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Analysis of extended warranties for medical equipment: A Stackelberg game model using priority queues

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ABSTRACT

Healthcare institutions make use of technology-intensive equipment that follows tight quality standards. These companies aim at ensuring service continuity and safety of patients. In this context, maintenance services are generally performed exclusively by the Original Equipment Manufacturer (OEM) because it detains the required expertise, tools and spare parts. Then, we here propose a model to analyze the interaction among hospitals and OEM. We consider the OEM can provide maintenance services for two different classes of hospitals, which have the option of either hiring an Extended Warranty (EW) or paying for each maintenance intervention on demand with or without priority. Class 1 customers are often large hospitals, whereas institutions of class 2 are generally small/medium ones, which have shorter budgets, and thus would choose a non-priority option. To that end, we adopt a Stackelberg game, where the OEM is the leader and the customer is the follower. Failures and repairs follow a 2-class G/M/1 priority queuing system. The OEM maximizes its expected profit by setting the EW and repair intervention prices, and selecting the optimal number of customers in each class. An application example is used to demonstrate the proposed model; a sensitivity analysis is also performed.

1. Initial remarks

A growing trend observed in recent years has been the outsourcing of maintenance services to achieve lower costs, higher service quality, and competitiveness. As pointed out by Jackson & Pascual [1] and Pascual et al. [2], the following characteristics contribute to the decision of hiring a service rather than performing it in-house: (i) access to high-level experts; (ii) better services due to provider's expertise; and (iii) fixed price contracts, which control extremely high costs. According to Dyro [3], the benefits of outsourcing maintenance services are: (i) the possibility of shrinking the maintenance department; (ii) access to resources (e.g., specialized maintenance team, parts, supplies, specific software) that are not easily available to the equipment owner; (iii) fast solution of an unavailability problem; and (iv) cost decrease with invoice processing and reduction of bureaucracy. Such reasons are also present in healthcare institutions, where the sense of urgency and the requirements for high quality and high availability of technology-intensive equipment are very rigorous as mentioned by Kazemi Zanjani & Nourelfath [4] and Mutia et al. [5].

In this context, the increasing technology complexity of some medical equipment is making it difficult to execute maintenance services in-house. As pointed out by De Vivo et al. [6], a significant amount of medical equipment is so complex that performing in-house maintenance becomes uneconomical, and hiring independent service agents is difficult as they may not be familiar enough with the equipment and may not have access to all necessary information such as manuals or complementary documentation, which are detained by the Original Equipment Manufacturer (OEM).

Indeed, in order to maintain market share, the OEM generally adopts protectionist actions such as not training possible external providers and not providing sufficient parts for the market, which makes it difficult for third-party providers to repair failed units, as mentioned by Cruz & Rincon [7]. In this context, OEMs become the only agents able to execute maintenance services adequately, granting them monopoly power.

Along with the purchase of a product, for instance a healthcare device, a warranty is usually provided. Failures and malfunctions over a coverage period are repaired or replaced by the OEM. After the ordinary warranty expires, two main options emerge to hospitals: (i) hiring the

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OEM, with whom an Extended Warranty (EW) is established through an additional payment, or (ii) paying for each maintenance intervention on demand.

According to Murthy & Djamaludin [8], the notion of post-sale support has become an important feature of product sale, and thus the use of warranties is widespread, serving as protection for both seller and buyer, against misuse and bad performance. Indeed, Lutz & Padmanabhan [9] point out a significant number of customers purchase extra protection against failures in the form of an EW. Bollapragada et al. [10] argue that long-term EW negotiated between customers and manufacturers of high-tech industries, such as aviation and medical, is positive for both sides. These authors also suggest that the OEM's possibility of negotiating with multiple customers allows scale gains that yield lower costs, which may be partially transferred to customers and it is one of the reasons why these instruments may be beneficial for both sides. Furthermore, customers benefit from the risk transferred to the manufacturer after an EW is hired. In fact, customers are exposed to risks such as poor quality maintenance (Damnjanovic & Zhang [11]). On the other hand, the maintenance agent is subject to the risks of a too long effort to complete a repair or of an early equipment replacement. Both cases may be associated to considerably high costs, which can be completely transferred to the OEM depending on the contract or EW terms. Su & Shen [12] study EW costs from the manufacturer's point of view, comparing different EW policies and different repair effectiveness scenarios. Bouguerra et al. [13] present a decision model for adopting an EW, considering different maintenance policies, from both the manufacturer's and the buyer's point of view. Chang & Lin [14] investigate the optimal length and policy of an EW to maximize the seller's total profit, analyzing effects of different maintenance costs and equipment reliability.

The interaction between customers and maintenance providers can be studied through Game Theory (GT). Indeed, GT is a tool that allows each player to maximize their own expected profit or utility considering the actions of the others. For instance, Murthy & Blischke [15] analyzed GT based models for drawing Maintenance Service Contracts (MSC), and DeCroix [16] proposed a GT approach for EWs of durable goods. Murthy & Yeung [17] adopt a Stackelberg Game (Osborne & Rubinstein [18]), in which one player (the leader) has more bargain power than the other (the follower), to model the negotiation of MSCs considering only one service agent and one customer. In Ashgarizadeh & Murthy [19], the agent services multiple customers, who are homogeneous regarding their attitude to risk, and thus Queueing Theory (Gross et al. [20]) is employed to model the failures and repairs. Jackson & Pascual [1], in turn, adopted a Nash Game (Gibbons [21]) in which both players have the same bargaining power, i.e., negotiate in equal conditions, differently from what takes place in monopolistic markets. In Esmaeili et al. [22], three different game formulations are used: a non-cooperative static game (Nash Game), a non-cooperative sequential game (Stackelberg Game), and a semi-cooperative game.

Although EWs have been widely discussed in literature, their study in the specific context of healthcare institutions is still scarce and, at the best of authors' knowledge, has only been found in De Vivo et al. [6], which provides an overview of the challenges and singularities of medical equipment maintenance and analyze alternative ways to reduce maintenance costs through the association of in-house and OEM's maintenance services in an Italian hospital.

Therefore, this paper proposes an approach based on GT and Priority Queueing Theory (PQT) to model the interaction between OEM and healthcare institutions to outsource maintenance services. Similarly to Ashgarizadeh & Murthy [19], Esmaeili et al. [22], Murthy & Asgharizadeh [23] and Murthy & Yeung [17], we also adopted a Stackelberg Game in which the OEM is the leader and the customers are the followers. In fact, the present paper focuses on an angiography (Fig. 1), which is a technology-intensive clinical equipment used for imaging exams. For this situation, maintenance services market is monopolistic since the necessary knowledge, tools and parts are only provided by the OEM. Given the purpose of healthcare equipment, failures and malfunctions become critical, since they might cause imprecise or incorrect diagnostics or even directly harm patients, and causing repair costs and loss of revenue due to downtime.

Fig. 1. Angiography. Source: Suri & Laxminarayan [24].

The remainder of this paper unfolds as follows: Section 2 presents the description of the problem. In Section 3, the Stackelberg Game and PQT approaches are characterized. In Section 4, the proposed model is presented. In Section 5, the proposed method is applied to a numerical example, results are shown and a sensitivity analysis is performed. In Section 6, we provide some concluding remarks.

2. Description of the problem

In the context of healthcare institutions, the OEM commonly negotiates with two different classes of hospitals (customers). In fact, class 1 customers are often large hospitals, whereas class 2 institutions are generally small and medium ones, which have shorter budgets, less money to invest in maintenance, and treat fewer patients, thus they are more likely to wait for longer periods to have their equipment repaired. Then, hospitals of class 1 choose either hiring an EW or paying for each maintenance intervention on demand, both with priority, whereas class 2 has the option of either hiring a standard EW or paying for each maintenance intervention on demand, but both with no priority.

Thus, the first class consists of hospitals that prioritize the availability of their equipment, and then may prefer to establish an EW with priority according to which the OEM must execute each maintenance intervention within a period τ_1 of time; otherwise, OEM will pay a fine to the hospital. The second class, in turn, corresponds to hospitals that are inclined to wait for a longer downtime because they cannot afford an EW with priority. Therefore, although the equipment should also be repaired within a period $\tau_2 > \tau_1$, if an EW without priority is hired, there is a risk of a much larger delay to have their equipment back into operation in comparison with hospitals of class 1. This situation occurs because hospitals of class 2 compete (get in line) with other institutions of the same class as well as with class 1 hospitals.

In general, under coverage of an EW, every failure should be repaired at no additional cost for the customer and within a period τ_c ($c = 1, 2$). If the equipment is not restored in these pre-established periods, the OEM is charged with a penalty that shall be paid to the customer. The amount of time that exceeds the limit, $y - \tau_c$, is defined here as overtime, where y is the time between the failure and completion of repair. If an EW is not hired, the customer will pay a fixed price for each intervention.

The abovementioned situation will be here tackled with PQT. As we consider multiple customers, and consequently multiple units of equipment to be repaired, we use PQT to model the dynamic of equipment failing, waiting for repair and being returned to operational state. Furthermore, in PQT, customers with the highest priorities are chosen for service ahead of those with lower priorities, independently of their arrival time into the system (Gross et al. [20]). In the case of 2 classes, customers of class 1 will always be selected to be served first. In fact, a queue will be formed by all the hospitals with failed equipment that either had signed an EW contract or requires on-demand maintenance interventions. However, customers with priority will have the preference to recover their equipment.

Thus, this paper proposes a methodology based on the combination of GT and PQT to model the strategic interaction between OEM and customers with and without priority regarding maintenance interventions. The game develops in three stages: first, the leader (OEM) observes the class of customer (hospital) that it is dealing with; next, OEM defines the EW's and maintenance intervention prices; and finally, the followers (customers) analyze the previously defined prices and decide whether to buy the equipment and whether to hire the EW, with or without priority. Then, the proposed model allows the OEM to determine EW and maintenance intervention prices and to select the optimal number of customers of each class to serve so that to maximize its expected profit. The proposed model also permits the incorporation of risk aversion preferences of the hospitals, which is an important characteristic as pointed out by Thomas & Rao [25], who argue that warranty decisions under risk and uncertainty are essential for manufacturers and customers.

Notation list

A^{c*}	optimal strategies for customers of class c.
A_k	customer's possible choices ($k = 0, 1, 2, 3, 4$);
C_b	equipment sale price;
C_r	average cost per repair;
C_s^1	price of maintenance service on demand for class 1;
C_s^2	price of maintenance service on demand for class 2;
$C_s^{c,max}$	maximum willingness to pay for maintenance services on demand for customers of class c;
k	shape parameter of the Weibull distribution;
m	number of servers (repair teams);
M	total number of customers ($M = M_1 + M_2$);
M_c	number of customers of class c, with $c = 1, 2$;
N_j	number of failures occurred over $[T_0, T]$ for the j^{th} equipment (j^{th} equipment is hold by the j^{th} customer (hospital));
P_w^c	EW price for customers of class c;
$P_w^{c,max}$	maximum willingness to pay for an EW for customers of class c;
q_0	probability that there is no failed equipment;
$q_{a,\,b,\,c}$	probability that there are a failed equipment of class 1, b failed equipment of class 2 and equipment of class c is being repaired;
r	repair duration, excludes queue waiting time;
R_1	revenue per hour for customers of class 1;
R_2	revenue per hour for customers of class 2;
t_0	equipment age;
T_0	basic warranty coverage period;
T	EW coverage period (starts after the end of the basic warranty coverage);
t_{a_c}	time of next arrival among available customers of class c;
t'_{a_c}	time of next arrival considering a single customer of class c;
t_d	time of next departure (completion of a maintenance intervention);
u	time between failure and beginning of repair, i.e. time spent in queue;
$U(w)$	utility associated to a wealth w;
$w(A_k)$	expected wealth (return) associated to action A_k;
X_{ji}	time to the i^{th} ($0 \le i \le N_j$) failure after the $(i-1)^{th}$ repair of the j^{th} equipment;
$\tilde{X}_j$	time range between the recovery from the last maintenance and T, over which no failures take place for the j^{th} equipment;
y	time between failure and completion of repair;
y_{ji}	time between failure and completion of repair the j^{th} equipment after the i^{th} failure ($0 \le i \le N_j$);
α_c	penalty per hour due to delays in repairing equipment associated to class c;
β	risk aversion parameter for customers;
θ	scale parameter of the Weibull distribution;
λ_c	arrival/failure rate of equipment of class c;
μ	service rate of a server;
μ_c	service rate of a server when the customer is of class c;
$\pi(\cdot)$	manufacturer's expected profit;
π_c	the total manufacturer's expected profit for all customers of class c;
τ_c	maximum time to repair equipment hold by a customer of class c;
υ_j^c	total overtime for customer j of class c, which is the sum of the portion of times where the wait in queue plus repair time surpass the limit τ_c.

