APPLICATIONS OF THE WIENER FILTER TECHNIQUE TO SOME ECONOMIC TIME SERIES

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ABSTRACT

The relationship -between the Argentinian prices and money supply, is examined by employing the 'one -sided' Wiener filter technique of estimating distribute lags.

EXTRACTO

En este estudio se examina la relación entre la oferta de dinero y precios en Argentina. Este análisis se realiza empleando la técnica del filtro de Wiener para estimar rezagos distribuidos.

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1. INTRODUCCION

The purpose of this paper is to illustrate the methodology involved in applying the "one–sided" Wiener filter technique of estimating distributed lag relationships to practical time series. This is a frequency domain technique and is an alternative to a "two–sided" technique suggested earlier by Hannan (1963). Both these techniques are also related to the notions of "causality" and "feedback" as defined by Granger (1969).

2. CAUSALITY, FEEDBACK AND THE WIENER FILTER

Let $\{y_t, x_t\}(t = 0, \pm, \ldots)$ be real–valued, jointly stationary time series with 0 means and absolutely summable covariance functions $R_{xx}(u) = E(x_{t+u}x_t)$, $R_{yy}(u) = E(y_{t+u}y_t)$, $R_{yx}(u) = E(y_{t+u}x_t)$, and let

$$f_{jk}(\lambda) = (2\pi)^{-1} \sum_{u=-\infty}^{\infty} R_{jk}(u) \exp(-iu\lambda) \qquad (j, k = x, y)$$

denote their power and cross spectral density functions. Assume that $f_{xx}(\lambda) \neq 0 \ (-\pi \leqslant \lambda \leqslant \pi)$.

Write the 'one–sided' distributed lag relationship between y_t and x_t as

$$y_t = \sum_{j=0}^{\infty} h(j) x_{t-j} + w_t, \tag{2.1}$$

where the coefficients $h(j)$ are chosen by least squares and satisfy the equations

$$R_{yx}(j) = \sum_{k=0}^{\infty} h(k) R_{xx}(k-j) \qquad (j = 0, 1, \ldots) \tag{2.2}$$

The analytical techniques needed for solving these equations are discussed, e.g., by Whittle (1963) and Bhansali and Karavellas (1982). Suffice it to say here that

$$h(j) = (2\pi)^{-1} \int_{-\pi}^{\pi} H(\lambda) \exp(ij\lambda)\, d\lambda \qquad (j = 0, 1, \ldots), \qquad (2.3)$$

where

$$H(\lambda) = (2\pi/\sigma^2)\, A(\lambda) \{ \sum_{u=0}^{\infty} d(u) \exp(-iu\lambda) \} \qquad (2.4)$$

$$d(u) = (2\pi)^{-1} \int_{-\pi}^{\pi} f_{yx}(\lambda)\, \overline{A(\lambda)} \exp(iu\lambda), \qquad (2.5)$$

and σ^2 and $A(\lambda)$ are 'factors' of $f_{xx}(\lambda)$ i.e. $f_{xx}(\lambda) = (\sigma^2/2\pi)|A(\lambda)|^{-2}$. Note that $2\pi d(u) = E(y_t\, \epsilon_{t-u})$, where ϵ_t is the 'innovation' process of x_t, and

$$E(w_t^2) = R_{yy}(0) - \frac{4\pi^2}{\sigma^2} \sum_{j=0}^{\infty} d^2(j) \qquad (2.6)$$

Similarly, let

$$y_t = \sum_{j=-\infty}^{\infty} g(j)\, x_{t-j} + z_t \qquad (2.7)$$

be the 'two–sided' relationship, where

$$g(j) = (2\pi)^{-1} \int_{-\pi}^{\pi} \frac{f_{yx}(\lambda)}{f_{xx}(\lambda)} \exp(ij\lambda)\, d\lambda.$$

We have (see Whittle, 1963; p. 69)

$$E(z_t^2) = R_{yy}(0) - \frac{4\pi^2}{\sigma^2} \sum_{j=-\infty}^{\infty} d^2(j) = E(w_t^2) - \frac{4\pi^2}{\sigma^2} \sum_{j=-\infty}^{-1} d^2(j). \qquad (2.8)$$

The following theorem is a direct consequence of (2.8)

Theorem 2.1. In equations (2.10) and (2.7),

$w_t = z_t$ (in quadratic mean)

if and only if $d(j) \equiv 0$, for all $j < 0$.

