Applied Soft Computing 47 (2016) 141–150

Contents lists available at ScienceDirect

Applied Soft Computing

journal homepage: www.elsevier.com/locate/asoc

Aggregation operators in economic growth analysis and entrepreneurial group decision-making

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ARTICLE INFO

Article history:
Received 11 August 2013
Received in revised form 27 March 2016
Accepted 23 May 2016
Available online 26 May 2016

Keywords:
Economic growth
Entrepreneurial group decision making
Aggregation systems
Uncertainty

ABSTRACT

An economic crisis can be measured from different perspectives. A very commonly used measure is that of a country's economic growth. When growth is lower than desired, the economy is assumed to be near stagnation or in an economic recession. This paper connects entrepreneurship and economic growth in decision-making problems assessed with modern aggregation systems. Aggregation techniques can represent information more comprehensively in uncertain and imprecise environments. This paper suggests several practical aggregation operators for this purpose, such as the ordered weighted average and the probabilistic ordered weighted averaging weighted average. Other aggregation systems based on macroeconomic theory are also introduced. The paper concludes with an application in an entrepreneurial uncertain multi-criteria multi-person decision-making problem regarding the selection of optimal markets for creating a new company. This approach is based on the use of economic growth as the fundamental variable for determining the preferred solution.

1. Introduction

An economic crisis is a situation where a country or a region's economy is near stagnation or has a negative economic growth rate. Currently, the world is in the grips of a severe economic crisis that has affected most of the developed countries for several years. In this environment, creating a new company or business is not an easy task as expected profits will be lower than foreseen, and where sometimes the initial investment are lost since the market does not provide enough income. Therefore, the incentives for embarking in entrepreneurship are low and analysis is required to find the most appropriate markets for entrepreneurial activities by either individual [1,2] or corporate entrepreneurs [3]. Prolonged economic crises tend to modify the cultural and institutional context of entrepreneurship [4–6] and, in such case, instruments that facilitate entrepreneurial action are needed to compensate for the negative environment.

A useful methodology for identifying optimal markets for entrepreneurship is to analyze a country's expected growth rate. If a country's economy is expected to expand the next year, it is assumed to be a good place for entrepreneurial activities and vice versa. The information used to develop such forecasts is usually affected by a wide range of imprecisions and uncertainties, and various criteria and attributes may influence the expected results. Thus, aggregation systems are required to assess the information [7,8].

A very popular aggregation operator in uncertain environments is the ordered weighted average (OWA) [9]. This aggregation operator provides a parameterized family of aggregation operators between the minimum and the maximum. Since its appearance, it has been studied by many authors [10,11]. A practical generalization of the OWA operator is integration with the weighted average. Several authors have studied this issue, including Torra [12] with the introduction of the weighted OWA (WOWA) operator, Yager [13] with the importance of OWA and Xu and Da [14] with the hybrid average. Recently, Merigó [15] has suggested a new generalization that integrates the OWA operator and the weighted average considering the degree of importance that each concept has in the

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http://dx.doi.org/10.1016/j.asoc.2016.05.031
1568-4946/

aggregation. He called it the ordered weighted averaging weighted average (OWAWA).

When dealing with imprecise environments an additional issue of interest is that the information cannot be assessed with exact numerical values, although other techniques can be used such as interval numbers [16], fuzzy systems [17,18] and linguistic variables [19,20]. The main advantage of interval numbers is that they represent numerical information that takes into account the minimum, the maximum and the most likely results. From this it is possible to develop uncertain aggregation operators [21–23] such as the commonly used uncertain OWA (UOWA) operator: an extension of the OWA operator to an uncertain environment that can be assessed with interval numbers. It is also possible to extend the OWAWA operator to an uncertain environment forming the uncertain OWAWA (UOWAWA) operator.

This paper presents a new approach for analyzing an economy's economic environment. It introduces several aggregation systems that analyze the expected economic growth rate of a country or a region. Several types of OWAWA operators are used to assess the information, considering subjective opinions and analysts' attitudes in the specific problem considered. Various properties and particular cases of this approach are presented, including an extension for situations where several experts provide their own opinions in order to form the decision. We introduce some examples including the economic growth OWA (EG-OWA) and the economic growth OWAWA (EG-OWAWA) operator. Observe that some other extensions are considered with uncertain information that can be assessed with interval numbers and the UOWAWA operator.

An illustrative example is presented that analyzes the optimal market for new entrepreneurial activities. It is assumed that a key variable for such activities is the economic environment of an economy that can be analyzed with a country's economic growth rate. Several scenarios and expert opinions are considered to provide indicative results regarding an economy's expected economic growth rate. Obviously, if the economic growth rate is positive, it should be easier to implement new entrepreneurial activities and conversely, if the rate is negative, it should be more difficult. Therefore, the optimal decision will be to select the market with a higher expected growth rate for the next period. However, it is worth noting that many other variables should be considered in the problem. In this example, the objective is to present a general overview of how a decision-maker should analyze the decision to develop new entrepreneurial activities in a new market according to a country's or a region's economic environment.

The paper is organized as follows. Section 2 reviews some basic preliminaries regarding aggregation operators, economic crises and economic growth. Section 3 develops a new framework for analyzing economic growth with aggregation systems. Section 4 introduces a multi-criteria multi-person decision-making problem for entrepreneurship assessed with economic growth. Section 5 presents an illustrative example and Section 6 summarizes the main conclusions of the paper.

2. Preliminaries

2.1. Interval numbers

Interval numbers [16] are a very useful simple technique for representing uncertainty. They have been used in a wide range of applications and can be defined as follows.

Definition 1. Let $a = [a_1, a_2] = \{x \mid a_1 \le x \le a_2\}$, then a is called an interval number. Note that a is a real number if $a_1 = a_2$.

Interval numbers can be represented in different ways. For example, assume a 4-tuple $[a_1, a_2, a_3, a_4]$, and let a_1 and a_4 represent the minimum and the maximum of the interval number, and a_2 and a_3 the internal values with the highest possibility of occurrence. Note that $a_1 \le a_2 \le a_3 \le a_4$. If $a_1 = a_2 = a_3 = a_4$, the interval number is an exact number. If $a_2 = a_3$, it is a triplet, and if $a_1 = a_2$ and $a_3 = a_4$, it is a simple 2-tuple interval number.

We review some basic operations below. Let A and B be two triplets, where $A = [a_1, a_2, a_3]$ and $B = [b_1, b_2, b_3]$.

1. $A + B = [a_1 + b_1, a_2 + b_2, a_3 + b_3]$.
2. $A - B = [a_1 - b_3, a_2 - b_2, a_3 - b_1]$.
3. $A \times k = [k \times a_1, k \times a_2, k \times a_3]$, for $k > 0$.
4. $A \times B = [\min(a_1 \times b_1, a_1 \times b_3, a_3 \times b_1, a_3 \times b_3), a_2 \times b_2, \max(a_1 \times b_1, a_1 \times b_3, a_3 \times b_1, a_3 \times b_3)]$, for R.
5. $A/B = [\min(a_1/b_1, a_1/b_3, a_3/b_1, a_3/b_3), a_2/b_2, \max(a_1/b_1, a_1/b_3, a_3/b_1, a_3/b_3)]$, for R.

