UNIVERSIDAD DE CHILE
FACULTAD DE CIENCIAS FÍSICAS Y MATEMÁTICAS
DEPARTAMENTO DE INGENIERÍA INDUSTRIAL

DYNAMIC EQUILIBRIUM IN MULTIPLE LIMIT ORDER MARKETS: OVERLAPPING

TESIS PARA OPTAR AL GRADO DE MAGÍSTER EN ECONOMÍA APLICADA

MEMORIA PARA OPTAR AL TÍTULO DE INGENIERO CIVIL INDUSTRIAL

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SANTIAGO DE CHILE
2017

RESUMEN DE LA MEMORIA PARA OPTAR AL TÍTULO DE: Ingeniero Civil Industrial y grado de Magíster en Economía Aplicada
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FECHA: 19/06/2017
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Desarrollamos un modelo de equilibrio dinámico para el fenómeno de “Overlapping”, donde agentes neutros al riesgo transan un activo de forma continua en libros de órdenes reales mediante órdenes de tipo “market” y “limit”. Encontramos que los principales oferentes de órdenes “limit” se ven beneficiados tanto en la apertura como en el cierre del “overlap”. Antes del inicio, cargan el libro con órdenes “limit” lejos de los precios de “bid” y “ask”. Al cierre del “overlap”, obtienen los mejores términos transaccionales del día, alcanzando así su máximo bienestar. Por otro lado, los demandantes de órdenes tipo “market” tienen su bienestar mínimo durante los primeros minutos de “overlap” entre los libros de órdenes.

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We develop a dynamic overlapping model where risk-neutral agents trade continuously in real limit order books via limit and market orders, and adapt their behavior to the time of the day they're in. We find the main suppliers of limit orders to take advantage of both the opening and closing transition of the overlap. Before the open, they load-up the book with limit orders away from the bid-ask quotes. At the close of the overlap, they get the best terms of transaction of the day, thus achieving their maximal welfare. Meanwhile, demanders of market orders have their lowest welfare during the first minutes after the open of the overlap.

DEDICATORIA

Dedico este trabajo a mi Madre y a mi Padre, como un regalo que parcialmente intenta recompensarlos por toda una vida de sacrificios llevados a cabo en mi nombre. Les debo todos mis privilegios, y siempre les estaré agradecido.

Olga y Germán, son la materialización del Amor verdadero; son lo mejor que un hijo podría pedir.

Los amo.

ACKNOWLEDGEMENTS

Agradezco por el apoyo financiero para la realización de esta tesis proveniente del proyecto Fondecyt # 11140628 y el Instituto Milenio para la Investigación en Imperfecciones de Mercado y Políticas Públicas ICM IS130002.

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1. INTRODUCTION

In the last decades, firms have increasingly cross-listed their shares at foreign exchanges according to Pagano, Randl, RoKell, & Zechner (2001). The New York Stock Exchange (NYSE) factbook states non-US firms generated approximately 10% of total traded volume in 2002, and NASDAQ lists even more non-US firms than the NYSE. Furthermore, Pagano et al. (2001) show that up to 50% of the stocks traded in Amsterdam, Brussels, Frankfurt and Switzerland come from non-domestic firms.

In conjunction with this phenomenon, the proportion of traders who use Smart Order Routing (SOR) technology, which allows them to access both trading venues with ease, has been estimated by Foucault and Menkveld (2008) to be 27% for the Dutch stock market after the entrance of the London Stock Exchange (LSE), and reaches 54% if only the most traded quintile of stocks is considered. Moreover, the proportion of traders who use SOR is shown to be highly correlated to a dummy equal to one if the stock is cross-listed in the United States, with a correlation coefficient of 0.63.

Limited attempts to model how agents behave when firms are cross-listed and markets overlap have been made, and as Halling, Moulton and Panayides (2011) state, there's little empirical evidence of how discretionary traders actively participate in multiple markets and why [1]. Our goal is to understand how traders dynamically change their behavior throughout the trading day when shares are cross-listed and there's overlapping hours.

Our work makes three main contributions to the literature regarding limit order markets, cross-listing of shares and overlapping hours. First, we develop an asynchronous single-agent trading algorithm which includes overlapping hours in the trading day with a cross-listed share, inducing a changing market structure and a rich set of stages which are recognized by traders throughout the trading day. Second, we find the main suppliers of liquidity to take advantage of both the opening and closing of the overlap. Before the open, they load-up the book with liquidity away from the bid-ask quotes, expecting a rise in demand in the next period. At the close of the overlap, they slightly increase their aggressiveness and get the best terms of trade of the day, due to a rise in shares demanded enhanced by the fear of traders with intrinsic motives for trade of getting-stuck in the less liquid non-overlap period. Third, we find there's a regime-shift in liquidity supply from the single to the multimarket scenario, which drives market quality measures.

Markets do not perfectly overlap in their trading hours, for example, the NYSE and LSE only coincide for 2 hours each day. Investors who are able to trade in both markets have additional decisions to make: whether to hold their trades until the foreign market opens, and on which market to trade if it's overlapping hours. As more firms cross-list their shares and more traders have access to SOR, these decisions become crucially important since they drive agents' behavior, and ultimately determine efficiency and quality of the markets involved during the trading day.

[1] A discretionary trade is able to choose in which market to submit his orders.

We develop a model where risk-neutral heterogeneous agents make endogenous sequential decisions based on market conditions and their private information as in Goettler, Parlour and Rajan (2009). Agents arrive stochastically following a continuous time Poisson process and have one share of the financial asset to trade in one of two limit order books. They might place a limit or market order and have to decide its price, direction (buy or sell), and book of submission if they have access to both books[2]. Traders might also decide to not place an order given an order submission cost. If a trader decides to place a limit order or no order at all, he's allowed to revisit the market following a random Poisson process and resubmit his order.

The trading day is divided in two periods where agents face different market configurations: in the first one, they're only allowed to trade and monitor one book to which they are assigned randomly, facing a single-market or non-overlapping scenario. On the second one, they're able to trade and monitor both books, facing a multimarket or overlapping scenario. Furthermore, each period is divided into three cycles, creating 6 different stages along the trading day which traders are able to recognize and incorporate into their decision making; namely, before and after the open or close of the overlap, and stable periods in-between.

Our model is flexible enough to test a variety of scenarios and policies that could arise in real markets. Even more important, it helps to understand and sheds light on how agents endogenously adapt to the stage of the trading day they're facing, having direct impact on the quality of the market and transaction terms.

Given the sheer complexity of the problem, an analytical equilibrium can't be found and we solve numerically to get a Markov perfect Bayesian Equilibrium, by using an asynchronous single-agent maximization algorithm based on Pakes and McGuire (2001). This strategy has been proven successful in modelling dynamic limit order books by Goettler et al. (2005), Goettler et al. (2009) and Bernales & Daoud (2013), while keeping the model computationally tractable.

We test a variety of scenarios by varying the length of the stable and transition periods of the trading day, while keeping every other characteristic of the model fixed to isolate how markets and period transitions respond to different overlap conditions. In our scenarios, every agent sees the fundamental value of the asset lagged a fixed number of periods, i.e., is uninformed, and has a private valuation for the asset coming from a discrete probability distribution P_α.

We find traders behavior to vary substantially between the overlap and non-overlap periods. Liquidity suppliers are considerably more aggressive during the overlapping hours, while having a lower probability of getting picked-off and limit orders executing considerably faster; effects attributed to increased competition. Moreover, we find liquidity suppliers to be hesitant to get executed in anticipation of the overlap, as the later represents a period of better terms of transaction overall. During the overlap, demanders of liquidity are willing to accept worse prices in exchange of a considerably faster execution time. Also, the overlap represents a regime-shift in liquidity

[2] A limit order is a commitment made today, to trade the share of the asset at the specified price on some future time t'. A market order is a request to trade immediately at the best price available (bid or ask quote). Since transactions are executed by pairs of agents, market orders always execute an outstanding limit order on the book.

supply, where agents with no-intrinsic motives for trade supply most of the liquidity, while traders who have private motives to trade become clear demanders of shares. Agents with the highest absolute valuation for the asset are found to submit fleeting limit orders during the non-overlap period, similar to what's described by Rosu (2009), given a shortage in liquidity supply under favorable conditions for trade and high execution times.

We compute welfare for each period of the trading day, both at the trader and aggregate level. As expected from the dynamic of each traders' behavior, the overlapping period rewards liquidity suppliers with increased welfare. We further decompose welfare following Bernales and Daoud (2013) into the waiting cost incurred and the money transfer made in each period. Liquidity demanders accept considerably worse terms of transaction during the overlap, but they cut their waiting cost in half. The opening phase of the overlap is characterized by liquidity demanders getting the worst prices of the day, which are enjoyed by suppliers. Nonetheless, being execution times close to the highest of the day, speculators ($\alpha = 0$ traders) get their benefits heavily discounted and actually reach their maximal welfare in the closing period of the overlap, where every trader incentives for trade are aligned: suppliers are more aggressive and demanders execute more market orders in fear of getting stuck in the less-liquid non-overlap phase.

With regards to market quality measures, we find the overlapping period to be beneficial to global liquidity, although detrimental to local markets depth. Moreover, overall execution times reach their minimal and liquidity gets consumed from the books, characterizing the overlap as a period of higher trading activity and an increased number of market orders. Notably, the opening and closing phases of the overlap are characterized by a higher demand for shares, condition which proves beneficial to traders with no-intrinsic motives for trades. Suppliers load-up the book with liquidity in anticipation of the overlap open, expecting a rise in demand and the best effective spread of the day.

We also find predictions made by Rosu (2009) to be present in our model. The books clearly present a hump-shape in anticipation of the overlap, as liquidity suppliers expect a rise in the number of shares demanded. What's more, when the book is full with liquidity and demanders aren't able to find the right conditions for trade with ease, they are forced to place fleeting or marketable limit-orders, for instance, limit sell orders a tick under the fundamental value, to get the lowest possible execution time and better terms of transaction. Intriguingly, we find higher spreads when the market is overlapping, period which represents a considerably more liquid market according to Rosu (2009). This is explained by the heterogeneity of the agents of our model and a regime-shift in liquidity supply during the overlap, which allows increased spreads and improved welfare.

Our study is organized as follows. Section 2 presents a review of the relevant literature. Section 3 describes the model, algorithm and the numerical parametrization used to simulate the market. Section 4 presents the main results and shows how traders' behavior, welfare and market quality dynamically change throughout the trading day. Finally, section 5 concludes.

2. LITERATURE REVIEW

Early theoretical works on microstructure focus on the finding that markets should consolidate and centralize in one single venue as economies of scale arise. Nevertheless, the increase in fragmented trading (as in cross-listed stocks) and mixed empirical evidence on the consolidation of trading venues, has given emphasis to models where fragmented markets can exist as an equilibrium, emerging from queue-jumping behavior and captive traders in local markets.

Several studies find consolidation to be the market equilibrium and improve market quality. Pagano (1989) develops a multimarket model and predicts that given the same cost structure across markets, trade concentrates in a single venue as economies of scale benefit trading. Multiple markets equilibria can arise if costs differ, although welfare decreases. Lescourret (2015) uses an inventory model and shows that even though order flow might be fragmented ex-ante, intermediaries consolidate it ex-post improving global liquidity. Arnold, Hersh, Mulherin and Netter (1999) empirically find that merged regional stock exchanges in the US attract higher market share and experience narrower bid-ask spreads. Amihud, Lauterbach and Mendelson (2003) show that consolidation improves liquidity and prices of the stock. Bennett and Wei (2006) find that consolidation improves market quality by reducing return volatilities, price reversals, quoted spreads and trading costs.

Nonetheless, several authors show consolidation to not be the unique equilibrium. Chowdhry and Nanda (1991) show that in presence of small and large liquidity traders with informational advantages, multiple-markets equilibria are feasible. Madhavan (1995) develops a model and shows that fragmented markets need not coalesce into a single-market setup unless trade disclosure is mandatory, since large traders and dealers benefit from fragmentation even though it increases price volatility and violations of price efficiency. Parlour & Seppi (2003) develop a model and find that neither a pure-limit order book market nor a hybrid specialist/limit order market structure is competition-proof, and equilibria where multiple competitors exist are feasible.

