Electrical Power and Energy Systems 73 (2015) 725–733

Contents lists available at ScienceDirect

Electrical Power and Energy Systems

journal homepage: www.elsevier.com/locate/ijepes

Stability region and radius in electric power systems under sustained random perturbations

Humberto Verdejo [a,*], Wolfgang Kliemann [b], Luis Vargas [c], Cristhian Becker [a]

[a] Departamento de Ingeniería Eléctrica, Universidad de Santiago de Chile, Santiago, Chile
[b] Department of Mathematics, Iowa State University, Ames, IA 50011, USA
[c] Departamento de Ingeniería Eléctrica, Universidad de Chile, Santiago, Chile

ARTICLE INFO

Article history:
Received 21 October 2014
Received in revised form 14 May 2015
Accepted 3 June 2015
Available online 17 June 2015

Keywords:
Power systems
Power system stability
Stochastic systems

ABSTRACT

Two concepts are proposed to characterize the behavior of stochastic systems under sustained random perturbations in time: Using Lyapunov exponents we define the region where an electric power system can be operated under random perturbations without losing stability; and we characterize the maximum perturbation size that a system can sustain. The proposed methodology is applied to international test systems of nine and thirty-nine buses.

Introduction

In their daily operation, electric power systems are subjected to a variety of random perturbations sustained in time, due to the dynamic behavior of consumption, temperature changes in the wires, errors in the measuring instruments, changes in the network's topology, etc. Therefore, the randomness is present at all times, and it is necessary to represent it as faithfully as possible to capture the stochastic behavior of real systems.

Traditionally, there have been attempts from probabilistic theory to analyze the stochastic dynamics of electric systems, orienting the study to the analysis of contingencies and safety, see [1], where the objective consists in assigning an occurrence probability to a set of predefined events. Then, the probability that the system will be stable is estimated from the probability distributions of the elements that represent the random behavior.

In the context of dynamic stability, Refs. [2–4] analyze small signal stability, assigning a probability value to the occurrence of certain events. In [5] it is considered that consumption varies permanently in time, and an index is presented that allows the determination of the vulnerability of a system in studies of voltage collapse from the time at which the system abandons the stability region.

With respect to the probabilistic analysis of stability of small perturbations, Refs. [6–12] show important advances in this area, but the random effect is considered according to a stepwise type of event, and the sustained variation in time is not considered. To account for the above, Ref. [13] shows a theoretical development based on Lyapunov exponents that allow the characterization of the random phenomenon in electric power systems. However, no numerical methods are presented for implementation in real systems. In [14] numerical methods are reported to evaluate stability in mechanical systems by means of Lyapunov exponents, but the results shown cannot be extrapolated to large systems such as electric power systems.

In the context of the model of random variations sustained in time, white noise or Brownian motion, see [15], has been used to represent the stochastic dynamics of electric systems. However, this process is adequate for applications at the microscopic level, and it is not a correct approximation to represent the macroscopic phenomena existing in electric networks.

The present paper models the random perturbations sustained in time which affect electric power systems, according to a particular stochastic process reported in [16]. It also proposes to use Lyapunov exponents and the gains of the PSS controllers, to characterize the stability of electric systems, defining the stability region and stability radius of a system subjected to random perturbations sustained in time.

The proposed methodology is applied to two IEEE test systems: the three generator – nine bus and the ten generator – thirty-nine bus systems. The rest of the paper is organized as follows: Section 'Literature review' presents the mathematical model of linear stochastic systems and the concept of Lyapunov exponents. Sec-

* Corresponding author. Tel.: +56 9 2 718 33 10.
E-mail address: humberto.verdejo@usach.cl (H. Verdejo).

http://dx.doi.org/10.1016/j.ijepes.2015.06.012
0142-0615/

tion ‘Model of the system’ introduces two indicators in order to characterize random perturbations in power system operation and presents a methodology for numerical estimation. Finally, in Section ‘Methods’ the proposed methodology is applied to two examples of multimachine power systems, highlighting the potential applications of the presented concepts.

Literature review

Different authors have made valuable proposals that allow analyzing the small signal stability of an electric power system subjected to random perturbations self-sustained over time, [13,14] and others. However, the fact is that the reported novel and important methods are not directly applicable to international testing systems, mainly because of the large number of dynamic variables that represent the systems, and therefore the simulation of those techniques presents numerical disadvantages. The work reported in [15] shows important advances in this aspect, but the application has been focused on the analysis of the stability of mechanical structures.

Ref. [18] shows a method for tuning controlling parameters in very large electric power systems, considering a stochastic approach. The main objective of this work is to evaluate the system’s response from the definition of performance indicators, considering that the perturbation that affects the system’s dynamics is represented by means of an additive model self-sustained over time. The purpose is to evaluate the impact of the gains of the controllers of the machines on the cost of the energy losses under permanent regime, and in this way determine a better fit of the parameters when required.

The work reported in [19] shows the results of analyzing the small signal stability of electric systems subjected to multiplicative stochastic perturbations through the calculation of Lyapunov exponents. Three numerical methods are shown that allow determining a single Lyapunov exponent that allows generalizing the analysis of classical deterministic eigenvalues.

Ref. [23] uses the Lyapunov exponent to define stability radii in electric systems subjected to random perturbations self-sustained over time, using the numerical methods reported in [19]. This work shows a methodology that allows determining the maximum perturbation size that a system can resist without losing stability. However, the analysis is made on a test system that considers a generator connected to an infinite busbar.

Ref. [26] uses the methods reported in [19] to define performance indicators in linear stochastic systems subjected to random perturbations that are represented by a multiplicative model.

The present paper follows the guidelines of previous papers. A method is shown that allows determining stability radii and regions in multimachine electric systems subjected to random perturbations self-sustained over time. The perturbations are represented by means of a multiplicative model in which the stability radii and regions are determined from the calculation of Lyapunov exponents.

