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INCOME CONVERGENCE WITHIN AND BETWEEN COUNTRIES*

By Ronald D. Fischer and Pablo J. Serra¹

We study how trade changes the rate of income convergence within and between countries in a model of endogenous growth. An externality in the production of human capital implies that inequality slows down growth under autarky. Eventually incomes converge, raising the growth rate. Trade accelerates (slows down) growth and the rate of income convergence in the poor (rich) country. In the long run trade ensures that countries grow at the same rate and that the ratio of their incomes tends to 1. Trade pattern reversals are possible since the initially wealthy country may be overtaken by the poor country.

1. INTRODUCTION

The purpose of this paper is to study how trade affects the pattern of income convergence within countries and between countries in an endogenous growth model. Moreover, we analyze the related issue of how income inequality alters the effects of trade. We use a dynamic 2×2 factor proportions model with endogenous growth to examine income convergence. The factors of production are unskilled labor and human capital. Growth proceeds through the accumulation of human capital by agents who are differentiated by their stocks of human capital. Agents live a single period during which they spend resources in their descendants' education. The expenditure in a descendant's education enters the utility function of each agent.

In our model, the accumulation of human capital is the result of both formal and informal processes. The formal process corresponds to schooling and depends on parental expenditure in education. The informal process of producing human capital corresponds to the education agents receive in their homes and through social interaction with neighbors and schoolmates. In the informal process, the first effect depends on the education of the parents, while the second corresponds to an externality acting through the average level of human capital in society.

We begin our study by examining income convergence in a closed economy. As the society's stock of human capital increases, the relative price of the equitatively distributed factor—unskilled labor—goes up, which tends to reduce long-term inequality. On the other hand, rich agents spend a higher proportion of their income in education, tending to raise inequality. We derive a sufficient condition for the former effect to outweigh the latter effect, that is, for inequality to disappear in the

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long run. In the absence of the externality this condition is also a necessary one. The existence of an externality makes income convergence more likely. Moreover, the stronger the externality, the faster the reduction in inequality. This occurs because the externality causes agents with above average human capital to find it relatively more difficult to acquire additional human capital than agents with below average human capital, as in Tamura (1991). This implies that it is less costly to increase aggregate human capital when it is widely spread than when it is highly concentrated.

Thus, in our model inequality in the presence of the externality is associated with slower growth. This result is in agreement with Persson and Tabellinis's (1994) empirical finding that there is a negative relation between income inequality and growth. However, these authors use a different theoretical approach based on the effects of inequality on policy to explain this fact.

Next, we examine income convergence in a two-country world. Both countries have identical preferences and technologies, but differ in their initial per-capita human capital stocks. We assume that the externality is local to each country and that trade leads to Factor Price Equalization. In this framework we derive conditions for the world economy to grow and for inequality to fall in both countries. The following results assume that these conditions are met.

We show that in each period, trade reduces the growth rate and the rate at which inequality declines in the rich country as compared with autarky. It has opposite effects in the poor country. The slowdown in the rich country is explained by factor price changes due to trade. Since the expenditure in education enters each agent's utility function and trade raises the relative price of human capital in the rich country, less education is provided to descendants. While this effect is independent of the externality, the externality has its own effect on growth. As trade leads wealthy agents to save a larger proportion of total savings in the rich country, and the externality causes their savings to be less productive, growth suffers. Trade lowers the rate at which inequality declines in the rich country because the change in relative factor prices exacerbates the differences in the savings of poor and rich agents. Thus the acquisition of education by poor agents suffers, slowing down convergence in the rich country.

If the poor country has no more inequality (in terms of the Lorenz ordering) than the rich country, the poor country grows faster. We show that the existence of an externality in the production of human capital in conjunction with income inequality raises the possibility of the initially poor country overtaking the initially rich country. In fact, less inequality is a necessary condition for the poor country to overtake the rich country. An interesting implication of this result is that our model admits the possibility of trade pattern reversals, even though both countries have identical technologies and preferences and there are no factor intensity reversals.

The main result of this paper is to show that the ratio of average incomes in the two countries converges to one under free trade. The intuition for this result is the following: Eventually, either the poor country has less inequality than the rich country or inequality becomes negligible in both countries. In either case the poor country grows faster than the rich country. It is possible for the rich country to be overtaken by the poor country, but in the long run inequality disappears and incomes converge.

If the wealthy economy is not overtaken by the poor country, it suffers long run losses from trade liberalization: it is poorer and the gains from trade become negligible as the ratio of incomes in the two economies converges. On the other hand, trade leads to short-run welfare gains for everybody, assuming lump sum compensations. Hence, given those compensations, a free trade regime is guaranteed by the myopic utility function of agents and the fact that, each period, the model behaves like a conventional static trade model (see, for instance, Dixit and Norman 1980).

Tamura (1991) obtains convergence of incomes in an endogenous growth model with two income classes and an externality in the accumulation of human capital. Fischer and Serra (1993) show that the concavity or convexity of the functional form of the externality makes a difference to the results on income convergence. In a model with production, they show that a concave externality leads to income convergence. This paper extends the analysis of income convergence to open economies with general distributions of income. Previous papers have dealt with the effects of trade and income inequality on growth separately.

There is empirical evidence of conditional convergence of incomes between countries. Barro (1991) finds a significantly negative partial correlation between the per-capita growth rate for a large sample of countries from 1960 to 1985 and the log of the 1960 per-capita income when other variables are held constant.² Levine and Renelt (1992), and Blomstrom, Lipsey and Zejan (1992) obtain similar results.³ There is also evidence of regional income convergence within the U.S.A. (Barro and Sala i Martin 1991) or within countries in the OECD (Dowrick and Nguyen 1989).

The next section presents the model and describes its equilibria. The following section shows how an initial distribution of human capital evolves in a closed economy over time. The fourth section studies income convergence between countries. The final section presents conclusions.

