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Optimization Modeling for Resource Allocation in the Chilean Public Education System

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Abstract

This article presents an economic decision model for planning in public education, considering the satisfaction of the total demand for education in a municipality and the annual cost involved in the system. Given a set of established schools in a municipality and possible new locations, the aim of the model is to find the optimal supply in terms of which schools should be kept open or closed, which grades (or year levels) should be made available and the number of classes that should be provided within each such grade to meet student demand. The model includes specific information about curriculum requirements in terms of the number of hours per year per subject to be provided for each grade level. The objective function essentially seeks to minimize the total fixed and variable costs to schools, which mainly relate to human resources (principal and direction, teaching, administrative, and support staff). We propose an integer optimization model for solving this problem and we apply it to two large municipalities in Chile.

Keywords

education planning, optimization models

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Introduction

In Chile, one of the main challenges for increasing competitiveness and advancing development of the country is to improve the standard and quality of education, particularly in the public education system. The public sector represents around 40 percent of total primary and secondary education infrastructure and, in general, caters to the poorest segment of the juvenile population.

The Chilean system of elementary and high school education can be decomposed into three well-identified sectors: the public sector, managed by local authorities (municipalities); the private subsidized system, managed by private sponsors with partial funding from the state; and the private nonsubsidized sector, privately managed by for-profit and nonprofit owners. The private subsidized system is distinguished by a specific agreement between a private sponsor and the national Ministry of Education. All three systems are competitive, seeking to attract the best students, but the public sector is completely free and open, in the sense that all candidates must be admitted and if there is no place available at a given public school in the district, then the authority must look for places at another school in the public system, even outside the municipality. In general, families choose schools in the vicinity of their homes, especially for primary school. In addition, there is no gender segregation, most schools being mixed, and racial balance is not an issue in the country. The Ministry of Education only provides general regulations for managing the education system. These are in the form of a minimum standard for the teaching curriculum which schools are required to meet. In the particular case of the public sector, the state also provides economic resources at the federal, regional, and local level, but it is incumbent on local authorities to take responsibility for all other facets of school operation (academic strategies, resource management, etc.). These state resources are often limited, which creates many difficulties, because the public system is required to satisfy current demand and rules dictate that no financial contribution may be asked of students' families (Valenzuela, Bellei, and De los Ríos 2010). Moreover, teachers having tenure positions must receive higher wages, but federal government funding is not sufficient to afford these larger costs. In this context, the optimal use of public resources becomes a critical concern in each municipality.

During the last ten years, household income in the country has experienced a systematic increase while demographic changes throughout the whole of Chile have been reducing the proportion of the population that is of schooling age, altogether with a migration of enrollment from public to private subsidized schools. The effect of this has been to lower public school market share of the national education system at a rate of 1 percent to 2 percent per annum (Bellei, González, and Valenzuela 2010). Management of public resources is also strictly limited by legal regulations, particularly in terms of human resource concerns. This represents a real challenge for the public sector: municipalities must propose an attractive program to students and their families, while at the same time using public funds in an efficient way.

The problem we address in this article can be described as follows: given the set of existing public schools and the possible new locations that could be opened in a municipality and knowing the expected evolution of the demand for the following school years, the aim is to decide how to structure the supply of student places and classes. This means that the model must decide, for a given time horizon, which schools must be kept open, the grade levels to be offered by each school, and the number of parallel classes within each grade. The main constraints are essentially fulfillment of demand, infrastructure availability (number of classrooms in each school), and compliance with minimum teaching curriculum requirements, expressed in terms of the number of teaching hours per week for each subject. This last variable is important because it determines the human resources that need to be allocated, which represents a very important part of the total budget of the municipality.

The location of schools can have a great economic and social impact, particularly in countries having centralized public education systems. The criteria for the location, size, or structuring of schools can vary due to factors ranging from spatial or demographic considerations to social integration and administrative or economic constraints. Different authors have proposed various models, depending on particular state regulations and main objectives. In 1993, Church and Murray proposed a model to address the problem of school consolidation. Their work is about the assignment of students to schools with constraints that take into account racial issues and school balancing. In the situation we study here, we select schools to remain in operation and decide how many classes of a given grade will be made available, looking to meet demand for student places. Another difference is that we also consider the balancing of classes within each cycle for each school, but not the balance between schools as they do in their paper (in the Chilean system, primary education is partitioned into two so-called cycles, which will be described later). Church and Murray also consider a fixed number of schools to operate, while in our case, this decision is made by the model. Also, our objective function considers the cost of operating a school, unlike in their work where the monetary cost of the system is fixed, and they endeavor to use resources in a balanced way.

In Antunes and Peeters (2001), the authors use the simulated annealing heuristic to tackle this kind of problem and report good behavior of the strategy in real applications. Also, Caro et al. (2004) deal with the problem of school redistricting in which certain, stipulated blocks of the region must be assigned to establishments, combining the use of a binary decision model with a geographic information system, and obtaining encouraging practical results in real-world case studies. The problem of assigning students to schools, considering both capacity and racial balance, and minimizing the total distance from houses to schools, is addressed in Schoepfle and Church (1989). The resulting integer linear problem is estimated by solving a sequence of network instances. A simple binary optimization model is used in Geiger and Wenger (2010) to assign students to subjects at university level, but in that case the criteria are essentially the preferences of the students. In Martínez, Tamblay, and Weintraub (2011), the authors present a partial static competitive equilibrium *logit* model specified for a given scenario of policies, yielding the expected equilibrium locations, prices of schools, and students' school choices. The demand-supply equilibrium is studied, and a fixed-point algorithm is used to find a unique solution. For the districting problem, in Ferland and Guénette (1990), the authors proposed a decision support system, which uses distances between the student population and schools as the main criteria for assignment.

