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THE EFFECT OF EXTRA DEGREES OF FREEDOM ON THE PRIMORDIAL STATISTICS OF THE UNIVERSE: PRIMORDIAL FEATURES AND AXION LANDSCAPE

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WALTER ESTEBAN RIQUELME CHAMBLAS

PROFESOR GUÍA:
GONZALO PALMA QUILODRAN

MIEMBROS DE LA COMISIÓN:
LUIS CAMPUSANO BROWN
JORGE NOREÑA SÁNCHEZ
DOMENICO SAPONE

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Resumen

El modelo estandar de la cosmología posee algunos problemas a la hora de describir las etapas tempranas del universo. La teoría de Inflación Cósmica, una fase de expansión exponencial acelerada, fue concebida para resolver los previos problemas en el modelo y terminó dando una explicación para las semillas primordiales del universo. En esta tesis, compuesta de dos partes, estudiamos el efecto de grados de libertad extra durante el periodo de inflación cósmica, y sus implicancias en la estadística primordial del universo.

En la primera parte estudiamos la creación de *features* en el espectro de potencias escalar primordial como resultado de desviaciones temporales de un fondo quasi de-Sitter. Específicamente, cómo podemos correlacionar estos features con los posibles features en el espectro de potencias tensorial primordial. Notamos que estas desviaciones sobre la invariancia de escala están relacionadas a través de los parámetros de *slow-roll*. Derivamos una relación general que conecta features en el espectro de perturbaciones de curvatura con features en el espectro de perturbaciones tensoriales. Concluimos que, incluso para grandes desviaciones en la invariancia de escala en el espectro de potencias de las perturbaciones de curvatura, el espectro de potencia tensorial primordial se mantiene invariante de escala para todos los propósitos observacionales.

La segunda parte es sobre el análisis del *paisaje* inflacionario caracterizado por un potencial de múltiples campos escalares con muchos mínimos locales. Si este es el caso, las fluctuaciones cuánticas de los campos escalares tienen la oportunidad de excursionar a través de los mínimos locales del paisaje potencial. Estudiamos esta situación analizando la dinámica de un campo *axion-like*, presente durante inflación, con un potencial dado por $v(\psi) = \Lambda^4(1 - \cos(\psi/f))$. Asumiendo que el valor de expectación del vacío (VEV) del campo se encuentra estabilizado en uno de los mínimos, digamos $\psi = 0$, calculamos cada función de correlación de n-puntos a primer orden en Λ^4 usando el formalismo *in-in*. Este cálculo, que requiere resumar todos los bucles debido a la naturaleza no lineal de $v(\psi)$, nos permite encontrar una función distribución que describe la probabilidad de medir ψ en un valor particular del campo durante inflación. Debido a que ψ puede tunelear entre las barreras del potencial, encontramos que la función distribución de probabilidad consiste de una distribución no-Gaussiana multi-modal tal que la probabilidad de encontrar ψ cerca de un mínimo de $v(\psi)$ dado, diferente de $\psi = 0$, aumenta con el tiempo.

Abstract

The standard model of cosmology has some problems in order to describe the early stages of the universe. The theory of Cosmic Inflation, a phase of accelerated exponential expansion, was conceived to solve the previous problems in the model and it ended up giving an explanation for the primordial seeds of the universe. In this thesis, which consists of two parts, we study the effect of extra degrees of freedom during the inflationary era and their implications for the primordial statistics of the universe.

In the first part, we study the creation of *features* in the primordial scalar power spectrum resulting from temporary deviations from a quasi de-Sitter background. Specifically, how we can correlate scalar features to the ones in the primordial tensor power spectrum. We notice that these deviations from scale invariance are related via slow roll parameters. We derive a general relation linking features in the spectrum of curvature perturbations to the features in the spectrum for the tensor perturbations. We conclude that, even with large deviations from scale invariance in the curvature power spectrum, the tensor power spectrum remains scale invariant for all observational purposes.

The second part is about the analysis of the inflationary landscape characterized by a multi-scalar field potential with many local minima. If this is the case, the quantum fluctuations of the scalar field had a chance to experience excursions traversing many local minima of the landscape potential. We study this situation by analyzing the dynamics of an axion-like field ψ present during inflation, with a potential given by $v(\psi) = \Lambda^4(1 - \cos(\psi/f))$. By assuming that the vacuum expectation value of the field is stabilized at one of its minima, say $\psi = 0$, we compute every n-point correlation function of ψ to first order in Λ^4 using the *in-in* formalism. This computation, which requires a resummation of all the loops due to the non-linear nature of $v(\psi)$, allows us to find the distribution function describing the probability of measuring ψ at a particular field-value during inflation. Because ψ is able to tunnel between the barriers of the potential, we find that the probability distribution function consists of a non-Gaussian multi-modal distribution such that the probability of finding ψ near a given minimum of $v(\psi)$, different from $\psi = 0$, increases with time.

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Introduction

The question of how the universe started and how particles were created was fundamental since the early stages of mankind. The search of a satisfactory answer has taken us to different conceptions of the universe, in where most of the popular solutions through history were that something greater took part in all the creation, ignoring the questions about *how, when* and *where*. All these questions, mostly metaphysical, where discussed by the promising philosophers of different ages. This generated a mixture of ideas and conceptions for the universe where now the ideas went from a creationist perspective to a fully mathematical and rational believing. The passionate searching to give a mathematical description of the phenomena in our universe is now the motivation for physics. Even though this had worked very well for classical objects, the picture gets foggier when we try to describe larger and larger objects such as stars, galaxies, or even the universe itself. This picture began to clarify in 1915 when Albert Einstein formulated his Theory of General Relativity [1] using the concepts of deformed space-time. With this new theory, he ended up giving a very satisfactory description of gravity, the force that rules the large-scale structures of our universe.

This new picture of the universe according to Einstein has its foundations in two fundamental pillars: The cosmological principle and the cosmological constant Λ. The first one tells us that the universe at very large scales should look the same independent of the direction of the sky that we are looking, this implies the properties of homogeneity and isotropy for our universe. The second pillar gives us an admissible mathematical way to describe an expanding universe, where the cosmological constant works as the energy content that produces the expansion.

The solution for the Einstein equations that respects the previous properties was found by Alexander Friedmann in 1922 [2]. He postulated a metric, or notion of distance, for an expanding universe which is both homogeneous and isotropic. Its evolution depends directly on the energy content, allowing us to describe the behavior of the universe as a dynamical object. This implies that the universe can be separated in different epochs during its evolution, mostly because the matter densities in our universe evolve differently with time. The same result was obtained by Georges Lemaître in 1927 [3] and its final geometrical consistency was developed by Howard Robertson and Arthur Walker in 1935 [4, 5], giving the uniqueness to a geometrical metric which is homogeneous and isotropic. Because of these developments, the metric of our universe is now known as the Friedman-Lemaître-Robertson-

Walker metric.

The introduction of these new ideas turned out to be the starting point of a new era of discoveries for cosmology, both theoretically and observationally. New telescopes were built to test the theoretical models that appeared to explain the new phenomena that were affecting the universe. In 1929, Edwin Hubble [6] established a relation between the distance and the red-shift of the spiral nebula already discovered by the astronomer Vesto Slipher in 1909 [7]. This relation is known as the Hubble's law, which is the first observational evidence of an expanding universe.

The early idea of the expansion of the universe was that this expansion would be decelerating due to gravitational attraction. The big discovery appeared in 1998 when the Supernova Cosmology Project [8] and the High-Z Supernova Search Team [9] used *type Ia supernovae* to measure the rate of deceleration. They found out, surprisingly, that our universe was accelerating. Now, in order to describe the increasing rate of expansion of our universe, the theory needs an extra type of energy to counteract the collapse coming from gravitation. This energy is known as *dark energy* and can be included in the theoretical framework of gravity by the addition of a positive cosmological constant Λ.

These discoveries were the foundations of the standard model of cosmology known as the ΛCDM model, or concordance model. This way of looking our universe encapsulates different concepts for its description: First, we have the Λ component which tells us about the expanding universe at late times, and second, we have the CDM which stands for *cold dark matter*. Under this model, our universe is made of $\approx 70\%$ of *dark energy*, encoded in the cosmological constant Λ, and around a 26% of *cold dark matter* which is used to explain the formation of galaxies. Its real composition and why it does not interact with ordinary matter is still a remaining mystery. The last part is the 4% containing all the *baryonic matter* of our universe, these are the particles contained in planets, stars and gas clouds.

The standard model of cosmology, besides the fact that it gives a good dynamical description of the universe, it runs into when dealing with earlier times. Those problems appeared when in 1964 Arno Penzias and Robert Wilson [10] discovered a microwave signal which was the first observational indication of a cosmological phase of the universe proposed in 1948 by George Gamow and Ralph A. Alpher known as big bang nucleosynthesis [11]. This phase started in the first 10 seconds of the universe, the lighter nuclei such as Hydrogen and Helium were produced and soon after they reached their lowest energy state by the releasing of primordial photons. The photons were detected as microwave signals and identified as the Cosmic Microwave Background (CMB) (1).

The Cosmic Microwave Background is a homogeneous black body radiation map with small temperature fluctuations from earlier times and it is usually called the first picture of our primordial universe. It allowed cosmologist of the time to set some fundamental properties of the early universe, but actually, they ended up finding more problems than actual solutions.

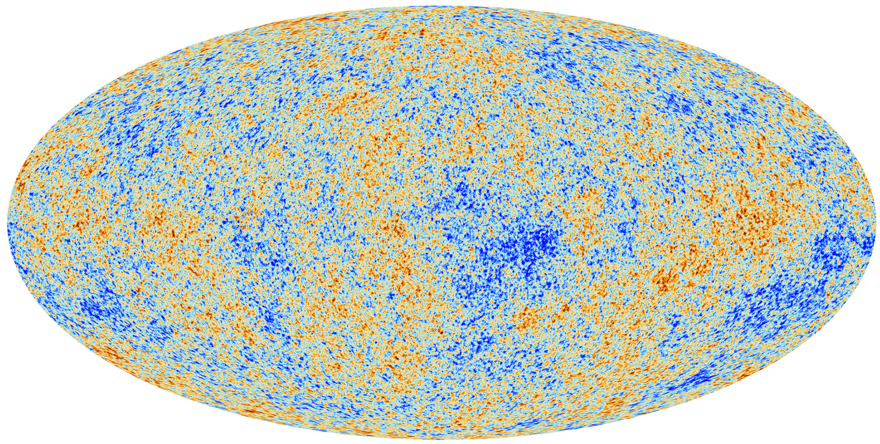

Figure 1: Temperature map of the Cosmic Microwave Background. Picture obtained by Planck [34].

The energy density distribution, encoded in the temperature, was almost homogeneous (up to the 5th decimal point), even for two very distant and causally disconnected points in the sky, see for example figure (2). If we consider this two points in the space as events from the past, given that both temperatures seem to be correlated due to the homogeneity of the CMB, this suggests that the temperature fluctuations should come from a primordial origin. The search for an explanation of this phenomenon is known as the horizon problem.

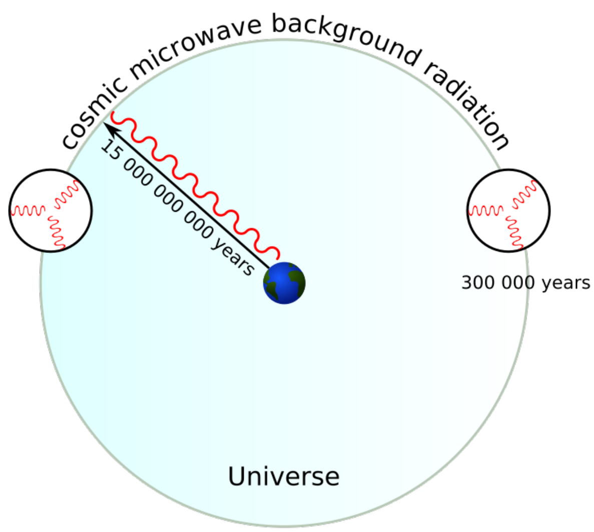

Figure 2: Horizon problem. Credit to Theresa knott at English Wikipedia

Along the same lines, the small observed inhomogeneities in the CMB also lack some explanation both for their origin and for their statistics. The second statistical moments between points seem to be characterized by a near scale-invariant power spectrum at large scales. Why this is so? was also an open question without explanation. To find a successful description of the early universe that takes into account these properties for the CMB was still an open question until 1981-1982 when a particular solution appeared: Cosmic Inflation.

The foundations for the theory of Inflation were set up by Alan Guth [12], Andrei Linde [13] and Andreas Albrecht, along with Paul Steinhardt, in 1982 [14]. It started as a solution to the horizon problem and the flatness problem, which postulates that the universe in its early stages was asymptotically flat. The developments of the theory were extended to address other problems of the ΛCDM model in its primordial stages. One particular surprise was that Inflation, apart from solving the problems previously described, it provides us with a mechanism for the early universe to create the observed small temperature fluctuations which serve as seeds to all the structures of the late universe.

The inflationary universe is well described as a *quasi de-Sitter* period of accelerated and exponential expansion encoded in its *scale factor*. The term "quasi" is added because since our late and present universe are well described by the principles of a ΛCDM model, we need a way to stop inflation. We can introduce control parameters which will act as clocks to stop the exponential expansion of the universe. The control quantities are called slow-roll parameters, they are defined as deviations from a perfect de-Sitter universe. Since inflation represents a controlled exponential expansion of the universe, we can use this theory to explain and solve the horizon problem, the scale-invariance of the CMB or even other problems.

On the other side of the picture, around the same time, the physical description of the baryonic matter of our universe was getting clearer. Most of the physicist of those years believed that the answers were written in the language of Quantum Field Theory, a relativistic description of quantum mechanics. The particles were seen as quantum states coming from fundamental objects known as fields, whose dynamics are given by an equation of motion coming from a Lagrangian which is constructed to respect the symmetries of our universe. This formalism gave a successful experimental description of the collision of particles in scattering processes and the theory was seen as a fundamental description of the particles in our universe.

The mechanism to create the energy fluctuations from Inflation was highly motivated by particle physics and, in fact, both share technical similarities. In the cosmic inflation frame, all the matter started as fluctuations of a primordial field know as the Inflaton. The Inflaton is a scalar field that fills the primordial universe and, as a scalar field, it has a similar physical behavior as the Higgs field which turned out to be the first and only elementary scalar field discovered in nature.

The theoretical challenges for the theory of inflation lay on the basis of the quantum theory of fields. The quantum fluctuations of the primordial inflaton create particles, or quantum states over a suitable vacuum, which after a period of evolution create an amount of energy that is transferred through decay to the particles of the standard model. One remarkable result of the fluctuations of inflaton is that, not only they evolve in the space-time, but they are connected to the quantum fluctuations of the space-time through the Einstein equations. It is just a matter of choice which type of perturbations we want to study. This allows inflation to be a topic where both general relativity and quantum theory intertwine for a good description of the early universe.

Once we describe the primordial universe with the fluctuations of a scalar field, then we can wonder about a different possibility. What about if we generate a strange mechanism that involves different kinds of interactions, in the same way that we do in particle physics and the Higgs field, whose interaction allows the particles to have mass. Interactions via couplings of the inflaton field to other types of fields could be an option. During this thesis, we will refer to the other type of fields as *extra degrees of freedom*. The nature of these extra fields could have different origins or motivations but its description is still based in using the quantum field theory formalism for objects that are also affected by gravity, in this case, an expanding universe. This gave enough motivation to rename the theory as Multi-field inflation, where the collection of fields and their interactions *draw* a new object known as *landscape*, which is the space where those fields live.

On the other hand, the fluctuations of the space-time produced by the merging of two black holes were discovered in 2015 by the group of LIGO (Laser Interferometer Gravitational-waves Observatory) [16]. They represented an empirical confirmation of the theory proposed by Einstein in 1915. Along the same lines, another remarkable result from the theory of inflation is that it provides a mechanism for the creation of primordial gravitational waves. Along with the evolution of the scalar degrees of freedom, there are also tensor modes which can be treated as fields, in the same way as the gravitational waves created by black hole mergings. The problem with primordial gravitational waves is that the only way that they can be detected is through the polarization of photons in the CMB, known as B-modes. This is a very different way to see the effect of gravitational waves in comparison with the ones detected by LIGO. Besides that difference, there are current efforts to look for the signal of the B-modes and their possible detection would be a smoking gun for the theory of inflation since it is the only theory that predicts primordial gravitational waves coming from an accelerated expansion.

The way to detect the effects of the inflationary era is based on statistics encoded in correlations functions of the temperature fluctuations. They represent a possible relationship between two or more variables, and for the case of the CMB, they allow us to compute relations between temperature fluctuations whose origin could be primordial. The correlations of fields can be used to check the effect of different interactions coming from them and from the evolution of these correlations we can connect them to the temperature fluctuations in the CMB. The collection of fields correlation functions, or even probability distribution functions, is what we will call as *primordial statistics* during this thesis. In particular, the two-point correlation function can be connected to the matter power spectrum, which is used to give a description of the energy density for Fourier modes expansion of the matter density of the universe. Despite the fact that the power spectrum is nearly scale-invariant, there are observational deviations whose explanation is still a mystery. This deviations from a scale-invariant power spectrum are known as *features*, and a successful description of them could reveal some important properties of the primordial universe.

In this thesis, we will be addressing the effect of extra degrees of freedom on the primordial statistics of the universe. As mentioned before, the extra degrees of freedom could appear in different contexts. During this work, we will focus on two different kinds of effects. First, we will use the time-dependent background of the de-Sitter universe, commonly encoded in the slow roll parameters, to create a self-interaction term for the curvature perturbations. One remarkable result is given in [17] where the features in the primordial power spectrum could be related with the self-interactions of the curvature perturbations produced by the time-dependent quantities. During this thesis, we want to analyze the effect of such background quantities in the power spectrum of the tensor modes and establish a possible relation between both power spectra.

The second context of the extra degrees of freedom is highly motivated by particle physics and string theory. Axion fields appear as a solution to the strong CP problem [18]. They are used to restore a missing symmetry hidden by topological terms that could be included in the Lagrangian that describes the strong interaction between quarks. One particular property of this fields is that its potential has a shift symmetry, which allows transforming the field by a periodic factor and its lagrangian remains unaffected by the transformation. This property is particularly interesting for the context of string theory and supergravity where such symmetry is related to geometrical aspects which, in some extensions, could create fields that started the period of inflation. Another possibility is that the axion-like fields can be used as candidates for dark matter, its shift symmetry allows them to be weakly interacting to the other matter content of the universe, which is a very important property for the dark matter description. In [19] there is a more complete discussion about the intriguing aspects of axions in the cosmological context. Either way, it seems that axion-like fields could play a role in the description of our universe, and because of this, we want to examine a scenario where an axion-like field evolves along with the inflaton during the primordial phase of the universe. A successful description of the behavior of the axion field evolving through the landscape could be given by the primordial statistics of the universe, and in this case, a probability distribution function for the axion field is one of the main objectives of this thesis.