2. THE MODEL

This is a dynamic model with two factors and two goods. The factors are labor and human capital. The stock of human capital is variable, while labor is in fixed supply.

Production. There is a continuum of agents indexed by $z \in [0, L]$. Each agent lives a single period. The production function for individual human capital at

 $^{^{2}}$ The set of conditioning variables consist of school enrollment rates in 1960, the average ratio of government consumption expenditure to GDP from 1970 to 1985, proxies for political stability, and a measure of market distortions.

 $^{^{3}}$ Quah (1993) argues that the economies across the world seem to be converging to a bimodal distribution with two distinct groups: the rich countries and the countries that remain poor.

time t is

(1)
$$h_{t+1}^z = f(h_t^z, \overline{h}_t, g_t^z),$$

where h_t^z is the amount of human capital owned by agent z, \bar{h}_t is the average level of human capital in the economy and g_t^z is the amount of human capital the agent spends in the education of his descendant. The production function explicitly takes into account the dependence of new human capital on formal education through parental spending on education, g_t^z .⁴ It also takes into account the effects of informal education through contact with parents, h_t^z , and through social interaction, \bar{h}_t .

The decision on how much human capital to acquire is made by the parent, rather than the more standard assumption that it is a decision made by the agent. This choice is governed by our need for analytic simplicity, but it is not a crucial assumption (Fischer and Serra 1993).

The production functions for the final goods are (we distinguish with upper case those variables referring to the whole economy)

(2)
$$X_{1t} = L_{1t}^{\alpha} H_{1t}^{1-\alpha}, \quad 0 \le \alpha \le 1,$$

 $X_{2t} = L_{2t}^{\beta} H_{2t}^{1-\beta}, \quad 0 \le \beta \le 1,$

where L_{it} and H_{it} are the labor and human capital used in industry *i* at time *t*, respectively. Assume that good 1 is human-capital intensive, so that $\alpha \leq \beta$. There are *L* agents in the economy, each of which supplies one unit of labor, so that $L_{1t} + L_{2t} = L$. Letting G_t denote the aggregate allocation of human capital for education of the next generation by agents living in period *t*, the aggregate stock of human capital in the economy satisfies $H_t = H_{1t} + H_{2t} + G_t$.

Consumption. Agents are myopic in the sense that they care about their expenditure in the education of their descendants, rather than caring about their utility.⁵ Their utility functions are given by:⁶

(3)
$$u(c_t, g_t) = c_{1t}^{\tau} c_{2t}^{\nu} (g_t + g^*)^{(1-\nu-\tau)}; \quad \nu, \tau \ge 0, \quad \tau + \nu \le 1, \quad g^* > 0$$

where c_{it} denotes the consumption of good *i* by an agent in period *t*, and g^* is a positive constant. This specification underscores the distinction between consump-

⁴ This does not mean that parents spend their human capital directly on the education of their descendants. A more plausible story is that there is a market for teachers and parents spend part of their income on education.

⁵ This type of utility function has often been used in the study of income distribution in dynamic models: Banerjee and Newman (1991), Galor and Zeira (1993), Karni and Zilcha (1989) and Fischer (1992). The other approach, where the utility function of the descendant is included in the utility of the parent, has become common since Barro (1974) and has been used by Fischer and Serra (1993) to study inequality and growth in a closed economy.

⁶ We omit the superscript z when there is no risk of confusion.

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tion and bequest in the utility function: households have a positive utility even if they do not spend on the education of their descendants. As a consequence, the propensity to consume is lower for high-income families. Although it is not a general utility specification, it raises possibilities which are absent when the marginal propensity to save is constant.

Let w_t be the wage, y_t be the income of the agent and let the return to human capital be normalized to one by appropriate choice of units. Income is given by⁷

$$(4) y_t = w_t + h_t.$$

Assuming an interior condition, that is, there is positive expenditure in education, the first-order conditions of an agent imply that

(5)
$$c_{1t} = \tau (y_t + g^*) / p_{1t}, \quad c_{2t} = \nu (y_t + g^*) / p_{2t},$$

 $g_t = (1 - \nu - \tau) y_t - g^* (\tau + \nu)$

where p_{ii} is the price of consumption good *i* in terms of human capital. Observe that wealthy agents spend more human capital in education.⁸ In what follows, we assume that the condition for an interior solution is met by all agents.

The closed economy equilibrium conditions imply (see the Appendix),

(6)
$$w_t = \frac{\alpha \tau + \beta \nu}{1 - \alpha \tau - \beta \nu} (\bar{h}_t + g^*)$$

so wages are increasing in the average stock of human capital. The explanation of this result is simple: as human capital grows, unskilled labor becomes relatively scarce, leading to a rise in wages.

Expressions (4) through (6) are used to obtain the amount of human capital that an agent allocates to the education of his descendant:

(7)
$$g_t = (1 - \tau - \nu) \left(h_t + \frac{\alpha \tau + \beta \nu}{1 - \alpha \tau - \beta \nu} (\overline{h}_t + g^*) \right) - (\tau + \nu) g^*.$$

 7 We implicitly assume that agents can rent their human capital endowment independently of their supply of unskilled labor, i.e., human capital is not embodied in labor. Specialists in human capital theory may find this an extreme assumption, but we believe it is a reasonable specification for our purposes.

⁸ In traditional models of human capital accumulation there are outside assets with a fixed return. Since marginal returns to investment in education are decreasing, agents invest all their savings in education up to the point at which the return to education becomes equal to that of alternative assets. Hence, poor agents invest a higher proportion of their savings in human capital accumulation.

In order to obtain closed-form results we consider a specific functional form for the human capital accumulation function:⁹

(8)
$$h_{t+1} = \overline{h}_t^{1-\delta} (\rho h_t + g_t)^{\delta},$$

where $(1 - \rho)$ measures the rate at which human capital depreciates and $(1 - \delta)$ measures the strength of the externality.

