Reward for failure and executive compensation in institutional investors

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\textbf{Abstract}

We propose a model of delegated portfolio management specialized in alternative investments, i.e., those with a high-return and high-risk profile. It is shown that in this context, as a reward for risk-taking scheme is optimal, a counter-intuitive reward for failure can also be desirable. This property emerges because it can be optimal to compensate extreme returns (even low ones) to encouraging managers to shape highly innovative portfolios. It is argued that this structure resembles compensation practices questioned in the context of the last financial crisis, such as golden parachutes and golden coffins. Implementation via equity and bonuses is also analyzed.

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\textbf{1. Introduction}

The last financial crisis has exposed executive incentive plans of investment banks and other institutional investors under a strong scrutiny, especially those compensation practices that seem to reward managers with generous benefits even though the performance of their institutions is clearly unsatisfactory.

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In contrast to these criticisms, this article proposes an agency model under which rewarding managers for low performance may indeed be a desirable property stemming from an optimal incentive scheme. We argue that our framework is particularly applicable to institutional investors, whose delegated portfolio management activity involves searching and selecting alternative investments, that is, investments with a high risk-high expected return profile.

Although counter-intuitive, it is shown that this “reward for failure” property can emerge as an element being part of a more general optimal reward for risk-taking scheme that compensates extreme returns and punishes (in relative terms) moderate ones. As a result, the ex ante promise of a reward to even low results may be an effective mechanism to encouraging managers to shape truly innovative portfolios.

The model here proposed is consistent with the optimal contracting approach (Jensen and Murphy, 1990; Hermalin, 2005; Cheng et al., 2013), and thus, their conclusions should be viewed as counter-acting the insights coming from the managerial power approach, summarized in Bebchuk and Fried (2004). Indeed, under the latter approach, practices such as golden parachutes, generous life insurance (golden coffins), entrenchment, and all-event bonuses are considered as evidence on weak corporate governance, and in particular, on the lack of independence between the board of directors and top executives (see also Bebchuk et al., 2010). Although we do not rule out that the managerial power approach can offer useful insights in some cases, we claim that in financial activities such as private banking (high-wealth investors), hedge funds and other alternative investments, a seemingly paradoxical practice like a reward for failure scheme can be, at least partly, understood under the lens of the optimal contracting approach.

Our paper is related to previous research on how other non-monotone and convex incentive schemes motivate managers to take desirable risk levels from the principal’s standpoint, especially option-like schedules (Carpenter, 2000; Garcia, 2001; Goetzmann et al., 2003; Ross, 2004; Kadan and Swinkels, 2007; Feltham and Wu, 2001; Hemmer et al., 1999; Dittmann and Maug, 2007; Duan and Wei, 2005; Bolton et al., 2010; Coles et al., 2006; Hirshleifer and Suh, 1992) and bonus payment structures (Starks, 1987; Leisen, 2014). At first glance, the convexity involved in the payoff function of this class of compensation schemes should incentivize risk-taking. However, a more-in-depth analysis of these incentive plans has delivered three results that weaken such a convexity argument.

First, in the context of a delegated risk-taking environment, in general these option and bonus schemes do not necessarily correspond to the optimal contract (see for instance, Carpenter, 2000; Ross, 2004; Kadan and Swinkels, 2007). Second, it is not true that such schemes always induce more managerial risk-taking, as other effects—different from convexity—may make the manager even more risk-averse or lead him to undertake more conservative investments (Carpenter, 2000; Chen and Pennacchi, 2009; Ross, 2004). Third, research on bonus and option plans has raised concerns about whether their convex payoff functions may encourage managers to adopt excessive risk-taking or risk-shifting practices (Carpenter, 2000; Green and Talmor, 1986; DeFusco et al., 1990).

Rather than to study a given incentive scheme, and in contrast to most of the above cited literature, our main goal is to characterize what is indeed the optimal scheme when risk-taking is involved. As a consequence, two main contributions arise from the present article. First, from the principal’s viewpoint, our reward-for-risk scheme effectively induces managers to choose the proper level of innovation (and risk), and thus, it is free from the concerns raised over option-like schemes as being ambiguous mechanisms to incentivize risk-taking. Furthermore, contrary to option and bonus payment structures, our optimal contract not only involves not to penalize low performance, but also to reward it. This property implies that, in contrast to the extant literature, we are able to provide an economic rationale for counter-intuitive managerial reward schemes, such as golden parachutes and other failure-compensation practices.

The present article is also related to research showing empirical evidence on how convex compensation schemes mitigate the incentive risk-taking problem faced by risk-averse managers, who have to invest in high-risk high-return projects on behalf of a risk-neutral principal. In general, this literature supports the idea that risk incentives involved in stock options do encourage managers to increase risk measures in profitable investments, such as exploration risk of oil and gas projects (Rajgopal and Shevlin, 2002); asset return variance after acquisitions by mergers and divestitures (Agrawal and Mandelker, 1987); and asset volatility in banking (Mehran and Rosenberg, 2007). In a complementary
line, other works conclude that there is no evidence that options and bonus schemes had led executives of financial institutions to excessive risk-taking even during the last credit crisis (Fahlenbrach and Stulz, 2011). Overall, this evidence is favorable to our model in two aspects. First, as we allow for the possibility that more innovation (and thus more risk) in portfolios brings jointly more expected return—technically, a no deterioration in a second-order stochastic dominance of the return distribution—, our distributional assumptions seem to be in accordance with the risk-return profile of some real-world innovative investments. Second, as our numerical results suggest that some alternative—although suboptimal—bonus plans can perform quite successfully relative to the optimal incentive scheme, the insights from our theoretical model seem to be consistent with the power that convex compensation plans have in practice to induce more aligned managerial risk-taking decisions.