Model of the system

Basic concepts

A system of linear differential equations, with constant coefficient matrix, can be written in the form

$$\Delta\dot{x} = A\Delta x \quad \text{in } \mathbb{R}^d. \tag{1}$$

To analyze the stability of the linear system (1) it is necessary to determine the real parts of the eigenvalues of the matrix A. The system will be asymptotically and exponentially stable if and only if all the real parts of the eigenvalues are negative. However, this result is not valid for systems that vary in time as follows (see [20])

$$\Delta\dot{x} = A(t)\Delta x \quad \text{in } \mathbb{R}^d. \tag{2}$$

In this context, it becomes necessary to consider a different approach to stability studies, and the theory of Lyapunov exponents allows this problem to be solved.

Let us consider a linear system in which the variation is stochastic and is sustained in time

$$\Delta\dot{x} = A(\xi_t)\Delta x \quad \text{in } \mathbb{R}^d, \tag{3}$$

where ξ_t represents the random and time-varying effect, by means of a Markov-type stochastic process. If we denote the solution of (3), for an initial condition $x_0 \in \mathbb{R}^d$, by $\varphi(t, x_0, \xi_t)$, then the exponential growth behavior of the linear system is given by the Lyapunov exponents

$$\lambda(x_0, \omega) = \limsup_{t\to\infty} \frac{1}{t} \log \|\varphi(t, x_0, \xi_t(\omega))\|. \tag{4}$$

In this case, ω is an element of the probability space on which the differential stochastic Eq. (3) is defined. Note that the trajectory $\varphi(t, x_0, \xi_t(\omega))$ is (exponentially) stable if and only if its Lyapunov exponent satisfies $\lambda(x_0, \omega) < 0$. In general, the stochastic linear system (3), with ergodic perturbation, will have up to d Lyapunov exponents.

Model of the stochastic perturbation

To model the perturbation ξ_t, use is made of the results of Refs. [16–18], where it was shown that the Ornstein–Uhlenbeck process can be used to represent random phenomena present in electric power systems. Considering that in general those perturbations are restricted in size, the model used here consists of

$$\xi_t^\rho = \rho \cdot \sin(\eta_t), \quad \rho \geqslant 0, \tag{5}$$

where

- η_t is a stationary solution of the stochastic differential equation known as Ornstein–Uhlenbeck equation

  $$d\eta_t = -\alpha\eta_t dt + \beta dW_t \quad \text{in } \mathbb{R}^1. \tag{6}$$

  Here W_t denotes the standard 1-dimensional Wiener process. The parameters α and β must be estimated from real measurements of the phenomenon that one wants to model. In this paper we use $\alpha = \beta = 1$, a particular case of the perturbation model reported in Ref. [16].

- ρ is a parameter that models the amplitude of the effect of the perturbation, i.e. for $\rho = 0$ we have the unperturbed system (1).
- W_t denotes Brownian Motion.

Uniqueness of the Lyapunov exponent

Let us consider the stochastic linear system (3) with perturbation given by Eqs. (5) and (6), under the conditions reported in Ref. [21]. Then the system has a unique Lyapunov exponent for each perturbation size $\rho > 0$, given by

$$\lambda(\rho) = \limsup_{t\to\infty} \frac{1}{t} \log \left\|\varphi(t, x_0, \xi_t^\rho(\omega))\right\|, \tag{7}$$

for every initial condition $x_0 \in \mathbb{R}^d \setminus \{0\}$, with probability 1 (almost surely). This means, in particular, that the system (3) is asymptotically (and exponentially) stable with probability 1 for the perturbation of size ρ if and only if $\lambda(\rho) < 0$.

Computation of the Lyapunov exponent

To compute numerically the Lyapunov exponent given by Eq. (7), we set a time interval $[0, T]$, and consider n initial conditions $\{x_0^i, i = 1, \ldots, n\}$ together with m realizations of the noise $\{\xi_t^j, t \in [0, T], j = 1, \ldots, m\}$. For every trajectory of the noise and for every initial condition we get an approximation of the Lyapunov exponent given by

$$\lambda^j(i) := \frac{1}{T} \log \left\| \varphi(T, x_0^i, \xi_t^j) \right\|. \tag{8}$$

We are using a second-order scheme to solve Eq. (6), and a fourth-order Runge–Kutta scheme to solve (3). Averaging Eq. (8) over the number of initial conditions and of realizations of the process, we get a numerical approximation of the Lyapunov exponent as

$$\lambda \approx \frac{1}{n \cdot m} \sum_{i=1}^{n} \sum_{j=1}^{m} \lambda^j(i). \tag{9}$$

For further details on the numerical method used see [19].

Model of the electric power system subject to sustained stochastic perturbations

An Electric Power System can be described by a set of differential algebraic equations (DAE) in the form:

$$\begin{aligned} \dot{x} &= f(x, y) \\ 0 &= g(x, y), \end{aligned} \tag{10}$$

where

- $f(x, y)$ represents differential equations: machines and controllers
- $g(x, y)$ represents algebraic equations: power flow and network equations

In order to obtain the equivalente linear system, we use the traditional approach given by [22]:

$$\Delta\dot{x} = A\Delta x + B\Delta y \tag{11}$$

$$0 = C\Delta x + D\Delta y, \tag{12}$$

where A, B, C and D are matrices defined in [22]. From a determinist point of view, all components in these elements are constant. The classical linear stability criteria consider compute eigenvalues from (13):

$$\Delta\dot{x} = (A - B \cdot D^{-1} \cdot C)\Delta x = A_{sys}\Delta x. \tag{13}$$

For a fixed steady state operation point matrix A_{sys} is constant and we have three options:

- When the eigenvalues have negative real parts, the original system (10) is asymptotically stable
- When at least one of the eigenvalues has a positive real part, the original system (10) is unstable
- When the eigenvalues have real parts equal to zero, it is not possible to say anything in the general

If we consider that any of matrices defined in (13) depends on time, the classical approach it is not useful. In order to analyze EPS under sustained random perturbations we will use Lyapunov exponents.

