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Optimal pricing for travelcards under income and car ownership inequities

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ABSTRACT

Travelcards are used in many parts of the developed world as a form of payment for public transport that is convenient for frequent users. In essence it involves a one-time payment T at the beginning of a period that covers all trips within that period. Carbajo (1988) applies the two-part tariff approach to find the optimal (welfare maximizing) value for T assuming a nil effect of T on the demand schedule of each and every individual (no income effect). Here we deal with an urban area where individual trips increase with income, but where car ownership – correlated with income – makes the public transport share diminish towards high income segments. A theoretical model is developed to find the optimal values (maximum social welfare with a budget constraint) for T and, simultaneously, for a single ticket P, considering the effect of T on available income as well as differences across individuals regarding car ownership. The model is applied using parameters associated with monthly travel in Santiago, Chile, where both income and car ownership are highly concentrated and correlated, and where travelcards do not exist. We obtain that the two richest segments choose to pay for the single ticket and the other eight choose to buy the travelcard, increasing equity. Sensitivity analysis regarding public transport quality, increased car ownership and poverty reduction show relatively marginal changes regarding optimal prices and preferred form of payment.

1. Introduction

Pricing public transport is an interesting tool for urban transport planning. Travelcards are used in many parts of the developed world as an alternative to the single ticket that consists of a fixed fee that allows unlimited trips within its validity period, a year, a month, a week, a day. It is a method to induce the use of public transport (PT) and to facilitate the access to transport services to captive users, like students or elders. The intention is also to take advantage of scale economies, environmental friendliness and the efficient use of the urban space, among other aspects. Generally, this ticket type is convenient for most of the city residents (and even for visitors if they stay long enough), because users compares the travelcard value (T) in a given period with the necessary expenditure associated with the expected number of trips during the same period using the single ticket (P); in addition, avoiding the time it takes to make the transaction on the vehicle is considered a plus. Once the travelcard is acquired, users will make more than the expected mandatory trips.

Experiences around the world have been studied by White (1981), FitzRoy and Smith (1998, 1999), Pucher and Kurth (1996), Matas (2004) and Gschwender (2007), among others. As evident, the larger the number of trips made by the

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Table 1
Comparison between single ticket and monthly travelcard.

City	Madrid	Paris	Rome	Berlin	London
Travelcard [€/month]	54.6	70	35	79.5	170
Single ticket [€/trip]	1.5–2	1.8	1.5	1.6–2.7	6.7
Equivalent trips	36–27	39	23	50–29	26
PT trips with travelcards [%]	64	77	56	85	83

Source: London Travel Report (2005), EMTA (2004), Pucher and Kurth (1996), ATAC (2016), and Verkehrsverbund Berlin-Brandenburg (2010).

individual the more attractive the travelcard is. Just to have an idea, the monthly price of a travelcard in some European capital cities is equivalent to 23–36 monthly trips, i.e. less than two trips per working day, such that most citizens would find it convenient to buy it (and use it).

Table 1 contains the monthly travelcard values (there are also daily, weekly and annual) and the single ticket price for the metropolitan area of five European cities; the fourth row shows the number of equivalent trips that users can make if they spent the value of the travelcard in single tickets, such that in all five cities the travelcard is worth buying even if individuals travel only twice a day. The last row confirms that usage of the travelcard dominates as form of payment in PT (most of the trips are paid with it).

Why this tool is not applied in most of the Latin American cities (only Sao Paulo has recently implemented it) is somewhat a mystery; if it gets implemented, though, there is a methodological problem that has not been yet solved: its effect depends on users' behavior, particularly on the marginal utility of income (how money is valued by different individuals) and car ownership, which vary significantly in the population. Prevailing theories to put optimal prices on travelcards do not take these aspects into account properly. The theoretical basis for travelcard pricing developed by Carbajo (1988) follows the two-part tariff theory (Oi, 1971; Feldstein, 1972; Brown and Sibley, 1986; Wilson, 1997), dismissing income effects. This theory is applied to the PT market to find the optimal P and T using a taste parameter to differentiate across users. We postulate that income should enter the theoretical framework to calculate optimal values for travelcards, and that this can be applied to a context of high income inequality where travelcards do not exist.

In this paper we develop and apply a theoretical model to find the optimal values (maximum social welfare with a budget constraint) for a travelcard and, simultaneously, for a single ticket, considering the effect of T on the available income as well as differences across individuals regarding car ownership. As these effects work in opposite direction, the single taste parameter approach is not applicable.

In the next section the approach developed by Carbajo (1988) is summarized, showing its limitations regarding income. In Section 3 an analytical model that captures both effects mentioned above is developed. Next the model is applied to Santiago, Chile, a city where both income and car ownership are highly concentrated and correlated. The final section contains a synthesis, conclusions and directions for further research.

2. Taste or income

2.1. Carbajo's (1988) model

A two part tariff (P, T) consists in separating the charge for a product into an entry fee T (to be allowed into the market) and a payment P for each unit consumed, as used in water, electricity and telecommunication markets. This structure allows gains in efficiency due to the possibility of lowering the price to get P near the marginal cost, but it also can induce consumers to exit the market because they refuse to pay the entry fee. With this approach, the lower the price P is, the higher T should be in order to cover the producers' costs, causing different reactions in consumers (assuming they are not all equal).

The differences among consumers are mainly treated in two forms in the two-part tariffs literature: by means of their income, as in Oi (1971) and Feldstein (1972), or by means of a taste parameter that is distributed in the population and that represents consumption intensity, as done by Brown and Sibley (1986) and Wilson (1997). In both treatments, however, the effect of T on the individual income (and, therefore, on the demand curve) is considered negligible; Oi assumes that demands are invariant to changes in income or in the entry fee and Feldstein considers that the effect is negligible because T is only a little fraction of the consumers income. No author considers the possibility that there could be markets (like public transport) or segments of the population where this assumption might not be reasonable.

Aiming at finding the optimal value of travelcards, Carbajo (1988) extends the Brown and Sibley model to the PT market, representing both the travelcard and the single ticket as special cases of a two part tariff (P, T): the travelcard as $(0, T)$ and the single ticket as $(P, 0)$. Under this consideration, each user will choose the alternative that yields the largest consumer's surplus. Following the demand scheme represented in Fig. 1a, the surplus CS_P gained by a user who chooses single ticket is A, with Q_P trips per period. With the travelcard the surplus CS_T will be $A + B - T$, making Q_T trips **regardless the value of** T; the choice will depend on the sign of $B - T$.

In Fig. 1b we represent three individuals (Active, Middle and Passive) with different PT demands schemes. For any single ticket price, their trips will fulfill the condition $Q_{act}(P) > Q_{mid}(P) > Q_{pas}(P)$. Then Carbajo notes that if $T = d + e$ each user will choose as shown in Table 2.