The outline of this thesis is the following: In Chapter 1, we discuss in more details the ΛCDM model of cosmology and how it arises from the theory of general relativity of Einstein. In Chapter 2, we present Inflation as a solution to the problems of the standard model of cosmology. In Chapter 3 we analyze the implications of the time-dependent background quantities that parametrize inflation for the *scale invariance of the tensor power spectrum.* In the next Chapter 4, we describe the statistics of the *excursions of an axion-like field evolving through the landscape* during the inflationary era.

Notation and Conventions

During this tesis the following conventions will be used:

- Natural units $c = \hbar = 1$.
- We use a reduced Planck mass: $M_{Pl}^2 \equiv 32\pi G_N$, with G_N Newton's constant.
- We use Einstein's summation over repeated indices. Greek and Latin indices ranges are given by: $\mu, \nu, ... = 0, 1, 2, 3$ and $i, j, ... = 1, 2, 3$.
- Sometimes we use $x^0 = t$ to simplify notations, where dots denote time derivatives, e.g. $\dot{A} = dA/dt$.
- We denote spatial 3-dimensional vectors in boldface, e.g. $\mathbf{x}$,$\mathbf{y}$,...
- The Minkowski metric is given by $\eta_{\mu\nu} \equiv \mathrm{diag}(-, +, +, +)$.
- Sometimes we use shortened notation $\int_{\mathbf{p}} \cdots \int_{\mathbf{q}} \equiv \int \frac{d^3\mathbf{p}}{(2\pi)^3} \cdots \int \frac{d^3\mathbf{q}}{(2\pi)^3}$ and $\int_{p_0} \equiv \int \frac{dp_0}{2\pi}$.

Chapter 1

The ΛCDM model of cosmology

In this chapter, we are going to describe the dynamics of the universe according to the standard model of cosmology. We start with Section 1.1, where we analyze the Einstein equation for general relativity in order to describe the dynamics of our universe. In Section 1.2, we introduce the metric that describes our universe, the Friedmann-Robertson-Walker metric, which is a homogeneous and isotropic solution to the Einstein's equation. In Section 1.3, we introduce Friedmann's solution to different matter sources. In the following Section 1.4, we present the Cosmic Microwave Background as one of the main observables for primordial cosmology. In the following two sections, we show that the standard model of cosmology has some problems in order to give a good description of our universe. We separate those problems into "old" problems, discussed in Section 1.5, that were fundamental for the paradigm shift that motivated the main topic of this thesis and some "new" problems, discussed in Section 1.6, that were used to reinforce that new paradigm.

1.1 Background dynamics

The discoveries about the expanding universe [24] or the recent discover of gravitational waves coming from different sources [16, 20–23], tell us that the space-time can be deformed, expand, and vibrate as a dynamical object. Those dynamics of the space-time are ruled by the *Einstein equations for General Relativity* that describe the evolution of the space-time given a metric $g_{\mu\nu}(\mathbf{x}, t)$, which has all the information about the geometry,

$$ds^2 = g_{\mu\nu}(\mathbf{x}, t) dx^\mu dx^\nu. \tag{1.1}$$

The energy content of the universe depends on the object that is deforming the space-time and its description is given by the energy-momentum tensor $T_{\mu\nu}$, where its components describe properties such as internal energy, shear stress and pressure of matter. Geometry and energy are combined via the Einstein equation, which describes the change in the geometry given the matter content that is deforming the space-time, the equation reads as follows:

$$G_{\mu\nu} = 8\pi G T_{\mu\nu}. \tag{1.2}$$

In the previous expression $G_{\mu\nu}$ is known as the *Einstein tensor* and is given by

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu}, \tag{1.3}$$

where we used the following definition of the Ricci tensor which tells us how curved is the space-time,

$$R_{\mu\nu} \equiv \partial_\lambda \Gamma^\lambda_{\mu\nu} - \partial_\nu \Gamma^\lambda_{\mu\lambda} + \Gamma^\lambda_{\lambda\rho}\Gamma^\rho_{\mu\nu} - \Gamma^\rho_{\mu\lambda}\Gamma^\lambda_{\nu\rho}. \tag{1.4}$$

From the previous expression $\Gamma^\mu_{\alpha\beta}$ are known as the Christoffel symbols whose definition is given by,

$$\Gamma^\mu_{\alpha\beta} \equiv \frac{1}{2} g^{\mu\lambda} \left(\partial_\alpha g_{\beta\lambda} + \partial_\beta g_{\alpha\lambda} - \partial_\lambda g_{\alpha\beta}\right), \tag{1.5}$$

where R is the Ricci scalar

$$R \equiv g^{\mu\nu} R_{\mu\nu}. \tag{1.6}$$

During this thesis, we will be dealing with extra degrees of freedom in the context of gravity, and because of that, we will present a more intuitive formalism to include them in the Einstein equation. The following action is called the *Einstein-Hilbert action*,

$$S = \frac{1}{8\pi G} \int d^4x \sqrt{-g} \left(R + \mathcal{L}_M\right), \tag{1.7}$$

which yields the Einstein equations through the principle of least action. Any extra degree of freedom can be added to the action through $\mathcal{L}_M$, which is the Lagrangian of the energy content and it could include different matter sources, or different kinds of fields, coupled to gravity.

1.2 The Friedmann-Lemaître-Robertson-Walker universe

Observations tell us that a large scales our universe is, to a first approximation, isotropic and homogeneous. This means that it doesn't matter in which direction are we looking to, our universe looks the same to all observers, and its matter content is distributed equally in the space. Now, in order to use the Einstein equations for our universe, we must include this features in the geometry of the space-time. The metric solution of those equations that includes the previous symmetries is called the Friedmann-Lemaître-Robertson-Walker metric (FLRW metric) and is given by, in spherical coordinates,

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu \tag{1.8}$$

$$= -dt^2 + a^2(t) \left[\frac{dr^2}{1 - kr^2} + r^2 d\Omega^2\right], \tag{1.9}$$

where $a(t)$ is called the scale factor which describes the time dependent expansion of the 3-dimensional space. Also, depending on the value of k our universe could be positively curved ($k = +1$), negatively curved ($k = -1$) or spatially flat ($k = 0$). Since we are dealing with and expanding universe, we need to take care on how the light signals propagate. Starting from this, we can define the *redshift* z of the light traveling through universe in terms of the expanding $a(t)$ parameter as follows,

$$1 + z \equiv \frac{a(t_0)}{a(t_1)}, \tag{1.10}$$

where the times t_1 and t_0 are selected as time of departure and arrival, respectively.
The rate of expansion can also be defined from the expanding parameter, this quantity is known as the *Hubble parameter* and it is defined as,

$$H(t) \equiv \frac{\dot{a(t)}}{a(t)}. \tag{1.11}$$

At this point, we can also define a new temporal quantity which scales with the expansion of the universe. We define the *conformal time* τ, as follows,

$$d\tau \equiv \frac{dt}{a(t)}. \tag{1.12}$$

Is straightforward to see that this redefinition of time allow us to write the FRW metric in a more compact way,

$$ds^2 = a^2(\tau)\left[-d\tau^2 + \frac{dr^2}{1 - kr^2} + r^2 d\Omega^2\right]. \tag{1.13}$$

We have now described the geometry of our cosmological space-time. In the other side of the picture, the fact that we are dealing with a homogeneous and isotropic universe also implies that the energy content of the universe should follow the same description, for this, the energy-momentum tensor of a perfect fluid is useful to describe how matter behaves in large scales of the universe,

$$T_{\mu\nu} = (\rho + P)U_\mu U_\nu - P g_{\mu\nu}, \tag{1.14}$$

where $U_\mu \equiv dX_{\mu/ds}$ is the relative four-velocity between the fluid and the observer, while ρ and P are the energy density and pressure in the rest-frame of the fluid. In particular, due to homogeneity and isotropy, we can consider that the relative velocity is the same as that of a comoving observer, $U^\mu = (1, 0, 0, 0)$, which gives a particular form to the energy-momentum tensor,

$$T^\mu_\nu = g^{\mu\lambda} T_{\lambda\nu} = \begin{pmatrix} \rho & 0 & 0 & 0 \\ 0 & -P & 0 & 0 \\ 0 & 0 & -P & 0 \\ 0 & 0 & 0 & -P \end{pmatrix}. \tag{1.15}$$

The main property of any energy-momentum tensor is that it is covariantly conserved,

$$\nabla_\mu T^\mu_\nu = \partial_\mu T^\mu_\nu + \Gamma^\mu_{\mu\lambda} T^\lambda_\nu - \Gamma^\lambda_{\mu\nu} T^\mu_\lambda = 0, \tag{1.16}$$

using (1.9) and (1.15) in the previous expression we obtain the *continuity equation* for a perfect fluid in a expanding FRW universe,

$$\dot{\rho} + 3\frac{\dot{a}}{a}(\rho + P) = 0 \tag{1.17}$$

This implies that the evolution of the energy content of the universe depends on how the universe expands.

Also, if we know the FRW metric (1.9), the energy-momentum tensor (1.15) and the Einstein equation (1.2), we can combine them to get the *Friedmann equations*, that describe the dynamics of a expanding space-time given the energy content of a perfect fluid,

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{k}{a^2}, \tag{1.18}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3P). \tag{1.19}$$

This gives a complete description of any kind of universe which is isotropic, homogeneous and its energy content is described by a perfect fluid.

1.3 ΛCDM model

Now we have the appropriate tools to present the model that describes our universe. According to the Friedmann equations, the evolution of our universe is ruled by its energy content. An important point here is that both ρ and P must be understood as the sum of all different kinds of contributions to the energy density and pressure of our universe. This is because different matter sources have their own dependence on the scale factor $a(t)$ given by the continuity equation (1.17). This implies that our universe can be separated into different epochs, where each epoch has a energy content which dominates during the evolution of the unvierse. We can solve (1.17) in terms of the *equation of state* $w = P/\rho$ in which is included matter ($w = 0$), radiation ($w = 1/3$) and vacuum energy ($w = -1$). So, we have the following expression for each part of the energy content in terms of its equation of state

$$\rho \propto a^{-3(1+w)}, \tag{1.20}$$

and hence;

$$\rho_m \propto a(t)^{-3}, \text{ for matter,} \tag{1.21}$$

$$\rho_\nu \propto a(t)^{-4}, \text{ for radiation,} \tag{1.22}$$

$$\rho_\Lambda \propto a(t)^{0}, \text{ for vacuum.} \tag{1.23}$$

We can rewrite the first Friedmann equation (1.18) as a function of the Hubble parameter in the following way,

$$H^2 = \frac{8\pi G}{3}\rho - \frac{k}{a^2}. \tag{1.24}$$

According to observations, we know that our universe at present time is approximately flat ($k \approx 0$), this allow us to compute the *critical density* ρ_{crit} in terms of the Hubble constant H_0, where the 0-subscripts denote that the time dependent quantities are valuated at the present time,

$$\rho_{crit} = \frac{3}{8\pi G} H_0^2. \tag{1.25}$$

We can use this quantity to define the dimensionless density parameters $\Omega_0 \equiv \rho_0/\rho_{crit}$. The previous argument allow us to write the Friedmann equation (1.18) as follows:

$$H(a)^2 = H_0^2 \left[\Omega_{r,0} \left(\frac{a_0}{a}\right)^4 + \Omega_{m,0} \left(\frac{a_0}{a}\right)^3 + \Omega_{k,0} \left(\frac{a_0}{a}\right)^2 + \Omega_{\Lambda,0} \right], \tag{1.26}$$

where we have defined the curvature density parameter, $\Omega_k \equiv -k/(aH)^2$ evaluated today. The observations show us that the dimensionless density parameters for our universe "today" are given by:

$$\Omega_{r,0} = 9.4 \times 10^{-5},\ \Omega_m = 0.32,\ |\Omega_k| \leq 0.01,\ \Omega_\Lambda = 0.68. \tag{1.27}$$

From this we can conclude that our universe is mostly filled with a 68% of vacuum energy, also know as dark energy, and a 32% of matter, where the matter content is divided in ordinary "baryonic" matter (5%) and Cold dark matter (26.8%).

1.4 Cosmic Microwave Background

In the earlier times of our universe, two different kinds of energy content, matter and radiation, were coupled together in a hot and dense state. In order to create the anisotropies from this primordial fluid, we need to induce deviations from the local thermal equilibrium. "How to?" is one of the main topics of the next chapter, for now, we are going to assume that we have these deviations in our model. The temperature fluctuations started to induce quantum interactions inside the primordial fluid and gave birth to the standard model particles, the physics of this transition lay on the theory of *reheating*, a topic which is out of the scope of this thesis. The different processes for these particles depend only in the temperature of the universe at a given time. This implies that as the universe cools down different kind of particles started to appear in our universe.

One of the main events in the history of our universe is called *recombination*. This epoch starts with the formation of neutral hydrogen through the reaction $e^- + p^+ \rightarrow H + \gamma$ leaving the photons created from this reaction to stream freely across the universe at the temperature of approximately $3000K$. Those primordial photons that were produced at the epoch of recombination left their imprint in one of the first pictures of our universe. As mentioned in the introduction, this first signal was discovered accidentally in 1964 by Arno Penzias and Robert Wilson [10] and the main picture was obtained in 1992 by the COBE mission [24].

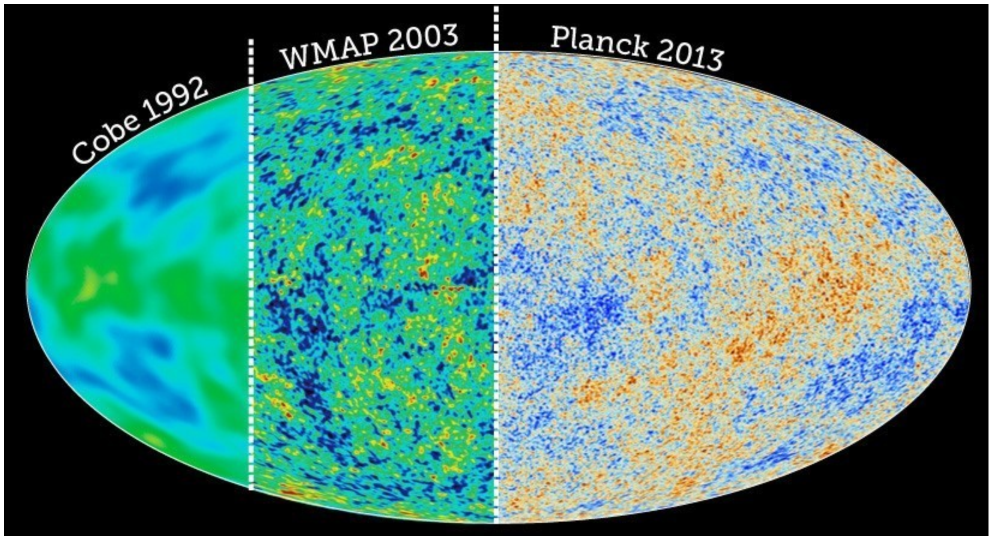

Figure 1.1: Comparison between the CMB picture obtained from COBE [24], WMAP [41] and PLANCK [43]. Credits for the picture: Brian Koberlein

The discovery from COBE also motivate different experiments which improved the resolution of the temperature fluctuations. This imprint of the recombination epoch of our universe is known as the *Cosmic Microwave Background* (CMB).

The principal feature of the CMB is that it is isotropic and homogeneous, just like our universe at large scales. The color temperature today of the decoupled photons (figure 1.1) has cooled down to $T = 2.7260K$ with fluctuations around this central value of magnitude $\Delta T = 0.0013K$. The measurement of these temperature fluctuations represents one of the main observations in cosmology and, in order to make a good description of the sky, we need to introduce an angular description of this map. The small anisotropies of the universe are well described using a spherical harmonics mapping $Y_{lm}(\theta, \phi)$ as follows,

$$\frac{\Delta T}{T} = \sum_{\ell=1}^{\infty} \sum_{m=-\ell}^{\ell} a_{\ell m} Y_{\ell m}(\theta, \phi). \tag{1.28}$$

The observed temperature fluctuations of the CMB are well described by a Gaussian distribution, this turned out to be one of the triumphs of the topic that will be presented in the next chapter. For now we have that for Gaussian distributions, the complete map of the temperature fluctuations is given by correlations of temperature C^{TT} at different angular points **n** and **n'**,

$$C^{TT}(\theta) \equiv \left\langle \frac{\Delta T}{T}(\mathbf{n}) \frac{\Delta T}{T}(\mathbf{n'}) \right\rangle, \tag{1.29}$$

where $\theta = \cos^{-1}(\mathbf{n} \cdot \mathbf{n'})$. This correlation between the temperatures is related to the multipole moments a_{lm} as it follows,

$$\langle a_{\ell m} a^*_{\ell' m'} \rangle = C_\ell^{TT} \delta_{\ell\ell'} \delta_{mm'}, \tag{1.30}$$

where C_ℓ^{TT} is the *angular power spectrum* of temperature fluctuations. The following plot shows how the angular power spectrum for the CMB temperature fluctuations depends on

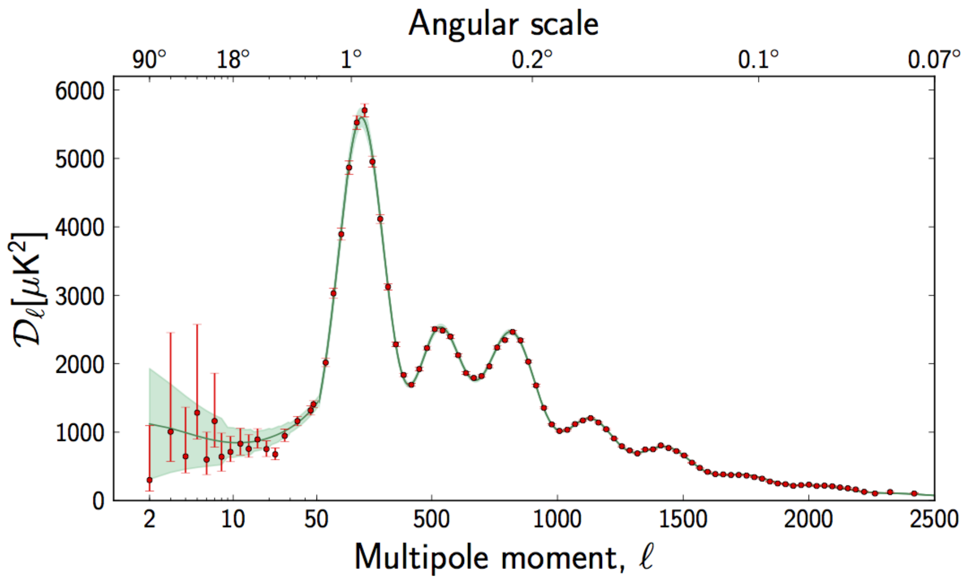

Figure 1.2: Plot of the angular power spectrum $D_\ell \equiv \ell(\ell+1)C_\ell^{TT}/(2\pi)$ in terms of the multipole moment ℓ and the angular scale. Plot obtained by Planck [34]

the scale k.