The intervals are sometimes difficult to rank because it is not clear which interval number is higher, so we must establish an additional criterion for ranking interval numbers [24]. For simplicity reasons, we use the following method throughout the paper. For 2-tuples, calculate the arithmetic mean: $(a_1 + a_2)/2$. For 3-tuples and above, calculate a weighted average that yields more importance to the central values: $(a_1 + 4a_2 + a_3)/6$. For 4-tuples, we calculate: $(a_1 + 4a_2 + 4a_3 + a_4)/10$, and so on. In the case of a tie between the intervals, we select the interval with the lowest difference, i.e., $(a_2 - a_1)$. For 3-tuples and above odd-tuples, we select the interval with the highest central value.

The main advantage of this method is that we can reduce the interval number to a representative exact number of the interval. As a general framework for dealing with ranking processes, let us assume the following methodology for two intervals $A = [a_1, a_2]$ and $B = [b_1, b_2]$:

1. If $a_1 > b_2$, then, $A > B$ always.
2. If not, then it is not 100% clear which interval is higher and one of the ranking methods explained above must be used. To simplify matters, a simple or weighted average of the intervals is recommended.
3. If there is still a tie, this implies that the interval provides similar values and more subjective methods are required. In this case, it is recommended to consider the highest interval as the one with lowest imprecision, i.e., A > B if $(a_2 - a_1) < (b_2 - b_1)$.

Note that other operations and ranking methods could be studied [16], but in this paper the focus is on those discussed above.

2.2. Aggregation systems

When analyzing information, it usually becomes necessary to deal with a wide range of different aspects that have to be collected and displayed in simple results. Aggregation operators can collect different information and provide a single or a practical summarized result. Aggregation systems can be seen as a generalization of the aggregation operators representing the entire structure involved in the aggregation process that may involve a wide range of aggregation operators. The literature contains many different types of aggregation operators [7,8,25]. Among others, the weighted average and the ordered weighted average (OWA) are some of the most popular ones [9]. The OWA operator provides a parameterized family of aggregation operators between the minimum and the maximum. It can collect information and provide the box-plot with interval numbers as outputs or similar results [26]. Moreover, the information can be represented so that it can be under- or overestimated, considering the decision-maker's attitudinal character. This can be defined as follows.

Definition 2. An OWA operator of dimension n is a mapping OWA: $R^n \rightarrow R$ that has an associated weighting vector W of dimension n with $\sum_{j=1}^{n} w_j = 1$ and $w_j \in [0, 1]$, such that:

$$\text{OWA}\,(a_1, a_2, \ldots, a_n) = \sum_{j=1}^{n} w_j b_j \tag{1}$$

where b_j is the jth smallest of the a_i.

The OWA operator is commutative, monotonic, bounded and idempotent. It is commutative because OWA $(a_1, a_2, \ldots, a_n) = f(e_1, e_2, \ldots, e_n)$ where $(a_1, a_2, \ldots, a_n)$ is any permutation of the arguments $(e_1, e_2, \ldots, e_n)$. It is monotonic because if $a_i \geq u_i$, for all a_i, then, OWA $(a_1, a_2, \ldots, a_n) \geq$ OWA $(u_1, u_2, \ldots, u_n)$. It is bounded because the OWA aggregation is delimitated by the minimum and the maximum. That is, $\text{Min}\{a_i\} \leq \text{OWA}\,(a_1, a_2, \ldots, a_n) \leq \text{Max}\{a_i\}$. It is idempotent because if $a_i = a$, for all a_i, then, OWA $(a_1, a_2, \ldots, a_n) = a$. It includes a wide range of particular cases including the minimum, the average and the maximum. Since being introduced, it has been studied by many authors [11,27,28].

Sometimes, the available information is imprecise and cannot be assessed with usual exact numbers. These situations need another approach, such as the use of interval numbers to represent imprecise information considering minimum and maximum potential results. Some very well-known approaches are the uncertain average (UA), the uncertain weighted average (UWA) and the uncertain OWA (UOWA) operator [14,29]. The UWA operator can be defined as follows.

Definition 3. Let Ω be the set of interval numbers. A UWA operator of dimension n is a mapping UWA: $\Omega^n \rightarrow \Omega$ that has an associated weighting vector W of dimension n with $w_i \in [0, 1]$ and $\sum_{i=1}^{n} w_i = 1$, such that:

$$\text{UWA}\,(\tilde{a}_1, \tilde{a}_2, \ldots, \tilde{a}_n) = \sum_{i=1}^{n} w_i \tilde{a}_i \tag{2}$$

where $\tilde{a}_i$ is an interval number.

Some other interesting aggregation operators are those that unify the weighted average and the OWA operator. In the literature, there are many approaches in this context including the weighted OWA (WOWA) operator [12], the hybrid average [14], the importance OWA [13] and immediate weights [30]. Recently, Merigó [15] has suggested a new approach that unifies both aggregations considering the degree of importance of each concept in the analysis. He calls it the OWA weighted average (OWAWA). When dealing with uncertain information that can be assessed with interval numbers, it becomes the uncertain OWAWA (UOWAWA) operator. It is defined as follows.

Definition 4. Let Ω be the set of interval numbers. A UOWAWA operator of dimension n is a mapping UOWAWA: $\Omega^n \rightarrow \Omega$ that has an associated weighting vector W of dimension n such that $w_j \in [0, 1]$ and $\sum_{j=1}^{n} w_j = 1$, according to the following formula:

$$\text{UOWAWA}\,(\tilde{a}_1, \tilde{a}_2, \ldots, \tilde{a}_n) = \sum_{j=1}^{n} \hat{v}_j b_j \tag{3}$$

where b_j is the jth largest of the $\tilde{a}_i$, each argument $\tilde{a}_i$ has an associated weight v_i with $\sum_{i=1}^{n} v_i = 1$ and $v_i \in [0, 1]$, $\hat{v}_j = \beta w_j + (1-\beta) v_j$ with $\beta \in [0, 1]$ and v_j is the weight (WA) v_i ordered according to b_j, that is, according to the jth largest of the a_i.

As we can see, if $\beta = 1$, we obtain the UOWA operator and if $\beta = 0$, the UWA. Note that if the weighting vector is not normalized, i.e., $W = \sum_{j=1}^{n} w_j \neq 1$, or $V = \sum_{j=1}^{n} v_i \neq 1$, the UOWAWA operator can be formulated as follows:

$$\text{UOWAWA}\,(\tilde{a}_1, \tilde{a}_2, \ldots, \tilde{a}_n) = \frac{1}{\hat{V}} \sum_{j=1}^{n} \hat{v}_j b_j \tag{4}$$

For practical simplification, it is assumed that the β parameter and the weights W and V are exact numbers. Nevertheless, it is also possible to consider situations where they are imprecise and represented with interval numbers.