Furthermore, recent studies show fragmentation to be the market-quality improving structure. O'Hara & Ye (2011) empirically find that fragmentation does not harm market quality. Higher fragmentation is associated with lower transaction costs, faster execution speeds and improved market efficiency, in that prices are closer to being a random walk. Degryse, Jong, & Kervel (2015) find fragmentation is beneficial to global liquidity, while detrimental to local liquidity. Boehmer & Boehmer (2003) show fragmentation in the market of ETFs leads to a dramatic improvement in liquidity: trading costs decrease, quoted depth increases and price impact declines. This effects are attributed to a sharp decrease in market-maker rents due to greater competition, as in Hengelbrock and Theissen (2009), who find better spreads in fragmented markets. Finally, Foucault & Menkveld (2008) find the consolidated limit order book to be deeper and conclude fragmentation enhances liquidity supply.

Several studies lay in-between the previous findings. Mendelson (1987) uses a model to predict that fragmentation reduces the quantity traded and expected gains from trade while increasing the price variance. On the other hand, consolidation imposes significant order-communication requirements on participants, increasing costs. Furthermore, price signals from a fragmented market are shown to be of a higher quality[3]. Biais (1993) theoretically predicts bid-ask spreads to remain unchanged between a centralized and a fragmented market, albeit more volatile in centralized ones. Based on Biais (1993) model, Frutos and Manzano (2002) show spreads in fragmented markets are actually smaller, thus preferred by market participants. In sharp contrast, Yin (2005) finds that if costs of searching for better quotes are introduced, then the opposite holds: narrower spreads are present in centralized markets.

With regards to cross-listing of stocks and its dynamics[4], Baruch, Karolyi, and Lemmon (2007) predict and find strong empirical support that the volume of a cross-listed stock it's higher on the exchange in which the stock returns have higher correlation with returns of other assets. Halling, Pagano, Randl, & Zechner (2008) show that the domestic turnover rate increases permanent and significantly for firms from developed countries that cross-list in the US. Foerster and Karolyi (1998) find firms that cross-list to enjoy up to a 29% increase in intraday trading volume and a 44 basis points reduction in effective spreads, although the price discovery process might not improve. Agarwal, Liu, and Rhee (2007) show that the home-market is the primary location for price discovery. Halling et al. (2011) find higher multimarket trading across markets with similar designs and strong enforcement of insider laws when cross-listing exists.

Cross listing gives rise to times during the trading day where shares can be traded in both the domestic and foreign market, i.e., overlapping periods, which are the core of our study. Menkveld (2008) develops a model of partially overlapped markets and predicts that given a sufficient quantity of small liquidity trading, traders concentrate their activity during the overlap and split orders. Analyzing British and Dutch ADRs, which overlap with the NYSE during a two and one hour interval, he finds empirical evidence of the effects described via increased volatility, volume, liquidity supply, reduced price impact and a positive correlation in order imbalance across markets during the overlap.

Other studies support the overlapping period as the one where trading concentrates. Menkveld, Koopman and Lucas (2007) compare Dutch stocks cross-listed in NYSE, which generate most of the volume from European stocks, and demonstrate that the overlapping hours are the most important for price discovery in the 24-hour day, followed by the domestic-market-only period (Amsterdam). Furthermore, they find that during the overlap cross-listed stocks underreact to their domestic market indexes and exhibit significant noise in their midquote, which is consistent with privately-informed traders splitting their orders across markets and through time to minimize price impact. Chelley-Steeley, Kluger, Steeley and Adams (2015) design a laboratory experiment of double-auction asset markets to show that when overlapping exists, transactions are shifted from the single-market period to the multimarket one and profits of informed traders increase. Moreover, quoted spreads are greater at the start of the overlap, decreasing as time passes.

Fargmentation, cross-listing and partially overlaped markets give rise to profitable risk-free transactions. Gagnon & Karolyi (2010) measure arbitrage opportunities from cross-listed shares in the US and find deviations from price parity average 4.9 basis points, but can reach large extremes

[3] Meaning they are more reliable. The effect arises since the "diversification" effect coming from every trading venue signal outweighs the liquidity "thinness" within submarkets

[4] See Gagnon & Karolyi (2010) for an extensive review on cross-listing studies.

given their high volatility. Price parity deviations are also positively correlated with holding costs and other barriers which can impede arbitrage.

Finally, limit order books are the dominant market structure and the one used in our study. Jain (2005) finds 85 out of the 120 leading stock exchanges around the world, including the NYSE and LSE, to be defined as entirely electronic limit order markets with no floor trading, while 101 exchanges combined both methods. Consequently, limit order books as a market design has received loads of attention to understand its underlying microstructure characteristics[5]. Hollifield, Miller, Sandas, & Slive (2006) estimate the gains from limit-order markets and provide empirical evidence that it's a good market design.

Few theoretical models exist to understand Limit Order Markets. Rosu (2009) models a dynamic limit order book and predicts that higher competition and trading activity causes smaller spreads and lower price impact, market orders lead to price overshooting, liquidity can cluster away from the bid-ask spread causing a hump-shaped order book, bid and ask prices display a comovement effect and, finally, predicts that when the order book is full, traders may submit quick or fleeting limit orders[6].

Methodologically related to our work are Goettler et al. (2005), Goettler et al. (2009) and Bernales & Daoud (2013), who develop microstructure models for limit order markets using an asynchronous single-agent maximization algorithm. Traders are heterogeneous and condition their actions on current information available and market conditions, progressively learning which action rewards the highest expected utility given the state of the world observed. Our study differs from previous ones in the rich dynamic of the trading day that's embedded in the six different stages traders are able to recognize and incorporate in their decision making. Therefore, our study adds a higher realism to previous models: not only arriving traders are heterogeneous, also the market structure they face varies throughout the day between a single and multimarket one.

[5] For a full survey and review of limit order markets, refer to Parlour & Seppi (2008) and Hasbrouck (2007).

[6] A quick or fleeting limit order, also called marketable limit order, is a limit order placed closely to the respective ask or bid quote so it significantly improves the current spread and executes extremely fast.

3. MODEL

We develop a dynamic model of trade in a single financial asset, considering two pure limit order books that overlap during a fraction of the trading day. Time is continuous and traders arrive randomly at the market with an endowment of one share to trade either on a limit or market order, deciding the price, direction and book of the order if both markets are overlapped. An unexecuted trader may reenter the market randomly, allowing him to update his order according to the new market conditions he faces. In the following sections, each of the model elements and its characteristics are described.

Limit Order Books: There exists an order book for each of the markets $m \in \{1,2\}$, which is represented at any moment in time by $L_{m,t}$ and is described by two elements that may differ across-books: a set of prices and a set of outstanding orders at each price.

Each book has a discrete set of feasible prices $P_m = \{p_m^i\}_{i=-\infty}^{\infty}$ at which traders can place orders, where d_m is the distance between two consecutive prices in book m (namely, the tick size of book m), and a backlog of outstanding orders at each possible price p_m^i in market m at time t, i.e., the depth at each price, $l_{m,t}^i$, where $l_{m,t}^i > 0$ for every buy order and $l_{m,t}^i < 0$ for sell orders. Therefore, the limit order book $L_{m,t} = \{l_{m,t}^i\}_{i=-\infty}^{\infty}$ is the vector of outstanding orders at each price p_m^i.

Therefore, given a limit order book $L_{m,t}$, the ask price will be $A(L_{m,t}) = \min\{p_m^i \mid l_{m,t}^i < 0\}$ and the bid price $B(L_{m,t}) = \max\{p_m^i \mid l_{m,t}^i > 0\}$; the lowest price at which there is a limit sell order and the highest price at which there is a buy order in the book, respectively.

Both books respect price and time priority as their order execution criteria when a suitable counterparty arrives: orders at better prices (higher for buy orders and vice versa) are executed first and, given the same price, earlier submitted orders are further ahead in the queue. Finally, limit orders are marketable by means of the price criterion; a limit sell order priced below the bid is executed as a market order (analogous for a limit buy order).

Asset: At any instant t the single financial asset has a fundamental value of v_t, considered to be the expected discounted value of future cash flows. The fundamental value changes randomly in time following a Poisson process with rate λ_v and in a fixed amount of k ticks up or down with equal probability (0.5). It's noteworthy to consider that different from Goettler et al. (2009), v_t might not coincide with a feasible trading price in one or both books, since each could have different tick sizes.

Traders: Traders arrive at the market following a Poisson process with intensity λ, and they are allowed to trade only one share. They might buy or sell the asset through a market or a limit order, or simply wait until market conditions change. Nonetheless, once they trade, they leave the market forever.

For a trader who's entered the market for the first time, his decision problem always involves whether to submit an order or not, and if so, to define its price and direction (buy or sell) [7]. However, the set of decision variables varies overtime as the market fluctuates between a single-market and a multimarket setup (Non-overlapping and overlapping respectively). When markets overlap, the agent also has to decide the book in which he's going to post his order.

Traders who've entered the market previously and placed a limit order or no order at all, and haven't yet traded their share, are allowed to revisit the market following a Poisson process with parameter λ_r. Therefore, the reentry period before they can revise their orders represents a friction that traders must take into account in their maximization problem[8], and there exists a random amount of active market participants who are waiting to revise their orders.

A returning trader has an additional decision to make if he's an active limit order in the market: To cancel it by paying a cancelation cost c_m or not. If he does, then his decision problems is identical to the one faced by a new trader, if he doesn't, he must wait before being able to revise his order again. The implicit benefit of not canceling his order is not losing his time priority in the queue, while the cost is the asset may have moved in a direction that reduces the expected payoff from his limit order.

Information: A fraction $u \leq 1$ of the agents are uninformed, and see the fundamental value with a lag of Δt periods ($v_{t-\Delta t}$). The remaining traders are considered to be informed and know the fundamental value exactly at each instant t.

Overlapping Dynamic: We divide the trading day in two major periods: non-overlapping, where traders are able to trade and monitor a single book to which they are assigned randomly; and overlapping, where agents can trade and monitor both books. We further subdivide each major period into three cycles each, which spawns 6 different stages throughout the trading day, labeled $\{OV_p\}_{p=1}^{3}$ and $\{NOV_p\}_{p=1}^{3}$ for the overlapping and non-overlapping periods respectively.

$OV_1(NOV_1)$ considers the moment in time immediately after trading in both books is allowed (denied) for new and unexecuted traders, by removing (activating) their restriction to trade and see only one book. $OV_2(NOV_2)$ are the longest periods, and during this time traders can trade and see both books (only one book). Periods $OV_3(NOV_3)$ come before trading in both books becomes denied (allowed), analog to how periods $OV_1(NOV_1)$ work. Figure 1 shows the different stages of the trading day.

[7] Recall the type of order, whether a limit or market order, is completely defined by its price and direction through the price priority criterion of limit order books.
[8] Frictions might include costs of monitoring the market continuously, cognitive constraints and technological barriers.

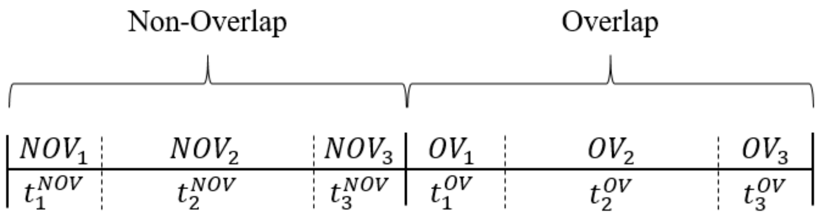

Figure 1: Stages of the trading day in the model. *The figure shows how the trading day is organized in our model. Both books run in parallel, always being in the same stage throughout the trading day. Two major periods divide the day: the non-overlap and the overlap. There's three different phases for each major period: Cycle 1 considers the first 30 minutes and Cycle 3 the last 30 minutes of each period. Cycle 2 is the time interval within the opening and close, and is considered a stable period of trading.*

Therefore, the pairs (NOV_3, OV_1) and (OV_3, NOV_1) are considered transition periods between a multimarket and a single-market setup, both at the opening and closing of the overlap, and thus they are the main interest of our study. Each period in the groups has a length of t_p^{ov} for the overlap periods or t_p^{nov} for the non-overlapping ones, where $p \in \{1,2,3\}$.