The angular power spectrum encodes the information of how the different scales of fluctuations are affected by the different contents of the universe. The great success of the ΛCDM model of cosmology is that it reproduces the shape of the power spectrum, giving a successful description of all the other cosmological phases of the universe.

1.5 "Old" problems in cosmology

Despite the fact that the ΛCDM model gives a good explanation to the evolution of the universe for different matter sources, there are still some limitations of the theory when we try to describe its origin. In this section, we will describe the first problems that motivated the construction of the theory of cosmic inflation.

1.5.1 Flatness problem

In the previous subsection in order to define the critical density of our universe we used the fact that our universe today is nearly flat ($k \approx 0$), but in fact, the standard ΛCDM predicts a different thing. We know that observations tell us, according to eq.(1.26), that the density related to the curvature of the universe today is given by:

$$|\Omega_{k,0}| = \frac{k}{(a_0 H_0)^2} \leq 0.01. \tag{1.31}$$

The problem starts when we decide to analyze the Friedmann equations of this curvature energy density Ω_k. First, let us define a new quantity called the *Comoving Hubble radius* χ_c which according to the Friedmann universe is given by,

$$\chi_c = \frac{1}{aH} = \frac{1}{H_0} a^{(3w+1)/2}, \tag{1.32}$$

where w is the equation of state for different matter sources. We can relate this comoving Hubble radius to the curvature density Ω_k via,

$$\Omega_k = k(\chi_c)^2, \tag{1.33}$$

which implies that for any energy source with an equation of state $w > -1/3$ the comoving radius grows. Now, if most of the energy content of universe meets the condition of a growing comoving Hubble radius, why then the curvature density Ω_k today is close to zero? This problem is known as the *flatness problem*.

1.5.2 Horizon problem

For this problem, we need to define two new quantities: The comoving distance and the particle horizon. We briefly define them as follows,

- **Comoving distance:** In principle, the definition of distances in an expanding universe in really "tricky" because particles take some time to travel from 1 to 2. Due to the fact that the scale factor is a function of time, it is not clear which dependence of time of the scale factor we should be using. One way to define distances is to consider that we have traveling photons which move in null geodesics ($ds^2 = 0$), this is known as the *comoving distance*,

$$\begin{aligned} \chi(\tau_1, \tau_2) &= \int_{\tau_1}^{\tau_2} d\tau \\ &= \frac{1}{(aH)_{\tau_2}} \frac{2}{3w+1} \left[\left(\frac{a}{a(\tau_2)} \right)^{(3w+1)/2} \right]_{a(\tau_2)}^{a(\tau_1)}. \end{aligned} \tag{1.34}$$

- **Particle horizon:** We can use the comoving distance to define the *particle horizon* χ_{ph}, which is the furthest distance between two objects that can be in causal correlation. From this, we only need to take the limit $\tau_1 \to \infty$ in the definition of comoving distance (1.34) which give us,

$$\chi_{ph}(\tau_\infty, \tau) = \begin{cases} \infty, & w < -1/3 \\ \dfrac{1}{aH} \dfrac{2}{3w+1}, & w > -1/3 \end{cases} \tag{1.35}$$

With these definitions, we can consider 2 points in the space at redshift z and compute how the comoving distance relates to the maximum causal connected distance of their respective light cones. If we select $w > -1/3$ (which describes most of the baryonic energy content), then the rate between the comoving distance and the particle horizon is given by,

$$\frac{\chi(z)}{\chi_{ph}(z)} = 2\sqrt{1+z}. \tag{1.36}$$

The problem appears when we take two points from the CMB at redshift $z_{CMB} \approx 1100$, this implies that the rate is $\chi(z_{CMB})/\chi_{ph}(z_{CMB}) \approx 66$. From this we can conclude that 2 points in the CMB are causally disconnected although their temperatures are almost the same, this contradiction is known as the *horizon problem*.

1.6 "New" problems of cosmology

In this subsection, we are going to describe some of the new problems of the ΛCDM model, which are mostly related to the behavior of perturbations in an FRW universe and they are the following.

1.6.1 Scale invariance

In order to describe this problem first, we need to specify what we mean by *scale-invariance*. For this, consider a correlation of fields as follows,

$$\langle R(\mathbf{x}_1)R(\mathbf{x}_2)...R(\mathbf{x}_n)\rangle, \tag{1.37}$$

where the brackets represent statistical correlations between the $R's$ at different points in space. This is the same as computing statistical moments of quantities using its probability distribution function. For a particular case when the quantities are treated quantum mechanically, this correlations could be computed using techniques such as the In-In formalism, which will be presented in the next chapter.
Scale-invariance tells us that we can perform a “scaling” in the coordinates $\mathbf{x}_i \rightarrow \lambda\mathbf{x}_i$ and that should leave our correlations invariant,

$$\langle R(\mathbf{x}_1)R(\mathbf{x}_2)...R(\mathbf{x}_n)\rangle = \langle R(\lambda\mathbf{x}_1)R(\lambda\mathbf{x}_2)...R(\lambda\mathbf{x}_n)\rangle. \tag{1.38}$$

Now, from the angular power spectrum plot (figure 1.2) we can notice that at very large scales, from $\ell = 0 \sim 50$, the power spectrum looks like a flat line, this implies that the C_ℓ^{TT} is “approximately” (and we stressed out the approximately because one of the main topics of this thesis, Chapter 3, involves the analysis of this approximation) given by:

$$C_\ell^{TT} \propto \frac{C}{\ell(\ell+1)}. \tag{1.39}$$

Using this we can roughly compute the correlator. If this correlation doesn't depend on the scaling, then we can say that it is scale-invariant. Let us compute the following, using the flat sky approximation,

$$\left\langle \frac{\Delta T}{T}(\mathbf{n})\frac{\Delta T}{T}(\mathbf{n'})\right\rangle = \int_{d\ell}\int_{d\ell'}\left\langle \frac{\Delta T}{T}(\hat{\ell})\frac{\Delta T}{T}(\hat{\ell}')\right\rangle e^{i\hat{\ell}\cdot\mathbf{n}}e^{i\hat{\ell}'\cdot\mathbf{n'}} \tag{1.40}$$

$$= \int_{d\ell}\int_{d\ell'}\delta(\hat{\ell}+\hat{\ell}')C_{\ell}^{TT}e^{i\hat{\ell}\cdot\mathbf{n}}e^{i\hat{\ell}'\cdot\mathbf{n'}} \tag{1.41}$$

where the delta function appears due to the homogeneity and isotropy of our universe, using (1.39),

$$\left\langle \frac{\Delta T}{T}(\mathbf{n})\frac{\Delta T}{T}(\mathbf{n'})\right\rangle = \int_{d\ell}\frac{C}{\ell(\ell+1)}e^{i\hat{\ell}\cdot(\mathbf{n}-\mathbf{n'})} \tag{1.42}$$

$$\approx \text{Constant}, \tag{1.43}$$

this implies that our universe is scale-invariant at large scales. Since the ΛCDM model doesn't explain why the CMB power spectrum is scale invariant at primordial scales we consider this a problem in the model.

1.6.2 Coherent Hubble perturbations

This particular idea is known not as a problem but a feature that the primordial theory of the universe should have in order to reproduce the primordial angular power spectrum. It was pointed out first by Scott Dodelson in [27]. In that work, he briefly describes that the characteristic peaks in Figure (1.2) are clear, and not just a flat line, because of all the Fourier modes coming from the primordial correlations are coordinating their phases when they re-enter the Hubble radius. How can the perturbations match their phases? This is another of the new problems of the ΛCDM model and we need to present a natural mechanism to create coherent Hubble perturbations.

Chapter 2

Inflation

In this chapter, we present the paradigm shift that allowed to solve the previous problems of the ΛCDM model that appeared when we asked ourselves the primordial questions of our universe. We introduce a new epoch in the early universe known as *cosmic inflation*. This theory started around 1973 by Alan Guth [12] as a solution of the two "old" problems presented before: The horizon and flatness problem. As years went by, people discovered that this inflationary universe not only solve the old problems, but its fluctuations gave a natural origin to the primordial seeds of our universe when we treated them quantum mechanically.

This chapter is presented as it follows. In Section 2.1, we describe the geometry of this epoch of the universe given by a primordial de-Sitter space-time. In Section 2.2, we present the topic of Single field slow-roll inflation as a solution to the old problems, also, we introduce the slow-roll parameters which keep in control inflation. In Section 2.3, we analyze how the primordial perturbations coming from the inflaton field behave during inflation. In Section 2.4, we describe how the statistics of the primordial fluctuations can reproduce the ones in the angular power spectrum described before. In Section 2.5, we present the quantization procedure for the fluctuations and introduce the mechanism to compute primordial correlation functions. In Section 2.6, we connect the correlation of quantum fluctuations to the ones in the angular power spectrum. In the last section 2.7, we briefly describe how inflation can be obtained considering more degrees of freedom in a topic called multi-field inflation.

2.1 de-Sitter space

A natural description of the primordial universe appeared when the de-Sitter geometry, worked around the 1920's by Willem de Sitter and Albert Einstein, was used as an exponential expanding solution to the Einstein equations. In this section, we are going to describe some important geometrical properties of de-Sitter geometry and discuss how these properties are naturally connected with the ones required for inflation.

The de-Sitter space is known as a maximally symmetric space, where the line element is given by, using the conformal time:

$$ds^2 = \frac{-d\tau^2 + d\mathbf{x}^2}{(H\tau)^2}. \tag{2.1}$$

In the previous expression, the scale factor is equal to,

$$a(t) = e^{Ht}; \quad H = \text{constant}. \tag{2.2}$$

Because a de-Sitter space-time behaves as a maximally symmetric space, it comes with 10 isometries described as follows:

- **3 Spatial translations:** These translations isometries give an explanation to the homogeneity of the CMB. Since all the fluctuations started from a de-Sitter universe, as we will show in this chapter, every perturbation has to respect the same symmetries as they evolve during inflation leaving us with a homogeneous imprint in the CMB.
- **3 Spatial rotations:** Rotation isometries can be used to describe the isotropy of the CMB since rotational invariance tells us that the temperature fluctuations in the CMB do not depend on the observational directions.
- **Dilation:** The dilation isometries are given by,

$$\tau \to \tau(1+\lambda), \quad \mathbf{x} \to \mathbf{x}(1+\lambda), \tag{2.3}$$

  which together with the previous isometries allow us to construct a *scale invariant* power spectrum.
- **dS-boost:** Just like the boost part of the Lorentz transformations, the de-Sitter space has its own boosts described as follows, in its infinitesimal form:

$$\tau \to \tau(1 - 2\mathbf{b}\cdot\mathbf{x}), \quad \mathbf{x} \to \mathbf{x} - 2(\mathbf{b}\cdot\mathbf{x})\mathbf{x} + (\mathbf{x}^2 - \tau^2)\mathbf{b}. \tag{2.4}$$

A detailed discussion of the previous symmetries in the context of inflation is given in [28].

This collection of symmetries describe most of the physical characteristics that we need for the primordial temperature fluctuations. Now, let us briefly describe how the "old" problems can be solved using a de-Sitter expanding universe.

- **Flatness and Horizon problems:** According to eq.(1.32) the solution to the flatness problem can be obtained in a period of a shrinking comoving Hubble radius which implies an equation of state with $w < -1/3$. For a de-Sitter universe, we have that the ratio of change of X_c is given by

$$\frac{d}{dt}\left(\frac{1}{a(t)H}\right) = -e^{-Ht} < 0; \quad H = \text{const} \tag{2.5}$$

  this implies a shrinking comoving Hubble radius and, indeed, a equation of state $w < -1/3$. Moreover, we can notice that, according to eq.(1.35), the previous equation of state also solves the horizon problem.

2.2 Single field slow-roll inflation

In the previous section, we described how a de-Sitter primordial expanding universe solves some problems of the ΛCDM model cosmology. Since after this exponential expansion we need to continue with the evolution of our universe, inflation has to end. The question now is how can we stop this evolution? For this we have one preferred choice, what about if we consider a cosmological constant Λ, where we know that according to the Einstein equations that lead us to an exponential expansion, but in this case, we put some "clock" ϕ in such a way that $\Lambda \to \Lambda(\phi)$. This clock will be used to turn off Λ and stop the exponential growth of the universe. We will call this clock the *inflaton field* which can have different origins depending on the primordial theory. There are two different approaches to this:

- The field ϕ could appear breaking the time diffeomorphisms for a de-Sitter symmtery. The symmetry breaking will induce Goldstone bosons which will be related to the perturbations of ϕ, being this perturbations the ones that will be treated quantum mechanically. This topic is covered by the Effective Field Theory of Inflation [64].
- The other option, the canonical one, is to consider the field ϕ with a particular almost-flat potential $V(\phi)$. The modifications from this could include changes in the kinetic terms and more exotic potentials, worth to mention that all of these different considerations are included in the Effective field theory approach.

During this introduction to the topic, we are going to focus on this last option and as we advance in this thesis we will make some comments on the Effective field theory of inflation point of view.

The most simple approach to the inflationary theory is given by the model known as *Single field slow-roll inflation* and is described by the following action:

$$S = -\frac{1}{2}\int d^4x\sqrt{-g}\left[M_{pl}^2 R + (\partial_\mu\phi\partial^\mu\phi) + 2V(\phi)\right], \tag{2.6}$$

this is the Einstein-Hilbert action described before (1.7) plus the action of a scalar field, the inflaton, which is minimally coupled to gravity. During the first chapter, we mentioned that the information about the energy content of some epoch in the universe was encoded by its energy-momentum tensor $T_{\mu\nu}$. In the inflationary era is straightforward to check that the energy-momentum tensor for a scalar field theory is given by:

$$T_{\mu\nu} = -\frac{2}{\sqrt{-g}}\frac{\delta S}{\delta g^{\mu\nu}} = \partial_\mu\phi\partial_\nu\phi - g_{\mu\nu}\left[\frac{1}{2}\partial_\mu\phi\partial^\mu\phi + V(\phi)\right]. \tag{2.7}$$

Also, the equation of motion for the inflaton can be obtained directly using the principle of least action for eq.(2.28),

$$\frac{1}{\sqrt{g}}\partial_\mu\left[\sqrt{g}g^{\mu\nu}\partial_\nu\phi\right] + \frac{\partial}{\partial\phi}V(\phi) = 0. \tag{2.8}$$

One of the requirements of the theory of inflation is that the inflaton field $\phi(\mathbf{x}, t)$ could be decomposed in a homogeneous part, which only depends on time, and perturbations around that background as follows,

$$\phi(t, \mathbf{x}) = \phi(t) + \delta\phi(t, \mathbf{x}), \tag{2.9}$$

where $\varphi(t, \mathbf{x})$ is a perturbation from the homogeneous background. For now, let us discard the perturbations and focus only on the homogeneous background solution $\phi \to \phi(t)$. Using the equation of motion (2.8) for the background inflaton, we get the following:

$$\ddot{\phi}(t) + 3H\dot{\phi}(t) + \frac{\partial}{\partial\phi}V(\phi) = 0. \tag{2.10}$$

Furthermore, we can relate the energy content given by the scalar field (2.7) with the energy-momentum tensor of a perfect fluid (1.15), this gives us the following relations between a perfect fluid and the scalar field's kinetic and potential energies,

$$\rho = \frac{1}{2}\dot{\phi}^2 + V(\phi), \tag{2.11}$$

$$P = \frac{1}{2}\dot{\phi}^2 - V(\phi). \tag{2.12}$$

Putting all these relations back together in the first Friedmann equation (1.18), we obtain:

$$3H^2 M_{pl}^2 = \frac{1}{2}\dot{\phi}^2 + V(\phi). \tag{2.13}$$

This is one of the master equations which relates the evolution of the background inflaton field to the expansion of the universe.

Now, let us think for a moment how can we stop this period of exponential expansion. Previously we have described that this expansion can be ruled by the geometry of a de-Sitter space-time, where its main feature is that the Hubble parameter H of the exponential expansion is constant. We have presented the inflaton field as a clock that stops the exponential evolution, in the following discussion we are going to describe how to connect this clock to the exponential evolution and how to stop it.

The first ingredient is to analyze the nature of this constant parameter H in the expanding universe. The fact that Inflation did end tell us that the parameter H is actually a function of time $H \to H(N)$ but with small variations from a constant value $H_0 = H(N_0)$, where N is a new type of time measure known as the *number of e-folds* defined as:

$$dN = d(\ln a) = H(t)dt. \tag{2.14}$$

Since we allow the Hubble parameter $H(N)$ to be slowly varying around a constant H_0 we can perform a taylor expansion around that value,

$$H(N) = H(N)\Big|_{N=N_0} + \frac{dH(N)}{dN}\Big|_{N=N_0}(N - N_0) + ... \tag{2.15}$$

$$= H_0\left(1 - \varepsilon(N)\Big|_{N=N_0}(N - N_0) + ...\right), \tag{2.16}$$

where we have defined $\varepsilon(t) \equiv -d\ln H/dN = -\dot{H}/H^2$, this function is known as the *First slow-roll parameter* and describes the deviations of $H(t)$ from a constant parameter H_0. This leads us to a fundamental requirement for the inflationary universe, ε should remain small in order to have a de-Sitter (or in this particular case a *Quasi de-Sitter*) expanding universe, and also, it should grow in order to stop Inflation.

Now that we've defined the first parameter that controls inflation, we have another requirement. The function ε, besides the fact that it should remain small, $\varepsilon \ll 1$, it also needs to be almost constant during all the period of Inflation to avoid sudden grows in its value. In order to achieve this, we can use the same argument for the Hubble parameter. Let us say that the first slow roll parameter is almost constant with small deviations from a constant value $\varepsilon_0 = \varepsilon(N_0)$ as follows,

$$\varepsilon(N) = \varepsilon(N)\Big|_{N=N_0} + \frac{d\varepsilon(N)}{dN}\Big|_{N=N_0}(N-N_0) + ... \tag{2.17}$$

$$= \varepsilon_0\left(1 - \eta(N)\Big|_{N=N_0}(N-N_0) + ...\right), \tag{2.18}$$

where $\eta(t) \equiv d\ln\varepsilon/dN = \dot{\varepsilon}/(H\varepsilon)$ is known as the *second slow-roll parameter* and parametrizes deviations for a constant (and small) ε_0. An important remark is that using the same procedure we can construct recursively every order of slow roll parameters, but since the two of them are already small we don't need to consider the other ones.