This article proceeds as follows. Section 2 presents a model of executive compensation under moral hazard and a delegated portfolio management environment. Section 3 characterizes the optimal incentive structure, which exhibits two salient properties: (i) a scheme involving a reward for risk-taking and a reward for failure, and (ii) a positive relationship between portfolio innovation and compensation of extreme returns. Section 4 presents some numerical exercises, showing that changes in preference and return parameters induce changes in the optimal scheme that are consistent with traditional agency models. Section 5 explores two issues related to implementing the optimal incentive scheme: the use of financial claims and the adoption of bonus-type structures. Finally, Section 6 discusses the principal conclusions. All the proofs are collected in the Appendix.

2. The model

We adapt the corporate governance framework of Hermalin and Weisbach (2005) to the fund management activity, and also extend it by examining the role played by aversion to risk and cost of innovation.

Consider the following agency relationship between an investor (the principal) and the portfolio manager (the agent). This relationship is essentially a delegated fund management process, in which the mandated task to the manager is to select an investment portfolio that maximizes expected returns (net of his compensation fees). We model this decision through \( e \), which represents the innovation degree of the portfolio the manager chooses.

2.1. Innovation and returns

We assume that \( e \in [0, 1] \), so that this innovation degree can be seen as the weight the portfolio manager assigns to alternative investments. This class of investments include, among others, innovative industrial sectors (high-tech, bio-tech, nanotech), emerging markets (BRICS, MINTs), alternative energy mutual funds, and in general, other investments with a profile of high risk and high expected return. By contrast, \((1 - e)\) can be understood as the weight put by the manager on more traditional sectors or markets. According to the financial terminology, one can also interpret \( e \) as the degree in which an investment strategy is “active”, that is, if it differs from a benchmark portfolio as, for example, a stock market index.

The degree of innovation (and so the portfolio decision) is not verifiable by the investor. However, she is able to verify the portfolio’s return. In this context, let us define \( x^e_i \) as the return yield by a portfolio with an innovation degree \( e \) when state of nature \( i \) occurs. Similarly, we denote \( p^e_i \) as the probability of observing return \( x^e_i \), with \( p^e_i > 0 \) for all \( e \in (0, 1) \) and \( i = 1, \ldots, n \). The last assumption implies that, from observing a given \( x_i \), it cannot be ruled out a priori a given level of positive innovation \( e \). For simplicity, we suppose that only three states of nature are possible (\( n = 3 \)), so that \( x^e_i \in X = \{x_1, x_2, x_3\} \) for all \( e \), and \( x_1 < x_2 < x_3 \).

Assumption 1 (A1). The probability distribution of \( x \) conditional on an innovation degree \( e \) (i.e. \( x^e \)), is described by

1 Biass and Casamatta (1999) propose a distribution function close to that analyzed here, but in the context of corporate financing and in which the agent is risk neutral. Hermalin and Weisbach (2005) and Loyola and Portilla (2010) also consider a similar distribution function, but they do not study the effects of either attitude to risk or cost of innovation.
\[ p_i^e \equiv \Pr(x = x_i | e) = \begin{cases} 
\gamma_1 e & \text{if } i = 1 \\
(\gamma_2 - 1)e + 1 & \text{if } i = 2 \\
\gamma_3 e & \text{if } i = 3 
\end{cases} \]

where \( \gamma_i > 0 \) for all \( i \) and \( \sum_{i=1}^{3} \gamma_i = 1 \).

Note that these restrictions on parameters \( \gamma_i \)'s guarantee that \( \partial p_i^e / \partial e > 0 \) for \( i = 1 \) and \( i = 3 \), but \( \partial p_i^e / \partial e < 0 \) for \( i = 2 \). Thus, the fact that a higher innovation degree \( e \) increases the probability of extreme events is consistent with the idea that \( e \) parametrizes the portfolio’s risk.

Also, this formulation implies that the so-called monotone likelihood ratio property (MLRP)—a classic condition in contract theory—is not satisfied. To see that, let us define the likelihood ratio as

\[ LR_i = \frac{\partial p_i^e / \partial e}{p_i^e} \]

from which it is easy to verify that \( LR_1 > LR_2 \) despite \( x_2 > x_1 \).

Recall that \( LR_i \) reflects how informative is the fund return \( x_i \) (verifiable) with respect to a portfolio selection decision \( e \) (unverifiable). Then, the larger \( LR_i \), the more likely that the portfolio manager had chosen an innovation degree \( e \). Thus, if the investor wants this innovation degree to be selected, the compensation should be higher whenever the return \( x_i \) is observed. Nevertheless, this fact does not guarantee that the compensation scheme will always be increasing in the portfolio’s return. The last property holds as long as the likelihood ratio is monotonically increasing in \( x_i \), that is, if the MLRP is met. Indeed, as it is established later, the no verification of this property in our model is crucial to attain the main result of the present work.