The production function for human capital is such that the costs of acquiring additional human capital for an agent are smaller the higher the society's average level of human capital. The cost of additional human capital increases when the individual stock of human capital is higher. This assumption corresponds to the notion that for agents close to the boundaries of knowledge, acquiring additional knowledge is more difficult (see Tamura 1991).

The concavity of the human capital production function is an important assumption. It implies that it is less costly to increase aggregate human capital when it is widely spread than when it is highly concentrated.

3. CLOSED ECONOMY GROWTH

Replacing the expression for parental expenditure (7) in the human capital accumulation function (8) we obtain the expression for the evolution of individual stocks of human capital as a function of individual and society stocks:

(9)
$$h_{t+1} = (1+\rho-\nu-\tau)^{\delta} \overline{h}_t^{1-\delta} (h_t + \mu \overline{h}_t - \gamma)^{\delta},$$

where

(10)
$$\mu = \frac{(1-\nu-\tau)}{(1+\rho-\nu-\tau)} \frac{(\beta\nu+\alpha\tau)}{(1-\beta\nu-\alpha\tau)}$$

and

(11)
$$\gamma = \frac{\tau(1-\alpha) + \nu(1-\beta)}{(1-\beta\nu-\alpha\tau)(1+\rho-\nu-\tau)}g^*.$$

An increase in g^* implies that fewer savings are required to attain a given level of utility, hence a higher g^* leads to slower human capital accumulation. It is simple to verify that larger values of the consumption parameters ν and τ lead to smaller savings, thus enhancing the effect of g^* . The effects of increases in ρ , α and β have the opposite effect on savings, since a larger value of ρ implies that past savings are more important, while larger coefficients of labor in the production function, α and β , imply that lower savings have a smaller impact on production.

 9 It is crucial to assume constant returns to scale in the production of human capital for the model to converge to a stable growth path.

With these definitions the expenditure in education (7) can be rewritten

(12)
$$g_t = (1 - \tau - \nu)h_t + (1 + \rho - \tau - \nu)(\mu \bar{h}_t - \gamma).$$

Hence a sufficient condition for all agents to have a strictly positive expenditure in education is $\mu \bar{h}_i \ge \gamma$. This condition will play an essential role in this model, as we show by examining the issue of income convergence.

PROPOSITION 1. A sufficient condition for income convergence is $\mu \bar{h}_t \ge \gamma$. This is a necessary and sufficient condition when there is no externality.

PROOF. The rate at which inequality falls in a country can be described by:

(13)
$$\frac{d(h_{t+1}/h_t)}{dh_t} = -(1+\rho-\tau-\nu)^{\delta} (\bar{h}_t)^{1-\delta} (h_t+\mu\bar{h}_t-\gamma)^{\delta-1} \frac{[(1-\delta)h_t+\mu\bar{h}_t-\gamma]}{h_t^2}$$

which can be rewritten as follows using the expression for h_{t+1} in equation (9):

(14)
$$\frac{h_t}{h_{t+1}/h_t} \frac{d(h_{t+1}/h_t)}{dh_t} = -\frac{\left[(1-\delta)h_t + \mu \overline{h}_t - \gamma\right]}{\left(h_t + \mu \overline{h}_t - \gamma\right)} \le 0.$$

Equation (14) measures the elasticity in the rate of growth of the human capital of an individual with respect to her own stock of human capital. If $\mu \bar{h}_t \ge \gamma$, the rate of growth of human capital declines with the stock owned by an agent, thus reducing inequality. Furthermore, the larger the difference $\mu \bar{h}_t - \gamma$, the faster the decline in inequality. Note that if there is no externality (i.e., if $\delta = 1$) the condition is necessary and sufficient. Q.E.D.

The intuition for this result is the following. As the stock of human capital increases, the relative price of labor goes up, hence reducing inequality by raising the reward of the factor that is equitatively distributed (the effect of $\mu \bar{h}_i$). On the other hand, the fact that the savings rate of wealthy agents is higher than that of poor agents has the opposite effect (the impact of γ). However, this negative effect on the rate of decline in inequality becomes less important as the economy grows and all agents save a larger share of their income, that is, the growth of \bar{h}_i makes the impact of γ less significant. The externality $(1 - \delta)$ also tends to reduce income inequality over time by raising the productivity of human capital investments of poor agents and having the opposite effect on rich agents. If the condition $\mu \bar{h}_i \geq \gamma$ is not satisfied, the difference in savings rates of rich and poor agents could be sufficiently high that incomes never converge.

In order to study the evolution of society's average income over time, we assume an initial distribution of human capital (equivalently, of income, given that all agents possess the same amount of labor). We then trace the evolution of income over time and its effect on the growth rate of the economy. Taking expectations in (9) results in an equation for the evolution of average human capital:

(15)
$$\overline{h}_{t+1} = (1+\rho-\nu-\tau)^{\delta}\overline{h}_{t}^{1-\delta}\int_{0}^{1} (h_{t}^{z}+\mu\overline{h}_{t}-\gamma)^{\delta}f(h_{t}^{z}) dz,$$

where $f(h_t^z)$ is the density of the distribution of human capital h_t^z in period t. Note that when there is no externality in education, that is, when δ equals 1, (15) becomes

(16)
$$\bar{h}_{t+1} = (1 + \rho - \nu - \tau) ((1 + \mu)\bar{h}_t - \gamma)$$

implying that in this case inequality has no impact on growth.

PROPOSITION 2. More inequality leads to a lower growth rate in the presence of the externality.

PROOF. Dividing (15) by \overline{h}_t results in:

(17)
$$\frac{\overline{h}_{t+1}}{\overline{h}_t} = (1+\rho-\nu-\tau)^{\delta} \int_0^1 \left(\frac{h_t^z}{\overline{h}_t} + \mu - \frac{\gamma}{\overline{h}_t}\right)^{\delta} f(h_t^z) dz.$$

Consider a Lorenz ordering of income distributions. An increase in inequality can be achieved by a sequence of mean preserving spreads (Atkinson 1970). Since the integrand in (17) is concave, by Theorem 2 in Rothschild and Stiglitz (1970), \bar{h}_{t+1}/\bar{h}_t is lower the higher the degree of inequality. Hence inequality is associated with slower growth in the presence of the externality. Q.E.D.

