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Retail consumers and risk in centralized energy auctions for indexed long-term contracts in Chile

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ABSTRACT

Centralized energy auctions for long-term contracts are commonly-used mechanisms to ensure supply adequacy, to promote competition, and to protect retail customers from price spikes in Latin America. In Chile, the law mandates that all distribution companies must hold long-term contracts—which are awarded on a competitive centralized auction—to cover 100% of the projected demand from three to fifteen years into the future. These contracts can be indexed to a series of financial parameters, including fossil fuel prices at reference locations. Drawing from portfolio theory, we use a simple example to illustrate the difficulties of selecting, through the current clearing mechanism that focuses on average costs and individual characteristics of the offers, a portfolio of long-term energy contracts that could simultaneously minimize the expected future cost of energy and limit the risk exposure of retail customers. In particular, we show that if the objective of the regulator is to limit the risk to regulated consumers, it could be optimal to include contracts that would not be selected based on individual characteristics of the offers and a least-cost auction objective, but that could significantly reduce the price variance of the overall portfolio due to diversification effects between indexing parameters.

1. Introduction

Chile was the first country to restructure its electricity market in the 1980's (Raineri, 2006). Today, all generation investments are made by private firms, which can engage in long-term contracts with large customers or distribution companies and sell their power in an audited cost-based spot market managed by an independent system operator. Further Latin-American countries followed a similar path such as Peru, Brazil, and Argentina. As in many other restructured markets, both transmission and distribution companies remain operating as regulated monopolies; however, unlike markets in the US, Europe, New Zealand, and Australia that present a competitive retail sector (Defeuilley, 2009), small consumers in Chile (with less than .5 MW of power) have no retail choice. In this context, the regulator mandates auctions where generators can make offers to obtain regulated long-term contracts to supply future demand of small consumers (Moreno et al., 2010). Such contracts are signed between awarded generators and distribution companies, which then pass through the contract price to consumers. In contrast, large consumers, such as mining companies, can negotiate bilateral contracts or Power Purchase Agreements (PPA) directly with generation companies.

In contrast, in electricity markets that allow for retail competition, consumers have the freedom to engage in alternative deals with a number of retailers. All the procured power is offered to customers as a menu of products that can be differentiated by price and risk—and, in theory, even by service quality levels—which are endogenously determined through consumer preferences and competition among all retailers available in a geographical region. Just as with internet cable providers, in competitive retail markets consumers are free to choose the retailer that offers the products and services that best suits their needs. In the UK market, for example, large retailers have historically offered hedged products while smaller retailers have offered more exposed/volatile ones (Vaughan, 2017). In this context, risk-averse consumers for whom electricity costs may represent a larger proportion of their budgets are more likely to choose hedged products at higher (fixed) prices than unhedged one. In contrast, there are consumers who prefer more exposure, seeking a less costly electricity supply in the longer term (note that hedged products include the cost of insurance) (Smith, 1989; Strauss and Oren, 1993).[1]

Clearly, customers in electricity markets where there is no retail choice—such as those in Chile—are captive. Hence, a customer that is unsatisfied with the particular mean-risk ratio offered or any further

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[1] Borenstein et al. (2007), for instance, discusses different hedging strategies for customers that are exposed to real-time electricity prices.

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aspects associated with the service provision by a franchised retailer has very limited options (e.g., producing part of their own power consumption through rooftop solar generation).

Despite the benefits of competitive retail markets, explained in detail in Littlechild (2000), there is no conclusive evidence that retail competition can actually benefit small customers in restructured electricity markets (Joskow, 2008). Furthermore, Green (2004) demonstrates that if prices are sufficiently volatile from year to year, the transition to retail competition in the electricity industry could lead to a significant reduction in long-term contracting, and higher prices overall. There is further empirical evidence that shows mixed levels of success in implementations of competitive retail markets (Defeuilley, 2009). Finally, in small systems such as those of interest in this paper, the exercise of market power and price manipulation are common concerns where mandated auctions of electricity contracts can represent a reasonable alternative.

In the particular case of Chile, auctions for long-term contracts, implemented for the first time in 2006, replaced the centralized retail price calculation based on mathematical models. In fact, prior to 2006, the regulated price was calculated by the authority every 6 months as the average value of future marginal costs in every node obtained through an SDDP-like model (Pereira and Pinto, 1991), plus the regulated costs of distribution companies that provided both network and retail services. In other words, generation and distribution companies signed long-term contracts that were priced by the authority on a 6-month basis; thus contract prices were passed through to the end consumer, on top of the regulated cost of service of distribution companies. Hence, retailed prices reflected the authority's (rather than the market) expectation of system marginal costs in the near future (average over a time horizon of 4 years), which also aimed at mitigating the potential exercise of market power and therefore reduce the overall price to the end consumer. Although this method worked reasonably well during the 80s and 90s, it presented major problems since 2004 due to the reduction of natural gas imports from Argentina that increased the actual (but not necessarily the modeled or foreseen) system marginal cost. This created a significant barrier for new (regulated) long-term contracts and thus generation investments that, during 2005, led to changes in the regulatory framework. It was then that the government decided to implement a new mechanism of auctions for long-term contracts that addressed the aforementioned problems by incorporating real cost expectations from actual market participants in the retail price, fostering the needed generation investments in the Chilean market.

For more than 10 years since their implementation auctions have been used to clear prices for regulated consumers through long-term contracts. These have been iteratively improved by the regulatory authority and, although auctions have presented several problems in the past (Moreno et al., 2010), they have represented, overall, a reasonable alternative to hedge retail prices, to promote generation investment and therefore ensure adequate capacity, and to make the market more contestable with substantial international attention from investors. These improvements, coupled with reductions in the cost of renewable energy technologies, have led to a significant decrease in the cleared prices of electricity in the last few years. In fact, in the last auction carried out in 2016, the average price was equal to 47 \$/MWh with a minimum offer price of 28 \$/MWh, offered by new solar power plants.[2] An additional benefit of the auction mechanism has attracted new generation firms to join the Chilean electricity market, which has reduced its concentration.

Although the success of the auction mechanism has been material, especially in the latest processes, there are various aspects that need to be improved, including the selection of an efficient portfolio of indexed long-term contracts while facing an uncertain future. In this vein, our goal is to demonstrate that a centralized auction mechanism for indexed long-term energy contracts, as implemented now in Chile, is not necessarily efficient when attempting to limit the risk exposure of retail customers. Note that, as explained earlier, limiting risks is critical for energy-intensive consumers, which are not necessarily large consumers, and thereby is an expected output from competitive retail markets. Therefore, we argue that not being able to properly balance expected costs and price risk is a major drawback of the current auction mechanism that requires immediate action. The problem arises given that the auctioneer determines the optimal portfolio based on individual characteristics of offers (e.g., expected present cost of it) that are indexed to a series of financial parameters. We quantify this effect using a simple model that emulates the current selection methodology in the Chilean long-term energy auctions. Our results indicate that ignoring uncertainty and correlations among offers that have synergistic effects in terms of diversification may significantly increase the risk exposure of retail consumers.

To our best knowledge, this is the first study that quantifies the limitations of selection mechanisms that solely focus on minimizing expected costs and compares their performance to more sophisticated approaches that explicitly consider price risk. Apart from Chile, auction mechanisms for long-term contracts applied in Latin America and beyond (e.g., Peru, Brazil, Colombia, Panama, South Australia, Vietnam, Philippines, Thailand, etc.) (Maurer and Barroso, 2011) present similar problems and thus the concepts and models analyzed in this paper can be of particular interest for regulators and policymakers who look for risk-averse portfolio solutions in these markets. Table 1 illustrates that, although the targeted markets present no retail competition, they can differ in many other characteristics. Furthermore, regardless of the market features and implementation details of auction mechanisms across jurisdictions (see Table 1), this paper discusses fundamental aspects to support authorities (as public auctioneers) find the most convenient portfolio solution for a desirable level of risk exposure, rather than selecting the one with the best "average" performance, but with the potential to negatively affect end consumers under the realization of adverse conditions.

We structure the rest of the paper as follows. In Section 2 we summarize some of the existing literature on risk allocation in competitive markets, contracts, and public-private partnerships. In Section 3 we provide a brief overview of the Chilean power system. In Section 4 we present different selection methods to construct a portfolio of indexed contracts. In this section we also describe a simple stochastic model of energy commodity prices that we later on use to compare the performance of different selection methods. In Section 5 we specify the data and framework used to test the methods, which is inspired in the current Chilean auction. In Section 6 we compare the performance of the selection methods both in terms of expected costs and price volatility. Finally in Section 7 we conclude and provide some policy recommendations.