A comparison of the payoff distribution adopted according to assumption (A1) and that considered by Biais and Casamatta (1999) (BC hereafter) deserves our attention. As the last paper studies two dimensions of moral hazard, namely effort and risk-taking, such comparison works only if we restrict the analysis exclusively to risk-taking. In that dimension, it should firstly be stressed that whereas BC consider a binary risk-shifting decision (i.e., from a low risk to a high risk project), our model assumes a continuous decision variable \( e \in [0, 1] \) that parametrizes in turn a continuum of portfolios with different levels of innovation and risk. This allows us to explore more-in-depth how risk taking and its associated optimal incentive schemes are affected by changes in preferences and returns parameters, which is either not possible or much more limited in the BC’s framework. Second, BC assume a priori that risk-shifting deteriorates the payoff distribution in a second-order stochastic dominance (SOSD) notion, which in the context of our model would be equivalent to assume that

\[ \gamma_3 (x_3 - x_2) \leq \gamma_1 (x_2 - x_1) \]  

In contrast, our setting does not assume a priori that more innovation (and thus more risk) induces necessarily a return distribution dominated in a SOSD sense. In fact, numerical simulations performed in Section 4 take parameter values for which the opposite of condition (1) holds true. Interestingly, empirical evidence suggests that our choice of not assuming condition (1)—unlike BC—is reasonable in the context of risk-taking decisions made by more sophisticated fund management institutions. Indeed, as compared to traditional portfolio managers (e.g., mutual funds or equity index funds), the empirical return distribution of more innovative investment vehicles (e.g., hedge funds specialized in emerging markets) exhibits in practice not only a larger second moment (variance) and four moment (more kurtosis or fatter tails), but also a higher first moment (mean return) (see Malkiel and Saha, 2005).3

Third, BC assume that the riskier project has a negative present value, which in the context of our setup would be equivalent to suppose that

\[ E(x_i^e) - I < 0 \]
for all $e > 0$, where $I$ represents the amount of funds delegated to the portfolio manager (see an explanation of this variable in the next subsection). Contrary to that, we do not assume that condition (2) is or not verified a priori, which allows us to study a richer set of cases than that analyzed by BC. Lastly, BC assume universal risk-neutrality and no costs of risk-taking. As a consequence, their setup is not able to examine the role played by the risk aversion and disutility of innovation on the part of the agent, as our model indeed does via numerical simulations in Section 4.

2.2. Preferences

As we are thinking of investors with high wealth and the ability to heavily diversify her wealth, we assume that the investor is risk neutral and her preferences are represented by the (ex post) utility function

$$B(x_i, w) = x_i - w,$$

where $w$ is the compensation (performance fee) paid to the portfolio manager. In turn, the manager’s preferences are described by the following (ex post) additively separable utility function$^4$:

$$U(w, e) = (1 - \delta)w^{1-\delta} - \frac{\lambda e^2}{2},$$

where the first term represents the manager’s payoff coming from his compensation, and the second term represents his disutility from choosing a level of positive innovation.

The utility function assumed to account for the manager’s compensation guarantees that his payoff always increases with performance fee (positive marginal utility), and that he is risk averse (strictly decreasing marginal utility). In particular, the parameter $\delta \in (0, 1)$ corresponds to the coefficient of (constant) relative risk aversion.

Moreover, as a higher innovation degree requires a higher level of effort, we assume that the innovation disutility function exhibits a positive and increasing marginal disutility. In this formulation, $\lambda > 0$ parametrizes the marginal cost to the manager of seeking more innovation. His reservation utility corresponds to $U$ so that $U > \frac{U}{2}$, as this assumption ensures that the optimal performance fee is a positive real number (see Proof of Proposition 1 in the Appendix).

In addition, $I$ corresponds to the amount of funds delegated to the portfolio manager according to a pre-defined criterion of the investor, and thus, it constitutes an exogenous variable in the model.

Assumption 2 ($A2$). Under asymmetric information, the optimal investor’s choice is never to avoid risky (innovative) portfolios. Formally, we suppose that:

$$\sum_{i=1}^{n} \beta_i^x (x_i^r - w_i^r) > x_2 - w^*(0),$$

where $w^*(0)$ represents the compensation related to the first-best contract that induces a null degree of innovation, and $w_i^r$ corresponds to the compensation related to the second-best contract that induces an optimal innovation degree $e^r \in (0, 1]$.

2.3. An alternative interpretation$^5$

We end this section by posing an alternative interpretation of the decision variable $e$. According to this view, the manager’s decision can be interpreted as fractions $(1 - e)$ and $e$ of an investor’s wealth.

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$^4$ Although we assume this specific functional form, our results hold without loss of generality for all utility function increasing and concave in $w$, and a disutility function increasing and convex in $e$.

$^5$ This subsection is due to a suggestion of an anonymous referee.
put into two classes of financial investments: a safe (or traditional) and a risky (or alternative) portfolio. Thus, different values of $e$ shape different return-risk profiles of a mixed portfolio.