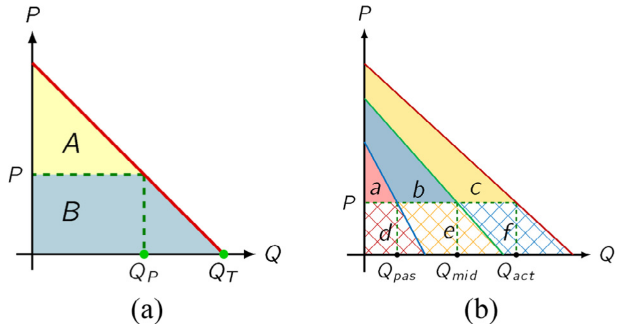

Fig. 1. Single ticket and travelcard surplus according to Carbajo (1988).

Table 2
Surpluses and choice.

User	CS_P	CS_T	Choice
Active	$a+b+c$	$a+b+c+f$	Choose T
Middle	$a+b$	$a+b$	Indifferent
Passive	a	$a-e$	Choose P

Carbajo assumes that users differ in terms of a taste parameter θ that represents the intensity of use of PT. For a given pair (P, T), there will be an indifferent user $\tilde{\theta}$, characterized by $CS_P = CS_T$ (as the Middle user in Table 2), that will establish the limit between users who choose P or T. Specifically, users with $\theta > \tilde{\theta}$ choose buying the travelcard and those with $\theta < \tilde{\theta}$ choose the single ticket. Carbajo assumes that we know the taste parameter density function $h(\theta)$, its cumulative distribution $H(\theta)$ and the range where it belongs, $\theta \in [\underline{\theta}, \bar{\theta}]$. With this information, the trips made by a user θ are $q(P, \theta)$ for a single ticket user and $q(0, \theta)$ for a travelcard user, independent of the value of T. The indifferent user is implicitly determined by the expression that equalize the surpluses obtained from T and P as shown in Eq. (1), which yields a function $\tilde{\theta}(P, T)$.

$$\int_0^\infty q(p', \tilde{\theta})dp' - T = \int_P^\infty q(p', \tilde{\theta})dp' \tag{1}$$

Considering a linear cost function $C = F + m \cdot Q$, where F is the fixed cost and m the trip marginal cost, the social welfare for given P and T is the sum of the users' consumer surplus and the operators' profits, that is,

$$\begin{aligned} SW = {} & \int_{\underline{\theta}}^{\tilde{\theta}(P,T)} \left[\int_P^\infty q(p', \theta)dp' \right] \cdot h(\theta') \cdot d\theta' + \int_{\tilde{\theta}(P,T)}^{\bar{\theta}} \left[\int_0^\infty q(p', \theta)dp' - T \right] \cdot h(\theta') \cdot d\theta' \\ & + \int_{\underline{\theta}}^{\tilde{\theta}(P,T)} (P - m) \cdot q(p, \theta') \cdot h(\theta') \cdot d\theta' + \int_{\tilde{\theta}(P,T)}^{\bar{\theta}} [T - m \cdot q(0, \theta')] \cdot h(\theta') \cdot d\theta' - F \end{aligned} \tag{2}$$

The first and the second terms represent benefits of the single ticket and travelcard users, respectively; the other three are the profits of the operators. Carbajo maximizes this SW function under a budget constraint to obtain the optimal values, P^* and T^*. Then the taste parameter for the indifferent user $\tilde{\theta} = \tilde{\theta}(P^*, T^*)$ and the corresponding trips for each user are obtained.

Carbajo says that θ may be thought of as a vector of characteristics or as the individual's income, but the consequences of including income are not discussed, particularly the fact that the expenditure in PT is not always negligible with respect to income and, in consequence, the two part tariff assumptions of no income effect and invariance of Q_T with respect to T could not be directly applicable to this model.

2.2. The role of income

If users are identified by their income, the role of this variable in PT demand has to be understood. Let us present a case that reflects the reality in most Latin American capital cities. In Fig. 2 the relationships between income and car ownership (CO, cars/1000 households) and trips in Santiago, Chile, are shown (Sectra, 2001). CO increases sharply with income, such that the richest segment exhibits nine times the CO of the poorest segment; this explains the dotted line (triangles) in Fig. 2b that shows the monthly car trips by income segment. The thick continuous line (squares) represents the total motorized trips, which also increase with income. Both curves present a similar shape (logistic), with trips growing faster at the beginning until a constant is reached asymptotically. The difference between these two curves is the thin continuous line (dots) that represents the PT trips, which decrease with income: poor people use PT more intensively. In other words, urban trips

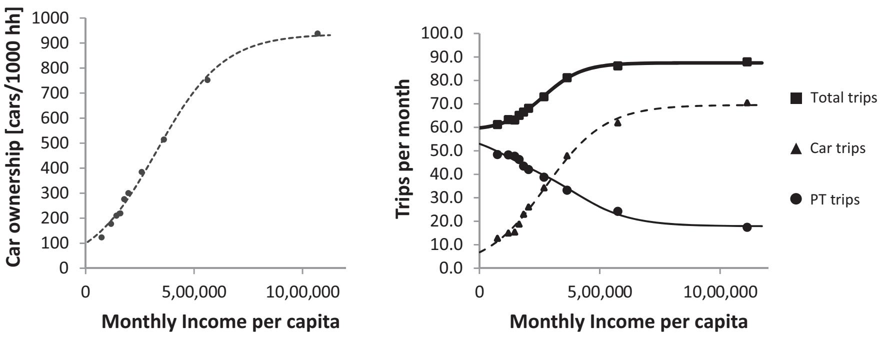

Fig. 2. Car ownership and trips variation with income. *Source:* 2002 Santiago OD survey (In the 2002 origin-destination survey made in Santiago, Chile, 15 thousand families were asked about their transport habits and socio-economic characteristics. People were classified into ten income segments.).

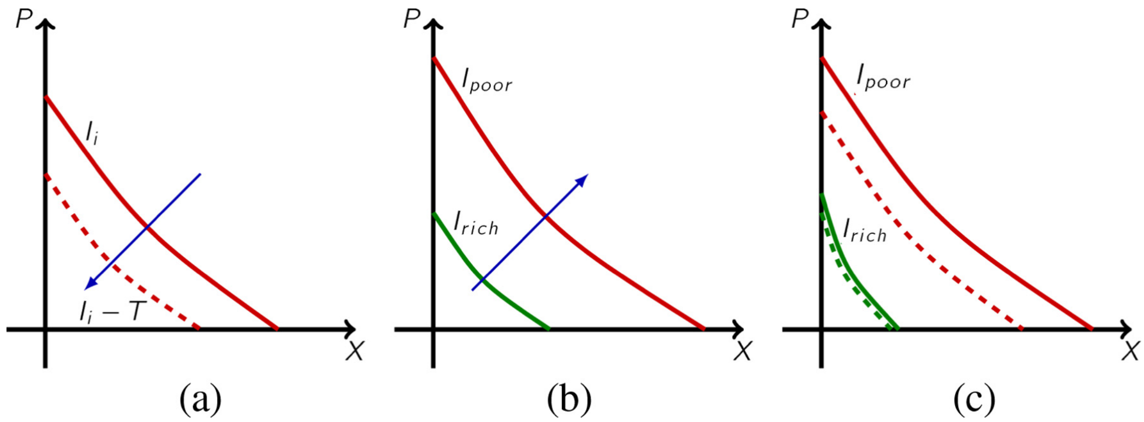

Fig. 3. Demand movements with income.

increase with income but a car ownership effect makes the demand for PT trips in higher income segments smaller than the demand of individuals in lower income segments, as represented in Fig. 3b.

