



## Editorial

## Preface: LAGOS'13: Seventh Latin-American Algorithms, Graphs, and Optimization Symposium, Playa del Carmen, México – 2013



This issue contains full-length versions of selected papers from those presented at the Seventh Latin-American Algorithms, Graphs, and Optimization Symposium LAGOS'13, held on April 22th–26th, 2013, in Playa del Carmen, Mexico. LAGOS keeps up a tradition of Latin-American discrete mathematics symposia, bringing together researchers from all over the world. It resulted from the merger of the symposia GRACO (held in 2001 in Fortaleza, and in 2005 in Angra dos Reis, Brazil) and LACGA (held in 2004 in Santiago, Chile). Since then, LAGOS has been held biannually: LAGOS'07 in Puerto Varas, Chile; LAGOS'09 in Gramado, Brazil, and LAGOS'11 in Bariloche, Argentina. All these conferences have special issues of DAM dedicated to expanded versions of a selection of the best papers. The most recent, LAGOS'15 took place in Beberibe, Brazil.

The papers in this issue represent only a small fraction of those presented in Playa del Carmen during the meeting. There are interesting contributions to both theoretical and practical aspects of graph theory, algorithms, and optimization. Below we offer a brief glimpse of their contents.

*Slash and burn on graphs – Firefighting with general weights*, by Vitor Costa, Simone Dantas, Mitre C. Dourado, Lucia Penso, and Dieter Rautenbach, shows that it is NP-hard to minimize losses in burning trees in a weighted version of Hartnell's firefighter game (1995), but also provides two approximation algorithms for that.

*A survey-based approach for designing the lines of a rapid transit network*, by L.F. Escudero and S. Muñoz, presents a novel approach for designing the lines of a rapid transit network that behaves better in the experiment than those previously known in the literature.

*Rainbow copies of  $C_4$  in edge-colored hypercubes*, by József Balogh, Michelle Delcourt, Bernard Lidický, and Cory Palmer, determines the maximum number of rainbow colored copies of  $C_4$  in a  $k$ -edge-coloring of the  $d$ -dimensional hypercube when  $4 \leq k < d$  and  $k \neq 5$ .

*Clique cycle-transversals in distance-hereditary graphs*, by Andreas Brandstädt, Simone Esposito, Loana T. Nogueira, and Fábio Protti, characterizes distance-hereditary graphs that contain a clique intersecting all the cycles of the graph. The characterization is given as a list of forbidden induced subgraphs and extends similar results for chordal graphs and cographs.

*Computational hardness of enumerating groundstates of the antiferromagnetic Ising model in triangulations*, by Andrea Jiménez and Marcos Kiwi, studies satisfying spin-assignments of triangulations of a surface which are states of minimum energy of the antiferromagnetic Ising model on triangulations which in turn correspond (via geometric duality) to perfect matchings in cubic bridgeless graphs. It shows that it is NP-complete to decide whether or not a triangulation of a surface admits a satisfying spin-assignment, and that it is #P-complete to determine the number of such assignments. Consequently the determination of even the entropy of the Ising model on triangulations at the thermodynamical limit is already #P-hard.

*Greedy flipping of pancakes and burnt pancakes*, by Joe Sawada and Aaron Williams, proves that a stack of  $n$  pancakes is rearranged in all  $n!$  ways by repeatedly applying the following rule: Flip the maximum number of pancakes that gives a new stack. This complements the previously known pancake flipping Gray code (S. Zaks, A New Algorithm for Generation of Permutations BIT 24 (1984), 196–204) which is also described as a greedy algorithm: Flip the minimum number of pancakes that gives a new stack. These maximum and minimum flip algorithms also rearrange stacks of  $n$  “burn” pancakes in all  $(2^n)n!$  ways.