More formally, let us denote the returns of the original portfolios as $y_i$ and $z_i$ for the traditional and alternative portfolio, respectively. Whereas the first portfolio yields a return $y_i = x_1$ with probability 1, the second portfolio’s return exhibits the following probability distribution:

$$z_i = \begin{cases} 
    x_1 & \text{with probability } \gamma_1 \\
    x_2 & \text{with probability } \gamma_2 \\
    x_3 & \text{with probability } \gamma_3
\end{cases}$$

where $x_1 < x_2 < x_3, \gamma_i > 0$ for all $i$, and $\sum_{i=1}^{3} \gamma_i = 1$.

Hence, the return of the mixed portfolio, conditional on a weight $e$, then becomes

$$x^e_i = (1-e)x_2 + ez_i,$$

so that

$$p^e_i = \Pr(x^e_i) = \begin{cases} 
    \gamma_1 & \text{if } i = 1 \\
    \gamma_2 & \text{if } i = 2 \\
    \gamma_3 & \text{if } i = 3
\end{cases}.$$  

In terms of conditional expected return, this formulation is equivalent to our original presentation of the model, as in both cases it can easily be verified that

$$E(x^e_i) = \gamma_1 ex_1 + ((\gamma_2 - 1)e + 1)x_2 + \gamma_3 ex_3.$$  

Despite this equivalence in terms of expected returns, notice that risk aversion on the part of the manager implies that, whereas perhaps the qualitative properties of the optimal incentives scheme are similar in both formulations of the model, they do not necessarily deliver the same specific optimal contract.

In addition, this alternative interpretation requires to justify why the weight $e$ may be unverifiable to the investor. As for this point, it can be argued that in practice it is usual that mutual funds and other institutional delegated fund managers send a report to their investors in which a decomposition of their managed wealth in different classes of funds is detailed. This decomposition is however, in most of cases, clearly insufficient to allow the investor to know the actual return-risk profile of the ultimate investments underlying each of these funds. This occurs as it is frequent that such funds are just presented under a generic name, which in turn, hides other collection of portfolios.

3. The optimal incentive scheme

In this section we characterize the optimal incentive scheme under the asymmetric information environment previously described. Then, if the investor wants to implement the innovation degree $e_o \in (0,1]$, the optimal contract $\{w^e_i\}_{i=1}^{3}$ must solve the following program$^6$:

$$\begin{align*}
\text{Max} & \quad \sum_{i=1}^{3} p^e_i(x_i - w_i) \\
\text{s.t.} & \quad \sum_{i=1}^{3} p^e_i((1-\delta)w_i^{1-\delta} - \frac{\delta e^2}{2}) \geq U \\
& \quad e_o \in \arg\max_e \sum_{i=1}^{3} p^e_i((1-\delta)w_i^{1-\delta} - \frac{\delta e^2}{2}),
\end{align*}$$

$^6$ For the sake of notation, we omit in this program the superscript 0 associated to the innovation degree $e_o$. 

where (4) and (5) correspond to the participation constraint and the incentive compatibility constraint, respectively.

After solving this program, we can establish the following result.

**Proposition 1.** Under assumptions (A1) and (A2), it is verified that

\[ w_0^\beta = w_1^\beta > w_2^\beta > 0, \]

so that

\[ w_1^\beta = w_3^\beta = \left[ \frac{1}{1-\delta} \left( U - \lambda \frac{e_0^2}{2} + \frac{\lambda e_0}{1-\gamma_2} \right) \right]^{\frac{1}{\gamma_1}}, \]

\[ w_2^\beta = \left[ \frac{1}{1-\delta} \left( U - \lambda \frac{e_0^2}{2} \right) \right]^{\frac{1}{\gamma_2}}. \]

This implies that the optimal compensation schedule is clearly non-monotone as it pays the same fee to the portfolio manager when the observed return is low \(x_1\) or high \(x_3\). A risk-taking reward thus emerges as a property of the optimal scheme. In formal terms and based on Proposition 1, one can define \(D_w^\beta\), the risk-taking reward that induces an innovation degree \(e_0\), as follows

\[ \Delta w^\beta \equiv w_1^\beta - w_2^\beta = w_3^\beta - w_2^\beta. \]

Then, when return \(x_1\) is observed, \(\Delta w^\beta\) represents a measure of the reward for failure. Similarly, when \(x_3\) takes place, \(\Delta w^\beta\) can be understood as a measure of the reward for success.

