Contents

1	Introduction

1	
-	

Ι	\mathbf{St}	rong C	Convergence of the Symetrized Milsteim Scheme	11
2 Numerical Schemes for Stochastic Differential Equations			Schemes for Stochastic Differential Equations	13
	2.1	Introdu	ction	13
	2.2	The Eu	ller-Maruyama Scheme	14
	2.3	The Mi	lstein Scheme	14
3	Stro	ong con	vergence of the SMS for some CEV-like SDEs	23
	3.1	Introdu	action and main result	23
		3.1.1	The symmetrized Milstein scheme	24
		3.1.2	Strong rate of convergence	26
	3.2Some preliminary results for \overline{X}		28	
			29	
		3.3.1	The Weighted Error	30
		3.3.2	Proof of Theorem 3.1.6	37
		3.3.3	Strong Convergence of the Projected Milstein Scheme	40
	3.4	Numeri	cal Experiments and Conclusion	41

Contents

		3.4.1	Empirical study of the strong rate of converge	41
		3.4.2	Application of the SMS in Multilevel Monte Carlo	49
		3.4.3	Conclusion	51
	3.5	Proofs	for preliminary lemmas	51
		3.5.1	On the Positive Moments of the SMS	51
		3.5.2	On the Local Error of the SMS	52
		3.5.3	On the Probability of SMS being close to zero	53
		3.5.4	On the Local Time of the SMS at Zero	55
		3.5.5	On the negative moments of the stopped increment process $(\overline{Z}_{t\wedge\Theta_\alpha})~$.	58
		3.5.6	On the corrected local error process	64
Π	S 1	tocnas	stic Morris Lecar Model	00
11 4	Sto	chastic	Morris-Lecar Model for Neurons	71
II 4	Stoo 4.1	chastic Introd	Morris-Lecar Model for Neurons	71 71
II 4	Stor 4.1 4.2	chastic Introd Single	Morris Lecar Model Morris-Lecar Model for Neurons uction Neuron Model	71 71 72
II 4	Stoo 4.1 4.2	chastic Introd Single 4.2.1	Morris-Lecar Model for Neurons uction Neuron Model Deterministic Model for a Single Neuron	71 71 72 72
II 4	Stoo 4.1 4.2	chastic Introd Single 4.2.1 4.2.2	Morris-Lecar Model for Neurons uction Neuron Model Deterministic Model for a Single Neuron The Hybrid Model for a Single Neuron	71 71 72 72 76
II 4	Sto 4.1 4.2	chastic Introd Single 4.2.1 4.2.2 4.2.3	Morris-Lecar Model for Neurons uction Neuron Model Deterministic Model for a Single Neuron The Hybrid Model for a Single Neuron Diffusive Approximation of the Hybrid Model and Perturbed Hybrid Model	 71 71 72 72 76 83
II 4	 Store 4.1 4.2 4.3 	chastic Introd Single 4.2.1 4.2.2 4.2.3 Intera	Morris-Lecar Model for Neurons uction Neuron Model Deterministic Model for a Single Neuron The Hybrid Model for a Single Neuron Diffusive Approximation of the Hybrid Model and Perturbed Hybrid Model etion between neurons	71 71 72 72 76 83 89
11 4	 Store 4.1 4.2 4.3 	chastic Introd Single 4.2.1 4.2.2 4.2.3 Interac 4.3.1	Morris-Lecar Model for Neurons uction Neuron Model Deterministic Model for a Single Neuron The Hybrid Model for a Single Neuron Diffusive Approximation of the Hybrid Model and Perturbed Hybrid Model Model etion between neurons The Hybrid Model for a Network of Interacting Neurons	71 71 72 72 76 83 89 91
II 4	 Store 4.1 4.2 4.3 	chastic Introd Single 4.2.1 4.2.2 4.2.3 Interac 4.3.1 4.3.2	Morris-Lecar Model for Neurons uction nuction Model Neuron Model Deterministic Model for a Single Neuron The Hybrid Model for a Single Neuron Diffusive Approximation of the Hybrid Model and Perturbed Hybrid Model Continuous Approximations for the hybrid models	71 71 72 72 76 83 89 91 98
11	 Store 4.1 4.2 4.3 4.4 	chastic Introd Single 4.2.1 4.2.2 4.2.3 Intera 4.3.1 4.3.2 Propag	Morris-Lecar Model for Neurons uction uction Model Neuron Model Deterministic Model for a Single Neuron The Hybrid Model for a Single Neuron Diffusive Approximation of the Hybrid Model and Perturbed Hybrid Model Model tion between neurons The Hybrid Model for a Network of Interacting Neurons continuous Approximations for the hybrid models gation of Chaos	71 71 72 72 76 83 89 91 98 100
11	 Store 4.1 4.2 4.3 4.4 	chastic Introd Single 4.2.1 4.2.2 4.2.3 Interac 4.3.1 4.3.2 Propag 4.4.1	Morris-Lecar Model for Neurons uction neuron Model Deterministic Model for a Single Neuron The Hybrid Model for a Single Neuron Diffusive Approximation of the Hybrid Model and Perturbed Hybrid Model Model The Hybrid Model for a Network of Interacting Neurons Continuous Approximations for the hybrid models Some preliminaries on Propagation of Chaos.	71 71 72 72 76 83 89 91 98 100 101

Contents

Bi	Bibliography			131
5	5 Final Remarks		127	
	4.6	Conclu	usions and Open Questions	125
		4.5.2	Synchronization	118
		4.5.1	Some known results from the literature	114
	4.5	Long '	Time Behavior	113
		4.4.3	Propagation of Chaos for the Diffusive Model	110