3. Theoretical background

3.1. Stackelberg game

In accordance with Greve [26], in a Stackelberg game, (i) there are two companies in a given market: one is well established and the other is a new entrant that produces homogeneous products; (ii) the players (companies) are rational and intend to maximize their profits; (iii) both companies are aware of all production costs; (iv) all production is consumed by the market; (v) the companies do not cooperate, then they make their decisions without negotiation with the other; vi) the game develops in two stages and the companies have different levels of bargain power. One company is called leader, as it is already established in the market and has more market share, while the other, the follower (new entrant), has a smaller market share; vii) before deciding how much to produce, the leader is aware that the follower observes its action and will react to the decision made in the first stage. Then, the leader anticipates the action of the follower and determines its production level to maximize its profit. It is possible to anticipate the other player's action due to the assumption of perfect information; and, viii) higher production volumes lead to shorter market prices, thus the companies determine a quantity to produce that maximizes their profits considering this relationship between supply and demand.

The profits generated by each company represent their payoffs. In order to determine the optimal solution of the game, backward-induction (Greve [26]) is employed, i.e., first the follower's decision problem is solved (player that decides in the second stage), and then the leader's problem is computed. The optimal solution found in the follower's decision problem is incorporated into the leader's problem as an extra constraint. By doing so, it is possible to determine the optimal solution for both companies and consequently the total quantity that will be offered to the market (Greve [26]).

Such structure is normally employed to model duopoly situations. However, it is possible to tailor the Stackelberg Game to consider more general negotiations/conflicts, which are characterized by one company having more bargain power than the other. It is exactly what happens in the case here studied. Then, a more careful explanation about adapting Stackelberg approach to tackle the problem of outsourcing maintenance services in the context of medical equipment is given in Table 1. At the best of authors' knowledge, it is the first time a Stackelberg model is tailored for the medical situation.

Table 1
Stackelberg game adaptation for EWs negotiation in the context of medical institutions.

	General Situation	Medical Context
Description	A new entrant must decide if it enters a market in which there is only one company (monopoly). Both companies need to determine their productions to maximize their profits	A hospital (customer) must decide if it buys a technology-intensive equipment for which it will be necessary to outsource maintenance actions provided only by the OEM (maintenance market is monopolistic)
Players	Two companies: leader and follower	Two companies: OEM (leader) and hospital (follower)
Bargain power structure	Leader is established in market and has more power than the follower	OEM acts in a monopolistic market, and thus has more bargain power than the hospital
Actions	Companies determine how much to produce	OEM determines the prices of each option offered. The hospital selects the option that maximizes its utility
Game development	The leader decides first. Then, the follower observes the action of the leader and determines whether it enters in the market and how much to produce	First, OEM determines the prices, considering the customer's maximum willingness to pay for each option. Then, the hospital observes such prices and chooses an action
Solution	Backward-induction	Backward-induction. First, the hospital's maximum willingness to pay for each option is determined. Then, OEM profit is maximized, while the optimal number of customers is determined. Then, OEM sets the prices and the customer's optimal choice is found
Equilibrium	The sum of the quantities produced by both companies determines the equilibrium	The set of decisions (prices and maintenance options chosen) determines the equilibrium

3.2. Priority queues

In queueing systems with priority disciplines, customers are divided into classes with different priority levels. Suppose there are 2 classes of customers (class 1 and class 2), each one with a given arrival rate (λ_1 and λ_2, respectively). We consider that users of class 1 have non-preemptive priority over customers of class 2. Thus, after a server finishes a job, the next user to be served belongs to the highest priority class among customers in queue. If there is more than one user of the same priority class, they will be served according to a first-come, first-served (FCFS) discipline.

The states of this system can be denoted as (*a, b, c*), where *a* and *b* are the number of class 1 and 2 customers in the system, respectively, and *c* is the class of the client being served (Gross et al. [20]). To illustrate this, (2, 5, 1) is the state at which there are two customers of class 1 and five customers of class 2 in the system, while one of the customers in class 1 is being currently served. This notation considers a queue system with exactly one server.

In queueing theory, steady state probabilities are the probabilities of occurrence for each possible state in the long run, that is, the mean proportion of time the system stays in each of the states. These probabilities can be denoted as $q_{a,\ b,\ c}$, where the subscripts are the same as previously defined. Note that q_0 indicates the probability that the system is empty.

To obtain these probabilities, it is necessary to define a set of equations based on the relations between the states of the system. These equations were obtained by equating, for a given set (*a, b, c*), the rate at which the system enters such state with the rate at which the system leaves the same state. These are the steady-state balance equations (Gross et al. [20], Hillier & Lieberman [27]).

For the priority system here described, each balance equation is related to a given state (a, b, c) and is labeled by $s_{a,\ b,\ c}$; M_1 and M_2 are the total numbers of customers and μ_1 and μ_2 are the service rates for each priority class, respectively. The resulting system of equations may be solved if one of them is disregarded and substituted by $q_0 + \sum_{a=1}^{M_1} q_{a,0,1} + \sum_{b=1}^{M_2} q_{0,b,2} + \sum_{a=1}^{M_1}\sum_{b=1}^{M_2}\sum_{c=1}^{2} q_{a,b,c} = 1$. An illustration example encompassing 4 customers (2 of each class) is described below by the following set of equations:

$$s_0 : (2\lambda_1 + 2\lambda_2)\ q_0 = \mu_1\ q_{1,0,1} + \mu_2\ q_{0,1,2};$$

$$s_{0,1,2} : (2\lambda_1 + \lambda_2 + \mu_2)\ q_{0,1,2} = 2\ \lambda_2\ q_0 + \mu_2\ q_{0,2,2} + \mu_1\ q_{1,1,1}$$

$$s_{0,2,2} : (2\lambda_1 + \mu_2)\ q_{0,2,2} = \lambda_2\ q_{0,1,2} + \mu_1\ q_{1,2,1};$$

$$s_{1,0,1} : (\lambda_1 + 2\lambda_2 + \mu_1)\ q_{1,0,1} = 2\ \lambda_1\ q_0 + \mu_2\ q_{1,1,2} + \mu_1\ q_{2,0,1};$$

$$s_{1,1,1} : (\lambda_1 + \lambda_2 + \mu_1)\ q_{1,1,1} = 2\ \lambda_2\ q_{1,0,1} + \mu_2\ q_{1,2,2} + \mu_1\ q_{2,1,1}$$

$$s_{1,1,2} : (\lambda_1 + \lambda_2 + \mu_2)\ q_{1,1,2} = 2\ \lambda_1\ q_{0,1,2};$$

$$s_{1,2,1} : (\lambda_1 + \mu_1)\ q_{1,2,1} = \lambda_2\ q_{1,1,1} + \mu_1\ q_{2,2,1};$$

$$s_{1,2,2} : (\lambda_1 + \mu_2)\ q_{1,2,2} = 2\ \lambda_1\ q_{0,2,2} + \lambda_2\ q_{1,1,2};$$

$$s_{2,0,1} : (2\lambda_2 + \mu_1)\ q_{2,0,1} = \lambda_1\ q_{1,0,1} + \mu_2\ q_{2,1,2};$$

$$s_{2,1,1} : (\lambda_2 + \mu_1)\ q_{2,1,1} = \lambda_1\ q_{1,1,1} + 2\ \lambda_2\ q_{2,0,1} + \mu_2\ q_{2,2,2};$$

$$s_{2,1,2} : (\lambda_2 + \mu_2)\ q_{2,1,2} = \lambda_1\ q_{1,1,2}$$

$$s_{2,2,1} : \mu_1\ q_{2,1,1} = \lambda_1\ q_{1,2,1} + \lambda_2\ q_{2,2,1}$$

$$s_{2,2,2} : \mu_2\ q_{2,2,2} = \lambda_1\ q_{1,2,2} + \lambda_2\ q_{2,1,2};$$

$$q_0 + q_{1,0,1} + q_{0,1,2} + q_{2,0,1} + q_{1,1,1} + q_{1,1,2} + q_{0,2,2} + q_{2,1,1} + q_{2,1,2} + q_{1,2,1} + q_{1,2,2} + q_{2,2,1} + q_{2,2,2} = 1.$$

Conversely to the equation system presented in Gross et al. [20], which describes a priority queue with infinite population, the one here developed has been tailored for populations of size $M = M_1 + M_2$. While an infinite population queue model has a constant rate of arrival at the system for each class, the rate of arrival of a queueing system with limited population has an arrival rate that varies depending on the number of clients currently in system. Therefore, the rate of arrival is the sum of the rates of all clients who may still arrive.

Note that the number of possible states and, consequently, the number of balance equations quickly grows as the number of customers in each class increases. For instance, the number of states of this model is $2\ M_1 M_2 + M_1 + M_2 + 1$. A problem with 5 customers of class 1, and 35 customers of class 2 results in 391 states, while a problem with 15 customers of class 1, and 90 customers of class 2 has 2806 states. Due to this situation, the analytical tractability of priority queues becomes burdensome.

Moreover, the description above fits into a two-class priority M/M/1/∞/M$_1$ + M$_2$/FCFS queue, with finite population of size $M_1 + M_2$. In the proposed model, we use a more general queue model by making times until arrivals (failures) for each customer follow a Weibull distribution. Such queue may be referred to as Weibull/M/1/∞/M$_1$ + M$_2$/FCFS, following a similar notation as used by Tamazian & Bogachev [28]. The use of Weibull-distributed times until failures for each equipment generalizes the case of a M/M/1 queue as well as incorporating characteristics such as equipment wear-out.

However, the change to Weibull-distributed times until failures causes the queue system to be even more complex. Indeed, the balance equations presented above become no longer valid. Therefore, we adopt a simulation-based algorithm to obtain the queue measures. Besides the complexity of the priority queue itself, the characteristics of the model proposed in Section 3 also require that we use a simulation approach. Detailed information on the simulation method can be found in Section 4.7.

4. Proposed model

4.1. Game characteristics

The proposed model deals with the problem of analyzing EWs for medical equipment, considering two classes of customers. Priority customers, normally large hospitals, belong to class 1, and non-priority customers, generally smaller hospitals, constitute class 2. The interaction between manufacturer and each customer will be modeled according to a Stackelberg Game formulation (Osborne & Rubinstein [18]) with perfect and complete information, where the OEM is a monopolist in the maintenance service market and has more bargain power than healthcare institutions. Moreover, both OEM and healthcare institutions (customers) are aware of their own alternatives and are interested choosing the one that maximizes their respective payoffs, which characterize rationality of the game players (Osborne & Rubinstein [18]). Thus, we adopt a static, non-cooperative and sequential game.

Moreover, the OEM is considered to be risk neutral and will pursue profit maximization through the sale of the medical equipment, EWs, and maintenance interventions on demand. Customers are considered to be risk averse (Boom [29]) and may belong to one of the two different classes. To model risk aversion, we use a utility function (Eq. (1)) that depends on the risk aversion parameter and on the profit obtained using the medical equipment:

$$U(w) = \frac{1 - e^{-\beta w}}{\beta}, \tag{1}$$

where w is the wealth, β is the risk aversion parameter, and the agent becomes more averse to risk as β grows; a similar utility function has been used by Ashgarizadeh & Murthy [19]. Other examples of utility functions, such as the Cobb-Douglas, used by Glickman & Berger [30] in the warranty context, may be found in Varian [31].