**Corollary 1.** The risk-taking reward \(\Delta w^\beta\) is increasing with the optimal innovation level \(e_0\).

Thus, as long as the investor wants to invest in a more innovative portfolio, the optimal contract should include a larger compensation to the portfolio manager when extreme returns \((x_1\) or \(x_3\)) are observed, and a lower compensation when a moderate outcome is observed \((x = x_2\). In other words, the higher the optimal innovation degree desired by the investor, the larger (smaller) the compensation of extreme (moderate) returns.\(^7\)

The model here developed illustrates thus how the violation of the MLRP is a sufficient condition for a non-monotone optimal incentive scheme to emerge. In the context of delegated portfolio management, this may occur, if for instance, an (ex post) low return is a better signal of the selection of a more innovative portfolio than an (ex post) moderate return. In fact, it is usual that more innovation brings together more expected returns, but also more volatility. Thus, it may be optimal for the investor to reward the portfolio manager when extreme returns are observed (either sufficiently high or sufficiently low), and to punish him (in relative terms) when intermediate returns take place. Hence, it may be efficient to reward for failure, as long as low returns be sufficiently suggestive of high effort and creativity to elaborate investment strategies with a better profile of risk and expected return.\(^8\)

Our analysis suggests that this reward-for-failure property could explain partly some executive compensation practices commonly adopted in the financial industry, which have been highly questioned in the context of the recent subprime crisis. These practices include, among others, golden parachutes and golden coffins.

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\(^7\) All this analysis is ceteris paribus, i.e., keeping constant the model’s parameters, especially \(\delta\) and \(\lambda\). A more complete static comparative analysis is performed in the next section, where a positive relationship between optimal \(e\) and \(\Delta w\) is confirmed.

\(^8\) A similar idea of “reward for failure” has also been posed to encourage processes of technological innovation and exploration. See, for instance, Manso (2011), and Manso and Ederer (2013).
4. Numerical analysis on the optimal scheme

This section presents the results of a numerical analysis performed to illustrate some static comparative exercises on the optimal contract. In particular, we characterize how the optimal innovation degree and its associated non-linear compensation depend on preference and portfolio parameters.9

We perform two analysis. The first one is a sensitivity analysis related to the preference parameters λ and δ, but keeping constant parameters γ’s. The second is a sensitivity analysis related to the portfolio parameters γ’s and the risk aversion coefficient δ, but with a fixed value of λ. In both exercises, the space of returns is assumed to be X = {0.1, 3}, and the reservation utility U = 0.3.

4.1. Preference parameters analysis

To this exercise, we fix γ2 = 0.2, and γ1 = γ3 = 0.4.10 Table 1 reveals properties of the optimal contract that are consistent with more traditional agency models. First, optimal innovation e∗ is decreasing with λ and δ. This is a quite intuitive result, as it means that when innovation is more costly for the portfolio manager or he is more risk averse, the optimal innovation degree the principal can induce is lower. Second, reward for risk-taking Δw∗ is in general decreasing with λ and δ.11 The latter property is especially intuitive, as it points out that if the manager is more risk averse, it is optimal to expose him to less variability in his compensation. This is consistent with a classic result in contract theory: the power of the incentives is smaller as the agent is more risk averse relative to the principal.

4.2. Portfolio parameters analysis.

In order to evaluate the impact of changes in the characteristics of the portfolio (those different from the innovation degree), we compare two structures of γ’s. Fixing λ = 0.5, this exercise yields two portfolios with different profiles of expected return and risk:

Portfolio A. For which γ2 = 0.2, and γ1 = γ3 = 0.4, so that its conditional expectation is given by E(xγ) = 0.4e + 1, and its conditional variance corresponds to V(xγ) = (2 − 0.16e)e.

Portfolio B. For which γ2 = 0.4, and γ1 = γ3 = 0.3, so that E(xγ) = 0.3e + 1 and V(xγ) = (1.5 − 0.09e)e.

Thus, whereas Portfolio A has a higher expected return than Portfolio B, it exhibits more variability too.12

Table 2 compares optimal couples e∗ and Δw∗ for both portfolios. Simulations suggest that the portfolio with more likely extreme returns (Portfolio A) demands a higher risk-taking reward than that with more concentrated returns (Portfolio B), as the former also induces a larger degree of innovation.

5. Implementation

In this section we explore different ways of implementing the optimal incentive scheme previously characterized. This analysis includes two kind of exercises. First, we examine theoretically a proposal of implementing the optimal contract via already existing financial claims, such as initial equity and potential issues of additional shares.

Second, we present numerical simulations to compare the optimal incentive scheme with two option-like managerial compensation plans used in practice, which although suboptimal, have been proposed by prior research as possible means to achieve more risk-taking on the part of risk-averse managers. These plans are: (i) a pure bonus scheme, and (ii) a bonus scheme plus a fixed salary. This

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9 This numerical exercise is needed as there not exist an analytical solution for the optimal contract e∗ and \{wγ\}.

10 The constellation of values considered for parameters λ and δ so that it excludes a negative investor’s expected payoff and corner solutions for e∗, as well as it ensures that optimal fees wγ are positive real numbers.

11 There is a slight non-monotonicity when δ is close to the upper bound 0.7, beyond which the investor’s expected payoff is negative.

12 Note however that Si, the Sharpe ratio of portfolio i, is so that Si < S0. This means that an investor, with mean–variance preferences and without delegation, would choose Portfolio B.
Although suboptimal, bonus-type plans may yield innovation and investor’s payoff levels close to those of the optimal risk-reward scheme. This conjecture is based on prior research showing that exercise seems pertinent, as the public outrage against golden parachutes generated after the recent crisis can make any reward-for-failure compensation practice unrealistic and unviable. Thus, those managerial pay schemes that, despite they do not reward a low return, they do not penalize it, look at least a priori as promising candidates to achieve innovation and investor’s payoff levels close to the optimal ones.