2. Risk allocation in competitive markets, contracts, and public-private partnerships

Economic theory states that if a market is complete and agents are risk averse, then the final allocation of risk is efficient (Arrow, 1964). A market is complete if there exist insurance options (i.e., Arrow-Debreu securities) for every possible state of nature, which means that all market participants can adjust their risk exposure by trading these securities. Consequently, the price of these securities and the final allocation of risk is endogenous (i.e., a result of the interactions among all market participants). However, if some securities do not exist due to information asymmetries, transaction costs, or enforceability issues, then a market is incomplete and the final allocation of risk is inefficient (i.e., some agents might bear too much or too little risk compared to an efficient allocation in a complete market) (Mas-Colell et al., 1995).

Naturally, market completeness is a rather strong assumption in the

[2] In this paper the currency is in US dollars.

Table 1
Comparison among countries where auctions for long-term contracts are applied.

Jurisdiction	Institutional arrangements	Wholesale electricity market	Discrimination and energy policy	Who decides auctioned volumes	Price cleared	Contract duration
Chile	Wholesale competition (no retail competition)	Cost-based dispatch, locational marginal prices, and financial contracts	All technologies and projects compete against each other	Distribution companies	Pay as bid	Up to 15 yrs
Brazil	Wholesale competition (no retail competition)	Cost-based dispatch and financial contracts	Discrimination among existing, new and renewable generation	Distribution companies (and authority for some cases)	2-phase process (final phase is pay as bid)	Up to 15 yrs
Peru	Wholesale competition (no retail competition)	Cost-based dispatch, locational marginal prices, and financial contracts	Discrimination for renewable generation	Distribution companies	Pay as bid	Up to 15 yrs
Colombia	Wholesale competition (no retail competition)	Bid-based dispatch and financial contracts	All technologies and projects compete against each other (although existing generators are price takers)	Authority	Descending clock auction (uniform marginal price)	Up to 20 yrs
Panama	Wholesale competition (no retail competition)	Cost-based dispatch and financial contracts	All technologies and projects compete against each other	Distribution companies	Pay as bid	Up to 15 yrs

electricity industry. Although in many electricity markets there exist a variety of financial securities, including financial transmission rights, options, swaps, and day-ahead markets (Deng and Oren, 2006), electricity markets are inherently incomplete (Wilson, 2002). Willems and Morbee (2010), for instance, use data from the German electricity market to show that welfare increases when options become available in addition to forward contracts (e.g., PPAs). Further analyses on modeling equilibrium problems with risk-averse agents in electricity markets are in Ehrenmann and Smeers (2011), Ralph and Smeers (2015), and in Munoz et al. (2017b).

The discussion of optimal risk allocation for retail customers has received much less attention in the literature. As we mentioned earlier, retailers that offer a menu of contracts differentiated by service quality levels—either in a deregulated or regulated context—allow consumers to select their optimal exposure to price risk (Smith, 1989; Strauss and Oren, 1993). Hypothetically, a diverse enough menu of products could lead to an efficient (and endogenous) allocation of risk, as in a complete market, but in practice there are very few markets that offer these types of products to retail customers.

Risk allocation is also a concern in other infrastructure projects that involve public-private partnerships (PPP), including water and wastewater utilities, roads, airports, and ports (Crampes and Estache, 1998). In these contexts, the principle is that risks should be allocated to those parties that can manage them in the most cost-effective manner. When these risks are not allocated properly, or when risks are not clearly allocated in contracts, infrastructure projects in PPP arrangements can be more costly than expected or even fail (Akintoye et al., 2008). For instance, Guasch et al. (2007) found that nearly 75% water-concession contracts were renegotiated after an average of 1.6 years of the initial agreement, which can be seen as a transfer of risk from the private partner to final customers.

Marques and Berg (2011) point out that there is an optimal allocation of risk in PPP arrangements. Naturally, a contract that allocates all risks to the public sector (i.e., consumers) does not incentivize the private partner to invest or operate infrastructure efficiently. On the other hand, allocating too much risk to the private partner could increase the cost of capital and, ultimately, the cost of infrastructure. In general, if the public partner controls an event that could lead to a negative outcome, such as an unilateral change in legislation, then that risk should be borne by the public partner. However, most of the remaining risks, including the construction, operation, and increase in prices as a result of raw material price rise should be borne by the private partner. Unfortunately, none of these studies make reference to selection methodologies when private parties (i.e., generation companies) make offers differentiated by base prices and indexing parameters, such as the ones used in the Chilean long-term energy auctions.

3. A brief overview of the Chilean power system

The Chilean power grid is composed of two main transmission interconnected systems, the Northern Interconnected System (SING) and the Central Interconnected System (SIC), in addition to further, isolated, medium systems in the southern area (Aysén and Magallanes). To date, the installed generation capacity in the country is approximately 20 GW, with 32% of hydro, 10% of biomass, solar, and wind, and 58% of coal, natural gas, and diesel generation (CNE, 2016). The current installed capacity is nearly twice of what it was available by the end of 2005.

Fig. 1 shows generation per technology in 2005 and 2015 in GWh. The main changes in the generation mix in the last decade are a large increase in the share of coal generation, a reduction in the share of hydropower (as a fraction of total generation), and an increase in the share of generation from biomass, wind, and solar. It is projected that more than 70% of the generation in 2050 will come from renewable energy resources (including large hydro), in line with the country's energy roadmap (Munoz et al., 2017a).

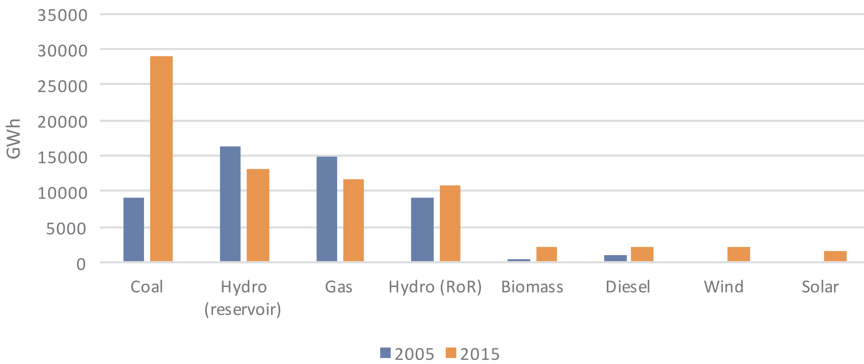

Fig. 1. Total energy generated per technology in 2005 and 2015 (CNE, 2016).

As we mentioned in the introduction, in terms of market organization, generation, transmission, and distribution services are unbundled. The generation segment is deregulated, but both transmission and distribution companies (including retail services) operate as regulated monopolies. Although generation investments are deregulated, operations are based on audited costs and on a centrally-planned allocation of water resources for large hydroelectric power plants (through an SDDP-like model) in contrast to bid-based designs in most deregulated markets elsewhere. This design has been justified due to the high concentration of the generation market, which is dominated by 3 main firms, and the potential incentives of large hydro units to exercise market power through an strategic water allocation (Arellano, 2004; Munoz et al., 2018).

4. Methodology

In this section we describe our methodology to compare the economic performance of long-term contracts selected based on three different approaches: a) the current criterion used by the Chilean regulator in centralized auctions, b) a more sophisticated approach that explicitly accounts for price risk and correlations between indexing parameters (mean-variance optimization), and c) a heuristic method that accounts for risk of individual offers, but that disregards correlations between indexing parameters (a modified Sharpe ratio). We divide this section into three parts. In Section 4.1 we present an abstract model to describe the structure of generation offers in the auction and explain the criteria used by the Chilean regulator to select the subset of winning ones used to supply power to distribution companies. In Section 4.2 we propose a mean-variance portfolio optimization model to select offers explicitly considering price risk. In Section 4.3 we define a third possible selection criterion based on the Sharpe ratio from finance. Finally, in Section 4.4 we present a simple stochastic model of energy commodity prices that we later use to generate scenarios and compare the performance of the selection methods described in Sections 4.1, 4.2 and 4.3.

