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Trust in cohesive communities ☆

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Abstract

This paper studies which social networks maximize trust and welfare when agreements are implicitly enforced. We study a repeated trust game in which trading opportunities arise exogenously and a social network determines the information each player has. The main contribution of the paper is the characterization of optimal networks under alternative assumptions about how information flows across a network. When a defection is observed only by the victim's connections, cohesive networks are Pareto efficient as they allow players to coordinate their punishments to attain high equilibrium payoffs. In contrast, when a defection is observed by the victim's direct and indirect connections, barely connected networks maximize the number of players that can punish a defection and are therefore efficient.

JEL classification: C73; D82; D85; Z13

Keywords: Repeated games; Social networks; Cohesiveness

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1. Introduction

The use of informal mechanisms of misconduct deterrence has been widely recognized and documented by economists (Milgrom et al., 1990; Dixit, 2006; Greif, 2006), political scientists (Ostrom, 1990; Fearon and Laitin, 1996), sociologists (Coleman, 1990; Raub and Weesie, 1990; Burt, 2001; Rauch, 2001), and legal scholars (Bernstein, 1992). Crucial to their use is the way in which trading partners get informed about mischievous actions. Several authors have argued that the social structure is a key determinant of information flows and of the sustainability of trust-based transactions. For instance, Greif (1993) studies contract enforcement between medieval Maghribi traders, and argues that a close-knit community can quickly disseminate information about its members' behaviors. Thus members' incentives can be aligned by employing community-based sanctions that punish behaviors deemed opportunistic.[1]

This paper formally explores optimal network architectures in the context of a repeated trust game in which the social network determines information flows and feasible punishments. Our main contribution is the characterization of optimal networks under alternative assumptions about how information flows across a network. Cohesive networks are efficient when information flows are slow, whereas barely connected networks are efficient when information spreads quickly within a component.

Our baseline model is a repeated game played by N investors and one agent. At each round $t \geq 1$, one of the N investors is randomly and uniformly selected to play a trust game with the agent. In the trust game, the investor decides whether or not to participate. If he participates, he also picks an action or investment level, then the agent chooses whether to cooperate or to defect. If the agent does not participate, stage game payoffs are 0 and the game moves to next round. The equilibrium of the stage game is inefficient as the agent will defect after an investment is made and, anticipating this behavior, the investor will not participate. The agent's temptation to defect may be curtailed by the existence of community sanctions governed by a social network of investors G. Our model exhibits *network monitoring* since after the agent misbehaves when facing investor i, then i and all his direct connections in G become aware of that, but players who are not connected to i learn nothing about it. We focus on perfect Bayesian equilibria that sustain cooperation on the path of play.

Our results characterize optimal networks. Theorem 1 establishes the Pareto optimality of any social network of equally sized complete components (i.e., networks in which all players have the same number of connections and if a player is connected to two other players, then all three are connected). Precisely, if G^* has equally sized complete components and G is such that no investor has more connections than in G^*, there exists an equilibrium σ^* under network G^* yielding higher expected payoffs to each community member than any equilibrium σ under network G. When the number of links is scarce and the number of connections per player is

[1] Other cases abound. In the automobile industry, for example, firms usually outsource large amounts of work and suppliers are routinely called upon to make specific investments. Hold-up problems are overcome by the threat of future business losses. As McMillan (1995) documents, one of the keys to deter opportunistic behavior in vertical relationships is the existence of cohesive business associations, such as Japanese keiretsus or Korean chaebols, that facilitate information exchange about parties' previous performances. McMillan (1995) observes that "the institutionalization of links among firms that is provided by the keiretsu system arguably serve as ... an information-provision device." He also notes that "by providing a mechanism for keeping track of any opportunistic behavior ..., the keiretsu provides a disincentive to such behavior."

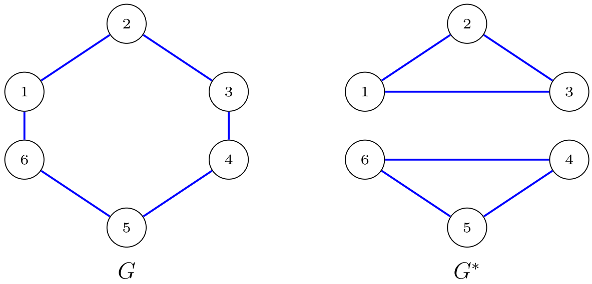

Fig. 1. Illustration of Theorem 1. Even when each investor has the same number of connections in each network, G^* yields higher payoffs than all equilibria of network G.

exogenously given, the formation of a fully cohesive social network is the best that all game players can hope for.[2]

The mechanism driving this result can be understood as follows. Consider networks G and G^* in Fig. 1 and note that in both networks each investor is connected to two other players. However, while in network G^* all players within a component can observe all defections, in G they cannot. Take, for example, investors 1 and 5 in network G and note that a defection against 1 is observed by 2 and 6 but not by 5. The agent can defect against 1 and then against investor 5, but investor 3 will still be willing to trade. In contrast, in network G^*, play within each component is observed by all players and punishments can be immediately implemented leaving no room for further off-path defections (defections against 1 and 5 leave the agent with no investor willing to participate in G^*). Thus, compared to G^*, network G allows the agent to defect twice (first against 1 and then against 5) by incurring a rather small loss in continuation value. Consequently, the network of complete components G^* results in higher equilibrium payoffs.

While characterizing optimal play in networks of complete components is simple, finding optimal equilibria in games with private monitoring (as the game with monitoring network G in Fig. 1) is difficult. In networks of incomplete components, optimal equilibria need not be in trigger strategies as, by ignoring some defections, an investor can strengthen the position of some of his neighbors and reduce the room for off-path gaming (see Proposition 3 and Example 2 in Appendix A.1). To prove Theorem 1, we sidestep the difficulties that private monitoring poses by studying relaxed equilibrium conditions. These conditions, combined with lattice theory tools (Topkis, 1998), yield upper bounds for investment paths that apply to all equilibria. A key intermediate result, Proposition 1, reveals the constraints that private monitoring imposes on equilibrium play by showing that such upper bounds cannot be attained in networks with some incomplete component. Theorem 1 then follows by comparing the upper bounds across different networks.

We also explore social networks that maximize the sum of players' equilibrium payoffs given an exogenous number of links. In this problem, a reassignment of links translates into utility transfers between different investors. As a result, the characterization of welfare maximizing networks will depend partly on the convexity properties of the map from the number of connections a player has to utility levels. Theorem 2 provides conditions under which welfare maximizing networks have complete components. Despite the fact that our upper bound on equilibrium in-

[2] Our results apply under the assumptions that the studied networks are feasible. For example, Theorem 1 applies when a regular network of complete components exists. Our results are silent otherwise due to integer problems.

vestments is slack in networks of incomplete components, the convexity of payoffs as a function of the number of links is a force towards an heterogenous distribution of connections that favors the formation of a star. A star can maximize social welfare since the central investor can leverage his connections and attain high equilibrium investments that can offset the low actions taken by other investors.

Section 4 studies our setting under a general network monitoring technology. It is assumed that, after a defection against some investor i, all investors within distance $d \geq 1$ of i observe it, and that an investor can observe whether one of his connections refused trade at t with probability $q \in [0, 1]$. This model reduces to our baseline model when $d = 1$ and $q = 0$.

Section 4.1 studies the model when a defection against i is immediately transmitted to all investors that are directly and indirectly connected to i (formally, here we impose $d = N$). Compared to the baseline model in Section 2, this alternative formulation can be seen as a polar assumption on the role of the social network – extremely slow information flows in the baseline model, very fast dissemination of news in Section 4.1. Theorem 3 shows that Pareto efficient networks of two or more components are minimally connected: in an optimal network, any two investors that are indirectly connected must be connected by a single path. Intuitively, with fast information flows, a minimally connected network maximizes the number of players that become aware of a defection and therefore more severe punishments are available. Theorem 4 nails down the sizes of the different minimally connected components of welfare maximizing networks.

Taken together, our results uncover a trade-off between networks of complete components and minimally connected networks. By doing so, we clarify two different sociological views on the merits of alternative social architectures (Coleman, 1990; Burt, 1992; Sobel, 2002). When information flows are slow, a network of complete components creates local common knowledge of play that facilitates the coordination of punishments (Theorem 1). In contrast, when information flows quickly within a component, minimally connected networks maximize the number of investors that can sanction a deviation (Theorem 3).

Section 4.2 explores the general model when $d = 1$ and $q > 0$. This is a complicated model, because on top of the incentive considerations, news percolate the network randomly and the agent must selectively decide whether or not to defect following a chain of defections. Theorem 5 shows that, under appropriate convexity restrictions, a star maximizes social welfare even if we impose no restriction on q. Thus, the star appears as a very robust welfare maximizing network structure under convexity restrictions.

Related literature Our paper belongs to the growing literature on repeated games in networks.[3] In our model, the network determines the monitoring technology as in Ahn and Suominen (2001) and Wolitzky (2013). Ahn and Suominen (2001) study a model similar to ours, in which the network of information transmission is drawn at the beginning of each round. They do not study optimal network design, nor do they explore how networks of complete components deter off-path gaming. Wolitzky (2013) studies a repeated public provision game in which the social network determines the monitoring technology. Since Wolitzky (2013) does not study optimal

[3] Kandori (1992) and Ellison (1994) are precursors to this literature. The literature is extensive and includes Haag and Lagunoff (2006), Bloch et al. (2008), Karlan et al. (2009), Mihm et al. (2009), Fainmesser (2012), Jackson et al. (2012), and Nava and Piccione (2013). We only discuss the papers that are closest to ours. See Nava (2015) for an overview of the literature.

networks, our research question is different. As he does, we use lattice theory techniques to prove our results, but in our model once a player defects whether or not defections keep occurring depends on the actions being implemented on the path of play. Wolitzky's (2013) analysis does not apply to our environment because in our model on-path actions are neither complements nor substitutes in general networks. As our examples show, a player's own maximal on-path action increases with the neighbors' actions, but decreases with the investment of more distant players. To prove our optimality results, we obtain estimates of continuation values and derive relaxed incentive constraints to bound equilibrium actions. A problem arising in our model but not in Wolitzky's (2013) is that the upper bound need not be tight. We explore conditions under which the bound is tight and use this result to derive optimal architectures under a variety of criteria.

When players only observe their own interactions, a social network of trading opportunities has also a key role spreading information about distant interactions. Lippert and Spagnolo (2011) and Ali and Miller (2013) show that cohesive networks facilitate information dissemination. Lippert and Spagnolo (2011) highlight the existence of cycles to spread punishments and attain cooperation. The main result in Ali and Miller (2013) establishes that a contagion-equilibrium in a network of complete components Pareto dominates all (on-path) stationary equilibria of networks having incomplete components. We need not restrict attention to stationary equilibria, and derive results about welfare maximizing networks (which need not be of complete components) under various assumptions on the monitoring technology. Our Theorem 1 highlights the fact that in networks of complete components, it is easier to coordinate punishments.[4] By disentangling the monitoring network from the network of interactions, we uncover a trade-off between networks of complete components and minimally connected networks that is absent in theses papers.

Organization Section 2 outlines the baseline model. Section 3 presents our optimal network design results. Section 4 studies our model under alternative information transmission technologies. Section 5 discusses extensions. All proofs are in the Appendix.

2. Set up

2.1. The environment

Our game has $N \geq 2$ investors and one agent. At the beginning of each round $t \geq 1$, an investor i^t is randomly and uniformly selected from the set $\{1, \ldots, N\}$. Investor i^t and the agent (hereinafter, player 0) may produce some surplus in round t by playing a trust game as in Kreps (1996) or Greif (1993). First, the investor decides whether to participate or not (P or NP). If he chooses not to participate, then period payoffs equal 0 and the game moves on to round $t + 1$. If investor i^t participates, he chooses an action $a \in \mathbb{R}_+$. After observing the investor's decision, the agent decides whether to cooperate (C) or to defect (D). Fig. 2 illustrates the game.[5]

[4] The fact that coordination is easier in networks of complete components is also observed by Chwe (2000) and Morris (2000). Our results emphasize how off-path coordination restricts on-path payoffs in a repeated game model of imperfect private monitoring.

