is determined by the optimal consumption set of goods and time decided at (n,1), denoted (X*,T*), and given by $V_M^{n,v} = U_M^{n,v}(X_k^{n*}, T_i^{n*})$; replacing optimal goods and time given in Eq. (4) yields $V_M^{n,v} \equiv V_M^{n,v}\left(\varepsilon_M^{n,1}, \varepsilon_m^{n,1}, \Delta\right)$.

Observe that although the optimal set (X*,T*) remains fixed for all v in the macro period n, the macro-scale utility may change along this period if the consumer's perceptions change; hence $V_M^{n,1}$ is not necessarily equal to $V_M^{n,v}, v > 1$. Second, observe that, except for variable perceptions, the macro-scale utility remains fixed depending only on the economy at the adjusting point $(n,1)$.

Similarly, we define the indirect micro utility function for the micro-time window $(n,\ v)$ as $V_m^{n,v} \equiv U_m^{n,v}(x_k^{n,v*},\ t_i^{n,v*})$, which becomes $V_m^{n,v} \equiv V_m^{n,v}(\varepsilon_m^{n,v},\ S^{n,v},\ \tau^n,\ X^n)$. Note that $V_m^{n,v}$ makes clear how the utility of micro decisions is conditional on the decisions taken at the macro level, by way of the availability of durables, fixed-time to long-term activities and monetary savings. Additionally, the indirect utility is conditional on the parameters of the current economy at (n,v).

Note that the global utility perceived by an agent in a period $(n,\ v)$ (excepting the adjusting point $v = 1$) is given by both the macro utility and the micro utility, which can aggregated by:

$$\alpha V_M^{n,v}\left(\varepsilon_M^{n,1}, \varepsilon_m^{n,1}, \Delta\right) + (1-\alpha)V_m^{n,v}\left(\varepsilon_m^{n,v}, S^{n,v}, \tau^n, X^n\right) \tag{9}$$

This follows from micro utility maximization problem (2) which provides $V_m^{n,v}$ and the fact that the maximum macro utility is constant at any time window $v > 1$. At $v = 1$ the micro and macro scales problems (1) and (2) are solved simultaneously, which yields:

$$\alpha V_M^{n,1}\left(\varepsilon_M^{n,1}, \varepsilon_m^{n,1}, \Delta\right) + (1-\alpha)V_m^{n,1}\left(\varepsilon_M^{n,1}, \varepsilon_m^{n,1}, \Delta\right) \tag{10}$$

The conditional indirect micro-level utility function increases monotonically with time and money budgets. Thus, we have the following important properties for $v > 1$ of the dynamics of the following key optimization parameters (see proof in Jara-Díaz and Gschwender 2008):

$$\lambda^{n,v}\left(S^{n,v}\right) = \frac{\partial V_m^{n,v}}{\partial S^{n,v}} \geq 0,\ \frac{\partial \lambda^{n,v}}{\partial S^{n,v}} = \frac{\partial^2 V_m^{n,v}}{\partial\left(S^{n,v}\right)^2} \leq 0. \tag{11}$$

$$\mu^{n,v}(\tau^n) = \frac{\partial V_m^{n,v}}{\partial \tau^n} \geq 0,\ \frac{\partial \mu^{n,v}}{\partial \tau^n} = \frac{\partial^2 V_m^{n,v}}{\partial(\tau^n)^2} \leq 0. \tag{12}$$

where $\lambda^{n,v}, \mu^{n,v}$ are the Lagrange multipliers associated with the income and time constraints in period (n,v). The conditions (11) and (12) indicate that if micro-scale income $(S^{n,v})$ or micro-scale time (τ^n) budgets grow, the marginal utility of income or time decreases. This result becomes important below because it supports the analysis of the sub-optimality in the transfer of macro-micro resources and the sub-optimality of long-term and short-time decisions.

Note that the formulation of the utilities (9 and 10) is similar to the function utility proposed by Bhat (2000) in an econometric context, where the author estimates separately the macro and the micro effects on mode choice, using concepts of hierarchical decision levels applied to a different definition of hierarchies based on the aggregation levels: macro scale is defined by clusters of individuals and zones for residential and work locations choices, while micro scale is defined at the individuals level for mode choice. Based on this similarity we can get some guidelines for the econometric estimation of (9 and 10) using a similar approach where micro-scale utilities are conditioned on macro-scale decisions (i.e., on the set of macro decisions), thus making estimates of micro utility dependent on the set of macro-scale decisions (e.g., car ownership, residential location, education).

Besides, the intertemporal nature of our model is represented by long-term decisions such as money savings and available time, plus variations in the economy, which affect the short-term utility maximization process. Thus, the required data for the econometric estimation of the utility function is a panel data containing observations of how the aforementioned macro scale factors change during the panel period, as well as records of how the selected individuals change their choices. We also conclude that our theory on hierarchical choice process provides theoretical support for defining how to cluster individuals in micro scale decisions, such as mode choice. This clustering must be according to their similar macro scale choices, such as: residential location, work place, car ownership, marriage status, child care. All these choices define a different set of conditions for the micro scales choices, e.g. income and time available, and durables.

The value of time

We now analyze the expression for the value of time, following Jara-Díaz (2003) and De Serpa (1971) to study the dynamics of such a measure and the influence of macro decisions on micro activities. We also study the willingness to pay by allocating either more or less time to specific activities.

The subjective value of time (VT) as a resource by definition represents the variation of utility at the margin caused by an increment in time compared with the same variation caused by an increment in income. This marginal behavior can be identified at the micro scale and calculated directly from (11) and (12) as:

$$VT^{n,v} = \frac{\mu^{n,v}}{\lambda^{n,v}} = \frac{\frac{\partial V_m^{n,v}}{\partial \tau^n}}{\frac{\partial V_m^{n,v}}{\partial S^{n,v}}} \geq 0, \quad \forall v \tag{13}$$

We assume that the value of $\lambda^{n,v}$ is nonzero, i.e., the individual exhausts the money budget for all v. Additionally, the value of $VT^{n,v}$ may vary along micro-time intervals because the economy and perceptions change; in particular the money transference may change due to price changes modifying the money budget along time. The VT calculated in any time window $v > 1$ is a micro-scale value conditional on macro-scale decisions, which implies that equivalent individuals who make different long-term choices have a different VT. This result further implies that a proper definition of clusters of agents in studies of consumer behaviors in short-term activities should include, in addition to traditional socioeconomic variables, the most-important long-term choices, such as location of residence and work and car ownership.