*Repetition-free longest common subsequence of random sequences*, by Cristina G. Fernandes and Marcos Kiwi, studies the asymptotic behavior of the length  $R$  of a longest common subsequence without repeated symbols of two random strings  $x$  and  $y$ , each of which have  $n$  symbols on a  $k$ -symbol alphabet. It establishes that there are three distinct regimes, depending on the relative speed of growth of  $n$  and  $k$  and it also establishes the limiting behavior in each of the three regimes.

*On the closest point to the origin in transportation polytopes*, by Gilberto Calvillo and David Romero, considers the problem of finding the point in the transportation polytope that is closest to the origin. Recursive formulas to solve it are provided, explaining how they arise from geometrical considerations, via projections, and a solution algorithm with linear computational complexity in the number of variables is provided.

*On the monotonicity of process number*, by Nicolas Nisse and Ronan Pardo Soares, considers the *processing game* in which a team of searchers aims at capturing a fugitive that is invisible, arbitrarily fast, and moves in the direction opposite to the arcs of a digraph, but only as long as it can access a strongly connected component free of searchers. It proves that the processing game is monotone which leads to its equivalence with a new digraph decomposition.

*The cost of perfection for matchings in graphs*, by E.V. Brazil, C.M.H. de Figueiredo, G.D. da Fonseca, and D. Sasaki, considers the ratio between the maximum weight of a perfect matching and the maximum weight of a general matching. It characterizes bridgeless cubic graphs that attain the extreme ratios and presents a lower bound for all bridgeless cubic graphs. This work is indeed a precursor of another article published in *Discrete Applied Mathematics* 207 (2016), 45–55.

*Diffuse reflection diameter in simple polygons*, by Gill Barequet, Sarah M. Cannon, Eli Fox-Epstein, Benjamin Hescott, Diane L. Souvaine, Csaba D. Tóth, and Andrew Winslow, proves a conjecture of Aanjaneya, Bishnu, and Pal that the minimum number of diffuse reflections sufficient to illuminate the interior of any simple polygon with  $n$  walls from any interior point light source is  $\lfloor n/2 \rfloor - 1$ .

*Algorithms to approximately count and sample conforming colorings of graphs*, by Sarah Miracle and Dana Randall, studies conforming colorings of multigraphs: vertex colorings avoiding a predefined color pair on each edge. Conforming colorings generalize many natural graph theoretic concepts, including independent sets, vertex colorings, list colorings, H-colorings, and adapted colorings. The paper then introduces Markov chains on the set of conforming colorings and provides general conditions for when they can be used to design efficient Monte Carlo algorithms for sampling and approximate counting.

*Component-cardinality-constrained critical node problem in graphs*, by M. Lalou, M.A. Tahraoui, and H. Kheddouci, studies the Component-Cardinality-Constrained Critical Node Problem (3C-CNP), which seeks to find a minimal set of nodes whose removal constrains the size of each connected component in the induced graph to a given bound. The paper proves several NP-hardness results for 3C-CNP and related problems.

*Exploring the concept of perfection in 3-hypergraphs*, by Natalia García-Colín, Amanda Montejano, and Deborah Oliveros, studies a natural extension to hypergraphs of the notion of perfection in graphs and finds that, contrary to the case of graphs, comparability hypergraphs (which are also a natural extension of comparability graphs) are not necessarily perfect.

*Hull number: P5-free graphs and reduction rules*, by J. Araujo, G. Morel, L. Sampaio, R. Soares, and V. Weber, proposes a fixed parameter tractable algorithm to compute the geodesic hull number of any graph. Here, the hull number is the minimum number of vertices of a hull set. Relevant definitions are as follows: For any two vertices  $u, v \in V$  of a connected undirected graph  $G = (V, E)$ , the closed interval  $I[u, v]$  of  $u$  and  $v$  is the set of vertices that belong to some shortest  $(u, v)$ -path. For any  $S \subseteq V$ , let  $I[S] = \bigcup_{u, v \in S} I[u, v]$ . A subset  $S \subseteq V$  is (geodesically) convex if  $I[S] = S$ . Given a subset  $S \subseteq V$ , the convex hull  $I_h[S]$  of  $S$  is the smallest convex set that contains  $S$ . We say that  $S$  is a hull set of  $G$  if  $I_h[S] = V$ . The size of a minimum hull set of  $G$  is the hull number of  $G$ , denoted by  $hn(G)$ .