[5] We could also formulate the model by assuming that the agent first chooses an action $a \in \mathbb{R}_+$ and then the investor and the agent play a trust game.

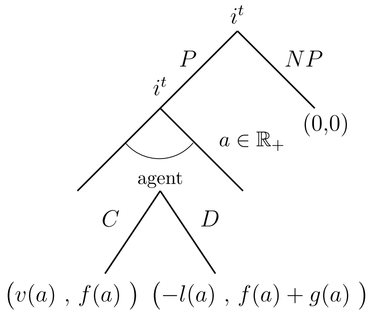

Fig. 2. The game between investor i^t and the agent. The first (resp. second) component of each payoff vector represents the investor's (resp. agent's) payoff.

Investors who are not selected get a per-period payoff equal to 0. Given a stream of period payoffs $(u_i^t)_{t\geq 1}$ for player $i \in \{0, \dots, N\}$, his utility function equals $\sum_{t\geq 1} \delta^{t-1} u_i^t$, where $0 < \delta < 1$ is the common discount factor.[6]

The following restrictions on payoffs are maintained throughout the paper.

Condition 1.

1. *For* $a \geq 0$, $f(a), v(a), l(a), g(a) \geq 0$ *with strict inequality if* $a > 0$.
2. g *is increasing,* f *is nondecreasing and* v *is increasing.*
3. g *is continuous and* f *is continuous and bounded.*
4. $g(0) < \frac{\delta}{1-\delta}\frac{1}{N} f(0)$ *and there exists* $\bar{b} > 0$ *such that* $g(a) > \frac{\delta}{1-\delta} f(a)$ *for all* $a \geq \bar{b}$.

Restriction 1 says that for any investment level, the agent's static best response is to defect. As a result, the investor decides not to participate and each player's payoff equals 0. This outcome is Pareto dominated by the outcome in which the investor participates and makes a positive investment and the agent cooperates. Restriction 2 implies that when the agent cooperates, both players benefit from a higher investment. Yet, the higher the investment, the larger the agent's temptation to defect. The third and fourth restrictions are mainly technical and are discussed after Lemma 1.

There is an undirected network of investors that determines the monitoring technology. More formally, a social network of investors is a symmetric matrix $G \in \{0, 1\}^{n\times n}$ such that $G_{ij} = 1$ if and only if i and j are linked. We also write $ij \in G$ whenever $G_{ij} = 1$ and assume $G_{ii} = 0$. Denote by $N(i, G) = \{j \mid ij \in G\}$ the set of i's neighbors in G and define the closed neighborhood of i as $\bar{N}(i, G) = N(i, G) \cup \{i\}$.

Players can monitor the agent's behavior. If i^t plays NP, then investors receive an empty signal $s_i^t = \emptyset$. If i^t chooses P, then all investors $j \in N(i^t, G)$ become aware of that and observe whether the agent cooperated or defected. More formally, if investor i^t participates and the agent played $x^t \in \{C, D\}$, player $j \in N(i^t, G)$ receives signal $s_j^t = (i^t, x^t)$, while if $j \notin \bar{N}(i^t, G)$, player j receives signal $s_j^t = \emptyset$. If i^t did not participate, then all players $j \neq i$ receive signal

[6] Our results also apply to a more general environment with N investors and $M \geq 1$ agents, as in Greif (1993), provided matching is random and uniform and that the incentive problem is one sided. In this case, there is no need to discipline the investors – so it is enough to focus on how they transmit information about each agent's behavior.

$s^t_j = \emptyset$. Player i^t perfectly observes play during round t. Players receive signals only about current interactions; in particular, we assume that information does not travel any further. A history h^t_i for investor i at the beginning of round t will consist of all the signals $(s^1_i, \dots, s^{t-1}_i)$ he has received during past play. We assume the agent has perfect information.

These assumptions capture the idea that the network determines the transactions each investor can observe. This modeling choice illustrates situations in which information is hard to transmit throughout the network, and other sources of information transmission (such as signaling through stakes or by refusing trade) are inexistent. One could also interpret these assumptions as saying that the victim (and only the victim) of a mischievous action let all his connections know that the agent is a miscreant. In on-line markets, for example, one is called upon to rate a seller only after trading with the seller. We postpone further discussion to Section 4.

Our repeated trust game can be interpreted broadly and captures key aspects of several repeated interactions. The agent can be seen as a firm that may or may not hold-up the specific investments made by suppliers (investors), as in Williamson (1979). The social network represents all business ties among the different suppliers (Greif, 1993; McMillan, 1995; Uzzi, 1996). The game can also be seen as a model in which consumers (investors) decide whether or not to buy experience goods from a monopolist (agent) who may be tempted to sell low-quality goods. The social network represents all sources of information on the monopolist performance (such as online feedback systems, word-of-mouth) as in Dellarocas (2003). The basic strategic structure of our repeated trust game also appears in models of relational contracting with multiple employees (investors), in which the employer may renege on payments to their employees.

We introduce some network terminology (for details consult Jackson, 2008). Let the set of investors that are at distance 2 of i be $N_2(i, G)$. This is formally defined as $N_2(i, G) = \{j \notin N(i, G) \mid jk \in G \text{ for some } k \in N(i, G)\}$. More generally, we define $N_d(i, G)$ as the set of investors that are at distance d of i: $N_d(i, G) = \left(\cup_{j \in N_{d-1}(i,G)} N(j, G) \right) \setminus \cup_{l=1}^{d-1} N_l(i, G)$. The set of players within distance $d \geq 2$ of i is written as $N_{\leq d}(i, G) = \cup_{l \leq d} N_l(i, G)$. The component of investor i, $C(i, G)$, will include i and the set of all j such that there exists a path $i_0, \dots, i_K$ with $i_n i_{n+1} \in G$ with $i_0 = i$ and $i_K = j$. It follows that if $j \in C(i, G)$, then $C(j, G) = C(i, G)$. Each network G has a finite number of components that partition the set of investors. If $C(i, G) = \{i\}$, we will say that the component is trivial. A network G has *complete components* if for any $j \in C(i, G)$, $ij \in G$; a network G *has some incomplete component* if there exists i, j such that $j \in C(i, G)$ but $ij \notin G$. We will say that G is κ*-regular*, for some $\kappa \geq 1$, if for all i such that $|N(i, G)| \geq 1$, we have $|N(i, G)| = \kappa$. A network is *regular* if it is κ-regular for some $\kappa \geq 1$. A network G is *minimally connected* if for any link $ij \in G$, $G \setminus \{ij\}$ has more components than G. When there is no risk of confusion, we omit the dependence of these sets on G (and write $N(i)$ instead of $N(i, G)$, for example).

2.2. Strategies and cooperation

A pure strategy for investor i is a family of functions $\sigma_i = (\sigma^t_i)_{t \geq 1}$ such that each σ^t_i maps private histories $h^t_i = (s^1_i, \dots, s^{t-1}_i)$ to stage game actions in $\{NP\} \cup (P \times \mathbb{R}_+)$.[7] A strategy σ_0 for the agent maps game histories, including the decision of investor i^t, to $\{C, D\}$. For any strategy σ and given any sequence of selected investors $(i^1, \dots, i^t)$, let $\alpha_\sigma(i^1, \dots, i^t) \in \{NP\} \cup$

[7] While we restrict attention to pure strategies, Theorems 1, 2, 3 and 4 can be extended to mixed strategies.

$(\{P\} \times \mathbb{R}_+)$ be the decision that player i^t makes in round t on the path of play.[8] We say that a strategy profile σ *sustains cooperation* if σ is a perfect Bayesian equilibrium of the game, and for all $(i^1, \ldots, i^t)$, $\alpha_\sigma(i^1, \ldots, i^t) \neq NP$ and, following a history

$$\big((i^1, \alpha_\sigma(i^1), C), \ldots, (i^{t-1}, \alpha_\sigma(i^1, \ldots, i^{t-1}), C), (i^t, \alpha_\sigma(i^1, \ldots, i^t))\big)$$

the agent's strategy σ_0 prescribes C. In words, a strategy sustains cooperation if on the path of play, in all encounters, the selected investor participates and the agent cooperates. Let $\Sigma(G)$ be the set of all strategies σ that sustain cooperation. We say that α_σ is *stationary* if the investment decision of player $i^t = i$ depends on i but not on the identity of players choosing in previous rounds. Formally, α_σ is stationary if $\alpha_\sigma(i^1, \ldots, i^t, i) = \alpha_\sigma(j^1, \ldots, j^{t'}, i)$ for all $i^1, \ldots, i^t, j^1, \ldots, j^{t'}, i \in \{1, \ldots, N\}$ and we simply write $\alpha_\sigma(i)$.

As can be expected, trigger strategies can sustain cooperation in networks of complete components. While trigger strategies can also sustain cooperation in networks of incomplete components, optimal equilibria need not be in trigger strategies as it could be beneficial that some players ignore some defections (Example 2 in Appendix A.1). A general characterization of strategies that sustain cooperation is beyond the scope of the paper, but Appendix A.1 discusses some new strategic effects arising in networks of incomplete components.

The equilibrium set depends on the monitoring network in subtle ways. Restricting attention to networks of complete components, more connections are always beneficial. However, in networks of incomplete components, more connections can hurt a player. Example 3 in Appendix A.1 shows that two investors may obtain higher payoffs in a network with smaller neighborhoods and components (even if we take the most favorable equilibrium in the more connected network). Intuitively, in networks of incomplete components some players may loose leverage after a defection has occurred. When a network has more connections, such leverage loss can be bigger and therefore equilibrium actions are smaller.

3. Characterization of optimal networks

In this section, we derive upper bounds on equilibrium actions and then characterize the architecture of Pareto dominant and welfare maximizing social networks. Note that when links are costless, the optimal network is the complete network in which all investors can punish any defection. However, in many applications links are costly and we therefore focus on optimal design given a fixed number of links.

3.1. Bounding equilibrium investments

Our first observation is that investments along an equilibrium path can be bounded above. Indeed, take any network G and $\sigma \in \Sigma(G)$ and assume, for now, that α_σ is stationary. Since $\sigma \in \Sigma(G)$, the agent prefers to cooperate in each encounter over defecting against i and cooperating in subsequent encounters with players outside $\bar{N}(i, G)$. Note that after a defection against i, investors $j \in \bar{N}(i, G)$ are aware of the defection and can inflict a punishment yielding period payoffs to the agent greater than or equal to 0.[9] Investors $j \notin \bar{N}(i, G)$ are still willing to participate and cooperation in those encounters remains feasible. This implies that

[8] This means that $\alpha_\sigma(i^1, \ldots, i^t)$ is the action played at t by player i^t when all players have followed σ.
[9] It will be 0 only if investors $j \in \bar{N}(i, G)$ use trigger-strategies, but this need not be the case.

$$f(\alpha_\sigma(i)) + \frac{\delta}{1-\delta}\frac{1}{N}\sum_{j=1}^{N} f(\alpha_\sigma(j)) \geq (f(\alpha_\sigma(i)) + g(\alpha_\sigma(i))) + \frac{\delta}{1-\delta}\frac{1}{N}\sum_{j\notin \bar{N}(i,G)} f(\alpha_\sigma(j)).$$

We thus deduce that

$$g(\alpha_\sigma(i)) \leq \frac{\delta}{1-\delta}\frac{1}{N}\sum_{j\in \bar{N}(i,G)} f(\alpha_\sigma(j)) \quad i = 1, \dots, N. \tag{3.1}$$

The left-hand side of (3.1) is the short-term gain from a defection against i, whereas the right-hand side is an upper bound for the losses in continuation value from the same defection.[10]

Equation (3.1) defines a system of inequalities, where the unknowns are actions $(a_i)_{i=1}^{N} \in \mathbb{R}_+^N$. Our restrictions on f and g in Condition 1 allow us to use Tarski's fixed point theorem (Topkis, 1998) to deduce that such system has a largest solution, $\bar{a}^G$, so that $\alpha_\sigma(i) \leq \bar{a}_i^G$ for all $i = 1, \dots, N$. Moreover, at $\bar{a}^G$, all inequalities must actually bind. While we have assumed that α_σ is stationary, the following result shows such restriction is not really needed.