We now consider the value of time for specific activities subject to technological constraints. Let us define $\kappa_{i(-)}^{n,v}, \kappa_{i(+)}^{n,v}$ as the Lagrange multipliers associated with constraints of minimum or maximum time allocated to activities, all with respect to micro activity i in time window (n,v). We can calculate the value of time at each point in time as a dynamic process; however, it is sufficient to analyze two characteristic points: any time point $v > 1$ (the general case) and the adjusting point $v = 1$.

General case (v > 1)

Assume that the value of $\lambda^{n,v}$ is nonzero, i.e., the individual exhausts the money budget. The first order conditions for the micro-scale consumption of good $k \in \Lambda^m$ in time window (n,v) are:

$$\frac{\frac{\partial U_m^{n,v}}{\partial x_k^{n,v}}}{\lambda^{n,v}} - p_k^{n,v} + \sum_{i\in\Omega^m}\left(\frac{\kappa_{i(+)}^{n,v}}{\lambda^{n,v}}\frac{\partial \bar{d}_i^{n,v}(x^{n,v},X^n)}{\partial x_k^{n,v}} - \frac{\kappa_{i(-)}^{n,v}}{\lambda^{n,v}}\frac{\partial \underline{d}_i^{n,v}(x^{n,v},X^n)}{\partial x_k^{n,v}}\right) = 0, \quad \forall k \in \Lambda^m, \tag{14}$$

This Eq. (14) makes evident that the optimal consumption of goods at the micro level is conditional on the consumption decisions at the macro level. Additionally, we derive micro-scale values of time as follows:

$$\frac{\mu^{n,v}}{\lambda^{n,v}} = \frac{\frac{\partial U_m^{n,v}}{\partial t_i^{n,v}}}{\lambda^{n,v}} + r_i^{n,v} + \frac{\kappa_{i(-)}^{n,v}}{\lambda^{n,v}} - \frac{\kappa_{i(+)}^{n,v}}{\lambda^{n,v}}; \quad i \in \Omega^m; v = 2,\ldots,V \tag{15}$$

which depends on the marginal valuation of the time allocated to activity *i*, the net income of the activity, and a factor that depends on the Lagrange multipliers of technological –upper and lower- constraints.

The calculation of the value of time implies the solutions of the system of Eqs. (14) and (15) plus the constraints set $F_m^{n,v}$, which implies that the value of time implicitly depends on an individual's macro utility valuation α]embedded in $X^n, S^{n,v}$ and τ^n, in (14) and (2)].

Note that our assumption that technological constraints depend on the specific agent's consumption of durables implies that $\kappa_{i(-)}^{n,v}, \kappa_{i(+)}^{n,v}$ may vary across agents. For example, the macro-scale decisions of residential and job locations define the minimum travel time. Thus, we conclude that it is likely to find individuals with identical conditions, i.e., the same wage rate r_i^n and the same valuation of the activity $\frac{1}{\lambda^{n,v}}\frac{\partial U_m^{n,v}}{\partial t_i^{n,v}}$ but with a different value of time as a short-term resource. Note also that from the complementary slackness conditions on the technological constraints we have the following:

$$\kappa_{i(-)}^{n,v}(t_i^{n,v} - \underline{d}_i^{n,v}(x^{n,v},X^n)) = 0, \kappa_{i(+)}^{n,v}(\bar{d}_i^{n,v}(x^{n,v},X^n) - t_i^{n,v}) = 0, \tag{16}$$

and $\kappa_{i(+)}^{n,v} \times \kappa_{i(-)}^{n,v} = 0, \forall i \in \Omega^m$.

The set of conditions of (16) yields two interesting cases.

Case (a) Lower-bounded time $t_i^{n,v} - \underline{d}_i^{n,v}(x^{n,v},X^n) = 0, \kappa_{i(+)}^{n,v} = 0, \kappa_{i(-)}^{n,v} > 0$

That is, the lower bound constraint for the time allocated to an activity is saturated in the micro-time window (*n,v*) indicating that the individual assigns the minimum time possible. As such, we obtain expression (17) for the individual's marginal willingness to pay for a marginal reduction in the time spent on activity *i:*

$$\frac{\kappa_{i(-)}^{n,v}}{\lambda^{n,v}} = \frac{\mu^{n,v}}{\lambda^{n,v}} - \frac{\frac{\partial U_m^{n,v}}{\partial t_i^{n,v}}}{\lambda^{n,v}} - r_i^{n,v} \tag{17}$$

Note that ${\kappa_{i(-)}}^{n,v} \neq 0,$ in the literature is interpreted as representing an unpleasant or mandatory activity because the time assigned is the minimum amount.

Case (b) Upper-bounded time $\bar{d}_i^{n,v}(x^{n,v},X^n) - t_i^{n,v} = 0, \kappa_{i(+)}^{n,v} > 0, \kappa_{i(-)}^{n,v} = 0.$

In this case, the upper bound constraint ($\bar{d}_i^{n,v}(x^{n,v},X^n) \geq t_i^{n,v}$) is active, thus

$$\frac{\kappa_{i(+)}^{n,v}}{\lambda^{n,v}} = \frac{\frac{\partial U_m^{n,v}}{\partial t_i^{n,v}}}{\lambda^{n,v}} + r_i^{n,v} - \frac{\mu^{n,v}}{\lambda^{n,v}} \tag{18}$$

which represents the individual's willingness to pay for a marginal increase in the time spent on the activity above the maximum capacity.