*Lift-and-project ranks of the stable set polytope of joined  $a$ -perfect graphs*, by S. Bianchi, M. Escalante, and M.S. Montelar, studies lift-and-project polyhedral operators defined by Lovász and Schrijver and Balas, Ceria and Cornuéjols on the clique relaxation of the stable set polytope of webs. The paper computes the disjunctive rank of all webs and consequently of antiwebs.

*Edge intersection graphs of L-shaped paths in grids*, by Kathie Cameron, Steven Chaplick, and Chinh T. Hoàng, studies the class of B1-EPG graphs (and subclasses of it), which are edge intersection graphs of the shapes in  $\{\perp, \sqcap, \sqcup, \sqsupset\}$  (and subsets of it). It obtains NP-completeness results for the recognition of these classes.

*Null and non-rainbow colorings of projective plane and sphere triangulations*, by Jorge L. Arocha and Amanda Montejano, applies homology theory to graphs and proves that for maximal planar graphs (sphere triangulations) of order  $n \geq 4$ , a vertex-coloring containing no rainbow faces uses at most  $\lfloor \frac{2n-1}{3} \rfloor$  colors, and this is best possible. For a maximal graph embedded on the projective plane, the analogous best bound  $\lfloor \frac{2n+1}{3} \rfloor$  is obtained.

*On Ramsey numbers of complete graphs with dropped stars*, by Jonathan Chappelon, Luis Pedro Montejano, and Jorge Luis Ramírez Alfonsín: Let  $r(G, H)$  be the smallest integer  $N$  such that for any 2-coloring (say, red and blue) of the edges of  $K_n$ ,  $n \geq N$ , there is either a red copy of  $G$  or a blue copy of  $H$ . Let  $K_n - K_{1,s}$  be the complete graph on  $n$  vertices from which the edges of  $K_{1,s}$  are dropped. This paper presents present exact values for  $r(K_m - K_{1,1}, K_n - K_{1,s})$  and new upper bounds for  $r(K_m, K_n - K_{1,s})$  in numerous cases.

*Almost empty monochromatic triangles in planar point sets*, by Deepan Basu, Kinjal Basu, Bhaswar Bikram Bhattacharya, and Sandip Das: For positive integers  $c, s \geq 1$ , let  $M_3(c, s)$  be the least integer such that any set of at least  $M_3(c, s)$  points in the plane, no three on a line and colored with  $c$  colors, contains a monochromatic triangle with at most  $s$  interior points. It is proved in this paper that the least integer  $\lambda_3(c)$  such that  $M_3(c, \lambda_3(c)) < \infty$  satisfies:

$$\left\lfloor \frac{c-1}{2} \right\rfloor \leq \lambda_3(c) \leq c-2,$$

where  $c \geq 2$ . Moreover, the exact values of  $M_3(c, s)$  are determined for small values of  $c$  and  $s$ .

*Generalized minor inequalities for the set covering polyhedron related to circulant matrices*, by Paola B. Tolomei and Luis M. Torres, studies the set covering polyhedron related to circulant matrices and presents a family of inequalities that generalizes the family of minor inequalities previously reported in the literature.

*A polyhedral study of the maximum stable set problem with weights on vertex-subsets*, by Manoel Campêlo, Victor A. Campos, Ricardo C. Corrêa, Diego Delle Donne, Javier Marenco, and Marcelo Mydlarz: Given a graph  $G = (V, E)$ , a family of nonempty

vertex-subsets  $S \subseteq 2^V$ , and a weight  $w : S \rightarrow \mathbb{R}_+$ , the maximum stable set problem with weights on vertex-subsets consists of finding a stable set  $I$  of  $G$  maximizing the sum of the weights of the sets in  $S$  that intersect  $I$ . A polyhedral study of this problem is performed in this paper, by introducing a natural integer programming formulation and studying the associated polytope. It addresses general facts on this polytope including some lifting results; it provides connections with the stable set polytope; and it presents three families of facet-inducing inequalities.