4.1. Design and selection criteria in energy auctions in Chile

We assume that there are M generation firms, indexed m, participating in the auction and that each firm m makes a single offer for a defined amount of energy Q_{mt} for each of the following T years, the latter indexed t.[3] The regulator in Chile allows generation firms to make price offers based on a base price P_0^m per unit of energy and indexation parameters, the latter of which allow generation firms to adjust the base price that will be charged to distribution companies—and ultimately to retail consumers—based on changes in factors such as fossil fuel prices and the Consumer Price Index (CPI). Assuming a set of J different factors, indexed j, the price P_t^m (per unit of supplied energy) of contract m at time t can be expressed as follows:

$$P_t^m \;:=\; P_0^m \sum_{j=0}^{J} \alpha_{mt} \frac{I_{jt}}{I_{j0}} \tag{1}$$

The parameter I_{jt} denotes the price of index j at time t (e.g., Henry Hub natural gas price in 2020) and the parameter α_{mj} is the weight that generation firm m gives to index j. Firms are free to chose how much weight to give to each index j in their offers, however, valid weights must satisfy $\sum_{j=1}^{J} \alpha_{mj} = 1 \;\forall\; m$.

For a better understanding of this formula, consider the following example. A generator makes an offer with a base price of 50 \$/MWh and chooses to index 100% of the contract to the Henry Hub natural gas price, which is currently $I_{j0} = 5\$/\text{MMBtu}$. Let's assume that this contract was selected in the auction and that, for simplicity, the first year of delivery is the current one. If next year the Henry Hub natural gas price went up to 10 \$/MMBtu, the contract price of electricity for the distribution company would double to 100 \$/MWh. In contrast, if next year the Henry Hub natural gas price went down to 3 \$/MMBtu, the contract price would decrease from 50 \$/MWh to 30 \$/MWh. Since distribution companies also operate as retailers under a regulated rate of return, these potential increments or decrements in costs would be directly passed on to retail customers.

One of the issues of selecting contracts from a pool of offers is that different indexation strategies make it difficult to compare them on a cost basis. The current approach used by the Chilean regulator is to rank them based on their so called expected *levelized costs* per unit of energy, denoted $\overline{C_m}$. To compute $\overline{C_m}$ we first define the levelized cost of an offer, denoted C_m, as follows:

$$C_m := \frac{\sum_{t=0}^{T-1} \frac{Q_{mt} P_{mt}}{(1+\delta)^t}}{\sum_{t=0}^{T-1} \frac{Q_{mt}}{(1+\delta)^t}} = P_{m0} \frac{\sum_{t=0}^{T-1} \frac{Q_{mt}}{(1+\delta)^t} \sum_{j=1}^{J} \alpha_{mj} \frac{I_{jt}}{I_{j0}}}{\sum_{t=0}^{T-1} \frac{Q_{mt}}{(1+\delta)^t}} \tag{2}$$

The parameter δ is the annual discount rate. The Chilean regulator defines the expected levelized cost of an offer as $\overline{C_m} = \mathbb{E}[C_m]$, where $\mathbb{E}[C_m]$ is the expected value of the levelized cost C_m considering different scenarios for the price indexes I_{jt}.[4] Note that the timing of the amount offered matters in $\overline{C_m}$, rewarding bigger amounts in the short term.

Let Q be the total demand for energy per year that must be satisfied using long-term contracts. Currently, the regulator selects the offers by sorting all offers from lowest to highest expected levelized cost $\overline{C_m}$ and later selecting the cheapest ones that satisfy $\sum_{m=1}^{M} x_m Q_{mt} = Q$ $\forall\; t = 0\,..\,T-1$.

[3] Note that in practice retail prices are indexed (according to contracts) on a monthly basis, but end consumers only observe changes every 6 months, when the authority publishes updated retail prices. Despite this, retail tariffs are set such that generators receive, in average, the same revenues as if retail prices were allowed to change on a monthly basis. For illustration purposes here we only consider annual changes in index prices, but extending the stochastic model for energy commodity prices and selection methodologies to incorporate monthly changes is straightforward.

[4] In practice, the regulator does not compute the expected value of the present cost of each candidate offer. Instead, it uses a Baseline or Reference scenario from projections of indexes from international sources, such as the Annual Energy Outlook (AEO, 2017). Here we make the implicit assumption that using a Baseline scenario is the equivalent to computing the expected value of the levelized cost of an offer.

4.2. A mean-variance optimization model

As we discussed it in the introduction, the current methodology does not consider price risk when selecting the subset of winning offers. There are two factors that can affect the level of risk in a portfolio of indexed contracts. The first one is the volatility of the indexes referenced in the winning contracts (e.g., interannual volatility of natural gas prices). The second one is the correlations that might exist between indexing parameters J. The methodology described in the previous section does not consider these factors and might, for instance, recommend selecting offers with low levelized costs, but high volatility, thus increasing the price risk for retail consumers of electricity. Here we develop a mean-variance optimization model to select indexed contracts explicitly considering the tradeoff between cost and price risk for end consumers.

The problem of selecting a group of contracts at minimum cost and with a limited exposure to price risk recalls the famous portfolio selection problem in finance, first introduced in Markowitz (1952). The goal of the problem introduced by Markowitz was to find a portfolio of assets that could reach a particular performance level or expected return with minimum volatility. The selection of indexed contracts in a long-term energy auction is similar to the portfolio selection problem, except that in the energy auction our goal is to simultaneously minimize (i) the present worth of supplying Q units of power for the next T years, i.e. the sum of the expected costs of the chosen contracts, and (ii) the risk exposure of retail consumers, which here we assume is proportional to the volatility or variance of the aggregate costs of the chosen contracts.

Inspired on Markowitz (1952) we propose the following optimization problem to select offers:

$$\min \sum_{m=1}^{M}\sum_{n=1}^{M} x_m Cov_{mn} x_n \tag{3}$$

$$s.\ t.: \sum_{m=1}^{M} x_m \mu_m \le C^{max} \tag{4}$$

$$\sum_{m=1}^{M} x_m Q_{mt} = Q \quad \forall\ t = 0..T-1 \tag{5}$$

$$x_m \in \{0, 1\} \quad \forall\ m = 1..M \tag{6}$$

Here x_m is the variable that equals 1 when the contract m is chosen and 0 otherwise. The expression $Cov_{mn} \coloneqq Cov(C_m, C_n)$ denotes the covariance matrix between the costs of contracts m and n and is equivalent to the variance of the selected portfolio, since $Var[\sum_{m=1}^{M} x_m C_m] = \sum_{m=1}^{M}\sum_{n=1}^{M} x_m Cov_{mn} x_n$. The parameter $\mu_m = \mathbb{E}[C_m]$ is the mean value or expected cost of contract m, by definition equal to the expected levelized cost of the contract. The parameter C^{max} is the maximum mean cost of the selected portfolio of contracts and can be estimated based on the price ceiling established by the regulator. However, in our experiments we solve the optimization problem for different values of C^{max}, which allow us to identify portfolios in the efficient or Pareto frontier.[5] This model, denoted MV heretofore, follows the same idea as the mean-variance portfolio problem for financial assets. Thus we are implicitly assuming that portfolios with more expensive contracts have lower volatility on average. MV results in an integer nonlinear optimization program that can be solved using commercial packages.[6]

[5] A portfolio of indexed contracts is said to be in the efficient or Pareto frontier if it is not possible to reduce its variance (or expected cost) without increasing its expected cost (or variance).

[6] Note that MV is also valid if the regulator or auctioneer could select only fractions of contracts. For that case we could relax (6) to $0 \le x_m \le 1$. In that case MV becomes a quadratic program, just like the standard mean-variance problem in finance.

4.3. An alternative selection methodology: the modified sharpe ratio

A common approach in finance to compare the performance of individual assets is to compute the ratio between the expected value of the excess of return with respect to a risk-free alternative to the standard deviation of it, known as the Sharpe ratio (Sharpe, 1994). The advantage of this metric is that it normalizes the asset returns for their underlying amount of risk (i.e., standard deviation). Here we consider the Sharpe ratio as a third possible selection criterion that does not require the use of a formal optimization program as the MV approach; however, we adapt it to our problem since in energy auctions the objective is to minimize cost instead of maximizing returns. We define the modified Sharpe ratio for a given contract m as the product between the mean of an offer (μ_m) and the standard deviation of costs (σ_m), $\mu_m \sigma_m$.