A dynamic multiperiod model is proposed in Antunes and Peeters (2000) for the evolution of school networks, allowing size reduction and/or expansion within certain predefined facility size limits. This generates a relatively complex mixed linear programming program, whose variables represent assignment, location, and capacity decisions. The objective is to minimize the total discounted cost. By using a variant of the *p-median* model, in Teixeira, Antunes, and Peeters (2007), the authors propose to maximize the accessibility of students to schools (through minimization of total distances) and they apply the resulting facility location model to real case studies.

In Malczewski and Jackson (2000), the authors deal with the existence of contradictory interests of all parties (administrators, teachers, parents, students, etc.). They propose using multicriteria models for spatial allocation of educational resources and give some critical comparison of the different multicriteria techniques that can be used: Pareto optimality, data envelopment analysis, parametric programming, and goal programming methods. In Pizzolato and Fraga da Silva (1997), the authors focus on the location problem, for the real case of two counties in the state of Rio de Janeiro, using heuristic procedures to solve the *p-median* problem. The criterion is to minimize the expected sum of overall home–school distances of students. In Araya et al. (2012), the authors address the location and sizing of rural schools in Chile (many of them are multigrade schools, having only one teacher for all students), essentially considering accessibility and costs, including transportation costs. They do not consider the internal balance in cycles nor the academic requirements in terms of minimal number of teaching hours for each subject. They also impose budget limits for construction/closing, operations, and transportation. In contrast, our work is oriented to relatively large municipalities, which are in general located in urban areas.

In general, models proposed in the literature consider only aggregated demands or capacities, whereas our model includes cycle/grade specification and internal balancing inside each school. Also, our model is more detailed because it takes account of pedagogic demand, expressed in terms of the minimum number of hours assigned to each subject. Further, the model proposed here measures demand per grade within each geographic zone, but this demand can be met by schools belonging to any of the predefined zones; this gives rise to an integrated nonseparable model with a staircase structure and a set of common constraints. Finally, the solution not only provides the total optimized cost for the system but also suggests a suitable structure for the schools in terms of cycles and number of parallel classes.

This article is organized as follows: in the second section, we give the main principles for the planning model and in the third section, we define the variables, parameters, and equations describing the optimization model. The fourth section is devoted to the application of the model to two real case studies, and in the fifth section, we give some general conclusions and ideas for further research.

The Conceptual Model

We begin by describing the main constraints of the system, most of which stem from legal and structural regulations. First, the total demand for each grade must be met because the public sector cannot deny enrollment to students. Second, the minimum number of hours per week of each subject is essentially imposed by the academic program established by the state authority. Furthermore, the specific academic plan of a school may fine-tune this number. Third, the maximum number of students in each class is limited by law, but a lower limit could be imposed depending on the education level, on the infrastructure available at the school and on preferences of local authorities related to improving average quality for a particular school or the whole local public educational system. Quality of education is not explicitly included as an endogenous factor in our model, but this issue can be addressed by tuning the classmate size and the number of teaching hours of different subjects.

Our purpose is to design an optimization model to suggest which schools should be opened or closed in a given area and, for each of these schools, to provide a forecast of the number of students at each level, the cycles that are to be set up, the number of classes that need to be created for each grade, the number of teaching hours to be allowed, and so on. In fact, we have to cope with an existing situation and to consider the possibility of closing/consolidating schools. Practically, it is not always possible to merge all sets of schools to optimize resources due to geographic restrictions, population distribution, and quality heterogeneities, among other factors. In order to implicitly take those restrictions into account, the model considers a partition of the geographic area of interest into zones, each zone containing a certain number of schools that can be opened, closed, or merged without restriction, based on location criteria and satisfying local demand. The zones are predefined by the decision maker and at least one school must be opened in each zone. The model allows any specific school to satisfy demand coming from different zones, so the general problem cannot be decomposed into separate subproblems.

The Chilean education system is made up of the following four cycles:

1. Preschool: two years (children being four and five years of age),
2. Primary 1: four years,

3. Primary 2: four years, and
4. High school: four years.

The inputs to our model are essentially the resources, school capacities, costs, and demands (estimated according to historical evolution and the preregistration process). This means the existing number of classrooms (infrastructure availability), unitary costs of teaching and other personnel (variable), other fixed and variable costs of operating each school, capacities (the maximum number of students who can be admitted to each class), and the total capacity of the school in each grade. The model assumes that the evolution of demand is known over the short to medium term.

The main assumptions of our model are as follows:

1. All inputs data to the model are specified per annum.
2. The evolution of demand is supposed to be known for each grade and geographic zone.[1] In this model, a specific school can satisfy demand coming from different zones, but we penalize this fact in the objective function.
3. If the model decides that a given school should be kept open, at least one cycle must be made available, but if two nonconsecutive cycles are to be provided, then any intermediate cycles must also be provided (e.g., if the first primary cycle and high school cycle are to be offered, then the second primary cycle must also be made available in order to offer continuity to students completing the first primary cycle).
4. Due to infrastructure availability, we also impose an upper limit on the number of parallel classes within every grade. If a grade is to be made available, then the entire cycle to which it belongs must also be made available.
5. The model also requires that the total number of classes in a given school be restricted to the available infrastructure.
6. Each school must have a coherent distribution of students and grades. This means that the number of parallel classes in the same grade must be as similar as possible for every grade throughout the same cycle. Note that we impose balancing inside a school and not between schools.
7. Concerning educational requirements, it is necessary to comply with the total number of teaching hours per week for every grade and subject. This is dictated by national regulations but schools can impose additional teaching hours according to their specific academic program.[2]
8. It is required that the number of students per classroom be equal or smaller than a specified limit, this limit being fixed according to legal, pedagogic, and infrastructure criteria. We note here that the class size is usually related to quality of education, but we do not explicitly include quality issues in our formulation.

