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6.6 Example of texture images ALOT used for the experiments. Fruit sprinkles, couscous, toilet paper, spearmint, bread (wholewheat), silver foil (heavily cramped), soya beans, wool knitwear, sand (wet), ricotta rice, sage, cotton (color orange), terry cloth (orange), rock salt, cork, mustard seed (yellow) ribbed cotton (yellow), pine seeds, scourer (rough side), lef (brown), poppy seed, rosemary leaves, stones (gray level), fake fur, moss (green), vitamin C pills (roter), flan (apple), snail poison, reed (plumes), lego (plates yellow and blue), lace, spaghetii (regularly ordered), ribbed cardboard, carpet (blue), wallpaper (beige motive), chamois (punched), carpet (beige), cotton (blue and purples stripes), sand paper (roughness 40), and rabbit food.

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6.10 First column shows the frequency partitions induced by the best WP solution, which is represented by the tree in the second column.