In the literature, leisure is defined as an unbounded activity ($\kappa_{i(-)}^{n,v} = 0$) . If we further assume that $\kappa_{i(+)}^{n,v} \geq 0$, then

$$\frac{\frac{\partial U_m^{n,v}}{\partial t_i^{n,v}}}{\lambda^{n,v}} \geq \frac{\mu^{n,v}}{\lambda^{n,v}} - r_i^{n,v}$$

which states that the marginal valuation of the individual's time allocated to a leisure activity can be higher than the cost per unit time ($r_i^{n,v} \leq 0$) plus the time value as a resource for the micro window (*n*, *v*) (see for example Jara-Díaz 2003).

The results (14)–(18) are consistent with those found by De Serpa (1971) and followers. The contribution of the hierarchical model is that values of time at the micro scale are dependent on the long-term choices made by the individual at $(n,1)$, including the location and the time allocated to home, work or study and the investments in durables such as a car. In this way, some results, descriptively introduced by Juster (1990), are formalized here with the hierarchical model that considers the influence of these durable and capital goods in the allocation and time valuation. Moreover, the micro-scale value of time changes within a macro-time window due to a change in money transfers caused by changes in exogenous prices and changes in the valuation of durables. This shows that the value of time is a dynamic and continuous measure, dependent on each individual's environment and can also vary according to the time point where it is computed, supporting the results of other works as Winston (1987).

The estimation of parameter for describing intertemporal dynamics of the time value as a resource as well as the willingness to pay for reducing the allocated time to activities requires an important econometric effort; besides, for the same purposes it is important to have an exhaustive collection of appropriate longitudinal data set related to consumers' choices, with long enough data to include observations of individuals making long term choices.

The specific model requires identifying the utility function, including the functional form, the set of variables and the set of parameters assumed varying along time and those parameters assumed static (if any). Additionally, the model specification requires defining the functional form describing the dynamics of preference' parameters. It is essential in our theory that micro utilities are specified dynamically dependent on long term choices; this effect may be represented by the impact of time and income budgets, but also on varying preferences parameters. In conclusion, the econometrics of this problem requires further research.

Case of the adjusting point

At the adjusting point (*n*, 1) macro and micro activities are optimized simultaneously. In this case the first-order conditions for the optimal micro-scale consumption $k \in \Lambda^m$ are as follows:

$$(1-\alpha)\frac{\frac{\partial U_m^{n,1}}{\partial x_k^{n,1}}}{\lambda^{n,1}} - p_k^{n,1} + \sum_{i \in \Omega^m}\left(\frac{\kappa_{i(+)}^{n,1}}{\lambda^{n,1}}\frac{\partial \bar{d}_i^{n,1}(x^{n,1},X^n)}{\partial x_k^{n,1}} - \frac{\kappa_{i(-)}^{n,1}}{\lambda^{n,1}}\frac{\partial \underline{d}_i^{n,1}(x^{n,1},X^n)}{\partial x_k^{n,1}}\right) = 0, \forall k \in \Lambda^m \tag{19}$$

where $\lambda^{n,1} = \alpha\frac{\partial V_M^{n,1}}{\partial I_C^{n,1}} + (1-\alpha)\frac{\partial V_m^{n,1}}{\partial I_C^{n,1}}$ and $I_C^{n,1} = I^{n,1} - C^{n,1}$.

And the first-order conditions for the optimal macro-scale consumption $k \in \Lambda^M$ are as follows:

$$\alpha\frac{1}{\lambda^{n,1}}\frac{\partial U_M^{n,1}}{\partial X_k^n}-P_k^{n,1}+\sum_{i\in\Omega^m}\left(\frac{\kappa_{i(+)}^{n,1}}{\lambda^{n,1}}\frac{\partial\bar{d}_i^{n,1}(x^{n,1},X^n)}{\partial X_k^n}-\frac{\kappa_{i(-)}^{n,1}}{\lambda^{n,1}}\frac{\partial\underline{d}_i^{n,1}(x^{n,1},X^n)}{\partial X_k^n}\right)$$
$$+\sum_{i\in\Omega^M}\left(\frac{K_{i(+)}^{n,1}}{\lambda^{n,1}}\frac{\partial\bar{D}_i^{n,1}(X^n)}{\partial X_k^n}-\frac{K_{i(-)}^{n,1}}{\lambda^{n,1}}\frac{\partial\underline{D}_i^{n,1}(X^n)}{\partial X_k^n}\right)=0, \tag{20}$$

where $K_{i(-)}^{n,1}$ and $K_{i(+)}^{n,1}$ are the Lagrange multipliers associated with minimum and maximum time allocated to macro activity i in time window $(n, 1)$. Equation (20) shows the marginal effect of macro-consumption decisions on short- and long-term choices.

The first order conditions associated with time allocated to micro activity are as follows:

$$\frac{\mu^{n,1}}{\lambda^{n,1}}=(1-\alpha)\frac{\frac{\partial U_m^{n,1}}{\partial t_i^{n,1}}}{\lambda^{n,1}}+r_i^{n,1}+\frac{\kappa_{i(-)}^{n,1}}{\lambda^{n,1}}-\frac{\kappa_{i(+)}^{n,1}}{\lambda^{n,1}};\ \forall i\in\Omega^m \tag{21}$$

Similarly, the first order conditions associated with time allocated to macro activity $i\in\Omega^M$ are as follows:

$$\frac{\mu^{n,1}}{\lambda^{n,1}}=\alpha\frac{\frac{\partial U_M^{n,1}}{\partial T_i^n}}{\lambda^{n,1}}+R_i^{n,1}+\frac{K_{i(-)}^{n,1}}{\lambda^{n,1}}-\frac{K_{i(+)}^{n,1}}{\lambda^{n,1}};\forall i\in\Omega^M \tag{22}$$

The Eqs. (21) and (22) are equivalent because if, at the adjusting point, macro and micro activities are simultaneously optimized, then money and time budgets are equal, implying that in this case the VTs are equal for both time scales. Hence, we define the long-term VT as not constrained by longer-term choices, which is revealed by the consumer's allocation of time at the adjusting point; i.e., when the optimal allocation of time of macro and micro activities is made. Moreover, we conclude that the short-term VT differs from the long-term value. This statement is valid in the context of the two examined time scales, micro and macro; the statement can be extended to more time scales, obtaining values of time for each scale. We also note that the long-term value of time (22) remains constant along $v>1$, while the short-term values of time change at every v.