The Mathematical Model

The model seeks to minimize the total cost, satisfying the structural and legal regulations of the education system, while using the notion of student allocation to schools for every grade by including the corresponding costs of those allocations in the objective function. We suppose that the demand is known for every grade and zone throughout the time horizon.

Let us define the following variables (that characterize a decision):

- x^t_{ik}: number of classes in grade $i = 1, \ldots, N_G$ at school $k = 1, \ldots, N_S$, period $t = 1, \ldots, T$.
- z^t_k: binary, 1 if school k is open at period t; 0 otherwise.
- u^t_{jk}: binary, 1 if cycle $j = 1, \ldots, N_C$ is to be provided by school k at period t; 0 otherwise.
- v^t_{mik}: total hours per year assigned to subject $m = 1, \ldots, N_M$, grade i, school k, period t.
- w^t_k: maximum difference between the number of classes of two consecutive grades, school k, period t.
- a^t_{ihk}: number of children in grade i from zone h who are allocated to school k, period t.

We also define the parameters (problem data):

- W^t_k: fixed annual cost of school k, period t.
- Q^t_{mi}: teaching cost per hour for subject m, grade i, period t.
- C_{ik}: capacity (maximum number of places per class) of grade i, school k.
- T^t_k: total number of available classrooms at school k, period t (capacity of the school).
- H_{mik}: number of hours of subject m required per year in grade i at school k.
- R_{ik}: maximum number of classes in grade i at school k.
- r_{ik}: minimum number of classes in grade i at school k.
- $\{C_j\}$: partition of grades into cycles (e.g., preschool, ..., high school).
- $\{Z_h\}$: partition of $\{1, \ldots, N_S\}$ representing geographic zones where schools are located.
- D^t_{ih}: demand for grade i in zone $h = 1, \ldots, N_Z$, period t.
- P^t_k: penalty coefficient for heterogeneity within school k, period t.
- E_0: set of existing schools at period 0.
- N_0: set of nonexisting schools at period 0 (possible new locations).
- ρ^t_{ihk}: penalty associated with a child in grade i, zone h, allocated to school k, at period t.
- K_{kt}: cost of closing an existing school k at period t.
- K'_{kt}: cost of opening a new school k at period t (to use a new location).

The mathematical model can be written as (P):

$$\min \sum_{mikt} Q^t_{mi} v^t_{mik} + \sum_{kt} W^t_k z^t_k + \sum_{kt} P^t_k w^t_k + \sum_{ihkt} \rho^t_{ihk} a^t_{ihk} + \sum_{k \in E_0, t} K_{kt}(z^t_k - z^{t+1}_k)$$
$$+ \sum_{k \in N_0, t} K'_{kt}(z^{t+1}_k - z^t_k),$$

$$\sum_{k \in Z_h} z^t_k \geq 1 \quad \forall h, t, \tag{1}$$

$$u^t_{jk} \leq z^t_k \leq \sum_{j'} u^t_{j'k} \quad \forall j, k, t, \tag{2}$$

$$z^t_k \geq z^{t+1}_k \quad \forall k \in E_0, t < T, \tag{3}$$

$$z^t_k \leq z^{t+1}_k \quad \forall k \in N_0, t < T, \tag{4}$$

$$u^t_{jk} + u^t_{j'k} - 1 \leq u^t_{lk} \quad \forall k, t, \ 1 \leq j \leq j' - 2 \leq N_C - 2, \ j + 1 \leq l \leq j' - 1, \tag{5}$$

$$-w^t_k \leq x^t_{ik} - x^t_{(i+1)k} \leq w^t_k \quad \forall k, t, \ 1 \leq i \leq N_G - 1, \tag{6}$$

$$r_{ik} u^t_{jk} \leq x^t_{ik} \leq R_{ik} u^t_{jk} \quad \forall i, j, k, t \ \text{ such that } i \in C_j, \tag{7}$$

$$\sum_i x^t_{ik} \leq T^t_k \quad \forall k, t, \tag{8}$$

$$\sum_h a^t_{ihk} \leq C_{ik} x^t_{ik} \quad \forall i, k, t, \tag{9}$$

$$v^t_{mik} \geq H_{mik} x^t_{ik} \quad \forall m, i, k, t, \tag{10}$$

$$\sum_k a^t_{ihk} \geq D^t_{ih} \quad \forall i, h, t, \tag{11}$$

$$x^t_{ik}, v^t_{mik}, a^t_{ihk} \quad \text{integer} \quad \forall m, i, k, h, t,$$

$$u^t_{jk}, z^t_k \quad \text{binary} \quad \forall j, k, t,$$

$$w^t_k \geq 0 \quad \forall k, t.$$

The index t in the last two terms of the objective function runs for $t = 1, \ldots, T - 1$. This objective function seeks to minimize total fixed and variable school operating costs, mainly related to human resources (teachers, administrative, and support staff). The variable a^t_{ihk} gives information about the optimal allocation but only represents the estimated supply not the real decision. To enable the calculation of allocation costs, we must define the corresponding parameters ρ^t_{ihk}

representing the cost of assignment of children to schools, but in fact it only gives an estimate of the total cost of the system since student assignment to schools is not mandatory and parents have the final decision about where to send their children.

Note that E_0 and N_0 represent a partition of the existing and new possible locations, which means that $E_0 \cup N_0 = \{1, \ldots, N_S\}$ and $E_0 \cap N_0 = \phi$. We must also note that the partition $\{Z_h\}$ is a data, not corresponding to a decision of the model.