4.4. A simple stochastic model of energy commodity prices

We now describe the stochastic model of energy commodity prices that we utilize to compare the performance of the portfolios selected using the methodologies described in the previous sections. For simplicity, we assume that the prices of indexes I_{jt} evolve over time following a multivariate geometric Brownian motion (MGBM), which is a continuous-time stochastic process (see (Karatzas and Shreve, 2012) for more details). Well known assets and derivatives pricing models, such as Black and Scholes (1973), are derived from a MGBM; however, other assumptions can be made (Deng, 2000; Geman, 2009). Under the MGBM, each index is driven by the following stochastic differential equation:

$$dI_{jt} = g_{jt} I_{jt} dt + \sigma_j I_{jt} dW_{jt} \tag{7}$$

Here the parameter g_{jt} denotes the drift of I_{jt}, σ_j the local volatility of the same, and W_{jt} is the Brownian motion associated with index j. Basic MGBM usually have drift parameters that are constant over time (i.e., g_j instead of g_{jt}). However, as we will see, empirical projections used for the indexes have a different structure in the short and long term. Hence, we assume that the drift can change over time.

One of the advantages of using this stochastic model is that we can compute closed-form solutions for μ_m and Cov_{mn}, both of which are used as parameters (i.e., inputs) in the optimization model described previously. For details on the close formulas of μ_m and Cov_{mn} please refer to section A in the Appendix.

Finally, note that since we are using a continuous-time stochastic process we adapt the original definition of C_m in Eq. (2) to the following one:

$$C_m = P_{m0}\left[\sum_{t=0}^{T-1} e^{-rt} Q_{mt} \sum_{j=1}^{J} \alpha_{mj}\frac{I_{jt}}{I_{j0}}\right]\cdot\left[\sum_{t=0}^{T-1} e^{-rt} Q_{mt}\right]^{-1} \tag{8}$$

where r is the continuous discount rate, i.e. r is such that: $e^{-rt} = \frac{1}{(1+\delta)^t}$

5. Data

In this section we describe our data assumptions to recreate an energy auction similar to the ones done in Chile. Inspired in the last energy auction in 2016, we assume that the regulator allows generation firms to index their contracts to the following five parameters taken from the U.S. Energy Information Administration (EIA):

1. Residual Fuel Oil (Fuel)
2. Brent Spot (Oil)
3. Henry Hub Natural Gas Spot Price (Gas)
4. Distillate Fuel Oil (Diesel)
5. All Region Average Bituminous Coal Prices (Coal)

All indexes are in real terms (i.e., inflation adjusted). Contracts can also be indexed to the U.S. Consumer Price Index (CPI), which results in

Table 2
Estimations for the drift g, local volatility (i.e., standard deviation) σ and correlations for the 5 indexing parameters allowed in the Chilean auction other than the CPI. Short term drift considers the first half of the projections shown in Table B.1 in the Appendix, while long term drift considers the second half of the same data. Projections shown in Table B.1 are taken from the 2016 Energy Outlook (AEO, 2017). Local volatility is taken from EIA database (EIA, 2017).

	Fuel	Oil	Diesel	Gas	Coal
Short term g [%]	17	14	8	9	0
Long term g [%]	2	3	2	2	0
σ [%]	23	25	21	29	26
P	Fuel	Oil	Diesel	Gas	Coal
Fuel		.93	.93	.69	.42
Oil			.95	.60	.52
Diesel				.73	.58
Gas					.44

Table 3
Base prices in \$/MWh and index weights for each of the 15 contracts.

Contract	P_{m0}	α_{Fuel}	α_{Oil}	α_{Diesel}	α_{Gas}	α_{Coal}	α_{CPI}
1	28	1	–	–	–	–	–
2	43	–	–	–	1	–	–
3	38	.30	.30	.20	–	–	.20
4	65	–	–	–	–	1	–
5	47	.17	–	.50	–	–	.33
6	48	–	.30	–	.30	–	.40
7	50	–	.40	–	–	–	.60
8	68	–	–	–	–	.60	.40
9	61	–	–	.20	.10	.20	.50
10	70	–	–	–	–	.40	.60
11	65	–	–	–	.20	–	.80
12	72	–	–	–		.20	.80
13	70	–	–	.10	–	–	.90
14	72	–	–	–	.05	.05	.90
15	75	–	–	–	–	–	1

a total of six indexing parameters ($J = 6$). We assume that the auction is performed to supply a determined amount of power Q per year, for the next 15 years ($T = 15$).

Table 2 shows a summary of relevant parameters used in the MGBM.[7] Note that the prices of all the commodities considered are highly correlated, particularly for oil, fuel, and diesel. This correlation structure suggests that there are low diversification opportunities in the Chilean auction, since the costs of all indexed contracts will tend to move to the same direction, unless a contract is indexed 100% to the CPI. However, we later introduce an additional index that is negatively correlated with the indexes shown in Table 2. We use this additional index to assess how robust is the performance of the current and other heuristic selection criteria depending on the types of correlations between the indexing parameters.

Note that the volatility of all indexes is similar and higher in magnitude to the drifts, which highlights the importance of considering risk in the selection of contracts. Since all volatilities are similar, in our simulations we consider a local volatility of 25% for all indexes other than the CPI. In terms of the drifts, we consider two different values per index, one for the short term and another one for the long term. We estimate the drifts based on the projections of the indexes prices shown in Table B.1 in the appendix. Fuel and oil are expected to have the highest increase in the short term, but all indexes have a small drift in the long term.

For illustration purposes, we create a pool of 15 different contracts and assume that all offers are for the same amount of energy per year, i.e. $Q_{mt} = Q/3$. This means that the regulator must select 3 contracts from the the centralized auction to supply all projected demand for retail customers for the next 15 years. The MV model does not depend on Q, since constraint (5) changes to $\sum_{m=1}^{M} x_m = 3$. Thus, we exclude the complexity in the selection process attributed to the energy quantities offered by each contract and measure only the impact of considering price risk.

Table 3 shows base prices and index weights for each of the 15 contracts. Fig. 2 shows that the first contracts have a lower base price and thus are cheaper on average. However they have a high volatility (i.e., standard deviation) of prices because they are referenced to more volatile indexes. Note that these contracts were purposely defined to cover different indexation structures and base prices P_{m0}. In other words, none of the proposed contracts is dominated in terms of mean and volatility, simultaneously, by another offer or otherwise they would never be selected as part of a portfolio. For example, contract 15 has the highest base price, but the lowest volatility since it is 100% indexed to the CPI. Contract 14 has a lower base price, but a higher volatility since only 90% of it is indexed to the CPI, the remaining 10% makes

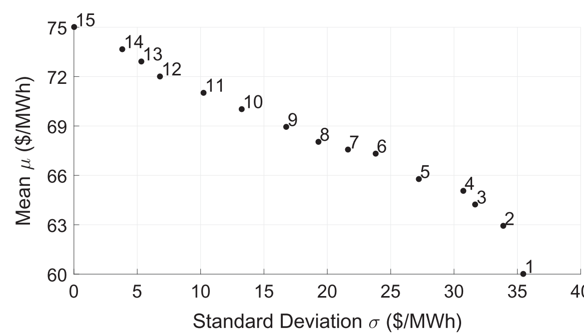

Fig. 2. Mean and standard deviation values for C_m of the 15 contracts.

reference to natural gas and coal prices. However, as we will see later, under other correlation structures (i.e., if there are negative correlations) it might be optimal to select contracts that are individually dominated by other offers due to diversification effects.

6. Results and discussion

We now compare the performance of different selection criteria. The most simple selection algorithm builds a portfolio using the contracts with the lowest expected cost,[8] as currently done in the Chilean energy auction. The most sophisticated model, the mean-variance portfolio approach (referred to as *MV* here), explicitly considers the tradeoff between expected cost and volatility. In addition, we also consider a third selection method based on individual characteristics of the offers, the modified Sharpe ratio. We show that the current selection criteria can leave retail customers locked up into a portfolio of generation contracts with high price volatility (i.e., risk) compared to what it would have been selected using a criteria that explicitly takes risk into account, such as the *MV*. As we will see, there can be contracts with high base prices that might not seem attractive when analyzed individually, but that provide a high value to a portfolio when combined with other offers.

6.1. Example 1

We solve *MV* for different values of C^{max} in order to identify portfolios that are in the Pareto frontier, each one with a different expected cost and volatility. We denote $MV_{C^{max}/S}$ the solution to each run of *MV* with a maximum expected cost C^{max}. Thus, C^{max}/S is the maximum expected cost of a portfolio normalized by the number contracts

[7] By definition, the index CPI has neither drift nor volatility, because it is in real terms. Thus, the CPI has no correlation with the rest of the indexes.

[8] Since the amount of energy offered per year is the same for all contracts, this is equivalent to select the contracts with lowest levelized costs.