The willingness to pay (WP) for reducing the time allocated to a macro activity is observed at the adjusting point $(n, 1)$ and given by the following:

$$WP_{i(-)}^{n,1}=\frac{K_{i(-)}^{n,1}}{\lambda^{n,1}}=\frac{\mu^{n,1}}{\lambda^{n,1}}-\alpha\frac{\left.\frac{\partial U_M^{n,1}}{\partial T_i^n}\right|_{T^{1*},X^{1*}}}{\lambda^{n,1}}-R_i^{n,1} \tag{23}$$

Note that although for all v, the allocations of time and consumption durables for long-term activities are fixed (at values, T^{1^*}, X^{1^*}), this willingness to pay $WP_{i(-)}^{n,1}$ can vary for $v>1$, according to (23), which can be calculated as follows:

$$WP_{i(-)}^{n,v}=\frac{\mu^{n,v}}{\lambda^{n,v}}-\alpha\frac{\left.\frac{\partial U_M^{n,v}}{\partial T_i^n}\right|_{T^{1*},X^{1*}}}{\lambda^{n,v}}-R_i^{n,v}. \tag{24}$$

Observe that $WP_{i(-)}^{n,v}\neq WP_{i(-)}^{n,1}$ is possible, because the value of time $\frac{\mu^{n,v}}{\lambda^{n,v}}$, the marginal utility yield by time and goods (T^{1*},X^{1*}) and income $(R_i^{n,v})$ may change along the macro-time period.

Similarly, the willingness to pay for increasing the time of a macro "leisure" activity (such as child care at home) in the micro-time window (n,v), $v>1$, is:

$$WP_{i(+)}^{n,v} = \alpha \frac{\left.\frac{\partial U_M^{n,v}}{\partial T_i^n}\right|_{T^{1*},X^{1*}}}{\lambda^{n,v}} + R_i^{n,v} - \frac{\mu^{n,v}}{\lambda^{n,v}} \tag{25}$$

Hence, the contribution of the hierarchical model to the VT theory is twofold: one contribution is the calculation of the time value, which, in contrast to the static model, depends on long-term choices. Our theoretical construct explains the origin of the differences observed in time values by characteristics such as gender, residence, age, which, according to our model, are caused by long-term decisions that condition the short-term choices where time values are observed. The second contribution is the calculation of the willingness to pay for specific activities, differentiating the activities by the time scale of the decisions and making the willingness to pay for short-term activities dependent on the long-term choices.

So far, we have assumed that the adjusting point is exogenous for the individual, i.e., the macro-time window is fixed. A different plausible assumption is that agents adjust macro decisions following changes in environmental and economic conditions, which could be introduced by an endogenous adjusting point based on a learning process with regards to the evolution of macro and micro utilities. For example, travel times and travel costs change over the macro-time window and can modify the work duration and the job location. Analogously to the literature on economic investments, immediately after the adjusting point, the endogenous model could assume that individuals incur costs associated with a change in durables (e.g., contracts); over time, these costs may increase or decrease, through the deterioration of durables or the capitalization of real estate, for example.

Hierarchical versus simultaneous decisions

We call a simultaneous microeconomic problem, or "*sim*", the classic static consumer problem, where all macro and micro variables are decided simultaneously without the associated adjustment costs. This choice process defines long-term optimal values for the consumption of goods and for the values of time in any micro period. Conversely, the hierarchical process only reveals long-term optimal choices and values at the adjusting point ($v = 1$), while in other time periods, choices define sub-optimal or short-term consumptions and time values. In this section, we analyze the differences between the hierarchical (HP) and simultaneous (SP) consumer problems assuming an exogenous adjusting point, particularly with regards to the sub-optimality associated with decisions made in the short term conditional on long-term choices. We also ignore adjustments costs.

The formulation of the simultaneous problem at any time period (n,v) is:

$$\max_{X,T,x,t} \left(\alpha U_M^{n,v}\left(X_k^{n,v}, T_i^{n,v}\right) + (1-\alpha) U_m^{n,v}\left(x_k^{n,v}, t_i^{n,v}\right)\right)$$

subject to

$$F_M^{n,v} \cup F_m^{n,v} \equiv \left\{ \begin{array}{c} R^{n,v}T^{n,v} - P^{n,v}X^{n,v} + I^{n,v} + r^{n,v}t^{n,v} - p^{n,v}x^{n,v} = 0 \\ \sum\limits_{i\in\Omega^M} T_i^{n,v} + \sum\limits_{i\in\Omega^m} t_i^{n,v} = \Delta \\ \underline{D}^n(X^{n,v}) \le T^{n,v} \le \bar{D}^{n,v}(X^{n,v}), \\ \underline{d}^{n,v}(x^{n,v}, X^{n,v}) \le t^{n,v} \le \bar{d}^{n,v}(x^{n,v}, X^{n,v}) \end{array} \right\} \tag{26}$$

And the corresponding indirect utility is $V_{sim}^{n,v} \equiv V_{sim}^{n,v}(\varepsilon^{n,v}, \Delta)$ where $\varepsilon^{n,v} \equiv (R^{n,v}, r^{n,v}\ P^{n,v}, p^{n,v}, I^{n,v}, \theta^{n,v}, \alpha)$, which is the complete set of macro and micro parameters without adjustment costs.