To summarize, we have related the deviations from perfect de-Sitter universe with two new functions that should remain small in order to describe Inflation, this the first ingredient of the inflationary theory and it will be one of the key arguments in the calculation from Chapter 3, where the relationship between these parameters allow us to connect scalar perturbations with tensor perturbations. Finally, the slow roll conditions are given by:

$$\varepsilon(t) \equiv -\frac{\dot{H}}{H^2} \ll 1; \quad \eta(t) \equiv \frac{\dot{\varepsilon}}{H\varepsilon} \ll 1. \tag{2.19}$$

The second ingredient is to relate the slow-roll parameters to our "ϕ clock" that stops inflation. For this we use the Friedmann equations (1.18-1.19) and the previous expressions (2.19). We can obtain the following relation between the background scalar field and the first slow-roll parameter,

$$\varepsilon(t) = \frac{1}{2}\frac{\dot{\phi}(t)^2}{M_{pl}^2 H^2}. \tag{2.20}$$

In the same way for the second slow-roll parameter we have,

$$\eta = \frac{\dot{\varepsilon}}{H\varepsilon} = 2\frac{\ddot{\phi}}{H\dot{\phi}} - 2\frac{\dot{H}}{H^2}. \tag{2.21}$$

Now, if we use the slow-roll conditions for the previous expressions we can simplify the master equations for the scalar field, for the Friedmann equation (1.18) we obtain:

$$H^2 \approx \frac{V}{3M_{pl}^2}. \tag{2.22}$$

Using the same procedure now for the continuity equation (1.17),

$$\dot{\phi} \approx -\frac{1}{3H}\frac{\partial V}{\partial \phi}.. \tag{2.23}$$

There are 2 remarkable things about the previous equations:

- The continuity equation (1.17) which started as a second differential equation ended up as a first order differential equation (2.23). This approximation allows us to lose the memory of one of the initial conditions and in principle could lead us to attractor solutions in the phase space for the field.
- The left-hand side knows about the dynamics of our universe while the right-hand side is purely field content given by the potential of the field. This implies that the amount of inflation only depends on the scalar potential $V(\phi)$.

Now we have all the tools needed to describe the inflationary era of our universe. We have defined the deviations from de-Sitter and we connected them with our "clock ϕ" in order to stop inflation. The next step is to analyze the end of inflation and the amount of time that we need in order to have solutions to the problems presented in Chapter 1.

Let us recall the rate of change of our comoving Hubble radius for a de-Sitter expansion,

$$\frac{d}{dt}\left(\frac{1}{aH}\right) = -\frac{\dot{a}H + a\dot{H}}{(aH)^2} = -\frac{1}{a}(1-\varepsilon) < 0. \tag{2.24}$$

This implies that, if we take $\varepsilon \geq 1$, then the condition of de-Sitter expanding universe breaks and Inflation stops. The duration of Inflation can be obtained through the counting of e-folds where, in an accelerated expansion, is given by the following expression:

$$N = \int d\ln a = \int_{t_i}^{t_e} H(t)dt, \tag{2.25}$$

where t_e is defined via $\varepsilon(t_e) = 1$, the end of inflation. Now, using the slow-roll conditions we can rewrite the number of e-folds as a function that depends on the dynamics of the inflaton field,

$$N = \int_{t_i}^{t_e} H(t)dt = \int_{\phi_i}^{\phi_e} \frac{H}{\dot{\phi}}d\phi \approx \int V\left(\frac{\partial V}{\partial \phi}\right)^{-1} d\phi \tag{2.26}$$

The number of e-folds needed to reproduce the seeds in the CMB is $N_{cmb} \gtrapprox 60$, this implies that any potential that could achieve this number of e-folds is a good candidate to expand our primordial universe.

2.3 Perturbation theory in Inflation

In the previous section, we described the foundations of the inflationary theory and we mentioned that the inflaton field had a background part which was only time dependent. In this section, we are going to discuss the behavior of the small fluctuations of the inflaton field $\delta\phi(\mathbf{x},t)$ from the following expression,

$$\phi(\mathbf{x},t) = \phi_0(t) + \delta\phi(\mathbf{x},t), \quad \delta\phi(\mathbf{x},t) \ll \phi(t). \tag{2.27}$$

Let us recall that the action of the inflationary era was previously discussed and it was given by:

$$S = -\frac{1}{2}\int d^4x\sqrt{-g}\left[M_{pl}^2 R + (\partial_\mu\phi\partial^\mu\phi) + 2V(\phi)\right], \tag{2.28}$$

where we have a coupling between the inflaton field $\phi(\mathbf{x},t)$ and the determinant of the metric of an expanding universe $\propto g = \det(g_{\mu\nu})$. This coupling makes the equation of motion for the field $\phi(\mathbf{x},t)$ highly nonlinear and its way to be solved is using perturbation theory around the background field $\phi_0(t)$, and around the background FRW metric.

Considering the following small perturbation $h_{\mu\nu}$ of the FRW metric $\bar{g}_{\mu\nu}$,

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}, \tag{2.29}$$

where the perturbations of the metric are coupled to the perturbations of the inflaton field $\delta\phi(\mathbf{x},t)$ via the Einstein equations. First, we are going to focus on a flat Friedmann-Robertson-Walker background, using the *conformal time*,

$$ds^2 = a^2(\tau)\left(d\tau^2 - \delta_{ij}dx^i dx^j\right). \tag{2.30}$$

A perturbation of the previous metric can be written as follows,

$$ds^2 = a^2(\tau)\left((1+2A)d\tau^2 - 2B_i dt dx^i - (\delta_{ij}+h_{ij})dx^i dx^j\right), \tag{2.31}$$

where, in order to respect symmetries of the background metric, the perturbation $h_{\mu\nu}$ must have some particular features. Isotropy and homogeneity, the key elements of the background, tells us that the perturbation of the metric can be decomposed into scalars, vectors, and tensors, which are decoupled at linear order during their evolution. For vectors is familiar that, due to *Helmholtz theorem*, we can split a vector B_i into the gradient of a scalar B and a divergenceless vector $\hat{B}_i$,

$$B_i = \partial_i B + \hat{B}_i. \tag{2.32}$$

In a similar way, there is a generalization of the Helmholtz theorem for any rank symmetric tensor. In our case we can decompose the rank-2 symmetric tensor h_{ij} into a scalar, vector, and tensor components as it follows:

$$h_{ij} = 2C\delta_{ij} + 2\partial_{\langle i}\partial_{j\rangle}\gamma + 2\partial_{(i}\gamma_{j)} + 2\gamma_{ij}, \tag{2.33}$$

where,

$$\partial_{\langle i}\partial_{j\rangle}\gamma \equiv \left(\partial_i\partial_j - \frac{1}{3}\delta_{ij}\nabla^2\right)\gamma, \tag{2.34}$$

$$\partial_{(i}\gamma_{j)} \equiv \frac{1}{2}\left(\partial_i\gamma_j + \partial_j\gamma_i\right). \tag{2.35}$$

Just as before, the decomposition put some features in the components of the original tensor. The vector and tensor part of the decomposition are now divergenceless and traceless respectively, i.e. $\partial^i \hat{\gamma}_i = 0$, $\partial^i \gamma_{ij} = 0$ and $\gamma^i{}_i = 0$. This implies that the perturbed quantities around inflation have three main features:

- Scalar degrees of freedom could appear as perturbations of the metric and/or perturbations of the inflaton field, this implies some kind of relationship between both of the scalar degrees of freedom.
- The perturbation of the metric involves the appearance of tensor degrees of freedom. These tensor perturbations are related to the production of primordial *gravitational waves* during inflation and they are essential for the discussion in Chapter 3.
- Vector degrees of freedom decay rapidly as the universe expands. One way to have significant amplitudes of these vector modes at the present is that they had very large initial amplitudes. This spoils the initial isotropy of the universe and because of this, we are going to neglect the vector perturbations of the metric in our description.

On the other hand, an important result appears when we consider a homogeneous FRW space-time with the following change of spatial coordinates,

$$x^i \to x'^i = x^i + \xi^i(\tau, \mathbf{x}), \tag{2.36}$$

if we assume that the change $\xi^i(\tau, \mathbf{x})$ is small, then the change in the coordinates looks just like a perturbation. This implies that

$$dx^i = dx'^i - \frac{\partial \xi^i}{\partial \tau} d\tau - \frac{\partial \xi^i}{\partial x^k} dx'^k, \tag{2.37}$$

which lead us to the following form of the metric,

$$ds^2 = a^2(\tau) \left[d\tau^2 - 2\frac{\partial \xi_i}{\partial \tau} dx'^i d\tau - (\delta_{ij} + 2\partial_{(i}\xi_{j)}) dx'^i dx'^j \right]. \tag{2.38}$$

The previous expression looks like we have induced metric perturbations of the form $B_i = \frac{\partial \xi_i}{\partial \tau}$ and $\hat{\gamma}_i = \xi_i$ in the equation (2.31), but in fact, we have just induced fictional *gauge* modes which can be eliminated going back to the previous coordinates. The question now is, how can we be sure that any perturbation of our metric isn't just a gauge mode that could be determined for a particular choice of coordinates? The solution to this is to fix the gauge liberty of the metric. Previously, we have discussed that the scalar degrees of freedom for the perturbations could come from two different places: direct perturbations of the scalar field, or from the perturbed scalar part of the metric. This leaves us with two particularly special gauge choices:

- **Spatially-flat gauge:** This gauge choice allow us to select the scalar degrees of freedom in our theory to come only from the perturbation of the field $\delta\phi$. This choice is the same to set $C = \gamma = 0$ in the equation (2.31)

- **Comoving gauge:** This is the contrary case of the previous gauge choice. In here, we chose $\delta\phi = 0$ which implies that the primordial seeds came from the scalar perturbations of the metric. This particular choice will be helpful and will lead us to remarkable results in Chapter 3.

With the previous arguments in mind, we are going to focus now on a particular kind of formalism which doesn't involve the vector degrees of freedom at the linear order and allows us to analyze the gauge effect more clearly, the *ADM formalism*. The *Arnowitt-Deser-Misner formalism* is a useful way to split the metric in $(3+1)$ dimensions, with this, we can write the unperturbed metric in the following way:

$$ds^2 = -N^2 dt^2 + g_{ij}(N^i dt + dx^i)(N^j dt + dx^j), \tag{2.39}$$

where N and N^i are known as the lapse and shift functions respectively, and g_{ij} is the 3-dimensional induced metric. The most important result of using this metric is that the Einstein-Hilbert action (1.7), without considering matter sources, is written as follows:

$$S_{E-H}[N, N^i, g_{ij}] = \frac{1}{2}\int d^4x N\sqrt{-g^{(3)}}(R^{(3)} - K^2 + K^{ij}K_{ij}), \tag{2.40}$$

where,

$$K_{ij} = \frac{1}{2N}[-\dot{g}_{ij} + N_{i;j} + N_{j;i}], \tag{2.41}$$

and

$$K = g^{ij}K_{ij}. \tag{2.42}$$

The great advantage of this form of the metric is that it allows us to work with the dynamics of the spatial part of the metric g_{ij} directly and it gives a clearer way to see the scalar spatial perturbations.

At this point, we should have noticed that we have two different sources for scalar degrees of freedom: the perturbations from the inflaton and the scalar perturbation of the metric. Now, we can introduce a gauge-invariant scalar variable called *comoving curvature perturbation* which relates the scalar part of the metric, described by C, and the perturbation of the inflaton $\delta\phi$ in the following way:

$$\mathcal{R} \equiv C + \frac{H}{\dot{\phi}_0}\delta\phi. \tag{2.43}$$

From the previous equation is clear that depending on our gauge choice, the comoving curvature perturbations can be described by either the inflaton fluctuations or the scalar perturbed part of the metric.

2.4 Primordial statistics of the early universe

In Chapter 1, we showed that the temperature fluctuations in the CMB can be described statistically using correlation functions between different points in space. This implies that, if inflation is the mechanism that created these fluctuations, then there should be a statistical way to describe this seeds, this is what we will call *primordial statistics*. The starting point for the statistical analysis is to work with a *probability distribution function* (PDF) which encodes all the statistical information of a random variable, which in our case are the curvature perturbations. The probability distribution function for the curvature perturbations $\rho(\mathcal{R})$, in a particular long-modes limit, is related to a statistical n-point correlation as it follows:

$$\langle \mathcal{R}_{\rm L}^n \rangle = \int d\mathcal{R}\ \mathcal{R}^n \rho(\mathcal{R}), \tag{2.44}$$

where the "L" subcript denotes that we are dealing with modes with long-wavelength, this is translated in a condition for the momenta in the correlation function, but more on this in Chapter 4.

Since we are dealing with close to Gaussian statistics, we can consider a perturbative description of this PDF instead of the full n-point correlation. The most important contribution is given by the second order expansion of the PDF called the *power spectrum* of $\mathcal{R}$, $P_{\mathcal{R}}$, defined as follows:

$$\langle \mathcal{R}_{\mathbf{k}} \mathcal{R}_{\mathbf{k}'} \rangle = (2\pi)^3 \delta(\mathbf{k} + \mathbf{k}') P_{\mathcal{R}}(k), \tag{2.45}$$

where the brackets represent the statistical two-points correlation, or average, between the comoving curvature perturbation $\mathcal{R}$ at different scales. Starting from this, we can define the dimensionless power spectrum by,

$$\Delta_s^2 \equiv \Delta_{\mathcal{R}}^2 = \frac{k^3}{2\pi^2} P_{\mathcal{R}}(k), \tag{2.46}$$

where we can see that the primordial power spectrum is a scale-dependent quantity. The deviations from scale-invariance are described by the *scalar spectral index*,

$$n_s - 1 \equiv \frac{d \ln \Delta_s^2}{d \ln k}, \tag{2.47}$$

where full scale-invariance corresponds to the value $n_s = 1$. From the previous expression, we can also define the running of the spectral index by

$$\alpha_s \equiv \frac{dn_s}{d \ln k}. \tag{2.48}$$

As we saw, the power spectrum is related to the 2-point correlation (or the second statistical moment) and, if the comoving curvature perturbations $\mathcal{R}$ are Gaussian, then the power spectrum gives us all the statistical information of the field. This argument is the starting point to talk about *primordial non-Gaussianity*. The primordial perturbations are well described by Gaussian distributions but with small room for signals of non-Gaussianity due to non-linear interactions. Signals of non-Gaussianity can be detected by computing higher-order correlation functions, and if they are different from zero, then we can say the primordial comoving perturbation could be non-Gaussian. This kind of effects arises in both single field and multi-field inflation. We will focus in how they appear in the context of multi-field inflation, a topic that will be briefly described in the following sections.

How can we connect these curvature perturbations to temperature fluctuations? As we saw in Chapter 1, the correlation between temperature fluctuations is described by the C_ℓ^{TT}. If these fluctuations are ruled by inflation, then C_ℓ^{TT} is defined by the following equation:

$$C_\ell^{TT} = \frac{2}{\pi}\int k^2 dk P_{\mathcal{R}}(k)\Delta_{T\ell}(k)\Delta_{T\ell}(k), \tag{2.49}$$

where $\Delta_{T\ell}(k)$ is known as the transfer function which, in general, has to be computed numerically using Boltzmann-codes such as CLASS [29] that describes the evolution of the curvature perturbations depending on the background cosmology for different matter sources.

The same statistical description used for the comoving curvature perturbations can be done for the tensor perturbations of the metric γ_{ij}, which are connected to the creation of primordial gravitational waves. The power spectrum for the polarization modes of γ_{ij}, *i.e* $h \equiv h^+, h^\times$ defined as,

$$\langle h_{\mathbf{k}} h_{\mathbf{k}'}\rangle = (2\pi)^3\delta(\mathbf{k}+\mathbf{k}')P_h(k), \tag{2.50}$$

can be used to compute the dimensionless power spectrum for the polarization modes:

$$\Delta_h^2 = \frac{k^3}{2\pi^2}P_h(k). \tag{2.51}$$

We define the tensor power spectrum as the sum of the power spectra for the two polarizations,

$$\Delta_t^2 \equiv 2\Delta_h^2, \tag{2.52}$$

and, in the same way as the one for scalars, its scale-dependence can be described by the tensor spectral index:

$$n_t \equiv \frac{d\ln\Delta_t^2}{d\ln k}. \tag{2.53}$$

Now that we have an idea of the objects that we need to compute, we are going to describe in the following section the formalism to compute the correlation functions for curvature perturbations and for primordial gravitational waves.

2.5 Quantum fluctuations in de-Sitter space

In the previous chapter, we discussed how the early universe perturbations can be related to the temperature fluctuations in the CMB. Now, we are going to describe the remarkable result that appears when we treat these perturbations, produced by either the inflaton or the scalar part of the metric, as quantum fields that can create the primordial seeds of our universe.