In the present paper it is considered that the perturbations affect the dynamics of the generating units. In particular, we have that the currents in the $d - q$ axes and the voltage of the excitation system vary according to the form

$$\begin{aligned} I_{d0} &= I_{d0}^{ss} \cdot (1 + \rho \cdot \sin(\xi_t)) \\ I_{q0} &= I_{q0}^{ss} \cdot (1 + \rho \cdot \sin(\xi_t)) \\ E_{fd0} &= E_{fd0}^{ss} \cdot (1 + \rho \cdot \sin(\xi_t)), \end{aligned} \tag{14}$$

where

- I_{d0}^{ss} and I_{d0}^{ss} are the currents in the permanent regime on the $d - q$ axes,
- E_{fd0}^{ss} is the voltage of the excitation system in the permanent regime,
- ρ is the size of the perturbation.

According to conditions defined in (14), the equivalent system can be described by a stochastic linear system in the form of (3).

Methods

For small perturbation stability studies, an electric system in a deterministic environment can be described according to Eq. (1) (see [22]). Considering that there are random variations sustained in time, around a stable operation point the electric system is described in the form of Eq. (3). In this context the stability analysis by means of the calculation of eigenvalues is not valid, since the system is variant in time. Using Lyapunov exponents as in (7) one obtains the following criterion for stability: The randomly perturbed system (3) is asymptotically (and exponentially) almost surely stable if and only if $\lambda < 0$, see, e.g., [20] or [21].

The Lyapunov exponent as a function of the parameters of the PSS controllers

In the operation of electric systems one of the applications of PSS (Power System Stabilizer) controllers has been to improve the permanent regime stability. This is achieved by introducing damping into the system, causing the real parts of the critical eigenvalues to be as far as possible from the origin. Given that the parameters of the PSS controllers have direct influence on the location of the real parts, and that the (unique) Lyapunov exponent expresses the largest exponential growth rate of the system, we expect that the tuning of the PSS will have an influence on the value of the Lyapunov exponent.

Considering that the system's stability will depend on the variation sustained in time and on the parameters of the PSS controllers, we have that the dynamic behavior will be given by

$$\Delta\dot{x} = A(\xi_t, K)\Delta x \quad \text{in } \mathbb{R}^d, \tag{15}$$

where

- ξ_t is the perturbation that characterizes the random behavior sustained in time, and
- K is the set of parameters that defines the PSS controllers. If the system has n PSS controllers, and if the gain of each PSS can be adjusted, then for the dimension of the parameter we have $K \in \mathbb{R}^n$.

To analyze the stability of system (15), the Lyapunov exponent will be determined numerically. In the present paper this concept is used to present the ideas of stability region and stability radius.

Stability region

Since the gains of the PSS controllers are expected to be the parameters that have a direct influence on the exponential growth behavior of the system, the stability region is determined by calculating the Lyapunov exponents for different combinations of gains and perturbation size. The gains are obtained by varying around the tuning obtained based on traditional techniques. As a function of the above, the following parameters are, therefore, considered (for $n = 1$)

$$K_{ij} = (K_{PSS_i}, \rho_j, \lambda_{ij}) \quad \text{with} \tag{16}$$
$$K_{PSS_i} = k_i \cdot K_{PSS_{nom}}, \quad k_i \in [k_{\min}, k_{\max}]$$
$$\rho_j \in [\rho_{\min}, \rho_{\max}],$$

where

- $K_{PSS_{nom}}$: nominal gain of the controller obtained from traditional tuning techniques,
- k_i: factor that modifies the nominal gain of the controller,
- $k_{\min}$: minimum value that modifies the nominal gain of the controller,
- $k_{\max}$: maximum value that modifies the nominal gain of the controller,
- ρ_j: perturbation size,
- λ_{ij}: Lyapunov exponent for gain K_{PSS_i} and size of the noise ρ_j.

Once the Lyapunov exponents are computed numerically, a surface in $\mathbb{R}^{n+2}$ is obtained using the values $\{(k_i, \rho_j, \lambda_{ij}), k_i \in [k_{\min}, k_{\max}]^n, \rho_j \in [\rho_{\min}, \rho_{\max}]\}$. Drawing the level surfaces for different values of the Lyapunov exponent, we obtain the stability regions of (15). In particular, the region under the surface for $\lambda = 0$ in $k - \rho$-space is the (asymptotic and exponential) stability region of the system, see the illustration in Fig. 1 for $n = 1$. This region consists of combinations of $(k_i, \rho_j) \in [k_{\min}, k_{\max}] \times [\rho_{\min}, \rho_{\max}]$ that lead to stable operating conditions despite sustained random perturbations of size ρ_j.

Stability radius

Theoretically, the stability radius of a linear stochastic system (3) is defined as $r := \inf\{\rho \geqslant 0, \lambda(\rho) > 0\}$, compare [23]. If the system depends on a parameter $K \in \mathbb{R}^n$ as in (15), one considers the stability radius

$$r^K := \inf\{\rho \geqslant 0, \lambda^K(\rho) > 0\},$$

for each parameter value K. Here $\lambda^K(\rho)$ denotes the Lyapunov exponent of the system (15) for each K. If the goal of the system design is to maximize the size of a random perturbation that the system can sustain without becoming unstable, then we are looking for

$$r_{\max} := \max\{r^K, K \in \mathbb{R}^n\}, \tag{17}$$

and we set the controller gains to the values K^* of the parameter $K \in \mathbb{R}^n$ at which this maximum is attained. In practice, the number of machines, and hence the dimension n of the parameter space can be relatively large, so that visualization and intuition about stability

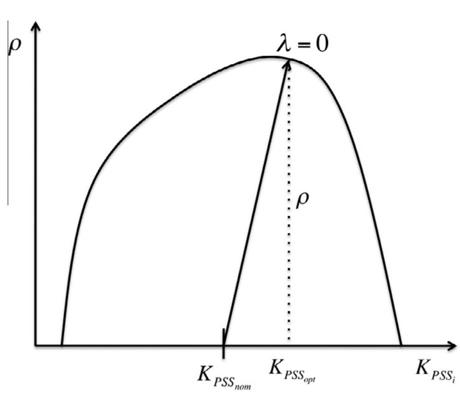

Fig. 1. Stability region and radius.

radii can be difficult to achieve. For this reason, this paper presents an approximation of the maximal stability radius as follows.