Let us consider now an individual within a given income segment I_i, whose PT trips demand is $X(P, I_i)$. If he chooses to travel paying with the single ticket P_0, he will make $X(P_0, I_i)$ trips, but, if he chooses to pay T_0 for the travelcard, demand in the (P, X) space will move towards the origin, as shown in Fig. 3a. In this case the user will make $X(0, I_i - T_0)$ trips,[1] which is less than what is assumed in the literature, $X(0, I_i)$. The effect of paying for a travelcard on the PT demand diminishes as I_i increases, as represented in Fig. 3c.

Therefore, two differences with Carbajo (1988) appear when introducing income in the travelcard analysis: (i) travelcard trips depend on the travelcard value – they are independent for Carbajo – because of the available income effect that can be relevant in some segments; and (ii) the effect at an individual level goes in opposite direction with respect to what happens across individuals belonging to different segments because of CO. The double effect of income makes it necessary to reformulate the travelcard problem.

3. Analytical model

3.1. General formulation and the role of the indifferent income

Let us consider an individual in an income segment (or CO) I with a given marginal utility of income, and whose trips in PT are represented by a function $X(P, T, I)$, that has associated an inverse $P = X^{-1} = P(X, T, I)$. Following Fig. 4 the user obtains a surplus A if he chooses to pay with single ticket (making X_P trips) or $A + B - T$ if he buys the travelcard, making $X_T = X(0, T, I) < X(0, 0, I)$ trips, that depend on the value of T. The analytical expressions for the consumer surpluses are shown in Eq. (3), valid under the assumption that the marginal utility of income is constant within each income segment as shown in Jara-Díaz (2007, pp. 97).

[1] This is in line with Van Vuuren and Rietveld (2002) in their econometric paper on train travel, which emphasizes that the demand for train trips depends on the price of the single ticket and the cost of the travelcard.

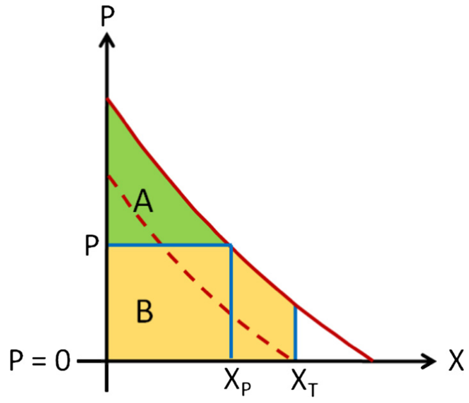

Fig. 4. Single ticket and travelcard surpluses for a given income segment.

$$CS_P = \int_P^\infty X(\phi, 0, I)d\phi \quad CS_T = \int_0^{X(0,T,I)} P(\phi, 0, I)d\phi - T \tag{3}$$

For given P and T, the equality $CS_P = CS_T$ defines an individual with income $\widetilde{I}(P,T)$ who is indifferent between both payment alternatives, that is when $T = B$ in Fig. 4, which will happen for only one income level as long as the demand curves do not intersect, as shown below. The indifferent user establishes the limit between users who choose P or T, assuming no liquidity effects. Let us analyze which alternative is chosen by individuals with $I_i \neq \widetilde{I}$.

Following Fig. 5, consider an individual with income I_i that is indifferent between P_0 and T_0, i.e. $T_0 = B + \delta_2$ such that $CS_{P_0} = A + \delta_1 = CS_{T_0}$. If the single ticket price is raised to $P_1 > P_0$ keeping T_0 constant, the user will obtain a surplus of A if he chooses P_1. With this, $CS_{P_1} = A < A + \delta_1 = CS_{P_0} = CS_{T_0}$ such that he will choose the travelcard over P_1. To keep the indifference of the individual it is necessary to raise the travelcard value in order to reduce its surplus and match it with the single ticket surplus. So, there is $T_1 > T_0$ that makes the individual with income I_i indifferent between T_1 and P_1. Analogously, when the single ticket price is lowered to $P_2 < P_0$ the individual chooses P_2 over T_0; it would be necessary to lower T to keep the indifference (point 2: T_2 equilibrates P_2). The three indifferent pairs are shown in the (P,T) plane in Fig. 5: (P_0,T_0), (P_1,T_1) and (P_2,T_2) represented by points 0, 1 and 2 respectively. Finally, it is easy to show that any individual is indifferent between the alternatives in the pair $(P,T) = (0,0)$, because both surpluses are represented by the whole area below the demand function. We conclude that there is an indifference curve IC in the (P,T) plane that increases from the origin where the individual with income I_i is indifferent between P and T.

The same exercise of raising or lowering the single ticket price can be made at any point of the IC, which shows that the IC divides the (P,T) plane in two regions: the individual of income I_i will choose the travelcard if he is offered any pair located above his IC, and the single ticket if he is offered a pair located below his IC, as shown in Fig. 6.

Individuals belonging to different income segments will have different IC. Following Fig. 7 consider two individuals from different income segments such that $I_{Poor} < I_{Rich}$.

Based on the analysis of Fig. 4, the travelcard value that equilibrates P_0 is represented by $B_{Rich} = T^0_{Rich}$ for I_{Rich} and by $B_{Poor} = T^0_{Poor}$ for I_{Poor}. As $B_{Poor} > B_{Rich}$, $T^0_{Poor} > T^0_{Rich}$, which implies that the IC of the rich individual is to the left of (or above)

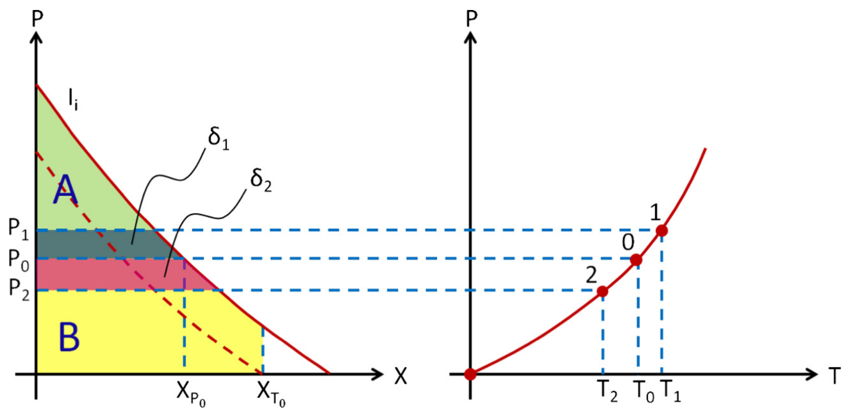

Fig. 5. Building an indifference curve.