It's important to consider the model runs continuously across trading days, so no opening or ending auction phase is implemented. The later represents a challenge by itself and it's out of the scope of this work. Our interest lays in the dynamics of the transition periods during the trading day. Also, as described, both books are open to trade every minute of the day and run in parallel: they are always in the same stage of the trading day. This simplification is made to reduce the state space compared to a setup where markets are individually open on different times of the day, keeping the model tractable.

Traders Optimization: Traders have a common rate ρ for discounting realized payoffs to their first arrival time, which represents the cost of delaying trades and not the time value of money. For instance, an agent might face an opportunity cost if he could use the share committed to the delayed limit order to perform a strategy across different assets, implicit costs if the fundamental value moves in an unfavorable direction (increasing the risk of being picked-off or lowering the probability of execution of the order), or explicit costs if there's a cost to monitoring the market.

Traders also have a private valuation α for the asset, which represents intrinsic motives to trade[9] and is drawn independently from a discrete probability distribution P_α. Agents are risk-neutral and maximize their expected discounted payoff. Since payoff is realized only if an order executes, his instantaneous utility at execution time t_e is:

$$u_{t_e} = \begin{cases} d\left(\alpha + v_{t_e} - p^i\right) & if\ he\ executes\ an\ order \\ 0 & if\ not \end{cases}$$

[9] Intrinsic motives for trade might include hedging needs and inventory risk management

Where p^i is the execution price and $d \in \{1, -1\}$ for a buy or sell order respectively. The expected discounted payoff depends on the information each trader has available, which includes his private valuation and the assets fundamental value at instant t (if informed) or $t - \Delta t$ (if uninformed).

Equilibrium: To find the equilibrium of the model, we solve for optimal trader strategies in the resulting trading game, based on an updating process of each actions' expected payoff until convergence criteria are met.

Each time a trader reaches the market, he observes a state of the world which consists of public and private information (such as his valuation of the asset and information about the fundamental value) and chooses an action that maximizes his expected discounted utility. Thus, optimal strategies are state-dependent and the traders' decision is Markovian.

Furthermore, the trading game is Bayesian since a traders' private valuation is unknown to other traders and each agent might have different information about the fundamental value of the asset. Then, the proper concept is a Markov perfect Bayesian Equilibrium, where agents play dynamically optimal strategies based on the current state of the market and their current beliefs. We focus on stationary symmetric equilibria, in which each trader chooses the same strategy independent of the time of his arrival and given the same state of the world.

Solving for Equilibrium: Given the sheer complexity of the problem, an analytic closed form solution is not feasible, and we numerically solve for the Markov perfect Bayesian Equilibrium using an extension of Pakes and McGuire (2001) algorithm, as in Goettler et al. (2005), Goettler et al. (2009) and Bernales & Daoud (2013). To keep the problem computationally tractable, restrictions are imposed on the state space and traders are required to tremble, i.e., choose suboptimal decisions to ensure they learn the payoffs rewarded by each available action. A detailed description of the trading game, the algorithm and the convergence criteria can be found in appendix A.

3.1. NUMERICAL PARAMETRIZATION OF THE TRADING GAME

- The arrival rate of new traders that characterizes the Poisson process, λ, is normalized to one through the trading day, so each unit of time represents the delay between new agent arrivals, and the liquidity injection to each book remains constant when transitioning to an overlapped (or non-overlapped) market[10]. In a single-market setup, traders are assigned with probability 0.5 to each book, so on average, a trader arrives to book m every two periods.

[10] An alternative parametrization was tested where the arrival rate of agents was set to $\lambda = 2$ during the non-overlap period, and then halved during the overlap ($\lambda = 1$), with the intention to model that the complete mass of agents might move to the currently opened exchange during the day if the markets are located geographically close to each other. However, the model is not able to capture this phenomenon. More details can be found on Appendix B.

- Reentry times for traders are determined by a Poisson process of rate $\lambda_r = 0.25$, so they are allowed to reenter the market every 4 units of time on average. This rate remains constant across types of traders and throughout the trading day.

- A tick represents an eight of a dollar.

- The support of the distribution for traders' private values α is $F_\alpha = \{-8, -4, 0, 4, 8\}$ with discrete probability distribution $P_\alpha = \{0.15, 0.20, 0.30, 0.20, 0.15\}$. The distribution is based on the findings of Hollifield et al. (2006). Traders with $\alpha = 0$ represent traders who are willing to buy or sell depending on the state observed, thus, they are called speculators, since they have no intrinsic motive for trade. Accordingly, traders with $\alpha > 0$ ($\alpha < 0$) are expected to be buyers (sellers), since they have a high (low) valuation of the asset, being mainly demanders of liquidity.

- Changes in the fundamental value of the asset are driven by a Poisson process of rate $\lambda_v = 0.125$, therefore, on average, a change in the assets fundamental value occurs every eight units of time with the same probability of going up or down, and having a magnitude of $k = 1$ tick.

- The discount rate ρ is the same for all agents and set to 0.05.

- The complete mass of traders is set to be uninformed ($u = 1$), so we can isolate the effect of the market transition periods from possible interactions between different types of traders. Moreover, the lag with which an uninformed trader sees the fundamental value is set to $\Delta_t = 8$ units of time.

- Traders may submit orders at any feasible price above or below 6 ticks from their expectation of the fundamental value v. Since in this case every trader sees the fundamental value with a lag of 8 units of time, their expectation is the best possible estimate they can make given the information available. A description of how agents assess the fundamental value can be found in Appendix A.

- The transparency of each book is limited to traders observing both quotes, their respective depths, the total depth on each side of the book and the last transaction made. This is imposed to allow for the maximum amount of information available, while keeping the state space finite and the model computationally manageable. It's important to consider that the information available to each trader effectively changes during the trading day: when there's a single-market setup, they are only able to see the information of the book they are allowed to trade. Once the market transitions to an overlapping phase, agents both new and active are allowed to trade and see the information on both books, which might be interpreted as an informational shock.

- The cancelation fee c_m is set to 0 in both markets, to reduce the frictions faced by traders and isolate the effects of market transitions.

- The overlapping period is set to have a length of 120 units of time. If a unit of time in the model is considered to be 1 minute long as in Goettler et al. (2009), the duration of the overlap is set accordingly to the NYSE and LSE 2 hours overlapping time. The non-overlapping stage is set to be 300 minutes, since different scenarios showed it was enough to reach reliable market quality measures and trading behavior during the stable single-market period. Therefore, the trading day consists of seven hours of activity.

- The three cycles of the overlapping period are set to be $(t_1^{ov}, t_2^{ov}, t_3^{ov}) = (30,60,30)$ units of time, while for the non-overlapping $(t_1^{nov}, t_2^{nov}, t_3^{nov}) = (30,240,30)$. Therefore, each intermediate period is considered a stable single (multi) markets scenario. The 30 units of time length for each of the periods which conform transition stages is chosen based on Comerton-Forde & Putniņš (2011) finding that most price manipulation and abnormal trading activity during the day happens 30 minutes before the closing and after the opening of each exchange. We checked with different lengths for the stable and transitions stages, and results hold qualitatively the same [11].

[11] We tested with lengths of 60 and 90 minutes for both of the stable phases of the overlap and non-overlap periods, and with a transition time of 10 minutes.

4. RESULTS

This section presents and analyzes the results obtained from the described parametrization of the model. Results are obtained by letting the agents learn for a few billion events their optimal action in each state of the world before checking for convergence in their expected payoffs. Once the algorithm has converged, 40 million trader arrivals are simulated and averaged to get the different statistics presented. Statistics for the Non-Overlapping period are computed using only results from 1 book, while for the overlapping period both books are used to get cross-book statistics[12]. Given the high amount of observations, standard errors are less than 10^{-3}, and are thus omitted in the following results.

Results are organized as follows. Section 4.1 considers trading behavior of agents in each overlapping stage. Section 4.2 presents welfare implications of an overlapping setup for the agents. Finally, section 4.3 analyzes a set of market quality measures throughout the trading day.

4.1. TRADING BEHAVIOR

4.1.1. Aggregate level

The following section presents how traders behave throughout the trading day at the aggregate level.

Observation 1

i. Traders limit orders execute faster in stable multi-markets periods than in stable single-market ones.

ii. The risk of being picked-off diminishes during the overlapping hours, nonetheless, traders are more aggressive.

Aggregate trader behavior is presented in Table 1. Traders execute 2.45 periods faster in a stable multimarket setup compared to a stable single-market one (cycles OV_2 and NOV_2), which is due to a higher competition effect: during the overlap, liquidity suppliers have to be significantly more aggressive, increasing the probability of placing orders at the ask quote from 32.85% in cycle NOV_2, to 41.95% on cycle OV_2, while reducing the probability of getting picked-off by 10.32% during the same interval. Thus, liquidity demanders get better terms of transaction overall, being able to execute outstanding limit orders 2.45 minutes faster.

[12] For example, when both books can be seen and traded by every agent, the bid-ask spread is computed as the difference between the best ask and the best bid available.

The probability of being picked-off sheds light on who are liquidity suppliers and demanders on each period, and on how the spread is behaving. Under non-overlapping conditions, there's a higher chance of getting picked-off by approximately 10.00%; therefore, narrower bid-ask spreads must exist. Given that agents are more aggressive during the overlapping hours by at least 5.02% more orders placed at the ask quote (cycles NOV_1 versus OV_2), a higher proportion of traders with private motives for trade are supplying liquidity during the non-overlap, narrowing the spread and placing barriers on agents with $\alpha = 0$ to compete. When the overlap opens, competition increases the aggressiveness of agents without intrinsic motives for trade, driving-off from the supply side a part of traders with $|\alpha > 0|$, which now get better terms of trade for market orders, thus reducing the probability of getting picked-off.

Table 1: Aggregate traders behavior. *The following table displays different statistics that reveal how traders behave on each cycle. Two rows are shown for each statistic: the upper one is the cross-book measure, while the lower row is the single book one. Cross-book measures are computed considering orders from both books when in overlap, while using a single-book otherwise. This is done to analyze market quality measures from the traders perspective, by using only the information they have available in each period. The probability of being picked-off is computed as the total amount of limit orders that were executed below (above) the fundamental value for sell (buy) orders divided by the total amount of limit orders traded. The time to execution of limit orders is presented in units of time from the model or minutes. All statistics are computed using limit orders submitted, canceled or executed on each of the respective periods.*

	Period Average		Overlap Cycle					
Measure type	NOV	OV	Non-Overlap			Overlap		
			1	2	3	1	2	3
			Prob. of being picked-off after submitting a limit order					
Cross-book	38.25%	28.50%	38.79%	38.23%	37.91%	28.11%	28.55%	28.80%
Single-book		28.54%				28.16%	28.67%	28.67%
			Number of limit orders 'submitted' per trader					
Cross-book	2.23	1.63	1.97	2.24	2.38	1.63	1.67	1.57
Single-book		1.63				1.63	1.66	1.58
			Number of limit order cancellations per trader					
Cross-book	1.72	1.15	1.43	1.74	1.87	1.19	1.17	1.08
Single-book		1.15				1.19	1.16	1.09
			Time between the instant in which a trader arrives and the execution of her limit order					
Cross-book	10.32	9.16	7.22	10.63	10.82	12.19	8.18	7.77
Single-book		9.16				12.16	8.16	7.83
			Prob. of submitting a limit sell order at the ask price (an aggressive order)					
Cross-book	33.05%	42.46%	36.93%	32.85%	31.38%	42.62%	41.95%	43.40%
Single-book		41.61%				41.63%	40.97%	42.92%

Observation 2

i. Liquidity suppliers are hesitant to trade in anticipation of the overlap.

ii. Out of the Non-Overlapping stages, traders' willingness to trade is higher on the opening phase.

