ESTIMATING LABOR SUPPLY AND PRODUCTION DECISIONS OF SELF-EMPLOYED FARM PRODUCERS

Ramon E. LOPEZ*

University of Maryland, College Park, MD 20742, USA

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This paper proposes a model oriented towards integrating farm households' production and consumption decisions into a unified theoretical and econometric framework. It is argued that, in contrast with other forms of economic organization, farm households' utility and profit maximization decisions are not likely to be independent.

Econometric estimation of a farm-household model using Canadian data suggests that utility and profit maximizing decisions are not indeed independent and, moreover, that there are significant gains in explanatory power and efficiency by estimating the consumption and production equations jointly.

1. Introduction

This paper is concerned with the analysis of labor supply and production decisions of households which also own and operate a farm. Distinctive features of these households are: (a) a significant proportion (often the whole) of the labor input used by the household farm is supplied by its proprietors, i.e., the household's members; (b) the returns from the family farm's operation constitute an important proportion of the household's income available for consumption and other purposes.

The objectives of this paper are twofold: (1) to estimate labor supply and production responses of farm households in Canada considering the interdependence between utility and profit maximization decisions which may arise from features (a) and (b), and (2) to formally test the hypothesis of independence of utility maximization and profit maximization decisions.

This study differs from previous analyses of farm households in the following respects. First, we explicitly consider the labor choice problem between farm and off-farm work, recognizing that such time allocations may have different utility connotations. Lau, Lin and Yotopoulos (1978) and Barnum and Squire (1979), for example, have used a unique exogenous price of operator and family labor, thus implicitly assuming that farm producers

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are indifferent between alternative allocations of their time in farm and offfarm activities. Second, previous studies have either estimated total labor supply or have estimated a single off-farm labor supply equation. Moreover, the only linkage between the consumption (i.e., labor supply) and production sides of the model is the effect of profits from the farm operation on the household income. In particular, Lau, Lin and Yotopoulos (1978) have used the predicted value of a profit function estimated independently by Yotopoulos, Lau and Lin (1976) as a component of the farm household income. In contrast, we jointly estimate the labor supply and the profit function (via the output supply and input demand equations) and show that important gains in explanatory power result from doing this.

Finally, the functional forms used by the above mentioned studies in specifying preferences and production technologies are rather restrictive. Preferences are assumed to be either homothetic to the origin or to a fixed point in the commodity space (affine homotheticity) and a Cobb-Douglas production function is used. We use a somewhat more general specification for preferences, which allows us to formally test whether the special forms of homotheticity used in previous studies hold, and we use a flexible functional form for representing the farm profit function.

2. Theoretical considerations

It is assumed that households maximize utility subject to a budget constraint. Utility is a function of the goods consumed, the number of hours of on-farm work, and the number of hours of off-farm work. The budget constraint indicates that the total expenditures on consumer goods cannot be greater than the total income obtained by the household. This income consists of the net income obtained from the family farm's operation, represented by a dual profit function conditional on the number of hours of work which the household members supply to their own farm, the non-labor income which includes the returns obtained from financial and real assets owned by the household, and the off-farm employment income.

More formally, the utility maximization problem of the farm-household is

$$\max_{L,X} f(L_1, L_2; X_1, \dots, X_N),$$
(1)

subject to

(i)
$$\sum_{n=1}^{N} p_n X_n \leq \pi(q; L_1) + w_2 L_2 + y,$$

(ii)
$$L_1 + L_2 \leq H$$
,

(iii) $X_n \ge 0, L_1, L_2 \ge 0,$

where

- f = households' utility function,
- $X = (X_1, \dots, X_N)$ is the N dimensional vector of consumption goods,
- L_1 = number of hours of work supplied to the family farm by household members,

 $L_2 =$ number of hours of off-farm work,

- p_n = rental price of commodity *n* consumed by household members,
- y = net non-labor income,
- q = price vector of the s net outputs the family farm produce (using the convention of representing outputs as positive quantities and inputs as negative quantities),
- H =total number of hours that household members have available for all activities,
- w_2 = wage rate received by household members when they work off-farm.

Further, $\pi(q; L_1)$ is the family farm's conditional profit function defined by¹

$$\pi(q;L_1) \equiv \max_{Q} \{q^T Q: (Q;L_1) \in \tilde{T}\},\$$

where $Q \equiv [Q_1, Q_2, \dots, Q_s]$ is a column vector of net outputs (outputs and inputs) and \tilde{T} is a closed, bounded and convex production possibilities set.

It is assumed that the utility function $f(L_1, L_2; X_1, ..., X_N)$ satisfies the following regularity conditions [Diewert (1974)]:

- A.1. defined and continuous from above for X, L_1 , $L_2 \ge 0$,
- A.2. quasi-concave in its arguments,
- A.3. non-decreasing in X,
- A.4. non-increasing in L_1 and L_2 .

The fact that the farm profit function is dependent on L_1 and that preferences are allowed to be affected differently by on-farm and off-farm time allocations signifies that farm household utility and profit maximization cannot in general be dichotomized. That is, labor supply and production decisions are interdependent mainly due to the fact that the shadow price of L_1 is endogenous, dependent on both the production and the consumption sides of the model. This interdependence is reduced if one assumes either that (1) households' utility depends on total labor supply and not on the allocation of that supply between on-farm and off-farm employment provided that households work off-farm, or (2) household labor and hired labor are perfect substitutes in production provided that some hired labor is used.

^{&#}x27;The properties of $\pi(q; L_1)$ are those of the variable profit function as discussed by Diewert (1974).

The use of either of these assumptions allows one to consider the shadow price of on-farm household work as exogenous equal to the off-farm rate if Assumption 1 is used or equal to the hired labor wage rate if Assumption 2 is used. In either of these cases the interdependence of utility and profit maximization decisions is reduced to the effects of farm profits on household income.

Given that both assumptions are likely to be unrealistic we do not rely on them. It has long been recognized that the disutility associated with diverse working activities is different.² Utility differences associated with different working activities are likely to be even greater when one of the activities involves self-employment with a large component of entrepreneurial work and the other is a wage earning activity. Assumption 2 is also dubious if one considers differences in supervision costs and differences in educational levels between farm operators and their families and hired labor. Additionally, the absence of perfect substitutability between hired and non-hired labor has been empirically established in studies applied to agriculture [e.g. Barichello (1979)].