Lemma 1. *For any G, the following hold.*

a. *The system of inequalities*

$$g(a_i) \leq \frac{\delta}{1-\delta}\frac{1}{N}\sum_{j\in \bar{N}(i,G)} f(a_j),\ i = 1, \dots, N$$

has a largest solution $\bar{a}^G \gg 0$. *At* $\bar{a}^G$, *all inequalities above bind.*

b. *For all* $\sigma \in \Sigma(G)$ *and all* $(i^1, \dots, i^t) \in \{1, \dots, N\}^t$, $\alpha_\sigma(i^1, \dots, i^t) \leq \bar{a}_{i^t}^G$.

The proof builds on arguments appearing in the proof of Theorem 1 in Wolitzky (2013). Wolitzky (2013) obtains a system of incentive constraints and proves such system has a largest solution. In this paper, however, we derive a system of relaxed incentive constraints that has a largest solution. More importantly, a key question (which does not appear in Wolitzky, 2013) is whether such largest solution can actually be implemented as an equilibrium outcome – a question answered in Proposition 1 (and Proposition 3 in the Appendix). We then build on these preliminary results to derive the architecture of optimal networks.

Restrictions 3 and 4 in Condition 1 are crucial to derive Lemma 1. Consider the case of only 1 investor and assume that $\frac{\delta}{1-\delta} f(a) = 1$. Condition 1, part 4, fails when either (i) g is above $f \equiv 1$ (for example $g(a) = 2 + a$) or (ii) g is always below $f \equiv 1$ (for example, $g(a) = 1 - \exp(-a)$). In the first case, Lemma 1 does not hold because the system of incentive constraints does not have a solution (and the only equilibrium of the repeated game is the repetition of the stage game equilibrium). In the second case, every action a can be an equilibrium and therefore $\bar{a}^G$ cannot be defined. The existence of a largest investment profile can also fail when the third restriction in Condition 1 is not satisfied. For example, if $g(a) = a^2$ for $a < 1$ and $g(a) = 1 + a$ for $a \geq 1$, any $a < 1$ can be an equilibrium but there is no largest equilibrium action.

We now turn to the problem of whether $\bar{a}^G$ is a tight bound for equilibrium actions.

10 Note that (3.1) does not apply to equilibria in which the agent defects on the path of play. When the agent is allowed to defect in some encounters, our bounds for equilibrium investments need not hold.

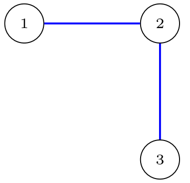

Fig. 3. A network where investors 1 and 3 are not connected.

Definition 1. Let G be a network and $a \in \mathbb{R}^N_{++}$. We say that the equilibrium $\sigma \in \Sigma(G)$ *implements a* if for all $(i^1, \dots, i^t) \in \{1, \dots, N\}^t$, $\alpha_\sigma(i^1, \dots, i^t) = a_{i^t}$.

The incentive constraints exploited to obtain the upper bound $\bar{a}^G$ need not be sufficient for equilibrium since optimal behavior following a defection by the agent could, for example, involve defections when facing some of the remaining investors. It is entirely possible that for some networks, no equilibrium can actually attain $\bar{a}^G$ on the path of play. The following result, which is the main contribution of this subsection, provides necessary and sufficient conditions to implement $\bar{a}^G$.

Proposition 1. *Let G be a network. The following are equivalent:*

a. G is a network of complete components.
b. There exists $\sigma \in \Sigma(G)$ that implements $\bar{a}^G$.

Proposition 1 shows that the action profile $\bar{a}^G$ can be implemented as an equilibrium outcome if and only if G is a network of complete components. When G is a network of complete components, it is relatively easy to construct σ to implement $\bar{a}^G$ along the path of play. Indeed, trigger strategies suffice: on the path of play, investor i participates and makes investment decision $\bar{a}_i^G$; if the agent ever defects against $j \in \bar{N}(i, G)$, investor i refuses trade in all subsequent interactions. The agent cooperates against any investor who decides to participate and who has not observed any defection. Since G is a network of complete components and (3.1) holds, $\sigma \in \Sigma(G)$.

The converse is more subtle and reveals some important constraints arising due to the fact that in networks with incomplete components, monitoring is private. Take, for example, network G in Fig. 3. Suppose that there exists $\sigma \in \Sigma(G)$ such that in all encounters $\alpha_\sigma(i) = \bar{a}_i^G$, $i = 1, 2, 3$. Since $\bar{a}^G$ is the largest solution to the system of relaxed incentive constraints, (3.1) binds and thus

$$f(\bar{a}_1^G) + \frac{\delta}{1-\delta}\frac{1}{3}\sum_{j=1}^{3} f(\bar{a}_j^G) = f(\bar{a}_1^G) + g(\bar{a}_1^G) + \frac{\delta}{1-\delta}\frac{1}{3} f(\bar{a}_3^G). \tag{3.2}$$

From the agent's perspective, cooperating in all encounters is as attractive as defecting against investor 1 and cooperating in all encounters with investor 3.

We argue that the strategy σ cannot be sequentially rational for the agent. Indeed, following an off-path defection against 1, both players 1 and 2 must refuse trade in all subsequent rounds. In the continuation game right after a defection against 1, player 3 has less leverage because he is uninformed and player 2 is already punishing the defection against 1. It is therefore in the interest of the agent to defect when facing 3. In other words, after a defection against 1, the agent strictly prefers to defect against the uninformed player 3. This implies that the total expected payoff the

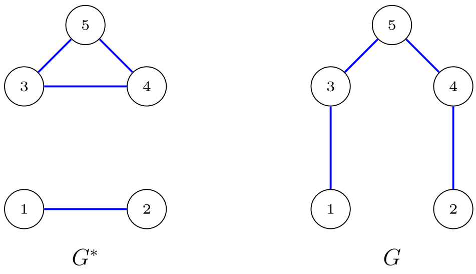

Fig. 4. The number of connections each player has is the same in both networks.

agent obtains after a defection against 1 is strictly higher than the payoff he would obtain by defecting against 1 and cooperating when facing 3 (which equals the payoff from cooperating always, as Equation (3.2) shows). Therefore, for the agent it is optimal to defect when facing investor 1 in the first place. The strategy profile σ cannot be sequentially rational and thus that $\bar{a}^G$ cannot be implemented in the network of incomplete components G.

3.2. The design of optimal networks

Given G and a strategy profile σ, let $u_i(\sigma, G)$ be the expected normalized sum of discounted payoffs for player i when play evolves according to σ and the social network is G.[11] We will say that G^* *Pareto dominates* G if there exists $\sigma^* \in \Sigma(G^*)$ such that for all $\sigma \in \Sigma(G)$, $u_i(\sigma^*, G^*) \geq u_i(\sigma, G)$ for all $i = 0, 1, \ldots, N$ with at least some strict inequality.

Theorem 1. *Let G^* be a regular network of complete components. Let G be any other network having some incomplete component such that $|N(i, G)| \leq |N(i, G^*)|$ for all $i = 1, \ldots, N$. Then, G^* Pareto dominates G.*

The proof of Theorem 1 exploits Lemma 1 and Proposition 1. The first observation is that, since each player has more connections in G^* than in G, $\bar{a}_i^G \leq \bar{a}_i^{G^*}$ for all i. Proposition 1 is then used to show that no equilibrium in network G can implement $\bar{a}^G$, whereas some equilibrium $\sigma^* \in \Sigma(G^*)$ implements $\bar{a}^{G^*}$. The result follows since payoffs are monotone in a_i.

As the following example shows, the assumption that G^* is regular in Theorem 1 cannot be dispensed with.

Example 1. Consider networks G^* and G in Fig. 4. Each investor has the same number of connections in each network. Network G^* has complete components but is not regular, while network G has a single component which is incomplete. Assuming that both f and g are strictly increasing and that $g(0) < \frac{\delta}{5} f(0)$, we will show that there exists an equilibrium in G that results in higher payoffs for player 1 than *all* equilibria in network G^*.

Take the upper bound $\bar{a}^{G^*}$ and note that $(\bar{a}_1^{G^*}, \bar{a}_2^{G^*})$ is the largest solution to

$$g(a_1) \leq \frac{\delta}{1-\delta} \frac{1}{5} \Big(f(a_1) + f(a_2) \Big) \tag{3.3}$$

[11] More formally, $u_i(\sigma, G) = \mathbb{E}_{\sigma,G}[\sum_{t \geq 1} \delta^{t-1} u_i^t]$, where u_i^t is player i's payoff at time t.

$$g(a_2) \leq \frac{\delta}{1-\delta}\frac{1}{5}\Big(f(a_1) + f(a_2)\Big). \tag{3.4}$$

Therefore, no equilibrium can implement investments higher than $\bar{a}_1^{G^*}$ for investor 1 in network G^*.

To construct an equilibrium for network G, take $\bar{a} = (\bar{a}_i)_{i=1}^5$ as the largest solution to

$$g(a_1) \leq \frac{\delta}{1-\delta}\frac{1}{5}\Big(f(a_1) + f(a_3)\Big) \tag{3.5}$$

$$g(a_3) \leq \frac{\delta}{1-\delta}\frac{1}{5}\Big(f(a_1) + f(a_3) + f(a_5)\Big) \tag{3.6}$$

$$g(a_i) \leq \frac{\delta}{5} f(a_i) \quad \text{for } i = 2, 4, 5 \tag{3.7}$$

We can construct an equilibrium that implements $\bar{a}$ in network G as follows. Along the path of play, investor i participates and chooses $\bar{a}_i$. If investor 1 or investor 3 observes a defection against either of them, both refuse trade in all subsequent rounds. Investor 3 ignores any defection against 5. Player 5 does not participate in subsequent trading opportunities if he observes a defection against 3. A defection against either 2, 4 or 5 is punished only by the victim by refusing trade only in the next round (which happens with probability 1/5) and then resuming play as no defection had occurred. The agent cooperates when facing any investor who, according to his strategy, should participate and invest (including off-path histories). This strategy profile is sequentially rational for all players. Indeed, the system (3.5)–(3.6)–(3.7) ensures it is in the agent's interest to cooperate on the path of play. Following a defection against either 1 or 3, it is still in the agent's interest to cooperate when facing players who still participate (2, 4 and 5 if the defection was against 1, 2 and 4 if the defection was against 3) as those players did not count on either 1 or 3 to punish defections against them. Defections against either 2, 4 or 5 do not affect agents' continuation values as those players punish for a single round. In sum, it is in the agent's interest to cooperate when facing any investor who participates.

We now claim that $\bar{a}_1 > a_1^{G^*}$. Consider the systems of equations (3.3)–(3.4) in (a_1, a_2) and (3.5)–(3.6) in (a_1, a_3), where $a^5 = \bar{a}^5$ is treated as a fixed parameter. By construction $(\bar{a}_1^{G^*}, \bar{a}_2^{G^*})$ is the largest solution to (3.3)–(3.4), whereas $(\bar{a}_1, \bar{a}_3)$ is the largest solution to (3.5)–(3.6). The two systems of equations coincide, save for the fact that the second equation in the second system has the constant term $f(\bar{a}^5)$. Since both systems of equations are fixed point conditions for monotonic operators and $f(\bar{a}^5) > 0$, $(\bar{a}_1, \bar{a}_3) \geq (\bar{a}_1^{G^*}, \bar{a}_2^{G^*})$. Now, if $\bar{a}_1 = \bar{a}_1^{G^*}$, equations (3.3) and (3.5) can be solved out to deduce that $\bar{a}_3 = \bar{a}_2^{G^*}$ (f is strictly increasing). Then $(\bar{a}_1, \bar{a}_3)$ is the largest solution to (3.5)–(3.6), but (3.6) is not binding. This is a contradiction (see the discussion and the proof of Lemma 1). It follows that $\bar{a}_1 > \bar{a}_1^{G^*}$.