Table 4
Optimal portfolios of contracts when solving MV for different values of C^{max}/S, with $S = 3$. Columns C1, C2, and C3 indicate the contract numbers that are part of the portfolio for each solution of MV. The parameters μ_p and σ_p denote the mean and volatility (standard deviation) of the selected portfolio, respectively. These measures are standardized (divided) by S to express the performance in terms of average costs per contract in $/MWh. Note that since we use binary variables to select offers in the MV approach, it is possible that in the optimum $\frac{\mu_p}{S} < C^{max}/S$.

Solution	C1	C2	C3	$\frac{\sigma_p}{S}$	$\frac{\mu_p}{S}$
MV_{62} (min. levelized cost)	1	2	3	31	62
MV_{64}	1	2	4	27	63
MV_{65}	1	2	10	23	64
MV_{66}	1	4	12	20	66
MV_{67}	1	8	12	17	67
MV_{68}	1	10	14	15	68
MV_{69}	1	12	15	13	69
MV_{70}	8	11	12	11	70
MV_{72}	10	11	13	8	71
MV_{73}	11	12	15	5	73
MV_{74} (min. modified Sharpe ratio)	13	14	15	3	74

selected S. Table 4 shows the results for an auction where the total number of contracts needed to supply projected demand is $S = 3$. As expected, increasing C^{max} results in portfolios with lower price volatility for retail customers.

We also identify the portfolio of contracts that would be selected in the Chilean energy auction. As we mentioned earlier, the current criterion is to select the S contracts with the lowest expected costs μ_m. Given our 15 available contracts, this selection rule would result in the same portfolio selected in MV_{62}, which includes contracts 1, 2, and 3. We also consider a third selection rule, a modified Sharpe ratio, that explicitly considers the compromise between mean and volatility of a selected contract by ranking them based on the product of these two individual features, $\mu_m \sigma_m$. The idea is to rank contracts by this product and choose the contracts with the S lowest values. From the example, the contracts with the three lowest values are 13, 14 and 15, which equals to MV_{74}, i.e. the portfolio in the Pareto frontier with the lowest volatility. Thus, the solutions given by these two criteria that only consider the individual characteristics of each contract, the current Chilean method and the modified Sharpe ratio, are in this case part of the Pareto frontier. In general, the portfolio of contracts selected based on the current method used in the Chilean auction is guaranteed to be the in the Pareto frontier for $MV_{C^*/S}$, where C^* the expected total cost of the portfolio formed by the S contracts available. However, the portfolio selected using the modified Sharpe ratio will not always be in the Pareto frontier as we will show in an example in Section 6.2.

To test each solution, we simulate 1000 outcomes (scenarios) for the prices of the N indexes, following the MGBM model described previously. The results from the simulation are displayed in Table 5. As expected, the current methodology used in the Chilean energy auction results in the portfolio of contracts with the lowest expected or levelized cost of all portfolios in the efficient frontier (MV_{62}). However, this portfolio is nearly 10 times more volatile (in terms of standard deviation) than the one that would be selected using the MV approach (MV_{74}) with an expected cost of 74 $/MWh instead of 64 $/MWh. In other words, selecting MV_{74} instead of MV_{62} reduces volatility by nearly 90% and increases expected costs by only 13.4%.

However, measuring volatility only using the standard deviation of prices of a portfolio is only one alternative. In Table 5 we also provide other standard metrics for risk, including the Value at Risk (VaR) and the Conditional Value at Risk ($CVaR$). The disadvantage of using the standard deviation or variance as a risk metric instead of the VaR and $CVaR$ is that the former penalizes both upside and downside risks equally, while the latter can focus only on downside risks (i.e., high-price scenarios), which might be more relevant for the auctioneer. For

Table 5
Simulation results of portfolios in the efficient frontier. All values are standardized (divided) by S. The $VaR_{90\%}$ is the 90^{th} percentile of the portfolio total cost (i.e, the 100^{th} worst value over the 1000 generated). The $CVaR_{90\%}$ is the average of the 100 worst case values generated.

Solution	Std Dev.	Mean	$VaR_{90\%}$	$CVaR_{90\%}$
MV_{62}	29	62	102	127
MV_{64}	26	62	98	119
MV_{65}	22	64	94	112
MV_{66}	19	66	90	108
MV_{67}	16	67	87	103
MV_{68}	14	68	87	100
MV_{69}	12	69	84	96
MV_{70}	10	70	84	94
MV_{72}	8	71	82	88
MV_{73}	5	73	79	83
MV_{74}	3	74	78	80

example, the $CVaR_{90\%}$ shows that the average cost per contract of the portfolio MV_{62} is 127 $/MWh in the 100 worst cases out of the 1000 simulated scenarios, which is nearly 4.4 times the expect cost of this portfolio. In contrast, for portfolio MV_{74} the average cost per contract is 80 $/MWh in the 100 worst cases, which is only 8.1% higher than its expected cost of 74 $/MWh per contract. Thus, selecting a portfolio using an approach such as the MV can result in a much more stable evolution of prices for retail consumers, regardless of how index prices evolve.[9]

Finally, note that the efficient frontier offers several alternatives of portfolios for different risk exposures, while the least-cost approach and the methodology based on the modified Sharpe ratio only return the portfolios that are in the two extremes of volatility (i.e., max and min variance, respectively). We highlight this result to show that, in contrast to the MV approach, the modified Sharpe ratio gives no control to the auctioneer to select a specific amount of risk exposure when constructing a portfolio of contracts.

6.2. Example 2: sensitivity in correlation structure

We now want to illustrate what occurs when we introduce new contracts indexed to a commodity price that is negatively correlated to the indexes defined previously. Up to this point we have assumed that the structure of correlations and volatilities for indexes is unique for the entire duration of the contracts (i.e., 15 years). However, empirical research in finance suggests that volatility and correlation structures for commodity prices can change over time. In fact, regime-switching models aim at identifying how these structures evolve over time. These models have been applied to commodities such as crude and natural gas prices (Choi and Hammoudeh, 2010; Fong and See, 2002; Chen and Forsyth, 2010), both of which are considered as indexes in energy auctions in Chile. Hence, it is important to see how different correlation structures can affect the performance of different selection methodologies. In our next example, we want to show that portfolios in the efficient frontier might include contracts that are not considered individually competitive in terms of expected costs or volatility, but that reduce the volatility of a portfolio when paired with other contracts.

Let's assume now that there is a new index, called “new” heretofore, that is negatively correlated with the rest of the indexes. For simplicity, we assume this new index has the same volatility as the rest of the indexes, i.e. 25%, and a drift equal to the oil index. Table 6 shows 4 new contracts that are indexed to the new index. Fig. 3 shows the individual expected and standard deviations of costs for of all 19 contracts. Note

[9] This statement is based on the assumption that the auctioneer has knowledge of the historical and future distribution of index prices.

Table 6
Base prices in \$/MWh and index weights for each of the new 4 contracts.

Contract	P_{m0}	α_{Fuel}	α_{Oil}	α_{Diesel}	α_{Gas}	α_{Coal}	α_{CPI}	α_{new}
16	30	–	–	–	–	–	–	1
17	33.5	.5	.25	–	–	–	–	.25
18	56	–	–	–	–	–	.75	.25
19	67	–	–	–	–	–	.9	.1

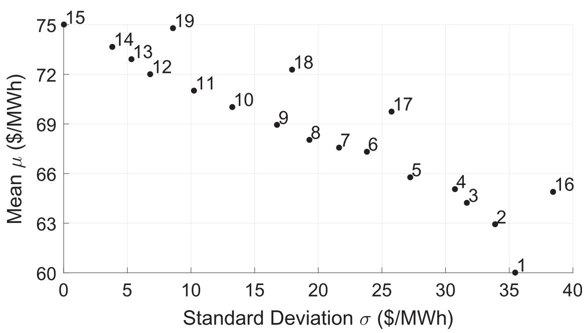

Fig. 3. Mean and standard deviation values for C_m of the 19 contracts.

that the 4 new contracts, 16–19, are all simultaneously worse in terms of volatility and expected costs when compared to several of the 15 contracts defined initially. For example, contract 19 is individually dominated by contracts 12, 13, and 14. Thus, it is unlikely that 16–19 will be selected as part of a portfolio when using methodologies that rank contracts based on individual characteristics.

In Table 7 we show the optimal portfolios of contracts identified using the MV approach (i.e., the efficient frontier). Note that the portfolio MV_{62} is still equivalent to the one that would be selected using the current methodology in the Chilean auction (i.e., min. levelized cost). However, all the other portfolios in the efficient frontier include at least one of the new contracts. Furthermore, ranking individual offers based on the modified Sharpe ratio $\mu_m\sigma_m$ in this case yields the same portfolio as in the previous example: contracts 13, 14, and 15. However, this portfolio is no longer part of the efficient frontier since it yields an expected cost of approximately 74 \$/MWh and a standard deviation of 3 \$/MWh. Therefore, this portfolio is now dominated by MV_{74}, which highlights the heuristic nature of the modified Sharpe ratio as a metric to rank contracts.