Traders submit and cancel an increasing number of limit orders as the overlapping period approaches, statistics that reach their maximal value just before the open of the overlap, when they submit on average 2.38 orders and cancel 1.87 of them. This high rate of order revision, in conjunction with a 31.38% probability of placing an aggressive order, the lowest during the trading day, and a 37.91% probability of being picked-off, the minimum out of the non-overlapping periods, reveals liquidity suppliers not wanting to execute, placing orders away from the quotes while widening the spread, and changing them accordingly when they revisit the market. Hence, they are hesitant to trade.

Execution times for limit orders present a concave shape throughout the trading day, reaching its lower levels at cycles NOV_1 and OV_3, and its longer interval in cycle OV_1. The minimum is reached on cycle NOV_1, driven by two parallel effects; first, the current cycle displays the best terms of transaction out of the non-overlapping ones: There's a 38.79% chance of suppliers being picked-off, the highest of the day, and the probability of them submitting aggressive orders reaches its single-market maximum of 36.93%; second, waiting proves to be harmful for liquidity demanders, since as they get closer to a stable single-market period (NOV_2), suppliers are decreasing it's aggressiveness in 4.08%, which lowers their probability of getting picked off in 0.56%. Moreover, suppliers increase their order revision rate by submitting 0.27 and canceling 0.31 more orders, ultimately increasing the execution time for limit orders in 3.41 units of time, which means liquidity demanders are finding the right market conditions for trade with significantly less frequency.

4.1.2. Trader level

The following results show how traders differentiated by their private valuation behave in each stage of the trading day.

Observation 3

i. Agents with no intrinsic motives for trade act as liquidity suppliers, while agents with $|\alpha| = 8$ act as liquidity demanders.

ii. Agents with a private valuation of 4 act as "hybrid traders": they supply liquidity during the Non-Overlapping phase, while they act as liquidity demanders during the overlap.

Table 2 shows the percent of limit orders that each private value type trader gets executed out of all the orders he executes, and the average time it takes for them to get a limit order executed. Clearly, agents with no intrinsic motives for trade, i.e., $\alpha = 0$, execute consistently more limit than market orders, reaching a minimum of 59.43% in cycle NOV_1 and a maximum of 76.68% in cycle OV_1, thus they are liquidity suppliers most of the time. On the other side, agents with $|\alpha| = 8$ act as liquidity demanders, executing as much as 4 times the number of market than limit orders on cycle OV_1 (21.28% of limits executed), and a minimum of 63.20% of market orders on stage NOV_1. Henceforth, $\alpha = 0$ and $|\alpha| = 8$ traders will be referred to as the main suppliers and demanders of liquidity, respectively.

Interestingly, traders with a private valuation of 4 cannot be easily labeled, since they exhibit a "hybrid" behavior: they execute more limit than market orders during the non-overlapping hours, with percentages of limit orders executed higher than 50.00%, while they turnaround their behavior during the overlapping phase, becoming liquidity demanders through the execution of a higher than 50.00% share of market orders. Moreover, suppliers and demanders role on the market becomes more clearly defined during the overlap, where traders with no intrinsic motives for trade increase their limit orders executed by at least 14.03% (NOV_2 versus OV_2); and traders with $|\alpha| = 8$ increase their market orders executed share by at least 10.77% (NOV_2 versus OV_2).

With regards to execution times for limit orders when supplying liquidity, type 0 agents wait the most in the transition period at the open of the overlap, with 21.42 and 19.57 units of time, and the least at the closing transition with 12.89 and 13.55 minutes. Type 4 agents, which supply liquidity before the open of the overlap, are also willing to wait the most on cycle NOV_3, reaching its' maximum of 6.34 units of time, behavior consistent with suppliers being reluctant to trade in anticipation of the overlap. On the other hand, at the closing phase of the overlap every agents' limit order execution time diminishes to its' day minimum, which suggests cycle OV_3 being a period of greater liquidity demand since suppliers are more aggressive than ever, as seen in table 1, with a 43.40% probability of placing orders at the ask quote.

Table 2: Percentage of limit orders executed and time to execution. *The following table presents the percentage of limit orders that got executed and the average waiting time until a limit order is executed on each cycle, both at the trader and aggregate level. The percentage of limit orders executed is computed as the ratio of total limit orders that a trader sent and got executed divided by the amount of orders he traded (limits plus markets). Hence, it measures the share of limit orders a trader executes out of all the orders executed, and its complement is the shate of market orders sent. The average time until a limit order executes is presented in units of time from the model or minutes. Every statistic is computed considering orders that got executed on the respective period.*

	% Limit orders 'executed' per trader type				Time between the instant in which a trader arrives and the execution of her limit order			
Overlap	Private value |α|				Private value |α|			
Cycle	0	4	8	Total	0	4	8	Total
NOV_1	59.43%	53.80%	36.80%	50.00%	13.55	4.92	2.49	7.22
NOV_2	62.03%	52.93%	34.21%	50.00%	19.94	6.31	2.86	10.63
NOV_3	60.37%	53.90%	35.13%	50.00%	21.42	6.34	2.82	10.82
OV_1	76.68%	46.92%	21.28%	50.00%	19.57	4.82	1.97	12.19
OV_2	76.07%	50.39%	23.44%	50.00%	14.08	3.75	1.74	8.18
OV_3	76.24%	49.35%	23.28%	50.00%	12.89	3.68	1.75	7.77
NOV_{avg}	61.64%	53.11%	34.56%	50.00%	19.53	6.18	2.81	10.32
OV_{avg}	76.29%	49.22%	22.85%	50.00%	15.41	4.00	1.80	9.16

Observation 4

i. Agents with no intrinsic motives to trade get better terms of transaction during the overlap, despite being more aggressive when acting as suppliers.

ii. Traders with $|\alpha| > 0$ pay a higher immediacy cost during the overlap.

Table 3 shows the price of all executed orders and the price of submitted limit sell orders as a measure of traders' aggressiveness, in ticks from the fundamental value. It's clear that during the overlap liquidity suppliers get a greater benefit per trading within 0.11 and 0.20 ticks, albeit they place limit orders between 0.43 and 0.68 ticks closer to the fundamental value. This effect is a consequence of traders executing faster during the overlap, increased competition, and traders with no intrinsic motives for trade supplying liquidity in approximately 76.00% of their executed orders, facing less barriers and competition from $|\alpha| > 0$ traders who, as explained in Observation 1, become more clearly defined as liquidity demanders.

Table 3: Price of executed orders. *The following table presents the price of all executed orders and the price at which limit orders are submitted on each cycle in ticks from the fundamental value, at the trader and aggregate level. Every statistic is computed using limit orders sent, submitted, canceled or executed on each of the respective periods.*

	Price of all executed orders (Benefit per trading)				Price of limit sell orders (order aggressiveness in ticks from the fundamental value)			
Overlap	Private value α				Private value α			
Cycle	0	-4	-8	Total	0	-4	-8	Total
NOV_1	1.03	-0.16	-0.70	0.00	2.81	0.13	-0.97	1.70
NOV_2	0.97	-0.20	-0.72	-0.01	2.82	0.16	-1.13	1.81
NOV_3	1.01	-0.18	-0.70	0.00	2.76	0.16	-0.95	1.81
OV_1	1.17	-0.35	-0.96	0.00	2.14	0.16	-0.86	1.53
OV_2	1.14	-0.23	-0.84	0.00	2.33	0.23	-0.74	1.60
OV_3	1.16	-0.26	-0.87	0.00	2.20	0.20	-0.82	1.48
NOV_{avg}	0.98	-0.19	-0.71	-0.01	2.82	0.16	-1.09	1.80
OV_{avg}	1.16	-0.27	-0.88	0.00	2.25	0.21	-0.79	1.55

When traders with private motives for trade are considered, two effects are seen by comparing the overlap to the non-overlap period: first, they are less aggressive when acting as liquidity suppliers, placing orders further away from the fundamental value within 0 and 0.10 ticks for traders with $\alpha = 4$, and 0.09 to 0.39 ticks for traders with $\alpha = 8$ [13], which is explained by their role as liquidity demanders being enforced during the overlap, thus limit orders become less profitable. Second, the price of all executed orders worsens in at least 0.03 ticks and 0.12 ticks for hybrid traders and type 8 traders during the overlap (NOV_2 versus OV_2). Since they send a higher share of market orders, they execute faster while accepting worse terms of trade, paying a higher immediacy cost and lowering their overall benefit per trading. Moreover, this leads to $\alpha = 0$ traders, main suppliers of liquidity, to substantially increase their gains by facing less competition on the supply side, and reducing the opportunity cost of placing limit orders through faster execution.

Observation 5

i. Liquidity demanders submit fleeting limit orders during the stable single-market phase.

[13] Since they place on average orders below the fundamental value, the closer to the fundamental value the less aggressive they are.

Table 3 shows traders with private valuation $|\alpha| = 8$ to reach their maximum aggressiveness during cycle NOV_2, submitting sell orders -1.13 ticks below the fundamental value, getting almost surely picked-off. Interestingly, their execution time for limit orders also reaches its highest value of 2.86 minutes, which forces them to be more aggressive when acting as suppliers. Moreover, they are forced to submit fleeting limit-orders since hybrid traders are in their least aggressive period of the non-overlap, placing orders 0.16 ticks away from the fundamental value, and type 0 traders are placing buy orders 2.82 ticks lower than the assets true value [14], making demanders look for alternatives to execute as soon as possible while getting better terms of trade, i.e., place marketable limit orders.

Observation 6

i. Liquidity suppliers have the highest risk of being picked-off in the stable single-market period, even though overall a higher share of orders gets picked-off in the opening phase.

ii. Traders with private valuation $|\alpha| = 4$ get picked-off the most, given their "hybrid" character and intrinsic motives for trade.

Table 4 shows picking-off risk statistics for each cycle of the trading day divided by traders' private valuation. The aggregate probability of being picked-off is downward sloping through the non-overlap, being reduced from 38.79% to 37.91%, decreases sharply when the overlap opens to 28.11% and rises steadily until the end of the overlap. In contrast, each individual trader has the highest probability of getting picked-off in cycle NOV_2, where $|\alpha| = 0$ traders supply more liquidity than in cycle NOV_1 by 2.60% more limit orders executed according to Table 2, and agents with a private valuation of 8 are considerably more aggressive by 0.16 ticks respectively as shown in Table 3, while reducing their limit orders executed in 2.59%. This effect is due to a higher proportion of speculators participating in trades in cycle NOV_2, consistent with Table 2.

Table 4 displays that hybrid traders get picked-off approximately on 40.00% of their limit orders throughout the trading day, nonetheless, as explained on Observation 3, given their hybrid character they supply liquidity in approximately half of their orders, and their private valuation for the asset allows them to place them between 0.13 and 0.20 ticks from the fundamental value [15], making them extremely vulnerable to being picked-off when the assets value moves in an unfavorable direction. Therefore, their share of the picked-off probability is extremely high, reaching a maximum of 21.86% in phase NOV_1 out of the 38.79% available, and a minimum of 15.64% out of the 28.55% available in cycle OV_2. What's more, hybrid traders represent more than 50.00% of the picked-off orders in every stage of the day.

[14] Since the model is symmetric, results for sell orders also apply for buy orders.
[15] The instantaneous utility function is $u = d(\alpha + v_{t_e} - p)$, where $d = 1$ for a buy order and -1 otherwise. Given the same transaction terms, having $|\alpha| > 0$ increases the expected utility of an agent compared to $\alpha = 0$, allowing them to place more aggressive orders.