Note that in this paper, the main assumption is that information is very uncertain and can only be assessed with interval numbers. However, it is possible to use other tools when dealing with imprecise and complex environments including fuzzy numbers [31] and linguistic variables [32]. By using fuzzy numbers, the operators used would be the fuzzy weighted average (FWA) [33], the fuzzy OWA (FOWA) [31] and the fuzzy OWAWA (FOWAWA) operator [33]. The mathematical formulation is straightforward by looking at the UWA and UOWAWA operators.

2.3. Economic crisis and economic growth

Economic crisis can be defined as a scenario where a region's economic situation is considered to show lower than desired or negative growth. Usually, this is assumed when the gross domestic product (GDP) growth rates are near stagnation or negative over time. Economists consider that an economic crisis can be considered significant when the GDP contracts for more than a half year. Throughout history, the world has experienced many economic crises and the current financial crisis has been severely affecting developed economies since 2007–2008 [34].

Using a mathematical notation, let us assume that a region is in an economic crisis if $\text{GDP}_t < \text{GDP}_{t-1}$, where GDP_t represents the GDP for year t and GDP_{t-1} for year $t-1$. There are obviously many explanatory factors. Before making a brief mathematical analysis, let us look into the concept of economic growth. Note that here we have assumed only one year for the analysis but it is possible to consider two $(t-2)$ or more $(t-n)$.

Economic growth (EG) is an economic variable that represents the evolution of the economy over time. It analyzes the increase or decrease in economic output over different periods of time. It can be studied for a wide range of entities including countries, regions, cities and companies. It is usually measured as the percentage rate of increase in the real GDP. That is:

$$\text{Economic growth} = \left(\frac{\text{GDP}_t}{\text{GDP}_{t-1}} - 1\right) \times 100 \tag{5}$$

$\text{GDP}_t > \text{GDP}_{t-1}$ indicates that EG > 0 and thus it is positive. An economy is usually assumed to be experiencing economic expansion if EG is well above 0. If $\text{GDP}_t < \text{GDP}_{t-1}$ growth is negative and thus the economy is in recession.

The literature contains many studies of EG by economists [35], especially since the mid-1950s when Solow [36] introduced his growth model. Since then, many economists have studied a wide range of different variables and methodologies in order to understand growth theory. Some of the key assumptions of growth are based on the inputs of capital, labor and technological development.

EG also depends on the levels of GDP and can therefore be analyzed from the variables that shape GDP [37]. Usually, it is assumed that the GDP in an open economy is formed by:

$$\text{GDP}(Y) = C + I + G + X - M \tag{6}$$

where C is consumption, I investment, G government spending, X exports and M imports. Therefore, all these variables can be analyzed in order to understand EG. That is:

$$\text{Economic growth} = \left(\frac{(C+I+G+X-M)_t}{(C+I+G+X-M)_{t-1}} - 1 \right) \times 100 \quad (7)$$

The equation above can be sub-divided by introducing in each variable the transformation C_{t-1}/C_{t-1}, I_{t-1}/I_{t-1}, and so on, in order to find the main changes in the observed period. Thus, EG becomes:

$$EG = \left(\frac{C_{t-1}}{GDP_{t-1}} \left(\frac{C_t}{C_{t-1}} - 1 \right) + \frac{I_{t-1}}{GDP_{t-1}} \left(\frac{I_t}{I_{t-1}} - 1 \right) + \frac{G_{t-1}}{GDP_{t-1}} \left(\frac{G_t}{G_{t-1}} - 1 \right) + \frac{X_{t-1}}{GDP_{t-1}} \left(\frac{X_t}{X_{t-1}} - 1 \right) - \frac{M_{t-1}}{GDP_{t-1}} \left(\frac{M_t}{M_{t-1}} - 1 \right) \right) \times 100 \quad (8)$$

It could be represented by a non-monotonic weighted average [38] as follows:

$$EG = \left(WA_C \left(\frac{C_t}{C_{t-1}} - 1 \right) + WA_I \left(\frac{I_t}{I_{t-1}} - 1 \right) + WA_G \left(\frac{G_t}{G_{t-1}} - 1 \right) + WA_X \left(\frac{X_t}{X_{t-1}} - 1 \right) - WA_M \left(\frac{M_t}{M_{t-1}} - 1 \right) \right) \times 100 \quad (9)$$

where WA is the weighted average for each considered variable. It is worth noting that it is nonmonotonic because it has a negative weight in the imports. Note that each WA should be calculated according to the importance of the variables in the GDP. For example, it could be calculated considering the GDP from the previous year or the expected GDP for the next period but other situations can also be considered in order to make different types of forecasts. Note that if the degree of importance is not known, some type of OWA, OWAWA or POWAWA aggregation [9,15,39] can be used in order to under -or -overestimate the expected results. Furthermore, any other type of aggregation operator could be used depending on the information available for the specific problem under consideration.

GDP can also be determined by the income method [40] that consists in adding up the total income earned by all domestic households and firms. Thus:

$$GDP(Y) = WS + CP + OI \quad (10)$$

where WS are wages and salaries, CP corporate profits and OI other incomes. In this case, it is possible to use a similar methodology to that used for EG. That is:

$$\text{Economic growth} = \left(\frac{(WS+CP+OI)_t}{(WS+CP+OI)_{t-1}} - 1 \right) \times 100 \quad (11)$$

The equation can also be divided using the transformations WS_{t-1}, CP_{t-1} and OI_{t-1} as follows:

$$EG = \left(\frac{WS_{t-1}}{GDP_{t-1}} \left(\frac{WS_t}{WS_{t-1}} - 1 \right) + \frac{CP_{t-1}}{GDP_{t-1}} \left(\frac{CP_t}{CP_{t-1}} - 1 \right) + \frac{OI_{t-1}}{GDP_{t-1}} \left(\frac{OI_t}{OI_{t-1}} - 1 \right) \right) \times 100 \quad (12)$$

Thus, we could use the weighted average (WA) in order to simplify the equation as follows:

$$EG = \left(WA_{WS} \left(\frac{WS_t}{WS_{t-1}} - 1 \right) + WA_{CP} \left(\frac{CP_t}{CP_{t-1}} - 1 \right) + WA_{OI} \left(\frac{OI_t}{OI_{t-1}} - 1 \right) \right) \times 100 \quad (13)$$

Furthermore, if the weights are not known, other aggregation operators such as the OWA and the POWAWA operator can be used. Note that the use of this type of aggregation operators becomes relevant when working with future forecasts where the degree of importance of each variable is not known, such as in $t+2$, $t+3$ and so on. See Section 3 for the explanation.