We can identify three kinds of constraints in the model: structural, capacity, and requirements constraints. First, inequalities (1) to (6) configure the general logic or regular structure of the solution, along the planning horizon: at least one school must be open at each zone and period; the specific school must be open when at least one cycle is on offer at that school and vice versa; if an existing school is closed, then it remains closed throughout the planning horizon; and if a new school is opened, then it also remains open during the remainder of the planning period; a coherent structure on the cycles must be provided by the school in the sense that, if two nonconsecutive cycles are supplied, then the intermediate cycles must also be supplied; the number of classes taught throughout the duration of a given cycle must be nearly constant.

Concerning capacities, inequalities (7) to (9) mean: if a given grade is opened at school *k*, then it must satisfy bounds on the number of classes in that grade (infrastructure availability); an upper bound on the number of classes to be taught at each school is imposed; a maximum number of students per grade and per school is allowed each year.

Finally, the third set of is expressed through inequalities (10) and (11), specifying the total of teaching hours, according to the requirements given by the academic program of the school, for each subject; we also impose that the total supply of the system in a given zone or area must be sufficient to satisfy demand for every grade.

The right-most inequality of (2),

$$z_k^t \leq \sum_{j'} u_{j'k}^t,$$

could be redundant because variable z_k^t has a positive coefficient in the objective function.

The costs depend in general on the period *t,* and this could take the form of a constant value weighted by a discount factor, for example, $W_k^t = W_k \varepsilon^t$, where $0 < \varepsilon < 1$. The coefficient ρ_{ihk}^t accounts for several factors. First, it may include the private or individual cost (to families), for example, transportation from area h to school k (which can be supposed to be zero for $k \in Z_h$) the social perception of this travel by the families living in area h, the perception of the value of the travel time, the age of children in grade i, and so on. Second, it may include the marginal cost to school k for admitting pupils from other zones. In this context, the objective function is a weighted additive combination of three criteria: the direct cost of education, the indirect cost due to heterogeneity within cycles, and the private cost to families of students coming from other zones. Evidently, this combination of factors could be handled by using multicriteria modeling and appropriate algorithms.

The model allows situations to be handled in which demand is overestimated, for example, as a result of preenrollment processes. We could ease this requirement by changing constraint (11) into a relaxed inequality,

$$\lambda^t_{ih} D^t_{ih} \leq \sum_k a^t_{ihk} \forall i, h, t.$$

This relaxation allows only a certain given proportion $\lambda^t_{ih} \in [0, 1]$ of the (local) demand in each zone to be met. To encourage demand to be satisfied, we should add to the objective function a term of the form

$$\sum_{iht} \tau^t_{ih} \left(D^t_{ih} - \sum_{k \in Z_h} a^t_{ihk} \right).$$

Here, τ^t_{ih} represents the penalty for not admitting a pupil in grade i from zone h to a school in his own zone. Note that the term $\sum_{iht} \tau^t_{ih} D^t_{ih}$ is a constant, but we keep it in the last expression to highlight the fact that failing to satisfy demand comes with a cost.

Since the time index t varies from 1 to T, we consider initial values $k = 1$, for $k \in E_0$, and $k = 0$, for $k \in N_0$, in order to obtain a consistent interpretation of constraints (3) and (4) and the corresponding terms in the objective function.

Finally, at the optimal solution, the real variables w^t_k take integer values, because in constraint (6) the term $x^t_{ik} - x^t_{(i+1)k}$ is integer valued while positivity of P^t_k allows w^t_k to be set to an integer value.

Case Studies

In this section, we present two case studies corresponding to two municipalities in Chile (designated here municipality 1 and municipality 2). Both have typical middle and upper-middle-class populations. Large shares of students from these families are enrolled in private subsidized schools, while public schools essentially accommodate students from poor and low-middle-class backgrounds. The decision makers decided a priori to divide each of both municipalities into eight zones, with different demands per grade. This choice could indeed be left to the model, but we defer this approach for future study. Municipality 1's system has fifteen preexisting schools and municipality 2 has twenty-four, with an estimated demand of 10,320 and 19,946 students, respectively.

The cost of a teaching hour is supposed to be a constant, but in general this value should depend on the subject and the specific teaching contracts. Although the scope of the general model is strategic, we only have reliable data for the first year and we can derive results for a one-year horizon. The availability of good information about the evolution of demand over the planning timeline is limited, but we can predict the evolution over five years, under realistic assumptions because we know that, in the two municipalities included in this study, the demand is decreasing at around 5 percent per annum.

Closing, merging, or changing the internal structure of schools is a critical decision in any municipality, but such decisions are made relatively frequently in Chile. An important number of schools are closed every year, and this occurs even some months before the opening of the academic year in March. From 2000 to 2006, the total number of public schools was reduced by 284 or 4.5 percent and between 2006 and 2012 by 457 or 7.3 percent, according to the official information of the Chilean Ministry of Education.[3] In most cases, it is necessary for local authorities and private or subsidized suppliers to handle the emotional and practical problems such decisions cause the families of pupils.

On the other hand, the two municipalities have incorporated the results of this study into their planning processes for 2011 and 2012, which also take into account the social impact and labor regulations; and the results from the model serve to provide an optimized structure for the system and the reduction in total costs to achieve under its adoption.

We use costs (expressed in US dollars) based on the 2010 academic year, while demand is estimated from the number of students applying for enrollment at each school at the end of 2010; that is to say, the demand is the expected student population for 2011. The annual fixed cost corresponds to the purchase and maintenance of infrastructure (buildings, equipment, etc.), and the total cost includes the fixed cost plus the wages of the teaching and nonteaching staff of the school. This includes the cost of teaching hours. Rational planning based on the application of our model should essentially reduce the human resource cost (variable component) as well as the fixed cost of opening and/or closing schools. The cost of opening and closing a school is explicitly included in the model through the coefficients K_{kt} and K'_{kt}.

