FACULTAD ECONOMÍA Y NEGOCIOS UNIVERSIDAD DE CHILE
DEPARTAMENTO DE ECONOMÍA

SERIE DE DOCUMENTOS DE TRABAJO

SDT 415

Ambiguity is Detrimental for Long-Run Cooperation

Autores:
Marco Rojas
Damián Vergara

Santiago, Septiembre de 2016

sdt@econ.uchile.cl
econ.uchile.cl/publicaciones

Ambiguity is Detrimental for Long-Run Cooperation[1]

Marco Rojas[a], Damián Vergara[b]

[a] *Department of Economics, University of Michigan*
[b] *Department of Economics, University of Chile*

Abstract

Why agents cooperate is an old question that has been widely studied in economics, as well as in other disciplines. In economics, the focus has been mainly put in analyzing whether cooperative behaviors may arise from repeated interactions between strategic individuals. This paper seeks to contribute to the literature by studying the effects of ambiguity, shortly defined as uncertainty about the realization of future states and the probabilities assigned to their realization, on long-run cooperation. Concretely speaking, the infinitely repeated Prisoner's Dilemma, and the consequent determination of conditions for sustaining long-run cooperative behaviors, is revisited in an ambiguity setting. Results suggest that ambiguity is detrimental to cooperation as it decreases (increases) the expected payoff of the cooperative (non-cooperative) equilibrium and, therefore, (1) it may change the game's structure from the Prisoner's Dilemma setting to scenarios in which the expected payoff associated with the cooperative equilibrium is no longer Pareto-superior, and (2) it demands more patient players for sustaining cooperative agreements when the game's structure remains unchanged. An application is made to the Cournot duopoly and the likelihood of sustaining a tacit collusion. In addition to previous results, it is found that the Cournot duopoly may not longer behave as a particular case of the Prisoner's Dilemma for high levels of ambiguity. A positive relation between ambiguity and competition is stated. Therefore, our results may serve as an additional theoretical input for antitrust policy design.

Keywords: Ambiguity, Neo-Additive Capacities, Long-Run Cooperation, Infinitely Repeated Prisoner's Dilemma, Cournot Duopoly, Tacit Collusion.

1. Introduction

Why agents cooperate is an old question that has been widely studied in economics, as well as in other disciplines (Nowak and Highfield, 2011). In economics, the focus has been mainly put in analyzing whether cooperative behaviors may arise from repeated interactions between strategic individuals. The analysis, given games' structures that inhibit short-run cooperation between rational players, is to determine conditions over the repeated game that allow long-run cooperative

[1] We would like to thank Juan Pablo Torres-Martínez for his great support on writing this paper and Andrés Carvajal, Alejandro Corvalán, Juan Pablo Gama-Torres, Flavio Menezes and the assistants of the internal seminar of the Faculty of Economics and Business of the University of Chile and the 16th SAET Conference in Rio de Janeiro, Brazil, for very helpful comments. We acknowledge financial support from FONDECYT project 1150207. A MATLAB code with the numerical calculations is available at https://goo.gl/rtxWhs.

Email addresses: `rojasmi@umich.edu` (Marco Rojas), `dvergara@fen.uchile.cl` (Damián Vergara)

This Version: September 14, 2016

relations (Mailath and Samuelson, 2006). In that context, a classical case analysis has been the infinitely repeated Prisoner's Dilemma, where rational players that are *patient enough* are able to sustain cooperative agreements. Industrial organization has applied this reasoning when analyzing potential tacit collusions between competing firms, given imperfect competition frameworks.

This paper seeks to contribute to the literature by studying the effects of ambiguity on long-run cooperation.[2] In general, when analyzing strategic interactions and potential long-run cooperative equilibria, standard theory assumes that once individuals begin cooperating, they cooperate forever. Hence, deviations to non-cooperative behaviors always take individuals by surprise, as deviation is not profitable and, therefore, is not considered a plausible scenario. Nevertheless, it may be reasonable to assume that individuals may anticipate potential deviations from expected behaviors, either from cooperative to non-cooperative behaviors, or vice versa. For example, individuals may distrust on the other player's rationality, may anticipate potential signalling actions, or may simply internalize that the counterpart could eventually make mistakes when executing the actions.

In that line, this paper revisits the infinitely repeated Prisoner's Dilemma in an ambiguity setting, with a focus on the analysis of the possibility of sustaining a cooperative equilibrium in the long-run. We follow Eichberger, Kelsey, and Schipper (2009) strategy to model ambiguity in strategic games based on Chateauneuf, Eichberger, and Grant (2007). In this framework, individuals anticipate that their counterparts may not behave in the way they are expected to, i.e. they face some degree of uncertainty about the other player's action. Formally, individuals partially distrust their own beliefs about other player's behavior and place themselves in the best and worst cases off the expected path, depending on their relative optimism and pessimism (i.e. depending on their attitude toward uncertainty). The model allows us to analyze how ambiguity affects long-run decisions regarding cooperative behaviors. The analysis is applied to the Cournot duopoly, which is usually understood as a particular case of the Prisoner's Dilemma. The application studies how ambiguity affects the competitive equilibrium and the likelihood of achieving a tacit collusive agreement in the long-run.

Results suggest that ambiguity is detrimental to cooperation. First, ambiguity may change the game's structure from the Prisoner's Dilemma setting to contexts in which a cooperative equilibrium is no longer Pareto-superior in expected value. In those cases, characterized by relatively *high* levels of uncertainty, it stops being relevant to ask for conditions for implementing a cooperative equilibrium, as the non-cooperative static equilibrium is Pareto-superior. In formal terms, expected payoffs associated with the cooperative equilibrium are lower than the minmax payoff. Second, in cases when asking for conditions to implement a cooperative equilibrium is still a relevant question (i.e. when the game's structure regarding equilibria expected payoffs remain unchanged), ambiguity decreases the likelihood of sustaining a cooperative equilibrium, as higher levels of patience are demanded. In formal terms, the critical discount factor needed for cooperating is increasing in the level of uncertainty.

[2] The concept of ambiguity extends the notion of risk, stating that not only the realization of future states is unknown, but also the probabilities assigned to their realization. Ambiguity has become an important topic in economic theory as it has been able to explain some facts that the standard theory has failed to (see, for example, Ellsberg, 1961, and Chen and Epstein, 2002). For a survey about ambiguity, see Etner, Jeleva, and Tallon (2012).

Moreover, when analyzing the application to the Cournot duopoly, a third channel through which ambiguity may affect long-run cooperation arises. In the Cournot duopoly game, payoffs are not exogenous: they come from the firms' optimization problem. As ambiguity affects the objective function, equilibrium quantities (and, therefore, profits) are perturbed. It is found that in some scenarios, characterized by *high* levels of ambiguity and relatively pessimistic firms, the Cournot duopoly stops behaving as a particular case of the Prisoner's Dilemma. New structures share the implications of the first result: the competitive equilibrium expected payoff is larger than the expected payoff associated with the collusive agreement. In cases in which the game's structure is unaffected, two previous results apply. Then, ambiguity reduces the likelihood of sustaining a tacit collusion. Considering that competitive equilibrium quantities are increasing in the level of ambiguity, results can be reinterpreted as a positive relation between ambiguity and competition. As is discussed in Section 4, this result can be used as a theoretical input for antitrust policy design.