2.5.1 Free Field action for scalars

First, let us consider the comoving gauge described in the previous subsection. This gauge choice allows us to set $\delta\phi = 0$ and relate directly the comoving curvature perturbation to the spatial part of the perturbed metric:

$$\delta\phi = 0; \quad g_{ij} = a^2[(1-2\mathcal{R})\delta_{ij} + h_{ij}], \quad \partial_i h_{ij} = h^i_i = 0. \tag{2.54}$$

In this gauge we can use the ADM formalism, presented in the Chapter 1, for this perturbed metric. The Einstein-Hilbert action in this formalism including the inflaton field ϕ is presented as follows:

$$S_{\text{infl}} = \frac{1}{2}\int d^4x N\sqrt{g^{(3)}}(R^{(3)} - K^2 + K^{ij}K_{ij}) + \frac{1}{2}\int d^4x\sqrt{g^{(3)}}\left[N^{-1}(\dot{\phi} - N^i\partial_i\phi)^2 - Ng^{ij}\partial_i\phi\partial_j\phi - 2V\right]. \tag{2.55}$$

Starting from this action we can analyze the dynamics of both lapse and shift functions, N and N^i,

$$\begin{aligned} \nabla_i\left[K^i_j - \delta^i_j K\right] &= 0 \qquad (2.56)\\ R^{(3)} - 2V - (K_{ij}K^{ij} - K^2) - N^{-2}\dot{\phi}^2 &= 0. \qquad (2.57) \end{aligned}$$

The previous functions are non-dynamical quantities, this is because if we substitute the first order solutions for N and N_i back in to the previous action they will not contribute to the equation of motion. Also, we can expand the previous action up to second-order in $\mathcal{R}$ to get the following simplified expression for the dynamics of the comoving curvature perturbation,

$$S^{(2)}_{\text{infl}} = \frac{1}{2}\int d^4x a^3\frac{\dot{\phi}^2}{H^2}\left[\dot{\mathcal{R}}^2 - a^{-2}(\partial_i\mathcal{R})^2\right]. \tag{2.58}$$

In order to simplify even more, let us define the *canonically normalized variable* or *Mukhanov variable*,

$$v \equiv z\mathcal{R}; \quad z^2 \equiv a^2\frac{\dot{\phi}^2}{H^2} = 2a^2\epsilon, \tag{2.59}$$

and start using conformal time τ. This leads us to the action for a canonically normalized scalar field:

$$S^{(2)}_{\text{infl}} = \frac{1}{2}\int d\tau d^3x\left[(v')^2 + (\partial_i v)^2 + \frac{z''}{z}v^2\right], \tag{2.60}$$

where the prime denotes derivatives respect to the conformal time $()' = \frac{d}{d\tau}$. From the previous action is straightforward to see that the equation of motion for the v-field is given by:

$$v'' + \partial^2 v - \frac{z''}{z}v = 0. \tag{2.61}$$

The solution of the previous differential equation is clearer if we treat the field in Fourier space as it follows,

$$v(\tau,\mathbf{x}) = \int\frac{d^3k}{(2\pi)^3}v_{\mathbf{k}}(\tau)e^{i\mathbf{k}\cdot\mathbf{x}}, \tag{2.62}$$

this leads us to the equation of motion for a free field,

$$v_k'' + \left(k^2 - \frac{z''}{z}\right) v_k = 0. \tag{2.63}$$

This equation is known as the *Mukhanov Equation* and, in general, is difficult to solve because of its explicit dependence on the background dynamics via z. Luckily for us, we can give a solution for the Mukhanov equation in a particular limit, the de-Sitter universe. In this limit, we can take $\epsilon \to 0$, or as we saw in the previous section, take $H =$ constant. This implies the following approximation for z''/z,

$$\frac{z''}{z} \approx \frac{a''}{a} = \frac{2}{\tau^2}, \tag{2.64}$$

and therefore the equation that we need to solve now is:

$$v_k'' + \left(k^2 - \frac{2}{\tau^2}\right) v_k = 0. \tag{2.65}$$

It can be verified that the exact solutions for this equation of motion are given by:

$$v_k = \alpha \frac{e^{-ik\tau}}{\sqrt{2k}} \left(1 - \frac{i}{k\tau}\right) + \beta \frac{e^{ik\tau}}{\sqrt{2k}} \left(1 + \frac{i}{k\tau}\right). \tag{2.66}$$

In the next section, we are going to describe some extra condition that we need to impose on this solution in order to describe the fluctuations quantum mechanically.

2.5.2 Canonical quantization

The next step in order to produce the primordial seeds of our universe is to promote the v-field to a quantum field, this will allow us to use the techniques from quantum field theory to describe any possible kind of primordial interaction. First, let us expand the v-field in plane waves solution as follows,

$$v(\mathbf{x}, \tau) = \int \frac{d\mathbf{k}^3}{(2\pi)^3} \hat{v}_{\mathbf{k}} e^{i\mathbf{k}\cdot\mathbf{x}}, \tag{2.67}$$

where,

$$\hat{v}_{\mathbf{k}} = v_k(\tau)\hat{a}_{\mathbf{k}} + v_{-k}^*(\tau)\hat{a}_{-\mathbf{k}}^\dagger. \tag{2.68}$$

In order to give a correct quantum mechanical description of this field, we must choose a vacuum state for our fluctuations. For this, the standard choice is to take a comoving observer in the far past, that is, taking $\tau \to -\infty$ where all the scales were far inside the Hubble horizon $aH \ll k$, in this limit the Mukhanov equation becomes,

$$v_k'' + k^2 v_k = 0. \tag{2.69}$$

This is the harmonic oscillator equation for the v-field and we require that its vacuum be the state of minimum energy. Then the initial condition for the solution should be ruled by the following condition,

$$\lim_{\tau\to-\infty} v_k = \frac{e^{-ik\tau}}{\sqrt{2k}}. \tag{2.70}$$

This condition defines the ground states of the fluctuations as a *Bunch-Davies vacuum* where, after imposing this on (2.66), we can see that the modes solutions are given by:

$$v_k = \frac{e^{-ik\tau}}{\sqrt{2k}}\left(1 - \frac{i}{k\tau}\right), \tag{2.71}$$

this solutions are known as the *Bunch-Davies mode functions.*

The next step is to impose the canonical commutation relation between the creation and annihilation operators, $\hat{a}^\dagger_{\mathbf{k}}$ and $\hat{a}_{\mathbf{k}}$:

$$[\hat{a}_{\mathbf{k}}, \hat{a}^\dagger_{\mathbf{k}'}] = (2\pi)^3\delta(\mathbf{k}-\mathbf{k}'). \tag{2.72}$$

The fact that the inflationary perturbations can be expanded into Fourier modes, where all of them are in phase, tells us that the inflationary theory is the mechanism to create the coherent Hubble perturbations presented in Section 1.6.2. This is one of the striking aspects of the perturbations generated during inflation.

2.5.3 Free Field action for tensors

A similar computation to the previous one can be done for the tensor degrees of freedom that came from the perturbation of the metric. Let us start using the conditions given in (2.54) and the ADM formalism action (2.55). From them we can obtain the following action for the tensor perturbation γ_{ij},

$$S^{(2)}_{\rm GW} = \frac{M_{pl}^2}{8}\int d\tau dx^3 a^2\left[(\gamma'_{ij})^2 - (\partial_l\gamma_{ij})^2\right]. \tag{2.73}$$

In here we can define the following Fourier expansion for the primordial gravitational waves,

$$\gamma_{ij} = \int \frac{d^3k}{(2\pi)^3}\sum_{s=+,\times} e^s_{ij}(k)h^s_{\mathbf{k}}(\tau)e^{i\mathbf{k}\cdot\mathbf{x}}, \tag{2.74}$$

where we have separated the tensor degree of freedom into its polarization modes, $e_{ii} = k^i e_{ij} = 0$ and $e^s_{ij}(k)e^{s'}_{ij}(k) = 2\delta_{ss'}$.
We can also define a canonically normalized field for the polarization modes as follows,

$$f_s \equiv \frac{M_{pl}}{2}a(\tau)h^s. \tag{2.75}$$

This leads us to the following form of the action (2.73),

$$S^{(2)}_{\rm GW} = \sum_s \frac{1}{2}\int d\tau d^3k\left[(f'_s)^2 + \left(k^2 + \frac{a''}{a}\right)f_s^2\right]. \tag{2.76}$$

We may notice that the previous action is the same presented before for the comoving curvature perturbation but now for every polarization mode. This implies that the procedure of canonical quantization can be used in the same way as in the previous section. The similarities between these two cases will be important in the next chapter, where we are going to discuss a possible connection that could lead to important consequences for the primordial gravitational waves.

2.5.4 in-in formalism

When we promoted the comoving curvature perturbation to a quantum field we extended the treatments of regular quantum field theory to this new object; as we showed, second quantization and plane wave expansion were just a part of the usual elements. What about if we have now a non-free theory involving interaction between different matter sources or higher order self-interacting terms? Just as in the usual quantum field theory, we can use a new quantum picture to describe them. This is known as the interaction picture, and in it, the action of these fields can be separated in the following way,

$$S_{\rm infl}^{(2)} = S_0^{(2)} + S_{\rm int}, \tag{2.77}$$

where $S_0^{(2)}$ correspond to the free part of the action that leads to the free equation of motion (in our case (2.60)), and $S_{\rm int}$ represents the interaction part, which in principle could involve any kind of interaction.

In the interaction picture, the fields evolve via an *interaction propagator* defined as follows:

$$U(\tau) = \mathcal{T}\exp\left\{-i\int_{-\infty^+}^{\tau} d\tau' H_I(\tau')\right\}, \tag{2.78}$$

where H_I is the interaction picture Hamiltonian which is obtained from $S_{\rm int}$. In the previous expression, we also have $\mathcal{T}$ which is known as the time ordering symbol, and $\infty^+ = (1+i\epsilon)\infty$ is the prescription of isolating the in-vacuum in the infinite past. This formalism allows us to write the solution fields as,

$$v(\mathbf{x},\tau) = U^\dagger(\tau) v_I(\mathbf{x},\tau) U(\tau), \tag{2.79}$$

where v_I is the interaction picture field whose mode solutions are given by the free-field equation, in our case,

$$v_k^I = \frac{1}{\sqrt{2k}}\left(1 - \frac{i}{k\tau}\right) e^{-ik\tau}. \tag{2.80}$$

This formalism will be crucial for the treatment of extra degrees of freedom during inflation, which is the main topic of this thesis. The complete and more profound computations will be shown in the next chapters.

2.6 From Quantum fluctuations to Power Spectrum

In the previous sections, we showed that the primordial perturbations can be treated quantum mechanically in terms of a field theory. Now we will see that most of the previous computations can be used to relate these quantum fluctuations with the computation of one of the main observables of inflation, the power spectrum. Since the power spectrum is related to the statistical two-point correlation for the comoving curvature perturbations $\mathcal{R}$, we can connect this with the correlation between quantum fields.

The correlation is particularly straightforward in a free theory. Using the definition of the canonically normalized fields (2.59) directly in Fourier space we get the following:

$$\begin{aligned} \langle \mathcal{R}_{\mathbf{k}}(\tau)\mathcal{R}_{\mathbf{k}'}(\tau)\rangle &= \frac{1}{2a(\tau)^2\epsilon(\tau)} \langle 0| \hat{v}_{\mathbf{k}}(\tau)\hat{v}_{\mathbf{k}'}(\tau) |0\rangle & (2.81)\\ &= \frac{1}{2a(\tau)^2\epsilon(\tau)}(2\pi)^3\delta(\mathbf{k}+\mathbf{k}')|v_k(\tau)|^2. & (2.82)\end{aligned}$$

At this point, we can select a specific instant for the value of the power spectrum. If we consider that all the modes that we want to correlate need to be evaluated at the time of horizon crossing, then we can set $a(t^*)H(t^*) = k$. This ensures us that the perturbations can re-enter the horizon at different times depending on the scale and interact with different matter sources during its evolution. With this in mind, we see that the correlation is given by,

$$\langle \mathcal{R}_{\mathbf{k}}(\tau)\mathcal{R}_{\mathbf{k}'}(\tau)\rangle = (2\pi)^3\delta(\mathbf{k}+\mathbf{k}')\frac{H_*^2}{2k^3}\frac{H_*^2}{\dot{\phi}_*^2}, \tag{2.83}$$

where, in order to derive the previous expression, we consider that the scales behave as *super-horizon scales*, $|k\tau| \ll 1$. This implies that the dimensionless power spectrum for comoving curvature perturbations is given by

$$\langle \mathcal{R}_{\mathbf{k}}(\tau)\mathcal{R}_{\mathbf{k}'}(\tau)\rangle = (2\pi)^3\delta(\mathbf{k}+\mathbf{k}')P_{\mathcal{R}}(k), \qquad \Delta^2_{\mathcal{R}}(k) \equiv \frac{k^3}{2\pi^2}P_{\mathcal{R}}(k). \tag{2.84}$$

This implies that the dimensionless power spectrum in terms of the inflaton field is given by:

$$\Delta^2_{\mathcal{R}}(k) = \frac{H_*^2}{(2\pi)^2}\frac{H_*^2}{\dot{\phi}_*^2}. \tag{2.85}$$

The same computation can be done for the tensor modes using (2.75) and evaluating them at horizon crossing. The power spectrum for tensor modes is given by:

$$\begin{aligned} \langle h^s_{\mathbf{k}}(\tau)h^{s'}_{\mathbf{k}'}(\tau)\rangle &= \frac{4}{M_{pl}^2 a(\tau)^2} \langle 0| \hat{f}^s_{\mathbf{k}}(\tau)\hat{f}^{s'}_{\mathbf{k}'}(\tau) |0\rangle & (2.86)\\ &= (2\pi)^3\delta(\mathbf{k}+\mathbf{k}')\delta^{ss'}\frac{4}{M_{pl}^2}\frac{H_*^2}{2k^3}. & (2.87)\end{aligned}$$

This gives the following expression for the dimensionless power spectrum for tensor modes in a de-Sitter background,

$$\Delta_t^2 = 2\Delta_h^2(k) = \frac{2}{\pi^2}\frac{H_*^2}{M_{Pl}^2}. \tag{2.88}$$

One remarkable property of the tensor modes is that they don't interact with matter sources because they can not be connected with the matter densities via the energy-momentum tensor to linear order. This implies that the tensor power spectrum does not need the same transfer functions in order to describe its present shape.

A more delicate treatment of the primordial power spectrum in a quasi-de Sitter background will be treated in the next chapter where the quantum mechanical description of the background quantities will induce some new results for the power spectra of both tensor and scalar modes.

2.7 Multi-field inflation

Throughout this entire chapter, we have told the story of how inflation driven by a single scalar field led to the primordial seeds of our universe. These seeds were first statistically described by quantum correlations and then evolved in order to give the plot of the primordial power spectrum of the CMB.

The scale of energy during inflation was extremely high, around $10^{15} GeV$. This is a lot higher than the energy that drives the collisions at the Large Hadron Collider at CERN, where they already have confirmed the existence of a particle that comes from a scalar field, the Higgs particle. Following this motivation, a lot of theories that go beyond the energy scale of the standard model of particle physics have been developed in the past years, and surprisingly, many of these theories predict multiple numbers of degrees of freedom. This is a good motivation to think that maybe during inflation there was more than one scalar field in the primordial universe. The theory that describes multiple primordial fields during the beginning of the universe is called *Multi-field inflation* and is described by the following action,

$$S = \frac{1}{2}\int d^4x\sqrt{-g}R - \int d^4x\sqrt{-g}\left[\frac{1}{2}\gamma_{ab}g^{\mu\nu}\nabla_\mu\phi^a\nabla_\nu\phi^b + V(\phi^a,\phi^b)\right], \tag{2.89}$$

where ∇_μ is the space-time derivative. Also, it is clear at this point that the first term corresponds to the usual Einstein-Hilbert action for the space-time metric $g_{\mu\nu}$. The γ_{ab} is the metric that describes the target space produced between the scalar fields. Furthermore, $V(\phi^a,\phi^b)$ is the potential that describes the behavior of the fields, the shape of this primordial potential is what we will call during this thesis the *landscape*, where the fields can experience different effects as they evolve through it as we will show in the last chapter.

Let us now describe the dynamics of this kind of action. First, is straightforward to show that the background dynamics are ruled by the following equations of motion:

$$\frac{D}{dt}\dot{\phi}_0^a + 3H\dot{\phi}_0^a + V^a = 0, \qquad 3H^2 = \frac{1}{2}\dot{\phi}_0^2 + V, \tag{2.90}$$

where $V^a = \partial V/\partial\phi^a$ is the partial derivative of V over the fields. The D/dt operator represents the covariant derivative with respect to the cosmic time t, whose action on a vector A^a is defined by $D_t A^a \equiv \dot{A}^a + \Gamma^a_{bc}\dot{\phi}^b A^c$, where Γ^a_{bc} is the Christoffel symbol of the induced metric γ_{ab}.

Now, to connect the perturbations of the field to the shape of the landscape, it is useful to define unit vectors tangent and normal to the trajectory described by the fields. In a particular case, for 2-fields models, they are respectively defined as:

$$T^a \equiv \frac{\dot{\phi}^a}{\dot{\phi}_0}, \qquad N_a \equiv \sqrt{\det\gamma}\epsilon_{ab}T^b \tag{2.91}$$

where we used the following definition, $\dot{\phi}_0 \equiv \sqrt{\gamma_{ab}\dot{\phi}_0^a\dot{\phi}_0^b}$, and ϵ_{ab} is the two-dimensional Levi-Civita symbol with $\epsilon_{11} = 1$. From the previous expression, one important result is that the scalar perturbations along the landscape can be described in terms of these two vectors in such a way that:

$$\delta\phi(\mathbf{x}, t) = T^a(t)\delta\phi_{||}(\mathbf{x}, t) + N^a(t)\sigma(\mathbf{x}, t). \tag{2.92}$$

Written in this way $\delta\phi_{||}(\mathbf{x}, t)$ correspond to the inflaton perturbations, parallel to the background trajectory, whereas $\sigma(\mathbf{x}, t)$ correspond to the perturbations normal to the trajectory. The σ-perturbations are called *entropy perturbations* and, in principle, could have imprints in *non-Gaussianities* and *non-Adiabicity*.

Now we have all the tools to describe the effect of different types of degrees freedom in the primordial statistics of the universe. In the following chapters, we are going the describe the problems that were the main motivations for this thesis, along with some interesting results.

Chapter 3

Scale invariance of the primordial tensor power spectrum

As we previously mentioned, the simplest models of cosmic inflation [12–14, 97] predict both scalar and tensor primordial fluctuations, characterized by a set of nearly scale invariant power spectra. While cosmic microwave background (CMB) observations have enabled us to tightly constrain the power spectrum of scalar perturbations, a detection of primordial gravity waves (in the form of B-modes) remains a pending challenge. Current efforts to observe the CMB polarization will reach the limits of cosmic variance, allowing us to either measure or constrain the tensor-to-scalar ratio r down to $r \sim 0.01$ - 0.002 [30–33]. The observation of B-modes in the CMB would give us access to the value of the Hubble expansion rate H during inflation, reinforcing the idea that the Hot Big Bang era was preceded by a stage of dramatic accelerated expansion.

Although current CMB observations are compatible with a nearly scale invariant power spectrum for curvature perturbations [34], there are some hints of scale dependent features present in the spectrum at certain multipoles [35–38]. The shape and size of such features could in principle allow us to discriminate the type of physics that played a role during inflation, since their appearance in the primordial spectra would invalidate the simplest models of inflation, forcing us to consider models in which non-trivial degrees of freedom interacted with primordial curvature fluctuations around horizon crossing [34, 39–55] (see also [56–58] for early work on features of the tensor spectrum and [59] for an up-to-date review). The prospects of unveiling physics beyond the single-field slow-roll paradigm has also propelled new ideas to analyze the presence of such features in 21 cm and Large Scale Structure observations [60–63].