We solved the mathematical model under two scenarios for both municipalities:

- Scenario 1 is defined by fixing the maximum number of students in every class at thirty-five.
- In scenario 2, we used a bound of thirty-five students per class at preschool levels, and forty-five for primary and high school levels.

For each scenario, we consider two instances:

- $\lambda^t_{ih} = 1$ (forcing complete satisfaction of the total demand) and
- $\lambda^t_{ih} = .5$ (relaxing this constraint to allow as little as 50 percent of the demand to be satisfied).

Data

In this study, we used some realistic values of the parameters. For the maximum number of children per class, C_{ik}, we consider two scenarios, already defined in the previous paragraph. Despite forty-five pupils being a legal class size, the municipalities could opt for a lower maximum to obtain better quality outcomes. The computational results could be dramatically different depending on this value, but here we use the legal bound and a tighter value, knowing that the current average class size in urban public primary schools is around thirty-one students (Bellei, González, and Valenzuela 2010).

The maximum number of parallel classes in a grade, R_{ik}, varies from 0 to 4 in municipality 1 and 0 to 3 in municipality 2. Setting this value to 0 means that the corresponding grade will not be offered by the school. For the minimum number of classes, r_{ik}, we set 0 everywhere.

Concerning to the demand D^t_{ih} in zone h for grade i, this is detailed per school in Tables A1 and A2 in Appendix. The number H_{mik} of teaching hours/week for subject m required in grade i at school k varies from 0 to 9 in municipality 1 and from 0 to 8 in municipality 2. This is not constant, because it depends on each subject and on the particular pedagogic choices made by each school. In Chile, the law establishes only minimum requirements for each subject.

For the costs, our choice is based on real data provided by the municipalities. The parameter Q^t_{mi} is fixed at constant value US$10.9. The cost of heterogeneity P^t_k is chosen at US$1, but the homogeneity could also be handled by limiting the number of parallel classes by an upper bound fixed ahead of time. In this case, there is a risk of infeasibility.

The annual cost of education per student ρ^t_{ihk} is fixed at a constant value of US$1,163. The same value is used for the penalty coefficient τ^t_{ih}. Under rational assumptions, it is expected that families would tend to choose the nearest schools, and we represent this fact by using $\rho^t_{ihk} = 0$, for $k \in Z_h$.

Finally, to evaluate the behavior of the dynamic model in the case of a decreasing demand, we run the model supposing that the demand decreases at a rate of around 5 percent per year for every grade, during a planning horizon of five years (this is quite realistic for the two municipalities, also for the whole public sector at national level, and the period considered in our study). For this evaluation, we also assume that the total demand must be satisfied and we apply scenario 2 for the maximum number of pupils per class: forty-five for primary and secondary schools and thirty-five for preschools. In the dynamic model, local planners can determine whether trends in change in demand be they decreasing or increasing, are stable and solve efficiently building new classrooms or schools, as well as determining the closure of a school and not setting courses, which is what actually happens, because it is politically costly to close schools and policy makers prefer to be very sure about such decisions.

In the following subsections, we show the main data and the results for our two case studies. The application of this mathematical model produces a significant reduction in total costs for both municipalities. These savings come from the optimal configuration of classes (satisfying legal and institutional regulations) and from the optimal allocation of teaching resources (personnel) to the teaching hours needed.

Tables A1 and A2 in Appendix display the main data for each case study. The first column contains an identifier for the school. The second column is the current number of available classrooms in each school. Here, we treat the classrooms as indistinguishable, but it is well known that in general, some classrooms are intended for preschool or primary school pupils, while others may be for the exclusive use of high school students. The next column is the identifier of one of the eight zones to which the schools belong. The next two columns contain the total fix cost (infrastructure, general services, etc.) and the total annual cost, which includes the variable cost, namely, teachers' salaries. The last column shows the total expected demand for the year 2011, but the calculations use the detailed distribution of the demand for every grade and school.

These instances correspond to medium size mixed linear programming problems, which can be solved by most existing commercial and academic software. All instances were run using CPLEX-9.1 on an IBM-iDataplex with two processors.

Results and Planning Implications

The general results are reported in Table A3 in Appendix. First, we observe that it is not possible to satisfy all of the demand in the case of scenario 1 (classroom capacity equal to thirty-five pupils per class in all grades), but in the relaxed version the model finds solutions leaving 3 percent and 5 percent of the demand unsatisfied in municipalities 1 and 2, respectively.

For comparison, we take the base case as the observed costs in year 2010. There, the total monthly hours, 81,496 for municipality 1 and 107,308 for municipality 2, can be decomposed into two components. The first is related to the "classroom hours," that is, the number of hours dedicated to teaching, and corresponds to 75,336 and 96,972 monthly hours, respectively. The second part represents the hours dedicated to support activities (e.g., the principal's hours and administrative staff hours are included in this figure) and corresponds to 6,160 and 10,336 monthly hours, respectively. Also it includes current costs for goods and services related to education, the salaries of other municipal staff that support educational services and conditional funding provided by central and regional governments to finance specific programs to support public education. The base case has been set up using the same items and cost definitions applied in the model. In particular, we used an average value for a teaching hour taking standard teachers' salaries into account.

Although this can vary between municipalities, teachers are generally paid for forty-four hours/week, but they must only provide thirty-three classroom teaching hours, a limit which is rarely reached in practice. We have considered this fact in our estimation of unitary costs.