This paper relates with three branches of the literature. First, there exists a wide literature of strategic interactions (and the potential achievement of cooperative equilibria) in repeated games with incomplete information. Kreps, Milgrom, Roberts, and Wilson (1982) and Conlon (2003) analyze finitely repeated games with incomplete information of counterpart's *type*. Ellison (1994), Watson (1994), Aoyagi (1996) and Chan (2000) do likewise for infinitely repeated games. Kandori (1992), Sekiguchi (1997), Compte (1998), Compte (2002), Ely and Valimaki (2002) and Piccione (2002) study infinitely repeated games with imperfect monitoring. Neyman (1999) and Normann and Wallace (2012) consider scenarios with incomplete information about game's termination rules. Second, industrial organization literature has analyzed tacit collusion in oligopoly models with stochastic environments (Green and Porter, 1984; Rotemberg and Saloner, 1986; Bagwell and Staiger, 1997; Kandori and Matsushima, 1998; Athey and Bagwell, 2001; Athey, Bagwell, and Sanchirico, 2004; Rojas, 2012). Therefore, our paper contributes to this literature by analyzing the likelihood of cooperation in repeated strategic interactions (and the application to oligopoly models) in an uncertainty context novel to the literature: players with ambiguous beliefs on counterpart's actions.

Third, this paper contributes to the ambiguity in strategic games literature. The literature has proposed several equilibrium concepts for games with ambiguity. And for illustrating the implications of uncertainty, different applications have been made on the Prisoner's Dilemma. Dow and Werlang (1994) analyze the twice repeated game, concluding that backward induction may not hold under uncertainty, thus existing the possibility of cooperation in the finite game. Marinacci (2000) shows that ambiguity has no effect on the static Prisoner's Dilemma's outcome. Haller (2000), Dimitri (2005) and Rothe (2011) also discuss the implications of uncertainty on the static and finitely repeated Prisoner's Dilemma outcomes. Nevertheless, none of these works has studied the infinitely repeated game and the conditions needed for sustaining a cooperative equilibrium in the long-run. On the other hand, scarce ambiguity applications have been made to oligopoly models. Fontini (2005) models the Cournot oligopoly under uncertainty using the Choquet Expected Utility model and concludes that, when uncertainty is *low*, optimistic firms make higher profits than pessimistic firms, and when uncertainty is *high*, only optimistic firms participate making losses. Eichberger, Kelsey, and Schipper (2009) model ambiguity in strategic games using the neo-additive capacities framework (Chateauneuf, Eichberger, and Grant, 2007) and find that in a static Cournot (Bertrand) duopoly model, given a *high* level of optimism, ambiguity increases (decreases) equilibrium quantities and decreases (increases) market prices, thus claiming that ambiguity may have different impact on the level of competition on a given market depending on the attitude that players have towards it. This paper extends Eichberger, Kelsey, and Schipper (2009) contribution, as using the same analytical framework 1) our *best* and *worst* outcomes are endogenously determined by the optimization problem of the firms, and 2) we extend the anaylisis to the possibility of sustaining a tacit collusion in the long-run between competing firms. Our economic interpreation differs, as we find a positive relation between ambiguity and competition, regardless the attitude towards uncertainty.

The rest of the paper is structured as follows. After this introduction, Section 2 specifies the strategy used to model ambiguity. Section 3 revisits the repeated Prisoner's Dilemma under this setting, while Section 4 makes the application to the Cournot's duopoly model. Finally, Section 5 concludes.

2. Preliminaries: Ambiguity and Strategic Games

We follow Chateauneuf, Eichberger, and Grant (2007) framework based on neo-additive capacities to model ambiguity.[3] A neo-additive capacity is a particular capacity which can be interpreted as a convex combination between an additive probability distribution and a capacity that only distinguishes if a state is possible, impossible or certain. Given a space of actions X, the Choquet integral of the function $f : X \to \mathbb{R}$ with respect to the neo-additive capacity $v : 2^X \to \mathbb{R}_+$ is defined by

$$\int f dv \quad := \quad \delta(\alpha M + (1-\alpha)m) + (1-\delta)E_\pi f, \tag{1}$$

where δ is the degree of ambiguity, α is the degree of optimism, E_π is the expectation induced by the probability distribution π (defined over X), $M = \max_{x \in X} f(x)$ and $m = \min_{x \in X} f(x)$. Chateauneuf, Eichberger, and Grant (2007) axiomatized (1) as a choice criterion under ambiguity. It can be noted that in a context of no ambiguity (i.e. $\delta = 0$), (1) is reduced to the standard Expected Utility model (Savage, 1954). On the other hand, for a completely ambiguous and pessimistic individual (i.e. $\delta = 1$ and $\alpha = 0$), (1) mimics the maxmin expected utility model (Gilboa and Schmeidler, 1989).

Eichberger, Kelsey, and Schipper (2009) follow this strategy to model ambiguity in strategic games. The authors propose a game in the form $G = \langle (S_i, u_i)_{i=1,2} \rangle$, where S_i and u_i are the strategies space and the utility function of player i, respectively. In this context, π (and, consequently, uncertainty) is defined over S_{-i}. Then, the expected utility under ambiguity of player i when choosing strategy s_i, is defined by

$$v(s_i; \delta, \alpha, \pi) := \delta(\alpha M_i(s_i) + (1-\alpha)m_i(s_i)) + (1-\delta)E_\pi u_i(s_i, s_{-i}), \tag{2}$$

where $s_i \in S_i$, $s_{-i} \in S_{-i}$, $M_i(s_i) = \max_{s_{-i} \in S_{-i}} u_i(s_i, s_{-i})$ and $m_i(s_i) = \min_{s_{-i} \in S_{-i}} u_i(s_i, s_{-i})$. In this context, ambiguity is understood as the uncertainty an individual faces regarding other

[3]For a technical discussion about capacities and neo-additive capacities, see Appendix A.

player's decisions.

This representation of ambiguity has three good properties. First, it has a clear intuition. The individual faces a subjective additive probability measure, π, but does not trust it fully. The ambiguity parameter, δ, measures the degree of distrust on π. Then, the *unassigned* probability is mapped to the best and worst possible outcomes, depending on the degree of optimism of the individual. Second, no assumptions are imposed about attitude towards ambiguity: the model can flexibly represent both optimistic and pessimistic individuals. Third, the setting fits well on the strategic games modelling. Concretely, it makes sense to assume the existence of π, because it can be derived endogenously from the game's equilibrium.

3. The Infinitely Repeated Prisoner's Dilemma

This section analyzes the infinitely repeated Prisoner's Dilemma in an ambiguity setting, using the neo-additive capacities based framework to model ambiguity. Consider the normal form representation of the Prisoner's Dilemma

	C	N
C	(c,c)	(e,d)
N	(d,e)	(n,n)

where C and N stand for *Cooperate* and *Non Cooperate*, $d > c > n > e$ and $\frac{d+e}{2} < c$. Given incentives for deviation, (N,N) is the only static Nash equilibrium, even though the cooperative equilibrium, (C,C), is Pareto-superior. In the repeated game, (C,C) can be implemented as a long-run equilibrium if the discounted benefits of always cooperating are larger than the discounted benefits of deviating from a cooperative equilibrium and then being punished for that. The analysis suggests that individuals have to be *patient enough* to achieve a cooperative equilibrium, i.e. individuals' subjective discount factors, β, have to be *large enough*. The aim of this paper is to analyze the effects ambiguity has on the possibility of implementing cooperative equilibria in these kind of games.