*Some results on the structure of kernel-perfect and critical kernel-imperfect digraphs*, by Hortensia Galeana-Sánchez and Mucuy-kak Guevara: A kernel  $N$  of a digraph  $D$  is an independent set of vertices of  $D$  such that for every  $w \in V(D) - N$  there exists an arc from  $w$  to  $N$ . The digraph  $D$  is said to be a kernel-perfect digraph when every induced subdigraph of  $D$  has a kernel. Minimal non kernel-perfect digraphs are called critical kernel-imperfect digraphs. The broadest sufficient condition for the existence of kernels in digraphs known so far is: (1) If  $D$  is a digraph such that every odd cycle has two consecutive poles, then  $D$  is kernel perfect. This paper studies the structure of critical kernel imperfect digraphs that belong to a very large special class of digraphs, and many structural properties are obtained. (1) is widely generalized in this class of digraphs, where the condition of the poles is requested only for odd cycles whose edges alternate in a set of arcs. As consequence, some classic results of kernel perfect and finite critical kernel imperfect digraphs are generalized for these classes of digraphs.

*Approximate robust optimization for the Connected Facility Location problem*, by M. Gisela Bardossy and S. Raghavan, considers the Robust Connected Facility Location (ConFL) problem within the robust discrete optimization framework introduced by Bertsimas and Sim (in *Mathematical Programming* 98 (2003), 49–71) and proposes an Approximate Robust Optimization (ARO) method that uses a heuristic and a lower bounding mechanism to rapidly find high-quality solutions.

*On basic chordal graphs and some of its subclasses*, by Pablo De Caria and Marisa Gutierrez, studies the connection between basic chordal graphs and some subclasses of chordal graphs with special clique trees, like  $DV$  graphs and  $RDV$  graphs, and studies the behavior of the clique operator over all the considered subclasses.

*Near-linear-time algorithm for the geodetic Radon number of grids*, by Mitre Costa Dourado, Vinícius Gusmão Pereira de Sá, Dieter Rautenbach, and Jayme Luiz Szwarcfiter: The Radon number of a graph is the minimum integer  $r$  such that all sets of at least  $r$  of its vertices can be partitioned into two subsets whose convex hulls intersect. Determining the Radon number of general graphs in the geodetic convexity is NP-hard. This paper shows the problem is polynomial for  $d$ -dimensional grids, for all  $d \geq 1$ . The proposed algorithm runs in near-linear  $\mathcal{O}(d \log d)^{1/2}$  time for grids of arbitrary sizes, and in sublinear  $\mathcal{O}(\log d)$  time when all grid dimensions have the same size.

*Edge separators for quasi-binary trees*, by Jorge Ramírez Alfonsín and Serge Tishchenko, investigates  $k$ -separator for quasi-binary trees, which are sets of  $k - 1$  edges of a vertex weighted tree such that, after removal of these edges, the weights of the  $k$  induced connected components are approximately the same. The paper shows that, under certain conditions on the total weight of the tree, a particular  $k$ -separator can be constructed such that the smallest (respectively the largest) weighted component is lower (respectively upper) bounded. Examples showing optimality for the lower bound are also given.

*A note on the middle levels problem*, by Andréia C.S. Gusmão, Letícia R. Bueno, Rodrigo A. Hausen, Celina M.H. Figueiredo, and Luerbio Faria, provides sufficient conditions for the existence of a Hamiltonian path in the Kneser graph  $K(2k + 3, k)$  for  $k \equiv 1$  or  $2 \pmod{3}$ .

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