Table 8 shows results for a Monte Carlo simulation for 1000 outcomes. As expected, we find that introducing new contracts to the original set of 15 offers results in diversification gains. For instance, for a maximum expected cost of 74 \$/MWh, the portfolio MV_{74} now results

Table 7
Optimal portfolios of contracts when solving MV for different values of C^{max}/S, with $S = 3$, considering the new contracts 16-19.

Solution	C1	C2	C3	$\frac{\sigma_p}{S}$	$\frac{\mu_p}{S}$
MV_{62} (min. levelized cost)	1	2	3	31	62
MV_{64}	1	4	16	14	63
MV_{65}	1	8	16	13	64
MV_{66}	2	10	16	12	66
MV_{67}	6	8	16	11	67
MV_{68}	6	12	16	10	68
MV_{70}	7	8	18	9	69
MV_{71}	8	11	18	7	70
MV_{72}	11	12	18	5	72
MV_{73}	12	13	19	3	73
MV_{74}	14	15	19	2	74

Table 8
Simulation results of portfolios in the efficient frontier considering the 4 new contracts. All values are standardized (divided) by S.

Solution	Std Dev.	Mean	$VaR_{90\%}$	$CVaR_{90\%}$
MV_{62}	29	63	102	127
MV_{64}	14	64	80	95
MV_{65}	12	65	80	92
MV_{66}	11	66	79	90
MV_{67}	10	67	78	90
MV_{68}	9	68	79	90
MV_{70}	9	70	80	90
MV_{71}	7	71	78	86
MV_{72}	5	72	77	82
MV_{73}	3	73	77	80
MV_{74}	2	74	77	80

in a standard deviation of 2 \$/MWh, instead of 3 \$/MWh as in the previous example. The main difference is that now MV_{74} is composed of contracts 14, 15, and 19, instead of 13, 14, and 15. Consequently, in this example the modified Sharpe ratio fails at identifying a portfolio in the efficient frontier. Although our results are not general, they illustrate the challenge of selecting a portfolio of indexed contracts with limited risk exposure for retail consumers using the current least-cost criterion or heuristics such as the modified Sharpe ratio.

6.3. Example 3: an application to the 2015/01 Chilean long-term energy auction

In this final example we want to show how our methodology can give new insights in an actual selection process using real data from the 2015/01 energy auction in Chile. All information, including the full list of offers and awarded contracts, is publicly available in EE (2016). Table 9 shows a sample of 10 of these offers. We want to highlight that we do not intend to replicate the real auction process since there are further auction rules that, for the sake of simplicity and clarification, we ignore. Instead, our goal is to illustrate through a simple example (but with realistic data) that the current methodology may overlook portfolios that could feature more attractive results for a planner who aims at striking a balance between minimizing expected costs and price risk.

We assume that in order to meet forecasted demand we must select 5 of these 10 offers. If we follow the current selection methodology, which ranks offers based on their levelized costs, the optimal portfolio includes 1, 2, 3, 4 and 6. As in the previous two examples, by solving

Table 9
Sample of 10 offers from the 2015/01 Chilean long-term energy auction. Base prices and expected levelized costs are in \$/MWh.

Offer number and firm name	P_{m0}	α_{Fuel}	α_{Oil}	α_{Diesel}	α_{Gas}	α_{Coal}	α_{CPI}	$\overline{C_m}$
1. Global Power Generation	46.1	–	–	–	–	.31	.69	46.1
2. Acciona Energía Chile S.A.	54.9	–	–	–	–	–	1	54.9
3. AES Gener S.A.	59.0	–	–	.48	–	–	.52	72.2
4. Colbún	60.0	–	–	–	–	–	1	60.0
5. SPV P4	64.0	.31	–	–	–	.16	.53	89.2
6. Engie Energía Chile S.A.	69.3	–	–	–	–	.13	.87	69.3
7. Empresa Eléctrica Puntilla S.A.	81.2	–	–	–	–	–	1	81.2
8. Parque Eólico los Cuturros	85.0	–	–	–	–	–	1	85.0
9. Empresa Eléctrica Rucutayo S.A.	91.8	–	–	–	–	–	1	91.8
10. Andes S.A.	96.8	.86	–	–	−.73	.16	.7	159.2

Table 10
Optimal portfolios when solving MV for different values of C^{max}/S, with $S = 5$, considering the 10 offers taken from Table 9.

Solution	C1	C2	C3	C4	C5	$\frac{\sigma_P}{S}$	$\frac{\mu_P}{S}$
MV_{60}	1	2	3	4	6	12	60
MV_{64}	1	2	4	6	8	9	63
MV_{67}	1	2	4	7	8	5	65
MV_{71}	2	4	6	7	8	3	70
MV_{75}	2	4	7	8	9	0	75

Table 11
Simulation results for portfolios in the efficient frontier considering the 10 offers taken from Table 9. All values are standardized (divided) by S.

Solution	Std Dev.	Mean	$VaR_{90\%}$	$CVaR_{90\%}$
MV_{60}	12	60	80	85
MV_{64}	9	63	77	82
MV_{67}	5	65	76	79
MV_{71}	3	70	76	77
MV_{75}	0	75	75	75

the MV model with different values of C^{max}/S we find different portfolios in the Pareto frontier that we show in Table 10. Note that the solution MV_{60}, which minimizes the expected cost of meeting forecasted demand using 5 offers, is equivalent to the portfolio selected using the current methodology that ranks contracts based on expected levelized costs.

Just as in the previous examples, we use the same 1000 simulated outcomes for the prices of the N indexes following the MGBM model. The results from the simulation are displayed in Table 11. These results illustrate the same effects observed previously. For this set of candidate energy contracts, the current methodology used in the Chilean energy auction selects the portfolio with the lowest expected cost (MV_{60}), but with the highest volatility of prices for retail consumers. Using the MV approach it is possible to identify other portfolios that could limit this risk in exchange for an increase in expected levelized costs. All portfolios in the Pareto frontier (Table 10) are indeed optimal for different risk profiles, including the one that minimizes expected cost (MV_{60}). Consequently, applying the MV approach would require the regulator to set a maximum tolerable amount of price risk for retail consumers that would ultimately determine the optimal portfolio of long-term energy contracts that minimizes expected costs subject to this constraint. Identifying the optimal amount if risk transfer in long-term contracts is qualitatively discussed by Marques and Berg (2011) for infrastructure projects that involve public-private partnerships. Yet, more research is needed to quantitatively determine what is the optimal level of risk transfer in long-term energy auctions with indexed contracts for regulated retail customers.

7. Conclusions and policy implications

Centralized auctions for long-term energy contracts have been in place for more than one decade in Latin America. In Chile and Brazil they were originally implemented as mechanisms to incentivize sufficient amounts of generation investments in the long term (i.e., resource adequacy). An additional benefit of auctioned forward contracts is that they can reduce market power in concentrated bid- (Allaz and Vila, 1993) and cost-based (Arellano and Serra, 2010) electricity markets. Market power is a rather large concern in Latin America, where systems are relatively small, concentrated, and with large hydroelectric power plants.

In Chile, centralized energy auctions are used to procure power to consumers that are served by distribution companies. These companies operate as regulated monopolies that can pass through all costs of energy contracts to retail consumers. Auctions have been modified several times since their initial implementation. For instance, the latest energy auction considered separate auctions for daily and nightly hours in order to reduce the risk exposure of renewable energy generation, such as wind and solar (Marambio and Rudnick, 2017). It is worth highlighting that in this auction contracts were awarded at record low prices, with an average of 47 $/MWh and a minimum offer price of 28 $/MWh for new solar power plants.

Although centralized energy auctions have been relatively successful at promoting investments in new generation capacity in the country, there is one concerning feature of their design that can have a negative impact on retail consumers. As it is common in long-term contracts (Kahn, 1991), offers can be indexed to fuel prices, the consumer price index (CPI), or other features such as the system's marginal operating cost[10] in order to limit risks for generation firms that might engage in up to 20-year commitments with distribution companies. Unfortunately, the current criterion used to select the winning subset of contracts in centralized energy auctions only considers a single projection of indexes for the future and ranks offers based on their individual (expected) present cost, also referred to as the expected *levelized* cost of a contract.