Finally, it is worth noting that there are many other approaches for analyzing GDP, such as the output method [40]. The usefulness of the aforementioned equations is that they can identify which variables have been strongly affected by an increase or decrease. But from a microeconomic perspective, other issues could be taken into account because many other variables may affect the problem.

3. Aggregation systems in the analysis of economic growth

Economic growth (EG) is a fundamental variable for measuring economic situations. Positive growth is usually assumed to indicate an economy in expansion and negative growth one in recession. As mentioned in the previous section, EG can be studied from different perspectives. In this Section, the focus is to present the use of a wide range of aggregation systems in EG in order to form more efficient measures for assessing the information.

First, let us look into the use of simple aggregation operators in EG. EG can be affected by attributes, criteria, states of nature and experts' opinions. Thus, EG may differ depending on the criteria considered. Therefore, in order to consider all the available information, aggregation operators are needed:

$$EG(EG_1, EG_2, \ldots, EG_c, \ldots, EG_n = \sum_{c=1}^{n} w_c EG_c \quad (14)$$

where EGc is the economic growth according to the cth criterion and w_c the cth weight that represents the importance of each criterion such that $w_c \in [0, 1]$ and $\sum_{c=1}^{n} w_c = 1$. Moreover, more variables can be included in the problem such as the opinions of several experts. In this case, Eq. (14) can be generalized as follows:

$$EG\left([EG_1{}^1, \ldots, EG_1{}^m], \ldots, [EG_n{}^1, \ldots, EG_n{}^m]\right) = \sum_{c=1}^{n} \sum_{e=1}^{m} w_c v_c^e EG_c^e \quad (15)$$

where $EG_c{}^e$ is EG according to each cth criterion and eth expert, w_c the cth weight with the importance of each criterion such that $w_c \in [0, 1]$ and $\sum_{c=1}^{n} w_c = 1$, and $v_c{}^e$ the weight for each expert e and criterion c such that $v_c{}^e \in [0, 1]$ and $\sum_{e=1}^{m} v_c^e = 1$.

Further generalizations could be developed by adding additional interpretations in the formula such as additional attributes, objectives and so on. Moreover, if the available information is imprecise, it is possible to use interval numbers. Thus, the expected economic growth could be denoted by an interval as EG = [EG_{Min}, EG_{Max}] and so on for any considered sub-variable according to criteria and experts.

Note that Eqs. (14) and (15) assume an aggregation process using the weighted average. However, many other types of aggregation operators can be used, including the OWA, the OWAWA, the POWAWA, the PWA and the UAO operator. Note that the OWA operator is useful when we do not know the importance of each variable but wish to aggregate them. Therefore, we may consider many methods for dealing with uncertainty such as the minimum, the maximum and the average. Hence the following expression:

$$\text{EG-OWA}(\text{EG}_1, \text{EG}_2, \ldots, \text{EG}_c, \ldots, \text{EG}_n = \sum_{j=1}^{n} w_j b_j \tag{16}$$

where b_j is the jth lowest expected EG and w_j is the jth weight that represents the attitudinal character (degree of optimism) such that $w_j \in [0, 1]$ and $\sum_{j=1}^{n} w_j = 1$. Note that if $w_n = 1$ and $w_j = 0$ for all $j \neq n$, the EG-OWA becomes the Maximum, that is, the most optimistic EG. If $w_1 = 1$ and $w_j = 0$ for all $j \neq 1$, the EG-OWA becomes the Minimum, that is, the most pessimistic EG.

More in general, other approaches could be used such as the OWAWA operator. By using the OWAWA it is assumed that the information is aggregated with two different sources of information based on attitudinal character and subjective belief regarding the importance of the variables. Hence the following formulation:

$$\text{EG-OWAWA}(\text{EG}_1, \text{EG}_2, \ldots, \text{EG}_c, \ldots, \text{EG}_n = \sum_{j=1}^{n} \hat{v}_j b_j \tag{17}$$

where b_j is the jth lowest expected EG_c, w_j is the jth weight that represents attitudinal character (degree of optimism) such that $w_j \in [0, 1]$ and $\sum_{j=1}^{n} w_j = 1$, each argument EG has an associated weight v_i with $\sum_{i=1}^{n} v_i = 1$ and $v_i \in [0, 1]$, $\hat{v}_j = \beta w_j + (1-\beta) v_j$ with $\beta \in [0, 1]$ and v_j is the weight v_i ordered according to b_j, that is, according to the jth lowest EG_c.

Observe that with the OWAWA operator it is also possible to consider many particular cases such as the minimum, the maximum and many others [15,41], in particular the following:

- Maximum: $w_n = 1$ and $w_j = 0$ for all $j \neq n$ and $\beta = 1$.
- Minimum: $w_1 = 1$ and $w_j = 0$ for all $j \neq 1$ and $\beta = 1$.
- Arithmetic mean: $w_j = 1/n$ for all j and $p_i = 1/n$ for all i.
- Max-WA: $w_n = 1$ and $w_j = 0$ for all $j \neq n$.
- Min-WA: $w_1 = 1$ and $w_j = 0$ for all $j \neq 1$ and $\beta = 1$.
- OWA: If $\beta = 1$.
- Weighted average: If $\beta = 0$.

Additionally, it is possible to extend this framework to uncertain environments with imprecise data assessed with interval numbers. This would form the economic growth UOWAWA (EG-UOWAWA) and the economic growth UOWA (EG-UOWA) operator, respectively.

Now let us look at the aggregation process when considering that economic growth is affected by the different constituent economic variables. As explained in Eq. (5), EG can be calculated using the GDP of the present year t and the previous period $t-1$. Thus, it is also possible to introduce Eq. (5) in Eqs. (16) and (17). Furthermore, by decomposing GDP into consumption, investment, government spending, exports and imports as shown in Eq. (6), it is also possible to introduce it in Eq. (16) and (17).

Following the transformations shown in Eqs. (8) and (9), it is also possible to study an additional aggregation framework when dealing with EG. For example, if the objective is to analyze EG in the time period $t+3$, that is, three years in the future, this can be studied with different types of aggregation operators. In these situations, the degree of importance of each variable that constitutes the GDP is often not known or only partially known. Therefore, an OWA or a POWAWA operator can be used:

$$\text{EG} = \left(\text{POWAWA}_C \left(\frac{C_{t+3}}{C_{t+2}} - 1 \right) + \text{POWAWA}_I \left(\frac{I_{t+3}}{I_{t+2}} - 1 \right) + \text{POWAWA}_G \left(\frac{G_{t+3}}{G_{t+2}} - 1 \right) + \text{POWAWA}_X \left(\frac{X_{t+3}}{X_{t+2}} - 1 \right) + \text{POWAWA}_M \left(\frac{M_{t+3}}{M_{t+2}} - 1 \right) \right) \times 100 \tag{18}$$

where POWAWA is weight $\hat{v}_i = \alpha_1 p_i + \alpha_2 v_i + \alpha_3 w_i$, with α_1, α_2 and α_3 being the degree of importance that the initial information (probability), subjective beliefs and attitudinal character (OWA) have in the specific aggregation considered, and w_i is the OWA weight w_j ordered according to the initial position i. It is worth noting that this type of aggregation is a nonmonotonic-POWAWA operator because it uses one negative weighting vector in the imports: $\hat{V} = (\hat{V}_C, \hat{V}_I, \hat{V}_G, \hat{V}_X, -\hat{V}_M)$ and the sum of the weights is equal to one. For further information on non-monotonic aggregations see Yager [38].