*Table 4: **Picking-off probability statistics.*** *The following table displays the probability of being picked-off both at the trader and aggregate level, and each types' of trader share on the aggregate picked-off probability. The probability of being picked-off is computed by dividing all limit sell (buy) orders that were executed bellow (above) the assets fundamental value, by the total amount of limit orders that were traded. Every statistic is computed using limit orders and traders that got executed on the respective period.*

	Prob. of being picked-off after submitting a limit order				Picked-off probability share by trader type			
Overlap	Private value |α|				Private value |α|			
Cycle	0	4	8	Total	0	4	8	Total
NOV_1	10.62%	42.45%	72.07%	38.79%	5.80%	21.86%	11.14%	38.79%
NOV_2	12.06%	43.31%	74.59%	38.23%	4.97%	20.99%	12.27%	38.23%
NOV_3	11.09%	42.80%	71.18%	37.91%	5.11%	20.77%	12.03%	37.91%
OV_1	8.08%	43.80%	69.16%	28.11%	4.22%	15.78%	8.10%	28.11%
OV_2	8.21%	38.88%	65.01%	28.55%	3.75%	15.64%	9.16%	28.55%
OV_3	8.11%	40.38%	67.17%	28.80%	3.83%	15.75%	9.22%	28.80%
NOV_{avg}	11.85%	43.18%	73.98%	38.25%	4.30%	18.45%	15.49%	38.25%
OV_{avg}	8.15%	40.48%	66.54%	28.50%	3.90%	15.71%	8.89%	28.50%

Observation 7

i. Liquidity suppliers ($\alpha = 0$) exhibit queue-jumping behavior.

Table 5 shows the amount of orders that were switched from one book to the other out of all the orders made each period. Queue-jumping behavior emerges as a benefit of fragmentation, as in Foucault and Menkveld (2008), and reaches its peak in the stable period of the overlap, with 2.53% of the orders being switched from one book to the other. Main liquidity suppliers are the ones who queue-jump the most, consistent with Foucault and Menkveld (2008), followed by hybrid traders and main liquidity demanders as would be expected, since most of the benefits offered by jumping the queue are enjoyed by suppliers of limit orders.

*Table 5: **Queue-jumping behavior.*** *The following table shows the percentage of orders that got switched from one book to the other during the overlapping period. The statistic is computed by dividing the amount of limit orders that were canceled from one book and then submitted to the alternative book, by the total amount of orders sent that period.*

	Percentage of orders switched from one to another market			
Overlap	Private value |α|			
Cycle	0	4	8	Total
NOV_1	0.00%	0.00%	0.00%	0.00%
NOV_2	0.00%	0.00%	0.00%	0.00%
NOV_3	0.00%	0.00%	0.00%	0.00%
OV_1	0.79%	0.34%	0.19%	0.02%
OV_2	1.42%	0.72%	0.39%	0.06%
OV_3	0.64%	0.34%	0.18%	0.01%
NOV_{avg}	0.00%	0.00%	0.00%	0.00%
OV_{avg}	2.84%	1.40%	0.75%	0.25%

4.2. WELFARE

Welfare is computed as the instantaneous payoff a trader with private valuation α receives at execution time t', discounted back to his arrival time t. Remember this discount factor represents an opportunity cost and not the time value of money. Let Π be the discounted payoff:

$$\Pi = d(\alpha + v_{t'} - \bar{p})e^{-\rho(t'-t)}$$

Where $d = 1$ if it's a buy order and $d = -1$ otherwise, $v_{t'}$ is the fundamental value at execution time and $\bar{p}$ is the transaction price.

Observation 8.

i. A multimarket scenario is welfare improving when only stable periods are considered.

ii. Main suppliers of liquidity achieve their maximum welfare in the pre-closing phase of the overlap.

Table 6 presents welfare divided by traders' private valuation and globally for each of the overlapping stages. Global welfare in cycles NOV_2 and OV_2 is 3.61 and 3.68 respectively, making the stable multimarket scenario welfare improving by 0.27 units compared to the stable single-market one. Furthermore, cycle OV_2 is preferred by main suppliers and hybrid traders to NOV_2, since it improves their welfare in 0.12 ticks. Liquidity suppliers are better off since agents with $|\alpha| > 0$ become more clearly defined as demanders, as described in Observation 3, removing competition and barriers for liquidity supply. On the other hand, hybrid traders enjoy lower waiting times until adequate conditions to execute, as described in Observation 1, sustained by higher competition from suppliers according to Observation 4. Main liquidity demanders slightly prefer cycle NOV_2 to OV_2 by 0.01 ticks [16].

The main suppliers of liquidity achieve their maximum welfare of 0.76 units 30 minutes before the end of the overlapping period. On cycle OV_3, Table 1 shows the order revision rate is at its day lowest value, with an average of 1.57 orders submitted and 1.08 orders canceled. At the same time, $\alpha = 0$ traders are supplying liquidity near their top, with a 76.24% of limit orders executed on 12.89 units of time on average, faster than in any other stage of the day according to Table 2. Likewise, data from Table 3 shows they are near their maximum benefit per trading, with 1.16 units and close to their maximal aggression, placing orders only 2.20 ticks away from the assets true value. Finally, Table 4 shows liquidity suppliers are near its minimum probability of getting picked-off, with 8.11%. Therefore, cycle OV_3 its characterized by better transaction terms than ever, only comparable to those of cycle OV_1. Nonetheless, they are allowed to be less aggressive

[16] More precisely, the difference is only 0.005 ticks. Table 6 shows 0.01 ticks due to rounding errors.

by 0.06 ticks than at the opening phase of the overlap while enjoying a significant reduction in their execution time of 6.68 minutes, thus discounting their benefit with considerable less strength.

Table 6: Welfare. *This table shows agents welfare both at the trader and aggregate level on each pahse of the trading day in ticks. Welfare is computed as the average discounted instantaneous payoff of orders executed on the respective period.*

Overlap Cycle	Average payoff per trader			
	Private value |α|			
	0	4	8	Total
NOV_1	0.69	3.37	6.97	3.76
NOV_2	0.61	3.29	7.00	3.61
NOV_3	0.61	3.28	6.96	3.65
OV_1	0.67	3.26	6.88	3.37
OV_2	0.73	3.41	6.99	3.68
OV_3	0.76	3.40	6.96	3.63
NOV_{avg}	0.61	3.30	6.99	3.63
OV_{avg}	0.72	3.37	6.96	3.59

Let $d = 1$ (buy order) for ease of exposition. Then, following Bernales and Daoud (2013), welfare can be decomposed as:

$$\Pi = \alpha + \alpha\left(e^{-\rho(t'-t)} - 1\right) + (v_{t'} - \bar{p})e^{-\rho(t'-t)}$$

Where $\alpha\left(e^{-\rho(t'-t)} - 1\right)$ is defined as the waiting cost incurred by the limit-order trader in the transaction, and $(v_{t'} - \bar{p})e^{-\rho(t'-t)}$ is the money transfer between agents.

Observation 9

i. During the stable and closing phase of the overlap, traders with $|\alpha| > 0$ get worse terms of trade, although drastically reducing their waiting cost.

ii. The open of the overlap is the worst period for demanders of liquidity.

Table 7 shows welfare decomposed in waiting cost and money transfer by traders' private valuation. Hybrid traders and main liquidity demanders get money transfers at least 0.07 and 0.18 ticks worse respectively comparing stable periods, gap which increases when opening and closing periods are compared. Nonetheless, traders with $|\alpha| = 4$ increase their overall welfare during the stable and closing phase of the overlap compared to cycles 1 through 4: even though they get worse transaction prices, they're able to execute much faster, reducing their waiting cost in at least 0.09 ticks. Main liquidity demanders cut their waiting cost in half during the overlap.

Table 7: ***Welfare decomposed by waiting cost and money transfer.*** *The following table displays welfare decomposed in the waiting cost incurred by traders and the money transfers made on each stage of the trading day in ticks, both at the trader and aggregate level. Every statistic is computed using limit orders and traders that got executed on the respective period.*

	Waiting cost per trader				Money transfer per trader			
Overlap	Private value |α|				Private value |α|			
Cycle	0	4	8	Total	0	4	8	Total
NOV_1	0.00	-0.50	-0.38	-0.32	0.69	-0.13	-0.64	-0.06
NOV_2	0.00	-0.57	-0.36	-0.34	0.61	-0.14	-0.64	-0.07
NOV_3	0.00	-0.59	-0.39	-0.36	0.61	-0.13	-0.65	-0.08
OV_1	0.00	-0.41	-0.19	-0.21	0.67	-0.32	-0.93	-0.15
OV_2	0.00	-0.38	-0.19	-0.21	0.73	-0.21	-0.82	-0.11
OV_3	0.00	-0.36	-0.19	-0.20	0.76	-0.24	-0.85	-0.11
NOV_{avg}	0.00	-0.56	-0.37	-0.34	0.61	-0.14	-0.64	-0.07
OV_{avg}	0.00	-0.38	-0.19	-0.21	0.72	-0.25	-0.85	-0.12

The opening phase of the overlap presents an interesting case during the trading day: main liquidity demanders and hybrid traders, which are now demanding more liquidity than ever as described in Table 1, get the worst money transfers of the whole day, -0.32 for $|\alpha| = 4$ traders and -0.93 for $|\alpha| = 8$ agents, while at the same time cutting their waiting cost in at least 0.07and 0.17 ticks respectively compared to the non-overlap. Interestingly, main liquidity demanders reduce their waiting cost as much as in the other stages of the overlap; consequently, their welfare reduction is driven by the low money transfers, i.e., they are getting the worst terms of trade of the day. This welfare is captured by liquidity suppliers, who are better off than on periods NOV_2 and NOV_3. It's noteworthy that even though the transaction terms liquidity suppliers get during cycle 4 are the best possible, substantially higher execution times for limit orders, as seen in Table 1 and Observation 8, dramatically increase the opportunity cost they face, heavily discounting their gains.

4.3. MARKET QUALITY

In the following paragraphs, relevant market quality measures are computed for each overlapping stage.

Observation 10

i. The overlapping period is beneficial to global liquidity, although detrimental to local depth.

ii. Trading concentrates in the overlapping hours.

Table 8 shows market quality measures for each cycle, both as single-book and cross-book measures. Through the overlapping period, depth at the ask rises to 1.21 orders at the opening phase, and remains slightly lower than the non-overlapping period on stages OV_2 and OV_3 (by 0.02 shares), nonetheless, depth on the sell side of the book increases substantially, staying within 2.58 and 3.11 outstanding orders, compared to 2.18 and 2.60 for the non-overlap period, since now traders have access to both pockets of liquidity. Therefore, demanders access to liquidity improves globally. When only one book is considered during the overlap, the depth at the ask is the same as when both books are considered, yet depth available on the sell side decreases substantially, from 1.96 to 1.60 orders at the end of the overlap, hence, local liquidity shrinks.