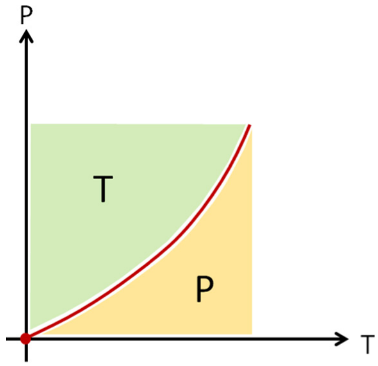

Fig. 6. Indifference curve and choice zones.

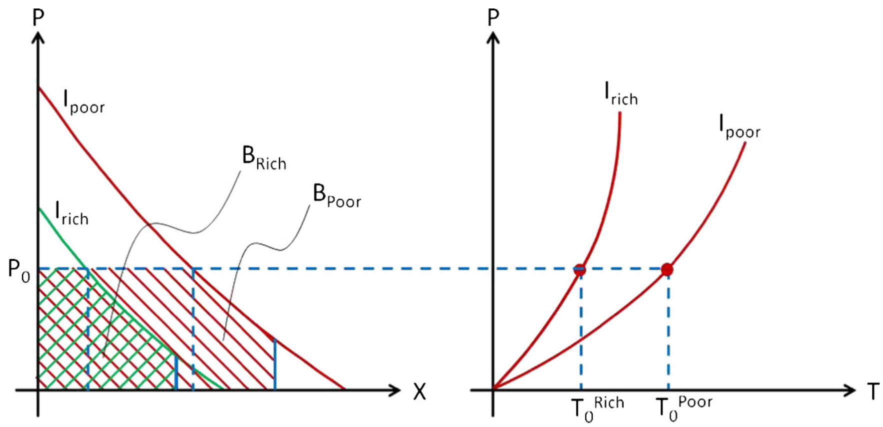

Fig. 7. Indifference curves for different income segments.

the *IC* of the poor. Note that this requires that the demand curves do not intersect, which is likely to be the case (as we have argued) for all reasonable price levels.

Finally, when a pair (P^*, T^*) is offered to the population there will be an indifferent user represented by the income $\widetilde{I}(P^*, T^*)$, that will have the pair (P^*, T^*) on his *IC*, as is shown in Fig. 8; only one income level will fulfill this condition as long as the indifference curves do not intersect (a property induced by the non-crossing demand curves assumed). As explained through Fig. 7, segments with $I > \widetilde{I}$ have their *IC* above that of $\widetilde{I}$. As the offered pair falls below their *IC*, they will choose to travel paying P^* for each trip, as obtained in Fig. 6. Analogously, segments with $I < \widetilde{I}$ will buy the travelcard.

If the indifferent income was known, the choices would be known for each income segment (CO). Then the expression for the social welfare *SW* would be

$$SW = \sum_{I>\widetilde{I}} N_i \cdot CS_P + \sum_{I<\widetilde{I}} N_i \cdot CS_T + (P - m) \cdot \sum_{I>\widetilde{I}} N_i \cdot X(P, 0, I) + \sum_{I<\widetilde{I}} N_i \cdot T - m \cdot \sum_{I<\widetilde{I}} N_i \cdot X(0, T, I) - F \tag{4}$$

where the surpluses are specified in Eq. (3), N_i is the number of individuals in the segment of income I_i, m is the marginal cost for trips and F is the operators' fixed cost. The first two terms are the consumers' surpluses and the last four, the operators' profits, π. Note that the income from the single tickets is associated to trips, while the income from the travelcards is associated to the number of individuals who buy it.

Maximizing *SW* subject to a budget constraint $\pi \geqslant 0$ for the transit operators leads to (see Appendix A)

$$\frac{P^* - m}{P^*} = \frac{\theta}{\sum_{I_i>\widetilde{I}} \alpha_{I_i} \cdot |\eta_{P,I_i}|} \tag{5}$$

and

$$T^* = \sum_{I_i<\widetilde{I}} \beta_{I_i} \cdot |\eta_{T,I_i}| \cdot X(0, T^*, I_i) \cdot \lambda^{-1} \cdot \{P[X(0, T^*, I_i), 0, I_i] - m \cdot (1 + \lambda)\} \tag{6}$$

where $\theta = \frac{\lambda}{1+\lambda}$ with λ the multiplier of the restriction, $\alpha_{I_i} = \frac{N_i \cdot X(P,T,I_i)}{\sum_{I>\widetilde{I}} N_i \cdot X(P,T,I_i)}$ and $\beta_{I_i} = \frac{N_i}{\sum_{I<\widetilde{I}} N_i}$.

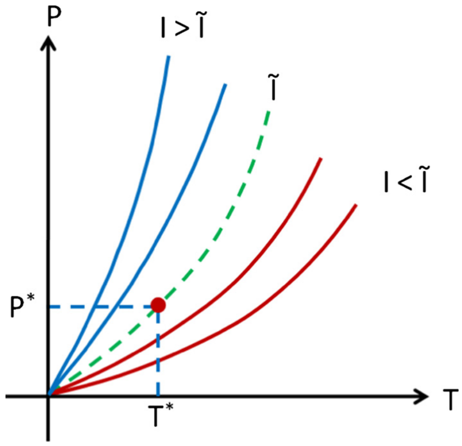

Fig. 8. Choice based on $\widetilde{I}(P^*, T^*)$.

Expression (5) shows that the optimal single ticket value makes the mark up ratio proportional to the inverse of the sum of the price elasticity of the demands ($\eta_{P,I}$), each weighted by a coefficient α_{I_i} that represents the PT trips proportion of each segment with respect to the total. Expression (6) indicate that the optimal value of the travelcard is the sum, over all segments that choose T, of the product of the proportion of individuals per segment (β_{I_i}), the elasticity of the demand with respect to T ($\eta_{T,I}$), and a term that depends on the PT demand with travelcard and the difference between its associated willingness to pay and the marginal cost amplified by the multiplier of the budget restriction; this suggests that T gets positive (exists) due to the presence of groups with strong propensity to use PT (low car ownership, strong income effect).