In our setting, ambiguity induces uncertainty on the other player's action through (δ,α,π). This affects the expected payoffs of playing C and N. For the individuals, the best (worst) scenario is the other individual to play C (N), as $c > e$ and $d > n$. The probability distribution, π, is derived from the game's outcome. Concretely, π states the probability an individual assigns to the counterpart to play C, p, and N, $1-p$. Then, expected payoffs of playing C and N are defined by

$$V(C;\delta,\alpha,\pi) = \delta\left(\alpha c + (1-\alpha)e\right) + (1-\delta)(pc + (1-p)e), \quad \text{and} \tag{3}$$
$$V(N;\delta,\alpha,\pi) = \delta\left(\alpha d + (1-\alpha)n\right) + (1-\delta)(pd + (1-p)n). \tag{4}$$

In the one-shot game, $p = 0$ holds as playing N is dominant strategy. Then, expected payoffs are reduced to $V(C;\delta,\alpha,\pi(p=0)) = \delta\left(\alpha c + (1-\alpha)e\right) + (1-\delta)e$ and $V(N;\delta,\alpha,\pi(p=0)) = \delta\left(\alpha d + (1-\alpha)n\right) + (1-\delta)n$. Since $d > c$ and $n > e$, $V(N;\delta,\alpha,\pi(p=0)) > V(C;\delta,\alpha,\pi(p=0))$ holds for every (δ,α) combination. Therefore, our ambiguity setting does not affect the static equilibrium (consistent with Marinacci, 2000).[4] However, in the infinitely repeated game it could exist

[4]Moreover, it remains being the only Nash equilibrium. Note that $V(N;\delta,\alpha,\pi(p=1)) > V(C;\delta,\alpha,\pi(p=1))$ and, therefore, incentives for deviating from a cooperative equilibrium remain existing in an ambiguity context.

an equilibrium with $p = 1$, if the patience conditions are consistent with individuals' subjective discount factors. In that context, ambiguity affects the discounted expected payoffs of playing the different strategies and, therefore, the possibility of implementing such equilibrium.

It is important to realize that the Prisoner's Dilemma analysis rises the question about the possibility of implementing a cooperative equilibrium in the long-run given that the cooperative equilibrium payoff, c, is larger than the non-cooperative equilibrium payoff, n. In our ambiguity setting, c and n are replaced by the following expected payoffs

$$\begin{aligned} V(C;\delta,\alpha,\pi(p=1)) &= \delta(\alpha c + (1-\alpha)e) + (1-\delta)c, \quad \text{and} && (5) \\ V(N;\delta,\alpha,\pi(p=0)) &= \delta(\alpha d + (1-\alpha)n) + (1-\delta)n. && (6) \end{aligned}$$

Note that ambiguity affects the equilibria's payoffs by decreasing the expected value of the cooperative equilibrium (as relatively pessimistic individuals internalize that the counterpart may deviate from the expected action to a non-cooperative behavior, thus losing $c - e$) and increasing the expected value of the non-cooperative equilibrium (as relatively optimistic individuals internalize that the counterpart may deviate from the expected action to a cooperative behavior, thus obtaining $d - n$). In fact, as $\delta \to 0$, $V(C;\delta,\alpha,\pi(p=1)) > V(N;\delta,\alpha,\pi(p=0))$, and as $\delta \to 1$, $V(N;\delta,\alpha,\pi(p=0)) > V(C;\delta,\alpha,\pi(p=1))$. In the later case, seeking to implement the cooperative equilibrium stops being desirable, as the static equilibrium is Pareto-superior. This intuition leads to Proposition I.

Proposition I: Given d, c, n and e,

$$V(N;\delta,\alpha,\pi(p=0)) > V(C;\delta,\alpha,\pi(p=1)) \quad \Leftrightarrow \quad \delta\left[\alpha(d-n) + (1-\alpha)(c-e)\right] > c - n. \tag{7}$$

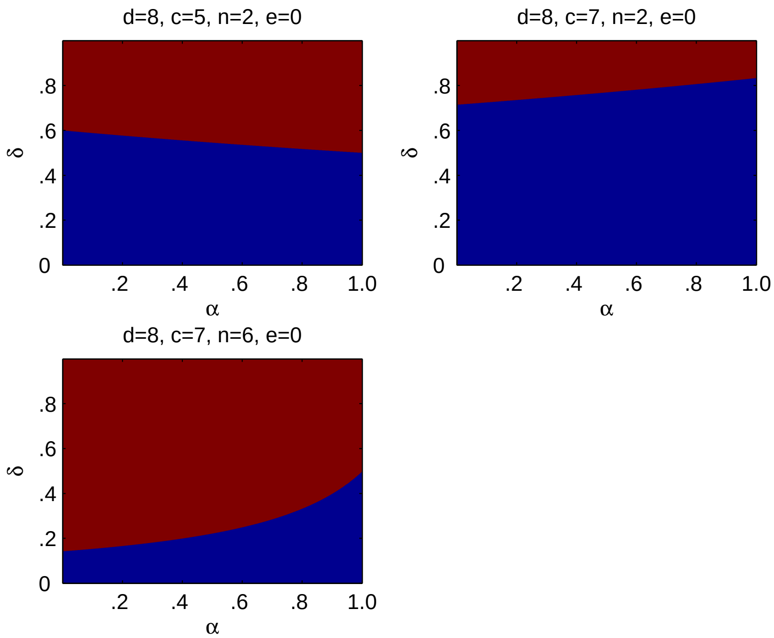
Proof: direct from (5) *and* (6).

From Proposition I, it follows that for some (δ, α) combinations characterized by *high* levels of ambiguity, uncertainty may change the game's structure to scenarios in which the expected payoff associated with the cooperative equilibrium is lower than the non-cooperative equilibrium payoff (minmax). This result suggests that, in those cases, asking for conditions for sustaining a cooperative equilibrium stops being relevant, given static non-cooperative equilibrium Pareto-superiority.

Figure 1 illustrates the first result. The three graphs represent Prisoner's Dilemma games with different exogenous payoffs. In each graph, the y-axis represents δ and the x-axis represents α. Therefore, each point represents an (α, δ) combination. Finally, blue and red zones represent parametric combinations in which $V(N;\delta,\alpha,\pi(p=0)) < V(C;\delta,\alpha,\pi(p=1))$ and $V(N;\delta,\alpha,\pi(p=0)) > V(C;\delta,\alpha,\pi(p=1))$, respectively. The basic idea is the following. Let $\delta = 0$. Given α, start increasing uncertainty. From a given point, ambiguity is large enough to make the cooperative equilibrium expected payoff no longer desirable. In those cases, asking for conditions for implementing cooperative agreements stops being a relevant issue. As ambiguity affects the relative payoffs through $\delta\alpha(d-n)$, $\delta(1-\alpha)(c-e)$ and their relation with $c - n$, the relation between the exogenous payoffs intuitively shapes the blue and red zones.

Now, let us restrict the attention to cases in which $V(C;\delta,\alpha,\pi(p=1)) > V(N;\delta,\alpha,\pi(p=0))$. In the repeated game, given a punishment scheme, cooperation is an equilibrium if the discounted

Figure 1: First Result



benefits of cooperating are larger than the discounted benefits of deviating from the cooperative agreement and then being punished for that. We look on the effects ambiguity may have on this long-run analysis.

We consider a grim trigger punishment scheme: an individual plays C until the other player deviates, punishing her by playing N forever. Given that, strategies considered are the same as in the standard case (i.e. non-ambiguity setting). The novelty in our analysis is that the discounted benefits of the different strategies are affected by ambiguity, as individuals internalize the possibility that the other player might choose an action different from the expected.

Concretely speaking, uncertainty about the other player's behavior (i.e. ambiguity) affects the computation of discounted benefits through three different channels. First, ambiguity induces changes on the expected payoffs of the different strategies, as payoffs depend on the other player's action which is (by definition) partially unpredictable given uncertainty. Second, potential future deviations from the cooperative equilibrium may induce potential future punishments. Third, whenever an individual plays an action different from the expected, counterparts update their beliefs. We follow Eichberger, Grant, and Kelsey (2010) to deal with that issue. While technical details about updating neo-additive capacities are discussed in Appendix B, the intuition is the following. Given the game's structure, π is always degenerated. That is, it assigns probability 1 to the counterpart to play C or N. Then, whenever the counterpart plays the expected action, the individual does not make any update, as *"everything went as expected"*. On the other hand, when the counterpart deviates from the expected action, the individual updates δ to 1, as the probability measure stops being trustworthy. That reaction makes sense, as the other player's action was expected to happen with probability 0. This affects the expected payoffs of the different strategies, given ambiguity parameters. That being said, we can derive the patience conditions for cooperation, which are summarized in Proposition II.