In this example, investor 1 can enjoy a higher equilibrium payoff in network G than in G^*, even when in both networks the distributions of connections coincide, $|N(1, G^*)| = |N(1, G)|$ and G^* has complete components.[12] In network G, investor 1 is connected to investor 3, who is also connected to 5. In contrast, in network G^* investor 1 is connected to investor 2 who is not connected to other players. What makes G attractive is a network effect that can be ruled out when $\bar{a}_i^G$ is determined by the number of connections i has in G. The next assumption is crucial to achieve that.

[12] Note however that $|C(1, G)| > |C(1, G^*)|$. This is needed for otherwise Theorem 1 would apply and G^* would Pareto dominate G.

Assumption 1. f attains its maximum.

This assumption says that the agent's payoff from cooperation does not grow beyond an upper bound. Under this assumption, let $\underline{a}$ be the smallest action attaining the maximum $\max f$.[13] Define

$$\bar{\delta} = \min\left\{\delta \mid \frac{\delta}{1-\delta}\frac{1}{N}\max f \geq g(\underline{a})\right\}$$

and note that $0 \leq \bar{\delta} < 1$.

Lemma 2. *Under Assumption 1, for $\delta > \bar{\delta}$, and all $i = 1, \ldots, N$,*

$$\bar{a}_i^G = g^{-1}\left(\frac{\delta}{1-\delta}\frac{\max f}{N}|\bar{N}(i, G)|\right).$$

When f is constant, the restriction $\delta > \bar{\delta}$ is equivalent to $g(0) < \frac{\delta}{1-\delta}\frac{1}{N}f(0)$.[14] A constant f means that regardless of the investment made in a given round, the way in which the investor and the agent divide the surplus leaves the investor with a constant amount that, for example, covers his effort costs plus a premium (like in efficiency wage models).

Lemma 2 allows us to simplify the design problem by focusing on the network effects that arise purely due to informational frictions. Intuitively, when $\delta > \bar{\delta}$, the i-th component of the upper bound $\bar{a}^G$ is large enough so that it lies in the flat portion of f and therefore, as a function of G, it is entirely determined by the number of connections investor i has.

The following optimality result applies to networks that are not necessarily regular. It shows how Assumption 1 and the restriction to $\delta > \bar{\delta}$ eliminate the network effects illustrated in Example 1.

Proposition 2. *Under Assumption 1, for any $\delta > \bar{\delta}$, any network of complete components G^* and any network G having some incomplete component with $|N(i, G)| \leq |N(i, G^*)|$ for all $i = 1, \ldots, N$, G^* Pareto dominates G.*

This result follows by combining Lemma 2 with the arguments in Theorem 1.

We are also interested in characterizing networks that maximize total welfare. We will say that G^* *gives strictly more total welfare* than G if there exists $\sigma^* \in \Sigma(G^*)$ such that for all $\sigma \in \Sigma(G)$, $\sum_{i=0}^N u_i(\sigma^*, G^*) > \sum_{i=0}^N u_i(\sigma, G)$. We will also write

$$U(x) = v \circ g^{-1}\left((x+1)\frac{\delta}{1-\delta}\frac{1}{N}\max f\right),$$

for $x \in \mathbb{N}$, where $v \circ g^{-1}(y) = v(g^{-1}(y))$ is the composition of v and g^{-1}. Under the conditions of Lemma 2, $U(x)$ can be interpreted as the period payoff an investor obtains when trading given a number x of connections when the upper bound on equilibrium actions is implemented.

[13] Formally, $\underline{a} = \min\{a \mid f(a) = \max f\}$ is well defined since f is continuous and attains its maximum.

[14] Assumption 1 and Condition 1 have different roles. Restrictions 3 and 4 in Condition 1 ensure that $g^{-1}\left(\frac{\delta}{1-\delta}\frac{\max f}{N}|\bar{N}(i, G)|\right)$ is well defined in that the range of g includes $\frac{\delta}{1-\delta}\frac{\max f}{N}|\bar{N}(i, G)|$. Assumption 1 and the restriction $\delta > \bar{\delta}$ ensure that $g^{-1}\left(\frac{\delta}{1-\delta}\frac{\max f}{N}|\bar{N}(i, G)|\right)$ is the largest solution to the system of constraints.

Theorem 2. *Under Assumption 1, let $\delta > \bar{\delta}$, G^* and G be networks such that $\sum_{i=1}^{N} |N(i, G)| \leq \sum_{i=1}^{N} |N(i, G^*)|$. The following hold:*

a. If U is linear and G^* is a network of complete components but G is not, then G^* gives strictly more total welfare than G.
b. If U is strictly concave and G^* is a network of equally sized complete components but G is not, then G^* gives strictly more total welfare than G.
c. If U is convex and satisfies

$$U(x+1) - U(x) \geq \max\left\{x(U(2) - U(0)), 2(U(x) - U(x-1))\right\},\ 2 \leq x \leq N-2 \quad (3.8)$$

and G^* is a star but G is not, then G^* gives strictly more total welfare than G.

Theorem 2 characterizes welfare-maximizing networks given a number of links to be freely allocated among the investors. The key determinants of the architecture of optimal networks are the convexity properties of $U = v \circ g^{-1}$. To see (b), note that when U is concave, an uneven assignment of links implies that the equilibrium actions of highly connected investors only slightly decrease when a link is reassigned to a barely connected investor, who can significantly increase investments due to the addition of a new link. The concavity of U is therefore a force towards equally sized components, that must be complete to avoid the implementation losses described in Proposition 1. Note that $v \circ g^{-1}$ is concave provided so are v and $-g$.

Part (a) follows since the linearity of U implies that the incremental gains and losses when links are reassigned do not depend on how connected the different players are. To attain optimality, therefore, it is enough to avoid the implementation losses of networks with incomplete components. The function $v \circ g^{-1}$ is linear provided $g(a) = \alpha v(a)$, where $0 < \alpha < 1$. This condition arises in models in which the agent can defect and appropriate a fraction α of the investors' payoffs $v(a)$.[15]

Part (c) is probably the most surprising one as it shows that a network of incomplete components –the star– can maximize social welfare. When assigning links to maximize total welfare, the convexity of U is a force towards an heterogenous distribution of connections and favors the formation of a star. On the other hand, as Proposition 1 shows, some implementation losses must be incurred when forming the network of incomplete components G^*. When U is "slightly convex", the implementation losses can be significant compared to the gains from a heterogenous distribution of links.[16] The strong convexity restriction (3.8) ensures that those implementation losses are small compared to the heterogeneity gains in the star.[17] Two different

[15] This happens in models where firms invest in non-contractible human capital and wages are determined ex-post by Nash-bargaining as in Acemoglu and Pischke (1999).

[16] Note that the problem of assigning a given number of links is different to the problem of assigning consumption among agents given a budget constraint. In the first problem, by reassigning a link one takes away one connection from two nodes but we cannot reassign both of them to a single node.

[17] Consider a model with $N = 4$, $\delta = 1/2$, $g(a) = a$, $f = 1$, and $v(a) = a^n$ with $n \geq 1$. Suppose that we must assign two links. Forming two complete components results in payoffs $\frac{1}{4}(1/2)^n$ for each player and welfare $W = 4\frac{1}{4}(1/2)^n = (1/2)^n$. Now, consider a star formed by players 1, 2 and 3, with player 2 being the center of the star. Let $a = (a_1, a_2, a_3, a_4)$ be a vector of equilibrium actions. From Proposition 3 in the Appendix, $\frac{1}{4} \leq 5|\frac{1}{2} - a_1| + |\frac{1}{2} - a_3|$. As a result, in any equilibrium $a_1^n + a_3^n \leq (1/2)^n + (1/4)^n$. The highest welfare that can be attained in the star is at most $\tilde{W} \leq \frac{1}{4}(3/4)^n + \frac{1}{4}(1/2)^n + \frac{1}{2}(1/4)^n$. It is therefore clear that for $n > 1$ close enough to 1, $W > \tilde{W}$. Thus, when $v \circ g^{-1}$

parameterizations of our model that satisfy (3.8) are (i) $v \circ g^{-1}(x) = \exp(x) - A$ when $A \geq 1$ and $\frac{\delta}{1-\delta}\frac{1}{N}\max f > \ln(2)$ and (ii) $v \circ g^{-1}(x) = x^p$ with $p > 1 + \frac{\ln(2)}{\ln(N/(N-1))}$.[18]

For any network G, we denote its degree distribution by

$$F^G(n) = \frac{|\{i \mid |\bar{N}(i,G)| \leq n\}|}{N}, \quad n = 0, 1, \ldots, N.$$

We say that network G^* is *more equally sized than* G if for all $n = 1, \ldots, N$

$$\sum_{n=1}^{\bar{n}} (F^G(n) - F^{G^*}(n)) \geq 0 \text{ for all } \bar{n} = 1, \ldots, N-1.$$

This definition is identical to the standard notion of second order stochastic dominance, restricted to a discrete distribution (Mas-Colell et al., 1995).

Corollary 1. *Under Assumption 1, let $\delta > \bar{\delta}$, G^* and G be networks such that $\sum_{i=1}^N |N(i,G)| = \sum_{i=1}^N |N(i,G^*)|$. If U is (weakly) concave, G^* is a network of complete components, and G^* is more equally sized than G, then G^* gives weakly more total welfare than G.*

This result shows that a concave map U favors the formation of more equally sized components. This result follows since G^* is a network of complete components and therefore its social welfare is entirely determined by the degree distribution induced by G^*.

4. Networks with richer information flows

Let $(d, q) \in \mathbb{N} \times [0, 1]$ parameterize the monitoring technology in the following way. Given G, histories are recursively determined as follows. The history i has at the beginning of $t = 1$ (before play) is $h_i^0 = \emptyset$. Let h_i^{t-1} be the history player i has at the beginning of t and let $i^t \in \{1, \ldots, N\}$ be the investor chosen at round t. If i^t chooses P, then all investors within distance d of i become aware of that and observe whether the agent cooperated or defected, whereas if i^t chooses NP investors j within distance d of i become aware of this decision with probability q. More formally, if investor i^t participates and the agent played $x^t \in \{C, D\}$, then player $j \in N_{\leq d}(i^t, G)$ receives a signal $s_j^t = (i^t, x^t)$, while if $j \notin N_{\leq d}(i^t, G)$ then j receives a signal $s_j^t = \emptyset$. If i^t chooses NP, then with probability $(1-q)$ (resp. q) each $j \in N_{\leq d}(i^t, G)$ receives $s_j^t = \emptyset$ (resp. $s_j^t = i^t$). Player i^t perfectly observes play during round t and knows all the observations that other investors in $N_{\leq d}(i, G)$ make about i^t's interaction with the agent. So, the signal that player i receives at t belongs to

$$\{\phi\} \cup \Big(N_{\leq d}(i,G) \times \{C, D\}\Big) \cup \Big(N_{\leq d}(i,G)\Big) \cup \Big(\{NP\} \cup (\mathbb{R}_+ \times \{C, D\} \times N_{\leq d}(i,G))\Big)$$

and the history investor i has at the beginning of $t+1$ is then defined as $h_i^t = \left(h_i^{t-1},\ s_i^t\right)$. We assume that the agent has perfect information and observes all information flows.

is convex but does not satisfy (3.8), the star need not be optimal. On the contrary, welfare is maximized by a network of complete components.

[18] Under these parameterizations, (3.8) holds with strict inequality. Small perturbations to these functions also satisfy (3.8). Note that if U_1 and U_2 satisfy (3.8), then $U \equiv U_1 + U_2$ also satisfy (3.8).

This model extends the setup analyzed in the previous section. When $q = 0$ and $d = 1$, we are back in the model of Section 2. When $d = 1$ and $q > 0$, we obtain a model in which investors can learn by observing the participation decisions of their neighbors. This is a model in which informed investors can signal their information through their participation decisions. (The results in this Section would also extend to allow investors to signal through actions a. In general, though, one should expect that signaling through investments should be more effective as more precise information can be transmitted.) When $d = N$, investors observe play of all other investors in the same component. In this model, once a player becomes informed, he can rapidly transmit his information to all his direct connections. Those direct connections can also transmit the new information to their own connections, and so on, and all these information flows occur fast enough so that the whole component becomes informed before a new trading opportunity arises.

