

Contents

List of Figures	vii
1 Introduction	1
1.1 Introduction	1
1.2 Introduction	12
1.3 Introducción	24
2 Preliminaries	36
2.1 Notation	36
2.2 Differential operators	37
2.3 Function spaces	38
2.4 Important theorems of functional analysis	40
I Mixture Theory	42
3 A mixture theory model for dense granular materials	43
3.1 Balance Laws for an heterogeneous mixture	45
3.1.1 General Framework	46
3.1.2 Homogenization process	47
3.1.3 Example of a laminated material	47
3.1.4 Hypotheses	49
3.1.5 Deduction of the dynamic equations	50
3.2 Constitutive equations	60
3.2.1 General settings	60
3.2.2 Entropy Principle	61
3.2.3 Constitutive equation using sub-differential inclusions	71
3.2.4 Examples of constitutive equations	73
3.3 Dynamic equations for a dense granular heterogeneous flow	83
3.3.1 Cauchy stress tensor for a dense granular mixture	83
3.3.2 Helmholtz free energy	87
3.3.3 Dynamics equations of a dense granular flow	89
3.3.4 Couette flow between two concentric cylinders	91
3.4 Discussion and conclusion	103

II	Multiphase models involving rigid structures	105
4	A Bingham fluid-rigid body system	106
4.1	Introduction and main result	107
4.2	Notation and preliminary results	112
4.2.1	Weak form and energy inequality	113
4.3	Approximated Problems	114
4.4	Passing to the limit $M \rightarrow \infty$ and $\varepsilon \rightarrow 0$	118
4.4.1	Weak convergences	118
4.4.2	Strong convergence of the velocity	119
4.4.3	A monotonicity argument	120
4.5	Passing to the limit $k \rightarrow \infty$	124
4.5.1	Strong Convergence of the velocity field	125
4.5.2	Passing to the limit in the velocity inequality	127
5	A Navier-Stokes-Fourier-rigid body system	129
5.1	Introduction and main result	130
5.2	Notation and preliminary results	137
5.2.1	Proof of Proposition 5.1.1	139
5.3	Approximated Problems	141
5.4	Proof of Lemma 5.3	144
5.4.1	Passing to the limit in N	148
5.4.2	Uniform estimates in M	154
5.4.3	Passing to the limit in M	158
5.4.4	Introduction of the pressure	161
5.5	Passing to the limit $k \rightarrow \infty$	163
5.5.1	Strong Convergence of the velocity field	164
5.5.2	Decomposition of the pressure	167
5.5.3	Strong Convergence of the temperature	169
5.5.4	Passing to the limit in the equations	171
A	Lemmas	180
A.1	Junction of solenoidal fields	180
A.2	Junction of scalar functions	187
A.3	Basis in a Hilbert space	188
A.4	Convergence of the characteristics	188
A.5	Material derivative of a fluid-solid mixture	190
A.6	Orthogonal projection over the set of rigid velocities	191
A.7	Convergence properties of the projection	191
A.8	Convergence of test functions	193
A.9	Material derivative in the solid phase	194
A.10	Rotation matrix	196
A.11	The norm of the symmetric gradient	197
B	MATLAB codes for Section 3.3	198
	Bibliography	214