In the definitions given by Granger (1969) the time series y_t is said to cause x_t if and only if the one-step mean square error of predicting x_n, say, is smaller by using the past of y_t and x_t than when only the past of x_t is used. Feedback is present when y_t causes x_t and x_t causes y_t. (Actually Granger's definition of causality is somewhat more general; however for the special case in which only linear predictions are used and the attention is confined to the two time series $\{x_t, y_t\}$ his definition simplifies to that described above).

Sims (1972) and Pierce and Haugh (1977), among others, have given several different characterizations of the Granger causality. Another characterization is given below in Theorem 2.2, a proof of which is implicit in Pierce and Haugh (1977, p. 275).

Theorem 2.2 The process y_t does not cause x_t in Granger's definition if and only if $d(u) = 0$, for all $u < 0$.

An implication of Theorems 2.1 and 2.2 is that the 'one-sided' and the 'two-sided' distributed lag relationships (2.1) and (2.7) are equivalent if and only if y_t does not cause x_t in Granger's definition; see Sims (1972), Caines and Chan (1975) and Pierce and Haugh (1977).

Finally, note that if y_t causes x_t in Granger's definition then the use of the 'two-sided' coefficients $g(j)$, but only for $j > 0$, in order to obtain a 'one-sided' relationship is not optimal in the linear least squares sense.

3. ESTIMATION OF THE WIENER FILTER

Suppose that we are given T observations $\{x_1, \ldots, x_T\}$, $\{y_1, \ldots, y_T\}$ from each of the series $\{x_t\}$ and $\{y_t\}$, and let $f_{xx}^{(T)}(\lambda)$ and $f_{yx}^{(T)}(\lambda)$, $\lambda = \lambda_j = j\pi/N_T$ $(j = 0, 1, \ldots, N_T)$ denote the 'window' estimates of $f_{xx}(\lambda)$ and $f_{yx}(\lambda)$, respectively, considered by Bhansali and Karavellas (1982), and put $P_T = 2N_T$. The estimate of $h(u)$ is given by

$$h^{(T)}(u) = P_T^{-1} \sum_{j=-N_T}^{N_T-1} H^{(T)}(\lambda_j) \exp(iu\lambda_j) \qquad (u = 0, 1, \ldots, N_T),$$

where

$$H^{(T)}(\lambda_j) = \frac{2\pi}{\hat{\sigma}^2} \Lambda^{(T)}(\lambda_j) \left\{ \sum_{u=0}^{N_T-1} d^{(T)}(u) \exp(-iu\lambda_j) \right\}$$

$$d^{(T)}(u) = P_T^{-1} \sum_{j=-N_T}^{N_T-1} f_{yx}^{(T)}(\lambda_j)\, A^{(T)}(\lambda_j) \exp(iu\lambda_j),$$

$$A^{(T)}(\lambda_j) = \exp\left\{- \sum_{v=1}^{N_T-1} c^{(T)}(v) \exp(-iv\lambda_j)\right\}$$

$$\hat{\sigma}^2 = 2\pi \exp\{ c^{(T)}(0) \},$$

$$c^{(T)}(v) = P_T^{-1} \sum_{j=-N_T}^{N_T-1} \log f_{xx}^{(T)}(\lambda_j) \exp(iv\lambda_j).$$

The corresponding estimate of g (u) is given by

$$g^{(T)}(u) = P_T^{-1} \sum_{j=-N_T}^{N_T-1} \{ f_{yx}^{(T)}(\lambda_j) / f_{xx}^{(T)}(\lambda_j) \} \exp(iu\lambda_j)$$

$$(u = 0, 1, \ldots, N_T);$$

see also Hannan (1963), e.g., for a study of closely related estimates of g (u).