Once the stability region has been obtained numerically, it is possible to define the (numerical) stability radius for each value K_{PSS_i} of the PSS gains as

$$r_i := \inf\{\rho_j \geqslant 0, \lambda_{ij} > 0\}.$$

In other words, for each value of the system parameter $K = k_i$ we obtain one stability radius r_i. In the illustration of Fig. 1 this maximal stability radius occurs for a value of $K_{PSS_i} = 1.4 \cdot K_{PSS_{nom}}$.

In practice, stability region and stability radius can be estimated by varying the parameters of a single PSS controller, keeping the others fixed. We define for PSS controller $l \in \{1, \dots, n\}$

$$r_{\max}(l) := \max\{r_i, k_i^l \in [k_{\min}, k_{\max}]\}$$
$$\text{where } k_i^l \text{ varies only for the } l\text{th controller.} \tag{18}$$

In other words, $r_{\max}(l)$ is the (computed) optimal stability radius if only the lth PSS controller is used, and the corresponding gain value k^{l*} achieves this optimal radius. To account for parameter settings at all n controllers, we approximate the overall optimal stability radius via the 'critical stability radius' $r_{\max_{global}}$ as

$$r_{\max_{global}} := \min\{r_{\max}(l), l = 1, \dots, n\}. \tag{19}$$

Intuitively, Eq. (19) gives the maximum perturbation size that the system can support when it is in the critical condition $\lambda_{ij} \approx 0$ for each PSS controller, using the gain values $(k^{1*}, \dots, k^{n*})$ obtained in (18).

Application to electric power systems

The following test systems were studied as applications

- System I: Three-machines nine-bus system described in [24]
- System II: Ten-machines thirty-nine bus system described in [25]

To get the nominal gain parameter values for Eq. (16) we use the values reported in Refs. [24,25]. Below we present the stability regions and stability radii obtained by applying the proposed methodology to the test systems considered.

Three machine – nine bus system

Adjustment of the controllers gains

As reported in [24], the nominal gains obtained for the controllers are the following

$$K^1_{PSS_{nom}} = 50, \; K^2_{PSS_{nom}} = 5, \; K^3_{PSS_{nom}} = 3.$$

Three cases are considered in this paper:

- Case A: vary K^1_{PSS} via $K^1_{PSS_i} = k_i \cdot K^1_{PSS_{nom}}$ with $k_i \in [0, 12]$, keeping $K^2_{PSS_{nom}}$ and $K^3_{PSS_{nom}}$ fixed, and consider $\rho \in [0, 1]$,
- Case B: vary K^2_{PSS} via $K^2_{PSS_i} = k_i \cdot K^2_{PSS_{nom}}$ with $k_i \in [0, 12]$, keeping $K^1_{PSS_{nom}}$ and $K^3_{PSS_{nom}}$ fixed, and consider $\rho \in [0, 1]$,
- Case C: vary K^3_{PSS} via $K^3_{PSS_i} = k_i \cdot K^3_{PSS_{nom}}$ with $k_i \in [0, 12]$, keeping $K^1_{PSS_{nom}}$ and $K^2_{PSS_{nom}}$ fixed, and consider $\rho \in [0, 1]$.

For each of these three cases, the Lyapunov exponents were computed according to Eq. (9), and the corresponding level curves were obtained. The Figs. 4–6 show the $\lambda^{K^l} \approx 0$ level curves for $l = 1, 2, 3$, the stability regions underneath these curves, and the

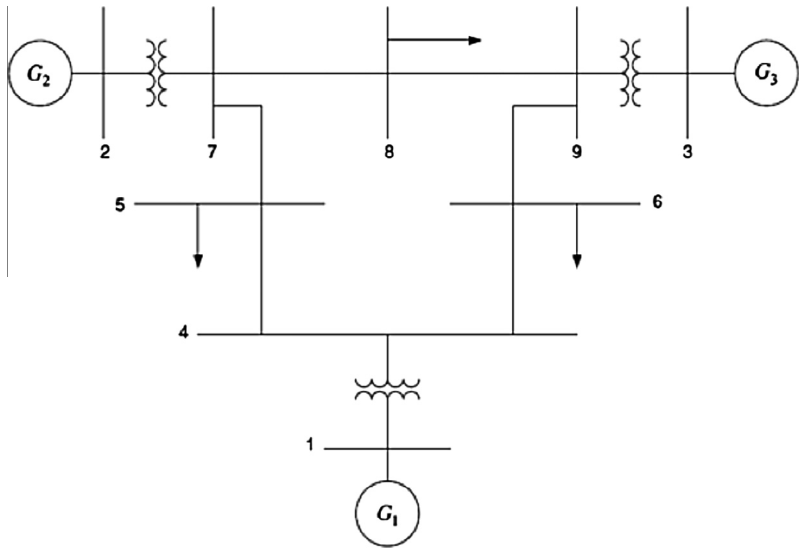

Fig. 2. Three-machine nine-bus system.

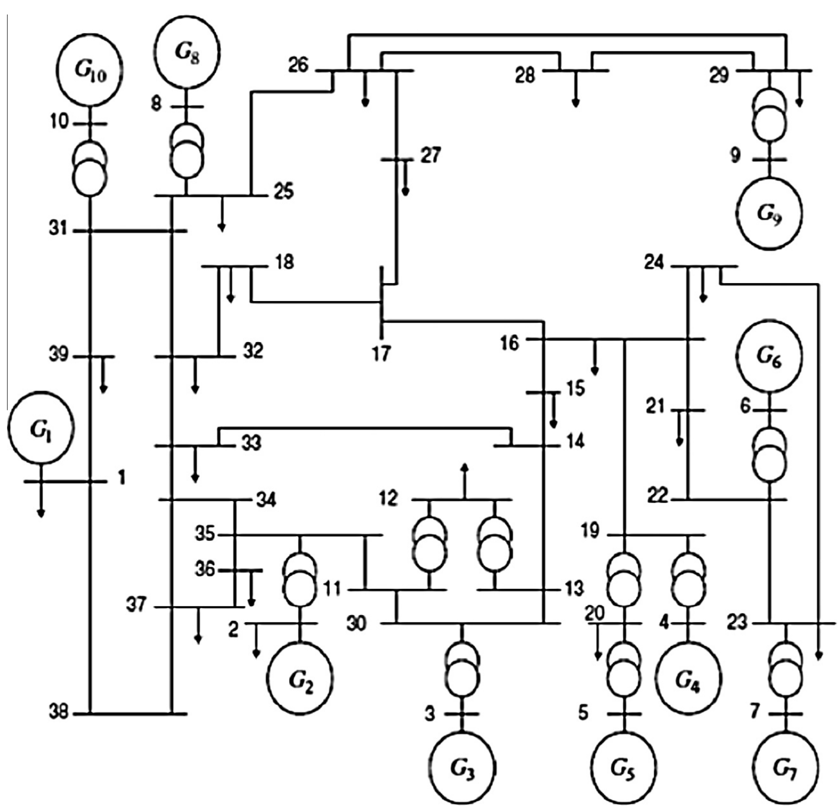

Fig. 3. Ten-machine thirty-nine bus system.