Table 8: Market quality measures. *The following table shows market quality measures for each cycle of the day. Two rows are shown for each statistic: the upper one is the cross-book measure, while the lower row is the single book one. Cross-book measures are computed considering orders from both books when in overlap, while using a single-book otherwise. This is done to analyze market quality measures from the traders perspective, by using only the information they have available in each period. The cross-book measures of the spread considers the best ask and bid availables from both books and are presented in ticks The number of effectively traded orders at the ask is computed as the average depth at the ask multiplied by the ratio of the total orders that were sent at the ask quote and not canceled, to the total amount of orders sent at the ask quote. The number of effectively traded orders on the sell side is computed as the average sell side depth multiplied by the ratio of total limit orders traded to the total amount of limit orders submitted. Depths and the number of effectively traded orders are presented in shares, while execution time in units of time from the model or minutes. Every statistic is computed using limit orders and traders that got executed on the respective period.*

	Period Average		Overlap Cycle					
Measure type	NOV	OV	Non-Overlap			Overlap		
			1	2	3	1	2	3
			Bid-ask spread					
Cross-book	2.39	2.51	2.43	2.37	2.50	2.56	2.48	2.53
Single-book		3.15				3.22	3.13	3.09
			Effective spread					
Cross-book	1.87	2.03	1.83	1.88	1.85	2.10	1.99	2.05
Single-book		2.51				2.59	2.48	2.48
			N. of limit orders at the ask					
Cross-book	1.14	1.15	1.12	1.15	1.15	1.21	1.13	1.13
Single-book		1.15				1.21	1.13	1.13
			N. of limit orders at the ask (effectively traded)					
Cross-book	0.38	0.43	0.41	0.37	0.38	0.44	0.43	0.44
Single-book		0.44				0.44	0.43	0.44
			N. of limit orders on the sell side of the book					
Cross-book	2.46	2.81	2.18	2.47	2.60	3.11	2.75	2.58
Single-book		1.80				1.96	1.77	1.70
			N. of limit orders on the sell side of the book (effectively traded)					
Cross-book	0.55	0.88	0.50	0.56	0.54	1.07	0.82	0.83
Single-book		0.57				0.67	0.53	0.55
			Time between the instant in which a trader arrives and her execution					
Cross-book	6.93	5.73	5.04	7.09	7.48	7.65	5.12	4.84
Single-book		5.73				7.64	5.11	4.87

With regards to trading activity, the number of limit orders effectively traded at the ask increases amidst 0.02 and 0.07 orders, while on the sell side the increment is substantial in the opening phase of the overlap, reaching 1.07 orders, and remaining considerably higher than in non-overlap periods on stages OV_2 and OV_3, with an average of 0.84 orders. In addition, the overlapping hours align both liquidity suppliers and demanders incentives to trade: Spread increases to at least 2.48 ticks, more importantly, effective spread increases in at least 0.11 ticks (NOV_2 versus OV_2), so suppliers are getting substantially better prices. On the other hand, demanders are getting increased liquidity available and better execution times, as revealed by the waiting cost analyzed in Observation 9.

Therefore, trading intensifies during the overlapping period, consistent with the findings of Chelley-Steeley et al. (2015) and Menkveld (2008).

Observation 11

i. Liquidity is highly demanded after the opening and before the closing of the overlap.

ii. $\alpha = 0$ agents take advantage of the opening transition of the overlap by loading-up the book with liquidity away from the bid and ask quotes.

iii. $\alpha = 0$ agents take advantage of the closing transition of the overlap by increasing their aggressiveness.

At the opening phase of the overlap, the number of limit orders effectively traded on the sell side of the book almost doubles, rising from 0.54 to 1.07 orders, and the number of limit orders traded at the ask increases by 0.06 orders. In addition, the effective spread reaches its maximal of 2.10 ticks, and as shown by Tables 2 and 3, $\alpha = 0$ agents are supplying more liquidity than ever, with 76.68% of limit orders executed on an average of 19.57 units of time, getting in fact the best prices of the day. Therefore, even though demanders are getting the worst terms of trade as described in observation 9, they drive a lot of transactions which allows for an increase in the spread and the amount of orders effectively traded, ultimately decreasing global depth to 2.75 shares on average on cycle OV_2. Thereupon, a substantial rise in the demanded quantity is said to happen at the opening stage of the overlap (cycle OV_1), mainly driven by suppliers of liquidity becoming 0.62 ticks more aggressive as shown in Table 3.

On the other hand, the closing phase of the overlap, namely, cycle OV_3, shows similar characteristics: the effective spread rises to 1.98 ticks from 1.95 in stage OV_2, both the number of orders effectively traded at and away from the ask slightly rise in 0.01 orders, and the depth available on the sell side of the book decreases from 2.84 to 2.58shares during the same period, so liquidity is getting consumed. More importantly, the average time between the arrival of a trader and his execution, whether via a limit or market order, reaches its lowest value of 4.84 units of time, and also suppliers of liquidity, divided by private valuation, are executing faster than ever as shown by Table 2, with $\alpha = 0$ traders executing limit orders in 12.89 minutes, $|\alpha| = 4$ in 3.68 and $|\alpha| = 8$ in 1.75 minutes.

Likewise, it’s important to note that Table 1 shows the aggregate execution time for limit orders reaches its minimal in cycle NOV_1, not in OV_3, which is explained by agents with intrinsic motives for trade supplying more liquidity during the open of the non-overlap, which diminishes aggregate limit order execution times. Therefore, a substantially higher amount of market orders is being executed on cycle OV_3 compared to the rest of the trading day, and the ending phase of the overlap is also characterized by a substantial rise in the quantity demanded, though this time driven by both suppliers and demanders, since they realize waiting to cycle NOV_1 decreases their welfare in 0.07 ticks for $\alpha = 0$ agents and 0.02 ticks for $|\alpha| = 4$ traders as shown by table 6, and the amount of liquidity available to trade decreases in 0.40 orders, almost doubling the waiting cost for traders with private motives for trade as shown by Table 7.

An important behavior from suppliers emerges when transition periods are analyzed. In the opening transition of the overlap, liquidity suppliers wait the most to execute their limit orders, 10.82 and 12.19 units of time as shown by Table 1. Even more important, Observation 2 states liquidity suppliers are hesitant to trade in anticipation of the overlap, and Table 8 sheds light on the motives behind this behavior. In fact, liquidity suppliers know the non-overlap it's coming to an end, and anticipate that in at most 30 units of time they'll be able to get a 2.10 effective spread, and that a positive demand shock can be triggered by increasing their aggressiveness.

Moreover, the main drivers of liquidity supply wait 21.42 units of time before executing their orders previous to the open of the overlap according to table 3. They don't want to get executed, and they revise their orders more than ever as described by Table 1, placing orders 2.76 ticks away from the fundamental value according to Table 4. Therefore, type 0 agents load-up the book with liquidity away from the quotes, waiting for the overlap to open, which drives sell side depth up to its maximum of 2.60 shares, forcing agents with intrinsic motives for trade into supplying limit orders. In OV_1, the expected time until execution of $\alpha = 0$ limit orders averages 19.57 seconds, which is explained by most of the liquidity available being outstanding orders from cycle NOV_3 still present on phase OV_1, which must be revised before being able to execute. This fact makes cycle OV_1 heavily discount its payoffs, thus pareto dominated by cycle OV_3, where every traders' incentives are aligned.

With regards to the closing phase of the overlap, agents with no intrinsic motives for trade increase their aggressiveness from 2.33 ticks in cycle OV_2 to 2.20 ticks as shown by Table 3, encouraging liquidity demanders. Nonetheless, they don't have to be as aggressive as in cycle OV_1, since the sharp rise in shares demanded is enhanced by liquidity demanders fear of getting stuck on the less liquid non-overlapping period, which has 0.40 less outstanding orders on the sell side and at least a 0.20 minute higher time to exit the market. In fact, market orders are so frequent that execution times for every trader reach 4.99 units of time, the fastest of the trading day.

Observation 12

i. Limit order books are hump-shaped in anticipation of the overlap.

ii. Fleeting limit orders are submitted by $|\alpha| = 8$ agents when the order book is full.

Two interesting facts lay in cycles NOV_2 and NOV_3 which are consistent with Rosu (2009). First, Table 8 shows that in anticipation of the overlap (cycle NOV_3), the number of limit orders at the ask is 1.15, while the depth on the sell side reaches 2.60, which leaves 1.45 shares outstanding away from the ask quote. Therefore, the book presents a prominently hump-shape, with liquidity clustering away from the quotes as described in observation 11. According to Rosu (2009), this happens if suppliers of liquidity expect a higher probability or large orders in the near future, which is similar to our current scenario: liquidity suppliers expect a rise in the demanded quantity, and hence a higher probability of executing at better prices in the next period (OV_1).

Cycle NOV_2 on Table 8 shows the book in a stable single-market period being with 2.47 outstanding shares on the sell side on average, almost at its maximum total depth of 2.60. What's more, being a stable period, there's no certainty the next period is coming, and liquidity suppliers are not being aggressive, as shown by Table 3 with $|\alpha| = 0$ traders placing orders 2.82 ticks away from the fundamental value, and hybrid traders 0.16 ticks. Therefore, similar to what's described by Rosu (2009), being the book full of liquidity and with unfavorable trading conditions, main demanders of liquidity are forced to place limit sell orders on average 1.13 ticks under the fundamental value, to execute almost instantly without submitting a market order at such unfavorable prices, i.e., submitting fleeting limit orders.

It's relevant to consider Rosu (2009) first prediction in light of the overlapping transition dynamic. According to Rosu (2009), an increase in competition should lead to liquidity suppliers being more aggressive, and higher trading activity to faster limit order execution, effects which reward demanders with lower spreads. Nonetheless, our scenario behaves differently when all the described conditions are met. As the overlap opens, competition, aggressiveness and trading activity increase dramatically, with execution times becoming faster than ever, as described in Observations 1 and 10. As no asymmetric information exists, this corresponds to a considerably more liquid market than the one present at the non-overlapping phase under the definition proposed by Rosu (2009), hence smaller spreads are to be expected.

Intriguingly, as shown by Table 8, the bid-ask spread rises to values within 2.48 and 2.56 ticks, and the effective spread increases substantially, from within 1.83 to 1.88 ticks up to 1.99 to 2.10 ticks during the overlap. This is due to our agents being heterogeneous not only in their waiting cost, as in Rosu (2009), but also in their private valuation for the asset, which explicitly enters their expected utility. Therefore, even though the market becomes more competitive, a regime-shift in liquidity supply is the main driving force behind increased spreads, as explained in Observation 3.

Observation 13

i. Multimarket scenarios are less informationally efficient than single-market ones.

ii. Price deviations from the fundamental value are greater and noisier at the opening and closing stages of the overlapping period.

Table 9 shows microstructure noise and its standard deviation. It's clear that microstructure noise increases substantially during the overlapping hours, rising from 1.11 ticks up to 1.18 to 1.24 ticks, given the increased demand for liquidity and wider prices of transactions as reflected on the effective spread. Therefore, trade prices pack less information about the true value of the asset, being less informationally efficient. On the other hand, microstructure noise becomes even 'noisier' during the overlapping hours, increasing to at least 1.52 ticks from 1.45 ticks during the non-overlapping period, given the higher amount of transaction prices that are seen in the market. As expected, microstructure noise is higher and has greater standard deviation when there's positive liquidity demand shocks, at the opening and ending phases of the overlap.

Table 9: Microstructure noise. *The following table shows the average microstructure noise and its standard deviation for each period. Two rows are shown for each statistic: the upper one is the cross-book measure, while the lower row is the single book one. Cross-book measures are computed considering orders from both books when in overlap, while using a single-book otherwise. This is done to analyze market quality measures from the traders perspective, by using only the information they have available in each period. Every statistic is computed using limit orders and traders that got executed on the respective period.*

	Period Average		Overlap Cycle					
Measure type	NOV	OV	Non-Overlap			Overlap		
			1	2	3	1	2	3
			Microstructure noise: Mean $\lvert v_t - p_t \rvert$					
Cross-book	1.11	1.20	1.11	1.11	1.11	1.24	1.18	1.20
Single-book		1.20				1.24	1.18	1.20
			Microstructure noise: Std. Dev. $(v_t - p_t)$					
Cross-book	1.45	1.54	1.45	1.45	1.45	1.58	1.52	1.54
Single-book		1.54				1.58	1.52	1.54

5. CONCLUSION

As more firms cross-list their shares and more traders have access to technology, investors who are able to trade in both markets have additional decisions to make: whether to hold their trades until the foreign market opens, and on which market to trade if it's overlapping hours. These decisions become crucially important since they are the drivers of agents' behavior, and ultimately determine efficiency and quality of the markets involved during the trading day.

Traders behavior varies substantially between the overlap and non-overlap periods. Liquidity suppliers are considerably more aggressive during the overlapping hours along with limit orders executing faster due to increased competition. During the overlap, demanders of liquidity are willing to accept worse prices in exchange of exiting the market earlier. Moreover, the overlap represents a regime-shift in liquidity supply, where agents with no-intrinsic motives for trade supply most of the liquidity, while traders who have private motives to trade become clear demanders of shares.