Finally, we discuss the conditions for growth. In an homogeneous economy average human capital grows according to (16), from which it follows that $(\mu + (\rho - \tau - \nu)/(1 + \rho - \nu - \tau))\overline{h_l} > \gamma$ is a necessary and sufficient condition for growth. Note that if ρ is greater than $\nu + \tau$, this condition for growth is less restrictive than the condition for income convergence. Since income inequality was shown to lead to slower growth, the growth condition above becomes a necessary condition in the case of nonhomogeneous economies. A necessary and sufficient condition for growth in nonhomogeneous countries is that (17) be larger than 1.

From the equation for the evolution of average human capital (15) and the definitions of γ and μ , it follows that the growth rate falls with the rate of depreciation of human capital, $(1 - \rho)$, and increases with greater altruism towards descendants (an increase in $1 - \nu - \tau$), indicating that less consumption results in higher growth. A change in tastes could also affect the rate of growth. Growth falls when altruism is constant but there is an increase in the desire for the human-capital-intensive good. The explanation is that education becomes more expensive if consumers increase their preferences for the human-capital-intensive good. Finally, increases in the productivity of labor (measured by α and β) raise the growth rate.

In what follows we concentrate on the case in which the conditions for growth and inequality reduction are met in the initial period. The other cases correspond to

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one or both conditions failing to hold. The next proposition proves that inequality disappears in the long run.

PROPOSITION 3. If the sufficient conditions for income convergence and growth are initially met, then the economy grows forever and inequality eventually disappears.

PROOF. If in a given period the conditions for income convergence and growth are met, they are also met in the following period. Consider two different agents: a and b, with $h_0^a > h_0^b$. From the equations for the evolution of individual human capital (9) we have

(18)
$$\frac{h_{t+1}^a}{h_{t+1}^b} = \left(\frac{h_t^a + \mu \overline{h}_t - \gamma}{h_t^b + \mu \overline{h}_t - \gamma}\right)^o.$$

When $\mu \bar{h}_t > \gamma$,

(19)
$$1 \le \frac{h_{t+1}^a}{h_{t+1}^b} \le \left(\frac{h_t^a}{h_t^b}\right)^a$$

hence

(20)
$$1 \leq \lim_{T \to \infty} \frac{h_T^a}{h_T^b} \leq \lim_{T \to \infty} \left(\frac{h_0^a}{h_0^b}\right)^{\delta^2} = 1$$

because $\delta < 1$ when there is an externality. If there is no externality ($\delta = 1$), equation (18) can be rewritten,

(21)
$$\frac{h_T^a}{h_T^b} = \frac{h_0^a + \mu \sum_{t=0}^T h_t - T\gamma}{h_0^b + \mu \sum_{t=0}^T \bar{h}_t - T\gamma}$$

and the result follows because \bar{h}_t is increasing.

This proposition does not imply that incomes converge, nor that the difference in incomes are bounded, but rather that income differences become negligible in comparison with incomes. In this case growth accelerates over time as wealthier agents tend to save more and as inequality declines, converging to a steady-state growth rate given by the homogeneous economy growth rate: $(1 + \rho - \nu - \tau)^{\delta}$ $(1 + \mu)^{\delta} - 1$.

There are other possibilities where one or both of the conditions for income convergence and growth fail to hold. In the case when neither the growth conditions nor the income convergence condition are met the economy will shrink and inequality will increase. If the condition for inequality reduction is met but the conditions for growth are not satisfied, the economy will shrink and inequality will decrease until either the growth conditions are satisfied or the inequality reduction condition is not met. Depending on which event occurs first, the economy will be either in the first or the second case above. Hence, if there is a significant degree of

Q.E.D.

initial income inequality, the economy may initially shrink until the dispersion of incomes becomes sufficiently low for growth to begin.

Finally, there is the possibility that only the growth conditions are satisfied. The economy will grow and inequality will increase until either the growth condition is not satisfied or the inequality reduction condition is met. Depending on which event occurs first, the economy will evolve according to the first or the second case above. In this case this model raises the possibility of a U-shaped Kuznet curve, where inequality increases at first and then declines with growth.

4. TRADE

Consider a world composed of two countries, denoted I = 1, 2. In what follows \bar{h}^I and f^I are, respectively, the average human capital stock and the density function for human capital in country I. Both countries have identical technologies and preferences given by (1) through (3). Without loss of generality, we normalize labor in both countries to 1.

In order to apply the previous model to study the effects of international trade, it is necessary to specify how the externality extends to a world of two countries. In particular we must decide whether the scope of the externality is local to the country or whether it is international—it depends on a measure of the world average human capital. The effects of trade on growth will be affected by the scope of the externality. Since our externality is derived from a social process that depends on the interaction of groups of people, we have assumed that the externality remains local after liberalization.

We study the case where trade leads to Factor Price Equalization, the case of no specialization. In the Appendix we derive the condition for no specialization. Following the reasoning leading to the wage equation (6) the world wage is given by:

(22)
$$w_t = \frac{\alpha \tau + \beta \nu}{1 - \alpha \tau - \beta \nu} (\bar{h}_t^w + g^*),$$

where \bar{h}_{t}^{w} denotes the world per-capita stock of human capital. Under the assumption that the externality is local to a country, equation (9) for the human capital of the descendant becomes:

(23)
$$h_{t+1} = (1+\rho-\nu-\tau)^{\delta} (\overline{h}_{t}^{I})^{1-\delta} (h_{t}+\mu \overline{h}_{t}^{w}-\gamma)^{\delta}.$$