In this paper we use three simple examples to illustrate how this selection approach disregards price risk for retail consumers, which have little or no alternatives to manage it. Inspired on the current design of the Chilean energy auction, we consider 15 competitive contracts for equal amounts of energy, but that differ in their base prices and indexation coefficients. We simulate the evolution of indexes using a simple model for energy commodity prices calibrated using historical and projected data and consider three selection methodologies to supply a projected demand for energy that requires a portfolio of 3 contracts. The simplest one emulates the current selection criterion in the Chilean auction and ranks contracts based on their individual expected levelized cost that depend solely on expected projections of fuel-price indexes. The most sophisticated one is a mean-variance (MV) portfolio optimization program that minimizes price risk, measured as the price variance, subject to a maximum expected or levelized cost of a selected portfolio. Thus, the MV gives a range of optimal results considering the tradeoff between price risk and expected costs (i.e., the efficient frontier). The third selection methodology is a variation of the Sharpe ratio, often used in finance to rank individual stocks based on risk-adjusted returns (i.e., the ratio of expected returns to their standard deviation).

As expected, we found that the current selection algorithm used in the Chilean auction results in the portfolio with the highest price risk in the efficient frontier under all risk metrics (i.e., standard deviation, Value at Risk, and Conditional Value at Risk). Portfolios with less risk exposure in the efficient frontier include contracts that would not be selected under the current auction design. In particular, the portfolio with minimum risk exposure is composed of the three available contracts with highest leveraged costs. In our first example we show that the portfolio that minimizes expected levelized costs is nearly 10 times more volatile (in terms of standard deviation) than the one that minimizes price risk; this reduction in price volatility comes at an increase of only 19.4% in expected levelized costs. Additionally, in this same example ranking offers based on the modified Sharpe ratio yields the portfolio of minimum price risk in the efficient frontier selected using the MV approach. However, we show that this latter result is not a general property of the Sharpe ratio itself; instead, it is a consequence of having positively correlated indexes.

As a sensitivity analysis, in our second example we introduce 4 new contracts that are indexed to a new seventh index that is negatively

[10] The marginal system cost in the Chilean power system is, in general, negatively correlated with the availability of water for hydro generation.

correlated to all fuel prices. Individually, these 4 new contracts are dominated in terms of expected costs and volatility by some of the 15 initial offers (i.e., there are contracts other than the new 4 that present both lower levelized or mean costs and standard deviation). We find that the portfolio that minimizes levelized or expected costs remains unaltered from the previous example. However, all of the other portfolios in the efficient frontier include at least one of these 4 new contracts. In this case, selecting contracts based on the modified Sharpe ratio also yields the same portfolio as in the previous example. Yet, when considering new contracts indexed to a variable that is negatively correlated to fuel prices, this portfolio is no longer part of the efficient frontier. This result illustrates the difficulties of constructing an efficient portfolio considering only individual characteristics of indexed contracts, disregarding the options to diversify a portfolio with offers that might not seem attractive on an individual basis. We also incorporate a third example that uses a sample of 10 indexed contracts from the 2015/01 long-term energy auction to illustrate that our observations from the first two examples remain unaltered.

Our main conclusion is that the current selection criterion disregards the volatility of indexing parameters (e.g., fuel prices) and might select contracts that are highly volatile in the long term, which can lead to a possibly inefficient allocation of risk between generators and retail consumers, with the potential to negatively affect end consumers under the realization of adverse conditions. Other heuristic selection methodologies, such as a modified Sharpe ratio, can, in some cases, lead to portfolios that are in the efficient frontier. However, unlike the MV approach, they have no adjustable parameter to control the amount of tolerable risk in the selected portfolio (i.e., location in the efficient frontier). Consequently, a regulator who is interested in limiting the risk exposure to regulated retail customers could benefit from using more sophisticated selection methodologies. Although this issue has been qualitatively discussed in previous research (Marques and Berg, 2011; Moreno et al., 2012), to our best knowledge this is the first paper that uses a stochastic model for energy commodity prices to quantify the economic performance of portfolios developed using simple methodologies against portfolios constructed with more elaborate models such as the MV approach.

We can think of two different alternatives to improve the final allocation of risk for retail consumers. The first and most obvious approach is to modify the current selection methodology in order to account for price risks. One alternative is to select offers based on an approach such as the MV that uses the variance as a metric to quantify risk. However, this selection methodology can be applied using other known risk measures, such as VaR or CVaR, in the case indexes' prices do not follow a multivariate geometric Brownian motion. Computational efforts for determining the optimal solution with such measures are similar to the MV approach. It is worth highlighting that an auctioneer that selects an optimal portfolio using any of these approaches (e.g., MV or CVaR) would have to justify its choice based on the solution of an optimization program and not on individual features of generation offers. This is somewhat similar to the selection of optimal dispatch schedules and pricing in day-ahead or real-time markets, which has to be done considering all generation and transmission assets simultaneously in the optimization (i.e., during certain hours it can be optimal to dispatch units out of the merit order due to transmission constraints or technical limits on generators).

A second alternative is the introduction of competition in the retail sector in Chile. In theory, this would incentivize retailers to offer a menu of different contract options for consumers with varying levels of risk. The advantage of this scheme compared to using a centralized energy auction is that the final allocation of risk among generation firms, retailers, and consumers will be endogenous, with little or no intervention from the regulator. However, the international experience shows, as explained earlier, mixed levels of success in the implementation of competitive retail markets (Defeuilley, 2009). This problem is compounded by the need to (possibly) rely on an additional mechanism (e.g., a capacity market) to ensure generation adequacy if competition in the retail sector is introduced in Chile since, currently, regulated auctions have supported both: (i) competitive energy prices for end consumers, and (ii) sufficient generation investment to keep an adequate level of generation capacity in the market. It can be argued that, without regulated auctions for long-term contracts, the current capacity payments in Chile might still provide the necessary price signals to incentivize adequate investments in new generation capacity. Yet, it has been argued that such payments only represent a small portion of the revenues needed to ensure supply adequacy and their role has proved to be very limited in the presence of uncertain energy prices, especially in hydro-dominated power systems (Moreno et al., 2010). Thus, in these types of electricity markets, getting rid of regulated auctions for long-term contracts could potentially leave the authority with very limited control on generation adequacy levels, particularly if the mechanism used to compute capacity payments is not designed carefully. However, more research is needed to support this statement. Furthermore, it can be argued that not only generacion adequacy, but also the overall composition of the generation mix may be affected by the regulated auctions for long-term contracts. In effect, if risks were accounted for in the selection criterion, the overall generation mix would include technologies that provide a hedge against uncertain factors, such as fossil fuel prices as presented in Inzunza et al. (2016), Matos et al. (2015), Cunha and Ferreira (2014), and Awerbuch and Berger (2003).

An important subject that we did not address in this paper is how different selection methodologies can affect indexation and price strategies for generators in centralized energy auctions. For instance, if generation firms were aware that the auctioneer will select contracts using the MV methodology, this might provide incentives for them to select indexation strategies other than the ones used under the current selection approach. One alternative to address this question is to utilize an equilibrium model such as the one proposed in Arellano and Serra (2010), but considering different clearing mechanisms for the selection of long-term contracts. This type of analysis is, however, beyond the scope of this paper and we leave this as a subject for future research.

Finally, we want to emphasize that the only risks analyzed in this paper refer to those that result from indexed prices that evolve over time. However, as pointed out by a referee, there are other important sources of risk for retail customers, including supply risk, that we do not address in our research, but that should be accounted for in the future.