Theorem 2.2 suggests that an examination of $d^{(T)}(u)$, or of $d^{(T)}(u)/d^{(T)}(0)$, may be helpful in deciding whether or not y_t causes x_t in Granger's definition; see Pierce and Haugh (1977) for a related time domain procedure.

For a summary of the large sample behaviour of the statistics $d^{(T)}(u)$, $h^{(T)}(u)$ and $g^{(T)}(u)$ see Bhansali and Karavellas (1982). Suffice it to say here that if there is no linear relationship between y_t and x_t then, under regularity conditions, as $T \to \infty$, a finite collection of the $h^{(T)}(u)$'s is asymptotically normally distributed with 0 means, and (approximate) covariances

$$T^{-1} \frac{1}{2\pi} \int_{-\pi}^{\pi} \frac{f_{yy}(\lambda)}{f_{xx}(\lambda)} \exp\{i(u-v)\lambda\} d\lambda,$$

and then the $g^{(T)}(u)$ and $h^{(T)}(u)$ are also asymptotically equivalent.

4. APPLICATIONS

4.1. Argentinian Prices and Money Supply

As a first example, consider the monthly series of Argentinian prices and money supply for January 1957–December 1976. The Wiener filter technique was actually applied to their second differences, hereinafter called y_t and x_t respectively, since these provide a measure of the rate of change, and are closer to stationarity. As in Sims (1972) the possibility of logarithmically transforming the series was also explored, but with negative results.

The modified Daniell window, which takes a weighted average of $2m + 1$ adjacent periodogram, or cross–periodogram, ordinates was utilised for estimating $f_{jk}(\lambda)$ ($j, k = x, y$). Four different values of m, namely $m = 2, 3, 4$ and 5, were used, but to save space only the results for $m = 4$ are presented below – the results for other values of m are similar.

An examination of the power spectra of x_t and y_t showed the presence of peaks at seasonal frequencies. However these peaks were broad and not inconsistent with the hypothesis of purely non–deterministic series. The spectral factorization procedure suggested the need for fitting a long autoregression for predicting x_t or y_t, from it own past.

TABLE I

ESTIMATED WIENER FILTER COEFFICIENTS

u	$h^{(T)}_{yx}(u)$	$h^{(T)}_{xy}(u)$	u	$h^{(T)}_{yx}(u)$	$h^{(T)}_{xy}(u)$
0	1.92	0.015	8	0.51	0.002
1	0.21	0.004	9	2.37	0.012
2	–1.89	0.003	10	2.19	0.007
3	-0.2	0.001	11	0.74	0.006
4	3.48	0.019	12	2.87	0.015
5	2.28	0.011	13	1.13	0.007
6	0.39	0.017	14	0.11	0.008
7	1.79	0.009	15	0.33	-0.004
95% limits	+1.92	+0.012			

The estimated Wiener filter coefficients, $h_{yx}^{(T)}(u)$ and $h_{xy}^{(T)}(u)$, in a one–sided distributed lag relationship of y_t on x_t and that of x_t on y_t respectively, are shown in Table I along with the 95% limits for testing the hypothesis of zero coefficients discussed in section 3.

On testing the coefficients individually, it is seen that for u = 0, 4, 5, 9 and 12 the $h_{yx}^{(T)}(u)$ lie outside the 95% limits. Similarly for u = 0, 4, 6, 9 and 12 the $h_{xy}^{(T)}(u)$ are also outside these limits. There would thus appear to be evidence for supposing that the past of x_t is helpful for predicting y_t and vice versa.

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