After optimization, the monthly "classroom hours" decrease by up to 47 percent for municipality 1 and by up to 34 percent for municipality 2 (depending on the scenario). This can be interpreted as the current percentage of overstaffing levels. This decrease results in a total annual savings of up to around US$4,500,000 to US$5,500,000. In fact, if we calculate the average daily number of teaching hours for both municipalities, the result is between nine and twelve hours, which does not reflect reality while, after optimization, this value decreases to 7 for the two municipalities; this is consistent with a typical academic curriculum in any school. Regarding the number of classes in the base case, 308 are needed in municipality 1 and 548 in municipality 2 (maybe not really needed, because of eventual overestimation). After optimization, this number can decrease by up to 13 percent for municipality 1 and by up to 14 percent for municipality 2. Note that, although the optimization efficiency varies from 25 percent to 47 percent in terms of teaching hours in both municipalities, the percentage of improvement in terms of number of classes is much smaller. This suggests that for these cases, the most significant inefficiency is found in the allocation of human resources. This is consistent with the behavior of municipalities in charge of public education, which have maintained the same total number of teaching hours over the last decade while total student enrollment has exhibited a systematic decline over the same period.

The model proposes a logical structure for both systems in the case of scenario 2, which is shown in Tables A4 and A5. These two tables contain the optimal configuration, number of classes per grade, number of places supplied per school, and the estimated annual costs, for both municipalities. It can be seen that the total supply is slightly higher than the demand, because the model specifies the number of classes that should be open.

Note that some schools are kept open, even with a very small population of students. For example, schools twenty-three and twenty-four in municipality 2 that cannot be merged, because in the corresponding zones (7 and 8) there are no other available locations. Certainly, this constraint expressed by inequality (1) could be easily relaxed by applying a penalty factor.

The enormous difference between the base and optimized cases can be explained by the overestimation of the required teaching staff (but here we include the salaries of the principal and other nonteaching staff of the school).

For the case of the dynamic model, the results are given in Table A6 and show that the model reacts to the decreasing demand by closing schools from fifteen to twelve, in the case of municipality 1, and from twenty-four to twenty in the case of municipality 2. The corresponding total costs (fixed plus variable costs) essentially follow the same tendency, given by the decreasing total number of teaching hours, which can reach around 50 percent in five years. This dramatic reduction is essentially explained by the rational structure proposed by the model, particularly in terms of the adjusted number of classes.

These results mean that substantial monetary savings are possible through strategic planning using the optimization model, which allows to municipalities to handle in a better way its current critical financial situation; in average, monetary transferences from municipalities into its educational services reaches 15 percent of its total annual budget and it represents more than its total resources allocated to investment. That savings through better planning could lead to improve low-quality public education services and permit them to compete in better conditions with private subsidized schools located in the same territories and support sustainability to educational local services for the long run.

Conclusions and Further Remarks

The model presented here provides a general framework for handling real instances of economic planning of an education system and permits planners to estimate the infrastructure and human resources needed in order to satisfy demand for the municipal school system and the corresponding costs of education at each school and for each grade. At the same time, the model gives the configuration of each school in terms of the size of the student population and the number of classes in each grade. The most significant extra costs of the base case from the optimal alternative come from the allocation of human resources, but it is sensitive to the definition of maximum classmate sizes, which can also be studied with the general model introduced here.

Some open questions for further research arise from this work. The model determines the total number of hours that are necessary to satisfy student demand, but not how many teachers are necessary to fulfill the teaching demands of each school. Determining the optimal number of teachers is a nontrivial function of the academic orientation, the pedagogic strategies, and the organizational structure of each school.

In this work, we have not considered the random behavior of demand, which is inherent to future demographic and economic evolution; this is especially true of Chile. To include the uncertainty of the demand needs a new model together with appropriate algorithms for resolution, for example, using stochastic or robust optimization techniques.

On the other hand, we did not consider the quality outcomes as a requirement of our model. In fact, the choice of variables and constraints to represent this factor remains an open question, but the classmate size and the number of teaching hours in the model could be interpreted as indirect quality indicators.

Closing a school may have a large impact; hence, the benefits or costs of this decision must be closely examined. First, significant public infrastructure is disaffected and the use of this spared resource must be envisaged. Next, it may drastically alter a neighborhood, for instance, a depreciated area of a city or a village. Incorporating these issues in the model is also a remaining open question of this work.

The current model only deals with the public sector, without considering the other two sectors of the Chilean education system, which are very dynamic and competitive. The challenge is to study how these sectors influence the demand for public education, in order to propose more general education models.

From the mathematical and algorithmic point of view, a big open question is how to take advantage of the staircase structure of the optimization model. In fact, we have not discussed the algorithmic aspects of solving the model, since the real instances we solved are only of moderate size, but in a more general case, it may be necessary to propose decomposition or heuristic methodologies. Larger instances appear because the evolution of demand over several years generates a much larger number of variables and constraints. Moreover, this problem has the general synthetic form:

$$(P') \min c_1^\top x_1 + \cdots + c_T^\top x_T,$$

$$E_1 x_1 + \ldots + E_T x_T \leq 0, \tag{12}$$

$$Ax_1 \leq b_1, \tag{13}$$

$$\ddots$$

$$Ax_T \leq b_T, \tag{14}$$

$$x_1, \ldots, x_T \in \{0,1\}^{n_1} \times \mathbb{N}^{n_2} \times \mathbb{R}_+^{n_3}. \tag{15}$$

Each vector $x_t, t = 1, \ldots, T$ is defined as follows:

$$x_t = \left(u_{jk}^t, z_k^t, x_{ik}^t, v_{mik}^t, a_{ihk}^t, w_k^t\right),$$

where u_{jk}^t, z_k^t are binary, $x_{ik}^t, v_{mik}^t, a_{ihk}^t$ are natural, and w_k^t are real positive variables.