The definition of strategy σ_i for player i is similar to that presented for the main model. We also define the set $\bar{\Sigma}(G, d, q)$ of equilibrium strategy profiles $\sigma = (\sigma_i)_{i=0}^N$ that sustain cooperation on the path of play. We also (with a slight abuse of notation) define $u_i(\sigma, G)$ as the expected sum of discounted payoffs given the network G and strategy profile σ.

Characterizing equilibria for the general game is extremely difficult; news travel through the network and, after a defection, the agent selectively decides whether or not to defect following a chain of defections. As a result, incentives and the process of information transmission are jointly determined in a non-trivial fashion.[19] Yet, as the following subsections show, some of the tools and insights from the previous sections can be used to nail down optimal networks in this general environment.

4.1. Social networks of rapid information transmission

We now study the general model under the assumption that $d = N$. This means that once the agent defects against i, all players in $C(i, G)$ become immediately informed.[20] In particular, for any network G, the system of incentive compatibility constraints becomes

$$g(a_i) \leq \frac{\delta}{1-\delta} \frac{1}{N} \sum_{j \in C(i)} f(a_j), \quad i = 1, \ldots, N$$

and has a largest solution $\hat{a}^G \in \mathbb{R}_{++}^N$. Moreover, $\hat{a}^G$ can be implemented as a trigger strategy equilibrium, and no equilibrium can implement higher actions.

Theorem 3. *Let G be a network which is not minimally connected having at least two components. Then, G is Pareto dominated by a network G^* having the same number of links.*

This result shows that Pareto efficient networks must be minimally connected, unless the number of links is large enough to form a single component. Intuitively, if G is not minimally connected, we can give an alternative use to one of the links in order to expand the number of players that become aware of any defection. In the new network more players can punish a defection and therefore on-path actions are higher.

19 Results on diffusion in networks can be found in Jackson (2008, Chapter 7), but those models apply to very special network structures. Results on rumor spreading (Pittel, 1987) typically focus on approximating the distribution of the number of periods that take a single rumor to reach the whole network.

20 In terms of the model of Section 2, this is similar to restricting attention to networks of complete components. The difference is that in this model once an investor is connected to some player, it becomes immediately connected to all players in the component at no cost.

To characterize welfare maximizing networks, note that $\hat{a}_i^G$ depends on G only through the size of the component i belongs to, $|C(i,G)|$. We thus write $\hat{a}_i^G = \hat{a}_i(|C(i,G)|)$. Up to a constant, the equilibrium payoff player i obtains in component C is $\hat{U}(|C|) = v\big(\hat{a}_i(|C|)\big)$. We also define the total welfare that members of component C obtain as $\Phi(|C|) = |C|\hat{U}\big(|C|\big)$.

Theorem 4. *Let G^* be a network that maximizes total welfare, and having $|E| = \sum_{i=1}^N \frac{|N(i,G^*)|}{2} \leq N-1$ links. Then, G^* is minimally connected. Moreover, the following hold:*

a. If Φ is strictly convex, G^* has a single non-trivial component.
b. If Φ is strictly concave, then G^* has $N-E$ components. Taking $\bar{n}$ as the only integer such that $\frac{N}{N-E} - 1 < \bar{n} \leq \frac{N}{N-E}$, a component of G^* is formed by $\bar{n}$ or $\bar{n}+1$ nodes. If $\bar{n} = \frac{N}{N-E}$, then all components have size $\bar{n}$.

The restriction $\frac{1}{2}\sum_{i=1}^N |N(i,G^*)| \leq N-1$ implies that the problem is nontrivial; if it did not hold welfare maximizing networks would have a single component of N or more links. When Φ is strictly convex, a network with two components can be improved by reassigning connections to form a single component involving all but one of the investors from the original components. This implies that in order to maximize welfare, a network must have a single nontrivial component. In contrast, when Φ is concave, two differently sized components can be improved by making their sizes more homogeneous. Therefore, maximizing total welfare requires that all components are of a similar size and therefore their sizes differ in at most one player. The rest of the characterization in part b is just an accounting exercise.[21]

Theorem 4 relates to results on cooperation and group size. Pecorino (1999) and Haag and Lagunoff (2007) explore how the addition of new players determines cooperation in repeated public good games with perfect monitoring. In contrast, we fix the total number of players and simply assign investors to be part of different components.

Discussion: closure versus structural holes There are two main sociological perspectives on the benefits of different social architectures for welfare (Burt, 2001; Sobel, 2002). On the on hand, the closure view set forth by Coleman (1990) emphasizes that in networks with high closure and cohesion, the enforcement of cooperative behaviors is more effective. In Coleman's (1990) argument, cohesion facilitates the implementation of sanctions and therefore leads to higher welfare. On the other hand, Burt's (1992) structural hole argument poses that the existence of players linking otherwise disconnected groups is socially beneficial as those players can bridge non-redundant sources of information.[22]

As our analysis makes clear, this seemingly conflicting views can be reconciled after observing that they apply to different models of information flows on networks. The closure view is consistent with the model presented in Section 2, in which once a player becomes aware of a defection against one of his direct connections, he cannot pass that information to other players. As shown in Section 3, the key game theoretical reason that makes cohesion desirable is that harsher punishments are available when, within a component, all players can observe and punish

[21] Under Assumption 1 and $\delta > \bar{\delta}$, $\hat{a}_i(c) = g^{-1}(\frac{\delta}{1-\delta}\frac{c}{N}\max f)$ and therefore $\Phi(c) = cv \circ g^{-1}(\frac{\delta}{1-\delta}\frac{c}{N}\max f)$. Determining the convexity properties of Φ is simple. Φ is convex when so is $v \circ g^{-1}$, whereas Φ is concave when $v \circ g^{-1}(x)$ is sufficiently concave (for example, $v \circ g^{-1}(x) = A - (\frac{1}{x})^{1+p}$, with $p > 0$ and $A > (\frac{\delta}{1-\delta}\frac{1}{N})^{1+p}$).

[22] See also Granovetter (1973).

a defection. Burt's (1992) perspective is supported by the model of Section 4.1, in which players can transmit any available information to their direct and indirect contacts. In this context, scattered networks are optimal as they maximize the number of players that become aware of a defection and result in more severe punishments.

These observations are similar to those made by Chwe (1999) and Sobel (2002). In particular, Sobel (2002) observes that "widely scattered weak links are better for obtaining information, while strong and dense links are better for collective action." Our results formalize this intuition in a full-fledged dynamic model of imperfect information in networks.

4.2. Design with intermediate information flows

We investigate the restrictions that incentive compatibility conditions put on equilibrium actions for the model with $d = 1$ and $q \in [0, 1]$. We show that the basic intuition behind the star result, Theorem 2 part c, survives when information flows are more general.

Theorem 5. *Under Assumption 1, let $\delta > \bar{\delta}$, and assume that U is such that*

$$U(x) - U(x - 1 + \delta) \geq x\big(U(x - 1 + \delta) - U(0)\big), \quad 1 \leq x \leq N - 1. \tag{4.1}$$

Let G^* and G be networks such that $\sum_{i=1}^{N} |N(i, G)| \leq \sum_{i=1}^{N} |N(i, G^*)|$. If G^* is a star but G is not, then G^* gives strictly more total welfare than G.

When $v \circ g^{-1}$ is sufficiently convex, forming a star is socially desirable as the incremental social gains from having a highly connected investor are substantial. Intuitively, the center of the star is in a very good position because in no network he could be part of a bigger component, and moreover, in the star he is directly connected to all nodes in the component.[23]

5. Concluding remarks

This paper studies optimal network design in a repeated game model in which the social network determines information flows. The main insight is that cohesive networks coordinate punishments when information travels slowly, while sparse networks increase the deterrence power when information flows rapidly within a component. These results clarify different sociological views on the merits of alternative social architectures.

We now discuss some generalizations and limitations of our results.

Non-uniform matching and asymmetries Our results are robust to small asymmetries between investors. But some of our results need not extend to models in which investors are extremely asymmetric. Suppose for example that investors are randomly (and independently) chosen according to a distribution $\pi \in \mathbb{R}^N_{++}$ where $\pi(1) = 1 - \epsilon$ and $\pi(j) = \epsilon/(N - 1)$ for $j \neq 1$, with ϵ close to 0. This means that a star centered at investor 1 maximizes total expected welfare when $v \circ g^{-1}$ is linear. This shows that Theorem 2 need not hold when investors are sufficiently heterogeneous. Theorem 1 need not hold because an investor $j \neq 1$ in a complete component that

[23] Some of the results from Section 3 do not extend to this general setup. In the Online Appendix, we extend this model and also show that a network of incomplete components can Pareto dominate a network of complete components. However, in the Online Appendix we also show that when q is close to 0, Theorems 1 and 2 are valid approximations.

excludes investor 1 could attain lower payoffs than in a network of incomplete components that includes investor 1. Theorem 3 remains valid.

Synchronous interactions We consider a variation of the model in which at each round a subset of investors is chosen to interact with the agent. Formally, there exists a set $\mathcal{I}$ containing subsets of $\{1, \dots, N\}$ and a distribution r over $\mathcal{I}$ such that at each t, $I^t \in \mathcal{I}$ is randomly selected according to r. We allow the possibility that $\emptyset \in \mathcal{I}$. All agents $i \in I^t$ interact with the investor simultaneously in round t as in our baseline model. We assume that a player $i \in I^t$ does not observe I^t. Players are equally likely to be selected: $\sum_{I \in \mathcal{I} \text{ st } i \in I} r(I)$ does not depend on i and we denote this probability by $\alpha < 1$. Focusing on profiles that sustain cooperation, it follows that an incentive condition similar to that in Lemma 1 holds (now an investor is selected with probability α instead of $1/N$), and all our results extend.

Pairwise stable networks Our results characterize optimal networks, but in reality links are formed in a decentralized way. One way to model such decentralized process is by exploring pairwise stable networks (Jackson and Wolinski, 1996). To follow such approach, it would be necessary to model how an equilibrium of the repeated game is selected when an arbitrary network is formed. Such selection process is not easy to model as in networks of incomplete components, there is no Pareto dominant equilibrium (see Example 3).

Appendix A

This Appendix consists of four parts. Appendix A.1 presents results to characterize equilibria in arbitrary networks and provides some comparative statics results. Appendix A.2 presents proofs for Section 3.1. Appendix A.3 presents proofs for Section 3.2. Appendix A.4 presents proofs for Section 4.

A.1. Equilibria in arbitrary networks

Characterizing the set of equilibria (or the Pareto-frontier of such set) in the model of Section 2 is beyond the scope of the paper. In this Appendix, we derive some estimates for equilibrium investments and use them to find optimal equilibria in networks.

Recall that a function $h : \mathbb{R} \to \mathbb{R}$ is Lipschitz-continuous with constant $L_h \geq 0$ in $X \subseteq \mathbb{R}$ if for all $x, y \in X$, $|h(x) - h(y)| \leq L_h |x - y|$. The following result is useful to characterize equilibria in arbitrary networks. All proofs can be found in the Online Appendix.

Proposition 3. *Assume that f and g are Lipschitz-continuous with constants L_f and L_g in $[0, \bar{b}]$ (where $\bar{b}$ was introduced in Condition 1). Let G be a network and $\sigma \in \Sigma(G)$ that implements $a \in \mathbb{R}^N_{++}$. Then, for all $i = 1, \dots, N$,*

$$\begin{aligned}
&\frac{\delta}{1-\delta}\frac{1}{N}\sum_{k\in N(i)} f(a_k)|N(k)\setminus N(i)| \qquad \text{(A.1)}\\
&\quad\leq \frac{N(1-\delta)+\delta|N_2(i)|}{\delta}\Big(\frac{\delta}{1-\delta}\frac{1}{N}\sum_{k\in\bar{N}(i)} L_f|a_k-\bar{a}_k^G|+L_g|a_i-\bar{a}_i^G|\Big)\\
&\quad\quad+\sum_{k\in N_2(i)}\Big(\frac{\delta}{1-\delta}\frac{1}{N}\sum_{l\in\bar{N}(k)} L_f|a_l-\bar{a}_l^G|+L_g|a_k-\bar{a}_k^G|\Big)
\end{aligned}$$

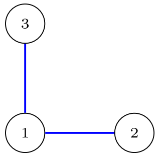

Fig. 5. There is an efficient equilibrium in which player 1 ignores defections against 3.