A multimarket scenario is welfare improving compared to a stable single-market one. Main liquidity demanders accept considerably worse terms of transaction during the overlap, however, they cut their waiting cost in half. The first 30 minutes of the overlap are characterized by liquidity demanders getting the worst prices of the day, which are enjoyed by suppliers. Nonetheless, execution times are close to being the highest of the day, hence suppliers get their benefits heavily discounted and actually reach their maximal welfare in the closing period of the overlap.

The overlap substantially improves global liquidity, being characterized as a period of higher trading activity and the lowest aggregate execution times. Demand for shares rises at the opening and closing phases of the overlap, which markedly benefits main suppliers of liquidity. Indeed, suppliers load-up the book with liquidity in anticipation of the overlap open, expecting the best effective spread of the day. On the closing phase, liquidity is consumed faster than ever and the effective spread slightly improves, hence, execution times are at the minimum of the day and every traders' incentives are aligned: suppliers get executed at nearly the best prices at the same time demanders are able to find liquidity and the right conditions for trade almost immediately.

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7. APPENDIX

7.1. APPENDIX A

1. Trading game

The trading game is based on Goettler et al. (2009). The type of a trader is given by $\theta = (\rho, \alpha, u)$, where ρ is the discount rate that represents the opportunity cost of the agent, α is its private valuation for the asset and u a binary variable that represents the information about the fundamental value of the asset available to the agent. If $u = 1$, the agent is considered to be uninformed, and sees the fundamental value with a lag of Δt periods, i.e., $v_{\mathrm{t}-\Delta t}$. On the other hand, if $u = 0$, the agent knows exactly the current fundamental value of the asset and is considered to be an informed trader.

When a trader enters the market at time t, whether it's his first time or not, he has to take an action $a = (p, q, d, m)$, where p denotes the price of his order, $q \geq 0$ his priority amongst orders at price p, d is an indicator variable which takes the value of 1 if the agent places a buy order, -1 for a sell and 0 if he decides to not place an order, and m is a binary variable which indicates if the trader wants to place the order on book 1 or 2. Note that if the market is in a non-overlapping stage, traders are forced to trade on a designed book and m becomes fixed, so the action only consists of $a = (p, q, d)$. For ease of exposition, m will be included in the notation without care for the period of the trading day traders are in. If in a single-market setup, simply consider m as an exogenously given parameter included in the traders' type set.

Recall that L_m denotes a limit order book, and therefore q is completely determined by (p, x, m). Nonetheless, his priority in a given price moves dynamically with the book and may change before he revisits the market. Hence, it becomes relevant as it determines the continuation payoff on reentry. If $x = 0$, meaning the trader did not place an order, the triplet (p, q, m) becomes irrelevant.

As described, market orders are distinguished from limit orders via the price priority rule. If on the opposite side of book m there's an outstanding order at price p or better, the limit order sent at price p in that market executes immediately at the best price available and is called a market order. Hence, if A_m and B_m are the ask and bid quotes of market m and a buy order is sent with $p \geq A_m$, it will execute at A_m, and vice versa for sell orders. If there's no outstanding order on the other side of the book at price p, the order is included in the book according to price and time priority. Therefore, recalling, with a slight abuse of notation, that l_m^p represents the depth at price p:

$$q(x, p, m) = \begin{cases} 0 \; if \; a \; market \; or \; no \; order \; is \; submitted \\ \left| l_m^p + x \right| \; if \; a \; limit \; order \; is \; submitted \end{cases}$$

When an agent enters the market at time t, he observes state $s(\theta)$, which includes all of its private information contained in θ, the history of play in the game as revealed by the book and the last transaction made in the market, and if the trader is revisiting the market, the state also includes his last action taken a'.

Let $\Omega(s)$ be the set of feasible actions a trader can take in state s. For computational tractability, the actions are restricted to prices above and below k ticks from the fundamental value, value which is chosen to be sufficiently large as to not influence the equilibrium of the game. Since uninformed agents might exist in the market, denote their current expectation of the fundamental value as $\bar{v}(s) = E(v|s)$. Then

$$\Omega(s) = \{(p, q, x, m) | x \in \{-1,0,1\} \wedge q = q(x, p, m) \wedge q \neq 0 \Rightarrow p \in [\bar{v}(s) - k, \bar{v}(s) + k] \cap P_m \}$$

Where P_m is the set of feasible prices on book m. Let $S(\theta)$ be the set of feasible states a type θ trader might encounter. Then, a mixed strategy for him is a map $\sigma(\theta): S(\theta) \to \Pi_{s \in S(\theta)} \Delta(\Omega(s))$, where $\Delta(\Omega(s))$ is the set of probability distributions over $\Omega(s)$.

Consider a type θ trader in state s. Each action of the feasible set $\Omega(s)$ that corresponds to a limit order gives rise to an expected payoff which consists of two components: first, a payoff conditional on the trader getting executed before he is able to reenter the market, and second, a value associated with his order not executing, allowing him to revisit the market in a new state s'. Then, the likelihood of the order being executed becomes relevant.

The probability of the order being executed depends on the strategies followed by other traders. Since only symmetric equilibria are considered, strategy σ is followed by every other player. Normalize the traders' entry time to the market to zero. Then, let $\phi(\tau, v|s, a, \sigma)$ be the probability that an action a taken in state s at time zero leads to execution at time $\tau > 0$ when the assets true value is v, given that all other agents play σ, and let $f(v|s, \tau)$ be the density function over v at time τ, given state s, which includes the traders beliefs if he is uninformed.

Consider $w > 0$ the reentry time of the agent. His expected payoff due to execution before he is able to revisit the market, at time $t < w$, is his discounted instantaneous utility to time zero, multiplied by the probability of executing at time t and the possible values the fundamental value may take at time t, given its direct influence in the realized payoff. Therefore, the expected payoff given execution before reentry is

$$\pi(s, a, w, \sigma) = \int_0^w \int_{-\infty}^{\infty} \left(e^{-\rho t} x(\alpha + v_t - p) \phi(t, v|s, a, \sigma)\right) f(v|s, t) dv dt$$

For a market order, $\phi(\cdot) = 1$, and $f(\cdot) = 1$ only if the agent is informed, i.e., $u = 0$. This allows us to define the continuation payoff of an action, and ultimately the Bellman equation for the agents' problem. Consider $G(\cdot)$ the probability distribution of the reentry time, and let

$\gamma(s'|s,a,w,\sigma)$ denote the probability of observing state s' on reentry, given the previous state s, last action taken a, elapsed time w since entry and players using strategy σ. Finally, let $J(s)$ denote the value of an agent of being in state s. Hence, the Bellman equation for the agents' problem is defined as:

$$J(s,\theta) = \max_{a\in\Omega(s)} \int_0^{\infty} \left\{ \pi(s,a,w,\sigma) + e^{-\rho w} \int_{s'\in S} J(s',\sigma)\, \gamma(s'|s,a,w,\sigma) ds' \right\} dG(w)$$

As defined previously, $\pi(\cdot)$ represents the expected payoff of executing before reentering the market. Therefore, the term on the right represents the continuation payoff from revisiting the market. In the algorithm, once a trader executes, its payoff is realized and the trader leaves the market forever, hence no continuation payoff exists.

As described in the model, when a trader reenters the market he is allowed to resubmit his order. If he sent a limit order a when he visited the market last time on state s, which hasn't been executed, the trader is able to cancel it and place a new order a'. The agent might also leave his order as it is, sending $a' = a$, since his last action might have evolved during the time he was absent from the market, improving its price and time priority through the actions of other peers. Hence, the action a taken at state s evolves to action a^* at the time of reentry w, which is captured by the outermost integral of the Bellman equation.

Given the action set $\Omega(s)$ is finite, the maximum over all feasible actions exists and is well defined. Therefore, fixing the strategies of other agents, a strategy $\delta^*(\theta)$ for a type θ trader is a best response if and only if for every possible state $s \in S(\theta)$ it holds that

$$\delta^*(\theta) \in \arg\max_{a\in\Omega(s)} \int_0^{\infty} \left\{ \pi(s,a,w,\sigma) + e^{-\rho w} \int_{s'\in S} J(s',\sigma)\, \gamma(s'|s,a,w,\sigma) ds' \right\} dG(w)$$

Note that the trader is required to act optimally in every feasible state, even though it might not be encountered in equilibrium. Also, as described by the Bellman equation, the trader takes into consideration the possibility of reentering the market on a new state s', in which an optimal action will be taken.

Finally, the equilibrium can be defined. A strategy $\delta^*(\theta)$ is a Markov perfect Bayesian Equilibrium of the described trading game if for each possible type (θ), $\delta^*(\theta)$ is a best response function in every feasible state $s \in S(\theta)$, given all other agents also using strategy $\delta^*(\theta)$.

2. Details of the numerical algorithm

To find the value of a given state $J(s)$ which satisfies the proposed Bellman equation, we use an iterative procedure based on Pakes and McGuire (2001). For that, the state-space needs to be reduced to keep the model computationally manageable.

We would like traders to condition their decisions on the complete history of the game, and since the fundamental value follows a random walk, the set of prices at which traders might trade is unbounded. Nonetheless, given the instantaneous utility defined as $u = d(\alpha + v_t - p)$, a trader only cares about the relative price with respect to the fundamental value, namely, $(v_t - p)$. Therefore, historical prices and lagged values of v_t can be expressed relative to the current fundamental value, reducing the state-space considerably up to the point where the set of recurrent states is finite [17].

Let m_t be the market conditions an agent type $\theta = (\rho, \alpha, u)$ observes at time t. Then, market conditions are defined as

$$m_t(u) = \{L_{1,t}, L_{2,t}, v_u, T\}$$

Where $L_{m,t}$ represents the information of each book according to their transparency, v_i is their information about fundamental value, with $u = t$ if the agent is informed and $u = t - \Delta t$ otherwise, and T contains the price, direction and book of execution of the last transaction made. Note that given the dynamic of the overlap, the information available for a trader changes throughout the day. In a single-market setup, only the information and last transaction of one of the books can be seen, while during the overlap they can see both books and the most recent transaction out of both books last trade.

To further reduce the state space, information from books L_m that can be seen by traders is limited, as described in the numerical parametrization of the game, to agents seeing bid and ask quotes, depths at these prices, and the total amount of outstanding orders on each side of the book, i.e., sell and buy side depths.

Since m_t is defined as the market observables, the state s_t the trader sees at time t is

$$s_t = \{\theta, m_t(u), a\}$$

[17] Recurrent states meet three conditions: first, the system always reaches the set of recurrent states at some point, second, once in the set of recurrent states, the probability of transitioning to a state outside of the recurrent set is null; and third, each state in the recurrent set of states is visited infinitely as t goes to infinity.

Where $a = (p, q, d, m)$ includes the current state of his last transaction made.

At time t, each action a taken in each state s found throughout the simulation has an associated real number $U_t(a|s)$ which represents the expected discounted payoff of taking action a on state s. Therefore, $U(\cdot)$ is the current belief of an agent visiting state s about the payoff of action a. Then, given current beliefs for every $a \in \Omega(s)$, the action a^* such that

$$a^* \in \arg\max_{a \in \Omega(s)} U_t(a|s)$$

Is the optimal action in state s and determines the value of state s: $J(s) = U_t(a^*(s)|s)$. Every optimal action taken in every possible state defines the optimal strategy profile y_t^*. The initial value for beliefs is set arbitrarily and its only relevance is to allow for faster convergence.

In the next paragraphs, the inner workings of the algorithm are explicitly described.

The simulation is driven by three possible exogenous events: a new trader arrival, an old trader who hasn't executed revisiting the market, and a change in the fundamental value. A time until each event triggers is assigned for each case. Let t_v denote the remaining time until the fundamental value changes, t_n the remaining time until a new trader arrives and t_r a vector containing all the remaining times for each old trader to revisit the market. From t_r, only the trader who's closer to visiting the market needs to be considered, therefore, let $\bar{t}_r = \min\{t_r\}$.