Note that the POWAWA operator seems to be more practical because the present weighted average regarding the degree of importance of each variable (seen as the probabilistic information) can be used and adjusted with some type of OWAWA that represents our belief concerning its future importance and our degree of optimism or pessimism that under- or overestimates the information. Following Merigó [42], the probabilistic weighted average (PWA) could be used assuming that the probabilistic information is the known information and the WA is our beliefs about the future. In this case, an OWA aggregation that assumes optimistic or pessimistic information is avoided. Furthermore, any particular type of POWAWA operator could be studied in the problem depending on the assumptions made in the analysis [39]. More complex structures could also be constructed considering expert opinions, criteria, attributes and so on. Note that this analysis could be run for any economy including supranational states, countries, enterprises, cities and any other type of region.

In order to show the usefulness of the POWAWA operator, let us look at a simple example.

Table 1
Available information.

Variable	$t-1$	t	$t+2$	$t+3$
GDP	100	105	115	120
C	65	68	73	75
I	12	13	15	16
G	18	19	21	22
X	15	16	18	20
M	10	11	12	13

Abbreviations: t = time; GDP = gross domestic product; C = consumption; I = investment: G = public spending; X = exportations; M = importations.

Example 1. Assume that a country is analysing its economic growth three years in the future. The available information they have for the future is shown in Table 1.

With this information, the degree of importance for each variable is formed for year t using C_{t-1}/GDP_{t-1} and so on. Experts on the country produce a new weighted average for year $t+3$ according to the weights found in year t and their beliefs regarding the increasing or decreasing importance of each variable. Note that the reason for doing so is that they are not sure that the first screening shown in Table 1 is strictly correct. Moreover, since imports are negative in the equation, the weighting vector is nonmonotonic [38]. The results are shown in Table 2.

Note that P is calculated from Table 1, while V is obtained as an approximation from $t+2$ and $t+3$. These results and the PWA operator, a particular case of the POWAWA, provide the following results shown in Table 3. The probabilistic information and the weighted average are assumed to have the same degree of importance ($\beta = 0.5$).

Note that EG-C is formed by using $\text{PWA}_C \times (C_{t+3}/C_{t+2})$, EG-I with $\text{PWA}_I \times (I_{t+3}/I_{t+2})$ and so on.

Table 2
Available information for the weighting vectors.

Variable	C	I	G	X	M	GDP
$P = \% \, t-1$	0.65	0.12	0.18	0.15	−0.10	1
$V = \% \, t+2$	0.62	0.13	0.19	0.16	−0.10	1
PWA	0.635	0.125	0.185	0.155	−0.10	1

The abbreviations are available in Table 1 except for: $P = (\{C, I, G, X, M\}_{t-1}/\mathrm{GDP}_{t-1}) \times 100$; $V = (\{C, I, G, X, M\}_{t+2}/\mathrm{GDP}_{t+2}) \times 100$.

Table 3
Aggregated results with the PWA operator.

Variable	EG-C	EG-I	EG-G	EG-X	EG-M	EG	ΔEG
PWA	0.652	0.133	0.193	0.172	0.108	1.042	4.2%

Abbreviations: EG – $\{C, I, G, X, M\}$ = part of the economic growth coming from the consumption, investment, public spending, exportations and importations.

Table 4
Aggregated results with the POWAWA operator.

Variable	EG-C	EG-I	EG-G	EG-X	EG-M	EG	ΔEG
POWAWA	0.606	0.16	0.199	0.17	0.089	1.046	4.6%

The OWA operator is introduced in the problem. Since the information is very uncertain, the OWA is used to represent different scenarios that may occur, from that which is the most pessimistic to that which is the most optimistic, and to select that which is in closest accordance with our interests and attitudes. This is a non-monotonic OWA aggregation with the weights $W = (0.5, 0.2, 0.2, 0.15, -0.05)$. Note that this weighting vector is implicitly conditioned by the initial information shown in Table 1 and we assume that it is equally as important as the weighted average and the probability. A simple average between them is used. The results are shown in Table 4.

As we can see, it is possible to construct intervals for each variable that represent all potential scenarios, from the minimum (pessimistic) to the maximum (optimistic). As no information is lost, this approach provides more comprehensive information that can be assessed more efficiently. In order to make decisions, note that the results found with the POWAWA operator should be our final assumption regarding the expected result for each variable. Therefore, it is an aggregated result that considers the initial degrees of importance of the variables, beliefs about the future and the degree of optimism or pessimism in the specific problem considered.

Similarly, this approach could also be extended to other methods for constructing the GDP. Thus, for the income method, EG for the time period $t+3$ is analyzed as follows:

$$\mathrm{EG} = \left(\mathrm{POWAWA}_{\mathrm{WS}} \left(\frac{\mathrm{WS}_t}{\mathrm{WS}_{t-1}} - 1 \right) + \mathrm{POWAWA}_{\mathrm{CP}} \left(\frac{\mathrm{CP}_t}{\mathrm{CP}_{t-1}} - 1 \right) + \mathrm{POWAWA}_{\mathrm{OI}} \left(\frac{\mathrm{OI}_t}{\mathrm{OI}_{t-1}} - 1 \right) \right) \times 100 \quad (19)$$

This equation can be used in analyses similar to Example 1. Among others, it is worth noting that we could use it in any of the particular cases of the POWAWA operator [39] including the weighted average, the OWA and the OWAWA operator.