optimal single controller stability radius $r_{\max}(l)$ with the corresponding optimal gain value k^{l*}, $l = 1, 2, 3$.

Figs. 4–6 show that the optimal gain settings are different from the nominal parameter values if maximal stabilization against sustained random perturbation is the design goal. Specifically, we obtain from these figures

$$K^{1^*} \sim 35,\ K^{2^*} \sim 31,\ K^{3^*} \sim 10.$$

But note that gain variations K^1_{PSS} and K^2_{PSS} for controllers 1 and 2 have a substantial influence on the stability region, i.e. on stability under sustained perturbation, while variation of K^3_{PSS} at controller 3 does not affect the stability region in a substantial way.

Figs. 4–6 show the (computed) maximal individual stability radii to be

$$r_{\max}(1) \sim 0.6,\ r_{\max}(2) \sim 0.61,\ r_{\max}(2) \sim 0.59,$$

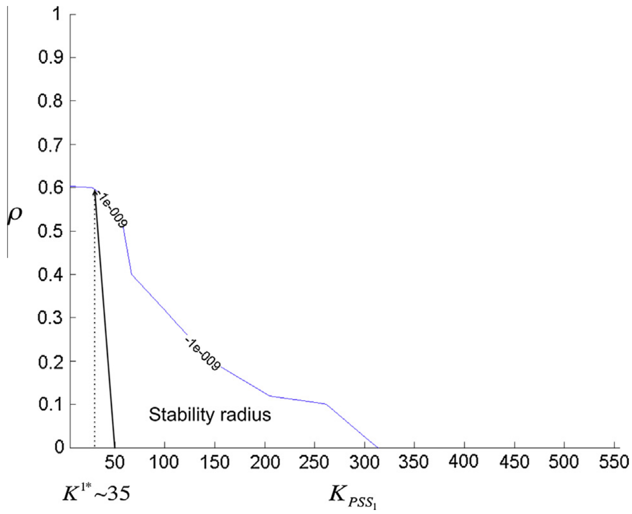

Fig. 4. Stability region and radius for Case A (three machine system).

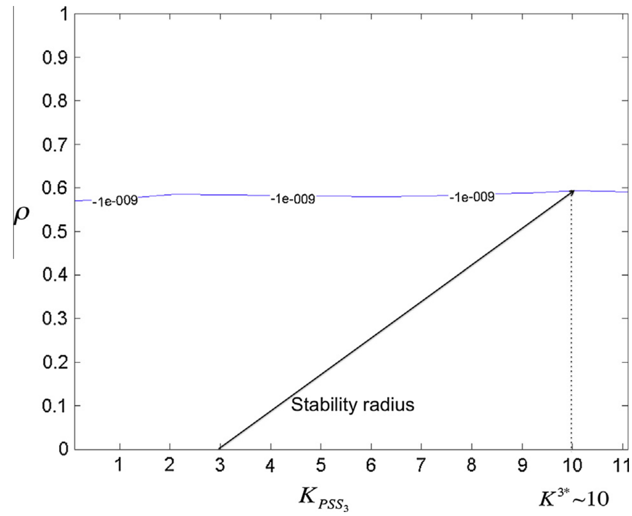

Fig. 6. Stability region and radius for Case C (three machine system).

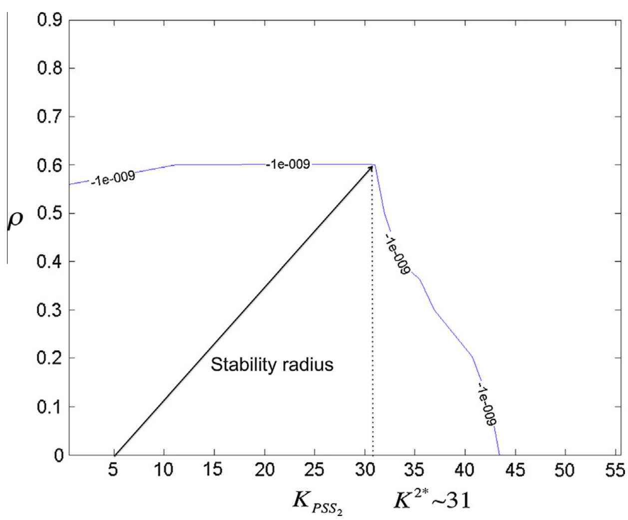

Fig. 5. Stability region and radius for Case B (three machine system).

hence Formula (19) leads to the common maximal stability radius $r_{\max_{global}}$ as

$$r_{\max_{global}} \sim 0.59.$$

This means that random perturbations of the kind (5) do not destabilize the system, as long as the noise size satisfies $\rho < 0.59$.

Analysis of the eigenvalues

If maximal stabilization against sustained random perturbation is the design goal, we have obtained:

$$K^{1^*} \sim 35,\ K^{2^*} \sim 31,\ K^{3^*} \sim 10.$$

The system is now subjected to light, normal and heavy loading conditions in the same way as in [24]. Then the eigenvalues are calculated and those closest to the origin are identified (critical values). Table 1 shows the real parts of the critical eigenvalues.

Table 1
Real part closest to the origin.

Case	Light load	Normal load	Heavy load
Original setting	−0.1007	−0.1007	−0.1005
New setting	−0.1016	−0.1016	−0.1013

Dynamic response analysis

This section presents the dynamic response of the system that is being studied under a three-phase fault, considering the two tuning options described above as 'Original Setting' and 'New Setting'.