By definition the following condition holds for simultaneous and hierarchical optimization problems with an exogenous adjusting point:

$$V_{sim}^{n,v}(\varepsilon^{n,v}, \Delta) \geq \alpha V_M^{n,v}\left(\varepsilon_M^{n,1}, \varepsilon_m^{n,1}, \Delta\right) + (1-\alpha)V_m^{n,v}\left(\varepsilon_m^{n,v}, S^{n,v}, \tau^n, X^n\right), \tag{27}$$

where equality necessarily holds at the adjusting point ($v = 1$), while inequality is expected to hold (though not necessarily) for $v > 1$.

The optimal expenditures of money and time for long-term activities in the simultaneous problem with no adjustment costs are:

$$S_{sim}^{n,v} \equiv R^{n,v}T_{sim}^{n,v*} - P^{n,v}X_{sim}^{n,v*} + I^{n,v}, \tag{28}$$

$$\tau_{sim}^{n,v} \equiv \Delta - \sum_{i\varepsilon\Omega^M} T_{sim,i}^{n,v*}, \tag{29}$$

where $T_{sim}^{n,v*}$ and $X_{sim}^{n,v*}$ are the solutions of the macro variables in the simultaneous problem. Now we can define a gap function between optimal (simultaneous) and sub-optimal (hierarchical) micro-scale expenditures or transfers:

$$GS^{n,v} = S_{sim}^{n,v}\left(\varepsilon_M^{n,v}, \varepsilon_m^{n,v}, \Delta\right) - S^{n,v}\left(\varepsilon_M^{n,1}, \varepsilon_m^{n,1}, \Delta\right), \tag{30}$$

$$G\tau^{n,v} = \tau_{sim}^{n,v}\left(\varepsilon_M^{n,v}, \varepsilon_m^{n,v}, \Delta\right) - \tau^n\left(\varepsilon_M^{n,1}, \varepsilon_m^{n,1}, \Delta\right), \tag{31}$$

A suboptimal choice set is characterized by not null GS or $G\tau$.

These gaps allow us to identify differences between the long-term valuation of time and the value of time for short-term activities. These differences arise from comparing the results of the hierarchical model - where the time budget and technological constraints are given for $v > 1$ and are conditional on the long-term decisions - with the results of the simultaneous model where these constraints are global. We discuss two cases of income variability to show that the simultaneous problem may over- or underestimate values of time. We consider only one macro-time window; thus, we drop the index n.

Case 1: Fixed income and overestimates:

The individual assumes long-term commitments X and faces a fixed income defined as $I \equiv Y \cdot T_{work}^{min}$, where Y is the wage rate and T_{work}^{min} is the fixed working time assumed equal to minimum time. Additionally, all short-term decisions generate net costs ($r_i^v < 0$, for all $i \in \Omega^m$). Then:

$$r^v t^v - p^v x^v < 0.$$

In the hierarchical model $S^v = I^v - P^v X^v$ and $\tau^v = \Delta - T_{work}^{min}$.

Now we introduce shocks in macro-scale prices P^v, e.g., land rents causing higher expenditures for macro-scale commitments, assuming prices increase compared to the adjusting point (P^1), while micro-scale prices remain fixed. Then, consumption of macro-scale goods decreases in the simultaneous problem while remaining fixed in the hierarchical problem: $X_{sim}^v \leq X^1$ and $P^v X_{sim}^v \leq P^v X^1$. Therefore, $S_{sim}^v \geq S^v$, while in this case $T_{sim,work}^v = T_{work}^v$ and $\tau_{sim}^v = \tau^1$. That is, because the individual does not have the optimum transfer of money for micro-activities in periods $v > 1$, then the optimal micro consumption is different than the simultaneous consumption in every period. Additionally, marginal utilities of the micro problem $\lambda^v(S)$ and $\mu^v(\tau)$ are decreasing functions, (see Eqs.

(11) and (12)), which yields $\lambda^v(S^v_{sim}) \leq \lambda^v(S^v)$ and $\mu^v(\tau^v_{sim}) \geq \mu^v(\tau^1)$. This result implies that

$$VT^v_{sim} = \frac{\mu^v(\tau^v_{sim})}{\lambda^v(S^v_{sim})} \geq \frac{\mu^v(\tau^1)}{\lambda^v(S^v)} = VT^v$$

Thus, we conclude that in this case, the individual's long-term willingness to pay for extra time for micro activities is higher than the value observed in the short term and calculated by the hierarchical problem. However, consider that the behavior follows the micro VT; the VT^v_{sim} is a theoretical construct.

Case 2: Variable income and underestimates:

Individuals make long-term contracts about working time T^v, while the wage rate varies for $v > 1$. If $\frac{\partial U}{\partial T^v} < 0$ and, for example, the wage rate increases, then the long-term optimal working time is reduced along with wage rates, which is captured by the simultaneous problem, compared to the hierarchical or short-term problem (if $T^v > T^{min}$). In this case, $S^v = Y^v T^v$ and $\tau^v = \Delta - T^v$.

An increase of I^v with respect to (I^1) induces $T^v_{sim} \leq T^v$, then $S^v_{sim} \leq S^v$ and $\tau^v_{sim} \geq \tau^1$. That is, in the HP model the individual does not decide the optimum transfer of money because he/she is working longer than the SP optimal. Then,

$$VT^v_{sim} = \frac{\mu^v(\tau^v_{sim})}{\lambda^v(S^v_{sim})} \leq \frac{\mu^v(\tau^1)}{\lambda^v(S^v)} = VT^v$$

In this case we conclude that the long-term value of time is overestimated by observing short-term decisions (HP).

To analyze these cases we define the sub-optimality gap in the time values by the instantaneous long- versus short-term VT values at period v, calculated as:

$$\Delta VT^{n,v} = \left| \frac{\mu^{n,v}_{sim}}{\lambda^{n,v}_{sim}} - \frac{\mu^{n,v}}{\lambda^{n,v}} \right|. \quad (32)$$

Note that again, at each adjusting point the gaps are equal to zero because the simultaneous and hierarchical solutions are equal.