Observe that when a country is in autarky, we use \bar{h}_t^I instead of \bar{h}_t^w in equation (23). Taking liberties with the notation, we can write that for a rich country, opening to trade is equivalent to a reduction in \bar{h}_t^w in this equation. Under free trade, equation (15) for the evolution of the average stock of human capital becomes:

(24)
$$\overline{h}_{t+1}^{I} = (1 + \rho - \nu - \tau)^{\delta} (\overline{h}_{t}^{I})^{1-\delta} \int_{0}^{1} (h_{t}^{z} + \mu \overline{h}_{t}^{w} - \gamma)^{\delta} f^{I}(h_{t}^{z}) dz.$$

Note that trade makes the condition for growth in the poor country less restrictive, raising the possibility that a country that was shrinking in autarky will start growing after opening to trade. The rate at which inequality falls in a country can be described by:

(25)
$$\frac{d(h_{t+1}/h_t)}{dh_t} = -(1+\rho-\tau-\nu)^{\delta} (\bar{h}_t^{I})^{1-\delta} (h_t+\mu\bar{h}_t^w-\gamma)^{\delta-1} \frac{\left[(1-\delta)h_t+\mu\bar{h}_t^w-\gamma\right]}{h_t^2}.$$

The sufficient condition for inequality reduction is the same in both countries, namely, $\mu \bar{h}_t^w > \gamma$. Note that the condition is very similar to the sufficient condition for a closed economy. As in the autarky case, if there is no externality, the condition is necessary and sufficient. A stronger externality accelerates the rate at which incomes converge.

We now examine the growth rate of the integrated world economy. The world stock of human capital evolves according to

(26)
$$\overline{h}_{t+1}^{w} = \frac{(1+\rho-\nu-\tau)^{\delta}}{2} \sum_{I=1}^{2} \left[\left(\overline{h}_{t}^{I}\right)^{1-\delta} \int_{0}^{1} \left(h_{t}^{z}+\mu\overline{h}_{t}^{w}-\gamma\right)^{\delta} f^{I}(h_{t}^{z}) dz \right].$$

If the condition for inequality reduction is met, a sufficient (but not necessary) condition for the world economy to grow forever is that in the period in which free trade is established:

(27)
$$(1+\rho-\nu-\tau)^{\delta} \int_{0}^{1} \left(\frac{h_{t}^{z}}{\overline{h}_{t}^{I}}\right)^{\delta} f^{I}(h_{t}^{z}) dz > 1, \qquad I = 1, 2.$$

It is simple to verify that if the above condition is ever satisfied, it is satisfied forever. Suppose the inequality reduction condition is met in period t and the growth condition (27) is also satisfied. Hence inequality is lower in both countries in the next period. Moreover, the world economy grows, so the inequality reduction condition is also met next period. Therefore, in the next period the growth condition (27) continues to hold. By induction, if condition (27) is satisfied in one period, it is satisfied forever. In the case of homogeneous countries condition (27) simplifies to: $(1 + \rho - \nu - \tau) > 1$. The following assumption simplifies the exposition:

ASSUMPTION. The world economy grows under free trade.

PROPOSITION 4. Trade raises inequality as compared to autarky in the rich country. Trade has the opposite effect on the poor country.¹⁰

PROOF. In any period, opening to trade lowers the return to labor in the rich country (compare equations 6 and 22). Given that all individuals have the same

 $^{^{10}}$ We define the rich country in any period as the country with the higher average human capital stock in that period. As we show later, the initially rich country can be overtaken by the poor country.

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amount of unskilled labor but different amounts of human capital, the result follows. Q.E.D.

PROPOSITION 5. Trade reduces the rate at which inequality declines in the rich country as compared with autarky. Trade has the opposite effect on the poor country.

PROOF. Differentiating (25) with respect to \overline{h}_t^w leads to:

(28)
$$\frac{d^{2}(h_{t+1}/h_{t})}{dh_{t}\,d\bar{h}_{t}^{w}} = -(1+\rho-\tau-\nu)^{\delta} \left(\bar{h}_{t}^{I}\right)^{1-\delta} \left(h_{t}+\mu\bar{h}_{t}^{w}-\gamma\right)^{\delta-2} \frac{\mu\delta\left[(2-\delta)h_{t}+\mu\bar{h}_{t}^{w}-\gamma\right]}{h_{t}^{2}} < 0.$$

The effect of trade on a rich country is to lower the relevant \bar{h}_{t}^{w} . Thus from equation (28) it follows that trade lowers the rate at which inequality declines in the rich country. Q.E.D.

Trade lowers the rate at which inequality declines in the rich country because the change in relative factor prices worsens the difference in the savings of poor and rich agents. Thus the acquisition of education by poor agents suffers, slowing down convergence in the rich country. An additional effect is due to the fact that agents with above-average human capital become relatively richer and they have the highest propensity to save.

PROPOSITION 6. Trade reduces the growth rate in the rich country as compared with autarky. Trade has the opposite effect on the poor country.

PROOF. Trade has the effect of lowering next period's human capital h_{t+1}^z of each agent in the rich country (compare equations (9) and (23)). It follows that trade lowers the rate at which the country grows. Furthermore, trade indirectly lowers the rate of growth in the rich country through its effect on inequality. Q.E.D.

The slowdown in the rich country due to trade is explained by factor price changes. Recall that all agents invest a fixed proportion of their income in education minus a constant term. As trade raises the relative price of human capital in the rich country, less education is provided to descendants. In addition, the effects of trade on growth are enhanced by the rise in inequality due to trade. The next proposition derives a condition for the poor country to grow faster than the rich country under trade.

PROPOSITION 7. If the poor country has no more inequality (in terms of the Lorenz ordering) than the rich country, then the poor country grows faster than the rich country under free trade.