Constraints (13)–(14) represent the requirements for every period, and constraint (12) links the periods between them (see constraints (3)–(4), the only set of constraints that involves different periods).

The matrix $[E_1, \ldots, E_T]$ is very sparse, containing only two nonzero coefficients 1 and -1 in each row. Moreover, it involves only the variables z_k^t. In a more general case having several planning periods, the structure of the optimization problem described here should play a crucial role in terms of resolution time.

Appendix A

Table A1. Main Data for Municipality 1.

School identificator municipality 1	Available classrooms, T_k^t	Zone, Z_h	Annual fixed cost, W_k^t	Annual total cost	Effective annual demand
1	20	8	US$340,158	US$948,428	723
2	18	8	US$159,858	US$717,439	439
3	38	6	US$391,602	US$1,534,137	992
4	32	2	US$365,143	US$1,371,324	929
5	6	3	US$48,444	US$247,652	70
6	29	1	US$384,239	US$1,276,876	834
7	18	4	US$315,956	US$897,361	545
8	38	7	US$593,062	US$1,734,583	1,192
9	55	6	US$834,686	US$2,421,765	1,885
10	20	8	US$195,865	US$628,751	429
11	16	7	US$201,030	US$437,242	331
12	23	7	US$319,585	US$925,828	641
13	20	5	US$243,611	US$809,302	507
14	14	7	US$71,888	US$222,942	89
15	22	7	US$323,172	US$942,087	714
Total	369		US$4,788,296	US$15,115,718	10,320

Table A2. Main Data for Municipality 2.

School identificator municipality 1	Available classrooms, T_k^t	Zone, Z_h	Annual fixed cost, W_k^t	Annual total cost	Effective annual demand
1	32	6	US$236,366	US$938,919	1,054
2	27	6	US$218,565	US$1,080,281	843
3	33	6	US$307,573	US$1,176,893	1,300
4	23	2	US$182,962	$650,316	793
5	23	2	US$200,763	US$751,755	860
6	17	2	US$147,358	US$597,985	608
7	19	2	US$154,281	US$625,691	603
8	20	2	US$163,182	US$649,291	614
9	18	2	US$154,281	US$600,346	584
10	14	6	US$120,656	US$502,346	467
11	20	4	US$165,160	US$580,811	622
12	16	2	US$170,105	US$608,060	707
13	22	2	US$194,829	US$729,094	905
14	18	3	US$180,984	US$680,272	777
15	24	1	US$205,708	US$763,290	929
16	14	6	US$124,612	US$620,352	492
17	37	3	US$308,562	US$1,109,958	1,361
18	25	2	US$225,488	US$854,034	833
19	39	2	US$330,320	US$1,148,444	1,563
20	28	2	US$248,234	US$922,908	1,092
21	27	3	US$234,388	US$820,356	1,004
22	40	5	US$356,033	US$1,441,796	1,667
23	9	7	US$89,008	US$297,848	184
24	6	8	US$53,405	US$220,173	84
Total	551		US$4,772,822	US$18,371,218	19,946

Table A3. Optimized Costs for Municipality 1 and Municipality 2.

	Base case	Scenario 1 (*P*)	Scenario 1 Relax. (*P*)	Scenario 2 (*P*)	Scenario 2 Relax. (*P*)
Municipality 1					
Demand	10,320	10,320	10,320	10,320	10,320
Satisfied demand	10,320	Infeasible	9,991	10,320	10,175
Number of schools	15		15	14	14
Total cost	$15,115,718		$10,583,595	$10,114,303	$9,542,702
Annual saving			$4,532,123	$5,001,415	$5,573,016
Reduction/base case (percent)			30	33	37
Total teaching hours/month	75,336		45,732	42,596	40,040

(continued)

Table A3. *(continued)*

	Base case	Scenario 1 (*P*)	Scenario 1 Relax. (*P*)	Scenario 2 (*P*)	Scenario 2 Relax. (*P*)
Reduction/base case (percent)			39	43	47
Total classes	308		305	284	268
Reduction/base case (percent)			1	8	13
Municipality 2					
Demand	19,946	19,946	19,946	19,946	19,946
Satisfied demand	19,946	Infeasible	19,014	19,946	19,849
Number of schools	24		24	23	24
Total cost	$18,371,218		$13,990,656	$12,991,598	$12,835,449
Annual saving			$4,380,562	$5,379,620	$5,535,769
Reduction/base case (percent)			24	29	30
Total teaching hours/month	96,972		72,740	66,156	63,624
Reduction/base case (percent)			25	32	34
Total classes	548		531	490	474
Reduction/base case (percent)			3	11	14

Table A4. Number of Classes per Grade in Optimized Structure for Municipality 1.