The utility function can be represented in a more convenient form by a transformation of the variables in $f(\cdot)$,

$$U(H - L_1, H - L_2; X_1, \dots, X_N) \equiv f(L_1, L_2; X_1, \dots, X_N).$$
⁽²⁾

The *r*-lvantage o $U(H-L_1, H-L_2; X_1, ..., X_N)$ is that it is defined over the non-negative orthant, and the corresponding budget constraint may be defined using non-negative prices and positive income [Diewert (1971)]. It is easy to verify that if f satisfies conditions A then U will satisfy A.1, A.2, and A.3. Condition A.4 for U will read 'non-decreasing in $H-L_1$, $H-L_2$ '.

Using (2) it is now possible to reformulate model (1),

$$\max_{H-L_1, H-L_2, X} U(H-L_1, H-L_2; X),$$
(3)

subject to

(i) $pX + w_2(H - L_2) \leq H \cdot w_2 + y + \pi(q; L_1),$ (ii) $(H - L_1) \geq 0, \quad (H - L_2) \geq 0, \quad X \geq 0,$ (iii) $(H - L_1) + (H - L_2) \geq H,$ (iv) $(H - L_1) \leq H, \quad (H - L_2) \leq H.$

From now on it is assumed that constraint (iii) is not binding. This implies that at all wage rates and commodity prices households will consume some leisure.

²See, for example, Benewitz and Zucker (1968), Diewert (1971), Fieldings and Hoseck (1973), and Rottenberg (1956).

Model (3) can be significantly simplified if the production technology exhibits constant returns to scale. If the production technology exhibits constant returns to scale and if there are no fixed factors of production then the profit function is homogeneous of degree one in L_1 and can be decomposed as follows:³

$$\pi(q;L_1) = L_1 \tilde{\pi}(q), \tag{4}$$

where $\tilde{\pi}(q)$ is non-negative, convex, continuous, and linear homogeneous in q.

Using (4) the utility maximization problem (3) may now be written as

$$\max_{H-L_1, H-L_2, X} U(H-L_1, H-L_2; X),$$
(5)

subject to

(i) $pX + \tilde{\pi}(q)(H - L_1) + w_2(H - L_2) \leq H(\tilde{\pi} + w_2) + y \equiv Z$, (ii) $(H - L_1) \geq 0$, $(H - L_2) \geq 0$, $X \geq 0$, (iii) $(H - L_1) \leq H$, $(H - L_2) \leq H$.

The advantage of using (5) rather than (3), is that (5) is a standard maximization problem with a linear constraint provided that $\tilde{\pi}(q)$ is known and that constraint (iii) is not binding. Thus, standard duality theory [see, for example, Diewert (1971)] can now be applied in order to derive equations for household commodity demand, labor supply to the household farm, and offfarm labor supply. The wage rate for on-farm work, $\tilde{\pi}(q)$, is determined by the farm production technology, output, and input prices.

An indirect utility function, $G(p, \tilde{\pi}, w_2; Z)$ can therefore be defined in the standard manner,

$$G(p, \tilde{\pi}(q), w_2; Z) \equiv \max_{H - L_1, H - L_2, X} \{ U(H - L_1, H - L_2; X):$$
(6)

(i) $pX + \tilde{\pi}(q)(H - L_1) + w_2(H - L_2) \leq Z$,

(ii)
$$H - L_1 \ge 0$$
, $H - L_2 \ge 0$, $X \ge 0$ },

where G is continuous, quasi-convex in p, $\tilde{\pi}$ and w_2 , non-increasing in p, non-decreasing in Z, and homogeneous of degree zero in p, $\tilde{\pi}$, w_2 and Z.

From (c) it is possible to derive the Marshallian demand functions for $H-L_1$, $H-L_2$ and X using Roy's identity,

(i)
$$H-L_1 = -(\partial G/\partial \tilde{\pi}(q))/(\partial G/\partial Z) = \phi(p, \tilde{\pi}, w_2, Z).$$

(ii)
$$H - L_2 = -(\partial G/\partial w_2)/(\partial G/\partial Z) = \Omega(p, \pi, w_2, Z),$$
 (7)

(iii)
$$X = -(\partial G/\partial p)/(\partial G/\partial Z) = \varepsilon(p, \tilde{\pi}, w_2, Z).$$

³The assumption of constant returns to scale in Canadian agriculture has often been not rejected. In a recent study by Char. (1981) the hypothesis of constant returns to scale was not, in fact, rejected.

Furthermore, the set of conditional net supply functions can be derived from the conditional profit function using Hotelling's lemma [Hotelling (1932)],

$$Q_i(q; L_1) = L_1 \cdot (\partial \tilde{\pi}(q) / \partial q_i), \qquad i = 1, \dots, S,$$
(8)

where Q_i is the conditional net supply of commodity *i*. The unconditional net supply functions are obtained by using (7.i) in (8),

$$Q_{i}(q; p, w_{2}, Z) = [H - \phi(p, \tilde{\pi}(q), w_{2}, Z)](\partial \tilde{\pi}(q)/\partial q_{i}), \qquad i = 1, \dots, S.$$
(9)

Eqs. (7) and (9) represent the set of supply and demand responses obtained from a model which considers consumption and production activities of the farm household within an integrated framework. Changes in the consumption side are transmitted to the net output supply functions via the function $\phi(p, \tilde{\pi}(q), w_2, Z)$ in (9). Similarly, changes in the production side affect utility maximization decisions not only via Z but also by changing the shadow price of L_1 , i.e., by changing $\tilde{\pi}(q)$ in (7). Thus if output prices increase, for example, then the household will reconsider its consumption and labor supply allocations, because the increased output prices imply a higher ievel for the shadow price of on-farm work $[\tilde{\pi}(q)]$.

3. The estimating model

In postulating a functional form for the indirect utility function (6) a major consideration was that the cross-sectional data used in the study are aggregated by census divisions (see section 4). This implies restrictions on the level of generality of the functional form postulated for the indirect utility function.