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Appendix A

Let $D = \sum_{t=0}^{T-1} Q_{mt} e^{-rt}$. The close formulas for μ_m and C_{mn} are:

$$\mu_m = \frac{P_{m0}}{D} \sum_{t=0}^{T-1} \sum_{j=1}^{J} \alpha_{mj} Q_{mt} e^{\bar{g}_{jt} - rt} \tag{A.1}$$

$$Cov_{mn} = \frac{P_{m0} P_{n0}}{D^2} \sum_{\substack{t=0\\s=0}}^{T-1} \sum_{\substack{j=1\\l=1}}^{J} \alpha_{mj} \alpha_{nl} Q_{mt} Q_{nt} e^{-r(t+s)} e^{\bar{g}_{jt} + \bar{g}_{ls}} (e^{\min(t,s)\sigma_j \sigma_l \rho_{jl}} - 1) \tag{A.2}$$

Proof:

Defining $\bar{g}_{jT} := \int_0^T g_{jt} dt$, the solution for the index model defined in (7) at t = 0 and t = T is:

$$I_{jT} = I_{j0} e^{\bar{g}_{jT} - \frac{1}{2}\sigma_j^2 T + \sigma_j W_{jT}} \tag{A.3}$$

The stochastic behavior of index j price at time t is characterized by the Brownian motion W_{jt}. An important property of a Brownian motion is that it has a normal distribution with mean 0 and variance t, i.e. $W_{jt} \overset{D}{=} \sqrt{t} z$, with z following a standard normal distribution. It is important to capture possible dependence between indexes in the model, since most of them are based in commodities that are empirically correlated, such as gas, crude oil, heating oil etc. The possible correlation between indexes is captured by the dependence between each Brownian motion. Let P be the correlation matrix between each index shown in Table 2. Using the normality property shown previously, then:

$$I_{jT} \overset{D}{=} I_{j0} e^{\bar{g}_{jT} - \frac{1}{2}\sigma_j^2 T + \sigma_j \sqrt{T} L'_j z} \tag{A.4}$$

where L_j is the j-th row of matrix L, which is the Cholesky decomposition of P, i.e. LL' = P and z a vector of i.i.d. standard normal distributions. Now:

$$\begin{aligned} E(e^{\sigma_j W_{jt}}) &= E(e^{\sigma_j W_{jt}}) = E(e^{\sigma_j \sqrt{t} L'_j z}) = \prod_{k=1}^{J} E(e^{\sigma_j \sqrt{t} L_{jk} z_k}) \\ &= \prod_{k=1}^{J} e^{\frac{1}{2}\sigma_j^2 L_j^2 t} = e^{\frac{1}{2}\sigma_j^2 t L_j L'_j} = e^{\frac{1}{2}\sigma_j^2 t} \end{aligned} \tag{A.5}$$

$$\begin{aligned} D\mu_m &= E(P_{m0} \sum_{t=0}^{T-1} e^{-rt} Q_{mt} \sum_{j=0}^{J} \alpha_{mj} \frac{I_{jt}}{I_{j0}}) = P_{m0} \sum_{t=0}^{T-1} e^{-rt} Q_{mt} \sum_{j=1}^{J} \alpha_{mj} e^{(\bar{g}_{jt} - \frac{1}{2}\sigma_j^2 t)} E(e^{\sigma_t W_{jt}}) \\ &= P_{m0} \sum_{t=0}^{T-1} \sum_{j=1}^{J} \alpha_{mj} e^{(\bar{g}_{jt} - rt)} Q_{mt} \end{aligned} \tag{A.6}$$

The last equality is due to (A.5). For Cov_{mn}

$$\begin{aligned} D^2 Cov_{mn} &= Cov\left(P_{m0} \sum_{t=0}^{T-1} e^{-rt} Q_{mt} \sum_{j=1}^{J} \alpha_{mj} \frac{I_{jt}}{I_{j0}}, P_{n0} \sum_{s=0}^{T-1} e^{-rs} Q_{ns} \sum_{l=1}^{J} \alpha_{nl} \frac{I_{ls}}{I_{l0}}\right) \\ & P_{m0} P_{n0} \sum_{\substack{t=0\\s=0}}^{T-1} \sum_{\substack{j=1\\l=1}}^{J} \alpha_{mj} \alpha_{nl} e^{-r(t+s)} Q_{mt} Q_{ns} Cov\left(\frac{I_{jt}}{I_{j0}}, \frac{I_{ls}}{I_{l0}}\right) \end{aligned} \tag{A.7}$$

The covariance equals:

$$\begin{aligned} Cov\left(\frac{I_{jt}}{I_{j0}}, \frac{I_{ls}}{I_{l0}}\right) &= e^{\left(\bar{g}_{jt} - \frac{1}{2}\sigma_j^2 t\right)} e^{\left(\bar{g}_{ls} - \frac{1}{2}\sigma_l^2 s\right)} (E(e^{\sigma_j W_{jt} + \sigma_l W_{ls}}) - E(e^{\sigma_j W_{jt}}) E(e^{\sigma_l W_{ls}})) \\ &= e^{\left(\bar{g}_{jt} - \frac{1}{2}\sigma_j^2 t\right)} e^{\left(\bar{g}_{ls} - \frac{1}{2}\sigma_l^2 s\right)} \left(E(e^{\sigma_j W_{jt} + \sigma_l W_{ls}}) - e^{\frac{1}{2}\sigma_j^2 t} e^{\frac{1}{2}\sigma_l^2 s}\right) \end{aligned} \tag{A.8}$$

Now, assuming since the Brownian motion has independent increments and $W_{jt} - W_{js}$ is equal in distribution to W_{jt-s} when t > s, then:

$$\begin{aligned} E(e^{\sigma_j W_{jt} + \sigma_l W_{ls}}) &= E(e^{\sigma_j (W_{jt} - W_{js}) + \sigma_l W_{ls} + \sigma_j W_{js}}) = E(e^{\sigma_j W_{jt-s}}) E(e^{\sigma_l W_{ls} + \sigma_j W_{js}}) \\ &= e^{\frac{1}{2}\sigma_j^2 (t-s)} E(e^{\sigma_l \sqrt{s} L'_l z + \sigma_j \sqrt{s} L'_j z}) = e^{\frac{1}{2}\sigma_j^2 (t-s)} \prod_{k=1}^{J} E(e^{\sqrt{s}(\sigma_l L_{lk} + \sigma_j L_{jk} z_k)}) \\ &= e^{\frac{1}{2}\sigma_j^2 (t-s)} \prod_{k=1}^{J} e^{\frac{1}{2}s(\sigma_l L_{lk} + \sigma_j L_{jk})^2} \\ &= e^{\frac{1}{2}\sigma_j^2 (t-s)} e^{\frac{1}{2}s\sigma_l^2} e^{\frac{1}{2}s\sigma_j^2} \prod_{k=1}^{J} e^{\frac{1}{2}s 2\sigma_l \sigma_j L_{lk} L_{jk}} = e^{\frac{1}{2}\sigma_j^2 t} e^{\frac{1}{2}s\sigma_l^2} e^{s\sigma_l \sigma_j \rho_{jl}} \end{aligned} \tag{A.9}$$

Then:

$$Cov\left(\frac{I_{jt}}{I_{j0}}, \frac{I_{ls}}{I_{l0}}\right) = e^{\bar{g}_{jt} + \bar{g}_{ls}} (e^{\min(t,s)\sigma_j \sigma_l \rho_{jl}} - 1) \tag{A.10}$$

Appendix B

Table B.1
The first five columns are the index projections used in the Chilean auction from the 2016 Energy Outlook done by the U.S. Energy Information and Administration (EIA, 2017). The last five columns show the log returns of the projections. These returns show how each index change rate decrease in time and the reason to consider 2 drifts for each index in Table 2. The short term drift is the mean of the log returns between 2017–2022, while the long term drift is the mean value from 2023 to 2031 period.

Y	Fuel	Oil	Diesel	Gas	Coal	Fuel	Oil	Diesel	Gas	Coal
2016	.72	36.84	2.58	1.80	50.37					
2017	.98	48.08	3.09	2.04	49.95	.31	.27	.12	.18	−.01
2018	1.17	57.01	3.62	2.27	49.75	.18	.17	.11	.16	0
2019	1.49	70.11	4.01	2.58	50.06	.25	.21	.13	.1	.01
2020	1.68	76.57	4.43	2.71	50.39	.12	.09	.05	.1	.01
2021	1.84	81.16	4.33	2.81	50.34	.09	.06	.04	−.02	0
2022	1.97	84.65	4.35	2.88	50.74	.07	.04	.03	.01	.01
2023	2.02	87.11	4.74	2.94	50.96	.03	.03	.02	.09	0
2024	2.07	89.15	5.00	2.99	51.03	.02	.02	.02	.05	0
2025	2.13	91.59	5.12	3.05	51.30	.03	.03	.02	.03	.01
2026	2.19	94.63	4.99	3.13	51.46	.03	.03	.02	−.03	0
2027	2.25	97.18	4.95	3.19	51.25	.02	.03	.02	−.01	0
2028	2.29	99.33	5.00	3.24	50.76	.02	.02	.02	.01	−.01
2029	2.35	102.23	5.05	3.31	50.29	.03	.03	.02	.01	−.01
2030	2.39	104.00	5.06	3.36	49.83	.02	.02	.01	0	−.01
2031	2.46	107.23	5.01	3.44	49.82	.03	.03	.02	−.01	0

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