A three-phase fault is applied to bus 9 (see Fig. 2) at $t = 1$ s, and it is cleared at $\Delta t = 0.1$ s. Fig. 7 shows the velocity curves of machines 1, 2 and 3 for this scenario. It is seen that the response of the system with 'New Setting' is a slightly better than using the 'Original Setting'.

The transient and steady state stability analyses verify that the combination of parameters of 'New Setting' presents better performance than the 'Original Setting'.

Ten machine – thirty-nine bus system

Adjustment of the controllers gains

As reported in [25], the nominal gains obtained for the controllers are the following

$$K^5_{PSS_{nom}} = 12.258,\quad K^7_{PSS_{nom}} = 10.981,\quad K^9_{PSS_{nom}} = 19.758.$$

Three cases are considered here:

- Case A: vary K^5_{PSS} via $K^5_{PSS_i} = k_i \cdot K^5_{PSS_{nom}}$ with $k_i \in [0.2, 4]$, keeping $K^7_{PSS_{nom}}$ and $K^9_{PSS_{nom}}$ fixed, and consider $\rho \in [0, 1]$,
- Case B: vary K^7_{PSS} via $K^7_{PSS_i} = k_i \cdot K^7_{PSS_{nom}}$ with $k_i \in [0.2, 4]$, keeping $K^5_{PSS_{nom}}$ and $K^9_{PSS_{nom}}$ fixed, and consider $\rho \in [0, 1]$,
- Case C: vary K^9_{PSS} via $K^9_{PSS_i} = k_i \cdot K^9_{PSS_{nom}}$ with $k_i \in [0.2, 4]$, keeping $K^5_{PSS_{nom}}$ and $K^7_{PSS_{nom}}$ fixed, and consider $\rho \in [0, 1]$.

As for the three machine system, for each of the three cases shown, the Lyapunov exponents were computed according to Eq. (9), and the corresponding level curves were obtained. The figures below show the $\lambda^{K^l} \approx 0$ level curves for $l = 5, 7, 9$, the stability

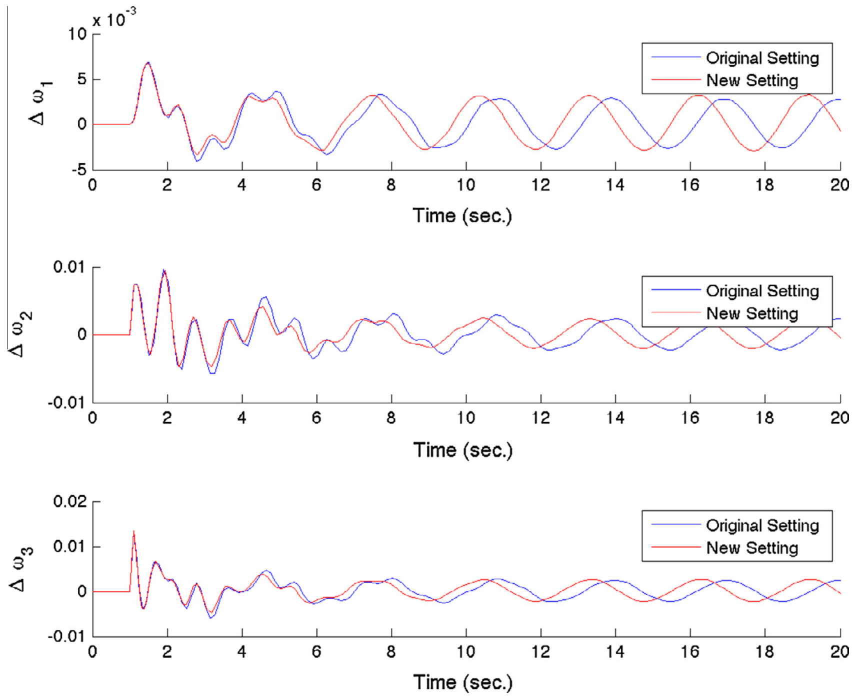

Fig. 7. Velocity evolution of the machines, in per unit, of the three-machine nine-bus system under a three-phase fault of bus 9.

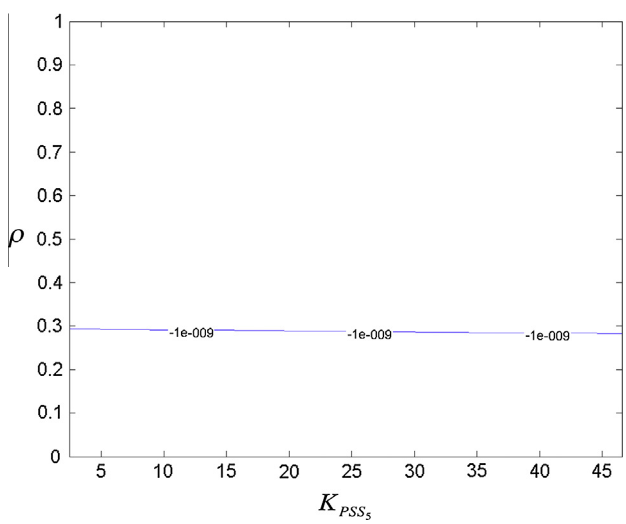

Fig. 8. Stability region and radius for Case A (ten machine system).

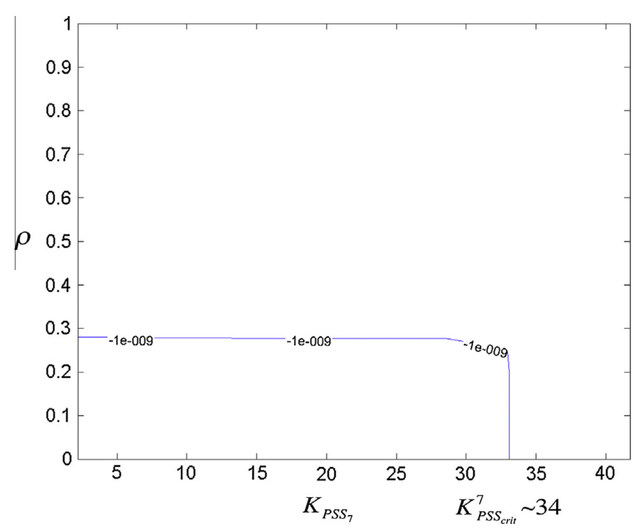

Fig. 9. Stability region and radius for Case B (ten machine system).

regions underneath these curves, and the optimal single controller stability radius $r_{\max}(l)$ with the corresponding optimal gain value $k^{l*},\ l = 5, 7, 9$.