This result proves that if an action profile $a \in \mathbb{R}^N_{++}$ can be implemented in network G, with a close to $\bar{a}^G$, then for any i the neighborhood $\bar{N}(i)$ must be sufficiently cohesive: the weighted sum of links that "leave" the neighborhood must be sufficiently small. As in Proposition 1, the basic force here arises since in networks of incomplete components, monitoring is imperfect. Intuitively, if some equilibrium in network G is implementing a profile close to $\bar{a}^G$ on some extended neighborhood of i, then the deterrence of double defections (first against i and then against investors within distance 2 of i) implies that investors $k \in N_2(i)$ cannot loose too much leverage following the first defection. As a result, investors $k \in N_2(i)$ cannot have many connections in common with i.

We apply this result to illustrate some novel effects in networks of incomplete components.

Optimal equilibria need not be in trigger strategies The following example shows that trigger strategies need not be optimal.

Example 2. Suppose that $N = 3$, $f(a) = 1$, $g(a) = a$, and consider network G in Fig. 5.

Let $a \in \mathbb{R}^3_{++}$ be implemented by some $\sigma \in \Sigma(G)$. From Lemma 1, $a_1 \leq \frac{\delta}{1-\delta}$, and $a_2, a_3 \leq \frac{\delta}{1-\delta}\frac{2}{3}$. Now, since f and g are Lipschitz with $L_f = 0$ and $L_g = 1$, we can use Proposition 3 to deduce that

$$\frac{\delta}{1-\delta}\frac{1}{3} \leq \frac{3(1-\delta)+\delta}{\delta}(\frac{\delta}{1-\delta}\frac{2}{3} - a_2) + (\frac{\delta}{1-\delta}\frac{2}{3} - a_3)$$

It follows that if $a_2 = \frac{\delta}{1-\delta}\frac{2}{3}$, then $a_3 \leq \frac{\delta}{1-\delta}\frac{1}{3}$. We claim that $a = (\frac{\delta}{1-\delta}, \frac{\delta}{1-\delta}\frac{2}{3}, \frac{\delta}{1-\delta}\frac{1}{3})$ can be implemented by a strategy σ which is not a trigger strategy. Such a strategy σ is such that on-path, investors play a. Any defection against 1 triggers all players to refuse trade in all subsequent rounds, a defection against 2 is punished by players 1 and 2, and a defection against 3 is punished only by player 3. Following steps similar to those in the proof of Theorem 2 (c), these outcome can be part of a perfect Bayesian equilibrium.

This example shows that in networks of incomplete components, a Pareto optimal equilibrium need not be in trigger strategies in that some players ignore defections. Moreover, there is no equilibrium in trigger strategies such that the Pareto efficient action profile $(\frac{\delta}{1-\delta}, \frac{\delta}{1-\delta}\frac{2}{3}, \frac{\delta}{1-\delta}\frac{1}{3})$ is played on the path of play. In network G, player 1 ignores defections against 3 to favor investor 2. If player 1 did not ignore defections against 3, investor 2 would be more vulnerable and the agent could profitable deviate against 3 and then against 2. To see this, suppose that player 1 refuses trade after a defection against 3. If the agent defects against 3, it will be in his interest to defect in a subsequence encounter with 2. This implies that the following incentive constraint must hold: $\frac{1}{1-\delta} \geq 1 + a_3 + \sum_{t=1}^{\infty} \delta^t (1/3)(2/3)^{t-1}(1 + a_2)$. When $a_2 = \frac{\delta}{1-\delta}\frac{2}{3}$ and $a_3 = \frac{\delta}{1-\delta}\frac{1}{3}$, this condition does not hold.

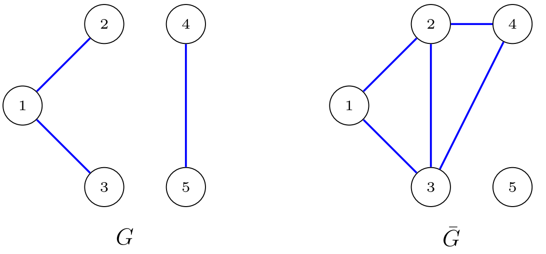

Fig. 6. Investors 1 and 4 have lower payoffs in network $\bar{G}$ even when both of them (a) have more direct connections in $\bar{G}$, (b) pertain to a larger component in $\bar{G}$, (c) have fully clustered neighborhoods in $\bar{G}$.

Comparative statics Proposition 3 suggests that investors can attain relatively high equilibrium actions only when their direct connections are not connected to more distant players. This implies that the impact of more connections or higher clustering coefficients (as defined in Chapter 2.2.3 in Jackson, 2008) on investors payoffs is ambiguous. The following example illustrates this observation.

Example 3. Suppose that $N = 5$, $f(a) = 1$ and $g(a) = a$. See Fig. 6.

In network G, investor 1 is connected to investors 2 and 3. Moreover, $\bar{a}_1^G = \frac{\delta}{1-\delta}\frac{3}{5}$ can be implemented by an equilibrium in which investor 1 plays $\bar{a}_1^G$ on the play-path, whereas 2 and 3 play $\frac{\delta}{1-\delta}\frac{1}{5}$. A defection against 1 triggers no further participation by 1, 2, and 3; a defection against 2 (resp. 3) triggers no further participation by 2 (resp. 3), investor 1 keeps participating choosing $\frac{\delta}{1-\delta}\frac{2}{5}$, investor 1 refuses trade only after a second defection. It is clear that it is in the interest of the agent to cooperate when facing either 1, 2, or 3. Since investors 4 and 5 belong to a different component (which is complete), we can construct a profile sustaining cooperation in which investor 1 invests $\frac{\delta}{1-\delta}\frac{3}{5}$ and investor 4 invests $\frac{\delta}{1-\delta}\frac{2}{5}$.

Take now network $\bar{G}$. At first glance, it could seem that investors 1 and 4 are in a more advantageous position (compared to G) since in $\bar{G}$ investor 4 has more direct connections and, moreover, both investors' neighborhoods are fully clustered. We show that this is not the case. Take $a \in \mathbb{R}^5_{++}$ that can be implemented by some $\sigma \in \Sigma(G)$ and apply Proposition 3 to deduce that

$$\frac{\delta}{1-\delta}\frac{2}{5} \leq \frac{5(1-\delta)+\delta}{\delta}\left|\frac{\delta}{1-\delta}\frac{3}{5} - a_1\right| + \left|\frac{\delta}{1-\delta}\frac{3}{5} - a_4\right|.$$

This inequality immediately shows that if $a_1 = \frac{\delta}{1-\delta}\frac{3}{5}$ then $a_4 \leq \frac{\delta}{1-\delta}\frac{1}{5}$ $(< \frac{\delta}{1-\delta}\frac{2}{5})$.

We finally present a result establishing some properties of the equilibrium set as links are added.

Proposition 4. *Under Assumption 1, let $\delta > \bar{\delta}$. The following hold:*

a. *Let G and G^* be networks such that for some i, $|N(i, G^*)| \geq |N(i, G)|$. Then there exists $\sigma^* \in \Sigma(G^*)$ such that investor i gets higher expected payoff under σ^* than in any $\sigma \in \Sigma(G)$.*
b. *Let G^* be a network formed of two or more complete components of size at least 2 and $\sigma^* \in \Sigma(G^*)$ be the Pareto optimal equilibrium implementing $\bar{a}^{G^*}$. Let l and m two investors in different components of G^*, and build a new network $G^* \cup \{lm\}$ by adding link lm to G^*. There is no $\sigma \in \Sigma(G^* \cup \{lm\})$ Pareto dominating the Pareto dominant equilibrium in G^*.*

This result shows that adding a link can always result in higher payoff for some player. The second part shows that the addition of a link need not result in Pareto improvements. The idea is that if one player strictly improves by adding a link, then some other player will lose leverage after a defection and therefore he should reduce his equilibrium investment.

A.2. Proofs for Section 3.1

Proof of Lemma 1. The proof is organized as follows. In Claim 1, we derive necessary conditions for implementability and then, in Claim 2, we show that those necessary conditions have a larger solution $\bar{a}^G$. Finally, in Claim 3 we show that the largest solution $\bar{a}^G$ is strictly positive.

Claim 1. *Let $\sigma \in \Sigma(G)$, $(i^1, \dots, i^T) \in \{1, \dots, N\}^T$ be any selection of randomly chosen investors, and a^T be the action chosen by i^T on the path of play. Then,*

$$g(a^T) \le \sum_{t=T+1}^{\infty} \delta^{t-T} \sum_{j \in \bar{N}(i^T)} \frac{1}{N} \mathbb{E}_\sigma\big[f(a^t_{i^t}) \mid i^1, \dots, i^T,\ i^t = j\big]. \tag{A.2}$$

To prove this Claim, denote by $V(C)$ (resp. $V(D)$) the continuation value accruing to the agent by cooperating (resp. defecting) against investor i^T at the on path history selecting $(i^1, \dots, i^T)$. Then,

$$g(a^T) \le V(C) - V(D).$$

Now, since σ sustains cooperation

$$\begin{aligned} V(C) &= \sum_{t=T+1}^{\infty} \delta^{t-T} \mathbb{E}_\sigma[f(a^t_{i^t}) \mid i^1, \dots, i^T] \\ &= \sum_{t=T+1}^{\infty} \delta^{t-T} \sum_{j=1}^{N} \frac{1}{N} \mathbb{E}_\sigma[f(a^t_{i^t}) \mid i^1, \dots, i^T, i^t = j]. \end{aligned}$$

Now, after a defection against i^T, it is still feasible for the agent to cooperate when facing players $i \notin \bar{N}(i^T)$. Thus, following a defection against i^T, the agent can secure a total discounted payoff

$$V(D) \ge \sum_{t=T+1}^{\infty} \delta^{t-T} \sum_{j \notin \bar{N}(i^T)} \frac{1}{N} \mathbb{E}_\sigma[f(a^t_{i^t}) \mid i^1, \dots, i^T, i^t = j].$$

It follows that

$$\begin{aligned} g(a^T) \le & \sum_{t=T+1}^{\infty} \delta^{t-T} \mathbb{E}_\sigma[f(a^t_{i^t}) \mid i^1, \dots, i^T] \\ & - \sum_{t=T+1}^{\infty} \delta^{t-T} \sum_{j \notin \bar{N}(i^T)} \frac{1}{N} \mathbb{E}_\sigma[f(a^t_{i^t}) \mid i^1, \dots, i^T, i^t = j] \\ = & \sum_{t=T+1}^{\infty} \delta^{t-T} \sum_{j \in \bar{N}(i)} \frac{1}{N} \mathbb{E}_\sigma\big[f(a^t_{i^t}) \mid i^1, \dots, i^T,\ i^t = j\big]. \end{aligned}$$

This completes the proof of the claim.

Consider now the set

$$\mathcal{A} = \Big\{\alpha \mid \alpha : \bigcup_{t=1}^{\infty} \{1, \dots, N\}^t \to \mathbb{R}_+\Big\}.$$

For any $\sigma \in \Sigma(G)$, we can generate the same distribution over on path actions by using a particular element $\bar{\alpha} = \bar{\alpha}_\sigma \in \mathcal{A}$.