5.1. Implementing via financial claims

A possible combination of financial claims replicating the optimal scheme includes giving the manager an initial share (or participation) $\alpha$ of the fund equity. Then, if the fund yields a return $x_j$ ($j = 2, 3$), an amount $\beta_j$ of additional shares (or claims) are issued and allocated to the investor, diluting thus the initial manager’s fund stake to $\frac{\alpha}{1+\beta_j}$.

Specific values of initial fund stake and potential new issues of shares are established in the next statement.

**Proposition 2.** The optimal incentive scheme can be implemented by giving the manager an initial fund stake of

$$\alpha = \frac{W_1}{x_1},$$

and:

(i) if $x_i = x_2$, by issuing and giving the investor an additional amount of shares equal to

$$\beta_2 = \frac{W_1 x_2}{W_2 x_1} - 1,$$

(ii) if $x_i = x_3$, by issuing and giving the investor an additional amount of shares equal to

$$\beta_3 = \frac{x_3}{x_1} - 1.$$
Table 3
Bonus scheme: innovation and reward for success.

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<th>( (\epsilon^b, \Delta w^b) )</th>
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<th>( \delta = 0.3 )</th>
<th>( \delta = 0.4 )</th>
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</table>

Option-like schemes can be an effective way to incentivize risk-averse managers to undertake riskier projects, as they do not penalize low performance. We explore two classes of such plans: (i) a pure bonus, and (ii) a bonus plus a fixed compensation.

A comparison between both alternative schemes and the optimal risk-taking structure is performed along three dimensions: (i) innovation levels, (ii) reward for risk (or success), and (iii) the investor’s payoff. In the last case, we specifically compute the investor’s optimal expected payoff as

\[
EB^b = \sum_{i=1}^{3} p^b_i (x_i - W^b_i),
\]

where \( k \) indicates the type of incentive scheme: optimal risk-taking reward (\( k = \ast \)), pure bonus (\( k = b \)), and bonus with fixed compensation (\( k = bf \)).

We begin our analysis with the pure bonus scheme \( b \). An optimal scheme of this class is obtained after imposing the additional constraint \( w_1 = w_2 = 0 \) on the problem characterized by Eqs. (3)–(5) in Section 3. This constraint is consistent with assuming that \( x_2 \) represents the return of a benchmark portfolio or asset, so that a positive fee is only paid as the portfolio return \( x_1 \) exceeds this benchmark. For instance, in the case of hedge fund managers, the typical benchmark is constant like the portfolio value at the beginning of the investment period (i.e. a zero return) or a Treasury yield (Brown et al., 1999), or it corresponds to the payoff of a more traditional portfolio like an index portfolio.

Table 3 reports optimal couples \( (\epsilon^b, \Delta w^b) \) resulting from the pure bonus scheme. A comparison between this table and Table 1 shows that whereas innovation levels of the bonus schedule are higher than those of the optimal scheme (in fact all of them are corner solutions), rewards \( \Delta w^b \) are much more higher as well. As a consequence, Table 4 reveals that the bonus plan exhibits a poor performance relative to the optimal scheme, as its investor’s expected payoff represents in general no more than 60% of that attained by the optimal scheme (with the exception of the cases with very low levels of risk aversion), becoming even negative for risk aversion coefficient sufficiently high (for instance with \( \delta = 0.5 \) for all values taken for \( \lambda \)).

We proceed our comparative analysis with the mixed scheme that includes a bonus and a fixed compensation, denoted by \( bf \). An optimal scheme of this type is obtained after imposing the additional constraint \( w_1 = w_2 = w \) on the problem characterized by Eqs. (3)–(5) in Section 3. Table 5 reports optimal couples \( (\epsilon^{bf}, \Delta w^{bf}) \) coming from such incentive plan. A comparison between Table 5 and Table 1 shows that whereas the innovation level of this mixed bonus scheme is always lower than that yield by the optimal scheme, reward for success \( \Delta w^{bf} \) is unambiguously higher than the risk-taking reward \( \Delta w^b \). As a result of this phenomenon, Table 6 reveals that this mixed bonus plan exhibits a fairly good performance relative to the optimal scheme, as its investor’s expected payoff represents in general more than 96% of that attained by the optimal scheme.

---

13 It is still a “gross” payoff as it is computed before investment \( l \).
14 This assumption is also consistent with the alternative interpretation of our model described in subSection 2.3.
15 As in the pure bonus scheme \( w^b_i = w^b = 0 \), the term \( \Delta w^b = w^b \) only represents a reward for success.
16 As in this mixed bonus scheme \( w^{bf}_i = w^{bf} = w \), the term \( \Delta w^{bf} = w^{bf} \) only represents a reward for success.
17 The remarkable performance of this bonus plus fixed compensation scheme in our model is consistent with the results of Gutiérrez and Salas-Pumás (2008). In fact, this work characterizes the second-best incentive scheme when uncertain output is governed by a random process with fat tails, showing that this scheme resembles a performance-standard contract that pays a fixed salary plus a capped bonus. Contrary to our work, however, they assume that the incentive zone is only that characterized by moderate performance and that density tails contain results that are much less informative on the effort provided.
6. Concluding remarks

This work studies optimal incentive schemes of institutional investors mandated to seek alternative investments, and in particular, it characterizes conditions under which these schemes allow for a reward-for-failure property. It is shown that this property emerges when the optimal contract has to reward extreme performance, that is, both sufficiently high and sufficiently low returns. In addition, a numerical analysis illustrates how changes in preference and return parameters affect this optimal scheme, revealing properties that in general are consistent with more traditional contract-theory models.