In the proposed model, all customers are assumed to be homogeneous regarding their risk aversion. The OEM, which decides first, determines the optimal EW price (P_w^c), and the optimal on-demand maintenance service price (C_s^c) for class c and the optimal number of customers of each class (M_1 and M_2). Then, customers decide whether to buy the equipment and which maintenance service option to hire. Given the complete and perfect information assumption, the OEM knows the customers' risk behavior, while customers can estimate their expected return given each alternative. Note that the OEM charges two different maintenance service prices, one for each class; this happens due to two factors. First, we consider that customers of class 1 generate more revenue per unit time than those of class 2, and thus they accept to pay higher prices for the service. Moreover, they will pay higher prices to have priority over the class 2, even when they do not hire an EW. Then, customers will decide among five options: A_0: not buying the equipment; A_1: hiring the priority EW; A_2: hiring the standard (no priority) EW; A_3: not hiring an EW, and thus paying for maintenance interventions on demand with priority; or A_4: not hiring an EW, paying for maintenance interventions on demand without priority. Thus, customers of class 1 choose among options A_0, A_1 and A_3, while class 2 customers decide among A_0, A_2 and A_4. Notice that class 1 customers will always have priority in recovering their equipment back into operation, since they prefer strategies with priority to without priority. Class 2 customers, on the other hand, favor less expensive options, even if they tend to wait longer.

4.2. Modeling failures and repairs

By using the referred equipment (an angiography in this case), class 1 hospitals generate revenue R_1 per time unit when it is on operational state, while class 2 hospitals generate R_2 per time unit. The equipment purchase price is C_b, which also covers the basic warranty period T_0; the EW coverage period is T, starting after the basic warranty expires. We consider that the time x between failures during the EW period, for each equipment, follows a conditioned Weibull distribution with scale parameter θ and shape parameter k, with probability density function described by Eq. (2).

$$f\left(x|t_0;\theta,k\right) = \frac{k}{\theta}\left(\frac{x+t_0}{\theta}\right)^{k-1} \exp\left[\left(\frac{t_0}{\theta}\right)^k - \left(\frac{x+t_0}{\theta}\right)^k\right], \tag{2}$$

where t_0 is the age of the equipment.

Notice that by conditioning the distribution of times until failures to the age t_0 of the equipment, it is implied that repairs do not restore equipment degradation; this is because we consider equipment to be subject to minimal repairs. Thus, the generation of times until failures is done by using Eq. (3), which returns a random time until next failure x that follows the age-conditioned Weibull distribution given in Eq. (2),

$$x = \theta\left[\left(\frac{t_0}{\theta}\right)^k - \ln\left(Unif(0,1)\right)\right]^{\frac{1}{k}} - t_0, \tag{3}$$

where *Unif*(0, 1) is a random number following a continuous uniform distribution between 0 and 1.

All equipment units are identical regarding their reliability and $M = M_1 + M_2$ represents the total number of customers in both classes. Failed units are repaired one at a time by the OEM with exponentially distributed times with rate μ. After a repair, the unit becomes operational again, being restored to the same condition it was before the failure, that is, repairs do not revert equipment degradation due to aging; thus, they are considered to be minimal.

The OEM offers four repair options to customers after the expiration of the basic warranty:

A_1) EW with priority: for a fixed price P_w^1, all failures occurred over $[T_0, T_0+T]$ are repaired at no additional cost for the customer. If the repair takes more than τ_1 (after failure) to be completed, the manufacturer is charged with a penalty of $\alpha_1(y-\tau_1)$ if $y>\tau_1$ and zero otherwise, where y is the time to repair that includes the time waiting in the queue and $y-\tau_1$ is the overtime, i.e., time that exceeds the limit of τ_1 to repair the equipment;

A_2) EW with no priority: for a fixed price P_w^2, all failures occurred over $[T_0, T_0+T]$ are repaired at no additional cost for the customer. If the repair takes more than τ_2 (after failure) to be executed, the manufacturer is charged with a penalty of $\alpha_2(y-\tau_2)$ if $y>\tau_2$ and zero otherwise, where $y-\tau_2$ is the overtime. Note that as class 1 customers have priority over class 2 customers, we have that $\tau_1 < \tau_2$ and $\alpha_1 > \alpha_2$;

A_3) No EW, with priority: each failure over $[T_0, T_0+T]$ is repaired at a cost C_s^1. As there is no legal agreement between both parties, there is no penalty in this option regarding the time to put the equipment back into operation. The total repair cost under this option is a random variable because it depends on the number of equipment failures over $[T_0, T_0+T]$, which is uncertain.

A_4) No EW, no priority: each failure over $[T_0, T_0+T]$ is repaired at a cost C_s^2. There is no penalty in this option due to the time to put the equipment back into operation. As it was the case with strategy A_3, the total repair cost under this option is a random variable because it depends on the number of equipment failures over $[T_0, T_0+T]$.

4.3. Customer's decision problem

Each customer's return $w(A_k)$ depends on the option $A_k (k = 0, 1, 2, 3, 4)$ the customers choose and is given as:

$$w\left(A_0\right) = 0, \tag{4}$$

$$w\left(A_1\right) = R_1\left(T_0 + \sum_{i=0}^{N_j} X_{ji} + \tilde{X}_j\right) + \alpha_1\left(\sum_{i=0}^{N_j} \max\left\{0, \left(y_{ji} - \tau_1\right)\right\}\right) - C_b - P_w^1, \tag{5}$$

$$w(A_2) = R_2\left(T_0 + \sum_{i=0}^{N_j} X_{ji} + \tilde{X}_j\right) + \alpha_2\left(\sum_{i=0}^{N_j} \max\{0, (y_{ji} - \tau_2)\}\right) - C_b - P_w^2, \tag{6}$$

$$w(A_3) = R_1\left(T_0 + \sum_{i=0}^{N_j} X_{ji} + \tilde{X}_j\right) - C_b - C_s^1 N_j, \tag{7}$$

$$w(A_4) = R_2\left(T_0 + \sum_{i=0}^{N_j} X_{ji} + \tilde{X}_j\right) - C_b - C_s^2 N_j, \tag{8}$$

where N_j is the number of failures occurred over $[T_0, T_0 + T]$ for the jth equipment; X_{ji} is the time to the ith $(0 \le i \le N_j)$ failure after the $(i{-}1)$th repair for the jth equipment; $\tilde{X}_j$ is the time interval during which jth equipment has been operational since the last repair to the end of the time horizon $T_0 + T$; note that $\tilde{X}_j = 0$ if the equipment is in failed state when it reaches $T_0 + T$; y_{ji} $(0 \le i \le N_j)$ is the total time to finish repairing the jth equipment since the occurrence of the ith failure, i.e., y_{ji} includes the waiting time in queue.

Thus, let $U(A_k| P_w^c, C_s^c, M_c)$ represent the customer's expected utility when option A_k $(0 \le k \le 4)$ is chosen by customers of class c; it can be obtained by using Eqs. (4)–(8) into Eq. (1). Therefore, the customer's optimal choice A^* is the one from the set $\{A_0, A_1, A_2, A_3, A_4\}$ that yields the maximum expected utility, which is a function of the OEM's decision variables.

We consider the cost of failures during the basic warranty period is covered by the equipment price (C_b), so these failures are not considered in this analysis. During this period, the OEM compensates the equipment revenue in case of failure, so that equipment downtime does not affect the customers' returns. Moreover, for $k > 1$, the number of failures during basic warranty coverage tends to be lower than during EW period, since failures tend to occur more often as time passes. For an angiography, these are reasonable assumptions.

4.4. OEM's decision problem

We assume the manufacturer is risk neutral and offers services to hospitals of classes 1 and 2. We also consider that, in each class, all customers are homogeneous, and thus they choose the same action with similar prices. Then, let the manufacturer's profit related to option A_k chosen by a given customer be denoted by π (P_w^c, C_s^c; A_k). Thus,

$$\pi(P_w^c, C_s^c|A_0) = 0, \tag{9}$$

$$\pi(P_w^1, C_s^1|A_1) = P_w^1 - C_r N_j - \alpha_1\left(\sum_{i=0}^{N_j} \max\{0, (y_{ji} - \tau_1)\}\right), \tag{10}$$

$$\pi(P_w^2, C_s^2|A_2) = P_w^2 - C_r N_j - \alpha_2\left(\sum_{i=0}^{N_j} \max\{0, (y_{ji} - \tau_2)\}\right), \tag{11}$$

$$\pi(P_w^1, C_s^1|A_3) = (C_s^1 - C_r) N_j, \tag{12}$$

$$\pi(P_w^2, C_s^2|A_4) = (C_s^2 - C_r) N_j, \tag{13}$$

where C_r is an average cost per repair.

The manufacturer's optimal choice for the decision variables P_w^c, C_s^c and M_c are obtained by maximizing the expected profit, considering the optimal choice A^* selected by the customers. Fig. 2 illustrates the game development by showing all possible actions in each decision stage for the OEM (determination of P_w^1 and C_s^1) and a given customer of class 1 (choice of A_0, A_1, and A_3), while Fig. 3 shows the game development by showing all possible actions in each decision stage for the OEM (determination of P_w^2 and C_s^2) and a given customer of class 2 (choice of A_0, A_2, and A_4).

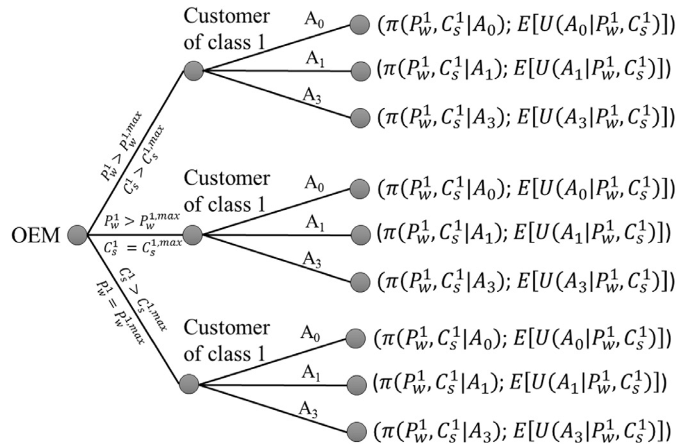

Fig. 2. Game tree for a given customer of class 1.

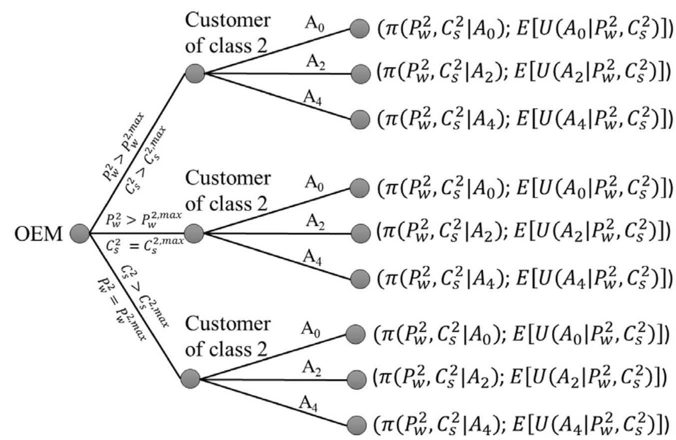

Fig. 3. Game tree for a given customer of class 2.