4. Using economic growth in entrepreneurial multi-person decision-making

Economic growth analysis is useful when applied on a wide range of problems, including general information on an economy, commercial policies with other countries, influences on other sectors including investment strategies and government policies. This section focuses on a group decision-making problem regarding entrepreneurial strategies. When new entrepreneurial activities are going to be developed, the different variables that may affect this process must be studied. Economic growth is a fundamental variable for these studies because it determines the environment of an economy, and consequently is a very important piece of information when searching for the most appropriate place for entrepreneurial activities. Several studies have analyzed entrepreneurial group decision-making problems [43–45]. This paper focuses on the analysis of the optimal place for creating a new company based on the economic environment. The analysis is performed by observing the expected economic growth in several countries. The country with the highest economic growth is assumed to be the optimal place for entrepreneurship. Note that this example considers a decision problem [46–48] with imprecise information that can be assessed with interval numbers. However, many other methodologies could be applied [49–51], including many other types of imprecise information [52,53]. In order to do so, let us present a procedure for entrepreneurial uncertain multi-criteria multi-person decision-making as follows:

Step 1: Let $A = \{A_1, A_2, \ldots, A_n\}$ be a set of finite alternatives, $T = \{T_1, T_2, \ldots, T_n\}$ a set of finite states of nature (or attributes), forming the payoff matrix $(\tilde{a}_{gi})_{p\times n}$. This matrix is formed for each criteria $\mathrm{CR} = (\mathrm{Cr}_1, \mathrm{Cr}_2, \ldots, \mathrm{Cr}_x)$. All these criteria are aggregated through the weighting vector $Y = (y_1, y_2, \ldots, y_h)$ with $\sum_{e=1}^{h} y_e = 1$ and $y_e \in [0, 1]$.

Step 2: Let $E = \{E_1, E_2, \ldots, E_p\}$ be a finite set of decision-makers. Let $Z = (z_1, z_2, \ldots, z_p)$ be the weighting vector of the decision-makers, such that $\sum_{k=1}^{p} z_k = 1$ and $z_k \in [0, 1]$. Each decision-maker provides his/her own payoff matrix $(\tilde{a}_{gi}{}^{(k)})_{m\times n}$. Use the weighted average to aggregate the information of decision-makers E using the weighting vector Z. The result is the collective payoff matrix $(\tilde{a}_{gi})_{m\times n}$. Thus, $\tilde{a}_{gi} = \sum_{k=1}^{p} z_k \tilde{a}_{gi}^{k}$.

Step 3: Calculate the weighting vector W and V to be used in the UOWAWA aggregation and the β parameter. Note that $W = (w_1, w_2, \ldots, w_n)$, such that $\sum_{j=1}^{n} w_j = 1$ and $w_j \in [0, 1]$ and $V = (v_1, v_2, \ldots, v_n)$, such that $\sum_{j=1}^{n} v_j = 1$ and $v_j \in [0, 1]$.

Step 4: Calculate the aggregated results using the UOWAWA operator explained in Eq. (3). Consider different particular manifestations of the UOWAWA using different expressions as explained after Eq. (17).

Step 5: Make decisions according to the results found in the previous steps. Select the alternative/s that provide(s) the best result/s. Rank the alternatives from the most to the least preferred alternative.

This approach uses several key components in order to build the decision process as shown in Fig. 1.

Furthermore, this decision-making process explained in the previous paragraphs can be summarized using the following aggregation system.

Definition 5. Let Ω be the set of interval numbers. An MC-MP-EG-UOWAWA operator of dimension n is a mapping MC-MP-EG-UOWAWA: $\Omega^n \times \Omega^p \times \Omega^h \to \Omega$ that has a weighting vector Z of dimension p with $\sum_{k=1}^{p} z_k = 1$ and $z_k \in [0, 1]$ and a weighting vector W of dimension n with $\sum_{j=1}^{n} w_j = 1$ and $w_j \in [0, 1]$, such that:

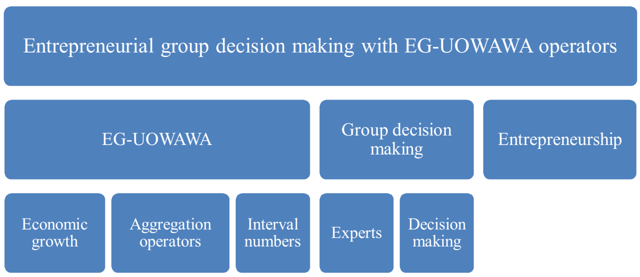

Fig. 1. Illustrative representation of the new approach.

$$f\left((\tilde{a}_1^1, \ldots, \tilde{a}_1^p), \ldots, (\tilde{a}_n^1, \ldots, \tilde{a}_n^p)\right) = \sum_{j=1}^{n} \hat{v}_j b_j \quad (20)$$

where b_j is the jth smallest of the $\tilde{a}_i$, each argument $\tilde{a}_i$ has an associated weight v_i with $\sum_{i=1}^{n} v_i = 1$ and $v_i \in [0, 1]$, $\hat{v}_j = \beta w_j + (1-\beta) v_j$ with $\beta \in [0, 1]$, v_j is the probability v_i ordered according to b_j, that is, according to the jth smallest $\tilde{a}_i$, and $\tilde{a}_{gi} = \sum_{k=1}^{p} z_k \tilde{a}_{gi}^k$ where $\tilde{a}_{gi}{}^k$ is the argument variable provided by each expert after having considered the different criteria, $\tilde{a}_i^k = \sum_{e_k=1}^{h} y_{e_k} \tilde{a}_i^{e_k}$, $a_i^{e_k}$ is the argument provided by each expert for each criterion and Y is a weighting vector of dimension h with $\sum_{e_k=1}^{h} y_{e_k} = 1$and $y_{ek} \in [0, 1]$.

Note that many properties and particular cases of the MC-MP-EG-UOWAWA operator could be studied following the usual literature on aggregation operators. Furthermore, more complex situations can be considered using other aggregation operators instead of the weighted average in the aggregation of the experts and the criteria.

5. Numerical example

In order to understand the procedure explained in Section 4, let us look at a numerical example regarding the use of the UOWAWA operator in an uncertain entrepreneurial multi-criteria multi-person decision-making problem using economic growth analysis. The literature contains a wide range of other decision-making methods [54–57], including problems assessed with linguistic [25,58,59] and fuzzy information [60]. Note that the aim of the example is to show numerically, how the new approaches suggested in the paper can be implemented. Therefore, it does not focus on the real world although the ideas mentioned in the example have strong connections with reality. The main reason for this is that the real world would include additional variables that would make it more difficult to understand the ideas suggested in the paper. Thus, the example is strictly focused on the ideas of the paper.

Step 1: Assume an investor wishes to develop entrepreneurial activities by opening a new company. However, the investor has not yet decided where to invest. After an initial screening, the investor considers five potential alternatives where he/she needs to analyze each country's expected economic growth as shown in Table 5.

In order to assess the problem, the investor uses two experts. They consider that the key variable, which will determine the success of the investment, is the economic situation of the country measured by the expected economic growth for the next period. Investment is advisable in countries with the most positive economic growth. Since the information is very uncertain, as experts have to develop forecasts for the next period, they use interval numbers to assess the information. Also note that experts could have used fuzzy numbers in order to better represent the information considering the possibility that the internal values would occur. In this example, the assumption is that the information is very uncertain and it is not easy to calculate the values between the minimum and the maximum. After careful analysis of the variables that may affect the problem over time, they assume that the information can be summarized in three criteria:

- Cr_1: European Union economic environment.
- Cr_2: World economic environment.
- Cr_3: Other variables.