PROOF. Dividing equation (24) by \bar{h}_t^I results in:

(29)
$$\frac{\overline{h}_{t+1}^{I}}{\overline{h}_{t}^{I}} = (1+\rho-\nu-\tau)^{\delta} \int_{0}^{1} \left(\frac{h_{t}^{z}}{\overline{h}_{t}^{I}} + \frac{\mu \overline{h}_{t}^{w}-\gamma}{\overline{h}_{t}^{I}}\right)^{\delta} f^{I}(h_{t}^{z}) dz.$$

The distribution of h_t/\bar{h}_t^I has the same mean in both countries, while the other term in the integrand, namely $(\mu \bar{h}_t^w - \gamma)/\bar{h}_t^I$, is positive and smaller for the rich country. From the fact that the integrand in (29) is concave, it follows that the poor country grows faster than the rich country if it has less inequality. Q.E.D.

Note that less inequality is a necessary condition for the poor country to overtake the rich country (see (24) for the evolution of average human capital). We study next the issue of income convergence between countries. This leads to our main result, which shows that growth under trade eliminates inequalities between countries as well as within countries. We begin by examining convergence between homogeneous countries.

LEMMA 1. In a world composed of homogeneous countries, the ratio of average income in the two countries converges to 1.

PROOF. When countries are homogeneous, (24) for the evolution of aggregate human capital becomes

(30)
$$h_{t+1}^{I} = (1 + \rho - \tau - \nu)^{\delta} (\overline{h}_{t}^{I})^{1-\delta} (h_{t}^{I} + \mu \overline{h}_{t}^{w} - \gamma)^{\delta}, \quad I = 1, 2.$$

Under free trade the initially wealthy country is never overtaken by the initially poor country because of our condition for inequality reduction $\mu \bar{h}_t^w \ge \gamma$. From this condition it also follows that:

(31)
$$\frac{h_{t+1}^1}{h_{t+1}^2} = \left(\frac{h_t^1}{h_t^2}\right)^{1-\delta} \left(\frac{h_t^1 + \mu \overline{h}_t^w - \gamma}{h_t^2 + \mu \overline{h}_t^w - \gamma}\right)^{\delta} < \left(\frac{h_t^1}{h_t^2}\right).$$

This sequence of ratios is decreasing. Using (30) and the assumption that the world economy grows, the ratio is bounded below by 1, so the sequence has a limit. Suppose the limit is $1 + \epsilon$, with $\epsilon > 0$. Then, taking the limits on both sides of the equals sign and simplifying leads to a contradiction, where we use again the assumption that the world economy grows. Q.E.D.

This inequality shows that income convergence is a consequence of the faster growth rate of the poor country and the fact that the growth rates do not converge so long as average incomes are different. Figure 1 shows the trajectory of the log of human capital in the two countries under autarky and trade in the case of homogeneous countries (in the figures, lh_{Ia} , I = 1, 2 indicates autarkic values of the log of human capital in each country, while lh_1 , I = 1, 2 indicates free trade

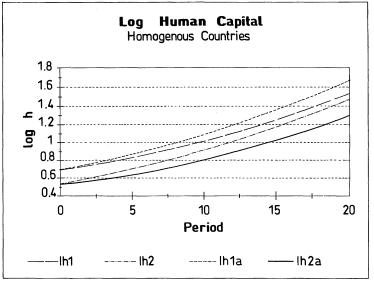


FIGURE 1

values).¹¹ Observe that the growth rate in the wealthier country suffers, while growth in the poor country accelerates. Moreover, in autarky there is no convergence in human capital stocks, while there is convergence under trade. The next lemma is also used in the proof of our main proposition.

LEMMA 2. If the sufficient condition for inequality reduction is satisfied, inequality disappears in both countries in the long run.

PROOF. Similar to Proposition 3.

PROPOSITION 8. The ratio of average income in the two countries converges to 1.

PROOF. Eventually, inequality in the poor country falls sufficiently that it starts growing faster than the rich country. Two possibilities arise. The first is that both countries converge smoothly. The other is that the poor country overtakes the initially rich country. If the rich country is overtaken by the initially poor country, it is the newly rich country that eventually grows slower. Ultimately, there is almost no inequality in the two countries, at which time income convergence follows as in the homogeneous countries case. Q.E.D.

If the initially rich country is overtaken by the initially poor country, we have a trade pattern reversal. Thus nothing precludes the initially labor-abundant country

¹¹ In all figures, $\delta = 0.5$, $\nu = 0.2$, $\tau = 0.3$, $\alpha = 0.6$, $\beta = 0.45$, $\rho = 0.5$ and $g^* = 0.75$. The starting values in Figure 1 are $h_0^1 = 2$ and $h_0^2 = 1.7$. The starting values for each agent in each country in Figure 2 are $h_0^{11} = 4$, $h_0^{12} = 0$, $h_0^{21} = 1.7$ and $h_0^{22} = 1.7$, where the first superscript indicates the country and the second superscript indicates the individual agent.

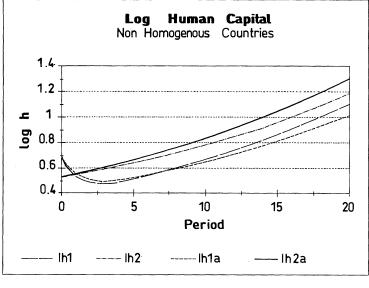


FIGURE 2

from eventually exporting human capital-intensive goods. Figure 2 shows a case in which the initially rich country is overtaken by the initially poor country. In this example, there is sufficient initial inequality in the wealthy country that it shrinks during the first few periods after trade is established. Once the initially wealthy country is overtaken, trade helps it to recover faster. In this particular case, the initially rich country being overtaken by the poor country is more likely when the former has more income inequality.

If there is no overtaking, the wealthy economy suffers long run losses from trade liberalization: it is poorer and the gains from trade become negligible as the ratio of incomes in the two economies converges. The reason for opening the economy is that there are short-run gains from trade.¹² These static gains from trade explain why both countries choose to liberalize trade. Their existence is guaranteed because each period the model behaves like a conventional static trade model. The myopic utility function of agents ensures trade, despite eventual long-run losses to the wealthy country.