4.5. Customer's optimal action

It is necessary to evaluate the expected utility for each of the possible decisions to determine the customers' optimal strategy. The expected utilities for the decisions A_0, A_1, A_2, A_3 and A_4 are given by the expressions (14), (15), (16), (17) and (18), respectively and, as stated in Section 4.3, are obtained by using Eqs. (4)–(8) into Eq. (1).

$$E[U(A_0|P_w^c, C_s^c)] = 0, \tag{14}$$

$$E[U(A_1|P_w^1, C_s^1)] = \frac{1}{\beta_1}\left(1 - e^{\beta_1[C_b + P_w^1]} E\left[e^{-\beta_1 R_1\left(T_0 + \sum_{i=0}^{N_j} X_{ji} + \tilde{X}_j\right) - \beta_1 \alpha_1 \upsilon_j^1}\right]\right), \tag{15}$$

$$E[U(A_2|P_w^2, C_s^2)] = \frac{1}{\beta_2}\left(1 - e^{\beta_2[C_b + P_w^2]} E\left[e^{-\beta_2 R_2\left(T_0 + \sum_{i=0}^{N_j} X_{ji} + \tilde{X}_j\right) - \beta_2 \alpha_2 \upsilon_j^2}\right]\right), \tag{16}$$

$$E[U(A_3|P_w^1, C_s^1)] = \frac{1}{\beta_1}\left(1 - e^{\beta_1 C_b} E\left[e^{-\beta_1 R_1\left(T_0 + \sum_{i=0}^{N_j} X_{ji} + \tilde{X}_j\right) + \beta_1 N_j C_s^1}\right]\right), \tag{17}$$

$$E[U(A_4|P_w^2, C_s^2)] = \frac{1}{\beta_2}\left(1 - e^{\beta_2 C_b} E\left[e^{-\beta_2 R_2\left(T_0 + \sum_{i=0}^{N_j} X_{ji} + \tilde{X}_j\right) + \beta_2 N_j C_s^2}\right]\right), \tag{18}$$

where υ_j^c is the total overtime during T regarding customer j of class c.

In the case that the customer decides not to buy the equipment, the expected utility will be zero, as in Eq. (14). Eqs. (15), (16), (17) and (18) use some outputs from the priority queue system, as N_j, υ_j^c and $\sum_{i=0}^{N_j} X_{ji} + \widetilde{X_j}$. Since these values are stochastic, they are obtained via simulation algorithm described in Section 4.7 . Finally, for given P_w^c and C_s^c, a comparison among the expected utilities of each of the options will define the optimal strategy for customers of class c.

4.6. OEM's optimal action

Besides setting the EW prices for each class of customers and the price for maintenance interventions on demand, the manufacturer also determines the optimal number of customers in each class. To this end, it is necessary to compare the expected profit of each combination of customers in classes 1 and 2.

In fact, let $P_w^{c,max}$ and $C_s^{c,max}$ represent the maximal prices the customer is willing to pay for each option. Then, in order to maximize the expected profit, the OEM should choose either $P_w^c > P_w^{c,max}$ and $C_s^c = C_s^{c,max}$ to force customers to decide for maintenance interventions on demand or $P_w^c = P_w^{c,max}$ and $C_s^c > C_s^{c,max}$ to force customers to decide for signing an EW. Thus, the OEM's expected profit for each choice of the customers is given by (19):

$$E\left[\pi\left(P_w^1, P_w^2, C_s^1, C_s^2, M_1^*, M_2^* | A^{1*}, A^{2*}\right)\right] = \pi_1 + \pi_2, \tag{19}$$

where π_c is the total manufacturer's expected profit for all customers of class c (π_1 and π_2 are given by (20) and (21)), A^{1*}, A^{2*} are the optimal strategies for customers of class 1 and 2, respectively, and M_1^*, M_2^* are the optimal number of customers of class 1 and 2, respectively.

$$\pi_1 = \begin{cases} 0, \text{ if } C_s^1 > C_s^{1,max},\ P_w^1 > P_w^{1,max} \\ \sum_{j=1}^{M_1^*}\left(P_w^1 - C_r E\left[N_j\right] - \alpha_1 E\left[\upsilon_j^1\right]\right), \text{if } C_s^1 > C_s^{1,max}, P_w^1 = P_w^{1,max} \\ \sum_{j=1}^{M_1^*}\left(\left(C_s^1 - C_r\right) E\left[N_j\right]\right), \text{ if } C_s^1 = C_s^{1,max},\ P_w^1 > P_w^{1,max} \end{cases} \tag{20}$$

$$\pi_2 = \begin{cases} 0, \text{ if } C_s^2 > C_s^{2,max},\ P_w^2 > P_w^{2,max} \\ \sum_{j=1}^{M_2^*}\left(P_w^2 - C_r E\left[N_j\right] - \alpha_2 E\left[\upsilon_j^2\right]\right), \text{ if } C_s^2 > C_s^{2,max}, P_w^2 = P_w^{2,max} \\ \sum_{i=1}^{M_2^*}\left(\left(C_s^2 - C_r\right) E\left[N_j\right]\right), \text{ if } C_s^2 = C_s^{2,max},\ P_w^2 > P_w^{2,max} \end{cases} \tag{21}$$

Eqs. (20) and (21) are also computed by using outputs from the priority queueing system. More specifically, the arrival, repair and departure times are checked and, when repair durations are greater than τ_c, they are analyzed to obtain the total expected overtime for each customer $E[\upsilon_j^c]$, which is used to estimate the penalties paid by the OEM. Multiplying this value by α_c, it is possible to obtain the expected amounts to be refunded to the customer by the OEM due to delays in returning the equipment to operational state for classes 1 and 2, i.e., $\alpha_1 E[\upsilon_1^c]$ and $\alpha_2 E[\upsilon_2^c]$, respectively. Note that an expected value is used to calculate the penalties for each customer. This happens due to the stochastic nature of the overtimes. Thus, an expected value of such overtimes must be computed over $[T_0; T_0 + T]$ to estimate how much the OEM shall refund each customer. The optimal strategies are given in Table 2.

4.7. Priority queue simulation

As mentioned in Section 2.2, the above formulation describes a queuing system with finite population (M), where users are split into two classes, with class 1 having priority over class 2 with no preemption.

Table 2
Optimal strategies.

Strategy	OEM decision	Customer decision
1	$P_w^c > P_w^{c,max}$ and $C_s^c > C_s^{c,max}$, $c = 1,\ 2$	$A^{c*} = A_0$, $c = 1, 2$
2	$C_s^1 > C_s^{1,max}$ and $P_w^1 = P_w^{1,max}$	$A^{1*} = A_1$
3	$C_s^2 > C_s^{2,max}$ and $P_w^2 = P_w^{2,max}$	$A^{2*} = A_2$
4	$P_w^1 > P_w^{1,max}$ and $C_s^1 = C_s^{1,max}$	$A^{1*} = A_3$
5	$P_w^2 > P_w^{2,max}$ and $C_s^2 = C_s^{2,max}$	$A^{2*} = A_4$

For the equipment unit of each client, times until failure follow a conditioned Weibull distribution; a service crew can repair one unit at a time with constant average rate, returning the equipment to operational state but not reverting aging degradation (minimal repairs). If there is more than one failed unit of a given class at a time, the queue follows a FCFS rule. Despite, upon completion of a repair, the next unit to be repaired is always of class 1 if there are class 1 units in queue.

In order to formulate the proposed model and determine the customers' and OEM's optimal actions, a discrete event simulation (DES) algorithm (Ross [32]) has been here developed. We resorted to DES because of the complexity involved in the priority queue equations (Section 2.2), as well as their use in the context of the present model (Sections 4.5 and 4.6) . Alrabghi & Tiwari [33] analyzed state of the art of methods and practices regarding simulation of maintenance systems; Yang et al. [34] used simulation to forecast the number of warranty claims of equipment subject to a generalized renewal process considering usage rate; Alrabghi & Tiwari [35] present an approach for modelling complex maintenance systems using DES; Li & Stanford [36] used simulation to validate a model of accumulating priority queue.

The algorithm here developed may be divided into 3 main portions: (i) the priority queue system module (Fig. 4); (ii) determination of maximum prices for maintenance services; (iii) and maximization of the OEM's expected profit. An overview of how the three modules are used is shown in Fig. 5 and a more detailed description is provided in Fig. 6.

Note that: (i) t_d, time of departure (end of a repair), is set to infinity in step 1.2, when there is no equipment being repaired so that it will never be smaller than t_{a_1} and t_{a_2}, which in turn refer to the times of arrival (occurrence of a failure) of equipment of class 1 and class 2, respectively; (ii) the algorithm generates two lists (one for each class) containing, for each client, number of failures, downtime, and overtime. These lists are built from the times for each failure/repair, and each of their entries is linked to the corresponding equipment. These are the main outputs of the simulation (step 3.2), later used to estimate the prices of EWs and repairs on demand.

Initially in Fig. 4, the first failure times are generated in step 1.1 from a Weibull distribution (Eq. (3) with $t_0 = 0$) for classes 1 and 2, and then it is checked in step 2.1 which class the first customer belongs to; as the queue will be empty, the customer will immediately be served. As the next events occur, it is checked if it is an arrival (failure) or a departure (repair end) based on which of the times t_{a_1}, t_{a_2} and t_d is the smallest. If a failure of a class 2 customer occurs and the system is not empty, this hospital will proceed to queue. However, if an arrival of a class 1 hospital happens, this customer will use its priority preference and go through all the class 2 customers in queue and wait until all the other class 1 customers are served or, if there are no other class 1 customers in queue, it will wait until the server finishes its current service.

During the simulation, all important information is saved, i.e., the time the server is idle, equipment downtime and overtime (by using failure, repair, and departure times). This process is repeated until $T_0 + T$ is reached. At the end of the simulation, the two aforementioned lists containing, for each equipment unit, the number of failures, downtime, and overtime are saved for estimating the model's variables. Steps 1, 2 and 3 are repeated until the desired number of samples (replications) is reached.

After the queue is simulated, it is necessary to determine the customer's optimal strategy. To this end, we estimate the maximum

Inputs: $\theta, k, \mu, m, \alpha_1, \alpha_2, \beta_1, \beta_2, \tau_1, \tau_2, R_1, R_2, C_b, C_r, T_0, T, samples$

1. Initialization
   - 1.1. Generate times of next arrivals (failures) for each equipment in classes 1 and 2 and set t_{a_1} and t_{a_2} equal to the times at which the first equipment fails for each class, respectively
   - 1.2. Set $t_d = \infty$, which is the time for the next departure (completion of repair) ($t_d = \infty$ because the queue starts empty, thus a departure will not be chosen in item 2.1)
   - 1.3. Set initial time: $t_0 = T_0$
2. LOOP: Simulate the model
   - 2.1. Is the next event a failure or a completion of repair (compare the time of the next failure in each class, respectively t_{a_1} and t_{a_2}, and the time of next departure t_d)?
     - 2.1.1. Arrival (failure) of client of class c
       - Store failure information
         - Increment number of failures for class c and
         - Increment number of equipment in queue
         - Store failure time
       - If the system was empty before this failure
         - Generate repair duration (r) and set $y = u + r$
         - $t_d = t + y$
         - Store repair duration and departure time
         - If $y > \tau_c$, then store overtime of $y - \tau_c$
       - Find the next failure time and store to t_{a_c} based on the times of next failure for the remaining number of operational equipment in class c
     - 2.1.2. Departure (completion of repair) of client of class c
       - Store repair information
         - Increment number of repairs for class c
         - Decrease number of equipment in queue
         - Generate a failure time t'_{a_c} for this equipment
         - If $t'_{a_c} < t_{a_c}$, then set $t_{a_c} = t'_{a_c}$
       - Is there any equipment waiting in queue?
         - Yes
           - Define u = time spent in queue
           - Generate repair duration (r) and set $y = u + r$
           - $t_d = t + y$
           - Store repair duration and departure time
           - If $y > \tau_c$, then store overtime of $y - \tau_c$
         - No
           - Set $t_d = \infty$
   - 2.2. Is time length $T_0 + T$ reached and is the queue empty?
     - 2.2.1. Yes
       - Exit LOOP and finish simulation
     - 2.2.2. No
       - Repeat LOOP
3. Outputs (for each class, each entry linked to the corresponding equipment):
   - 3.1. Number of failures/repairs
   - 3.2. Downtime
   - 3.3. Overtime