Numerical experiments

The objective of the following experiment is to give an illustrative example of the model showing the impacts of the hierarchical approach in values of time. The setting considers an extension to the proposed model by adding choices on discrete goods, which introduces realistic discontinuities in the choice process. We use simulation tests built on a hypothetical rational agent that maximizes utility in a deterministic choice process. The agent decides on macro-scale activities, including discrete choices on residential (l_{home}) and job (l_{work}) locations and continuous decisions on housing size (q_{home}) and time assigned to work (t_{work}). At the micro scale, he/she decides leisure activities, including free time at home (t_{home}), time assigned to shopping activities (t_{shop}), and transport modes. For each activity, the agent also decides on the consumption of goods (x). The simulation considers discrete sets of location choices for activities and transport mode options.

For simplicity, we assume one spatial dimension, which is a straight line of 25 km, where the individual decides on locations for activities. To consider the dynamic behavior of decisions in the long term, we use 40 temporal snapshots. Analytically, the following simultaneous (macro-micro) microeconomic consumer problem for the adjusting point associated with time windows $v = 1$, V, $2\ V, \ldots$ (in this notation, v is omitted for simplicity) extends the previous formulation presented in this paper by incorporating discrete mode and location choices, as well as time allocations:

$$\max_{l,m} \max U(t,x,q) = \alpha * (\ln(t_{work}+1) + ln(q_{home}+1)) + (1-\alpha) * [(\ln(t_{home}+1) + ln(t_{shop}+1) + \ln(x_{shop}+1) + \ln(x_{home}+1) - \ln(tt_m(l_{home}, l_{work})+1) - ln(tt_m(l_{home}, l_{shop})+1)] \tag{33}$$

subject to

$$y_{work}t_{work} - p \cdot x - r(l_{home})q_{home} - tc_m(l_{home}, l_{work}) - tc_m(l_{home}, l_{shop}) = 0, \tag{34}$$

$$t_{work} + t_{home} + t_{shop} + tt_m(l_{home}, l_{work}) + tt_m(l_{shop}, l_{home}) = 24, \tag{35}$$

and technological constraints

$$\boldsymbol{t_{work}} \geq tmin_{\boldsymbol{work}}, \tag{36}$$

$$t_{home} \geq \gamma x_{home}, \tag{37}$$

$$t_{shop} \geq \beta x_{shop}, \tag{38}$$

$$q_{home} \geq q_{min} \tag{39}$$

Here, a Cobb-Douglas utility, with exponent for the macro-micro substitution effect, is maximized. The solution identifies the optimal bundles of the activities' duration times (t), goods (x), housing size (q), locations (l) and travel times by mode m (tt_m). Utility variables ($t,x,q,l,\ tt$) are deviated in one unit to avoid taking the logarithm of zero. The income budget (34) balances the working income and expenditures on goods, residential rents and travel costs; housing prices, denoted r, are per size unit and depend on the location. Additionally, there is a set of technological constraints. Equation (36) indicates a minimum time for labor supply, either due to legal restrictions or representing features inherent to the specific type of job. Constraints (37) and (38) limit the minimum time allocated to home and shopping regarding the level of goods consumption. Finally, there is a survival constraint on the minimum amount of residential housing (39).

This setting defines discrete-continuous optimization problems, and the decision variables are defined per time snapshots (e.g., per day). Regarding the discrete variables, the long-term choices are home and job locations, while short-term choices are transport mode and shopping location. Note that for simplicity, and to reduce the computational effort, we evaluate specific discrete options for the location choices of activities $(l_{home}, l_{shop}, l_{work})$ along the linear city. With regard to the continuous decision variables, in the long term they are t_{work} and q_{home} in the short term they are: t_{home}, t_{shop}, x_{shop} and x_{home}. In addition, travel times (tt) and travel costs (tc) depend on the location choices with the optional transport modes Bus ($m = 1$) and Car ($m = 2$).

To explain the concepts of $S^v$and τ for this particular numerical example, we assume for simplicity that the adjusting point is $v = 1$; then, the macro transfers of money (S^v) and time (τ) for a time window v that is different from the adjusting point $(v > 1)$ can be computed:

$$S^v = y^v_{work} t^1_{work} - r^v\left(l^1_{home}\right) q^1_{home}$$

$$\tau = 24 - t^1_{work}$$

That is to say, the available money to make short-term activities is calculated as the income (obtained by the wage rate at v and the time allocated to work, which is fixed at adjusting point $v = 1$) minus the rent associated with housing during each period v, where residential location l^1_{home} and the housing size q^1_{home} are fixed in $v = 1$.

On the other hand, the available time for travel and short-term activities depends directly on the time allocated to work that is previously determined at the adjusting point ($v = 1$).

Thus, the income and time budgets for each micro consumer problem ($v > 1$) are mathematically represented by:

$$p^v \cdot x + tc_m\left(l^1_{home}, l^1_{work}\right) + tc_m\left(l^1_{home}, l^v_{shop}\right) = S^v$$

$$t^v_{home} + t^v_{shop} + tt_m\left(l^1_{home}, l^1_{work}\right) + tt_m\left(l^v_{shop}, l^1_{home}\right) = \tau$$

Costs and travel time are micro variables that depend on the mode (m) and location of shopping activities (l^v_{shop}). Note that for $v \neq 1$, technological constraints (36) and (39) are not used because they correspond to macro variables which are fixed in v.

On the other hand, the variable micro utility for $v > 1$ is

$$\ln\left(t^v_{home} + 1\right) + ln\left(t^v_{shop} + 1\right) + \ln\left(x^v_{shop} + 1\right) + \ln\left(x^v_{home} + 1\right)$$
$$- \ln\left(tt_m\left(l^1_{home}, l^1_{work}\right) + 1\right) - ln\left(tt_m\left(l^1_{home}, l^v_{shop}\right) + 1\right)$$

Next, we use this experimental design to show how exogenous shocks in the economy and macro-scale choices modify the computed values of time and time allocation.