Two ways regarding implementation of the optimal scheme are explored. First, this scheme can be implemented by providing the manager with an initial fund stake subject to a dilution process in favor of the investor. Also, a numerical exercise analyzes the performance of option-like schedules, showing that a mixed structure involving a bonus plus a fixed payment, although suboptimal, achieves fairly good results on the investor’s expected payoff.

Table 4
Optimal scheme vs. bonus.

<table>
<thead>
<tr>
<th>$\delta$</th>
<th>$\beta = 0.2$</th>
<th>$\beta = 0.3$</th>
<th>$\beta = 0.4$</th>
<th>$\beta = 0.5$</th>
<th>$\beta = 0.6$</th>
<th>$\beta = 0.65$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda = 0.30$</td>
<td>0.9204</td>
<td>0.7876</td>
<td>0.4564</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$\lambda = 0.35$</td>
<td>0.8874</td>
<td>0.7097</td>
<td>0.2863</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$\lambda = 0.40$</td>
<td>0.8466</td>
<td>0.6260</td>
<td>0.1117</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$\lambda = 0.45$</td>
<td>0.8013</td>
<td>0.5392</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$\lambda = 0.50$</td>
<td>0.7533</td>
<td>0.4508</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$\lambda = 0.55$</td>
<td>0.7039</td>
<td>0.3619</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

* In fact, the investor’s expected payoff from the bonus scheme is zero.

Table 5
Bonus with fixed compensation scheme: Innovation and reward for success.

<table>
<thead>
<tr>
<th>$(\alpha_b, \alpha_f)$</th>
<th>$\delta = 0.2$</th>
<th>$\delta = 0.3$</th>
<th>$\delta = 0.4$</th>
<th>$\delta = 0.5$</th>
<th>$\delta = 0.6$</th>
<th>$\delta = 0.65$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda = 0.30$</td>
<td>(0.7927, 0.8173)</td>
<td>(0.5914, 0.7554)</td>
<td>(0.4246, 0.7071)</td>
<td>(0.2853, 0.6756)</td>
<td>(0.1680, 0.6716)</td>
<td>(0.1170, 0.6886)</td>
</tr>
<tr>
<td>$\lambda = 0.35$</td>
<td>(0.6717, 0.8135)</td>
<td>(0.5014, 0.7516)</td>
<td>(0.3607, 0.7041)</td>
<td>(0.2430, 0.6735)</td>
<td>(0.1435, 0.6704)</td>
<td>(0.1000, 0.6877)</td>
</tr>
<tr>
<td>$\lambda = 0.40$</td>
<td>(0.5831, 0.8108)</td>
<td>(0.4354, 0.7490)</td>
<td>(0.3136, 0.7020)</td>
<td>(0.2117, 0.6720)</td>
<td>(0.1252, 0.6694)</td>
<td>(0.0874, 0.6870)</td>
</tr>
<tr>
<td>$\lambda = 0.45$</td>
<td>(0.5152, 0.8088)</td>
<td>(0.3847, 0.7471)</td>
<td>(0.2775, 0.7004)</td>
<td>(0.1875, 0.6708)</td>
<td>(0.1111, 0.6687)</td>
<td>(0.0776, 0.6865)</td>
</tr>
<tr>
<td>$\lambda = 0.50$</td>
<td>(0.4616, 0.8073)</td>
<td>(0.3447, 0.7456)</td>
<td>(0.2488, 0.6991)</td>
<td>(0.1683, 0.6699)</td>
<td>(0.0998, 0.6681)</td>
<td>(0.0697, 0.6861)</td>
</tr>
<tr>
<td>$\lambda = 0.55$</td>
<td>(0.4181, 0.8061)</td>
<td>(0.3122, 0.7444)</td>
<td>(0.2255, 0.6981)</td>
<td>(0.1526, 0.6692)</td>
<td>(0.0906, 0.6676)</td>
<td>(0.0633, 0.6857)</td>
</tr>
</tbody>
</table>

Table 6
Optimal scheme vs. bonus with fixed compensation.