Fig. 4. Priority queue system module.

customer's willingness to pay for each option offered by the OEM: $P_w^{1,max}$, $P_w^{2,max}$, $C_s^{1,max}$, and $C_s^{2,max}$, which are given by Eqs. (22)–(25). As these expressions denote the customer's maximum willingness to pay for each option, they are calculated by equalizing their expected utility to zero. Note that the amount that customers are willing to pay for each of the offered options depends on the magnitude of their risk aversion β.

$$P_w^{1,max} = -C_b - \frac{1}{\beta_1}\left(\ln E\left[e^{-\beta_1 R_1\left(T_0+\sum_{i=0}^{N_j} X_{ji}+\tilde{X}_j\right)-\beta_1\alpha_1 v_j^1}\right]\right), \tag{22}$$

$$P_w^{2,max} = -C_b - \frac{1}{\beta_2}\left(\ln E\left[e^{-\beta_2 R_2\left(T_0+\sum_{i=0}^{N_j} X_{ji}+\tilde{X}_j\right)-\beta_2\alpha_2 v_j^2}\right]\right), \tag{23}$$

$$\beta_1 C_b + \ln E\left[e^{-\beta_1 R_1\left(T_0+\sum_{i=0}^{N_j} X_{ji}+\tilde{X}_j\right)+\beta_1 C_s^{1,max} N_j}\right] = 0. \tag{24}$$

$$\beta_2 C_b + \ln E\left[e^{-\beta_2 R_2\left(T_0+\sum_{i=0}^{N_j} X_{ji}+\tilde{X}_j\right)+\beta_2 C_s^{2,max} N_j}\right] = 0. \tag{25}$$

Note also that it is difficult to determine $C_s^{c,max}$ analytically by using Eqs. (24) or (25). Thus, a numerical method is used. Along with the analytical difficulties commented in Section 2.2 due to the more general Weibull arrival times, the use of simulation is also required by the presence of the stochastic quantities N_j, $\sum_{i=0}^{N_j} X_{ji} + \tilde{X}_j$ and v_j^c in Eqs. (22)–(25).

Figs. 5 and 6 describe how each module of the proposed model interacts and how the OEM's expected profit is maximized. In fact, the queueing system is simulated through the algorithm described in Fig. 4 for every pair (M_1, M_2) such that $M_1 + M_2 = M$. For each case, $P_w^{c,max}$ and $C_s^{c,max}$ are computed via Eqs. (22)–(25) and the optimal strategies for the customers, which maximize the OEM's expected profit, are defined. Then, the population M of customers is incremented so that the simulation is repeated for the new values of (M_1, M_2). After the optimal population size is reached, subsequent increments on M will result in

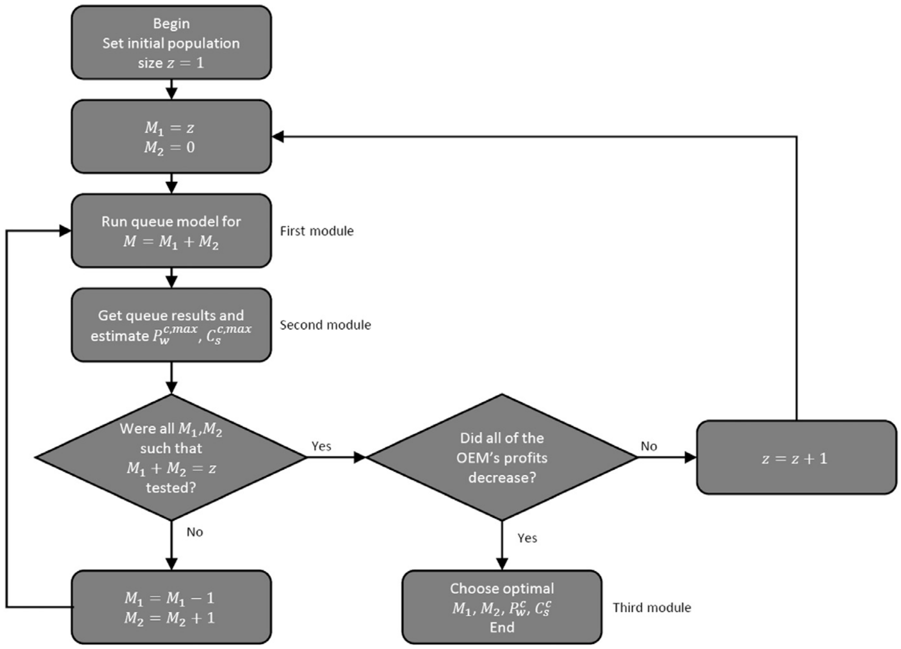

Fig. 5. Model simulation graphical overview.

Inputs: θ, k, μ, m, α_1, α_2, β_1, β_2, τ_1, τ_2, R_1, R_2, C_b, C_r, T_0, T, $samples$

1. Initialization (beginning of first module)
    - 1.1. Set initial population size ($z = 1$)
    - 1.2. Set initial class combination
        - 1.2.1. $M_1 = z$
        - 1.2.2. $M_2 = 0$
2. LOOP
    - 2.1. Run queue model for the current population $M = M_1 + M_2$ (Figure 4)
    - 2.2. Get queue results
        - 2.2.1. Estimate $C_s^{c,max}$ and $P_w^{c,max}$ using (22), (23), (24) and (25) (second module)
        - 2.2.2. Choose the best strategy for each class
        - 2.2.3. Store manufacturer profit with chosen strategies
    - 2.3. Have all class combinations M_1, M_2 for M been tested?
        - 2.3.1. No
            - ▪ Decrease the number of clients in class 1 and increment number of clients in class 2 ($M_1 = M_1 - 1, M_2 = M_2 + 1$)
            - ▪ Go back to the beginning of the loop
        - 2.3.2. Yes
            - ▪ Did OEM's profits decrease for all class combinations and service options?
                - No
                    - Increment population size ($z = z + 1$)
                    - Set new class combination
                        - $M_1 = z$
                        - $M_2 = 0$
                - Yes
                    - Exit LOOP
3. Define final results (third module)
    - 3.1. Choose population size and class combination which resulted in highest manufacturer profit
    - 3.2. Define C_s^c and P_w^c accordingly

Fig. 6. Model simulation description.

lower expected OEM's profits for all possible (M_1, M_2) and service options. If such behavior is observed, the simulation can be interrupted and optimal values can be defined.

4.8. Algorithm convergence

As a form of validation of the results presented in this paper, we present a comparison between the outputs from our simulation algorithm against the theoretical values given in Gross et al. [20], who present analytical formula for priority queues with exponentially distributed times. We make this comparison considering $k = 1$ in the simulation algorithm of Fig. 6. The following parameters were also used: $\theta = 2000$ h; $\mu = 0.05$ repairs per hour; $T_0 = 8760$ (hours); $T = 8760$ h. Results were compared for different numbers of replications (*samples*) = 1000, 10,000, 100,000 and 1000,000. The comparison can be found in Tables 3–6 and in Figs. 7, 8 for the expected number of failures and downtime. As it has been expected, for higher numbers of replications expected and theoretical values show close agreement. In this way,

Table 3
Comparison between theoretical and simulated results with 1000 samples.

M	M_1	M_2	Expected downtime			Expected number of failures		
			Simulated	Exact	Difference	Simulated	Exact	Difference
10	3	7	94.91	95.02	0.12%	4.316	4.332	0.38%
10	10	0	94.71	95.02	0.32%	4.331	4.332	0.04%
20	5	15	105.65	106.25	0.57%	4.313	4.327	0.32%
20	20	0	106.50	106.25	0.24%	4.317	4.327	0.23%
30	10	20	121.09	120.38	0.59%	4.344	4.320	0.56%
30	30	0	120.23	120.38	0.12%	4.319	4.320	0.02%
40	10	30	138.94	138.63	0.22%	4.330	4.311	0.45%
40	40	0	138.45	138.63	0.13%	4.303	4.311	0.18%
50	10	40	162.71	163.02	0.19%	4.306	4.298	0.17%
50	50	0	161.88	163.02	0.70%	4.301	4.298	0.06%

Table 4
Comparison between theoretical and simulated results with 10,000 samples.

M	M_1	M_2	Expected downtime			Expected number of failures		
			Simulated	Exact	Difference	Simulated	Exact	Difference
10	3	7	94.86	95.02	0.17%	4.341	4.332	0.19%
10	10	0	95.04	95.02	0.02%	4.337	4.332	0.10%
20	5	15	105.89	106.25	0.34%	4.322	4.327	0.10%
20	20	0	106.22	106.25	0.03%	4.322	4.327	0.12%
30	10	20	119.62	120.38	0.62%	4.313	4.320	0.17%
30	30	0	120.19	120.38	0.15%	4.322	4.320	0.04%
40	10	30	138.52	138.63	0.08%	4.307	4.311	0.08%
40	40	0	138.47	138.63	0.12%	4.315	4.311	0.10%
50	10	40	162.04	163.02	0.60%	4.296	4.298	0.05%
50	50	0	162.44	163.02	0.35%	4.301	4.298	0.05%

Table 5
Comparison between theoretical and simulated results with 100,000 samples.

M	M_1	M_2	Expected downtime			Expected number of failures		
			Simulated	Exact	Difference	Simulated	Exact	Difference
10	3	7	94.88	95.02	0.15%	4.331	4.332	0.03%
10	10	0	94.93	95.02	0.10%	4.332	4.332	0.02%
20	5	15	106.11	106.25	0.13%	4.325	4.327	0.04%
20	20	0	106.21	106.25	0.04%	4.327	4.327	0.01%
30	10	20	120.21	120.38	0.14%	4.322	4.320	0.04%
30	30	0	120.34	120.38	0.03%	4.322	4.320	0.04%
40	10	30	138.24	138.63	0.28%	4.310	4.311	0.03%
40	40	0	138.18	138.63	0.33%	4.311	4.311	0.01%
50	10	40	162.48	163.02	0.33%	4.299	4.298	0.02%
50	50	0	162.48	163.02	0.33%	4.300	4.298	0.03%

Table 6
Comparison between theoretical and simulated results with 1000,000 samples.

M	M_1	M_2	Expected downtime			Expected number of failures		
			Simulated	Exact	Difference	Simulated	Exact	Difference
10	3	7	94.91	95.02	0.12%	4.331	4.332	0.04%
10	10	0	94.98	95.02	0.05%	4.333	4.332	0.00%
20	5	15	106.15	106.25	0.10%	4.327	4.327	0.00%
20	20	0	106.17	106.25	0.08%	4.327	4.327	0.01%
30	10	20	120.16	120.38	0.18%	4.320	4.320	0.00%
30	30	0	120.18	120.38	0.16%	4.320	4.320	0.01%
40	10	30	138.32	138.63	0.23%	4.312	4.311	0.02%
40	40	0	138.31	138.63	0.24%	4.311	4.311	0.02%
50	10	40	162.42	163.02	0.37%	4.299	4.298	0.02%
50	50	0	162.41	163.02	0.37%	4.299	4.298	0.02%

we show the simulation algorithm that is here proposed is a general case of the developments of Gross et al. [20].

Moreover, we performed a comparison using different number of samples for $k = 2$ (Weibull distribution), $\theta = 5800$; $k = 2$; $\mu = 0.05$; $T_0 = 8760$ and $T = 8760$. In this case, we could not compare our results against the analytical findings of Gross et al. [20], but it is possible to notice how the outputs converge to narrower ranges as the number of samples increases. Also, this analysis confirms that the expected values for the single-class cases are equivalent to the expected values for the 2-class systems with the same number of devices M, i.e., the expectations converge to the same values when M is the same, even when M_1 and M_2 are different between the cases. Note, however, that expectations of each priority class are different, as is seen in Section 5.2 . The results can be found in Tables 7, 8, Figs. 9 and 10.