Since g is continuous, its range $g(\mathbb{R}_+)$ is connected. From the fourth restriction in Condition 1, for any $\alpha \in \mathcal{A}$,

$$\begin{aligned} g(0) &\leq \frac{\delta}{1-\delta}\frac{f(0)}{N} \\ &\leq \sum_{t=T+1}^{\infty} \delta^{t-T} \sum_{j \in \bar{N}(i^T)} \frac{1}{N}\mathbb{E}\big[f(\alpha(i^1, \dots, i^T, i^{T+1}, \dots, i^t)) \mid i^1, \dots, i^T,\ i^t = j\big]. \end{aligned}$$

Now, if $\sup_{a \in \mathbb{R}_+} g(a) = \infty$, then it immediately follows that

$$\sum_{t=T+1}^{\infty} \delta^{t-T} \sum_{j \in \bar{N}(i^T)} \frac{1}{N}\mathbb{E}\big[f(\alpha(i^1, \dots, i^T, i^{T+1}, \dots, i^t)) \mid i^1, \dots, i^T,\ i^t = j\big] \in g(\mathbb{R}_+). \tag{A.3}$$

If $\sup_{a \in \mathbb{R}_+} g(a) < \infty$, then the fourth restriction in Condition 1 implies that

$$\sum_{t=T+1}^{\infty} \delta^{t-T} \sum_{j \in \bar{N}(i^T)} \frac{1}{N}\mathbb{E}\big[f(\alpha(i^1, \dots, i^T, i^{T+1}, \dots, i^t)) \mid i^1, \dots, i^T,\ i^t = j\big] \leq \sup_{a \in \mathbb{R}} g(a)$$

and therefore (A.3) holds. We can define $\mathcal{T}^G : \mathcal{A} \to \mathcal{A}$ by

$$\begin{aligned} &\Big(\mathcal{T}^G(\alpha)\Big)(i^1, \dots, i^T) \\ &\quad = g^{-1}\Big(\sum_{t=T+1}^{\infty} \delta^{t-T} \sum_{j \in \bar{N}(i^T)} \frac{1}{N}\mathbb{E}\big[f(\alpha(i^1, \dots, i^T, i^{T+1}, \dots, i^t)) \mid i^1, \dots, i^T,\ i^t = j\big]\Big). \end{aligned}$$

Claim 2. *For any network G, there exists $\bar{a}^G \in \mathbb{R}_+^N$ which is the largest solution to the equation $a = \Phi(a, G)$, with*

$$\Phi_i(a, G) = g^{-1}\Big(\frac{\delta}{1-\delta}\frac{1}{N}\sum_{j \in \bar{N}(i,G)} f(a_j)\Big).$$

Moreover, any solution $\alpha \in \mathcal{A}$ to the system $\alpha \leq \mathcal{T}^G(\alpha)$ satisfies $\alpha(i^1, \dots, i^T) \leq \bar{a}^G_{i^T}$ for all $(i^t)_{t=1}^T \in \{1, \dots, N\}^T$ and for all T.

To prove this claim, use the third and the fourth restrictions in Condition 1 to define

$$\bar{A} = \sup_{a \geq 0} g^{-1}\Big(\frac{\delta}{1-\delta} f(a)\Big) = g^{-1}\Big(\frac{\delta}{1-\delta} \sup f\Big) \in \mathbb{R}.$$

Note that for any α, $\mathcal{T}^G(\alpha)(i^1,\dots,i^T) \le \bar{A}$ for all $(i^1,\dots,i^T) \in \{1,\dots,N\}^T$ and all T. Define $\mathcal{A}^* = \mathcal{A} \cap \{\alpha \mid \alpha(i^1,\dots,i^T) \le \bar{A}\}$. In order to find solutions to the system $\alpha \le \mathcal{T}^G(\alpha)$, it is enough to restrict the domain and range of $\mathcal{T}^G$ to $\mathcal{A}^*$ and, abusing notation, we write $\mathcal{T}^G\colon \mathcal{A}^* \to \mathcal{A}^*$. Let $\alpha^* \in \mathcal{A}^*$ be the largest element in $\mathcal{A}^*$ defined as $\alpha^*(i^1,\dots,i^T) = \bar{A}$ for all $(i^1,\dots i^T)$. Since $\mathcal{T}^G$ is non-decreasing, Tarski's fixed point theorem implies the existence of a largest fixed point $\bar{\alpha} \in \mathcal{A}$, which can be computed as the limit point of the sequence $((\mathcal{T}^G)^n(\alpha^*))_{n\ge 1}$. Observing that $(\mathcal{T}^G)^n(\alpha^*)(i^1,\dots,i^T)$ is only a function of i^T (and not of the vector $(i^1,\dots,i^{T-1})$), we can actually represent the sequence $((\mathcal{T}^G)^n(\alpha^*)) \subseteq \mathcal{A}^*$ as a sequence of vectors $(\alpha^n)_{n\ge 1} \subseteq \mathbb{R}^N$ with $(\mathcal{T}^G)^n(\alpha^*)(i^1,\dots,i^T) = \alpha^n_{i^T}$. Denote $\bar{a}^G = \lim_{n\to\infty} \alpha^n$. By construction, the sequence (α^n) satisfies $\alpha^n = \Phi(\alpha^{n-1}, G)$ with $\alpha^1 = \bar{A}$ an upper bound for the set of fixed points of $\Phi(\cdot, G)$. As a result, $\bar{a}^G$ is the largest fixed point of the non-decreasing function $\Phi(\cdot, G)$. Since $\bar{a}^G$ actually represents the largest fixed point of $\mathcal{T}^G$, it readily follows that for any $\alpha \le \mathcal{T}^G(\alpha)$, $\alpha(i^1,\dots,i^T) \le \bar{a}^G_{i^T}$ for all $(i^1,\dots,i^T)$. □

Claim 3. $\bar{a}^G \gg 0$.

To prove this claim, we argue by contradiction. Assume that $\bar{a}^G_i = 0$ for some i. Then,

$$0 = \Phi_i(\bar{a}^G_i) = g^{-1}\Big(\frac{\delta}{1-\delta}\frac{1}{N}\sum_{j\in\bar{N}(i,G)} f(\bar{a}^G_j)\Big)$$

and therefore $g(0) = \frac{\delta}{1-\delta}\frac{1}{N}\sum_{j\in\bar{N}(i,G)} f(\bar{a}^G_j)$. From the fourth restriction in Condition 1, $g(0) < \frac{\delta}{1-\delta}\frac{1}{N} f(0) \le \frac{\delta}{1-\delta}\frac{1}{N}\sum_{j\in\bar{N}(i,G)} f(\bar{a}^G_j)$, which is a contradiction.

Proof of Proposition 1. Proving $a \Rightarrow b$ is immediate. We prove the converse.

Let $\sigma \in \Sigma(G)$ and suppose that $\bar{a}^G$ is played on-path. Fix i and let $V(C)$ (resp. $V(D)$) be the continuation value after playing C (resp. D) in an encounter with i. Thus,

$$f(\bar{a}^G_i) + V(C) \ge f(\bar{a}^G_i) + g(\bar{a}^G_i) + V(D)$$

Using the fixed point characterization for $\bar{a}^G$, and the fact that $V(C) = \frac{\delta}{1-\delta}\frac{1}{N}\sum_{j=1}^N f(\bar{a}^G_j)$, we deduce that the continuation value after a defection against i equals

$$V(D) \le \frac{\delta}{1-\delta}\frac{1}{N}\sum_{j\notin\bar{N}(i,G)} f(\bar{a}^G_j).$$

Now, decompose the continuation value $V(D)$ following the defection against i as payoffs accruing from encounters with investors $j \in \bar{N}(i,G)$, $\bar{v}$, and payoffs accruing from encounters with investors $j \notin \bar{N}(i,G)$, $\hat{v}$. By definition $V(D) = \bar{v} + \hat{v}$. Since agents' payoffs are nonnegative, $\bar{v} \ge 0$. Moreover, following a defection against i, it is feasible for the agent to cooperate in all remaining encounters against $j \notin \bar{N}(i,G)$ and therefore $\hat{v} \ge \frac{\delta}{1-\delta}\frac{1}{N}\sum_{j\notin\bar{N}(i,G)} f(\bar{a}^G_j)$. It follows that $\bar{v} = 0$ and $V(D) = \hat{v} = \frac{\delta}{1-\delta}\frac{1}{N}\sum_{j\notin\bar{N}(i,G)} f(\bar{a}^G_j)$. Moreover, following a defection against i, continuation play must be such that (i) all investors $j \in \bar{N}(i,G)$ do not participate, and (ii) there is no contagion: investors $j \notin \bar{N}(i,G)$ keep investing according to $\bar{a}^G_j$ and the agent cooperates in all those encounters.

Now, assume that G is not of complete components and let i and k be within distance 2. Consider a defection against i in $t = 1$ and consider the incentives the agent faces if he meets k

in $t = 2$. From the previous paragraph, the path of play in encounters $j \notin \bar{N}(i)$ should involve cooperation. Following an argument similar to that in the proof of Lemma 1, we derive the incentive constraint for cooperation when facing k after a defection against i

$$g(\bar{a}_k^G) \le \frac{\delta}{1-\delta}\frac{1}{N}\Big(\sum_{j\notin \bar{N}(i)} f(\bar{a}_j^G) - \sum_{j\notin \bar{N}(i)\cup\bar{N}(k)} f(\bar{a}_j^G)\Big)$$

Since i and k are within distance 2, it follows that $\sum_{j\notin \bar{N}(i)} f(\bar{a}_j^G) - \sum_{j\notin \bar{N}(i)\cup\bar{N}(k)} f(\bar{a}_j^G) < \sum_{j\in \bar{N}(k)} f(\bar{a}_j^G)$ and therefore $\bar{a}_k^G < \Phi_k(\bar{a}^G)$, yielding a contradiction and concluding the proof. □

A.3. *Proofs for Section 3.2*

Proof of Theorem 1. Take G and G^* as in the statement of theorem. From the proof of Claim 2, $\bar{a}^G$ and $\bar{a}^{G^*}$ can be computed by iterative applications of $\Phi(\cdot, G)$ and $\Phi(\cdot, G^*)$. Starting both iterative procedures from a common upper bound $\bar{A}$, denote by $\bar{a}^n$ and $\bar{a^*}^n$ the corresponding sequences for $n = 0, 1, 2, \ldots$. Since G^* is regular and has complete components, it follows that $\bar{a^*}_i^n = \bar{a^*}_j^n$ if $|N(i, G^*)| = |N(j, G^*)|$, whereas $\bar{a^*}_i^n > \bar{a^*}_j^n$ if $|N(i, G^*)| > |N(j, G^*)|$.[24] In particular, if $N(j, G^*) \neq \emptyset$,

$$\bar{a^*}_j^n = \max_{k=1,\ldots,N} \bar{a^*}_k^n. \tag{A.4}$$

We argue that $\bar{a}^n \le \bar{a^*}^n$ using induction. Assume the claim is true for some n. Now,

$$\begin{aligned}\bar{a}_i^{n+1} &= \Phi_i(\bar{a}^n, G) = g^{-1}\Big(\frac{\delta}{1-\delta}\frac{1}{N}\sum_{j\in\bar{N}(i,G)} f(\bar{a}_j^n)\Big) \le g^{-1}\Big(\frac{\delta}{1-\delta}\frac{1}{N}\sum_{j\in\bar{N}(i,G)} f(\bar{a^*}_j^n)\Big)\\ &\le g^{-1}\Big(\frac{\delta}{1-\delta}\frac{1}{N}\sum_{j\in\bar{N}(i,G^*)} f(\bar{a^*}_j^n)\Big) = \bar{a^*}_i^{n+1}.\end{aligned}$$

The first inequality follows by the induction hypothesis. The second inequality follows from equation (A.4) and the fact that $|N(i, G^*)| \ge |N(i, G)|$ for all i. Thus, $\bar{a}^{n+1} \le \bar{a^*}^{n+1}$. We deduce that $\bar{a}^n \le \bar{a^*}^n$ for all $n = 0, 1, \ldots$ and, by passing to the limit, that $\bar{a}^G \le \bar{a}^{G^*}$.