Proposition II: (C, C) is an equilibrium of the repeated game if and only if the subjective discount factor, β, meets the following condition:

$$\frac{c^* - n_u^*}{1 - \beta\phi_c} + \frac{n_u^*}{1-\beta} \quad > \quad d^* + \frac{\beta\phi_c(n^* - n_u^*)}{1 - \beta\phi_n} + \frac{\beta n_u^*}{1-\beta}, \tag{8}$$

where

$$\begin{aligned} d^* &= \delta(\alpha d + (1-\alpha)n) + (1-\delta)d \\ c^* &= \delta(\alpha c + (1-\alpha)e) + (1-\delta)c, \\ n^* &= \delta(\alpha d + (1-\alpha)n) + (1-\delta)n, \\ n_u^* &= \alpha d + (1-\alpha)n, \\ \phi_c &= 1 - \delta(1-\alpha), \qquad \textit{and} \\ \phi_n &= 1 - \delta\alpha. \end{aligned}$$

Proof: See Appendix C.

Define $\beta^*(\delta, \alpha)$ as the critical subjective discount factor, i.e. the discount factor that meets (8) with equality

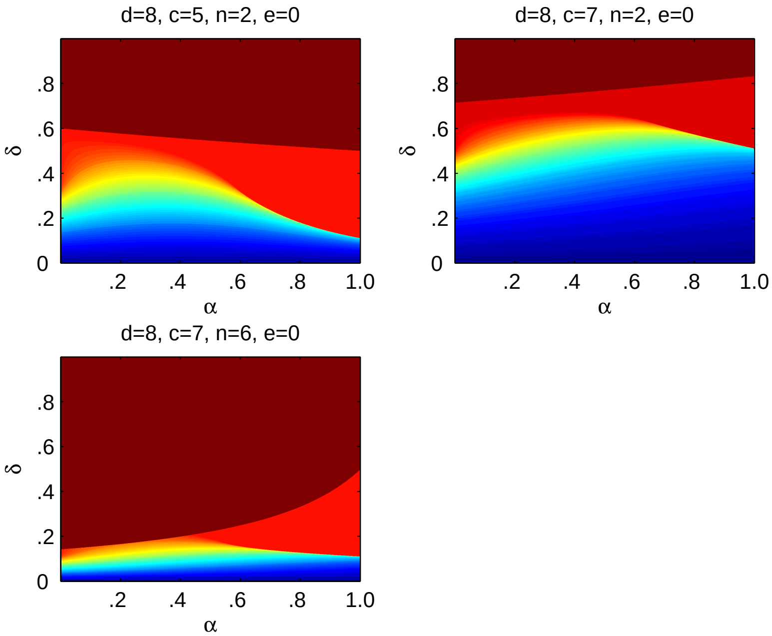
$$\frac{c^* - n_u^*}{1 - \beta^*(\delta,\alpha)\phi_c} + \frac{n_u^*}{1 - \beta^*(\delta,\alpha)} \quad = \quad d^* + \frac{\beta^*(\delta,\alpha)\phi_c(n^* - n_u^*)}{1 - \beta^*(\delta,\alpha)\phi_n} + \frac{\beta^*(\delta,\alpha)n_u^*}{1 - \beta^*(\delta,\alpha)}. \tag{9}$$

The (highly) non-linear nature of (9) prevents us for making an analytical characterization of $\beta^*(\delta, \alpha)$. Nevertheless, the critical discount factor can be numerically calculated for given parameters. A numerical analysis leads us to conclude that in cases in which the game's structure remains unchanged, i.e. when finding conditions to implement a cooperative equilibrium in the infinitely repeated game is profitable for players, ambiguity decreases its likelihood, as it demands more patient individuals. In formal terms, this result suggests that the larger the ambiguity, the larger the subjective discount factor needed to sustain a cooperative equilibrium.

Figure 2 illustrates the second result. The three graphs are the same games as in Figure 1. The red zones still account for parametric combinations in which the cooperative equilibrium is irrelevant for players. Nevertheless, the former blue zone is replaced by a characterization of the critical subjective discount factor needed for sustaining a cooperative equilibrium. The bluer the color, the smaller the critical discount factor needed. It can be seen that, for a given α, the higher the ambiguity, the higher the critical discount factor. More striking, for some (δ, α) combinations, the critical discount factor needed may be higher than one (orange zones), thus being cooperation potentially infeasible even when it is still profitable for the players.

Taken together both results, the analysis suggests that ambiguity is detrimental for cooperation. In words, uncertainty reduces the relative expected gains from cooperating and increases the patience requirements to achieve cooperative agreements. In the next section, the analysis is applied to the Cournot duopoly, which have been usually interpreted as a particular case of the Prisoner's Dilemma.

Figure 2: Second Result



4. Application: Cournot Duopoly

The standard Cournot duopoly has been usually seen as a particular case of the Prisoner's Dilemma. That is, given incentives for deviation, firms competing in quantities cannot achieve a static collusive agreement, although it is profitable to do it. We extend the previous analysis to this framework to study the effects of ambiguity on the standard Cournot duopoly's equilibrium. This case is slightly more complex than the analysis developed in the previous section, due to the fact that payoffs are no longer exogenous: they come from the firms' optimization problem.

Assume the existence of two firms competing in quantities, producing an homogeneous product with constant marginal cost k and facing an inverse demand function $P(Q) = A - bQ$, where $A > k$ and $b > 0$. Given the existence of ambiguity on the competitor's action, firms internalize that the other firm might produce a quantity different from the expected, i.e. different from the standard Cournot equilibrium. Then, following (2), the objective function of firm i is defined by

$$\begin{aligned}\max_{q^N} V_N &= \delta\left[\alpha(A - b(q^N + q^M))q^N + (1-\alpha)(A - b(q^N + q_j))q^N\right] \\ &\quad + (1-\delta)(A - b(q^N + q_j))q^N - kq^N, \end{aligned} \tag{10}$$

where q^N is the optimal competitive quantity and q_j is the quantity optimally produced by the other firm. Here, relatively *optimistic* firms consider that the counterpart may produce the collusive quantity, q^M. At the same time, the collusive quantity comes from maximizing the following problem

$$\begin{aligned}\max_{q^M} V_M &= \delta\left[\alpha(A-2bq^M)q^M+(1-\alpha)(A-b(q^M+q^D))q^M\right]\\ &+(1-\delta)(A-2bq^M)q^M-kq^M, \end{aligned} \tag{11}$$

where relatively *pessimistic* firms consider the possibility of the other firm deviating from the agreement and optimally choosing q^D, which in turn comes from the following problem

$$\begin{aligned}\max_{q^D} V_D &= \delta\left[\alpha(A-b(q^D+q^M))q^D+(1-\alpha)(A-b(q^D+q_j))q^D\right]\\ &+(1-\delta)(A-b(q^D+q^M))q^D-kq^D, \end{aligned} \tag{12}$$

where relatively *pessimistic* firms consider the possibility of the other firm taking the same decision simultaneously. Then, by taking first order conditions an applying symmetry, we can derive the reaction functions to then compute the equilibrium quantities. Proposition III summarizes those results.

Proposition III: In the scenario considered above, the equilibrium quantities are given by

$$\begin{aligned} q^N(\delta,\alpha) &= \frac{(A-k)}{b(3-\delta\alpha)}\left[\frac{2+(1-\delta(1-\alpha))(6-\delta(3-\alpha))}{2+3(1-\delta(1-\alpha))(2-\delta(1-\alpha))}\right],\\ q^M(\delta,\alpha) &= \frac{2(A-k)(1-\delta(1-\alpha))}{b(2+3(1-\delta(1-\alpha))(2-\delta(1-\alpha))},\\ q^D(\delta,\alpha) &= \frac{(A-k)(2-\delta(1-\alpha))(3-\delta(1-\alpha))}{b(2-\delta(1-\alpha))(2+3(1-\delta(1-\alpha))(2-\delta(1-\alpha)))}. \end{aligned}$$

Proof: See Appendix D.