5. Application example

5.1. Model's parameters

We apply the proposed method to a numerical example. The device here considered, an angiography, is used for imaging examinations with help of a contrast agent, allowing doctors to visualize blood vessels and blood flow. Failures might cause diagnostic errors, imprecise readings and even prevent it from functioning at all. In any case, failures are critical to the hospitals' ability to adequately make use of the angiography, diagnose and treat patients, therefore the importance of returning medical equipment into operation quickly.

When new, the device is more reliable than after one or more years of operation, which means it suffers degradation with age, and fails with increased rate over time. Therefore, failures during the basic warranty coverage period $[0, T_0]$ are expected to be fewer than during the EW coverage period $[T_0, T]$. The costs of failures during the basic warranty period $[0, T_0]$, from the OEM's point of view, are covered by the equipment price C_b; the customers' lack of revenue during downtime in that period are refunded by the OEM, also covered by C_b. Due to these assumptions, starts at time T_0, which is the beginning of the EW period and the starting age of all devices being simulated.

The model's parameters are the following: $\theta = 5800$ h; $k = 2$; $\mu = 0.05$ repairs per hour; $\alpha_1 = 1.5$ (10^3 \$ per hour); $\alpha_2 = 0.2$ (10^3 \$ per hour); $\beta_1 = \beta_2 = \beta = 0.2$; $\tau_1 = 48$ h; $\tau_2 = 96$ h; $C_b =$ \$ 1476.5 (10^3); $T_0 = 8760$ hours; $T = 8760$ h; $C_r =$ \$ 5.4 (10^3); $R_1 =$ \$ 0.110 (10^3 per hour); $R_2 =$ \$ 0.105 (10^3 per hour); samples = 1000,000, which is the number of iterations for simulating the priority queue system.

5.2. Analysis of the results

The inputs given in Section 5.1 were used to feed up the method proposed in this paper by following the algorithms given in Figs. 4–6. First, the algorithm described in Fig. 4 is executed for a number of samples for each combination of M_1 and M_2; next, $P_w^{c,max}$ and $C_s^{c,max}$ are estimated for each case, and finally the scenario that results in the greatest profit for the OEM is found. To that end, it is necessary to analyze which set of possible actions maximizes the OEM's profit: (i) EWs to both classes; or (ii) EWs to class 1 and services on demand to class 2; or (iii) services on demand to class 1 and EWs to class 2; or (iv) services on demand to both classes.

Given that and by using the estimates of the model's parameters, the optimal number of customers is $M = 48$, where $M_1 = 14$ and $M_2 = 34$. The optimal strategy for the OEM is to offer EWs with priority for class 1 ($A^{1*} = A_1$) and EWs without priority for class 2 customers ($A^{2*} = A_2$), as this strategy maximizes the OEM's expected profit. Thus, the manufacturer should set the prices as follows: $P_w^1 = P_w^{1,max} =$ \$ 441,216 and $C_s^1 > C_s^{1,max} =$ \$ 21,329 for class 1; $P_w^2 = P_w^{2,max} =$ \$ 329,864 and $C_s^2 > C_s^{2,max} =$ \$ 11,921 for class 2. Such strategy generates an expected total profit $E[\pi]$ of \$ 11,888,594.

We also considered two scenarios where all customers were in a single-class queueing system. In one case, all customers are from class 1 ($c = 1$), while for the other case all customers are from class 2 ($c = 2$). In this situation, the optimal number of customers to be served is $M = 35$ when $c = 1$, a reduction of 13 in the total number of customers, while $M = 49$ when $c = 2$, one customer more than the 2-class system. This happens because class 1 customers receive more penalties, causing the OEM to serve a lower number of customers when all are in class 1,

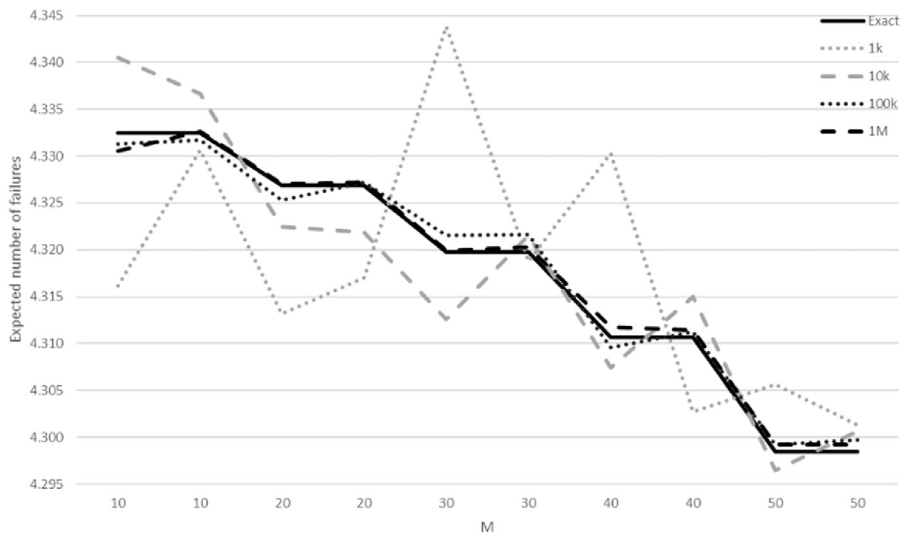

Fig. 7. Theoretical and simulated results for the expected number of failures.

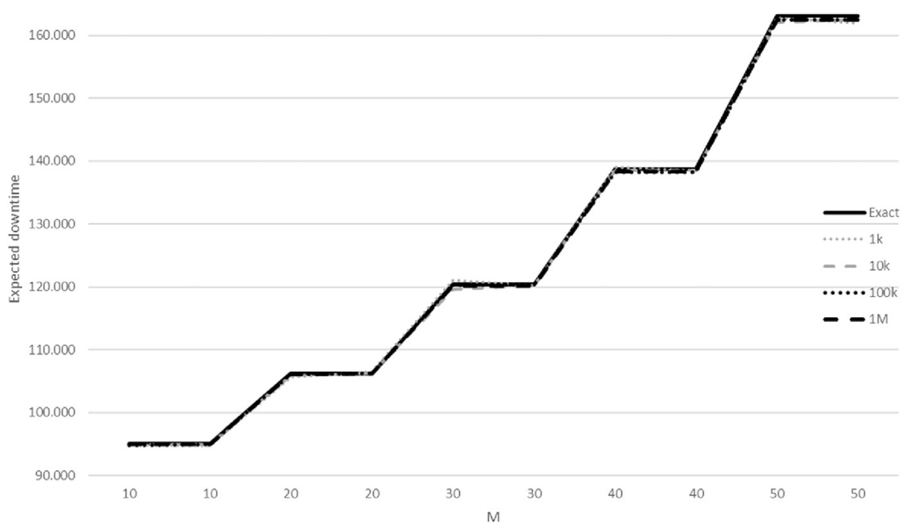

Fig. 8. Theoretical and simulated results for the expected downtime.

Table 7
Simulated results for the expected number of failures with different numbers of repetitions.

M	M_1	M_2	Expected number of failures			
			1000	10,000	100,000	1000,000
10	3	7	6.676	6.685	6.682	6.682
10	10	0	6.667	6.684	6.681	6.683
20	5	15	6.672	6.647	6.652	6.650
20	20	0	6.656	6.647	6.650	6.649
30	10	20	6.611	6.606	6.598	6.600
30	30	0	6.598	6.607	6.601	6.600
40	10	30	6.529	6.522	6.522	6.520
40	40	0	6.509	6.526	6.520	6.521
50	10	40	6.386	6.387	6.388	6.387
50	50	0	6.382	6.385	6.388	6.387

Table 8
Simulated results for the expected downtimes with different numbers of repetitions.

M	M_1	M_2	Expected downtime			
			1000	10,000	100,000	1000,000
10	3	7	155.07	155.71	155.42	155.32
10	10	0	155.32	154.82	155.42	155.37
20	5	15	187.43	188.09	188.38	188.32
20	20	0	188.37	187.93	188.13	188.32
30	10	20	238.48	238.60	237.88	237.90
30	30	0	236.73	239.00	237.85	237.91
40	10	30	320.06	319.08	317.79	318.07
40	40	0	314.47	318.39	318.02	318.07
50	10	40	457.85	455.92	454.61	455.21
50	50	0	452.11	454.61	455.06	455.20

while class 2 customers receive less in penalties, allowing the OEM to serve more customers before penalties put its profit at risk. Thus, the 2-class system with different contract terms allows OEM to expect a higher profit, while customers also benefit from having better availability levels (class 1) or from paying less for the services (class 2).

Indeed, when $c = 1$, considering the maximum prices that could be charged, the manufacturer should set $P_w = P_w^{max}$ = \$ 444,783 which is slightly higher than $P_w^{1,max}$ because customers in the single-class system receive more in penalties, becoming willing to pay a slightly higher EW price. Moreover, $C_s > C_s^{max}$ = \$ 20,460 (4% lower than $C_s^{1,max}$ and 71.6% higher than $C_s^{2,max}$) and the customers must choose the action A_1 and hire EWs. Then, the OEM yields an expected profit $E[\pi]$ = \$ 9836,003 (17.3% lower than the expected profit considering two classes).

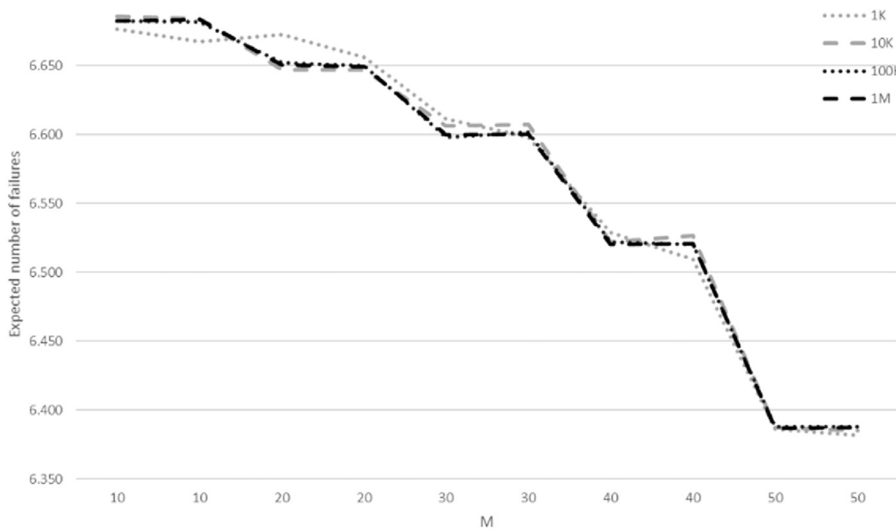

Fig. 9. Simulated results for the expected number of failures.

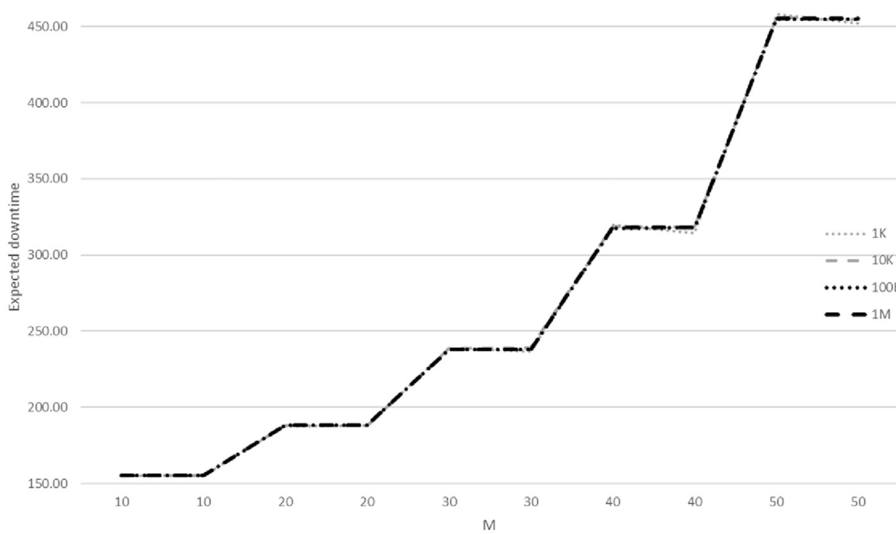

Fig. 10. Simulated results for the expected downtime.