The growth rate of the integrated world economy may increase or fall following trade. We conjecture that the effects of trade depend on how it changes "world income inequality." The intuition is the following: we know that an increase in inequality within a country reduces the growth rate in the country. If we consider

¹² Determining whether welfare over time is higher under trade requires the specification of an intertemporal welfare function. The results will depend on how future utility is discounted. In the present model, agents are incapable of evaluating the long term and behave myopically.

the integrated world economy as a single country, then if trade raises world income inequality, world growth should slow down.

In the specific case of a world with homogeneous income countries, trade reduces the rate of capital accumulation in the world. Consider a world with homogeneous countries. Let the parameter $\epsilon > 0$ measure the deviations from the worldwide average human capital stock, in other words, average human capital in the rich country is $\bar{h}_t^1 = (1 + \epsilon)\bar{h}_t^w$, while in the poor country it is $\bar{h}_t^2 = (1 - \epsilon)\bar{h}_t^w$. The world stock of human capital evolves according to

(32)
$$\overline{h}_{t+1}^{w} = \frac{(1+\rho-\tau-\nu)^{\delta}}{2} \times \left[(1+\epsilon) \left(1+\frac{\mu \overline{h}_{t}^{w}-\gamma}{(1+\epsilon)\overline{h}_{t}^{w}} \right)^{\delta} + (1-\epsilon) \left(1+\frac{\mu \overline{h}_{t}^{w}-\gamma}{(1-\epsilon)\overline{h}_{t}^{w}} \right)^{\delta} \right] \overline{h}_{t}^{w}.$$

Rearranging (32) and differentiating $(\bar{h}_{t+1}^w/\bar{h}_t^w)$ with respect to ϵ leads to

$$(33) \quad \frac{\left(d\bar{h}_{t+1}^{w}/\bar{h}_{t}^{w}\right)}{d\epsilon} = \frac{\left(1+\rho-\tau-\nu\right)^{\delta}}{\bar{h}_{t}^{w}}$$
$$\times \left(\frac{\left(1+\epsilon\right)\bar{h}_{t}^{w}+\left(1-\delta\right)\left(\mu\bar{h}_{t}^{w}-\gamma\right)}{\left(\left(1+\epsilon\right)\bar{h}_{t}^{w}+\mu\bar{h}_{t}^{w}-\gamma\right)^{1-\delta}\left(1+\epsilon\right)^{\delta}} - \frac{\left(1-\epsilon\right)\bar{h}_{t}^{w}+\left(1-\delta\right)\left(\mu\bar{h}_{t}^{w}-\gamma\right)}{\left(\left(1-\epsilon\right)\bar{h}_{t}^{w}+\mu\bar{h}_{t}^{w}-\gamma\right)^{1-\delta}\left(1-\epsilon\right)^{\delta}}\right)$$

which is negative, showing that the larger the difference between the countries, the lower the average growth rate, and that our conjecture is true in the case of homogeneous countries.

This result is a consequence of the way we extend the externality in the accumulation of human capital to the case of trade. Our extension is consistent with the idea of an informal component in the process of human capital accumulation that depends on the local average level of human capital. If instead, the externality were based on the dispersion of ideas, the effects of trade on the growth rate of the wealthy country would depend on whether the trading partners were similar or different. In this case, if the countries are similar, it is probable that trade would lead to higher growth in both countries.¹³

5. CONCLUSIONS

This paper examines income convergence in an endogenous growth model of countries with income inequality. It presents a clear illustration of the potential of

¹³ Consider, for example, the case in which the externality takes the form $h_{t+1}^{I} = (\bar{h}_{t}^{w})^{1-\delta}(1+\rho-\nu-\tau)^{\delta}(h_{t}+\mu\bar{h}_{t}^{w}-\gamma)^{\delta}$. This functional form would be appropriate if the externality depended on the stock of ideas and trade increases the stock of ideas available to a country (see Rivera-Batiz and Romer (1991)). This model leads to different dynamics following trade.

studying the interaction between inequality, externalities and trade. The existence of an externality in the accumulation of human capital enriches the analysis by allowing a wider variety of outcomes.

First, when an externality is present, inequality reduces the growth rate of the closed economy, a result with clear policy implications. Second, the externality increases the likelihood of a reduction of inequality. The most interesting implication of the externality, however, is that it raises the possibility that the initially poor country overtakes the initially rich country, a possibility that does not arise when the rich country is homogeneous. If the rich country gets overtaken, there is a trade pattern reversal.

If the rich country is not overtaken, our model shows that trade has three undesirable effects on the rich country: First, it raises inequality; second, it lowers the growth rate of its economy; and finally, it reduces the rate at which inequality declines over time. The rich country does benefit from the traditional gains from trade, but these disappear in the long run as the economies converge. The overall welfare effect of trade depends on the rate at which the economy discounts future consumption. In the case of the poor country, all four effects are positive: trade is an unalloyed blessing for poor countries.

The fact that trade raises inequality in the rich country is a standard result in the Stolper-Samuelson tradition. Trade liberalization raises the relative price of human capital, which is the unequally distributed factor, thus increasing inequality. This result has some empirical support in the observation that trade liberalization has coincided with increased income differentials in developed countries.

The second effect is that trade lowers the rate of growth in the rich country. The intuition for this result comes from the fact that opening the economy raises the price of human capital, so education becomes more expensive and less human capital is invested in providing education. This is a robust result since its only requirement is that parents invest less human capital in education when its price rises.

The fact that trade lowers the rate at which inequality falls is a consequence of the decline in the growth rate. In our model growth raises the ratio of human capital relative to unskilled labor. As labor becomes scarcer relative to human capital, wages increase in real terms, reducing the income gap between agents who have little human capital and those who are rich. Hence a fall in the growth rate lowers the rate at which inequality declines over time.