Example 1: Wage rate shock

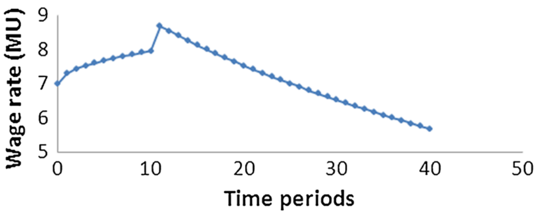
In this first example, for simplicity we assume a value of $\alpha = 0.5$, and job and residential location are fixed. In addition, we assume that only the wage rate varies along time according to Fig. 1 and described by the following function:

$$y^v_{work} = \begin{cases} 7 + 0.3\sqrt{v}; 1 \le v \le 10 \\ 12 - \sqrt{v}; 11 \le v \le 40 \end{cases}$$

At time snapshot 10 there is a sudden economic shock, changing the dynamics of wages from increasing to decreasing along time. We solve three different problems: two hierarchical problems (HP), one with adjusting points at $v = 9$ and $v = 18$ (denoted HP-9) and another one at $v = 15$ (HP-15); we also solve the simultaneous problem (SP). In Fig. 2 we show the computed variation of time values for the three cases.

The dynamics of the value of time (VT) in the simultaneous problem (SP) follow the changes assumed for the wage rate because this value is the only factor changing over time in the model. While the wage rate increases in the first period before $v = 9$, the VT obtained from the hierarchical model overestimates the long-term VT given by the SP model because in the HP model the individual maintains longer working hours (according to low wage rates in $v = 1$ when the working time was decided on), while in the SP model

Fig. 1 Changes in wage rates over time (in monetary units MU)



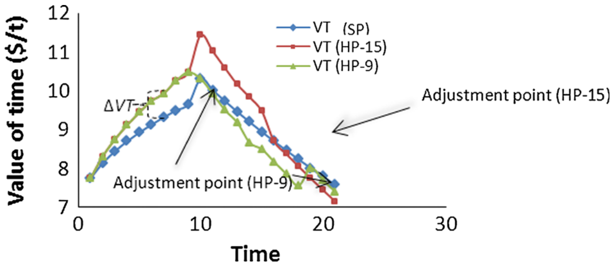


Fig. 2 Value of time for Example 1: simultaneous versus hierarchical models

working hours are reduced as income increases: thus, the time constraint is relaxed, reducing VT. In the second period, between $v = 9$ and $v = 15$, the wage rate decreases and we observe that the HP-9 model adjusts VT underestimating the SP values; conversely, the HP-15 model keeps overestimating VT values until its adjusting point at $v = 15$.

In addition, in the figure, we show the simultaneous versus hierarchical gap (ΔVT), indicating sub-optimality in the VT of the hierarchical problems. Note that the VT obtained by the SP problem represents the value associated with the hypothetical behavior where the agent adjusts long-term activities at every point in time, while the HP model calculates the VT consistent with the slow motion of long-term decisions.

Below in Fig. 3, we show the allocation of time to work obtained from these models. Because the marginal valuation of labor is assumed to be negative, and because the wage rate increases during the first macro-time window, in the SP model, the individual decides to work less time than in the HP model, adjusting income to expenditures and time to increase leisure. Conversely, if the wage rate is reduced, then he/she works more time.

Finally, if $\alpha < 0.5$, then the time assigned to work is reduced not only in the simultaneous problem but also in the hierarchical models, which is reflected in more time available for micro activities. From this result, t_{home} and t_{shop} increase, although the income decreases at each temporal snapshot; thus, the individual has more time but less budget available for short-term consumption, which means that the value of time is reduced in all time windows, obtaining parallel curves below the time valuations in Fig. 2. In the case of $\alpha > 0.5$, the valuation of time is larger with respect to Fig. 2 because the income and work time increase, which is reflected in less time available for micro activities.

Example 2: Residential location, value of time and transport mode choice

In this case, we consider an individual whose job is located at point 0, while assuming that the long-term residential location choice varies in the range [0,25]. We assume that bus travel costs are fixed and car travel costs vary with distance, as follows:

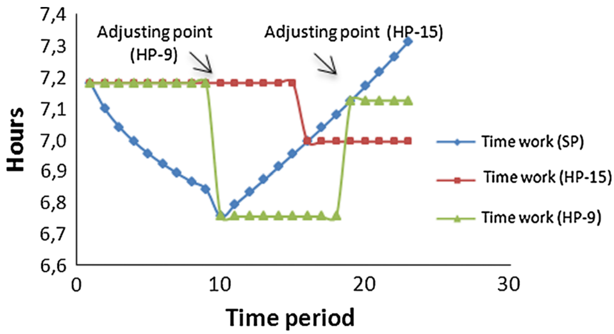


Fig. 3 Time allocated to work for Example 1: simultaneous and hierarchical problems

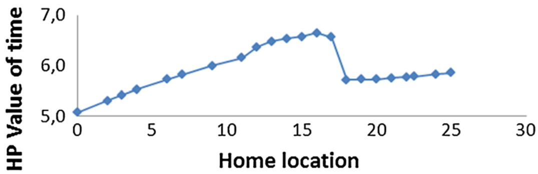


Fig. 4 Residential location versus value of time and transport mode

$$tc_{bus}(l_{home}, l_{work}) = tc_{bus}\left(l_{home}, l_{shop}\right) = 10$$

$$tc_{car}(l_{home}, l_{work}) = 2 * |l_{home} - l_{work}|, tc_{car}\left(l_{home}, l_{shop}\right) = 2 * \left|l_{home} - l_{shop}\right|$$

travel times are also proportional to distance as follows:

$$tt_m(l_{home}, l_{work}) = \frac{1}{a}|l_{home} - l_{work}|, tt_m\left(l_{home}, l_{shop}\right) = \frac{1}{a}\left|l_{home} - l_{shop}\right| ,$$

$$\text{where } a = \begin{cases} 2 \, if \; m = BUS \\ 4 \, if \; m = CAR \end{cases}$$

In this example, for any individual who chooses a residential location between 0 and 17 the mode CAR is more convenient; beyond that limit the high travel costs by car make BUS the optimal choice. The large drop observed in the hierarchical value of time shown in Fig. 4 at 17 is the result of switching the transport mode, which is compensated by increasing the housing size and rents, hence reducing the money transfer to micro-scale activities. Then, the observed micro-scale behavior of those living beyond reflects the lower disposable income implying a lower value of time. The most important result of this example is the clear visualization of the influence of the location of long-term activities on the values of time.