School ID	Pre school 1	Pre school 2	Primary school 1	2	3	4	5	6	7	8	High school 1	2	3	4	Total supply	Annual cost, US$
1	1	1	2	2	2	2	2	2	2	2	2	0	0	0	880	667,610
2	0	0	0	0	0	0	0	0	1	1	2	3	4	2	585	411,276
3	1	1	0	0	1	1	1	1	2	2	2	5	7	4	1,240	943,101
4	1	2	1	2	2	2	2	2	2	2	2	3	5	2	1,320	920,190
5	0	0	0	0	0	0	0	0	1	1	1	1	1	0	225	129,547
6	1	1	2	2	2	2	2	2	2	2	3	2	3	2	1,240	898,228
7	0	1	1	2	2	2	2	2	2	2	2	0	0	0	800	610,460
8	2	2	2	3	2	3	4	4	4	4	4	0	0	0	1,490	1,164,836
9	3	4	4	4	4	5	5	5	4	4	4	2	4	1	2,315	1,749,626
10	2	2	1	1	1	1	1	2	1	1	1	0	0	0	590	440,187
11	1	1	1	1	1	1	1	1	1	1	1	0	0	0	475	361,208
12	0	1	2	2	2	2	2	2	1	1	1	0	0	0	710	606,993
13	1	2	2	2	2	2	2	2	2	1	1	0	0	0	825	585,763
14	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
15	2	2	1	1	2	2	2	2	2	2	2	0	0	0	860	625,279
Total	15	20	19	22	23	25	26	27	27	26	28	16	24	11	13,555	10,114,303

Note: Scenario 2 and full demand satisfaction.

Table A5. Number of Classes per Grade in Optimized Structure for Municipality 2.

School ID	Pre school		Primary school								High school				Total supply	Annual cost, US$
	1	2	1	2	3	4	5	6	7	8	1	2	3	4		
1	0	1	0	1	1	2	2	1	2	3	4	3	2	2	1,070	705,749
2	1	1	1	1	1	1	1	1	2	1	3	3	2	2	925	626,613
3	0	1	1	1	1	1	2	2	2	2	6	5	5	4	1,475	960,450
4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
5	1	1	2	2	2	3	3	3	3	3	0	0	0	0	1,015	612,360
6	1	1	1	1	2	2	2	2	2	2	0	0	0	0	700	452,507
7	1	2	1	1	2	2	2	2	2	3	0	0	0	0	780	423,441
8	2	2	2	2	1	1	2	2	2	2	0	0	0	0	770	475,427
9	0	0	1	1	1	1	2	2	2	2	0	0	0	0	540	397,082
10	0	0	1	1	1	1	1	1	1	1	1	1	1	1	540	333,551
11	1	1	1	1	1	2	2	2	2	1	2	1	1	1	835	367,917
12	0	2	2	2	2	2	1	2	1	2	0	0	0	0	700	411,385
13	2	2	2	3	2	2	2	3	1	1	0	0	0	0	860	558,271
14	1	1	1	1	2	1	2	3	1	0	0	0	0	0	565	418,209
15	1	3	2	3	3	2	2	3	3	2	0	0	0	0	1,040	619,332
16	0	0	0	0	0	0	0	0	0	0	4	4	3	3	630	380,085
17	2	4	4	4	4	4	4	4	4	3	0	0	0	0	1,605	881,857
18	1	1	2	3	3	3	3	3	3	3	0	0	0	0	1,105	601,602
19	0	4	5	4	4	4	5	5	5	3	0	0	0	0	1,715	927,945
20	2	2	3	3	3	3	3	3	3	3	0	0	0	0	1,220	662,872
21	1	2	3	3	3	3	3	3	3	3	0	0	0	0	1,185	638,888
22	0	0	0	0	0	0	0	0	5	7	7	7	8	6	1,800	1,179,733
23	0	0	0	0	0	0	0	0	3	2	0	0	0	0	225	195,456
24	0	0	0	0	0	0	0	0	1	1	1	1	1	1	270	160,866
Total	17	31	35	38	39	40	44	47	53	50	28	25	23	20	21,570	12,991,598

Note: Scenario 2 and full demand satisfaction.

Table A6. Optimized Evolution of Municipality 1 and Municipality 2, in a Five-Year Horizon.

		Years				
	Base case	1	2	3	4	5
Municipality 1						
Demand	10,320	10,320	9,860	9,419	9,004	8,612
Satisfied demand	10,320	11,720	11,470	10,770	10,420	10,260
Number of schools	15	14	14	13	12	12
Total cost	US$15,115,718	US$10,460,264	US$10,334,434	US$9,993,221	US$9,855,488	US$9,772,736
Annual saving		US$4,655,454	US$4,781,284	US$5,122,497	US$5,260,230	US$5,342,982
Reduction/base case (%)		31	32	34	35	35
Total teaching hours/month	75,366	40,028	39,140	36,732	35,760	35,176
Reduction/base case (%)		47	48	51	53	53
Total classes	308	268	262	246	238	234
Reduction/base case (%)		13	15	20	23	24
Municipality 2						
Demand	19,946	19,946	18,986	18,073	17,213	16,395
Satisfied demand	19,946	20,385	19,640	18,895	18,195	17,585
Number of schools	24	23	22	21	20	20
Total cost	US$18,371,218	US$12,227,942	US$11,923,004	US$11,662,843	US$11,635,636	US$11,299,524
Annual saving		US$6,143,276	US$6,448,214	US$6,708,375	US$6,735,582	US$7,071,694
Reduction/base case (%)		33	35	37	37	38
Total teaching hours/month	96,972	52,612	50,460	48,624	48,432	46,060
Reduction/base case (%)		46	48	50	50	53
Total classes	548	463	446	429	413	399
Reduction/base case (%)		16	19	22	25	27

Note: Scenario 2 and full demand satisfaction.

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Notes

1. This assumption is consistent with the Chilean education system in which families must apply to schools for their children between August and October each year and formally enroll them in December. With this information, the local planner then takes decisions to open or close grades and schools. This strategy solves the problem of the local planner not knowing the exact allocation for each family since preknowledge of family preferences can be used to coordinate a supply strategy.
2. In the Chilean system, in both public and private schools, contracts are based on the number of hours per week, which is completely flexible. In this sense, the average contract is for thirty-two hours per week rather than a full-time contract, which is based on forty-four hours per week.
3. Statistics Annual Book, MINEDUC 2012.

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