3.2. Solution to the problem with many income segments and unknown indifferent income

So far the indifferent income has been assumed known and, therefore, the payment alternative chosen by every income segment is known. However, the indifferent income depends on the solution (P^*, T^*). What to do? Assume that there are k income segments, with I_1 the poorest and I_k the richest, and let $I_j < \widetilde{I} < I_{j+1}$. This means that the IC associated to $\widetilde{I}$ is below the IC of the segment of income I_{j+1} and above the IC associated to I_j. This implies that income segments with $I \geqslant I_{j+1}$ will choose to travel paying P per trip, while segments with $I \leqslant I_j$ will buy the travelcard. As curves intersect only at the origin, the k segments define $k+1$ zones in the (P, T) space. In each of those cases an optimal pair (P_j^*, T_j^*) can be found by solving

$$\begin{array}{ll} Max & SW \\ s.to & \pi \geqslant 0 \qquad (\lambda) \\ & P > P(T, I_j) \\ & P(T, I_{j+1}) > P \end{array} \tag{7}$$

where $P(T, I_j)$ is the IC of the segment with income I_j. This way the second and third constraint imposes that the solution belongs to the space between ICs that is compatible with the choices assumed in the SW expression. Solving the $k+1$ cases, the one with the largest SW among those that are feasible will be the optimal solution of the problem.

3.3. Linear demand functions, welfare measure and indifference curves

The monthly PT trips made by a user with income I_i can be represented linearly as

$$X(P, T, I_i) = A_i - B_i \cdot \frac{P + \Delta_i}{I_i - T} \tag{8}$$

where the parameter A_i is imposed as the total trips made (including private and public transport) by any individual with income I_i. As shown in Fig. 9, B_i is related to the elasticities with respect to the single ticket and to the travelcard value, such that a decrease in B_i will move demand away from the origin. Δ_i is associated with those variables different from price that influence the choice between car and PT, which depends on car ownership, such that even if public transport was free there will still be people who would use the car anyway.

From Eq. (8) the consumers' surpluses and the social welfare can be obtained as

$$\begin{aligned} SW = & \sum_{I_i > \widetilde{I}} \frac{N_i I_i}{2B_i} \left(A_i - \frac{B_i (P + \Delta_i)}{I_i} \right)^2 + \sum_{I_i < \widetilde{I}} \frac{N_i I_i}{2B_i} \left(A_i - \frac{B_i \Delta_i}{I_i} \left(1 - \frac{T}{I_i - T} \right) \right) \left(A_i - \frac{B_i \Delta_i}{I_i - T} \right) \\ & - \sum_{I_i < \widetilde{I}} N_i \cdot T + (P - m) \cdot \sum_{I_i > \widetilde{I}} N_i \left(A_i - \frac{B_i (P + \Delta_i)}{I_i} \right) + \sum_{I_i < \widetilde{I}} N_i \cdot T - m \cdot \sum_{I_i < \widetilde{I}} N_i \left(A_i - B_i \frac{\Delta_i}{I_i - T} \right) - F \end{aligned} \tag{9}$$

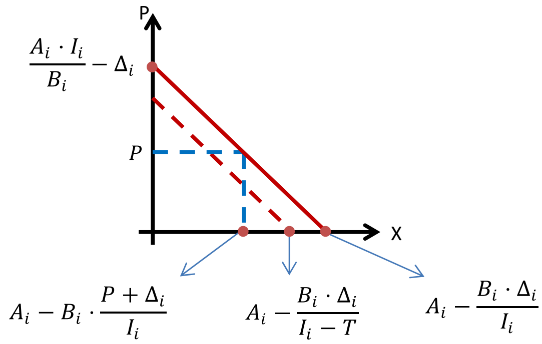

Fig. 9. Interpretation of the travel demand parameters.

The first three terms are the consumers' surpluses and the other three are the operators' profits. The terms associated to money spent on travelcards cancel out because it is a welfare transfer. Finally, from $CS_P = CS_T$ an expression for the IC of the segment with income I_i is obtained:

$$P(T, I_i) = \frac{A_i \cdot I_i}{B_i} - \Delta_i - \frac{I_i}{B_i}\sqrt{\left(A_i - \frac{B_i \cdot \Delta_i}{I_i}\left(1 - \frac{T}{I_i - T}\right)\right)\left(A_i - \frac{B_i \Delta_i}{I_i - T}\right) - \frac{2TB_i}{I_i}} \tag{10}$$

Knowing the social welfare and the IC of every segment, problems (7) can be solved.

4. Application

4.1. Case parameters and solution for P and T

The method was applied to the case of Santiago, Chile, where no travelcard system has been yet implemented and the single ticket is the only alternative, as in most of the Latin American cities. As seen in Section 2, the impact of a travelcard on PT usage will vary across income segments, particularly if income distribution is far from evenly distributed, which is exactly the case of Chile, as shown in Fig. 10, where it can be seen that only the two richest segments have larger income than the average, while the other eight are less or equal. Particulary, the tenth decile's income is more than 35 times the income of the first decile; even if compared with the ninth decile the difference is significant.

To provide a context, Santiago has some six million inhabitants with four long subway lines (soon six) and a feeder-trunk bus system. A mandatory smartcard (BIP!) is needed to access the public transport system for a single flat fare within a period (up to two transfers within two hours). Only students and retired people pay a reduced tariff (one third of the ticket). Presently, there are 11.4 million daily motorized trips, with roughly 50% by PT, a significant drop of 9% in the last ten years that is causing serious congestion in the city.

In order to obtain representative parameters for the demand of each income segment we used the most recently validated data from an OD survey (Sectra, 2001), where 15 thousand families were randomly selected to study their travel patterns in Santiago during weekdays, Saturdays and Sundays. Using a matching procedure we were able to construct weekly and monthly trip patterns representative of each of the 10 income segments. As explained earlier, A_i is obtained as the total trips made. Next we used the observed single ticket price and the number of trips made by each income segment as one point of the demand curve in Eq. (8). A second point was obtained by estimating the maximum price that the users were willing to

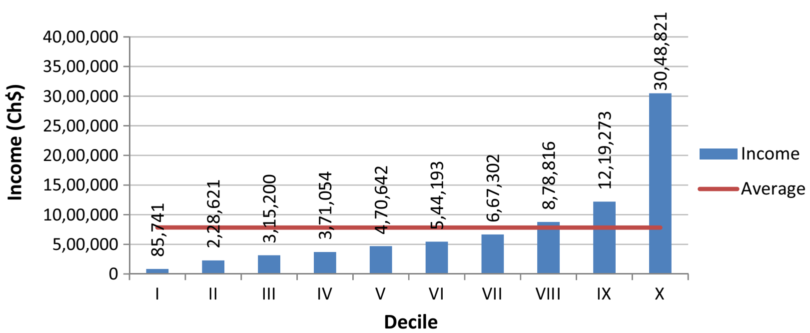

Fig. 10. Chilean income distribution.

Table 3
Parameters of the demand functions.