With the equilibrium quantities, we can compute the game's payoffs

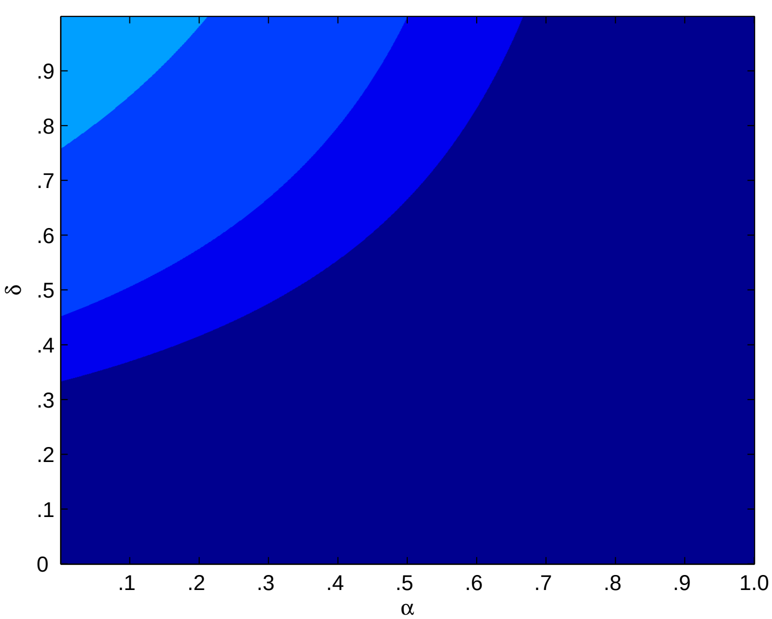
$$\begin{aligned} d(\delta,\alpha) &= (A-b(q^M(\delta,\alpha)+q^D(\delta,\alpha))-k)q^D(\delta,\alpha),\\ c(\delta,\alpha) &= (A-2bq^M(\delta,\alpha)-k)q^M(\delta,\alpha),\\ n(\delta,\alpha) &= (A-2bq^N(\delta,\alpha)-k)q^N(\delta,\alpha),\\ e(\delta,\alpha) &= (A-b(q^M(\delta,\alpha)+q^D(\delta,\alpha))-k)q^M(\delta,\alpha). \end{aligned}$$

The usual analysis regarding the possibility of sustaining a tacit collusion is analogous to the one developed in the previous section: although profits earned with the collusive agreement are larger than profits earned from competing, incentives for deviating from the collusive agreement make the collusion infeasible in the short run. Therefore, collusion is only feasible in the infinitely repeated game under some patience conditions. Nevertheless, in our ambiguity setting, game's payoffs depend (in a highly non-linear way) on the ambiguity parameters and, therefore, is not obvious that payoffs are consistent with a Prisoner's Dilemma structure. In other words, it is worth asking if the Cournot duopoly remains being a particular case of the Prisoner's Dilemma for every parametric combination.

A numerical analysis allows us to conclude that some (δ, α) combinations, characterized by *high* levels of ambiguity and relatively pessimistic firms, induce game structures different from the Prisoner's Dilemma. In the new structures, the competitive equilibrium expected payoff is larger than the collusive equilibrium expected payoff. Therefore, a third channel arises through which ambiguity may affect negatively the possibility of achieving a collusive agreement, again by making irrelevant the search for conditions for sustaining a cooperative equilibrium.

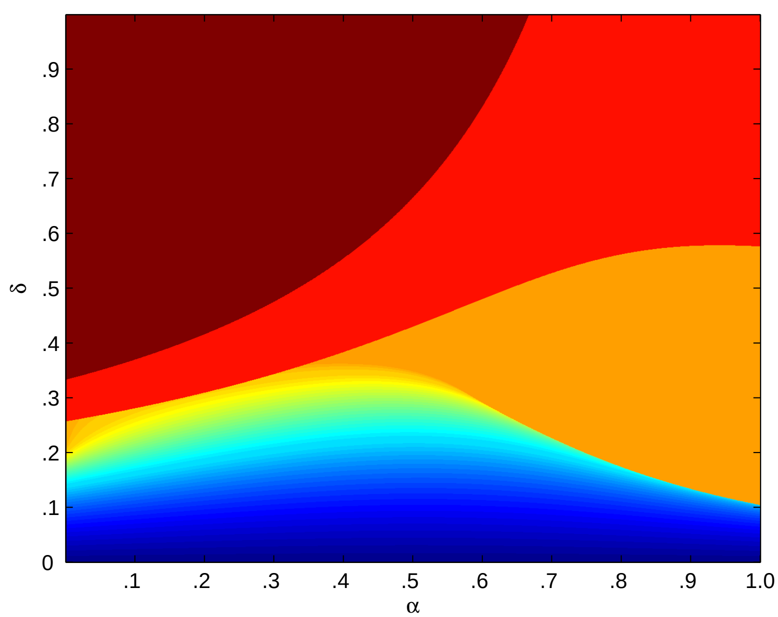
Figure 3 illustrates the third result. The dark blue zone represents (δ, α) combinations where the Prisoner's Dilemma structure holds, i.e. parametric combinations in which $d(\delta, \alpha) > c(\delta, \alpha) > n(\delta, \alpha) > e(\delta, \alpha)$. In those cases, previous results apply, being ambiguity negatively related with the possibility of achieving a cooperative equilibrium in the repeated game. Nevertheless, light blue zones are associated with different games that differ from the Prisoner's Dilemma structure. From darker to lighter, payoffs' relation of those games is characterized by $c(\delta, \alpha) > d(\delta, \alpha) > n(\delta, \alpha) > e(\delta, \alpha)$, $c(\delta, \alpha) > n(\delta, \alpha) > d(\delta, \alpha) > e(\delta, \alpha)$ and $n(\delta, \alpha) > c(\delta, \alpha) > d(\delta, \alpha) > e(\delta, \alpha)$, respectively. While each game displays its own particular features (new structures appear to mimic *classic* games), the three structures share one important characteristic: expected payoffs associated with the competitive equilibrium always Pareto-dominate the expected payoffs associated with the collusive agreement, for the given (δ, α) parameters. In other words, new games always behave as the red zone of Figure 1. As in each game competing is still a short-run Nash equilibrium, seeking to implement a cooperative equilibrium is again a non-relevant question. Thus, the unattractiveness of the cooperative equilibrium is achieved by a third channel when analyzing the application to the Cournot duopoly: the perturbation of the optimization problem of the firms and, consequently, the equilibrium outcomes and payoffs.

Figure 3: Third Result



As it was argued before, when the Prisoner's Dilemma's structure holds, previous results are valid for the Cournot duopoly application. With that in mind, Figure 4 summarizes all results. The main message of the paper can be read from this figure: ambiguity is detrimental for cooperation due to several reasons. Let $\delta = 0$. Given α, firms compete in quantities except when their subjective discount factor is large enough to guarantee a tacit collusive agreement in the repeated game. Therefore, patience demanded appears as a central restriction for the possibility of cooperating in the long-run. As ambiguity increases, the subjective discount factor needed for sustaining the tacit collusion also increases, thus being less probable to meet the condition for cooperating (the lighter the color, the higher the minimum subjective discount factor needed). Depending on the firms' optimism, the subjective discount factor may be even larger than one (orange zone), making collusion infeasible. Moreover, from a certain point, the collusive agreement stops being desirable (in expected value) given uncertainty (light red zone). Finally, for relatively pessimistic firms, this state can also be achieved by the alteration of the equilibrium outcomes (dark red zone).