The effective field theory (EFT) approach to inflation [64, 65] is particularly useful to understand the appearance of features in the primordial spectra. This formalism allows one to study models of inflation beyond the canonical single field paradigm by incorporating the sound speed at which curvature fluctuations propagate, as a parameter in the Lagrangian for perturbations. Within this framework, features are the consequence of time variations of background quantities appearing in the Lagrangian describing the dynamics of the lowest energy fluctuations. These time variations break – in a controlled way – the standard behavior required in single field slow-roll inflation, producing localized features in the spectra, though without invalidating inflation as a mechanism to explain the origin of primordial fluctuations in a way compatible with observations. Given that the source of features may be traced back to background parameters that affect the evolution of all perturbations, features appearing in different n-point correlation functions would be necessarily correlated [17, 66–76]. In the case of scalar perturbations, a powerful way to study such time-dependent departures from slow-roll is the joint estimator analysis of two- and three-point correlation functions [76], since a detection of correlated signals in the power spectrum and bispectrum would increase the statistical significance of these features.

In this chapter we explore the possibility of establishing a novel class of cross correlation between spectra. Specifically, the questions we wish to address are the following: *If features in the primordial scalar power spectrum are confirmed, would they also show up in the tensor power spectrum? In addition, if the scale suppression of the angular power spectrum in the multipole range* $4 \leq \ell \leq 50$ *is found to be of primordial origin, what type of signal should we expect in the angular power spectrum of B-modes?* To that end, we study the effect of time dependent backgrounds on the dynamics of fluctuations in order to correlate features in the power spectra of scalar and tensor modes. Our main result is that features $\Delta\mathcal{P}_T/\mathcal{P}_T$, appearing in the tensor power spectrum $\mathcal{P}_T$, are correlated to features $\Delta\mathcal{P}_S/\mathcal{P}_S$, appearing in the scalar spectrum $\mathcal{P}_S$ in Fourier space, in the following way

$$\frac{d^2}{d\ln k^2}\left(\frac{\Delta\mathcal{P}_T}{\mathcal{P}_T}\right) = 6\epsilon_0\frac{\Delta\mathcal{P}_S}{\mathcal{P}_S}, \tag{3.1}$$

where ϵ_0 is the (constant) average value of the slow-roll parameter $\epsilon = -\dot{H}/H^2$. This expression tells us that any feature appearing in the tensor spectrum is in general suppressed with respect to those appearing in the scalar spectrum [77]. This suppression is two-fold: On the one hand, ϵ_0 must be small in order to keep inflation valid as a mechanism to produce fluctuations over a large range of scales. On the other hand, the $\ln k$-derivatives must be large in order for features to be observable in the scalar power spectrum.[1] Note that this approach is model independent since it takes the scalar power spectrum data as an input without reference to the mechanism that produces the features.

Our results show that any strong departure of scale invariance in the scalar spectrum must come together with a consequential departure in the tensor spectrum, but at a level that is too small to be observed. As a corollary, any future observation of scale invariance departures in the tensor spectrum cannot be of primordial origin, unless some exotic mechanism underlies their origin. For example, models where the only background quantity experiencing rapid

[1]As we shall see in the next section, observable features in the spectra must have an identifiable structure over a range of scales smaller than $\ln k$. This implies that $\ln k$-derivatives acting on either $\Delta\mathcal{P}_T$ or $\Delta\mathcal{P}_S$ must be large.

variations is the tensor sound speed will have features only in the tensor spectrum [78]. On the other hand, non Bunch-Davies initial conditions may lead to features in the two spectra with the same amplitude [79]. In this chapter, however, we are interested in predicting the scale dependence of the tensor spectrum from the scalar power spectrum, highlighting the perspective of a joint analysis of the two spectra. Having this in mind, in the particular case of the observed deficit of the angular power spectrum around $\ell \sim 20$, we conclude that coming CMB polarization experiments should not encounter any scale dependence of the spectrum around that region.

This chapter is organized as follows: In Section 3.1 we present the method used and derive the correlation of the two power spectra for the cases where *i*) features appear due to sudden variations of the Hubble scale, and *ii*) variations in both the Hubble scale and the sound speed are responsible for features. Finally, in Section 3.2, we present results for the tensor power spectrum in the low ℓ region, modeling the features in the scalar signal with a Gaussian and a cosine function.

3.1 Correlation of power spectra

In this section we apply the methods elaborated in [17, 68] to correlate features appearing in the tensor and scalar power spectra. Our method is based on the *in-in* formalism to study the evolution of quantum fluctuations on a time dependent quasi-de Sitter background [80, 81]. Another widely used method to study features is the so called generalized slow-roll formalism [82–85].

3.1.1 Preliminaries

Let us set the ground for the computation by first writing down the quadratic actions for the scalar and tensor perturbations in Fourier space. For the scalar part we will consider the primordial curvature perturbation $\mathcal{R}$ in comoving gauge. On the other hand, for the tensor part we will work with the traceless and transverse perturbation γ_{ij} as:

$$\gamma_{ij}(\mathbf{k},\tau) \equiv h_+(\mathbf{k},\tau)e^+_{ij}(\mathbf{k}) + h_\times(\mathbf{k},\tau)e^\times_{ij}(\mathbf{k}), \tag{3.2}$$

where $\mathbf{k}$ is the wave vector (or momenta), and $e^+_{ij}(\mathbf{k})$ and $e^\times_{ij}(\mathbf{k})$ are the elements of a time independent basis for tensors satisfying $\delta^{ij}e_{ij} = 0$ and $k^i e_{ij} = 0$. We may further define canonically normalized fields u and $f_{+,\times}$ as

$$u = z\mathcal{R}, \qquad f_{+,\times} = a(t)h_{+,\times}, \qquad z \equiv \sqrt{2\epsilon}\frac{a}{c_s}, \tag{3.3}$$

where $a(t)$ is the scale factor, c_s is the sound speed of the curvature perturbations and $\epsilon = -\dot{H}/H^2$ the first Hubble slow-roll parameter. In these variables, the quadratic actions for scalar and tensor modes in conformal time τ are found to be

$$S_S^{(2)} = \frac{1}{2}\int d\tau\, d^3k \left[(u')^2 + c_s^2 k^2 u^2 + \frac{z''}{z}u^2\right], \tag{3.4}$$

$$S_T^{(2)} = \frac{1}{2}\int d\tau\, d^3k \left[(f')^2 + k^2 f^2 + \frac{a''}{a}f^2\right], \tag{3.5}$$

where we have chosen units such that $m_{\rm Pl} = 1$, while keeping only one polarization mode for simplicity. Notice that primes ($'$) represent derivatives with respect to τ. The background quantities z''/z and a''/a may be written as

$$\frac{z''}{z} = (aH)^2\left(2 - \epsilon + \frac{1}{2}\eta - s\right)\left(1 + \frac{1}{2}\eta - s\right) + aH\left(\frac{\eta'}{2} - s'\right), \tag{3.6}$$

$$\frac{a''}{a} = (aH)^2\,(2-\epsilon)\,, \tag{3.7}$$

where $\eta = \epsilon'/\epsilon aH$ and $s = c_s'/c_s aH$.

3.1.2 Rapidly time varying backgrounds

To describe the origin of features, we may split each action into a zeroth order term, that describes the evolution of fluctuations in a quasi-de Sitter spacetime, and an interaction term, that contains the rapidly varying contributions of the background. To do so, we will assume that the background is such that ϵ remains small ($\epsilon \ll 1$) throughout the whole relevant period where features are sourced. To model this behavior we will take ϵ to be of the form:

$$\epsilon = \epsilon_0 + \Delta\epsilon, \qquad |\Delta\epsilon| \ll \epsilon_0, \tag{3.8}$$

where ϵ_0 is (for any practical purpose) a constant, and $\Delta\epsilon(\tau)$ contains information about the sudden variations of the background. One could consider that $\epsilon_0 = -\dot{H}_0/H_0^2$, where H_0 is the slowly varying part of the Hubble expansion rate. In the same manner, η will have two contributions:

$$\eta = \eta_0 + \Delta\eta, \qquad \Delta\eta = -\frac{1}{\epsilon_0}\tau\Delta\epsilon', \tag{3.9}$$

where $\eta_0 = -\dot{\epsilon}_0/H_0\epsilon_0$. Given that we are taking ϵ_0 as a slowly varying function, we may neglect η_0 against $\Delta\eta$ and simply take

$$\eta = -\frac{1}{\epsilon_0}\tau\Delta\epsilon'. \tag{3.10}$$

We will additionally assume that η remains small at all times:

$$|\eta| \ll 1. \tag{3.11}$$

However, given that we are interested into understanding the effects of rapidly varying backgrounds, further derivatives of η could be large, and the following hierarchy may be satisfied:

$$|\eta| \ll |\tau\eta'| \ll |\tau^2\eta''|. \tag{3.12}$$

On the other hand, we may also consider rapid variations of the sound speed c_s admitting departures from the slowly varying value $c_0 = 1$:

$$\theta \equiv 1 - c_s^2 \ll 1, \qquad |\theta| \ll |\tau\theta'| \ll |\tau^2\theta''|. \tag{3.13}$$

The hierarchies (3.12) and (3.13), together with eqs. (3.8) and (3.11), reflect what we mean by having a rapid varying background near a quasi-de Sitter state.

The previous assumptions allow us to rewrite z''/z and a''/a in the following way

$$\frac{z''}{z} = \frac{2}{\tau^2}\left(1 + \frac{1}{2}\delta_S(\tau)\right), \qquad \frac{a''}{a} = \frac{2}{\tau^2}\left(1 + \frac{1}{2}\delta_T(\tau)\right), \tag{3.14}$$

where we have used $\tau \simeq -(aH)^{-1}(1+\epsilon)$, and introduced the quantities $\delta_S(\tau)$ and $\delta_T(\tau)$ to parametrize the rapid variations of the background:

$$\delta_S(\tau) = 3\epsilon + \frac{1}{2}\eta - \frac{\tau}{2}\eta' - 3s + \tau s', \qquad \delta_T(\tau) = 3\epsilon. \tag{3.15}$$

By plugging these expressions back into the actions of eqs. (3.4) and (3.5) and treating the rapidly varying parts as interaction terms, we may split the theory as:

$$S_S^0 = \frac{1}{2}\int d\tau d^3k \left[(u')^2 + k^2u^2 + \frac{2}{\tau^2}u^2\right], \qquad S_S^{\rm int} = \frac{1}{2}\int d\tau d^3k \left[\frac{\delta_S(\tau)}{\tau^2}u^2\right], \tag{3.16}$$

$$S_T^0 = \frac{1}{2}\int d\tau d^3k \left[(f')^2 + k^2f^2 + \frac{2}{\tau^2}f^2\right], \qquad S_T^{\rm int} = \frac{1}{2}\int d\tau d^3k \left[\frac{\delta_T(\tau)}{\tau^2}f^2\right]. \tag{3.17}$$

Notice that eq. (3.12) implies a further hierarchy of the form

$$|\delta| \ll |\tau\delta'| \ll |\tau^2\delta''|, \tag{3.18}$$

where δ stands for both δ_S and δ_T. Given that a change in e-folds dN is related to a change in conformal time by $dN = -d\tau/\tau$, the previous hierarchies simply tell us that δ_S and δ_T vary rapidly over an e-fold:

$$|\delta| \ll \left|\frac{d\delta}{dN}\right| \ll \left|\frac{d^2\delta}{dN^2}\right|. \tag{3.19}$$

As we shall see, these are the rapidly varying functions that source the appearance of features in the spectra.

3.1.3 In-in formalism

We may now use the standard *in-in* formalism (see [86] for a review), which provides a way to compute the effects of the rapid time varying background on n-point correlation functions. To simplify the discussion, let us focus our attention on the scalar sector of the theory (*i.e.* the u fluctuations), and then come back to the case of tensor modes. Firstly, the complete solution $u(\mathbf{k},\tau)$ can be written in terms of interaction picture fields $u_I(\mathbf{k},\tau)$ as

$$u(\mathbf{k},\tau) = U^\dagger(\tau)u_I(\mathbf{k},\tau)U(\tau), \tag{3.20}$$

where $U(\tau)$ is the propagator, given by

$$U(\tau) = \mathcal{T} \exp\left[-i\int_{-\infty_+}^{\tau} d\tau' H_I(\tau')\right].$$

Here $\mathcal{T}$ is the time ordering symbol, and $\infty_+ = (1+i\epsilon)\infty$ is the usual prescription to choose the right vacuum in the infinite past. In addition, $H_I(\tau)$ is the interaction Hamiltonian, given by

$$H_I = -\frac{\delta_S(\tau)}{\tau^2}\frac{1}{2}\int d^3k\, u_I^2. \tag{3.21}$$

The interaction picture fields $u_I(\mathbf{k},\tau)$ are given by free field solutions of the zeroth order action (*i.e.* with $\delta_S = 0$), written in terms of creation and annihilation operators $a_{\mathbf{k}}^\dagger$ and $a_{\mathbf{k}}$ as:

$$u_I(\mathbf{k},\tau) \equiv a_{\mathbf{k}} u_k(\tau) + a_{-\mathbf{k}}^\dagger u_k^*(\tau). \tag{3.22}$$

The creation and annihilation operators satisfy the standard commutation relation $\left[a_{\mathbf{k}}, a_{\mathbf{k}'}^\dagger\right] = (2\pi)^3\delta^{(3)}(\mathbf{k}-\mathbf{k}')$, whereas the mode functions $u_k(\tau)$ are given by mode solutions respecting Bunch-Davies initial conditions:

$$u_k(\tau) = \frac{1}{\sqrt{2k}}\left(1 - \frac{i}{k\tau}\right)e^{-ik\tau}. \tag{3.23}$$

Furthermore, the vacuum state $|0\rangle$ is defined to satisfy $a_{\mathbf{k}}|0\rangle = 0$. By expanding the propagator $U(\tau)$, we may compute corrections to the two point function as

$$\langle u(\mathbf{k},\tau)u(\mathbf{k}',\tau)\rangle = \langle 0|\, u_I(\mathbf{k},\tau)u_I(\mathbf{k}',\tau)\,|0\rangle + i\int_{-\infty_+}^{\tau} d\tau'\, \langle 0|\left[H_I(\tau'), u_I(\mathbf{k},\tau)u_I(\mathbf{k}',\tau)\right]|0\rangle\,. \tag{3.24}$$

The power spectrum $\mathcal{P}_{\mathcal{R}}(k,\tau)$ of the primordial curvature perturbation $\mathcal{R}$ (evaluated at a given time τ) is related to the two point function $\langle u(\mathbf{k},\tau)u(\mathbf{k}',\tau)\rangle$ as follows:

$$\frac{1}{z^2}\langle u(\mathbf{k},\tau)u(\mathbf{k}',\tau)\rangle \equiv \frac{2\pi^2}{k^3}\delta^{(3)}(\mathbf{k}-\mathbf{k}')\mathcal{P}_{\mathcal{R}}(k,\tau). \tag{3.25}$$

We are interested in the power spectrum of super horizon modes at the end of inflation $\mathcal{P}_{\mathcal{R}}(k)$, which corresponds to the $\tau \to 0$ limit of $\mathcal{P}_{\mathcal{R}}(k,\tau)$. By taking into account the splitting of the theory into the zeroth order quasi-de Sitter part and the interaction part, we finally obtain

$$\mathcal{P}_{\mathcal{R}}(k) = \mathcal{P}_S^0 + \Delta\mathcal{P}_S(k), \qquad \mathcal{P}_S^0(k) = \frac{H_0^2}{8\pi^2\epsilon_0}, \tag{3.26}$$

where $\mathcal{P}_S^0$ corresponds to the standard power spectrum for curvature perturbations in a quasi-de Sitter space-time, and $\Delta\mathcal{P}_S(k)$ contains the deviations from scale invariance induced by the rapidly varying background[2] [17]

$$\Delta_S(k) \equiv \frac{\Delta\mathcal{P}_S}{\mathcal{P}_S^0} = \frac{i}{4k^3}\int_{-\infty}^{\infty} d\tau\left[\frac{\theta''''}{8} + \frac{\delta_H''}{2\tau^2} - \frac{\delta_H}{\tau^4}\right]e^{2ik\tau}, \tag{3.27}$$

[2]Notice that in eq. (3.27) time derivatives may be interchanged by factors of $-2ik$. Therefore, the appearance of four derivatives in θ might be deceiving, as the original expression [17] leading to eq. (3.27) had no time derivatives acting on θ. Having time derivatives acting on both θ and δ_H in eq. (3.27) allows one to have a single function of time being Fourier-transformed at the right hand side of the equation.

where θ is defined in (3.13) and δ_H is given by

$$\delta_H(\tau) = 3\epsilon + \frac{1}{2}\eta - \frac{\tau}{2}\eta'. \tag{3.28}$$

Notice that the integration in eq. (3.27) is performed over the whole real line $(-\infty, +\infty)$, which from now on will be omitted. To derive eq. (3.27) we did the following trick [68]: We extended the τ-integration domain from $(-\infty, 0)$ to $(-\infty, +\infty)$ by imposing that both θ and δ_H are antisymmetric functions with respect to the interchange $\tau \to -\tau$.

We may now repeat all of the previous steps to compute the way that features appear in the tensor power spectrum. We find

$$\mathcal{P}_T(k) = \mathcal{P}_T^0 + \Delta\mathcal{P}_T(k), \qquad \mathcal{P}_T^0(k) = \frac{H_0^2}{2\pi^2},$$

where $\Delta\mathcal{P}_T(k)$ is given by

$$\Delta_T(k) \equiv \frac{\Delta\mathcal{P}_T}{\mathcal{P}_T^0} = \frac{i}{4k^3}\int d\tau \left[\frac{\delta_T''}{2\tau^2} - \frac{\delta_T}{\tau^4}\right] e^{2ik\tau}. \tag{3.29}$$

Equations (3.27) and (3.29) are the basic equations that we will exploit to obtain the desired correlation between the two sectors of the theory. Before deducing such a relation, let us notice that the hierarchy of eq. (3.18) necessarily implies a hierarchy in Fourier space affecting the spectra, that reads

$$|\Delta(k)| \ll \left|\frac{d\Delta(k)}{d\ln k}\right| \ll \left|\frac{d^2\Delta(k)}{d\ln k^2}\right|, \tag{3.30}$$

where $\Delta(k)$ stands for both $\Delta_S(k)$ and $\Delta_T(k)$.