Seg	Income	A_i	B_i	Δ_i	N_i	Seg	Income	A_i	B_i	Δ_i	N_i
1	74,333	69.9	2173.9	390.8	67,593	6	203,535	75.7	5162.7	983.5	159,398
2	119,650	71.3	3346.5	549.6	234,593	7	267,811	80.5	6268.0	1440.5	218,319
3	145,970	72.4	3962.0	665.8	210,603	8	363,701	87.2	7363.3	2306.8	85,567
4	164,288	73.3	4370.3	756.5	205,612	9	574,484	91.1	7809.6	4698.6	65,917
5	182,966	74.3	4755.7	859.9	214,250	10	1,112,151	90.8	11835.1	6528.1	54,760

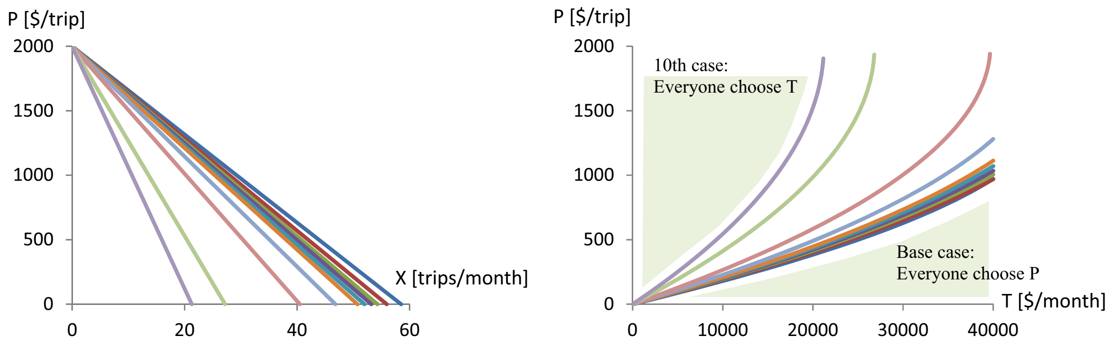

Fig. 11. Demands and indifference curves.

Table 4
Results.

Case	Choose P	Choose T	P [Ch$]	T [Ch$/month]	ΔSW[%]
All P	1–10	Nobody	556	–	0.000
1	2–10	Only 1	551	27,075	0.006
2	3–10	1 and 2	539	25,784	0.024
3	4–10	1–3	534	24,857	0.038
4	5–10	1–4	530	24,227	0.050
5	6–10	1–5	529	23,654	0.061
6	7–10	1–6	534	23,252	0.069
7	8–10	1–7	567	22,566	0.078
8	**9 and 10**	**1–8**	**663**	**22,176**	**0.080**
9	Only 10	1–9	1121	21,705	0.078
All T	Nobody	1–10	–	–	

pay for PT. Data showed that the 78.4% of all the taxi trips cost less than 2000 Chilean pesos (Ch$); we imposed this figure as the maximum willingness to pay for PT, assuming that a user would travel by taxi if the fare was higher. For short, the point $(0, 2000)$ belongs to the demand of each income segment. From these points the parameters B_i and Δ_i were obtained; they are shown in Table 3 along with the size of each segment. The corresponding demands and *IC* are shown in Fig. 11. The ten *IC* determine eleven zones where problem (7) must be solved.

Data to estimate a representative cost function was obtained from the transport authority in Santiago (CGTS, 2012) as every operator has reported its cost and trips every three months since the implementation of a new system in February 2007. With information until March 2013 we estimated that F was Ch$23,011 million per month and $m = Ch\$132.6$ per trip. With this parameters we have all the information required to obtain the optimal values for P and T.

Table 4 shows the results obtained for each of the eleven cases solved and the variation of social welfare (ΔSW) with respect to the base case, where the only payment alternative is the single ticket. In case 10 there is no value for T that fulfills the conditions of the problem. The optimal solution with the largest associated *SW* is case 8 (in bold), where only the two richest segments choose the single ticket.

Fig. 12 represents the solution. The green and blue curves are the *IC* of the 8th and 9th income segments, respectively. The slashed horizontal line represents the marginal cost and the nearly vertical one is the locus of all pairs that make the operators' profit zero, such that every pair located to the right side fulfills the budget constraint. The continuous red curves are the *SW* levels corresponding to the 8th case; note that the unconstrained maximum *SW* is outside the feasible region delimited by the *IC* s. The optimal point (P^*, T^*) marked in red is the closest to the unconstrained optimum that fulfills the budget constraint within the region determined by the 8th case, i.e. between the *IC*s; it lies on the *IC* of the 8th income segment, which means that individuals who belongs to it will be indifferent between P^* and T^*.

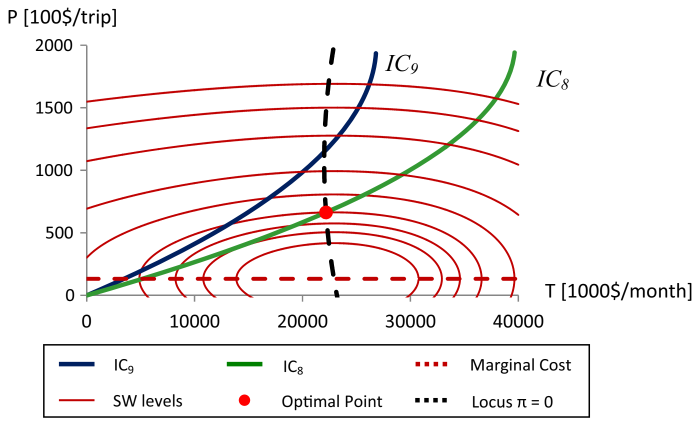

Fig. 12. Optimum pair.

Table 5
Public transit trips variation with respect to the single ticket case [%].

Cases	Income segments									
	1	2	3	4	5	6	7	8	9	10
All P	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
1	23.0	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3
2	24.1	28.0	1.1	1.1	1.1	1.1	1.1	1.1	1.1	1.1
3	24.9	28.5	29.0	1.5	1.5	1.5	1.5	1.5	1.5	1.5
4	25.4	28.8	29.3	29.4	1.8	1.8	1.8	1.8	1.8	1.8
5	25.9	29.1	29.6	29.7	29.6	1.8	1.8	1.8	1.8	1.8
6	26.2	29.3	29.7	29.8	29.8	29.7	1.5	1.5	1.5	1.5
7	26.7	29.6	30.0	30.1	30.1	30.0	29.3	**−0.8**	**−0.8**	**−0.8**
8	27.0	29.8	30.2	30.3	30.3	30.2	29.5	28.1	**−7.4**	**−7.4**
9	27.3	30.0	30.4	30.5	30.5	30.4	29.7	28.3	25.7	**−39.1**
All T	–	–	–	–	–	–	–	–	–	–

Table 5 shows the relative change of PT trips in each case with respect to the base. Users who buy the travelcard (low and mid income segments) always increase their PT trips by around 30%, while single ticket users either maintain or reduce them (highlighted in bold) because the new single ticket price is always larger (equal in one case) than in the base case. This means that the introduction of a welfare maximizing travelcard and associated single ticket price will act in practice as a highly equitable transport policy that favors eighty percent of the population (all of them with income below the average).