Fig. 8 shows that the gain of the PSS controller at machine 5 has almost no effect on the stability region. The level curve for $\lambda^{K^5} \approx 0$ is flat and the stability radius has a value of $r_{\max}(5) \sim 0.29$, independent of the values for K^5_{PSS}.

Figs. 9 and 10 show that for the PSS controllers at machines 7 and 9 the stability region (and hence the stability radii) do not depend on the values for K^7_{PSS}, and for K^9_{PSS}, respectively, within certain gain intervals. These intervals are $[0, K^7_{PSS_{crit}}]$ for PSS controller 7, and $[0, K^9_{PSS_{crit}}]$ for controller 9. Beyond those critical values, however, the stability radii drop rapidly: The critical values are $K^7_{PSS_{crit}} \sim 34$, and $K^9_{PSS_{crit}} \sim 28$, with stability radii $r_{\max}(7) \sim 0.28$ and $r_{\max}(9) \sim 0.28$, leading to $r_{\max_{global}} \sim 0.28$. Note that the nominal values $K^5_{PSS_{nom}}$ and $K^7_{PSS_{nom}}$ are well within the gain intervals for which the level curves $\lambda^{K^5} \approx 0$ and $\lambda^{K^7} \approx 0$ are flat, while for the PSS controller at machine 9 the nominal gain $K^9_{PSS_{nom}} = 19.758$ is quite close to the critical value $K^9_{PSS_{crit}} \sim 28$. For higher values of this controller gain the system has no stability reserve against random perturbations.

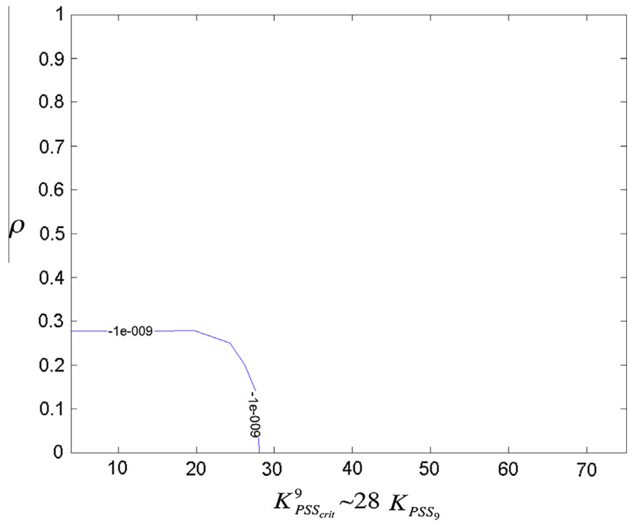

Fig. 10. Stability region and radius for Case C (ten machine system).

Table 2
Real part closest to the origin.

Case	Light load	Normal load	Heavy load
Original setting	−0.0073	−0.0073	−0.0072
New setting	−0.0074	−0.0074	−0.0073

Analysis of the eigenvalues

From the previous section we know that the PSS controller at machine 5 has almost no effect on the stability region, and at machines 7 and 9 the stability regions do not depend on the values for K^7_{PSS}, and for K^9_{PSS}, as long as the gains are smaller than the critical gains $K^7_{PSS_{crit}} \sim 34$, and $K^9_{PSS_{crit}} \sim 28$. As an example, we consider

$$K^{5^*} \sim 18,\ K^{7^*} \sim 15,\ K^{9^*} \sim 25.$$

The system is now subjected to light, normal and heavy loading conditions in the same way as previous system (three machines). Then the eigenvalues are calculated and those closest to the origin are identified (critical values). Table 2 shows the real parts of the critical eigenvalues.

The results shown in Table 2 are consistent with the fact that for this example gain parameters (within certain intervals) do not affect the stability regions and hence the stability radii of the system.

Dynamic response analysis

This section presents the dynamic response of the system that is being studied under a three-phase fault, considering the tuning of 'Original Setting' and 'New Setting'.

A three-phase fault is applied to bus 29 (see Fig. 3) at $t = 1$ s, and it is cleared at $t = 1.1$ s. Fig. 11 shows the velocity curves of machines 5, 7 and 9 for this scenario. It is seen that the response of the system with 'New Setting' is very similar to the 'Original Setting'.

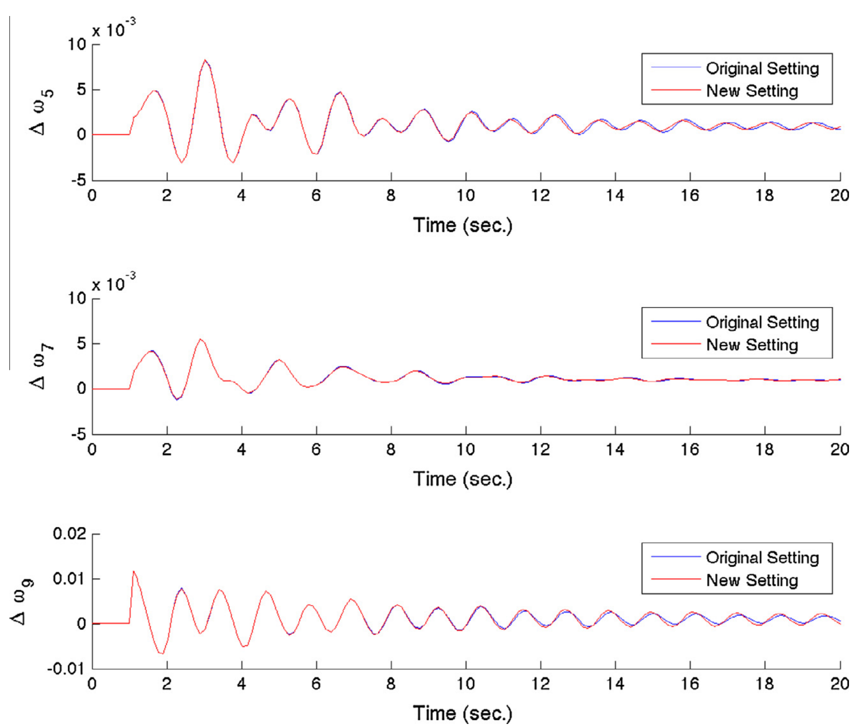

Fig. 11. Velocity evolution of the machines, in per unit, of the 10-machine 39-bus system under a three-phase fault of bus 29.