The experts assume that five potential states of nature may occur in the general economic environment:

- S_1: Very good economic situation.
- S_2: Good economic situation.
- S_3: Fair economic situation.
- S_4: Bad economic situation.
- S_5: Very bad economic situation.

For each criterion, the experts formulate the expected economic growth for each country. The results for the first expert are given in Tables 6–8. They are presented in the form of interval numbers, where the left value indicates the lowest value that can occur and the right value shows the highest result that this scenario could bring about. The numerical data represent evaluations given by the experts from 0 to 10 where 0 is very low benefits and 10 is very high.

Table 5
Initial strategy.

	Objective	Analysis
A_1	Create a new company in Poland	Economic growth in Poland
A_2	Create a new company in the Czech Republic	Economic growth in the Czech Republic
A_3	Create a new company in Hungary	Economic growth in Hungary
A_4	Create a new company in Romania	Economic growth in Romania
A_5	Create a new company in Bulgaria	Economic growth in Bulgaria

Table 6
Expert 1—criteria 1.

	S_1	S_2	S_3	S_4	S_5
A_1	[6, 7]	[7, 8]	[5, 7]	[3, 4]	[1, 2]
A_2	[7, 8]	[6, 7]	[3, 5]	[2, 4]	[1, 3]
A_3	[5, 7]	[5, 8]	[4, 6]	[3, 4]	[2, 3]
A_4	[3, 5]	[6, 7]	[4, 5]	[2, 3]	[1, 2]
A_5	[4, 5]	[7, 9]	[6, 7]	[3, 4]	[2, 3]

Table 7
Expert 1—criteria 2.

	S_1	S_2	S_3	S_4	S_5
A_1	[5, 6]	[8, 9]	[5, 6]	[3, 5]	[1, 3]
A_2	[6, 7]	[7, 8]	[5, 7]	[3, 5]	[1, 2]
A_3	[6, 8]	[4, 7]	[3, 6]	[2, 4]	[1, 3]
A_4	[7, 8]	[5, 8]	[5, 6]	[4, 6]	[3, 5]
A_5	[6, 7]	[5, 9]	[4, 7]	[2, 7]	[1, 5]

Table 8
Expert 1—criteria 3.

	S_1	S_2	S_3	S_4	S_5
A_1	[6, 8]	[7, 8]	[5, 7]	[4, 5]	[3, 4]
A_2	[7, 9]	[6, 7]	[5, 8]	[3, 5]	[2, 4]
A_3	[2, 8]	[1, 7]	[1, 5]	[0, 4]	[0, 3]
A_4	[5, 8]	[6, 7]	[4, 6]	[3, 5]	[2, 5]
A_5	[6, 8]	[5, 6]	[4, 7]	[3, 5]	[2, 4]

Table 9
Expert 1—collective results.

	S_1	S_2	S_3	S_4	S_5
A_1	[5.7, 6.9]	[7.3, 8.3]	[5,6.7]	[3.2, 4.5]	[1.4, 2.7]
A_2	[6.7, 7.9]	[6.3, 7.3]	[4,6.2]	[2.5, 4.5]	[1.2, 2.9]
A_3	[4.7, 7.5]	[3.9, 7.5]	[3.1, 5.8]	[2.1, 4]	[1.3, 3]
A_4	[4.6, 6.5]	[5.7, 7.3]	[4.3, 5.5]	[2.8, 4.3]	[1.8, 3.5]
A_5	[5,6.2]	[6,8.4]	[5, 7]	[2.7, 5.1]	[1.7, 3.8]

Table 10
Expert 2—criteria 1.

	S_1	S_2	S_3	S_4	S_5
A_1	[6, 9]	[7, 8]	[6, 7]	[4, 5]	[1, 3]
A_2	[5, 7]	[6, 7]	[5, 6]	[3, 4]	[1, 3]
A_3	[7, 8]	[6, 7]	[5, 6]	[4, 5]	[2, 4]
A_4	[6, 9]	[5, 8]	[4, 6]	[4, 5]	[3, 4]
A_5	[6, 7]	[6, 7]	[5, 6]	[4, 5]	[4, 5]

Table 11
Expert 2—criteria 2.

	S_1	S_2	S_3	S_4	S_5
A_1	[7, 8]	[5, 7]	[4, 7]	[4, 5]	[2, 4]
A_2	[6, 9]	[6, 7]	[5, 6]	[5, 6]	[1, 3]
A_3	[8, 9]	[5, 8]	[4, 6]	[4, 5]	[3, 5]
A_4	[7, 8]	[6, 7]	[4, 6]	[4, 5]	[3, 4]
A_5	[6, 8]	[6, 7]	[3, 7]	[3, 6]	[2, 4]

Table 12
Expert 2—criteria 3.

	S_1	S_2	S_3	S_4	S_5
A_1	[6, 8]	[6, 8]	[4, 6]	[4, 5]	[1, 3]
A_2	[5, 9]	[6, 7]	[4, 5]	[4, 5]	[1, 3]
A_3	[7, 8]	[5, 7]	[3, 6]	[4, 5]	[1, 4]
A_4	[6, 9]	[6, 8]	[4, 6]	[3, 5]	[3, 4]
A_5	[5, 8]	[6, 7]	[2, 7]	[3, 5]	[1, 5]

Table 13
Expert 2—collective results.

	S_1	S_2	S_3	S_4	S_5
A_1	[6.3, 8.3]	[6, 7.7]	[4.6, 6.6]	[4, 5]	[1.3, 3.3]
A_2	[5.3, 8.4]	[6, 7]	[4.6, 5.6]	[4, 5]	[1, 3]
A_3	[7.3, 8.3]	[5.3, 7.3]	[3.9, 6]	[4, 5]	[1.9, 4.3]
A_4	[6.3, 8.7]	[5.7, 7.7]	[4, 6]	[3.6, 5]	[3, 4]
A_5	[5.6, 7.7]	[6, 7]	[3.2, 6.7]	[3.3, 5.3]	[2.2, 4.7]

Expert 1 aggregates this information to form his/her collective results as shown in Table 9. He/she assumes that the criteria have the following degree of importance: $Y_1 = (0.5, 0.3, 0.2)$.

A similar methodology is used for expert 2. He/she assumes that the criteria can be represented with the following degrees of relevance: $Y_2 = (0.3, 0.3, 0.4)$. The results are shown in Tables 10–13.

Step 2: After forming the collective results for each expert, both opinions can be integrated into another collective structure. It is assumed that both experts are equally important, that is, $Z = (0.5, 0.5)$. The results are presented in Table 14.

Step 3: This general matrix represents the expected economic growth considering the different criteria and opinions of the two experts. In order to make decisions, the information must be aggregated according to the different states of nature that may occur. The UOWAWA operator is used for this purpose as it considers subjective opinions on the possibility that each state of nature will occur and the attitudinal character of the decision-maker. The following weighting vectors are assumed: $W = (0.1, 0.2, 0.2, 0.2, 0.3)$ and $V = (0.1, 0.2, 0.4, 0.2, 0.1)$.

