

Contents

1	General introduction	1
1.1	Navier-Stokes equations for an incompressible fluid	1
1.2	Some aspects of the controllability in PDE's	9
1.3	Controllability for the Navier–Stokes equations	12
1.4	Inverse problems in PDE's	15
1.5	Inverse source problems for the Navier-Stokes equations	18
1.6	Contribution of the thesis	19
2	Controllability for the Navier-Stokes with Navier-slip boundary conditions	22
2.1	Introduction	22
2.2	Preliminary results	24
2.3	Carleman inequality for the adjoint system	34
2.4	Null controllability of the linear system	44
2.5	Proof of the main result	49
2.5.1	Nonlinearity on the boundary conditions.	49
2.5.2	Nonlinearity in the main equation.	52
3	First inverse source problem for the Stokes system	54
3.1	Introduction	54
3.2	Uniqueness and reconstruction with one missing component	58
3.3	Convergence of two-parametric optimal controls to null controls with one vanishing component	61
3.4	Numerical examples	64
4	Second inverse source problem for the Stokes system	69
4.1	Introduction	69
4.2	Preliminary results	70
4.2.1	Carleman inequalities	70
4.2.2	Degenerate elliptic equations	71
4.3	Main result	73
A	Degenerate Sobolev spaces	78
A.1	Introduction	78
A.2	Some results in linear degenerate operators	79
	Bibliography	86