Conclusions

This paper proposes using Lyapunov exponents to study the stability of electric power systems that are subjected to random perturbations sustained in time. Based on the numerical computation of the Lyapunov exponent for different sizes of the perturbation and for different values of the PSS controller gains, two concepts called 'stability region' and 'stability radius' are proposed: the former involves determining an operating zone in which the system, subjected to random perturbations sustained in time, can operate without losing its stability; the latter corresponds to the maximum size of the perturbation that a system can withstand without becoming unstable. These two new concepts allow for more flexible operation of electric power systems, considering that operators would have available a range of possible parameters and perturbations under which the system would operate without collapsing.

In future work we propose to calibrate the Ornstein–Uhlenbeck perturbation model (6) with real measurements of the perturbations that affect the dynamics of the generators.

Acknowledgements

This research was financed by Fondecyt Project 11130169 of the Comision Nacional de Investigacion Cientifica y Tecnologica (CONICYT) of the Government of Chile.

References

[1] Schilling MT, Billinton R, Groetaers dos Santos M. Bibliography on power systems probabilistic security analysis 1968-2008. Int J Emerg Electr Power Syst 2009(3):1–48.
[2] Billiton R, Kuruganty PRS. A probabilistic index for transient stability. IEEE Trans Power Apparatus Syst 1980;99(1):195–206.
[3] Billiton R, Kuruganty PRS. Probabilistic assessment of transient stability in a practical multi-machine system. IEEE Trans Power Apparatus Syst 1981;100(7):3634–41.
[4] Billiton R, Chu K. Transient stability evaluation in an undergraduate curriculum-a probabilistic approach. IEEE Trans Power Syst 1986;1(4):171–7.
[5] De Marco C, Bergen A. A security measure for random load disturbances in nonlinear power system models. IEEE Trans Circ Syst 1987;34(12):1546–57.
[6] Pang CK, Dong ZY, Zhang P, Yin X. Probabilistic analysis of power system small signal stability region. In: International conference on control and automation (ICC2005), June 27–29, Budapest, Hungary; 2005.
[7] Burchett RC, Heydt GT. Probabilistic methods for power system dynamic stability studies. IEEE Trans Power Apparatus Syst 1978;97(3):695–702.
[8] Nwankpa CO, Shahidehpour SM, Schuss Z. A stochastic approach to small disturbance stability analysis. IEEE Trans Power Syst 1992;7(4):1519–28.
[9] Seo YB, Choi JW. Stochastic eigenstructure assignment with S-stability. In: SICE 2002 proceedings of the 41st SICE annual conference, vol. 5; August 2002. p. 3031–4.
[10] Tse CT, Chung CY, Wang KW, Niu R. Power-system stabilizer (PSS) design by probabilistic sensitivity indexes (PSIs). IEEE Trans Power Syst 2002;17(3):688–93.
[11] Cheng S, Yi HZH, Hou Y, Chen G. Power system probabilistic small signal stability analysis using two point estimation method. In: Universities power engineering conference 2007. UPEC 2007. 42nd international, Brighton; 2007. p. 402–7.
[12] Anderson PM, Bose A. A probabilistic approach to power system stability analysis. IEEE Trans Power Apparatus Syst 102 1983(8):2430–9.
[13] Namachchivaya NS, Pai MA, Doyle M. Stochastic approach to small disturbance stability in power systems. In: Arnold L, Crauel H, Eckmann J-P, editors. Lyapunov exponents. Springer Verlag; 1991. p. 292–308.
[14] Wei DQ, Luo XS. Noise-induced chaos in single-machine infinite-bus power systems. Europhys Lett 2009;5:50008.
[15] Xie WC. Dynamic stability of structures. Cambridge University Press; 2006.
[16] Loparo KA, Blankenship GL. A probabilistic mechanism for small disturbance instabilities in electric power systems. IEEE Trans Circ Syst 1985;32(2):177–84.
[17] Perninge M, Knazkins V, Amelin M, Soder L. Modeling the electric power consumption in a multi-area system. Eur Trans Electr Power 2010;21(1):413–23.
[18] Verdejo H, Vargas L, Kliemann W. Improving PSS performance under sustained random perturbations. Gener Transm Distrib IET 2012;6(9):853–62.
[19] Verdejo H, Vargas L, Kliemann W. Stability of linear stochastic systems via Lyapunov exponents and applications to power systems. Appl Math Comput 2012;218(22):11021–32.
[20] Arnold L. Random dynamical systems. Springer-Verlag; 1998.
[21] Arnold L, Kliemann W, Oeljeklaus E. Lyapunov exponents of linear stochastic systems. Springer Lect Notes Math 1986(1186):85–125.
[22] Sauer P, Pai MA. Power system dynamics and stability. Prentice Hall; 1998.
[23] Verdejo H, Vargas L, Kliemann W. Stability radii via Lyapunov exponents for large stochastic systems. Procedia IUTAM 2013:188–93.
[24] Gurrala G, Indraneel S. Power system stabilizers design for interconnected power systems. IEEE Trans Power Syst 2010;25(2):1042–51.
[25] Abdel-Magid YL, Abido MA, Mantawy AH. Robust tuning of power system stabilizers in multimachine power systems. IEEE Trans Power Syst 2000;15(2):735–40.
[26] Verdejo H, Kliemann W, Vargas L, Becker C. Stability indices for randomly perturbed power systems. Appl Math Comput 2014;231(15):386–94.