Figure 4: Three Results in One Graph

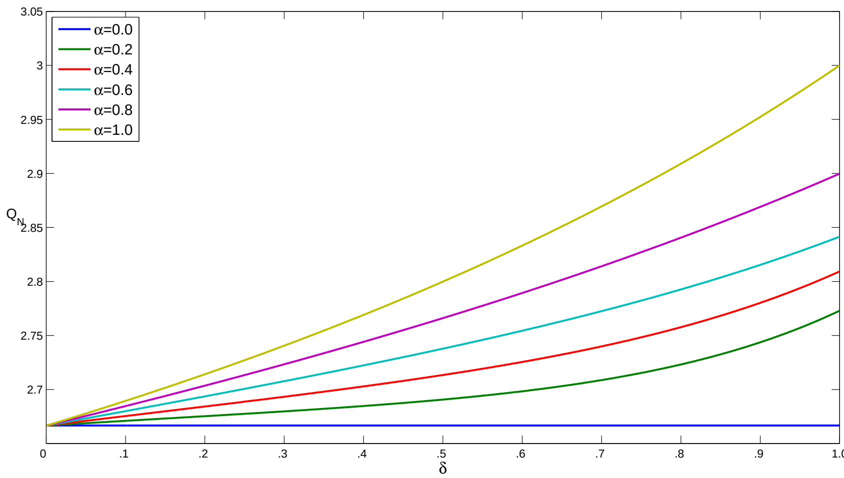


Finally, let us consider the following analysis. If ambiguity decreases the likelihood of sustaining a tacit collusion, then it increases the probability of firms engaging in competitive dynamics. Given that, how is the competitive outcome compared to the non-ambiguity case? In other words, how does (13) depend on the ambiguity parameter? Figure 5 shows the non-linear relation between $Q^N \equiv 2q^N$ and δ, for different values of α. It can be seen that, except for completely pessimistic firms, ambiguity increases produced quantities.[5] Moreover, the relation is increasing in α. This is intuitive: as optimistic firms anticipate that the counterpart may produce the collusive quantity,

[5]From (10), it can be seen that for completely pessimistic firms, ambiguity has no effect on the competitive outcome, as players always think that the counterpart will compete.

which is certainly smaller, they exploit that scenario by producing a higher quantity. The higher the ambiguity and the optimism, the higher relative importance firms assign to that possibility. Then, it is possible to conclude that ambiguity increases competition, as 1) it decreases the likelihood of sustaining tacit collusions, and 2) it increases equilibrium competitive quantities. If we assume that δ is determined by the firms' relationship, which in turn depends on firms' possibility to communicate with each other, then results suggest that the inhibition of communication between firms is desirable from an antitrust perspective. This is consistent with Kandori and Matsushima (1998), Athey and Bagwell (2001) and Fonseca and Normann (2012) findings.

Figure 5: Quantities of the Competitive Equilibrium: α fixed



5. Conclusions

This paper studies the effects of ambiguity on long-run cooperation. Concretely, the infinitely repeated Prisoner's Dilemma and its application to the Cournot duopoly, are revisited in an ambiguity setting. Ambiguity is modelled by using neo-additive capacities, which is a well-suited model for the paper's question.

In the model, uncertainty is defined over the other player's actions. Intuitively, it is assumed that individuals anticipate that the counterparts may not behave in the way they are expected to. Formally, individuals partially distrust their own beliefs about other player's behavior and place themselves in the best and worst cases depending on their relative optimism and pessimism. As the concepts of *ambiguity* and *optimism* are intuitively and flexibly parametrized, we can theoretically explore the effects ambiguity has on the probability of sustaining a cooperative equilibrium in the long-run.

Results suggest that ambiguity is detrimental for cooperation. When analyzing the infinitely repeated Prisoner's Dilemma, ambiguity is found to lower the expected payoff of the cooperative equilibrium and to increase the expected payoff of the non-cooperative equilibrium. If ambiguity is *high enough*, then cooperative agreements may not longer be Pareto-superior and, therefore, they stop being desirable for individuals. In formal terms, minmax payoff starts dominating the cooperative equilibrium payoff (in expected value), thus making the search for conditions to sustain a cooperative equiibrium no longer a relevant question. Moreover, when it is still relevant to ask for conditions to sustain cooperative agreements, ambiguity decreases the likelihood of achieving a cooperative equilibrium, as higher levels of patience are required.

While making the application to the Cournot duopoly (viewed as a particular case of the Prisoner's Dilemma) imposes additional technical complexities (as payoffs are endogenous, i.e. come from the optimization problem of the firms), similar conclusions hold. In words, ambiguity decreases the probability of sustaining a tacit collusion in the long-run. Furthermore, as competitive equilibrium quantities are increasing in the level of ambiguity, it can be concluded that ambiguity is positively related with market competition. This result has important implications for antitrust policy design.

Appendix A. Capacities and Neo-Additive Capacities

Given a finite space X and its correspondent power set 2^X, a capacity $v : 2^X \to \mathbb{R}_+$ is a function that satisfies

$$\begin{aligned} v(\phi) &= 0, \\ v(A) &\leq v(B) \quad \text{if } A \subseteq B, \\ v(X) &= 1. \end{aligned}$$

A capacity is said to be convex if $v(A) + v(B) \leq v(A \cup B) + v(A \cap B)$ (concave if the relation holds with $\geq$). Hence, capacities not necessarily comply the additivity law of probabilities. In this setting, integrating a function $f : X \to \mathbb{R}$ with respect to a capacity v (the analogous of an expectation in the additive probability framework) is done by using Choquet integrals (Choquet, 1954). When the capacity is additive, the Choquet integral is equivalent to the Riemann integral.

Intuitively, capacities can capture ambiguous beliefs as, given their non-additivity, the sum of the likelihood assigned to the realization of the different states does not necessarily sum one. For example, weight assigned to the union of two excluding acts may be greater than the sum of the weights assigned to each act individually. In that case, associated with a convex capacity, it is said that the individual is ambiguity averse.

A neo-additive capacity, proposed by Chateauneuf, Eichberger, and Grant (2007), is a particular type of capacity defined by

$$v(A) \;:=\; (1-\delta)\pi(A) + \delta\mu_\alpha^{\mathcal{N}}(A),$$

for all $A \subset X$, where $\delta \in [0,1]$, π is an additive probability distribution defined over X and $\mu_\alpha^{\mathcal{N}}$ is a Hurwicz capacity exactly congruent with $\mathcal{N} \subset X$ with an $\alpha \in [0,1]$ degree of optimism, defined by

$$\mu_\alpha^{\mathcal{N}}(A) = \begin{cases} 0 & \text{if } A \in \mathcal{N}, \\ \alpha & \text{if } A \notin \mathcal{N} \text{ and } S \setminus A \notin \mathcal{N}, \\ 1 & \text{if } S \setminus A \in \mathcal{N}, \end{cases} \tag{A.1}$$

where S is the set of all possible states and $\mathcal{N} \subset X$ is the set of *null* events, i.e. the set of states whose realization is impossible. Chateauneuf, Eichberger, and Grant (2007) show that the Choquet integral of a neo-additive capacity is given by (1) and axiomatize the functional form as a utility function under ambiguity.