In has been shown that homotheticity to the origin of preferences is a sufficient but not necessary condition for consistent aggregation [Gorman (1953)]. The Gorman Polar Form (GPF) is a more general restriction on preferences which allows for consistent aggregation and where the demand system satisfies the integrability conditions. Considering this, a GPF for (8) is used in this study,

$$G(\tilde{\pi}, w_2, p; Z) = (Z - A(\tilde{\pi}, w_2, p))/\psi(\tilde{\pi}, w_2, p),$$
(10)

where Λ and ψ are continuous, concave, non-decreasing and positively homogeneous of degree one in $\tilde{\pi}$, w_2 , and p.⁴ A number of empirical studies have concluded that demographic characteristics such as family size and

⁴For a description of the aggregation properties of the GPF, see Blackorby et al. (1978).

education substantially affect labor supply and commodity demand [Huffman (1977) and Wales and Woodland (1977)]. An approach commonly used has been to separate households into groups of approximately homogeneous characteristics and then proceed with the estimation of preferences for each homogeneous group separately. The approach followed here makes use of the property of the GPF which allows for different households' preferences via changes in the function $\Lambda(\cdot)$. It is assumed that households' educational level (E) and number of dependents (F) affect preferences by changing the reference or base surface, i.e., affecting $\Lambda(\cdot)$. Thus, instead of estimating a function like G in (10) for various homogeneous households groups, it is preferred to estimate (10) using $G(\tilde{\pi}, w_2, p; E, F)$ considering all households at the same time. Given that E and F do not affect the function $\psi(\cdot)$, the households' expansion paths are parallel even if these characteristics vary within a group. Thus, it is assumed the education and number of dependents affect optimal commodity or leisure ratios, but that the marginal propensity to consume is not affected.

The postulated functional form for (1) consists of a CES form for $\psi(\tilde{\pi}, w_2, p)$ and a generalized Leontief form for the $A(\tilde{\pi}, w_2, p; E, F)$ function [Blackorby et al. (1978)],

$$G = \frac{Z - \sum_{i=1}^{3} \sum_{j=1}^{3} \delta_{ij} p_{i}^{\frac{1}{2}} p_{j}^{\frac{1}{2}} + \sum_{i=1}^{3} l_{i} p_{i} E + \sum_{i=1}^{3} b_{i} p_{i} F}{\left[\sum_{i=1}^{3} \alpha_{i} p_{i}^{\rho}\right]^{1/\rho}}, \quad i, j = 1, 2, 3, \quad (11)$$

where $\delta_{ij} = \delta_{ji}$, l_i , b_i , α_i and ρ are parameters to be estimated, and $p_i \equiv \tilde{\pi}$, $p_2 \equiv w_2$, and $p_3 \equiv p$.

Using Roy's identity one can derive the demand equations in expenditure form,

$$S_{i} = \frac{\alpha_{i} p_{i}^{\rho} \left[Z - \sum_{i=1}^{3} \sum_{j=1}^{3} \delta_{ij} p_{i}^{\frac{1}{2}} p_{j}^{\frac{1}{2}} - \sum_{i=1}^{3} l_{i} p_{i} E - \sum_{i=1}^{3} b_{i} p_{i} F \right]}{\sum_{i=1}^{3} \alpha_{i} p_{i}^{\rho}} + p_{i} \left[\sum_{j=1}^{3} \delta_{ij} (p_{j}/p_{i})^{\frac{1}{2}} + l_{i} E + b_{i} F \right], \quad i = 1, 2, 3,$$
(12)

where

$$S_1 \equiv p_1(H - L_1), \qquad S_2 \equiv p_2(H - L_2), \qquad S_3 \equiv p_3 X_1$$

Note that it is possible to test for homotheticity to the origin, by testing if

all $\delta_{ij} = 0$. If $\delta_{ij} = 0$ for $i \neq j$, then preferences would be homothetic to a fixed point in the positive orthant.

Given that the total expenditures cannot exceed the after tax income rather than the gross income it is necessary to modify model (12) in order to consider taxes. The budget constraint in (8) now considering taxes can be expressed as [Wales and Woodland (1977)]

$$px + \tilde{\pi}(H - L_1) + w_2(H - L_2) \le H(\tilde{\pi} + w_2) + y - \tau(Y^T),$$
(13)

where $\tau(Y^T)$ are the total taxes paid as a function of the household's taxable income, $Y^T \equiv \tilde{\pi}L_1 + w_2L_2 + y - Ex$, where Ex are the tax exemptions. The tax function can be approximated by

 $\tau(Y^T) = \tau_i + \tilde{\beta}_i (Y^T - Y_i^T), \tag{14}$

where

 $Y_i^T =$ smallest taxable income in tax bracket *i*,

 $\tau_i = \text{taxes paid at income } Y_i^T$,

 $\vec{\beta}_i$ = marginal tax rate in tax bracket *i*.

Hence, using (13) and (14) and defining $\tilde{\pi}_i \equiv (1 - \tilde{\beta}_i)\tilde{\pi}$ and $w_{2i} \equiv (1 - \tilde{\beta}_i)w_2$, the after tax budget constraint is

$$px + \tilde{\pi}_{i}(H - L_{1}) + w_{\perp i}(H - L_{2}) \leq H(\tilde{\pi}_{i} + w_{2i}) + (1 - \tilde{\beta}_{i})y + \tilde{\beta}_{i}Y_{i}^{T}$$
$$-\tau_{i} + \tilde{\beta}_{i}Ex \equiv Z_{i}.$$
(15)

Thus, eqs. (12) are estimated using the after tax values $\tilde{\pi}_i$, w_{2i} , and Z_i as defined above.

In order to estimate the production side of the model we estimate the conditional profit function of the household's firm. The conditional profit function is dependent on a vector of net output prices, q_2 , on the lev 1 of family labor used by the firm and on the production technology. The prices considered arc one aggregate output price (q_1) and the following factor prices: rental price of land and structures (q_2) , hired labor wage rate (q_3) , rental price of livestock capital (q_4) , and rental price of other forms of capital (q_5) . In a cross-sectional framework, differences in the production technology among the observations might arise because of:

- (a) differences in the educational levels of farm households,
- (b) regional differences in climate and soil quality.

Factor (a) may lead to improvements in productive efficiency by affecting the technology which farmers select. Education may affect farm profits and the supply of net outputs in a non-neutral way. Thus, the variable education is considered as a factor affecting profits, and hence net output supply, allowing for measuring differential effects of education on the demand for the different inputs.