Similar behavior is observed when $c = 2$, but now results are closer to what is found for the class 2 customers. The OEM should set $P_w = P_w^{max}$ = \$ 324,957, which is slightly lower than $P_w^{2,max}$; $C_s > C_s^{max}$ = \$ 13,229 (38% lower than $C_s^{1,max}$ and 11% higher than $C_s^{2,max}$). The OEM has profit $E[\pi]$ = \$ 11,684,100 (1.7% lower than the expected profit considering two classes).

Thus, offering different terms for different classes of customers allows the OEM to serve either hospitals that do not want to wait or hospitals that do not need to rush to have their equipment repaired, offering different prices for different service levels that can be delivered simultaneously. The 2-class priority system also results in higher expected profit for the manufacturer. Between the single-class models, the OEM's profit is higher when $c = 2$ than when $c = 1$, due to the amount of penalties paid by the OEM. When $c = 2$, penalties are reduced, but customers are not willing to pay as much as class 1 customers. Thus, the 2-class system is able to service both hospital classes and still improve the OEM's profits; see Table 9.

The performance of the queueing system is another important source of information. First, considering the 2-class priority model, it has been found that the server's mean idle time is 2686.68 h (30.7% of T). The mean total time the equipment stays in the system, i.e., time the equipment spends in failed state (downtime) is 247.95 h for class 1 equipment and 492.99 h for class 2 equipment. Additionally, the mean overtime is 54.75 h for a customer of class 1 and 158.05 h for a hospital of class 2 (see Table 10). Class 2 equipment has considerably higher expected downtime; also, the expected overtime for each equipment unit is more than doubled for class 2 customers, although class 2 has a longer time limit before penalties occur ($\tau_2 = 96 > \tau_1 = 48$). Within such arrangement, the total amount the manufacturer pays in penalties is \$ 1149,750 for class 1 customers and \$ 2686,850 for class 2 customers, averaging \$ 82,125 for each class 1 customer and \$ 79,025 for each class 2 customer.

Considering now the queueing system with only one class, it has been found the server's mean idle time is 4201.76 h when $c = 1$ (48% of T and 56.4% higher than the 2-class priority case), and 2581.15 h when $c = 2$ (29.5% of T and 3.9% higher than the 2-class priority case). The mean time the equipment stays in the system (downtime) is 272.88 h when $c = 1$ (10% more than class 1 and 44.6% less than class 2), and 437.72 h when $c = 2$ (76.5% more than class 1 and 11.2% less than class 2). Yet, the mean overtime is 85.47 h when $c = 1$ (56.1% more than class 1 and 45.9% less than class 2), and 104.39 h when $c = 2$ (90.7% more than class 1 and 34% less than class 2); see Table 10.

All such results are in accordance to what is stated in Gross et al. [20] that the priority queue is beneficial for the class of higher priority, as it waits less and stays for a shorter period in system if compared

Table 9
Comparison between 2-class and single-class systems.

	Class 1	Class 2	2-class (total)	Single-class System (c = 1)	Single-class System (c = 2)
M_c	14	34	48	35	49
$P_w^{c,max}$ $	441,216	329,864	–	444,783	324,957
$C_s^{c,max}$ $	21,329	11,921	–	20,460	13,229
A^{c*}	A_1	A_2	–	A_1	A_2
E[π] $	4529,414	7359,180	11,888,594	9836,003	11,684,100

Table 10
Performance differences between priority and non-priority systems.

	2-class model		Single-class model (c = 1)	Single-class model (c = 2)
Server mean idle time (h)	2686.68		4201.76	2581.15
	Class 1	Class 2		
Mean n. of failures (N_j)	6.59	6.35	6.56	6.40
Mean downtime (h)	247.95	492.99	272.88	437.72
Mean overtime $E[v_j^c]$ (h)	54.75	158.05	85.47	104.39
Expected penalties ($)	1149,750	2686,850	4487,175	2557,555

Table 11
Optimal solution changes due to θ variations.

θ	M	M_1	$P_w^{1,max}$ $	$C_s^{1,max}$ $	A^{1*}	M_2	$P_w^{2,max}$ $	$C_s^{2,max}$ $	A^{2*}	E[π] $
5400	41	10	439,272	19,180	A_1	31	324,160	11,239	A_2	9752,573
5600	44	10	439,918	21,168	A_1	34	326,096	12,163	A_2	10,796,320
5800	48	14	441,216	21,329	A_1	34	329,864	11,921	A_2	11,888,594
6000	52	13	441,593	21,670	A_1	39	331,947	12,260	A_2	13,070,610
6200	55	17	442,525	23,061	A_1	38	334,938	12,399	A_2	14,301,400

Table 12
Optimal solution changes due to k variations.

k	M	M_1	$P_w^{1,max}$ $	$C_s^{1,max}$ $	A^{1*}	M_2	$P_w^{2,max}$ $	$C_s^{2,max}$ $	A^{2*}	E[π] $
1.8	65	23	444,092	25,070	A_1	42	339,063	12,283	A_2	17,509,530
1.9	56	16	442,479	22,782	A_1	40	334,232	12,454	A_2	14,434,970
2.0	48	14	441,216	21,329	A_1	34	329,864	11,921	A_2	11,888,594
2.1	42	6	438,544	19,694	A_1	36	323,437	12,276	A_2	9806,516
2.2	36	6	437,015	18,554	A_1	30	318,397	11,298	A_2	8062,281

Table 13
Optimal solution changes due to μ variations.

μ	M	M_1	$P_w^{1,max}$ $	$C_s^{1,max}$ $	A^{1*}	M_2	$P_w^{2,max}$ $	$C_s^{2,max}$ $	A^{2*}	E[π] $
3.0 (10^{-2})	27	0	–	–	A_0	27	328,749	11,547	A_2	5606,730
4.0 (10^{-2})	38	0	–	–	A_0	38	326,121	12,299	A_2	8594,872
5.0 (10^{-2})	48	14	441,216	21,329	A_1	34	329,864	11,921	A_2	11,888,594
6.0 (10^{-2})	59	23	439,353	21,456	A_1	36	332,075	11,703	A_2	15,544,420
7.0 (10^{-2})	69	34	438,357	21,093	A_1	35	333,872	11,240	A_2	19,336,350

to the single-class. On the other hand, the PQT leaves the non-priority class in a worse situation when compared to a single class system, as customers belonging to this class tend to wait more in queue to get their equipment repaired. In short, comparing the priority against the single-class model, it may be observed that class 1 customers face a much better situation regarding the mean downtime and the mean overtime, while class 2 customers are penalized and tend to face a worse situation in comparison to a single-class system. In order to enrich the analysis and provide managerial insights for decision makers, next we present a sensitivity analysis for the model's parameters.

5.3. Sensitivity analysis

A sensitivity analysis is performed to investigate how the optimal values change due to marginal variations of some parameters. Variations on the following parameters were considered: θ (Table 11), k (Table 12), μ (Table 13), β (Table 14), α_1 (Table 15), α_2 (Table 16), τ_1 (Table 17), τ_2 (Table 18), R_1 (Table 19), R_2 (Table 20) and m (Table 21).

Table 11 presents how the optimal solution changes due to variations in the Weibull distribution scale parameter (θ). For lower values of θ, failures occur more often; maintenance expenses tend to grow as well as the expected penalties. Thus, when θ is raised, it may be observed that the optimal number of customers increases along with the manufacturer's expected profit. In practical terms, more reliable equipment results in better availability to hospitals; thus, the OEM spends less with maintenance while receiving more revenue.

Table 12 shows the effect of variations on the Weibull distribution shape parameter. As k increases, equipment degrades at faster rates over time causing failures to occur more often, resulting in a greater expected number of failures over the EW. This causes customers' devices to be more unavailable and receive more penalties, as well as increasing OEM's costs due to repairs when an EW is hired, thus reducing the OEM's expected profit along with the number of customers being serviced, especially class 1 customers. Notice that the effects of changes on θ and k are analogous, since both parameters affect the frequency of failures over time. When more failures occur, equipment availability

Table 14
Optimal solution changes due to β variations.

β	M	M_1	$P_w^{1,max}$ $	$C_s^{1,max}$ $	A^{1*}	M_2	$P_w^{2,max}$ $	$C_s^{2,max}$ $	A^{2*}	E[π] $
1.0 (10^{-1})	49	10	449,129	23,882	A_1	39	344,434	15,561	A_2	12,581,710
1.5 (10^{-1})	49	10	444,135	22,438	A_1	39	336,492	13,124	A_2	12,221,950
2.0 (10^{-1})	48	14	441,216	21,329	A_1	34	329,864	11,921	A_2	11,888,594
2.5 (10^{-1})	47	13	437,959	20,671	A_1	34	323,449	11,501	A_2	11,660,650
3.0 (10^{-1})	47	15	435,672	20,366	A_1	32	317,855	11,184	A_2	11,414,430

Table 15
Optimal solution changes due to α_1 variations.

α_1 ($10^3)	M	M_1	$P_w^{1,max}$ $	$C_s^{1,max}$ $	A^{1*}	M_2	$P_w^{2,max}$ $	$C_s^{2,max}$ $	A^{2*}	E[π] $
0.5	49	49	447,845	17,040	A_1	0	–	–	A_0	14,967,580
1.0	47	26	442,513	20,838	A_1	21	334,411	9872	A_2	12,442,820
1.5	48	14	441,216	21,329	A_1	34	329,864	11,921	A_2	11,888,594
2.0	49	5	440,544	22,210	A_1	44	327,227	12,637	A_2	11,722,260
2.5	49	1	440,399	24,279	A_1	48	325,561	12,775	A_2	11,670,850

Table 16
Optimal solution changes due to α_2 variations.

α_2 ($10^3)	M	M_1	$P_w^{1,max}$ $	$C_s^{1,max}$ $	A^{1*}	M_2	$P_w^{2,max}$ $	$C_s^{2,max}$ $	A^{2*}	E[π] $
0.1	63	19	443,080	20,682	A_1	44	275,676	0	A_2	13,833,690
0.3	54	11	440,879	21,405	A_1	43	325,802	8916	A_2	13,084,120
0.5	48	14	441,216	21,329	A_1	34	329,864	11,921	A_2	11,888,594
0.7	45	11	440,551	21,333	A_1	34	330,603	13,436	A_2	11,122,900
0.9	41	16	441,199	21,507	A_1	25	332,412	14,130	A_2	10,545,520

Table 17
Optimal solution changes due to τ_1 variations.

τ_1	M	M_1	$P_w^{1,max}$ $	$C_s^{1,max}$ $	A^{1*}	M_2	$P_w^{2,max}$ $	$C_s^{2,max}$ $	A^{2*}	E[π] $
24	50	0	–	–	A_0	50	325,433	12,890	A_2	11,672,540
36	48	3	450,024	22,855	A_1	45	325,938	12,487	A_2	11,687,100
48	48	14	441,216	21,329	A_1	34	329,864	11,921	A_2	11,888,594
60	47	19	432,591	21,135	A_1	28	331,436	12,092	A_2	12,211,460
72	47	25	424,867	20,064	A_1	22	334,031	10,241	A_2	12,521,630

Table 18
Optimal solution changes due to τ_2 variations.

τ_2	M	M_1	$P_w^{1,max}$ $	$C_s^{1,max}$ $	A^{1*}	M_2	$P_w^{2,max}$ $	$C_s^{2,max}$ $	A^{2*}	E[π] $
48	45	15	441,248	22,217	A_1	30	361,018	12,403	A_2	11,199,380
72	47	13	440,999	21,173	A_1	34	345,916	12,594	A_2	11,663,350
96	48	14	441,216	21,329	A_1	34	329,864	11,921	A_2	11,888,594
120	49	13	441,173	21,785	A_1	36	313,151	11,284	A_2	11,973,570
144	49	16	441,703	21,105	A_1	33	297,094	10,835	A_2	11,907,572

Table 19
Optimal solution changes due to R_1 variations.