Table 14
Collective results.

	S_1	S_2	S_3	S_4	S_5
A_1	[6, 7.6]	[6.65, 8]	[4.8, 6.65]	[3.6, 4.75]	[1.35, 3]
A_2	[6, 8.15]	[6.15, 7.15]	[4.3, 5.9]	[3.25, 4.75]	[1.1, 2.95]
A_3	[6, 7.9]	[4.6, 7.4]	[3.5, 5.9]	[3.05, 4.5]	[1.6, 3.65]
A_4	[5.45, 7.6]	[5.7, 7.5]	[4.15, 5.75]	[3.2, 4.65]	[2.4, 3.75]
A_5	[5.3, 6.95]	[6, 7.7]	[4.1, 6.85]	[3, 5.2]	[1.95, 4.25]

Table 15
Aggregated results.

	Min	Max	UA	UWA	UOWA	UOWAWA
A_1	[1.35, 3]	[6.65, 8]	[4.48, 6]	[4.70, 6.27]	[3.95, 5.5]	[4.32, 5.88]
A_2	[1.1, 2.95]	[6,8.15]	[4.16, 5.78]	[4.31, 5.85]	[3.67, 5.26]	[3.99, 5.55]
A_3	[1.6, 3.65]	[6,7.9]	[3.75, 5.87]	[3.69, 5.89]	[3.31, 5.44]	[3.5, 5.66]
A_4	[2.4, 3.75]	[5.7, 7.5]	[4.18, 5.85]	[4.22, 5.86]	[3.85, 5.47]	[4.03, 5.66]
A_5	[1.95, 4.25]	[6,7.7]	[4.07, 6.19]	[4.16, 6.44]	[3.66, 5.84]	[3.91, 6.14]

Table 16
Ranking of the investments.

	Ranking		Ranking
Min	$A_5 \}A_4 \}A_3 \}A_1 \}A_2$	UWA	$A_1 \}A_5 \}A_2 \}A_4 \}A_3$
Max	$A_1 \}A_2 \}A_3 \}A_5 \}A_4$	UOWA	$A_5 \}A_1 \}A_4 \}A_2 \}A_3$
UA	$A_1 \}A_5 \}A_4 \}A_2 \}A_3$	UOWAWA	$A_1 \}A_5 \}A_4 \}A_2 \}A_3$

Step 4: With this information, the aggregated results can be calculated using the UOWAWA operator and some of its main particular cases. It is assumed that $\beta = 0.5$. This example uses the minimum, the maximum, the average, the UWA, the UOWA and the UOWAWA operator. The results are shown in Table 15.

Step 5: As we can see, the optimal choice is A_1. That is, develop entrepreneurial activities in Poland. However, due to uncertainty, the future is not perfectly known. Therefore, in some exceptional and unexpected situations some other alternative may be optimal. In order to see the different alternatives that could be optimal, we present a ranking of all the alternatives for each aggregation method. The rankings are shown in Table 16.

Although A_1 appears to be the optimal choice for the maximum, UA, UWA and UOWAWA, A_5 (Bulgaria) may be optimal when considering pessimistic scenarios such as the minimum and the UOWA operator (with pessimistic weights). Therefore, an important issue when dealing with uncertainty is to be aware that the calculations provide results that recommend an alternative, but many other decisions could be taken depending on events that occur in the future. The main advantage of the new approach presented in this paper is that it provides a broader picture of all the scenarios that may appear in the future and it recommends a particular alternative according to the attitudinal character and subjective beliefs of the decision-maker.

6. Conclusions

This paper has presented a new approach for dealing with entrepreneurship under uncertain environments that can be assessed by a country's economic growth. A main assumption is that positive economic growth is very helpful for entrepreneurial activities because it indicates that the market is expanding. The approach proposed relates the concepts of entrepreneurship and economic growth in decision-making processes where entrepreneurs look for the optimal market and it is important to detect countries in which a higher economic growth is expected. Since the information is uncertain and nobody knows exactly what the future will bring, several methods have been presented for forecasting economic growth using a wide range of aggregation systems. A key method for doing this is by using the classical methods available in macroeconomic theory. The main formulation compares GDP in one year with the previous year in order to calculate economic growth. It has been demonstrated that decomposing GDP into its main variables can provide a new framework for assessment with aggregation systems. Several aggregation methods have been presented including a wide range of POWAWA operators. Their main advantage is that they include as particular cases the classical methods based on the arithmetic mean and the weighted average. Therefore, the new methods presented in this study can always be reduced to the classical framework if the problem considered is simple. However, in complex environments with a lot of uncertainties, this approach is more complete than the classical ones because it can represent the information in a more complete way.

A different approach, focused exclusively on the end result of economic growth has also been suggested. This analysis has proven to be very similar to the common methodologies used when dealing with averaging aggregation operators, such as the OWA and the OWAWA operator. Therefore, it is possible to analyze a wide range of possible future scenarios, from the most pessimistic to the most optimistic. The use of aggregation systems in this context has also proven to be very efficient, as they provide a more complete overview of the problem. A more general application in multi-criteria multi-person decision-making has been developed forming the MC-MP-EG-UOWAWA operator. This approach has been applied in an entrepreneurial decision-making problem regarding the selection of the optimal market for entrepreneurship. Moreover, it has been linked with economic growth, assuming that economic growth stands as a key variable that stimulates entrepreneurship. Obviously, from a general perspective, it seems better to create new companies when economic growth is expected to be positive, as it implies that the market is expanding and therefore easier to consolidate and make profits.

This work has been able to link aggregation systems with entrepreneurship and economic growth by using decision-making processes. However, there are several limitations to be considered. First, the use of economic growth is a key variable that conditions entrepreneurship from a theoretical point of view. But many other variables should be considered in order to assess the problem from a more comprehensive real-world perspective. Furthermore, this link may sometimes not be directly found due to the company's position and strategy. For example, in a long-term strategy, it may be better to invest during a period of negative economic growth because it will be cheaper to establish the company. Then, after some years, the market will expand again and it will be easier for the company to make profits. Therefore, in this context, it is important to highlight that the new approach presented in this paper is expected to be very helpful for analyzing these problems. Yet it is important to recall that the real world is very complex and many variables and strategies may condition the optimal solution in a different way. In future research, we expect to develop more aggregation systems focused on these and other economic variables and by using a wide range of techniques commonly used in soft computing.

Acknowledgements

We would like to thank the editor and the anonymous reviewers for valuable comments that have improved the quality of the paper. Support from the University of Chile, the European Commission through the project PIEF-GA-2011-300062 and from the Universitat Politècnica de València through the project Paid-06-12 (Sp 20120792) is gratefully acknowledged.

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