Factor (b) may also affect the level of profit and the net output supply functions in a non-neutral manner. It was therefore decided to add dummy variables for four regions to the conditional profit function.

Consequently, assuming constant returns to scale and specifying a Generalized Leontief conditional profit function we have

$$\pi(q;L_i) = L_1 \left[\sum_{i=1}^5 \sum_{j=1}^5 b_{ij} q_i^{\frac{1}{2}} q_j^{\frac{1}{2}} + \sum_{i=1}^5 a_i q_i E + \sum_{i=1}^5 \sum_{k=1}^4 C_{ik} D_k q_i \right],$$
(16)

where $b_{ij} = b_{ji}$, a_i and C_{ik} are parameters, and D_k is the dummy corresponding to region k.

Given (16) the net output supply responses per unit of family labor can be obtained using Hotelling's lemma. Thus, the net output supply equations are

$$Q_i/L_1 = \sum_{j=1}^5 b_{ij}(q_j/q_i)^{\frac{1}{2}} + a_i E + \sum_{k=1}^4 C_{ik} D_k, \qquad i = 1, \dots, 5,$$
(17)

where

 Q_1 = output supply, $-Q_2$ = demand for land structures, $-Q_3$ = demand for hired labor, $-Q_4$ = demand for animal stocks, $-Q_5$ = demand for farm capital.

3.1. The econometric moael

To estimate eqs. (12) and (17) it is necessary to assume a stochastic structure. It is assumed that the disturbances are additive and normally distributed with zero means and positive semi-definite variance-covariance matrix Σ . Thus, if (12) and (17) are written in a more compact notation and if disturbance terms are added then the estimating model is

(i)
$$S_1 = f^1(\tilde{\pi}, w_2, p, Z; E, F) + \mu_1,$$

(ii) $S_2 = f^2(\tilde{\pi}, w_2, p, Z; E, F) + \mu_2,$
(iii) $S_3 = f^3(\tilde{\pi}, w_2, p, Z; E, F) + \mu_3,$
(iv) $Q_i/L_i = \phi^i(q; E) + \sum_{k=1}^4 C_{ik}D_k + v_i, \quad i = 1, ..., 5.$
(18)

The system of eqs. (18) is jointly estimated, after dropping the consumption goods expenditure equation, using a Full-Information Maximum Likelihood Method (FIML). Thus, the parameters of the utility and profit functions which maximize the logarithm of the concentrated likelihood function, L, are chosen,

$$L = -(kT/2)(\ln 2\pi + 1) - (T/2)\ln |\hat{\Sigma}| + \sum_{N=1}^{T} \ln (abs|B_N|), \qquad (19)$$

where

k =number of equations, T =number of observations.

 $abs|B_N|$ = absolute value of the determinant of the matrix of derivatives of the disturbances with respect to the endogenous variables.

Given that the number of households varies across the different census divisions, the variances of the disturbance terms will be different for the different observations even if the individual household's disturbances are assumed to be constant. Thus, one may expect that the disturbances of the grouped estimates are heteroscedastic. To correct this we multiply through eqs. (18) by the square root of the number of farms in each census division.

3.2. Testing for independence of utility and profit maximization decisions

To empirically test the hypothesis of independence we use as a reference a model based on this hypothesis similar to the one used by Lau et al. (1978). This model avoids the problem of interdependence by assuming that households are indifferent between working on their own farms and off-farm as wage earners. This allowed the authors to use the off-farm wage rate as the unique exogenous price of leisure under the implicit assumption that households do off-farm work. Thus, such a model is the following:

$$\tilde{G}(p, w_2, \tilde{Z}; E, F) = \max_{H - L_1 - L_2, X} \{ U(H - L_1 - L_2, X):$$
(20)

(i)
$$px + w_2(H - L_1 - L_2) \leq \pi(q, w_2; E) + w_2H + y \equiv \tilde{Z}$$
,
(ii) $X \geq 0$, $H - L_1 - L_2 \geq 0$, $L_1 \geq 0$, $L_2 \geq 0$ },

where $\pi(q, w_2; E) \equiv \{\max_{Q, L_1} q^T Q - w_2 L_1; Q, L_1 \in \tilde{T}(E)\}\$ is the unconditional profit function, $\tilde{G}(\cdot)$ is the indirect utility function and all other variables have previously been defined.

Note that in this model the assumption of constant returns to scale needs to be relaxed in order to obtain a well defined (unconditional) profit function, $\pi(q, w_2; E)$. Using Roy's identity one can derive the estimating utility

maximizing equations from $\tilde{G}(\cdot)$ and using Hotelling's lemma the unconditional net output supply responses are obtained from $\pi(q, w_2; E)$. Thus, the estimating model is

(i)
$$H - L_1 - L_2 = g^2(p, w_2, \tilde{Z}; E, F) + \tilde{\mu}_1$$
,

(ii)
$$Q_i = h'(q, w_2; E) + \tilde{v}_i, \quad i = 1, ..., 5,$$

(iii)
$$L_1 = h^6(q, w_2; E) + \tilde{v}_6$$
,

(iv)
$$X = g^{3}(p, w_{2}, \tilde{Z}; E, F) + \tilde{\mu}_{2}$$
,

where the g^i and h^i are functions solving (20).

Model (21) is estimated using the same functional forms for the indirect utility function (Gorman Polar Form) and for the profit function (Generalized Leontief) used in estimating the model based on the hypothesis of interdependence defined by (18). As in model (18), it is necessary to drop one of the equations of the consumption side in (21). It is arbitrarily chosen to drop the equation corresponding to the demand for goods (21.iv)

Before proceeding with a description of the testing procedure it is convenient to comment on the structural differences between the two models. The central difference is that while in model (20) the tabor supply and consumption goods demand equations jointly reflect the household's preferences and the firm's production technology, in model (21) they are solely determined by the household's preferences. Furthermore, in model (20), although the net output supply responses conditional on L_1 are not affected by the household's preferences, the implicit unconditional net output supply responses (i.e., when L_1 is considered variable) are also jointly determined by the household's preferences and the firm's production technology. This is in contrast with model (21) where the unconditional net output supply equations are defined independently of the household's preferences.