3.1.4 Features from varying Hubble parameters

In this subsection we consider the case where $c_s = 1$ for all times, so that $\delta_S = \delta_H$, and any observable feature is the outcome of sudden variations of $H(t)$. Firstly, because of the hierarchy (3.18) satisfied by δ_T, eq. (3.29) may be simplified as:

$$\Delta_T(k) = \frac{i}{8k^3}\int d\tau \frac{\delta_T''}{\tau^2} e^{2ik\tau}. \tag{3.31}$$

Furthermore, because of eq. (3.15), we see that eq. (3.31) may be rewritten in terms of η as:

$$\Delta_T(k) = -\frac{3i\epsilon_0}{8k^3}\int d\tau \frac{\eta'}{\tau^3} e^{2ik\tau}. \tag{3.32}$$

This expression may now be Fourier inverted, leading to a formal expression for η' in terms of $\Delta_T(k)$ as

$$\eta' = \frac{1}{3\epsilon_0}\int dk \left[\frac{d^3}{d\ln k^3}\Delta_T(k)\right] e^{-2ik\tau}. \tag{3.33}$$

Next, we may use the hierarchy of eq. (3.18) satisfied by δ_S to rewrite eq. (3.27) as

$$\Delta_S(k) = -\frac{i}{16k^3}\int d\tau \frac{1}{\tau}\eta''' e^{2ik\tau}, \tag{3.34}$$

where we used the fact that $\delta_H \simeq -\tau\eta'/2$. As a last step, we may insert the expression for η' in eq. (3.33) back into eq. (3.34), to obtain the main result of this work:

$$\frac{d^2}{d\ln k^2}\Delta_T = 6\epsilon_0 \Delta_S. \tag{3.35}$$

This equation offers the desired link between features in the tensor and scalar spectra. Notice from eq. (3.32) that even though we have assumed that $\epsilon \ll 1$, the piece $\Delta_T(k)$ could in principle be large. However, from eq. (3.35), we see that features in the tensor power spectrum are highly suppressed with respect to those in the scalar spectrum. This is not only due to the presence of ϵ_0 [77], but also due to the double $\ln k$-derivative acting on $\Delta_T(k)$, on account of the hierarchy (3.30).

In the next subsection we extend this result to the more general case in which rapid variations of the sound speed are also allowed. As we shall see, in this case too, tensor features remain generically suppressed.

3.1.5 Including the effects of a varying sound speed

In the EFT of inflation [64, 65], the quadratic part of the action may exhibit a non-trivial sound speed for the perturbations, which could also lead to the presence of features in the scalar power spectrum [87, 88]. In general the evolution of $c_s(t)$ is independent of the evolution of H. That means that if features are generated by the simultaneous rapid variation of both c_s and H, then the scalar and tensor power spectra would exhibit uncorrelated oscillatory features. This is because $\mathcal{P}_S$ would have features sourced by both c_s and H while $\mathcal{P}_T$ would have features sourced by H alone. We would then have a relation of the form

$$\Delta_S = \frac{1}{6\epsilon_0}\frac{d^2}{d\ln k^2}\Delta_T + \Delta_c, \tag{3.36}$$

where Δ_c represents the features sourced by variations of the sounds speed c_s.

There are however intuitive reasons to expect that, at least in certain classes of models, variations of c_s and H happen in synchrony. An example of such a situation is the case where the inflationary valley admits turns, which is typical in multifield inflation [87]. In these scenarios, as the inflaton traverses a curve in the field space, there are instant deviations from slow-roll produced by “centrifugal" effects. Furthermore, the existence of such turns is responsible for a non-trivial sound speed [89]. The two quantities should thus be related since they stem from the same source. Another situation where c_s and H vary simultaneously is in $P(X, \varphi)$ models, where the kinetic term of the inflaton has a non-trivial structure. In these cases a reduction of the rapidity of the vacuum expectation value of the inflaton would inevitably induce a change in both c_s and H.

To capture the aforementioned situations, in [72], a one parameter relation between the Hubble slow-roll parameter η and the sound speed was proposed. This had the form

$$\eta = \eta_0 - \frac{\alpha}{2}\tau\theta', \tag{3.37}$$

with $\alpha \in \mathbb{R}$ and $\theta = 1 - c_s^2$. It was also shown to hold within several classes of models including $P(X, \varphi)$ and multifield models, with α admitting specific values for each case.

Using this fact, one may now relate θ to η in eq. (3.27) and follow the exact same steps to obtain a generic relation between the scalar and tensor power spectra in the case where both the sound speed and the Hubble radius experience sudden variations:

$$\frac{d^2}{d\ln k^2}\Delta_T = 6\epsilon_0 \frac{\alpha}{1+\alpha}\Delta_S, \quad \alpha \neq -1, \tag{3.38}$$

and for the special case of $\alpha \simeq -1$:

$$\frac{d}{d\ln k}\Delta_T = -\frac{6}{5}\epsilon_0\Delta_S. \tag{3.39}$$

We see that in these set-up's too, deviations of the tensor power spectrum from scale invariance are suppressed by the slow-roll parameter ϵ as well as a double and a single momentum integral which smoothes out any acute variation of the scalar spectrum.

Before discussing quantitative features of these results, let us stress once more that the simple forms of eqs. (3.35), (3.38) and (3.39) are leading order expressions based on the assumption that any observable feature satisfy the following: *i*) it is sharp, in the sense that any departure from scale invariance should take place within few e-folds, and *ii*) it doesn't disrupt inflation, that is, ϵ remains small through out the whole dynamics.

3.2 A quantitative discussion

We now discuss the results of the previous section in two interesting situations. First, we consider the case in which resonant features are present throughout the whole spectra, and second, the case of the low ℓ power deficit observed in the scalar power spectrum. For this discussion, it will be useful to write concrete expressions relating features in the spectra and the rapidly varying contributions to the slow-roll parameters $\Delta\epsilon$ and $\Delta\eta$. By Fourier inverting eq. (3.34) for the general case where the sound speed also contributes to features, these are found to be given by [17]

$$\Delta\eta(\tau) = \frac{i}{\pi}\frac{\alpha}{1+\alpha}\int dk \left[\frac{d}{dk}\frac{\Delta\mathcal{P}_S}{\mathcal{P}_S^0}\right] e^{2ik\tau}\,, \quad \Delta\epsilon(\tau) = \frac{i\epsilon_0}{\pi}\frac{\alpha}{1+\alpha}\int dk \left[\frac{1}{k}\frac{\Delta\mathcal{P}_S}{\mathcal{P}_S^0}\right] e^{2ik\tau}, \tag{3.40}$$

with $\Delta\epsilon$ following from the relation $\Delta\eta = -\tau\Delta\epsilon'/\epsilon_0$. Note that the coefficient $\frac{\alpha}{1+\alpha}$ in eq. (3.40) is an $\mathcal{O}(1)$ number for any α so it's specific value has no impact on the results. We thus set it to one in what follows and work with eq. (3.35). The only case where it plays a role is when $\alpha \simeq -1$, in which the next to leading time derivative dominates in the RHS of eq. (3.27) leading to the following expressions:

$$\Delta\eta(\tau) = -\frac{i}{5\pi}\int dk\, k\left[\frac{d^2}{dk^2}\frac{\Delta\mathcal{P}_S}{\mathcal{P}_S^0}\right]e^{2ik\tau}\,, \qquad \Delta\epsilon(\tau) = -\frac{i\epsilon_0}{5\pi}\int dk\left[\frac{d}{dk}\frac{\Delta\mathcal{P}_S}{\mathcal{P}_S^0}\right]e^{2ik\tau}. \tag{3.41}$$

3.2.1 Resonant features

This type of scale dependence is relevant in models of inflation where the potential is periodic or semi-periodic, such as axion monodromy inflation [90], or models like Natural Inflation [106]. Inflationary scenarios involving axions usually require super-Planckian field range, and hence, they are good candidates for the production of primordial gravitational waves [92, 93].

To acquire an idea of the possible impact of resonant features on the tensor power spectrum, we model the resonant part of the scalar power spectrum as

$$\Delta_S(k) = A\cos\left(\Omega\log(k/k_*) + \phi\right), \tag{3.42}$$

where A parametrizes the amplitude of the feature, while Ω and ϕ denote the frequency and the phase of the oscillation, respectively. To be concrete, we will consider the following values $A = 0.028$, $\Omega = 30$ and $\phi/2\pi = 0.634$, which were found to constitute the best fit in the analysis of resonant features by Planck [113]. In addition, we set $k_* = 0.05$ [Mpc]$^{-1}$ as a reference scale.

Case for $\alpha \neq -1$

Using the parametrization (3.42) as a input, we numerically obtain the shape of the tensor spectrum feature via eq. (3.35), while the slow-roll parameters are reconstructed from eq. (3.40). The results are shown in the plots of figure 3.1. There we see that features in the tensor power spectrum are present, albeit with an amplitude of $\Delta_T \sim 10^{-6}$ making them observationally irrelevant. This is a complementary argument in support of the claim that tensor features stemming from axionic potentials should be suppressed due to the smallness of the decay constant of the axion [95].

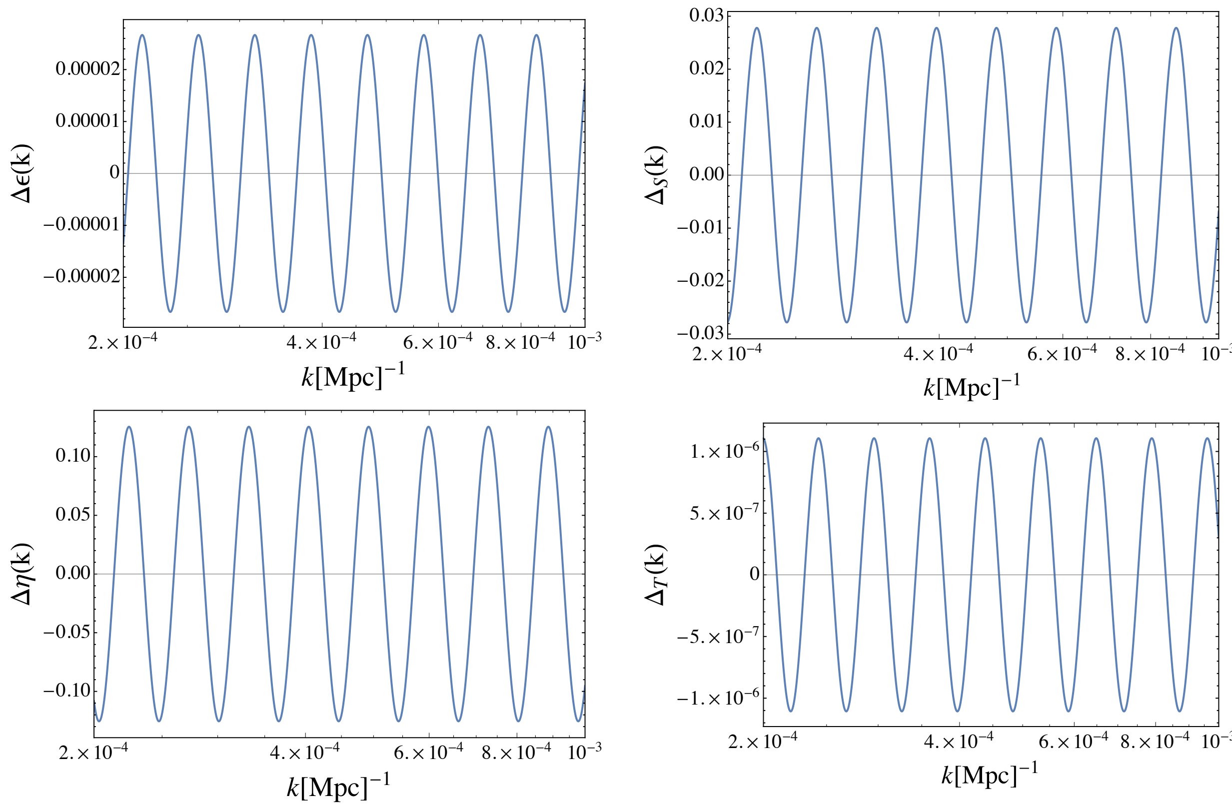

Figure 3.1: Plot of the first two slow-roll parameters $\Delta\eta$ and $\Delta\epsilon$ (left panels) using eq. (3.40) and $\frac{\Delta P_S}{P_S^0}(k)$, $\frac{\Delta P_T}{P_T^0}(k)$ (right panels) related by eq. (3.35), in the case of the resonant feature (3.42). We have used $A = 0.028$, $\Omega = 30$, $\phi/2\pi = 0.634$, $k_* = 0.05[\text{Mpc}]^{-1}$ and $\epsilon_0 = 0.0068$.

Case for $\alpha \simeq -1$

Next, we consider the special case of $\alpha \simeq -1$ for the resonance features. We numerically solve eqs. (3.39) and (3.41) and plot the results in figure 3.2. As can be seen, even though there is an order of magnitude enhancement with respect to the general case, the amplitude of the deviation from a scale invariant spectrum still remains extremely small. Furthermore, in this case η can reach values up to $\eta \sim 0.8$. This does not invalidate the hierarchy (3.12), as to go from eq. (3.6) to eq. (3.14) one really requires $\eta/2$ to be much smaller than 1.

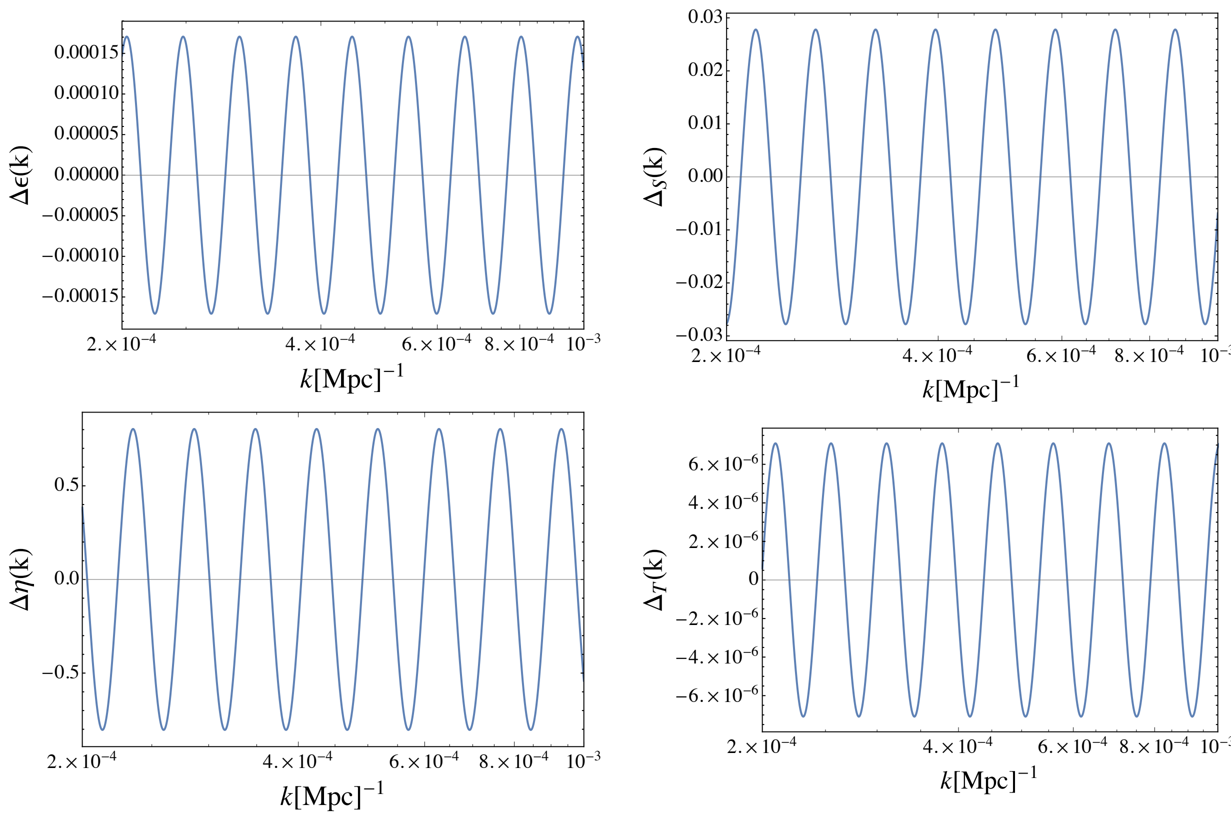

Figure 3.2: Plot of the first two slow-roll parameters $\Delta\eta$ and $\Delta\epsilon$ (left panels) using eq. (3.41) and $\frac{\Delta P_S}{P_S^0}(k)$, $\frac{\Delta P_T}{P_T^0}(k)$ (right panels) related by eq. (3.39), in the case of the resonant feature (3.42). We have used $A = 0.028$, $\Omega = 30$, $\phi/2\pi = 0.634$, $k_* = 0.05[\text{Mpc}]^{-1}$ and $\epsilon_0 = 0.0068$.

3.2.2 Predictions for the low ℓ tensor power spectrum

The low ℓ multipole region is the main observational window into CMB polarization since it is not contaminated by lensing effects. In addition, it is where the low ℓ deficit takes place in the scalar power spectrum [34, 39–45]. We focus in the $\ell < 50$ region, roughly corresponding to $0.0002 \lesssim k \lesssim 0.004$ [Mpc]$^{-1}$, which is the band that CMB polarization observatories focus on.

In order to get a quantitative look into the tensor power spectrum we model the $\ell \sim 20$ dip in the angular power spectrum as a sharp Gaussian:

$$\Delta_S(k) = -Ae^{-\lambda(\ln(k/k_*))^2}, \tag{3.43}$$

where k_* determines the location of the feature. We set $A = 0.15$, $\lambda = 15$ and $k_* = 0.002$ [Mpc]$^{-1}$, which are chosen to have a rough fit with the observed power deficit. In addition, we choose $\epsilon_0 = 0.0068$ [113].

Case with $\alpha \neq -1$

We solve eqs. (3.35) and (3.40) with the parametrization (3.43) as an input, with the results shown in the plots of figure 3.3. We see that for a realistic amplitude A the tensor power spectrum exhibits a feature of amplitude $\Delta_T \sim 10^{-9}$.