This paper suggests a straightforward policy option for rich countries. The rich country could reap some of the gains of trade while preserving its growth rate by subsidizing education. This policy leads to a higher standard of living without compromising growth. The model predicts that poor countries that open to trade tend to converge to rich countries, while countries that remain closed, especially countries with much unequality, will observe that their income gap with rich countries grows bigger. The obvious policy implication is that free trade is superior to autarky for poor, small economies.

The results in this paper reflect the assumptions we have made. The conclusions need to be taken with care, considering the fact that there are other sources of externalities in international trade, whose effects may be different. For instance,

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some authors assume that the human-capital-intensive industry also generates a public good which raises productivity in this industry.¹⁴ In these models trade leads to an increase in the size of the human-capital-intensive industry in the rich country. This effect of trade generates a positive externality that leads to a higher growth rate under trade. An extension to the present model in which these two approaches are merged is left for future research.

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APPENDIX

The equalization of factor prices implies

(A.1)
$$w_{t} = \alpha p_{1t} L_{1t}^{\alpha-1} H_{1t}^{1-\alpha} = \beta p_{2t} L_{2t}^{\beta-1} H_{2t}^{1-\beta},$$
$$1 = (1-\alpha) p_{1t} L_{1t}^{\alpha} H_{1t}^{-\alpha} = (1-\beta) p_{2t} L_{2t}^{\beta} H_{2t}^{-\beta}.$$

Using (2) and the fact that in autarky consumption equals production,

(A.2)
$$w_t = \alpha p_{1t} C_{1t} / L_{1t} = \beta p_{2t} C_{2t} / L_{2t},$$

where C_i denotes the consumption of good *i*. Using (5) and (A.2) we obtain

(A.3)
$$w_{t} = \alpha \tau \frac{Y_{t} + g^{*}L}{L_{1t}} = \beta \nu \frac{Y_{t} + g^{*}L}{L_{2t}},$$

which leads to

(A.4)
$$L_{1t} = \frac{\alpha \tau}{\beta \nu} L_{2t}.$$

Analogously, we have that:

(A.5)
$$H_{1t} = \frac{(1-\alpha)\tau}{(1-\beta)\nu} H_{2t}.$$

Using $L_{1t} + L_{2t} = L$ we obtain

(A.6)
$$L_{1\iota} = \frac{\alpha \tau}{\beta \nu + \alpha \tau} L$$

¹⁴ This in accordance to the strand of literature which assumes that some sectors produce more positive externalities than others. See Grossman and Helpmann (1991, chapter 8) for a standard model of this type.

From equations (4) and (A.3)

(A.7)
$$Y_t = \alpha \tau \frac{Y_t + g^* L}{L_{1t}} L + H_t$$

Solving for *Y* and using (A.6):

(A.8)
$$Y_t = \frac{H_t + (\alpha \tau + \beta \nu)g^*L}{1 - \alpha \tau - \beta \nu}.$$

Replacing (A.6) and (A.8) in (A.3) results in:

(A.9)
$$w_t = \frac{(\alpha \tau + \beta \nu)(\bar{h}_t + g^*)}{1 - \alpha \tau - \beta \nu}.$$

From (12), the average human capital allocated to education as a fraction of the average stock of capital is:

(A.10)
$$\bar{g}_t = \frac{1-\nu-\tau}{1-\alpha\tau-\beta\nu}\bar{h}_t - (1+\rho-\tau-\nu)\gamma,$$

hence

(A.11)
$$G_t = \frac{1-\nu-\tau}{1-\alpha\tau-\beta\nu}H_t - (1+\rho-\tau-\nu)\gamma L.$$

Now, as $H_{1t} + H_{2t} + G_t = H_t$, (A.5) and (A.11) imply that:

(A.12)
$$H_{1t} = \frac{(1-\alpha)\tau}{1-\beta\nu-\alpha\tau} (H_t + Lg^*)$$

$$H_{2t} = \frac{(1-\beta)\nu}{1-\beta\nu-\alpha\tau} (H_t + Lg^*).$$

From (A.6) and (A.12) it follows:

(A.13)
$$\frac{H_{1t}}{L_{1t}} = \frac{1-\alpha}{\alpha} \frac{\alpha\tau + \beta\nu}{1-\beta\nu - \alpha\tau} (\bar{h}_t + g^*)$$
$$\frac{H_{2t}}{L_{2t}} = \frac{1-\beta}{\beta} \frac{\alpha\tau + \beta\nu}{1-\beta\nu - \alpha\tau} (\bar{h}_t + g^*)$$

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where we have dispensed with the t subscript. From (A.1) and (A.13) we have that

(A.14)
$$p_{1t} = \frac{(1-\alpha)^{\alpha-1}}{\alpha^{\alpha}} \left(\frac{\alpha\tau + \beta\nu}{1-\beta\nu - \alpha\tau}\right)^{\alpha} \left(\overline{h}_{t} + g^{*}\right)^{\alpha}$$
$$p_{2t} = \frac{(1-\beta)^{\beta-1}}{\beta^{\beta}} \left(\frac{\alpha\tau + \beta\nu}{1-\beta\nu - \alpha\tau}\right)^{\beta} \left(\overline{h}_{t} + g^{*}\right)^{\beta}$$

The condition for no specialization (assuming good 1 is more *H*-intensive, i.e., $\beta > \alpha$) is that

(A.15)
$$(H_{1t}^w/L_{1t}^w) > (H_t^I - G_t^I)/L_t^I > H_{2t}^w/L_{2t}^w, \quad I = 1, 2, \text{ all } t.$$

Using (A.11) and (A.13), we get the inequalities that imply that countries are non-specialized,

(A.16)
$$\frac{1-\alpha}{\alpha} (\alpha\tau + \beta\nu) (\bar{h}_t^w + g^*) > ((1-\alpha)\tau + (1-\beta)\nu) (\bar{h}_t^I + g^*)$$
$$> \frac{1-\beta}{\beta} (\alpha\tau + \beta\nu) (\bar{h}_t^w + g^*).$$

Note that when the economies eventually converge, if this condition is met in the first free-trade period, it holds forever.

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