The problem in formally testing the null hypothesis of independence, i.e., that model (21) holds, against the alternative hypothesis of no independence using model (18), is that the parameter space of one of the models is not contained in the parameter space of the other. That is, we are dealing with separate families of hypotheses and the standard tests cannot be employed [Golfdeld and Quandt (15^{72})]. There are a number of alternative formal tests designed to discriminate between separate families of hypotheses. We use here the Hoel-Davidson-Mackinnon (HDM) test, which allows one to test the truth of a linear or non-linear and multivariate regression model, when there exists a non-nested alternative hypothesis.⁵

The HDM procedure for testing the null hypothesis of independence represented by eqs. (21.i) to (21.iii) against the alternative hypothesis

(21)

⁵For a detailed description of the test and its asymptotic properties, see Davidson and Mackinnon (1981).

embodied in eqs. (18.i), (18.ii) and (18.iv) suggests the estimation of the following equation system:

(i)
$$L_{1} = (1 - \beta_{1})h^{6}(\cdot) + \beta_{1}[H - \hat{f}^{1}(\cdot)/\tilde{\pi}] + \tilde{\mu}_{1},$$

(ii) $L_{2} = (1 - \beta_{2})[H - g^{2}(\cdot) - h^{6}(\cdot)] + \beta_{2}[H - \hat{f}^{2}(\cdot)/w_{2}] + \tilde{\mu}_{2},$
(iii) $Q_{i} = (1 - \beta_{i+2})h^{i}(\cdot) + \beta_{i+2}[\hat{\phi}^{i}(\cdot)(\hat{f}^{1}(\cdot)/\tilde{\pi}) + \hat{cov}(\mu_{1}/\tilde{\pi}, v_{i})] + \tilde{\mu}_{2+i},$ (2?)
 $i = 1, \dots, 5,$

where a hat (^) above the functions indicates expected or predicted values.

Note that the second terms of the right-hand sides represent the predicted or expected values [obtained from model (18)] of L_1 , L_2 and Q_i rather than of S_1 , S_2 and Q_i/L_1 . The null hypothesis that utility and profit maximization decisions are independent [i.e., that model (21) is the true model] is tested against the alternative hypothesis of interdependence represented by model (18) by jointly testing whether $\beta_k = 0$ for k = 1, ..., 7. It is clear that if H_0 is true then all β_k will vanish.

The first terms of the right-hand side correspond to model (21) modified in order to obtain a specific equation for L_2 from (21.i) and (21.iii). The interpretation of L_1 and L_2 in (22) should be carefully considered: the model based on independence does not provide two labor supply equations. It only defines one aggregated labor supply equation, and a demand equation for L_1 is determined at the firm level. Hence the equation for L_2 [i.e., $H-g^2(\cdot)-h^6(\cdot)$] has been obtained from model (21) as a residual reduced form, only for the purpose of making model (22) comparable to model (18). Thus, the equation for L_2 , obtained after some transformations of model (21) have been made, does not correspond to a household's behavioral equation. The only household's behavioral equation in model (21) is the *total* labor supply function.

The assumption of constant returns to scale used in the alternative hypothesis may cause some problems in the interpretation of the test. If the true production technology does not approximately exhibit constant returns to scale, then it is possible that neither the null hypothesis nor the alternative hypotheses are true. In this case the asymptotic properties of the test are generally unknown and hence it would be difficult to interpret the result of regression (22). However, Davidson and Mackinnon (1981) have shown that if H₀ is true then the plim $\hat{\beta}_k = 0$ (for all k) and the variance of $\hat{\beta}_k$ is consistently estimated by (22). This implies that the confidence interval for $\hat{\beta}_k$ is correctly estimated if H₀ is true and hence the probability of a type I error is correctly given by the level of significance chosen.⁶

⁶The roles of the alternative and null hypotheses were also reversed, thus using interdependence as the null hypothesis tested.

3.3. The data

The data used were obtained from the 1971 agricultural and population censuses which report data corresponding to 1970. It is not possible to have access to household data, because of tax confidentiality problems. However, aggregated data are available at the census division level. There are approximately 240 agricultural census divisions in Canada, and data are available in the form of total values per census division. Since data on the number of farm households per census division are also available, one can transform the data into averages per household.

The required data for this study are the number of days of off-farm work by household members, the number of days worked on-farm, the off-farm wage rate, the household's non-labor income, output and input prices faced by the household's firm, the farm operator's years of schooling, and the number of family dependents. An aggregated output price index and three input price indices, namely, the hired labor wage rate, an animal stocks rental price index, and a land rental price index, are needed. The price index of farm capital (machinery, implements and other intermediate inputs) is not available and is assumed constant across the observations. Farm machinery fertilizers and spray materials, in contrast with other farm inputs (such as labor, land, and livestock), are traded by large firms which operate at a national scale. It is reasonable to assume that these firms charge approximately homogeneous prices for their products in the different regions of the country, and thus the above assumption may not be too unrealistic.^{7.8}

4. Empirical results

4.1. Hypothesis testing

The main hypothesis tested is that utility and profit maximization decisions are independent, i.e., that $\beta_k = 0$ for k = 1, ..., 7 in (22). Other hypotheses are concerned with the restrictions on the functional form of preferences, the effects of education, and the effects of household dependents. In order to carry out the hypothesis tests, asymptotic likelihood ratio tests were performed.⁹ Table 1 shows the estimated χ^2 values and the critical values for the 5% and 1% levels of significance (LOS) for the corresponding

⁷For a detail description of the data used, see Lopez (1980b).

⁸An importance issue is whether 1970 was a 'normat year'. If 1970 was indeed a normal year from the point of view of weather, input prices, and output prices, then one can interpret the results obtained as being related to long-run equilibrium responses. In general, it appears that, although 1970 was not a perfectly normal year in relation to the previous 5 to 10 year period, at least this year cannot be singled out as a notoriously abnormal one. Hence, the interpretation of the results obtained as long-run equilibrium supply and demand responses is not totally inappropriate [Lopez (1980b)].

⁹See Theil (1971) for details regarding the likelihood ratio test.