Appendix B. Updating Neo-Additive Capacities

Eichberger, Grant, and Kelsey (2010) analyze different rules for updating neo-additive capacities. They recommend the use of the Generalized Bayesian Updating Rule for capacities as it i) preserves the utility index, and ii) remains unchanged the ambiguity aversion parameter. The later property is important, as the ambiguity aversion parameter, α, is usually seen as an individual's intrinsic parameter and, therefore, it is reasonable to assume that is fixed over time.[6]

Fix a conditioning event $E \subseteq S$ and an unconditional neo-additive capacity v defined by (δ, α, π). The authors show that the Generalized Bayesian Updating rule for neo-additive capacities implies that v_E is also a neo-additive capacity defined by $(\delta_E, \alpha_E, \pi_E)$, where

$$\begin{aligned} \delta_E &= \frac{\delta}{(1-\delta)\pi(E)+\delta}, \\ \alpha_E &= \alpha, \\ \pi_E(A) &= \frac{\pi(A \cap E)}{\pi(E)}, \quad \forall A \in S. \end{aligned}$$

In our setting, $E = \{C, N\}$ and π is always degenerated, as p takes value 0 or 1. Given that, the updating rule implies that

- When the other individual plays the expected action (i.e. plays E such $\pi(E) = 1$), then $\delta_E = \delta$ and $\pi_E(A) = \pi(A)$. Therefore, all parameters remain unchanged.
- When the other individual deviates from the expected action (i.e. plays E such $\pi(E) = 0$), then $\delta_E = 1$ and the probability measure becomes irrelevant. This condition is stationary, as with $\delta = 1$, $\delta_E = 1$.

Appendix C. Proof of Proposition II

Given the strategies described, (C, C) is an equilibrium of the repeated game if the present value of always cooperating is larger than the present value of deviating from the cooperative agreement and then being punished for that. Uncertainty on other player's behavior will affect the computation of the expected profits of the different strategies.

We first look on the discounted benefits of the cooperative strategy. In the first period, $t = 0$, the expected payoff of the cooperative agreement is given by

$$c^* = \delta(\alpha c + (1-\alpha)e) + (1-\delta)c.$$

[6] Other rules analyzed by the authors for updating neo-additive capacities implied shiftings of α to 1 (optimistic updating rule) or 0 (Dempster-Shafer rule).

In $t = 1$, the individual sees what the counterpart played on $t = 0$. If the counterpart played C in the previous period, then the individual keeps playing C and makes no update on the parameters. Therefore, the expected payoff is again c^*. However, if the counterpart deviated on the previous period (i.e. played N), then the individual punishes the counterpart by playing N and updates the ambiguity parameter, δ, to 1. This situation gives an expected payoff of

$$n_u^* \quad = \quad \alpha d + (1-\alpha)n.$$

The later scenario, which the individual predicts that is achieved with probability $\phi_c := (1-\delta) + \delta\alpha = 1 - \delta(1-\alpha)$ is stationary, as the individual will play N forever and will not make any further update, regardless the other player's future actions. Adding up, the expected payoff of the cooperative agreement in $t = 1$, seen from $t = 0$, is given by $\phi_c c^* + (1-\phi_c)n_u^*$.

A similar logic is applied recursively for future periods. If the counterpart played C in $t = 0$ and, therefore, the individual keeps on playing C in $t = 1$, in $t = 2$ sees what the counterpart played in $t = 1$ and decides how to behave following the rule described in the previous paragraph. Hence, the expected payoff of the cooperative agreement in $t = 2$, seen from $t = 0$, is given by $\phi_c^2 c^* + \phi_c(1-\phi_c)n_u^* + (1-\phi_c)n_u^* = \phi_c^2 c^* + (1-\phi_c^2)n_u^*$. Straightforward calculations allow to conclude that the expected payoff of the cooperative equilibrium in $t = T$, seen from $t = 0$, is given by $\phi_c^T c^* + (1-\phi_c^T)n_u^*$.

Thereby, given the subjective discount factor, β, the present value of playing the cooperative strategy is given by

$$\begin{aligned}
PV(C) &= c^* + \beta\left[\phi_c c^* + (1-\phi_c)n_u^*\right] + \beta^2\left[\phi_c^2 c^* + (1-\phi_c^2)n_u^*\right] + ... \\
&= c^*\left[1 + \beta\phi_c + (\beta\phi_c)^2 + ...\right] + n_u^*\left[\beta(1-\phi_c) + \beta^2(1-\phi_c^2) + ...\right] \\
&= c^*\sum_{s\geq 0}(\beta\phi_c)^s + n_u^*\sum_{s\geq 0}\beta^s - n_u^*\sum_{s\geq 0}(\beta\phi_c)^s \\
&= \frac{c^* - n_u^*}{1-\beta\phi_c} + \frac{n_u^*}{1-\beta}. \qquad \text{(C.1)}
\end{aligned}$$

Now, let us turn our attention to the deviating strategy. In $t = 0$, the expected payoff of deviating from the cooperative agreement is given by

$$d^* \quad = \quad \delta(\alpha d + (1-\alpha)n) + (1-\delta)d.$$

After deviating, the individual knows that the counterpart will punish her by playing N forever, so she will play N forever as well. Nevertheless, other player's actions may induce updating on the individual if they do not match the expected behavior. Other player's expected behavior in $t = 0$ is to play C (which the individual predicts it will happen with probability ϕ_c), and in $t \geq 1$ is to play N (which the individual predicts it will happen with probability $\phi_n := (1-\delta) + \delta(1-\alpha) = 1 - \delta\alpha$).

In $t = 1$ the individual sees what the counterpart played on $t = 0$. If the counterpart played C in the previous period, then the individual makes no update and perceives an expected payoff of

$$n^* \quad = \quad \delta(\alpha d + (1-\alpha)n) + (1-\delta)n.$$

However, if the counterpart played N in the previous period, then the individual updates the ambiguity parameter to 1 and perceives an expected payoff of n_u^*. Again, the later situation is stationary. Adding up, the expected payoff of the deviating strategy in $t=1$, seen from $t=0$, is given by $\phi_c n^* + (1-\phi_c)n_u^*$.

Again, a recursive argument is followed. If the counterpart played C in $t=0$ and, therefore, the individual made no update in $t=1$, in $t=2$ sees what the counterpart played in $t=1$ and decides how to behave. If the counterpart played N, then the individual makes no update and perceives an expected payoff of n^*. Conversely, if the counterpart deviated, then the expected payoff is n_u^*. Hence, the expected payoff of the cooperative agreement in $t=2$, seen from $t=0$, is given by $\phi_c\phi_n n^* + \phi_c(1-\phi_n)n_u^* + (1-\phi_c)n_u^* = \phi_c\phi_n n^* + (1-\phi_c\phi_n)n_u^*$. Straightforward calculations allow to conclude that the expected payoff of the cooperative equilibrium in $t=T$, seen from $t=0$, is given by $\phi_c\phi_n^{(T-1)}n^* + \left(1-\phi_c\phi_n^{(T-1)}\right)n_u^*$.

Thereby, given the subjective discount factor, β, the present value of playing the deviating strategy is given by

$$\begin{aligned}
PV(D) &= d^* + \beta\left[\phi_c n^* + (1-\phi_c)n_u^*\right] + \beta^2\left[\phi_c\phi_n n^* + (1-\phi_c\phi_n)n_u^*\right] + ... \\
&= d^* + \beta\phi_c n^*\left[1+\beta\phi_n + (\beta\phi_n)^2 + ...\right] + \beta n_u^*\left[1+\beta+\beta^2+...\right] - \beta\phi_c n_u^*\left[1+\beta\phi_n+(\beta\phi_n)^2+...\right] \\
&= d^* + \beta\phi_c n^*\sum_{s\geq 0}(\beta\phi_n)^s + \beta n_u^*\sum_{s\geq 0}\beta^s - \beta\phi_c n_u^*\sum_{s\geq 0}(\beta\phi_n)^s \\
&= d^* + \frac{\beta\phi_c(n^*-n_u^*)}{1-\beta\phi_n} + \frac{\beta n_u^*}{1-\beta}. \qquad \text{(C.2)}
\end{aligned}$$

Putting together (C.1) and (C.2) yields to (8). ■

Appendix D. Proof of Proposition III

Reordering terms in (10), (11) and (12), we yield to

$$\begin{aligned}
\max_{q^N} V_N &= \left(A - bq^N - b\left[(1-\alpha\delta)q_j + \alpha\delta q^M\right] - k\right)q^N, \\
\max_{q^M} V_M &= \left(A - bq^M - b\left[(1-\delta(1-\alpha))q^M + \delta(1-\alpha)q_j\right] - k\right)q^M, \\
\max_{q^D} V_D &= \left(A - bq^D - b\left[(1-\delta(1-\alpha))q^M + \delta(1-\alpha)q_j\right] - k\right)q^D.
\end{aligned}$$