To finally prove Theorem 1, let $\sigma \in \Sigma(G)$, and note that $\alpha_\sigma(i^1, \ldots, i^T) \le \bar{a}_{i^T}^G$ from Lemma 1, where the inequality is strict for some history due to Proposition 1. Noting that $\bar{a}^{G^*}$ can actually be implemented using trigger strategies in G^* and $\bar{a}^G \le \bar{a}^{G^*}$, the result follows since v is strictly increasing and f is nondecreasing. □

Proof of Lemma 2. Define $\tilde{a}$ as $\tilde{a}_i = g^{-1}\big(\frac{\delta}{1-\delta}\frac{|\bar{N}(i,G)|}{N}\bar{f}\big)$, where $\bar{f} = \max f$. Note that

$$\bar{a}_i^G = g^{-1}\Big(\frac{\delta}{1-\delta}\frac{1}{N}\sum_{j\in\bar{N}(i,G)} f(\bar{a}_j^G)\Big) \le \tilde{a}_i$$

for all i. Since $\delta > \bar{\delta}$,

[24] Note that in this last case, $|N(i, G^*)| = \kappa$ and $|N(j, G^*)| = 0$.

$$\tilde{a}_i \geq g^{-1}\big(\frac{\delta}{1-\delta}\frac{1}{N}\bar{f}\big) \geq \underline{a} = \min(\arg\max f)$$

for all i. Therefore $\Phi_i(\tilde{a}) = g^{-1}(\frac{\delta}{1-\delta}\frac{|\bar{N}(i,G)|}{N}\bar{f}) = \tilde{a}_i$. It follows that for $\delta > \bar{\delta}$, $\tilde{a}$ is a fixed point of Φ. Since $\bar{a}^G$ is the largest fixed point, $\tilde{a}_i \leq \bar{a}_i^G$ for all i and we conclude that $\tilde{a} = \bar{a}^G$. □

Proof of Theorem 2. Let $\rho = \sum_{i=1}^N |N(i,G^*)|$ and define $\bar{f} = \max f$. From Lemma 1, for a social network G and $\sigma \in \Sigma(G)$

$$\alpha_\sigma(i^1,\ldots,i^t) \leq g^{-1}\big(\frac{\delta}{1-\delta}\frac{\bar{N}(i^t,G)}{N}\bar{f}\big) \quad \forall\,(i^1,\ldots,i^t)$$

with at least some inequality strict if G has some incomplete component. Taking expectations and summing

$$\sum_{i=1}^N u_i(\sigma,G) \leq \frac{1}{1-\delta}\frac{1}{N}\sum_{i=1}^N v\circ g^{-1}\big(\frac{\delta}{1-\delta}\frac{\bar{N}(i^t,G)}{N}\bar{f}\big)$$

with strict inequality for G having some incomplete component.

Take G^* having complete components (as in parts a and b). From Proposition 1, there exists $\sigma^* \in \Sigma(G^*)$ such that

$$\sum_{i=1}^N u_i(\sigma^*,G^*) = \frac{1}{1-\delta}\frac{1}{N}\sum_{i=1}^N v\circ g^{-1}\big(\frac{\delta}{1-\delta}\frac{\bar{N}(i,G^*)}{N}\bar{f}\big).$$

Part a follows immediately. To prove b, consider the relaxed problem

$$\max_{(x_1,\ldots,x_N)}\sum_{i=1}^N v\circ g^{-1}\big(\frac{\delta}{1-\delta}\frac{x_i+1}{N}\bar{f}\big),$$

subject to $x_i \in \{1,2,\ldots,N\}$, $\sum_{i=1}^N x_i \leq \rho$. This problem can be thought of as the problem of assigning a total of ρ pennies between N persons with the purpose of maximizing total utility. Since $v\circ g^{-1}$ is strictly concave and all components of G^* are of the same size $\gamma \geq 2$, this problem has a single solution $x_i^* = \gamma - 1$ for all i. Noting that $(x_i)_{i=1}^N$ and $(x_i^*)_{i=1}^N$ defined as $x_i = |N(i,G)|$ and $x_i^* = |N(i,G^*)| = 2\rho/N$ are, respectively, feasible and optimal for the maximization above, it follows that

$$\sum_{i=1}^N v\circ g^{-1}\big(\frac{\delta}{1-\delta}\frac{\bar{N}(i,G)}{N}\bar{f}\big) < \sum_{i=1}^N v\circ g^{-1}\big(\frac{\delta}{1-\delta}\frac{\bar{N}(i,G^*)}{N}\bar{f}\big).$$

It remains to prove part c. Observe first that in the star G^*, we can construct a strategy profile σ^* that yields average total welfare equal to $U(E) + (N-1)U(0)$, where $E = \rho/2$ is the number of available links. Take investor 1 as the center of the star and investors $2,\ldots E+1$ as the leaves of the star. On the path of play, investor 1 invests $g^{-1}(\frac{\delta}{1-\delta}\frac{E+1}{N}\bar{f})$, whereas all other investors take action $g^{-1}(\frac{\delta}{1-\delta}\frac{1}{N}\bar{f})$. A defection against player 1 implies all other star members refuse trade in subsequent rounds (and after any second deviation). A defection against any $i \neq 1$ implies only player i refuse trade in subsequent rounds, while player 1 keeps participating and playing $g^{-1}(\frac{\delta}{1-\delta}\frac{E}{N}\bar{f})$ (more generally, after a number n of defections against leaves, 1 chooses $g^{-1}(\frac{\delta}{1-\delta}\frac{E+1-n}{N}\bar{f})$). The agent cooperates on the play path and, after a defection, keeps cooperating when facing any investor who according to his strategy should participate (defecting otherwise). This strategy profile sustains cooperation and results in total average welfare equal to $U(E) + (N-1)U(0)$.

Now, consider the maximization problem

$$\bar{W} = \max_G \sum_{i=1}^{N} U(|N(i,G)|) \tag{A.5}$$

subject to $\sum_{i=1}^{N} |N(i,G)| \le \rho$, and G is not a star. Problem (A.5) yields an upper bound for total equilibrium welfare over all networks G different from a star and having less than 2ρ links. Let $\bar{G}$ be optimal for (A.5) and let $i^* \in \arg\max_i |N(i,\bar{G})|$.

We first note that if there exists $ij \in \bar{G}$, with $i \ne i^* \ne j$, such that the new network formed from $\bar{G}$ by deleting ij and using that link to connect some isolated player k to i^*, then $\bar{G}$ cannot be optimal. Indeed, the net gain from forming the new network would be

$$\begin{aligned} U(|N(i^*,\bar{G})|+1) - U(|N(i^*,\bar{G})|) + U(|N(k,\bar{G})|+1) - U(|N(k,\bar{G})|) \\ + U(|N(i,\bar{G})|-1) - U(|N(i,\bar{G})|) + U(|N(j,\bar{G})|-1) \\ - U(|N(j,\bar{G})|) \end{aligned}$$

Define $\Delta(x) = U(x) - U(x-1)$ and rewrite the inequality above as

$$\Delta(|N(i^*,\bar{G})|+1) + \Delta(1) \ge \Delta(|N(i,\bar{G})|) + \Delta(|N(j,\bar{G})|).$$

As Δ is increasing, the inequality holds as $\Delta(x+1) + \Delta(1) \ge 2\Delta(x)$. It follows that if $\bar{G}$ has two or more components, then only two of them are nontrivial, one of the nontrivial components is a star formed by $E-1$ links, and the other nontrivial component is a line formed by two nodes. Such network yields total welfare which is less than the welfare from the star. Indeed,

$$U(E-1) + (E-1)U(1) + 2U(1) + (N-E-2)U(0) \le U(E) + (N-1)U(0)$$

if and only if $U(E) - U(E-1) \ge (E-1)(U(1) - U(0))$ and the inequality above holds true. It is therefore enough to consider the case where $\bar{G}$ has a single nontrivial component.

Define $I = \{i \mid |N(i,\bar{G})| \ge 2\}$. Using the construction above, it follows that $|I| \le 3$. Consider first the case $|I| = 3$ and let $I = \{i^*, j_1, j_2\}$. We claim that $N(j_n,\bar{G}) = \{i^*, j_{3-n}\}$ as otherwise, and following the construction above, deleting $j_n l$, with $l \notin I$, to form $i^* m$, with $N(m,\bar{G}) = \emptyset$, would be worthwhile. If $|I| = 3$, the value of (A.5) equals

$$\bar{W} = U(E-1) + (E-3)U(1) + 2U(2) + (N-E+1)U(0).$$

Now, assume $|I| = 2$ and let $I = \{i^*, j\}$. Observe that $|N(j,\bar{G})| = 2$ as otherwise $|N(j,\bar{G})| \ge 3$ and we could remove one of the links connecting j and use that link to connect i^* to a new player. If $j \in N(i^*,\bar{G})$, then it is relatively easy to see that the objective function at $\bar{G}$ is less than or equal to $U(E-1) + (E-3)U(1) + 2U(2) + (N-E+1)U(0)$ and therefore $j \notin N(i^*,\bar{G})$. Take $l, m \in N(j,\bar{G})$ and note that deleting jm and connecting j to i^*, deleting lj and connecting l so that in the new network the distance between l and i^* equals 2, one can increase the objective. We deduce that $|I| = 3$ and no equilibrium in a network which is not a star can give total payoffs above $\bar{W} = U(E-1) + (E-3)U(1) + 2U(2) + (N-E+1)U(0)$. By assumption $\Delta(E) \ge (E-2)(U(1) - U(0)) + \Delta(2) + U(2)$ and thus

$$U(E) + (N-1)U(0) \ge U(E-1) + (E-3)U(1) + 2U(2) + (N-E+1)U(0)$$

proving the result. □

A.4. Proofs for Section 4

Proof of Theorem 5. Let $|E|$ be the number of links. Note that in the star we can always attain total welfare of $U(|E|) + (N-1)U(0)$ by constructing strategies like the ones in the proof of Theorem 2 part c. In a network which is not a star, for at most $|E|+1$ players

$$g(a_i) \leq \frac{\delta}{N}|E| + \frac{\delta^2}{1-\delta}\frac{|E|+1}{N},$$

and therefore for any σ that sustains cooperation and any such i

$$u_i(\sigma, G) \leq \frac{1}{1-\delta}\frac{1}{N} v \circ g^{-1}\Big(\frac{\delta}{1-\delta}\frac{\max f}{N}(|E|+\delta)\Big).$$

To prove the result, it is therefore enough to show that

$$U(|E|) + (N-1)U(0) \geq (|E|+1)U(|E|-1+\delta) + (N-|E|-1)U(0)$$

which is equivalent to $U(|E|) - U(|E|-1+\delta) \geq |E|\big(U(|E|-1+\delta) - U(0)\big)$. Since this must hold for any $1 \leq |E| \leq N-1$, the result is established. □

Proof of Theorem 3. For any G, if i and j belong to the same component C, $\hat{a}_i^G = \hat{a}_j^G$, which is purely determined by the size of the component $|C|$. Now, for G as in the statement of the Theorem, take the component which is not minimally connected, C_1, and link $ij \in C_1$ such that $C \setminus \{ij\}$ is connected. Take a second component C_2 and any $k \in C_2$, and use the link ij to form a new connection ik. We therefore obtain a new network G^* such that, for all i, $\hat{a}_i^{G^*} \geq \hat{a}_i^G$ for all i, with strict inequality for $i \in C_1 \cup C_2$. It follows that G^* Pareto dominates G. □

Proof of Theorem 4. Since $\frac{1}{2}\sum_{i=1}^N |N(i, G^*)| \leq N-1$, G^* has two or more components and from Theorem 3, G^* is minimally connected. The characterization of part a follows by noting that if G^* has two non-trivial components of sizes n_1 and n_2, each component must be minimally connected and we can form two components of size $n_1 + n_2 - 1$ and 1. This new network yields strictly more total welfare because Φ is convex.

For part b, the same convexity argument shows that if a network has two or more components of sizes n_1 and n_2 and $n_1 > n_2$, then $n_1 = n_2 + 1$. Let m be the number of components in the optimal network, and assume that m_1 components have size n_1 and m_2 components have size m_2, with $n_1 = n_2 + 1$ and $m_1 + m_2 = m$. The following feasibility restrictions hold:

$$n_1 m_1 + n_2 m_2 = N$$
$$(n_1 - 1)m_1 + (n_2 - 1)m_2 = E.$$

Combining both equations, $m_1 + m_2 = m = N - E$. From the first equation, $(n_2+1)(N-E) \geq N$ and $n_2(N-E) \leq N$. Thus, $\frac{N}{N-E} - 1 \leq n_2 \leq \frac{N}{N-E}$. Since $n_1 = n_2 + 1$, the result follows. □

Appendix B. Supplementary material

Supplementary material related to this article can be found online at http://dx.doi.org/10.1016/j.jet.2017.05.005.

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