	χ ² value	_	Critical values	
Null hypotheses		Degrees of freedom	5% LOS	1% LOS
1. Independence of production				
and consumption decisions	127.20 ⁵	7	14.07	18.48
2. Affine homotheticity	9.51ª	3	7.81	11.34
3. Homotheticity to the origin	202.76 ^b	6	12.59	16.81
4. No effect of education on				
labor supply	28.63 ^b	3	7.81	11.34
5. Neutral effect of education				
in production	53.62 ^b	4	9.49	13.28
6. No effect of number of family				
dependents on labor supply	62.86 ^b	3	7.81	11.34

Table 1				
Chi-square statistics for the various hypothesis	tests.			

*Significance at the 5% LOS.

"Significance at the 1% LOS.

degrees of freedom. The first row of table 1 shows the χ^2 value for the null hypothesis that utility and profit maximizing decisions are independent against the alternative hypothesis of interdependence, i.e., that $\beta_k = 0$ for all k = 1, ..., 7 against the alternative hypothesis that not all β_k coefficients are zero. The calculated χ^2 is 127.20, which is higher than the critical values at both the 5% and 1% LOS. Hence, the hypothesis that production and consumption decisions are independent is categorically rejected. However, when the roles of the null and alternative hypotheses were reversed, i.e., when the null hypothesis was interdependence under constant returns to scale, the calculated χ^2 value was not sufficiently large to reject it at the 1% LOS. This result together with the rejection of the hypothesis of independence suggest that indeed an interdependent model is more appropriate than the conventional dictotomized model.

The hypothesis of affine homothetic preferences can be rejected at the 5% LOS but is not rejected at the 1% LOS. Thus, one can conclude that Canadian farm household preferences are not homothetic to the origin, and hence that imposition of this restriction may induce serious specification errors and inconsistent estimates.

Hypotheses (4) and (5) in table 1 are related to the effect o' education on habor supply and production decisions, respectively. Both h potheses are rejected at the 1% LOS which implies that education significantly affects labor supply decisions (and hence it affects the indirect utility function) and that education plays a non-neutral role in determining factor demands. Finally, hypothesis (6) confirms the results obtained in previous studies [Barichello (1979), Huffman (1980), Lau et al. (1978), etc.] regarding the importance of the number of family dependents on labor supply decisions.

4.2. Supply and demand responses

The parameter estimates obtained by the joint estimation of the consumption and production sides of the model are presented in table 2. The asymptotic standard errors of the coefficients are presented in parentheses under the coefficients. Most coefficients in the consumption and production sectors are significant. There is one degree of freedom for the parameters of the CES function which can be exhausted by any suitable normalization [Blackorby et al. (1978)]. The normalization chosen is that the share parameter, α_2 , is equal to one.

Table 2

Parameter estimates of the consumption and production equations ($\tilde{R}^2 = 0.994$).						
I. Consumption	sector [eqs. ([12]]			<u> </u>	
		0.980, $\alpha_2 =$ (0.086)	$\begin{array}{c} 1, \alpha_1 = 1.124, \\ () (0.222) \end{array}$			
δ_{ij}, l_i, b_i	Leisure 1	Leisure 2	Consumption goods	Education	Number of family dependents	R ²
Leisure 1 $(H-L_1)$	612.5 (4.591)	-9.111 (3.746)	4.749 (9.603)	-14.83 (3.205)	160.5 (7.149)	0.886
Leisure 2 $(H-L_2)$	~	829.3 (15.94)	60.86 (6.16)	- 24.88 (2.534)	166.3 (5.835)	0.812
Consumption expenditures			- 2418 (10.59)	- 2.812 (1.055)	42.76 (1.078)	the set

II Production sector [eqs. (17)]

b_{ij}, a_i	Output	Land	Hired labor	Animal stocks	Farm capital	Educa- tion	R ²
Output supply	113.6 (7.044)	147,4 (2.562)	- 99.09 (7.455)	- 39.61 (2.755)	-233.17 (2.858)	- 38.77 (2.276)	0.835
Demand for land	- 147.4 (2.562)	- 160.1 (1.969)	68.71 (4.562)	-2.584 (1.683)	i 50.2 (4.366)	15.56 (1.414)	0.427
Demand for hired labor	99.09 (7.455)		102.6 (22.21)	- 37.01 (4.702)	- 88.86 (9.743)	-9.124 (1.199)	0.801
Demand for animals stocks	39.61 (2.755)			7.518 (4.795)	2.359 (3.499)	1.011 (0.346)	0.645
Demand for farm capital	233.17 (2.858)				235.68 (4.674)	- 32.48 (1.783)	0.874

In exploratory estimation attempts .he coefficients of the regional dummy variables were found to be insignificant and, moreover, it was found that the inclusion of these variables distorts the values of other coefficients. Given that the use of dummies is essentially an ad hoc procedure, it was decided not to use them in the model actually reported. The results obtained when the regional dummics were used were less consistent with economic theory and, in general, provided elasticity estimates which appeared to be quite unlikely.

The goodness-of-fit measure used is the 'generalized R^2 ' which was originally proposed by Baxter and Cragg (1970). The coefficient obtained (\tilde{R}^2) is very close to 1, indicating that the goodness-of-fit of the estimation is very good. Raw-moment R^2 coefficients are provided for the individual equations as complementary information.

In order for the estimated function $G(\cdot)$ to be a valid indirect utility function, the functions $A(\cdot)$ and $\psi(\cdot)$ should be concave and monotonically increasing in prices. These properties were checked using the estimated coefficients. The monotonicity property was satisfied by both functions at each sample point. The function $\psi(\cdot)$ is globally concave, that is, it is concave for all $p \ge 0$, since all α_i coefficients are positive. Unfortunately, the function $A(\cdot)$ is not globally concave and, moreover, it does not satisfy this property for 62% of the observation points. The calculated matrix of elasticities of substitution exhibits negative own substitution effects for approximately 55% of the observations.

The conditional profit function reported in part II of table 2 should also possess certain properties which have been checked for each of the sample points. Firstly, the conditional profit function has the correct gradients with respect to prices, that is, conditional profit increases with increases in the output price and is a decreasing function of input prices. Secondly, the estimated conditional profit function is positive at each of the sample points. Thirdly, the estimated conditional profit function should satisfy the required convexity property. The signs of the determinants of the principal minors associated with the Hessian matrix of the estimated conditional profit function were checked. Although this matrix is not positive semi-definite for 60% of the observations, its diagonal elements are all positive for more than 80% of the sample points, which implies that the own price net output supply elasticities have the correct signs when evaluated at most of the observations.