<table>
<thead>
<tr>
<th>$\rho_b^N$</th>
<th>$\delta = 0.2$</th>
<th>$\delta = 0.3$</th>
<th>$\delta = 0.4$</th>
<th>$\delta = 0.5$</th>
<th>$\delta = 0.6$</th>
<th>$\delta = 0.65$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda = 0.30$</td>
<td>0.9690</td>
<td>0.9689</td>
<td>0.9729</td>
<td>0.9788</td>
<td>0.9848</td>
<td>0.9862</td>
</tr>
<tr>
<td>$\lambda = 0.35$</td>
<td>0.9751</td>
<td>0.9746</td>
<td>0.9775</td>
<td>0.9821</td>
<td>0.9870</td>
<td>0.9881</td>
</tr>
<tr>
<td>$\lambda = 0.40$</td>
<td>0.9791</td>
<td>0.9784</td>
<td>0.9806</td>
<td>0.9845</td>
<td>0.9887</td>
<td>0.9896</td>
</tr>
<tr>
<td>$\lambda = 0.45$</td>
<td>0.9819</td>
<td>0.9811</td>
<td>0.9830</td>
<td>0.9863</td>
<td>0.9899</td>
<td>0.9908</td>
</tr>
<tr>
<td>$\lambda = 0.50$</td>
<td>0.9840</td>
<td>0.9832</td>
<td>0.9848</td>
<td>0.9877</td>
<td>0.9910</td>
<td>0.9917</td>
</tr>
<tr>
<td>$\lambda = 0.55$</td>
<td>0.9856</td>
<td>0.9849</td>
<td>0.9863</td>
<td>0.9889</td>
<td>0.9918</td>
<td>0.9924</td>
</tr>
</tbody>
</table>
Appendix A

Proof of Proposition 1. In the optimal contract program, thanks to the First-Order Approach, one can substitute constraint (5) with the FOC of the manager’s problem that solves the optimal innovation degree. \(^{18}\) This condition is given by

\[
(1 - \delta)\left[ \gamma_1 w_1^{1-\delta} - (1 - \gamma_2)w_2^{1-\delta} + \gamma_3 w_3^{1-\delta} \right] - \lambda e = 0. 
\]  

(6)

Moreover, at the optimal contract, the participation constraint is binding:

\[
(1 - \delta)e\left[ \gamma_1 w_1^{1-\delta} - (1 - \gamma_2)w_2^{1-\delta} + \gamma_3 w_3^{1-\delta} \right] + (1 - \delta)w_2^{1-\delta} - \lambda \frac{e^2}{2} = U. 
\]  

(7)

After combining (6) and (7), we obtain that

\[
w_0^2 = \left[ \frac{1}{(1 - \delta)} \left( U - \frac{\lambda e^2}{2} \right) \right]^{\frac{1}{1-\delta}}, 
\]  

(8)

which is always a positive real number given that \(\delta \in (0, 1)\) and assumption \(U > \frac{\lambda}{2}\). After replacing (8) into constraint (6), and some algebra, we get that

\[
w_0^3 = \left[ \frac{(1 - \gamma_2)\left( U - \frac{\lambda e^2}{2} \right) - (1 - \delta)\gamma_1 w_1^{1-\delta} + \lambda e_0}{(1 - \delta)\gamma_3} \right]^{\frac{1}{1-\delta}} \equiv w_3(w_1). 
\]  

(9)

Then, the program that solves the optimal contract is equivalent to

\[
\min \gamma_1 w_1 + \gamma_3 w_3(w_1). 
\]

Using (9) in the FOC of this problem, it follows that

\[
w_0^3 = w_0^2 = w_0^1 = \left[ \frac{1}{(1 - \delta)} \left( U - \frac{\lambda e_0^2}{2} + \frac{\lambda e_0}{1 - \gamma_2} \right) \right]^{\frac{1}{1-\delta}}, 
\]  

(10)

which is also a positive real number given that by assumption \(U > \frac{\lambda}{2}\) and \(\gamma_2 < 1\). A comparison between (8) and (10) reveals finally that

\[
w_0^2 = w_0^3 > w_0^1 > 0, 
\]

which completes the proof. \(\square\)

Proof of Corollary 1. First, after taking derivative of (10) with respect to \(e_0\), we have that

\[
\text{sign} \frac{\partial w_0^1}{e_0} = \text{sign} \frac{\partial w_0^2}{e_0} = \text{sign} \left( \frac{1}{1 - \gamma_2} - e_0 \right) > 0, 
\]

(11)

where the inequality holds because by assumption \(e_0 \leq 1\) and \(\gamma_2 < 1\). Second, a similar derivative of (8) implies that

\[
\text{sign} \frac{\partial w_0^3}{e_0} = \text{sign} \left( - \frac{\lambda e_0}{1 - \delta} \right) < 0, 
\]

(12)

which completes the proof. \(\square\)

Proof of Proposition 2. First, if \(x_i = x_1\), the manager receives from his initial stake in the fund a payoff \(ax_1\), which after substituting \(a\), becomes

\[
ax_1 = w_1^*.
\]

\(^{18}\) The concavity of \(p_e^i\) guarantees the validity of the first-order approach.
Second, if $x_1 = x_2$, the dilution process implies that now the manager’s payoff is $\frac{w_2}{1 + \beta_2}$, which after substituting $x$ and $\beta_2$ corresponds to

$$\frac{2x_2}{1 + \beta_2} = w_2.$$ 

Third, if $x_1 = x_3$, the dilution process implies now that the manager’s payoff is $\frac{2x_3}{1 + \beta_3}$, which after substituting $x$ and $\beta_3$, becomes

$$\frac{2x_3}{1 + \beta_3} = w_3,$$

which proves the validity of the implementation via shares.  

References