R_1($10^3)	M	M_1	$P_w^{1,max}$ $	$C_s^{1,max}$ $	A^{1*}	M_2	$P_w^{2,max}$ $	$C_s^{2,max}$ $	A^{2*}	E[π] $
0.106	49	0	–	–	A_0	49	325,250	12,610	A_2	11,689,030
0.108	49	0	–	–	A_0	49	325,250	12,610	A_2	11,689,030
0.11	48	14	441,216	21,329	A_1	34	329,864	11,921	A_2	11,888,594
0.112	46	22	477,420	22,559	A_1	24	332,882	11,414	A_2	12,483,920
0.114	46	26	512,985	23,633	A_1	20	333,745	10,948	A_2	13,292,910

Table 20
Optimal solution changes due to R_2 variations.

R_2($10^3)	M	M_1	$P_w^{1,max}$ $	$C_s^{1,max}$ $	A^{1*}	M_2	$P_w^{2,max}$ $	$C_s^{2,max}$ $	A^{2*}	E[π] $
0.101	43	28	444,024	20,885	A_1	15	266,004	7514	A_2	10,259,980
0.103	46	23	442,999	20,619	A_1	23	299,529	9344	A_2	10,906,720
0.105	48	14	441,216	21,329	A_1	34	329,864	11,921	A_2	11,888,594
0.107	50	0	–	–	A_0	50	359,270	13,828	A_2	13,367,080
0.109	52	0	–	–	A_0	52	393,851	15,442	A_2	15,077,260

Table 21
Optimal solution changes due to m variations.

m	M	M_1	$P_w^{1,max}$ $	$C_s^{1,max}$ $	A^{1*}	M_2	$P_w^{2,max}$ $	$C_s^{2,max}$ $	A^{2*}	$E[\pi]$ $
1	48	14	441,216	21,329	A_1	34	329,864	11,921	A_2	11,888,594
2	102	59	438,714	20,913	A_1	43	333,092	11,104	A_2	29,540,950
3	154	112	438,189	20,832	A_1	42	335,047	11,216	A_2	48,302,200

is reduced and more penalties are incurred. Consequently, customers become less willing to pay for the services, forcing the OEM to reduce the number of customers served, cutting its expected profits.

Table 13 presents how the optimal solution changes due to variation in the service rate (μ). For low values of μ, the service team is able to repair fewer items per time unit, and thus the optimal number of customers decreases. When $\mu \leq 0.04$, the OEM decides not to service class 1 customers; this happens due to the low equipment availability and the occurrence of more penalties, since the OEM is not able to repair failed equipment as quickly as needed. On the other hand, when μ increases, the service team can complete repairs at faster rates and the optimal number of customers increases. Indeed, as μ is raised, the number of class 1 customers increases, while the number of class 2 customers remains about the same; since class 1 customers are willing to pay more for the services, the OEM tends to service more clients of this class for greater values of μ. In general, higher μ translates into faster repairs, allowing the OEM to service a higher number of hospitals while maintaining good levels of equipment availability.

In Table 14, it is shown how the optimal solution changes due to variations in the risk aversion parameter rate (β). As β increases, hospitals become more averse to risk, causing their expected utilities to reach lower values faster, thus they tend to pay lower prices for the services; this causes the OEM's expected profit to decrease. When customers are more averse to risk, their willingness to pay for the services is more affected by the randomness of costs. On the other hand, for low values of β, the OEM can service more class 2 customers than when β is higher, since they accept to pay more for maintenance services in these scenarios.

Tables 15 and 16 present how the optimal solution changes due to variations on the penalty per time unit parameters (α_1 and α_2) for classes 1 and 2 respectively. For higher α_1 values, class 1 customers receive more in penalties when there is overtime, causing the OEM to service fewer class 1 customers and more class 2 customers to compensate. For instance, when $\alpha_1 = 0.5$ (\$$10^3$), the OEM services 49 class 1 customers and does not sell the equipment to class 2 customers. On the other hand, when $\alpha_1 = 2.5$ (\$$10^3$) only one class 1 customer is served, while 48 customers of class 2 are served; for different values of α_2, a similar behavior is observed, but due to the differences in revenue between both classes, sometimes the OEM also serves more class 1 customers when α_2 is low, which is the case when $\alpha_2 = 0.1$ (10^3). In general, when α_c is high, customers of class c receive more in penalties due to equipment unavailability. Therefore, the OEM can serve fewer hospitals, especially of class c, due to the increased costs of overtime.

Tables 17 and 18 show how the optimal solution changes due to variations in the maximum times to repair τ_1 and τ_2. The effects of these variations are similar to the ones caused by changes in α_1 and α_2, because both α_c and τ_c influence the amount of penalties incurred. However, while α_c affects the rate at which overtime increases penalties, τ_c drives how long the equipment may wait before penalties occur. The OEM tends to service more customers of class c as τ_c increases; in general, the OEM's expected profit increases as τ_c is increased. However, for $\tau_2 = 144$, the number of customers M remains the same, M_1 increases, and M_2 decreases, in relation to the values for $\tau_2 = 120$; see Table 18. This situation occurs because the amount of penalties paid by the OEM to class 2 customers decreases, resulting in class 2 customers paying less for EWs. Thus, the OEM increases the number of class 1 customers and decreases the number of class 2 customers; the OEM's profit is slightly reduced, since class 2 customers do not pay as much as with lower values of τ_2.

Note that, among all the analysis here performed, strategies A_3 and A_4 were not chosen. We found this situation is due to the risk aversion behavior of customers, along with the fact that the OEM always seeks to maximize profit. Indeed, if customers choose strategies A_3 or A_4, they are more susceptible to unexpectedly high maintenance costs since their equipment may suffer too many failures. Thus, such a situation is avoided by choosing EWs since this option offers less risk to hospitals. Consequently, customers are willing to pay significantly less for repairs on demand, strongly decreasing OEM's expected profit.

In cases where penalties are very high, because τ_c is small (especially when $\tau_c < 1/\mu$) and/or α_c is high, it is expected that A_3 is also chosen since such scenarios would cause EWs to be financially unfeasible for the OEM. For instance, when $\alpha_1 = 4$, $\alpha_2 = 3$, $\tau_1 = 18$ and $\tau_2 = 24$, class 1 customers choose strategy A_3, buying the equipment without EW, and class 2 customers decide not to buy a device: $M_1 = 46$ and $M_2 = 0$. The prices set by the OEM are as follows: $P_w^1 > P_w^{1,max}$ = \$ 473,426, $C_s^1 = C_s^{1,max}$ = \$ 18,320. $E[\pi]$ = \$ 3832,864. Since both classes have modified values for α_c and τ_c, the model behaves differently from when there are changes in parameters for each class individually (Tables 15–18). In those cases, the OEM avoids the class for which penalties are incurred more easily, while changes in parameters for both classes at once result in EWs being less profitable for the OEM. Thus, A_3 (or A_4) becomes a better option in comparison with A_1 (or A_2). Yet, note that class 2 customers do not buy the equipment. Since class 1 customers receive higher revenue than class 2 customers, they are willing to pay higher values of C_s^c, resulting in higher profit for the OEM.

Changes on the strategies caused by the difference in revenue for each class can be seen in Tables 19 and 20. When class 1 customers have only slightly higher revenue during operational time in relation to class 2 customers, the OEM decides to service only class 2 customers, since class 2 customers receive less in penalties in relation to the revenue they generate. On the other hand, when class 1 customers' revenues are raised, the OEM services more class 1 customers, since they are willing to pay significantly more for services. Note that the model is highly sensitive to variations in R_c. In practical terms, if class 1 hospitals can generate much more revenue from using the equipment, the OEM should prefer selling EWs to that class. However, if class 2 hospitals are able to generate a revenue similar to that of class 1 hospitals, the OEM should service only class 2 hospitals.

Finally, Table 21 shows how the optimal solution changes when it is considered that the OEM has more than one service team. More specifically, the optimal results for queueing systems with $m = 2$ and $m = 3$ servers in parallel are presented. As m increases, the number of customers and the manufacturer's expected profit significantly increases to 102 and \$ 29,540,950, respectively, when the number of servers is $m = 2$; and to 154 and \$ 48,302,200 for $m = 3$. This is an expected consequence of the possibility of serving more customers when there are more service teams available. For $m > 1$, class 1 and class 2 hospitals choose to hire an EW. Since the OEM can repair up to 2 or 3 units simultaneously, the occurrence of penalties due to longer waiting is much less probable and it is possible to service a much larger number of class 1 customers, thus generating greater revenue. Note that the increase in the number of customers occurs in a slightly higher rate than that of the increase in m. This happens because besides a higher m results in a higher number of simultaneous repairs, it also decreases the negative impact of long repairs, resulting in shorter and less variable equipment downtime, and consequently causing penalties to be even lower.

6. Concluding remarks

The model developed in this work illustrates the situation in which a healthcare institution needs to acquire and maintain a technology-intensive equipment to service patients. Broadly speaking, the main developments presented here in relation to previous models were: (i) studying the specific field of healthcare institutions that deal with the OEM to maintain equipment for which it is difficult to find other service providers, and (ii) considering a 2-class priority queueing system through which the OEM offers two kinds of EWs: one with priority for class 1 customers, normally large hospitals, and a non-priority EW for class 2 customers, normally small hospitals, where class 1 has priority over class 2. The arrival times were modeled by a conditioned Weibull distribution, which is a generalization of the Exponential distribution often applied in queuing systems. Moreover, we assumed minimal repairs are performed to get the equipment into operation.

Customers of a given class were considered to be homogeneous regarding their attitude to risk, thus they make similar decisions. In this way, OEM and hospitals interacted via a Stackelberg Game. The OEM sets the prices of the EWs and of the maintenance interventions on demand to maximize the expected profit. The hospitals, in turn, intend to maximize their expected utility and decide among the different options offered by the OEM. Due to the game characteristics, the OEM was able to extract all the hospitals' surplus and leaves them with zero utility.

Regarding the application example, the option that maximizes profit is to offer EWs with priority for 14 customers of class 1 and EWs without priority for 34 customers of class 2. Then, 48 customers should be serviced. With such arrangement, the OEM yields an expected profit of $ 11,888,594. When it is considered that the OEM has more than one service channel, it is observed that both the optimal number of customers served and the OEM's expected profit significantly increases. For example, for $m = 2$ they reach 102 and $ 29,540,950, respectively; for $m = 3$ they are equal to 154 and $ 48,302,200, respectively. Generally, it was observed that the priority system clearly brings benefits to the priority class. On the other hand, non-priority hospitals need to wait for a longer time to get their equipment restored.

As an issue of our ongoing research, we may quote that our model can be extended in some ways. For example, (i) using a principal-agent game formulation (Jiang et al. [37], Jin et al. [38]) to consider information asymmetry, and (ii) considering heterogeneous customers as Padmanabhan & Rao [39] to turn the negotiation process more realistic; (iii) considering two-dimensional warranties (Samatli-Paç & Taner [40]) to take two performance parameters such as equipment reliability and availability into account in the negotiation process instead of only one, when an one-dimensional warranty is considered; (iv) considering the possibility of renewing the EW and analyzing a time period longer than one year based on the performance of the OEM and the frequency of failure of the equipment; (v) incorporating and evaluating different maintenance policies and actions to reduce warranty costs, (Yun et al. [41], Zio & Compare [42], Mun et al. [43]); (vi) considering imperfect repair, as brought to the warranties context by Yeo & Yuan [44], for instance by using Generalized Renewal Process proposed by Yanez et al. [45]; Tanwar et al. [46] present a survey and analysis of several approaches to modeling generalized renewal processes; and (vii) considering the effect of subsystems and components on equipment degradation (Lugtigheid et al. [47]).

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