The fact that the required curvature conditions are not met at several of the data points may be due to various reasons. It is possible that it might be related to simple random sample errors and that the 'true' underlying functions do satisfy the appropriate curvature conditions. This is supported by the fact that the concavity-convexity properties are missed by extremely narrow margins in most of the data points not showing the required curvature. Another reason could be related to the use of aggregate data. It is well known that the curvature properties of the behavioral equations are derived for individual firms or households. Only under special circumstances will aggregate behavioral equations satisfy the individual concavity-convexity conditions. Most empirical studies use aggregate data and, in fact, our data is less aggregated than the data typically used by most empirical studies. A third reason may be related to the functional forms used. Although relatively general, the functional forms utilized may be too restrictive. That is, if the 'true' indirect utility and profit functions are not well represented by the functional forms used in the analysis then, obviously, the estimated functions will not necessarily satisfy the properties of the true functions. This is naturally a risk which any parametric approach has to face. Finally, a fourth possible reason may be that producers simply do not maximize utility or profits. In this case the application of duality or, more generally, the use of the hypotheses of utility and profit maximization in the specification of the model is unjustified.

Table 3 contains the on-farm and off-farm labor supply elasticities with respect to on-farm returns to farm household labor, the off-farm wage rate received by household members, and the household's non-labor income. The own wage elasticities of labor supply are both positive when evaluated at mean values, with the off-farm labor supply elasticity substantially larger than the on-farm elasticity. However, the on-farm supply elasticity is negative at 8% of the observations and the off-farm elasticity is negative at 19%. These estimates are not comparable with previous studies, because previous studies provide estimates for aggregate labor supply. The elasticity of total labor supply with respect to a simultaneous change in the on-farm labor returns and the off-farm wage rate is approximately 0.024, which is substantially lower than the labor supply elasticities obtained in Lau et al. (1978) using farm household data from Taiwan (0.16) and by Barnum and Squire (1979) who used data from Malaysia (0.08). Huffman (1980), using U.S. farm household data, obtained off-farm labor supply elasticities of 0.33 for husbands and -0.06 for wives. On the other hand, Wales and Woodland (1977) using a sample of U.S. households also found small positive supply elasticities for some households and negative elasticities for others. The average supply elasticities for husbands was 0.11 for those on the upwardsloping section of the supply curve and -0.32 for those on the downwardsloping part.

	On-farm labor returns	Off-farm wage rate	Non-labor income
On-farm labor		<u> </u>	
supply	0.119	-0.107	-0.612
Off-farm labor supply Total labor	-0.259	0.180	-0.539
supply	0.043	- 0.049	-0.237

 Table 3

 Labor supply elasticities (at mean values of the variables).

Table 3 also shows the cross-wage effects on labor supply. A 1% increase in the off-farm wage rate induces a 0.1% decrease in the number of days of on-farm work by the household members. The effect of on-farm labor returns on off-farm work is stronger. A 1% increase in farm labor returns induces a 0.25% decrease in the off-arm supply of labor. The effect of non-labor income on total labor supply is approximately -0.23. This can be compared with estimates obtained by Ashenfelter and Heckman, who found elasticities of -0.112 for males and -0.594 for females using U.S. cross-sectional data [Heckman et al. (1979)].

The effect of education on both off-farm and on-farm labor supply calculated using the coefficients reported in table 2 is positive, but its effect on off-farm work is substantially larger than that on on-farm work. In fact, while a 1% increase in formal years of schooling induces a 0.35% expansion of on-farm work, a similar increase in education leads to a 1.25% expansion of off-farm work. The effect of education on off-farm work can be compared with Huffman's (1980) estimated elasticity of off-farm work with respect to the farm operator's educational level, which was 1.03.

Table 4 presents the estimated labor supply elasticities with respect to output price changes evaluated at mean values.¹⁰ As might be expected, changes in the output price have the largest effect in terms of absolute values on off-farm and on-farm labor supply. A 1% increase in output price increases the on-farm labor supply by 0.39% and decreases the off-farm supply of labor by approximately 0.85%.

Table 5 presents the supply and demand elasticities conditional on L_1 for outputs and inputs evaluated as the mean prices. The conditional elasticities (S_{ij}) are defined as

 $S_{ii} = (\partial (Q_i/L_1)/\partial q_i)(q_i/(Q_i/L_1)).$

These elasticities can be interpreted as net output supply responses assuming that operator and family labor remain constant after a net output price has changed. The diagonal elements in table 5 show the own price elasticities which, as can be expected, are positive for output and negative for all inputs. The off-diagonal elements are the conditional cross-elasticities of supply for output and of demand for inputs.

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<sup>10</sup>The effect of net output price q<sub>i</sub> on labor supply L<sub>j</sub> is 

∂L<sub>j</sub>/∂q<sub>i</sub> = (∂L<sub>j</sub>/∂π̃)(∂π̃/∂q<sub>i</sub>) + (∂L<sub>j</sub>/∂Z)(∂Z/∂q<sub>i</sub>), j = 1, 2, i = 1,...,5. Using Hotelling's lemma we have that 

∂π̃/∂q<sub>i</sub> = Qi/L<sub>1</sub>. Hence, we can express ∂L<sub>j</sub>/∂q<sub>i</sub> in elasticity terms as 

ε<sub>L,q<sub>i</sub></sub> = ε<sub>Ljx</sub>(q<sub>i</sub>Q<sub>i</sub>/L<sub>1</sub>π̃) + ε<sub>LiZ</sub>(q<sub>i</sub>Q<sub>i</sub>/Z)(H/L<sub>1</sub>).
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Table	.4
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Labor supply elasticities with respect to net output prices (at mean values of the variables).

	Output price	Land price	Hired labor wage rate	Animal stock price	Farm capital price
On-farin labor supriy	0.390	-0.046	-0.027	-0.015	-0.145
Off-farm labor Supply	0.849	0.101	0.059	0.033	0.315

Та	ble	5
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Conditional net output supply elasticities (at mean values of the variables).