The corresponding first order conditions are:

$$\begin{aligned}
A - bq^N - b((1-\alpha\delta)q_j + \alpha\delta q^M) - k - bq^N = 0, \\
A - bq^M - b((1-\delta(1-\alpha))q^M + (1-\alpha)\delta q_j) - k - b(1+(1-\delta(1-\alpha))q^M) = 0, \\
A - bq^D - b((1-\delta(1-\alpha))q^M + (1-\alpha)\delta q_j) - k - bq^D = 0.
\end{aligned}$$

Applying symmetry, we can derive the reaction functions

$$\begin{aligned} q^N\left(q^M\right) &= \frac{A-k-b\delta\alpha q^M}{b(3-\delta\alpha)}, \\ q^M\left(q^D\right) &= \frac{A-k-b\delta(1-\alpha)q^D}{2b(2-\delta(1-\alpha))}, \\ q^D\left(q^M\right) &= \frac{A-k-b(1-\delta(1-\alpha))q^M}{b(2-\delta(1-\alpha))}. \end{aligned}$$

Finally, solving the system yields the final expressions for q^N, q^M and q^D. ■

References

Aoyagi, M. (1996): "Reputation and Dynamic Stackelberg Leadership in Infinitely Repeated Games," *Journal of Economic Theory*, 71(2), 378–393.

Athey, S., and K. Bagwell (2001): "Optimal Collusion with Private Information," *RAND Journal of Economics*, 32(3), 428–65.

Athey, S., K. Bagwell, and C. Sanchirico (2004): "Collusion and Price Rigidity," *Review of Economic Studies*, 71(2), 317–349.

Bagwell, K., and R. Staiger (1997): "Collusion Over the Business Cycle," *RAND Journal of Economics*, 28(1), 82–106.

Chan, J. (2000): "On the Non-Existence of Reputation Effects in Two-Person Infinitely-Repeated Games," Economics Working Paper Archive 441, The Johns Hopkins University,Department of Economics.

Chateauneuf, A., J. Eichberger, and S. Grant (2007): "Choice under uncertainty with the best and worst in mind: Neo-additive capacities," *Journal of Economic Theory*, 137(1), 538–567.

Chen, Z., and L. Epstein (2002): "Ambiguity, Risk, and Asset Returns in Continuous Time," *Econometrica*, 70(4), 1403–1443.

Choquet, G. (1954): "Theory of capacities," in *Annales de l'institut Fourier*, vol. 5, pp. 131–295. Institut Fourier.

Compte, O. (1998): "Communication in Repeated Games with Imperfect Private Monitoring," *Econometrica*, 66(3), 597–626.

——— (2002): "On Failing to Cooperate When Monitoring Is Private," *Journal of Economic Theory*, 102(1), 151–188.

Conlon, J. R. (2003): "Hope springs eternal: Learning and the Stability of Cooperation in Short Horizon Repeated Games," *Journal of Economic Theory*, 112(1), 35–65.

Dimitri, N. (2005): "Cooperation with Strategy-Dependent Uncertainty Attitude," Quaerni 457, Universita degli Studi di Siena, Dipartimento di Economia Politica.

Dow, J., and S. R. D. C. Werlang (1994): "Nash Equilibrium under Knightian Uncertainty: Breaking Down Backward Induction," *Journal of Economic Theory*, 64(2), 305 – 324.

Eichberger, J., S. Grant, and D. Kelsey (2010): "Comparing Three Ways to Update Choquet Beliefs," *Economics Letters*, 107(2), 91–94.

Eichberger, J., D. Kelsey, and B. C. Schipper (2009): "Ambiguity and social interaction," *Oxford Economic Papers*, pp. 355–379.

Ellison, G. (1994): "Cooperation in the Prisoner's Dilemma with Anonymous Random Matching," *Review of Economic Studies*, 61(3), 567–588.

Ellsberg, D. (1961): "Risk, Ambiguity and the Savage Axioms," *Quarterly Journal of Economics*, 75, 643–669.

Ely, J. C., and J. Valimaki (2002): "A Robust Folk Theorem for the Prisoner's Dilemma," *Journal of Economic Theory*, 102(1), 84–105.

Etner, J., M. Jeleva, and J.-M. Tallon (2012): "Decision theory under ambiguity," *Journal of Economic Surveys*, 26(2), 234–270.

Fonseca, M. A., and H. T. Normann (2012): "Explicit vs. Tacit Collusion. The Impact of Communication in Oligopoly Experiments," *European Economic Review*, 56(8), 1759–1772.

Fontini, F. (2005): "Cournot Oligopoly Under Strategic Uncertainty With Optimistic And Pessimistic Firms," *Metroeconomica*, 56(3), 318–333.

Gilboa, I., and D. Schmeidler (1989): "Maxmin expected utility with non-unique prior," *Journal of Mathematical Economics*, 18(2), 141–153.

Green, E. J., and R. H. Porter (1984): "Noncooperative Collusion under Imperfect Price Information," *Econometrica*, 52(1), 87–100.

Haller, H. (2000): "Non-Additive Beliefs in Solvable Games," *Theory and Decision*, 49, 313–338.

Kandori, M. (1992): "The Use of Information in Repeated Games with Imperfect Monitoring," *Review of Economic Studies*, 59(3), 581–593.

Kandori, M., and H. Matsushima (1998): "Private Observation, Communication and Collusion," *Econometrica*, 66(3), 627–652.

Kreps, D. M., P. Milgrom, J. Roberts, and R. Wilson (1982): "Rational Cooperation in the Finitely Repeated Prisoners' Dilemma," *Journal of Economic Theory*, 27(2), 245–252.

Mailath, G., and L. Samuelson (2006): *Repeated Games and Reputations: Long-Run Relationships*. Oxford University Press.

Marinacci, M. (2000): "Ambiguous Games," *Games and Economic Behavior*, 31(2), 191 – 219.

Neyman, A. (1999): "Cooperation in Repeated Games when the Number of Stages is Not Commonly Known," *Econometrica*, 67(1), 45–64.

Normann, H. T., and B. Wallace (2012): "The Impact of the Termination Rule on Cooperation in a Prisoner's Dilemma Experiment," *International Journal of Game Theory*, 41(3), 707–718.

Nowak, M., and R. Highfield (2011): *Altruism, Evolution, and Why We Need Each Other to Succeed*. Free Press.

Piccione, M. (2002): "The Repeated Prisoner's Dilemma with Imperfect Private Monitoring," *Journal of Economic Theory*, 102(1), 70–83.

Rojas, C. (2012): "The Role of Demand Information and Monitoring in Tacit Collusion," *RAND Journal of Economics*, 43(1), 78–109.

Rotemberg, J., and G. Saloner (1986): "A Supergame-Theoretic Model of Price Wars during Booms," *American Economic Review*, 76(3), 390–407.

Rothe, J. (2011): "Uncertainty Aversion and Equilibrium in Extensive Games," in *Contributions to Game Theory and Management, Vol. IV: The Fourth International Conference Game Theory and Management*, ed. by L. Petrosyan, and N. Zenkevich, pp. 389–406. St. Petersburg University, Russia.

Savage, L. (1954): *The foundations of statistics*. New-York: John Wiley.

Sekiguchi, T. (1997): "Efficiency in Repeated Prisoner's Dilemma with Private Monitoring," *Journal of Economic Theory*, 76(2), 345–361.

Watson, J. (1994): "Cooperation in the Infinitely Repeated Prisoners' Dilemma with Perturbations," *Games and Economic Behavior*, 7(2), 260–285.