For synthesis, the optimal single ticket–travelcard pair for Santiago, for the parameters and functions that were used in this application, is $(P^*, T^*) = (663; 22, 176)$. This pair implies that the two richest segments of the city will travel paying P for each trip made and the other eight segments, the most intense in PT trips, will buy the travelcard.

4.2. Sensitivity analysis

The optimal values (P^*, T^*) were obtained for given parameters of the demand and cost functions representing the individuals and the PT system in Santiago. Now we examine the effect of potential changes in the prevailing conditions on the optimal fares. We will analyze three scenarios that will be represented through suitable variations in the parameters of the problem.

In the first scenario, a **better public transport system** will be represented. Improvements in the services (air conditioning, more comfortable seats, higher frequency, and so on) imply a cost increase and induce a larger willingness to pay for each trip by the users. This scenario will be represented through an increase by ten percent in the maximum price that users are willing to pay for the public transport (such that the new demands will be above the originals), and through a ten percent increase in the marginal cost.

Secondly we will examine the effect of an **increase in car ownership in the middle income segments**. Under these conditions PT is less attractive for individuals in those segments, diminishing their willingness to pay for each trip and, therefore, a contraction of the demand is expected. To represent this scenario the monthly car trips of the fourth to the seventh income segments are increased by ten percent in Fig. 2 and the demands were adjusted accordingly, maintaining the intersection with the price axis. It is important to note that this movement is bounded, because the contraction of any demand cannot be such that two demands match.

Table 6
Parameters for each scenario.

Scen	Segment	1	2	3	4	5	6	7	8	9	10
1	A_i	69.0	70.4	71.5	72.5	73.5	74.9	79.8	86.6	90.6	90.4
	B_i	1945.2	2993.4	3543.7	3908.3	4253.1	4616.9	5605.5	6583.8	6982.8	10582.0
	Δ_i	436.7	614.4	744.4	845.9	961.5	1099.8	1610.8	2579.9	5254.8	7301.1
2	A_i	69.9	71.3	72.3	73.0	73.9	75.2	79.9	87.2	91.1	90.8
	B_i	2173.9	3346.5	3962.0	4174.9	4514.1	4862.6	5734.4	7363.3	7809.6	11835.1
	Δ_i	390.8	549.6	665.8	871.1	996.6	1148.6	1732.0	2306.8	4698.6	6528.1
3	N_i	–	302,186	210,603	205,612	214,250	159,398	218,319	85,567	65,917	54,760

Table 7
Sensitivity results.

Sce	Base			1. Better PT			2. Greater CO for medium			3. No poverty		
Case	P	T	SW [$\times 10^{10}$]	P	T	SW [$\times 10^{10}$]	P	T	SW [$\times 10^{10}$]	P	T	SW [$\times 10^{10}$]
All P	556	–	3.93	563	—	4.51	576	–	3.70	557	–	3.91
1	551	27,075	3.95	560	27,294	4.53	571	27,822	3.72	539	25,778	4.00
2	539	25,784	4.02	550	26,118	4.59	557	26,463	3.80	534	24,851	4.06
3	534	24,857	4.08	546	25,242	4.64	550	25,480	3.86	530	24,221	4.11
4	530	24,227	4.12	543	24,649	4.68	559	24,170	3.90	529	23,647	4.15
5	529	23,654	4.17	543	24,109	4.72	560	23,501	3.95	533	23,245	4.18
6	534	23,252	4.20	549	23,727	4.74	568	23,027	3.98	567	22,559	4.22
7	567	22,566	4.23	583	23,058	4.78	620	22,183	4.02	**663**	**22,169**	**4.23**
8	**663**	**22,176**	**4.24**	681	22,666	4.79	654	21,938	4.04	1120	21,697	4.22
9	1121	21,705	4.23	1129	22,176	4.78	1101	21,471	4.03	–	–	–
All T	–	–	–	**–**	**22,338**	**4.80**	**–**	**21,190**	**4.06**			

The third scenario will consider a **successful fight on poverty** by moving all the individuals of the poorest income segment (1st) to the 2nd segment. To represent this scenario, it is only necessary to change the quantity of individuals such that the new N_2 is the sum of the old N_2 plus N_1, and the new N_1 becomes zero.

The parameters representing each new scenario are shown in Table 6; a new optimal pair (P^*, T^*) was obtained for each one and the results are shown in Table 7. As evident, in the third scenario only ten cases were solved.

The optimal case for each scenario is highlighted in bold. In scenarios 1 and 2 the optimal case is when everyone chooses to travel with a monthly travelcard. On the contrary, when poverty diminishes, the two richest segments choose the single ticket and the other eight buy the travelcard.

When a better PT system is represented and when car ownership of the medium income segments is increased, the case when everyone chooses T becomes feasible; this means that there are some values for the travelcard that fulfill the budget constraint and, simultaneously, would be chosen by the individuals of the richest segment. In the first scenario this occurs because an increase in quality induces more trips in this mode, increasing the fare box by a greater proportion than costs (we only increase marginal cost, not fixed), and because, with the expansion of the demands, people willingness to pay for a travelcard increased. Then, when the travelcard is available and PT is better, because of the increase of trips for each individual, this payment alternative is convenient for everyone.

In scenario 2, when the car ownership is increased in the medium income segments, fewer trips in PT are expected. This means that there will be lower costs and therefore, lower incomes are needed to cover them. As lower incomes are needed, the case when everybody chooses T becomes feasible, because a cheaper travelcard can fulfill the budget constraint and would be chosen by the richer segment. As in scenario 1, when the travelcard is available for the ten segments, everybody would choose it.

In scenario 3 the optimal case is when only the two richest income segments choose the single ticket and the other segments buy the travelcard. Here, the optimal travelcard is lower than the base case and the single ticket remains constant. This is explained because when individuals from the poorest income segment become part of the second income segment, they move to a segment with a less intensive demand and, therefore PT trips decrease. This implies that costs decrease and a lower revenue is needed. As we can see in Table 7, the variation is almost negligible, principally because the first segment is the one with fewer individuals (4% of the total). If the three poorest segments were put together we could observe a decrease in both fares.

5. Conclusions

We have developed and applied a model to obtain, simultaneously, the optimal (welfare maximizing) values for a monthly travelcard (T) and the price of a single ticket (P) for PT trips under a relatively high concentration of income and car ownership. Individuals are characterized by their car ownership (correlated with income) and income as purchasing power. This induces a double effect of income on PT trips demand, as lower income groups use PT more intensively than richer groups for the same price, but the income effect is stronger. So the demand for PT trips not only depends on the price, but also on the travelcard value and the income segment of the individual, something that has been ignored in the literature. For any given pair (P,T) the user will choose the alternative with the largest associated surplus (we do not consider the possible existence of liquidity effects). An indifferent user (with income $\widetilde{I}$) is defined as the one that receives the same surplus from both alternatives. This user is shown to be such that individuals with $I_i > \widetilde{I}$ will choose the single ticket and those with $I_i < \widetilde{I}$ will buy the travelcard for a given pair (P,T). If $\widetilde{I}$ was known the choice of each user would be known and both P^* and T^* that maximizes the social welfare under a budget constraint could be found. But $\widetilde{I}$ depends on the offered pair (P^*,T^*) and, therefore, the choice is unknown. Solving $k+1$ cases (where k is the number of income segments) assuming $\widetilde{I}$ known within a restricted region of the (P,T) plane, the overall optimum can be found by comparison.