	Output price	Land price	Hired labor wage rate	Animal stock price	Farm capital price
Output	0.332	0.113	-0.126	- 0.049	- 0.269
Land	-0.912	-0.418	0.458	-0.016	0.888
Hired labor	1.557	0.797	-0.420	-0.600	- 1.334
Animal stocks	1.103	-0.053	-1.107	-0.006	0.063
Farm capital	0.626	0.298	-0.260	0.005	-0.660

Table 6

Unconditional net output supply elasticities (at mean values of the variables).

	Output price	Land price	Hired labor wage rate	Animal stock price	Farm capital price
Output	0.732	0.066	-0.153	-0.064	-0.414
Land	-0.522	0.464	0.430	-0.031	0.743
Hired labor	1.947	0.750	-0.447	-0.66	- 1.479
Animal stocks	1.493	-0.099	-1.134	-0.021	-0.082
Farm capital	1.016	0.251	-0.287	-0.010	-0.835

Table 6 contains the unconditional supply and demand elasticities.¹¹ These elasticities measure the actual market net output supply responses after the

¹¹The unconditional effect of a change in net output price q_j on output Q_i can be readily derived using (8),

$$\partial Q_i / \partial q_i = (\partial^2 \tilde{\pi} / \partial q_i \, \partial q_j) L_1 + (\partial \tilde{\pi} / \partial q_i) (\partial L_1 / \partial q_j).$$

This equation can be expressed in elasticity terms,

$$\varepsilon_{ij} = S_{ij} + \varepsilon_{L,\tilde{\pi}}(q_j Q_j / \tilde{\pi})$$

where ε_{ij} is the output *i* elasticity with respect to q_j , S_{ij} is the elasticity of Q_i/L_1 with respect to q_j , and $\varepsilon_{i,1\tilde{\pi}}$ is the elasticity of L_1 with respect to $\tilde{\pi}$.

effects of output or factor price changes on family and operator labor supply have been considered. The output supply elasticity obtained is 0.73, which is somewhat lower than supply elasticities obtained in previous studies for agriculture. For example, Tweeten and Quance (1969), using different procedures, obtained estimates of 0.31, 1.79, and 1.52 for long-run aggregate output supply elasticities in U.S. agriculture. The effects of factor price changes on output supply are generally small with the exception of the farm capital price. A 1% increase in the farm capital price induces a 0.4% decrease in output supply. Changes in the land price index have a small effect on the demand for all inputs with the exception of hired labor. Factor demands are not very responsive to changes in their own prices. All factors present rather inelastic demand schedules. These estimates can be compared with previous results for U.S. and Canadian agriculture. Binswanger's (1974) own factor demand elasticity estimates for U.S. agriculture are -0.34 for land, -0.91for labor. -1.089 for machinery, and -0.95 for fertilizers. Lopez' (1980a) estimates for Canadian agriculture are -0.52 for labor, -0.35 for farm capital, -0.42 for land, and -0.41 for intermediate inputs. Thus, although the results are not entirely comparable because the disaggregation of inputs is different and because these studies estimated compensated price elasticities (i.e., for a constant level of output), the general pattern of inelastic factor demands is consistent in the three studies.

The fact that the effect of the price of land on output supply is slightly positive or, equivalently, using the symmetry conditions, the fact that the effect of output price on demand for land is negative is quite surprising and may suggest that land is an inferior input. Although economic theory does not prevent the existence of inferior inputs, this result is indeed unlikely. One possible reason could be that as the output price increases and hence as output levels are expanded, the output composition changes towards outputs which are less intensive users of land, i.e., from crops to poultry and hog production. Thus, although the pure output scale effect may be positive, the negative effect on demand for land due to changes in the composition of outputs might predominate. It is also possible that the positive effect of land prices on output supply may be due to insufficient adjustment for land quality differences. Thus part of the land price variability may be associated with changes in land quality. Higher land prices may also imply better land and, hence, this may have a positive effect on output supply.

The joint significance of education on net output supply implies that the effect of the operator's education on resource allocation is non-neutral. The effect of education is biased toward livestock forms of capital and land and against all other factors. Education has a negative effect on output levels, which is quite surprising. A 1% increase in education induces a 0.09% reduction in farm output when this effect is evaluated at mean values. This may be due to increases in other investment opportunities outside

agriculture. Education would allow farmers to consider alternative, perhaps more profitable, sources of investment. Thus, the main effect of education would be to induce cost savings rather than output expansion. A 1% increase of non-labor income would lead to a reduction in the on-farm labor supply and the scale of production of 0.16% (table 3).¹² Thus, increasing the assets of farmers which yield higher non-labor returns leads to quite an important contraction in the scale of agricultural production (including output supply and input demand). This effect has been ignored in previous studies which have assumed that changes on the consumption side have no effects on net output supply.

Under the assumptions used, the impact of the off-farm wage rate on the scale of production is identical to its effect on on-farm labor supply. Thus, the elasticity of on-farm labor supply with respect to the off-farm wage rate is -0.107 and therefore, the elasticity of net output supply with respect to a change in w_2 will be the same. Hence, a 1% increase in off-farm wages received by farmers will cause a contraction in net output supply of approximately 0.1%.

5. Conclusions

A number of important conclusions emerged from the study:

- (1) The hypothesis of independence between utility maximizing and profit maximizing decisions was categorically rejected. Moreover, it was shown that important gains in explanatory power result from estimating the production and consumption sectors jointly.
- (2) The cross-effects between the unconditional net output supply equations and the labor supply responses were quantitatively very strong.
- (3) The frequently used hypothesis of homotheticity of preferences has been rejected. However, the test of the hypothesis of affine homotheticity provided less conclusive results; affine homotheticity was rejected at a 5% level of significance but not at 1% level.
- (4) It has been shown that farm operator's educational level has a significant non-neutral effect on the demand for inputs. Moreover, education also has a significant effect on labor supply responses and induces a reallocation of the household's labor from on-farm to olf-farm work.

Additionally, it is noted that the model estimated explains farm household's consumption and production decisions reasonably well. The model generates results which are generally consistent with economic theory and it represents a substantial improvement with respect to previous studies.

 12 Using eq. (8) the effect of an increase in non-labor income on net output surply can be obtained as

$$\partial Q_j / \partial y = (\partial L_1 / \partial Z) (\partial Z / \partial y) (\partial \tilde{\pi} / \partial q_j) = (\partial L_1 / \partial y) (\partial \tilde{\pi} / \partial q_j).$$

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