Conclusions

The paper presents an extension of the classical theory of time allocation, introducing a feature observed in several dynamic systems: the temporal hierarchy of choice processes. The model developed considers two levels, the macro- and micro-temporal levels, to describe the process of the individual's allocation of time and wealth to long- and short-term activities. No other dynamic feature was included; except for this extension, the model setting is identical, and thus comparable with the well-known one-level-simultaneous approach. We do not identify relevant shortcomings caused from choosing only two levels; in fact, we claim that the simple macro-micro framework is the nucleus of more complex hierarchies because it can represent the most-relevant effects of moving from one to multiple levels.

Despite the simplicity of the model, our analysis yields some relevant conclusions that contribute to the theory of the value and allocation of time. First, the simultaneous and the new approaches are consistent because both collapse to the same model once we assume that all choices are made and adjusted simultaneously; that is, there is no hierarchy in the process of deciding on activities and in this sense, the hierarchical model is a generalization of the former one. Second, the hierarchical model explains why estimated values of time from real data differ significantly according to socioeconomic characteristics, such as gender, residence and job locations, availability of durables (e.g., car, house), and education level. For all these conditions, our model provides a unique answer: the value of time observed is yielded by short-term activities but determined by different long-term choices. This finding implies that two identical persons, clones in everything except for having made a different choice in at least one long-term choice, are expected to have different time values. Third, we conclude that observed values of time change over time for the same individual and that usually, these values represent short-term values (conditional on long-term choices), except for at the adjusting point, where long-term preferences are revealed.

Therefore, an important concept in the hierarchical model is the adjusting point, when the macro-time window elapses and long-term choices are adjusted to initiate the next macro-time window. The adjusting point is considered exogenous to the individual's decisions, an assumption which may be extended in future work to make this point endogenous.

One major feature worth mentioning regarding the proposed formulation, is that the hierarchical model considers deterministic parameters together with a myopic behavior premise, i.e. decisions are made only on the current state of the economy in each time window (n, v). However, the results associated with the dependencies between long and short term decisions are extensible to other contexts in decision-making. For example, if at the adjusting point (n, 1) individuals make long-term decisions with parameters associated with their rational expectations ("perfect foresight") then, both long-term variables as well $S^{n,v}$ and τ^n will be conditional on the individual's expectation on economic state, all the same, the results of conditionality of long-term decisions (macro) over short-term decisions (micro), remain the same.

The proposed hierarchical model also illuminates an arising question regarding rigorous economic assessments of transport projects. Indeed, we have concluded that there is no unique value of time for a given person. In fact, there are two (long and short-term) values in the simple macro-micro model (and multiple in a multi-time-scales model), and both of them have a dynamic behavior, changing along time according to external economic conditions. Then, the question arises on how to evaluate time saving yield by transport projects lasting for a long time horizon. Of course, if the dynamics of external economic conditions are known, we can simulate short-term and long-term activities of each individual and assess the associated values of time over time. Assuming we can accurately make such calculations, should we use the short-term or long-term dynamics of time values along the time horizon? We know, from our examples above, that short-term values can either over- or underestimate the long-term values, depending on whether the individual reduces or increases income along the macro-time window. We argue that the right value of time to use for project appraisals is the short-term value of time, regardless of the gap with long-term values. This argument is supported by the fact that individuals perceive short-term values; the long-term value of time is a theoretical construct affected by the slow motion of long-term decisions, whose associated benefit are potential but not realized due to unavoidable long-term commitments and adjustment costs.

Thus, the most general conclusion is that the theory of time allocation is enriched by the hierarchical structure of activities; conversely, the assumption of one temporal level obscures important features that explain consumers' behavior with relevant implications by modeling travel demand and time allocation. The main contribution of the hierarchical approach is that it makes clear how the behavior at the micro level is conditional on the decisions made at the macro level. This finding has an impact on the methodologies to estimate demand and on the understanding of the formation of the time value. Some shortcomings of the traditional one-level model can be partially overcome by clustering the population with regards to certain long-term decisions, such as car ownership and income level, based on intuitive arguments. Our model provides theoretical support to such intuitions; however, it also provides a theoretical framework to identify a complete set of variables to define the population clusters. More importantly, our results show that clustering can be avoided, at least partially, by specifying the indirect utility function at the micro level explicitly dependent on the macro-level choices. The benefit of this approach is the reduction of parameters in the estimation of the demand model, which reduces the number of observations required to obtain a given precision on the parameter estimates.

Practical studies of aggregate levels of population and their choices only require average time values; in that case the contribution of the hierarchical model may be considered irrelevant. However, the more disaggregated the study of individuals' behavior, the more relevant the understanding of the variability of these values across individuals. Additionally, average values are composed of individuals' values and the population distribution; hence the evolution of these values over time is better understood (thus better predicted) if the diversity of values and the evolution of the population are explicit in the model.

The theory of the value and allocation of time is complemented by the hierarchical model. This model explains how macro-level activities and their value of time depends on the expected effect of other decisions at the same macro level and at the micro level (what we call the adjusting point), while decisions at the micro level are dependent on the resources allocated to activities at the macro level. Thus, the estimate of the value of time inferred from consumers' behavior depends on the type of activity and whether the activity is decided at a macro or micro level.

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