Therefore, the next exogenous event happens at instant $t_e = \min\{t_n, \bar{t}_r, t_v\}$, where three things happen with regards to time: first, a global time counter from the algorithm t is updated to the value $(t + t_e)$, second, times until the remaining events happen are updated by subtracting t_e to each one of them, including every outstanding agents' waiting time for reentry to the market, and third, a new value for the remaining time until the triggered event happens again is drawn from the corresponding Poisson distribution.

If the triggered event is a change in the assets fundamental value, then v is increased or decreased in k ticks with equal probability (0.5). Since all prices are set relative to the fundamental value to greatly reduce the state-space, both books need to be updated to reflect the new price of the assets true value and how market conditions changed.

If the triggered event is the arrival of a new trader, he observes the state s he's in as defined previously, and chooses the expected payoff maximizing action based on his beliefs $U_t(\cdot)$. If an older trader revisits the market and didn't send any order previously, the problem he faces is identical to a new trader. If he has an outstanding limit order a, he is allowed to cancel it and resubmit a new order a'. To do this, he compares his beliefs about the expected payoff of leaving the order unchanged $U_t(a|s)$, to the expected payoff off canceling the order and sending a new order a', $U_t(a'|s')$. Note that when he evaluates removing his order, the agent is effectively changing market conditions, and thus, the state he's in; therefore, he evaluates action a' on the new state s' which doesn't include his order in any of the limit order books.

Note that an important element of an uniformed traders' information to decide his optimal action is his expectation of the fundamental value. Suppose an uninformed agent arrives at market conditions m_t, and define $\delta_t(m_t) = E(v_t|m_t) - v_{t-\Delta t}$, i.e., the expected change in the since time $(t - \Delta t)$. Then, the estimate of the fundamental value of the trader becomes $E(v_t|m_t) = v_{t-\Delta t} + \delta(m_t)$. Therefore, the agent belief about the fundamental value of the asset is completely defined by assessing $\delta(m)$. Since stationary equilibriums are considered, the time subscript is dropped.

A counter $r(m)$ is kept for each market condition m, which is incremented by one every time market condition m is visited. Also, an initial belief for each m is set as $\delta_0 = 0$. Then, $\delta(m)$ in visit number $r(m)$ is estimated by traders as

$$\delta_r(m) = \frac{r-1}{r}\delta_{r-1}(m) + \frac{1}{r}(v_t - v_{t-\Delta_t})$$

Now consider agents' beliefs about the payoff for taking action a in state s, $U_t(a|s)$, which also need to be updated so traders learn the actual payoff from each action and reach equilibrium. Suppose an agent places a limit order a at state s that doesn't get executed when he revisits the market at instant t'on state s', having his action evolved to a'. Then, a induces a continuation value $J(s')$, and his beliefs about payoffs are updated as

$$U_{t'}(a|s) = \frac{n}{n+1}U_t(a|s) + \frac{1}{n+1}e^{-\rho(t'-t)}J(s')$$

Where $n(a, s)$ is a counter which is incremented by one each time action a is taken on state s, similar to how $r(m)$ works. An initial value n_0 is set for each action-state pair, and the value is bounded to allow for faster and better convergence. Now consider a trader who submits a limit order a that gets executed at time τ, while he's waiting to revisit the market, then, the instantaneous payoff is realized for both agents in the transaction, and the payoff for the limit order is updated according to

$$U_\tau(a|s) = \frac{n}{n+1}U_t(a|s) + \frac{1}{n+1}e^{-\rho(\tau-t)}d(\alpha + v_\tau - p)$$

Where $d \in \{1, -1\}$ for a buy and sell order respectively, α is the agents' private valuation for the asset and p the transaction price. Note that for a market order, the payoff is known at each moment in time for the updating process, being only relevant the expected fundamental value of the asset for the decision of an uninformed trader.

An important detail has to be mentioned. Since every trader takes the optimal action given a visited state, a possibility exists for the algorithm to freeze in a non-equilibrium state: every trader of a given type would take the same action in state s, therefore, they aren't able to learn the payoffs rewarded by other possible actions, making their beliefs erred and their actions suboptimal. To ensure the correct beliefs for every trader and state, trembles are introduced. With probability ϵ a trader trembles and chooses randomly amongst all suboptimal limit orders he may submit. Then, the algorithm by itself updates the beliefs on the suboptimal action as described in the previous paragraphs.

Finally, beliefs are tested to meet convergence criteria to ensure the desired Markov perfect Bayesian Equilibrium has been reached.

3. Convergence

We run the model until the desired number of new traders arrive to the market[18]. Once there, the payoff of each action and the amount of times it has been taken for a given state are saved, and the model is run for another billion times, to check for convergence by computing $|U_{t_2}^{k_2}(a|s) - U_{t_1}^{k_1}(a|s)|$ for each pair of action taken on a state (a, s) that occurs throughout the simulation. k_1 is the number of times the action a has been taken under state s when the simulation has run for the desired arrivals, and k_2 for when an extra billion new traders reached the market.

The criterion weights each absolute difference by $(k_2 - k_1)$ to get a weighted absolute difference of the utilities assigned to each action. If this value is small enough, below 0.03 in our case, the algorithm is considered to have converged, since traders already learned the expected payoff of each action in a given state. Note that the convergence criterion is checked for each cycle of the model independently, so a weighted absolute difference exists for each of the periods, and the criteria requires the maximum of these metrics to be less than 0.03.

If the criterion is met by every phase, the learning process of traders stops and the beliefs about $U(a|s)$ are fixed. Finally, 40 million new trader arrivals are simulated and its optimum actions, characteristics, information and market conditions are saved to get the results described here.

[18] In this case, the model is run until 30 billion traders arrive.

7.2. APPENDIX B

Alternative model parametrization results

An alternative parametrization was tested to model the fact that in exchanges geographically close to each other, the complete mass of agents might move to the currently open market in each stage of the day (or both during the overlap). In this case, the rate for the Poisson process that governs the arrival of new traders changes throughout the day: During the non-overlap period, the rate λ is set to 2, while during the overlap $\lambda = 1$.

The following figure shows how the depth available to a trader varies throughout the day on a minute-by-minute basis. During the non-overlap period he's only available the outstanding shares on a single-book, while during the overlap both books limit orders are available to him.

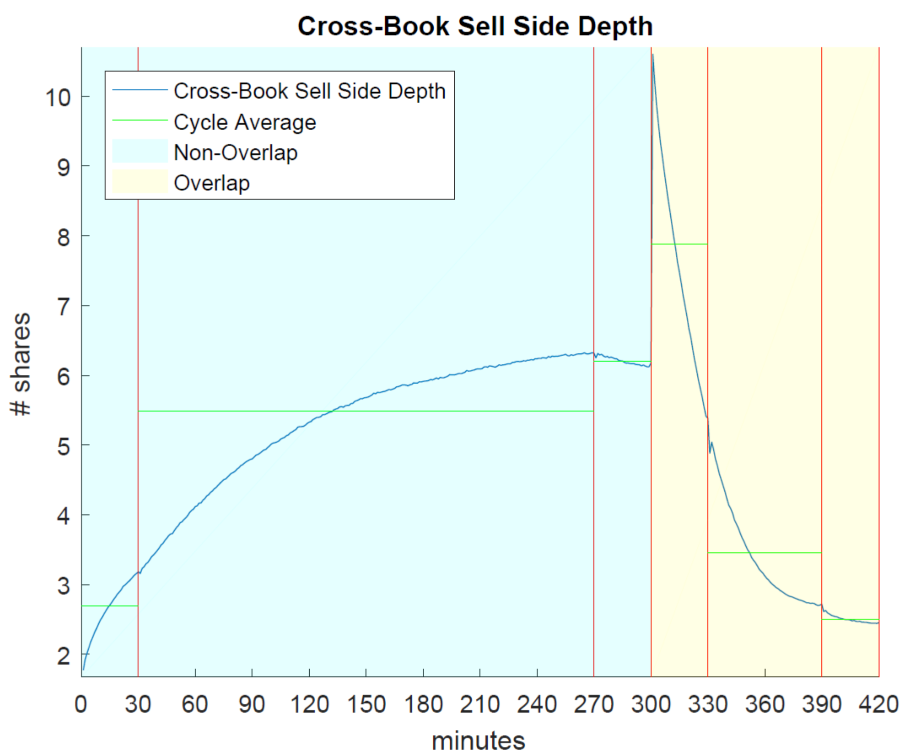

Figure 2: ***Cross-Book Average Sell Side Depth per Minute for the alternative parametrization.*** *The figure displays how the amount of limit orders (depth) available to a trader varies throughout the day, and is computed as the average depth available to an agent on each minute. In this case the arrival rate of agents is set to 2 during the non-overlap, and 1 during the overlap.*

The figure displays a sharp rise on the available depth to a trader at the open of the overlap, displaying a relevant fact about the model: it's unable to capture the phenomenon of every trader migrating to the currently open exchange. At the start of the overlap, the conjunction of both books represents a market were the amount of agents arriving has been doubled compared to the overlap, thus, the model artificially represents an arrival rate of $\lambda = 2$ during the opening transition, making the results not reliable.

Moreover, Table 10 shows market quality measures for the alternative model. As can be seen, most measures are consistent with the original model, albeit dynamics are more pronounced, and a controversial result appears: Agents wait almost 3 minutes more to execute and leave the market during the most liquid period of the day, the overlap. As Figure 2 shows, these results are not reliable, since the model is incapable of capturing the described phenomenon and an artificial doubling of the arrival rate is created during the open of the overlap.

Table 10: Market Quality measures for alternative model. *The following table displays various market quality measures for the alternative model. Two rows are shown for each statistic: the upper one is the cross-book measure,while the lower row is the single book measure. Cross-book measures are computed considering orders from both books when in overlap, while using a single-book otherwise. This is done to analyze market quality measures from the traders perspective, by using only the information they have available on each period.*

	Period Average		Overlap Cycle					
Measure type	NOV	OV	Non-Overlap			Overlap		
			1	2	3	1	2	3
					Bid-ask spread			
Cross-book	1.54	2.68	1.95	1.52	1.51	2.60	2.79	2.62
Single-book		3.35				3.31	3.48	3.16
					Effective spread			
Cross-book	1.36	2.24	1.62	1.33	1.34	2.31	2.25	2.13
Single-book		2.70				2.73	2.73	2.55
					N. of limit orders at the ask			
Cross-book	1.62	1.43	1.32	1.61	1.79	1.73	1.25	1.14
Single-book		1.44				1.76	1.26	1.14
					N. of limit orders at the ask (effectively traded)			
Cross-book	0.56	0.57	0.58	0.56	0.55	0.69	0.49	0.45
Single-book		0.57				0.71	0.50	0.45
					N. of limit orders on the sell side of the book			
Cross-book	5.41	4.96	2.69	5.49	6.21	7.88	3.46	2.50
Single-book		3.00				4.47	2.16	1.68
					N. of limit orders on the sell side of the book (effectively traded)			
Cross-book	1.13	1.95	0.77	1.14	1.16	4.83	1.18	0.84
Single-book		1.18				2.74	0.73	0.56
					Time between the instant in which a trader arrives and her execution			
Cross-book	7.69	10.55	3.50	7.79	10.61	15.45	9.74	5.31
Single-book		10.56				15.48	9.74	5.30

Therefore, none of the measures here presented should be taken seriously when studying the transition of markets from a non-overlap to an overlapped setup, when exchanges are geographically close to each other and every agent moves to the open exchange.

Finally, this creates a need for a new model that represents real world markets in a more precise manner, by not making the books run in parallel. In this model, the trading day should be split into 4 periods: first, only exchange A is open, second, both markets overlap during some hours, third, exchange A closes and only market B remains open, and lastly, both markets are closed during night hours. The development of this model is left as future work.