The model was applied to Santiago, Chile, a city where both income and car ownership are highly concentrated and correlated, where there is no travelcard in the sense of a season ticket described here, and where the available technology (touch card) would make it relatively simple to implement this tool. Considering the very uneven distribution of wealth in the country, ten income segments were considered and data from the last validated origin-destination survey and from the operators' reports was used to estimate the PT demand parameters, and the marginal and fixed costs of the system. An optimal pair $(P^*,T^*) = (663; 22,176)$ was found; only the two richest income segments would choose the single ticket and the other eight would buy the travelcard, increasing the benefits of the poorest and middle income segments.

In order to analyze how the optimum (P^*,T^*) changes under variations in the initial conditions of the problem, three scenarios were represented through changes in the parameters of the demand functions, costs and/or the number of individuals in each income segment: a better PT system, an increment in the car ownership of the middle segments and a redistribution of income. In the three of them we were able to obtain and explain the results, in terms of the trip generation and its influence in the operator costs.

There are many possible directions for further research. One is finding a way to represent a continuous distribution of income capturing both effects. Also, differences among tastes can be considered for individuals within the same income segment, coupling our general approach with Carbajo's (1988). A third direction is the inclusion of mode choice, giving the model the possibility to capture behavior of users in a better way and to include travel time as a decision factor.

Acknowledgements

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Appendix A. Derivation of Eqs. (5) and (6) for P^* and T^*

We need to maximize social welfare subject to a budget constraint (non-negative profit of the public transport operators, π).

$$\text{Max } SW = \sum_{I>\widetilde{I}} N_i \cdot CS_P + \sum_{I<\widetilde{I}} N_i \cdot CS_T + (P-m)\cdot \sum_{I>\widetilde{I}} N_i \cdot X(P,0,I) + \sum_{I<\widetilde{I}} N_i \cdot T - m \cdot \sum_{I<\widetilde{I}} N_i \cdot X(0,T,I) - F$$

Subject to

$$\pi = (P-m)\cdot \sum_{I>\widetilde{I}} N_i \cdot X(P,0,I) + \sum_{I<\widetilde{I}} N_i \cdot T - m \cdot \sum_{I<\widetilde{I}} N_i \cdot X(0,T,I) - F \geqslant 0$$

Let λ be the Lagrange multiplier of the budget constraint. The Lagrangian is

$$L = SW + \lambda \cdot \pi = CS + \pi + \lambda \cdot \pi = CS + (1+\lambda)\cdot \pi$$

Replacing the social welfare and the profits in this expression, leads toce:display>

$$L = \sum_{I>\widetilde{I}} N_i \cdot CS_P + \sum_{I<\widetilde{I}} N_i \cdot CS_T + (1+\lambda)\cdot \left[(P-m)\cdot \sum_{I>\widetilde{I}} N_i \cdot X(P,0,I) + \sum_{I<\widetilde{I}} N_i \cdot T - m \cdot \sum_{I<\widetilde{I}} N_i \cdot X(0,T,I) - F\right]$$

First order condition for P is

$$\frac{\partial L}{\partial P} = \sum_{I_i>\widetilde{I}} N_i \cdot \frac{\partial EMC_P}{\partial P} + (1+\lambda)\cdot \left[\sum_{I_i>\widetilde{I}} N_i \cdot X(P,0,I_i) + (P-m)\cdot \sum_{I_i>\widetilde{I}} N_i \cdot \frac{\partial X(P,0,I_i)}{\partial P}\right] = 0$$

Which yields

$$\lambda \cdot \sum_{\widetilde{I_i > I}} N_i \cdot X(P,0,I_i) + (1+\lambda)\cdot(P-m)\cdot \sum_{\widetilde{I_i > I}} N_i \cdot \frac{\partial X(P,0,I_i)}{\partial P} = 0$$

Calling η_{P,I_i} to the price elasticity of the demand of the stratum of income I_i and $\theta = \frac{\lambda}{1+\lambda}$ we get

$$\frac{(P-m)}{P} \cdot \sum_{\widetilde{I_i > I}} N_i \cdot |\eta_{P,I_i}| \cdot X(P,0,I_i) = \theta \cdot \sum_{\widetilde{I_i > I}} N_i \cdot X(P,0,I_i)$$

Finally, defining α_i as the ratio between the total trips of the segment with income I_i and the total trips with single ticket, the equation for P^* is

$$\frac{(P-m)}{P} = \frac{\theta}{\sum_{\widetilde{I_i > I}} \alpha_i \cdot |\eta_{P,I_i}|}$$

The first order condition for T is

$$\frac{\partial L}{\partial T} = \sum_{\widetilde{I_i < I}} N_i \cdot \frac{\partial EMC_T}{\partial T} + (1+\lambda) \cdot \left[\sum_{\widetilde{I_i < I}} N_i - m \cdot \sum_{\widetilde{I_i < I}} N_i \cdot \frac{\partial X(0,T,I_i)}{\partial T}\right] = 0$$

Which yields

$$\sum_{\widetilde{I_i < I}} N_i \cdot P[X(P,0,I_i),0,I_i] \cdot \frac{\partial X(P,0,I_i)}{\partial T} = -(1+\lambda) \cdot \left[\sum_{\widetilde{I_i < I}} N_i - m \cdot \sum_{\widetilde{I_i < I}} N_i \cdot \frac{\partial X(0,T,I_i)}{\partial T}\right] = 0$$

Calling η_{T,I_i} the demand elasticity of segment i with respect to the travelcard value, T, and manipulating terms we get

$$\sum_{\widetilde{I_i < I}} N_i \cdot P[X(P,0,I_i),0,I_i] \cdot |\eta_{T,I_i}| \cdot X(0,T,I_i) = (1+\lambda) \cdot \left[T \cdot \sum_{\widetilde{I_i < I}} N_i + m \cdot \sum_{\widetilde{I_i < I}} N_i \cdot |\eta_{T,I_i}| \cdot X(0,T,I_i)\right]$$

Finally, calling β_i the ratio between the number of individuals in segment i and the total number of travelcard users, the equation for T is,

$$T = \sum_{\widetilde{I_i < I}} \beta_i \cdot |\eta_{T,I_i}| \cdot X(0,T,I_i) \cdot \left\{\frac{1}{1+\lambda} \cdot P[X(P,0,I_i),0,I_i] - m\right\}$$

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