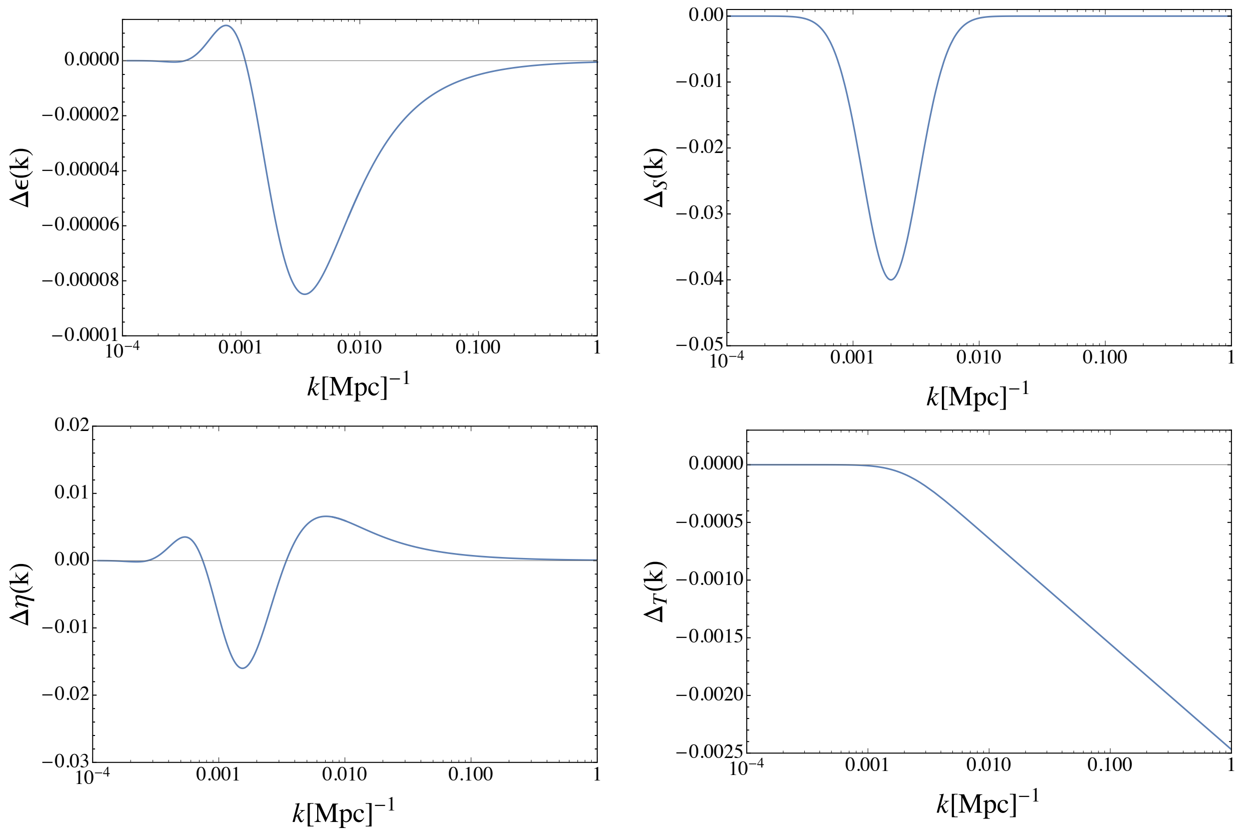

Figure 3.3: Plot of the first two slow-roll parameters $\Delta\eta$ and $\Delta\epsilon$ (left panels) using eq. (3.40) and $\frac{\Delta P_S}{P_S^0}(k)$, $\frac{\Delta P_T}{P_T^0}(k)$ (right panels) related by eq. (3.35), in the case of the Gaussian feature (3.43). We have used $A = -0.15$, $\lambda = 15$, $k^* = 0.002[\text{Mpc}]^{-1}$ and $\epsilon_0 = 0.0068$.

Case with $\alpha \simeq -1$

In the special case of $\alpha \simeq -1$, we see that the tensor spectrum and the slow-roll parameters, now given by eqs. (3.39) and (3.41) respectively, exhibit a feature which is enhanced by an order of magnitude compared to the previous case. However, as seen in figure 3.4, the amplitude still remains extremely small.

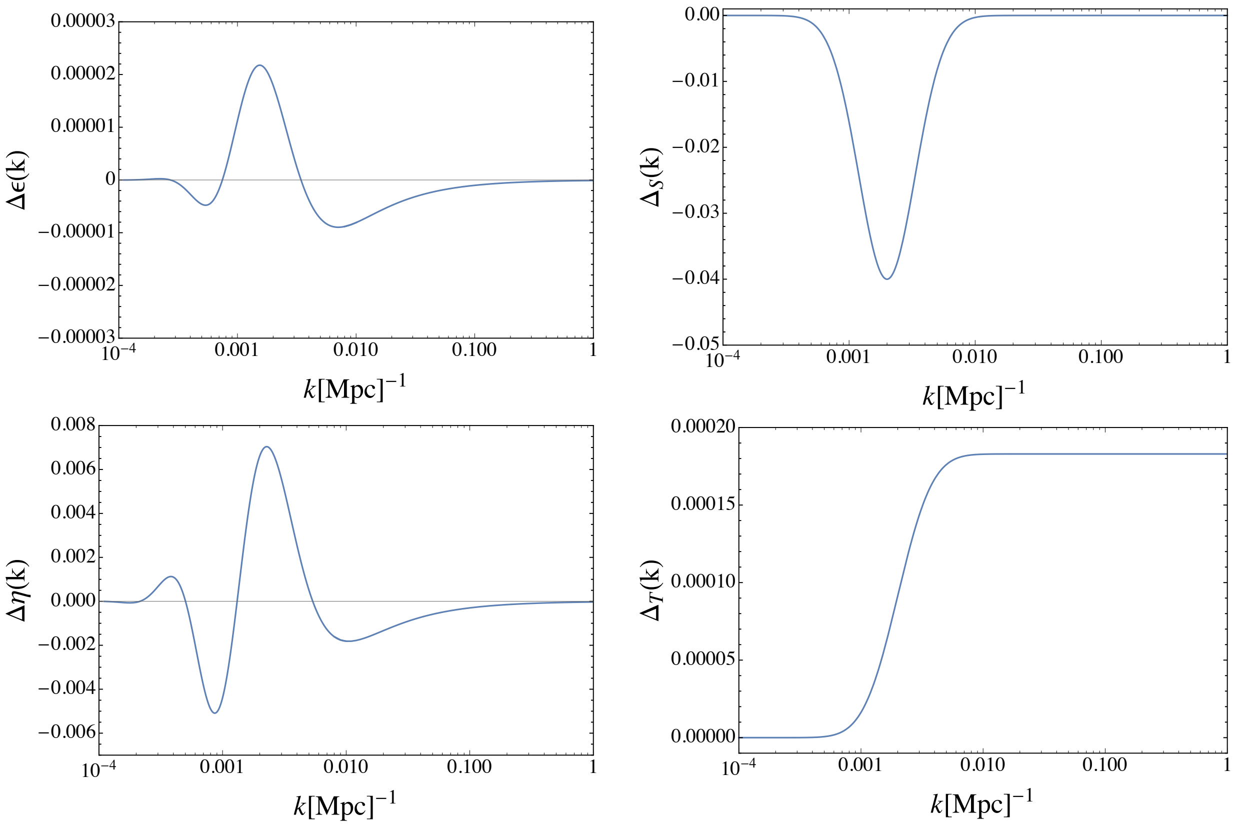

Figure 3.4: Plot of the first two slow-roll parameters $\Delta\eta$ and $\Delta\epsilon$ (left panels) using eq. (3.41) and $\frac{\Delta P_S}{P_S^0}(k)$, $\frac{\Delta P_T}{P_T^0}(k)$ (right panels) related by eq. (3.39), in the case of the Gaussian feature (3.43). We have used $A = -0.15$, $\lambda = 15$, $k^* = 0.002[\text{Mpc}]^{-1}$ and $\epsilon_0 = 0.0068$.

Chapter 4

Axion excursions of the landscape during inflation

In the previous chapter, we have dealt with interaction terms that appeared as deviations from the background quantities of a de-Sitter universe. In that case the interaction was between degrees of freedom that already part of the canonical inflation paradigm. During this chapter, we are going to describe how a particular extra scalar field, which does not drive inflation, could behave in the inflationary era.

As a motivation, we may think the following: How did cosmic inflation could influence the structure of the standard model of particle physics? The string-landscape picture [99]—in which the standard model (SM) is just one of many possible vacua—provides us with a useful framework to address this compelling question. In it, the inflationary history plays a crucial role in selecting the properties of our Universe, as we observe it [100].

This selection could have been classical if the inflationary trajectory followed by the scalar fields respected classical equations of motion. In this case, the SM vacuum would have been determined by certain initial conditions specifying the starting point of the inflationary attractor trajectory. In these situations the probability distribution functions (PDFs) describing any scalar degrees of freedom remained nearly Gaussian, and the analysis of their role on cosmological observables is limited to the computation of two- and three-point correlation functions.

Another more intriguing possibility is that, during inflation, tunneling across barriers separating different classical trajectories was relevant. In this case the vacuum selection was intrinsically quantum mechanical, and the quantum to classical transition during inflation [101–104] would have played a decisive role in determining the properties of the standard model. Moreover, the PDFs of extra scalar fields would be highly non-Gaussian, and an analysis based on the computation of two- and three-point correlation functions is found to be insufficient to understand their role at the end of inflation.

In this chapter, we study the role of quantum fluctuations in determining the final state of inflation and so the properties of particle physics within our observable Universe. To do this, we analyze a very simple inflationary setup containing an axion spectator field [18] with a sinusoidal periodic potential. This setup is simple enough to allow us perturbative computations leading to nontrivial results that are representative of more complicated systems, containing other classes of fields and/or potentials. We will show that the quantum fluctuations of axions are able to traverse many local minima of their periodic potentials, leading to a non-Gaussian multimodal PDF. In consequence, the final value at which they stabilize remains indeterminate during inflation, leading to a universe filled with patches characterized by different vacuum expectation values (VEVs) of the axion. Our results should lead to the derivation of new constraints on axions (and spectator scalar fields in general) connecting current cosmological and/or astrophysical constraints with primordial cosmology.

4.1 Axions in inflation

Axions may have an important role in connecting particle physics with cosmology [105]. Axions are natural candidates for the inflaton [106, 107], but this requires them to have super-Planckian decay constants, unless some alignment effect underlies their potentials [108]. In addition, thanks to axions, inflation may play a role in the existence of hierarchies in the SM [109].

In this paper, we will not bother about the specific nature of the inflaton. We will consider a model in which there is an axion field ψ that remains decoupled (at tree level) from the inflaton field ϕ, in charge of driving inflation. This class of models has been studied in the past [110–112], with an emphasis on the production of Gaussian isocurvature perturbations. The action describing this system is given by

$$S = S_{\rm EH}[g_{\mu\nu}] + S_{\rm infl}[g_{\mu\nu}, \phi] + S_{\psi}[g_{\mu\nu}, \psi], \tag{4.1}$$

where $S_{\rm EH}[g_{\mu\nu}]$ corresponds to the Einstein-Hilbert action describing the dynamics of the metric $g_{\mu\nu}$ and $S_{\rm infl}[g_{\mu\nu}, \phi]$ and $S_{\psi}[g_{\mu\nu}, \psi]$ are the respective actions for ϕ and ψ, both of them minimally coupled to gravity. The action $S_{\rm infl}[g_{\mu\nu}, \phi]$ includes a potential $V_0(\phi)$ that produces inflation. The inflationary background is well described by a Friedman-Robertson-Walker metric of the form $ds^2 = -dt^2 + a^2(t)d\mathbf{x}^2$, where $a(t)$ is the scale factor. If $V_0(\phi)$ is flat enough, the scalar field ϕ rolls down the potential slowly, and the background is well approximated to a de Sitter geometry, with deviations of the order of $\epsilon \equiv -\dot{H}/H^2$, where $H = \dot{a}/a$ is the Hubble expansion rate. The latest cosmic microwave background (CMB) observations tell us that $H_* < 8.8 \times 10^{13}$GeV [113], where H_* is the Hubble expansion rate at the time when fluctuations of wave number $k_* = 0.05\text{Mpc}^{-1}$ crossed the horizon.

The axion ψ has a potential $v(\psi)$ that we take to be

$$v(\psi) \equiv \Lambda^4 \left[1 - \cos\left(\frac{\psi}{f}\right)\right], \tag{4.2}$$

where f is the axion decay constant. This potential is the result of nonperturbative effects due to the coupling between the axion and gauge fields. It is convenient to write the action for ψ in conformal coordinates and in terms of a canonically normalized field $u = a\psi$. We may define conformal time τ through $d\tau = dt/a$. Then, the action S_ψ of Eq. (4.1) takes the form

$$S_\psi = \int d^3x\, d\tau \left[\frac{1}{2}(u')^2 + \frac{1}{2}(\nabla u)^2 - a^4 v(u/a)\right]. \tag{4.3}$$

In the de Sitter limit $\epsilon \to 0$ one has H =const. In this case one has $a(\tau) = -1/H\tau$. The conformal time covers the range $-\infty < \tau < 0$, and the limit $\tau \to 0^-$ corresponds to $t \to +\infty$.

4.2 The in-in formalism

In what follows, we focus on the computation of n-point correlation functions of ψ to first order in Λ^4. We shall use the shorthand notations $\int_x = \int d^3x$ when integration takes place in coordinate space and $\int_k = (2\pi)^{-3} \int d^3k$ when integration takes place in momentum space. In addition, as in the previous chapter, we shall use the following convention to relate fields in both spaces:

$$u(\mathbf{x}, \tau) = \int_k \hat{u}(\mathbf{k}, \tau) e^{i\mathbf{k}\cdot\mathbf{x}}. \tag{4.4}$$

The computation of n-point correlation functions in coordinate space, using the *in-in* formalism, takes the form

$$\langle u(\mathbf{x}_1, \tau) \dots u(\mathbf{x}_n, \tau)\rangle = \langle 0|U^\dagger u_I(\mathbf{x}_1, \tau) \dots u_I(\mathbf{x}_n, \tau) U|0\rangle, \tag{4.5}$$

where $U = U(\tau)$ is the propagator and $u_I(\mathbf{x}, \tau)$ is the interaction picture field. This field has a momentum-space representation given by

$$\hat{u}_I(\mathbf{k}, \tau) \equiv a_{\mathbf{k}} u^I_k(\tau) + a^\dagger_{-\mathbf{k}} u^{I*}_k(\tau) \tag{4.6}$$

where $a^\dagger_{\mathbf{k}}$ and $a_{\mathbf{k}}$ are creation and annihilation operators satisfying the standard commutation relation $\left[a_{\mathbf{k}}, a^\dagger_{\mathbf{k}'}\right] = (2\pi)^3 \delta^{(3)}(\mathbf{k} - \mathbf{k}')$. On the other hand, the mode functions $u_k(\tau)$ satisfy the equations of motion of a free field (that is, with $\Lambda = 0$), in the same way that for the fluctuations of the inflaton field. This implies that the mode functions are given by mode solutions respecting Bunch-Davies initial conditions:

$$u^I_k(\tau) = \frac{1}{\sqrt{2k}} \left(1 - \frac{i}{k\tau}\right) e^{-ik\tau}. \tag{4.7}$$

The vacuum state $|0\rangle$ is normalized $\langle 0|0\rangle = 1$, and it is defined to satisfy $a_{\mathbf{k}}|0\rangle = 0$. The propagator may be written in terms of $u_I(\mathbf{x}, \tau)$ in the following way:

$$U(\tau) = \mathcal{T} \exp\left\{-i \int_{-\infty^+}^{\tau} d\tau' H_I(\tau')\right\}, \tag{4.8}$$

where $H_I(\tau)$ is the Hamiltonian in the interaction picture. In the previous expression, $\mathcal{T}$ stands for the standard time ordering symbol, and $\infty^+ = (1 + i\epsilon)\infty$ is the prescription isolating the in vacuum in the infinite past. In the particular case of (4.2), the Hamiltonian $H_I(\tau)$ takes the form

$$H_I(\tau) = \frac{\Lambda^4}{H^4\tau^4}\int_z \left[1 - \cos\left(\frac{H\tau}{f}u_I(\mathbf{z},\tau)\right)\right]. \tag{4.9}$$

We may expand U to first order in Λ^4 to compute n-point correlation functions. Then, in momentum space Eq. (4.5) takes the form

$$\begin{aligned}\langle \hat{u}(\mathbf{k}_1,\tau)\dots\hat{u}(\mathbf{k}_n,\tau)\rangle &= \langle 0|\hat{u}_I(\mathbf{k}_1,\tau)\dots\hat{u}_I(\mathbf{k}_n,\tau)|0\rangle \\ &-i\int_{-\infty}^{\tau}d\tau'\langle 0|\left[H_I(\tau'),\hat{u}_I(\mathbf{k}_1,\tau)\dots\hat{u}_I(\mathbf{k}_n,\tau)\right]|0\rangle.\end{aligned} \tag{4.10}$$

We have dropped the prescription involving ∞^+ which is irrelevant for computations up to first order in the interaction.

It is important to notice that the Bunch-Davies initial condition of Eq. (4.7) does not share the periodicity of the potential. For this initial condition to be valid, we have to assume that the wall domain number N associated to the axion field is much larger than 1.

Before analyzing the entire theory in detail, let us briefly examine the trivial case in which $\Lambda = 0$. In this case the theory corresponds to a massless field in a de Sitter space-time, and the distribution function describing the probability of measuring a specific amplitude is Gaussian. The variance of such a distribution is given by the two-point correlation function $\sigma_0^2(\tau) \equiv \langle\psi^2(\mathbf{x},\tau)\rangle$ (with $\mathbf{x}$ evaluated at any desired value), which in terms of $u_{\mathbf{k}}$ has the form

$$\sigma_0^2 = H^2\tau^2\int_k u_k(\tau)u_k^*(\tau). \tag{4.11}$$

Notice that σ_0^2 is time independent, which is possible to verify by absorbing the combination $-k\tau$ into a single integration variable. This time independence comes from the fact that we are dealing with fluctuations in a de Sitter space-time. On the other hand, σ_0^2 is formally infinite on account of (4.7). This indeterminacy may be eliminated by introducing both infrared and ultraviolet cutoffs. These details will turn out to be irrelevant in our analysis.

4.3 Computing n-point functions

We now proceed to compute the second line in Eq. (4.10), which corresponds to the non-Gaussian contribution to the n-point correlation function. Notice that this computation will take into account the leading-order contribution in terms of Λ^4 which involves the whole function $1 - \cos(\psi/f)$. In this computation, we will encounter the following function:

$$\Delta(\tau',\tau,k) \equiv u_k^I(\tau')u_k^{I*}(\tau), \tag{4.12}$$

which may be thought of as a propagator in momentum space connecting vertices characterized by τ and τ'. Now, to compute (4.10), we first expand the cosine function appearing in the potential. This leads to a Hamiltonian of the form

$$H_I(\tau) = -\frac{\Lambda^4}{H^4\tau^4}\sum_{m=1}^{\infty}\frac{(-1)^m}{(2m)!}\int_z\left(\frac{H\tau}{f}u_I(\mathbf{z},\tau)\right)^{2m}. \tag{4.13}$$

By plugging this expression back into Eq. (4.10), we find that the contribution proportional to Λ^4 is given by

$$\langle\hat{u}(\mathbf{k}_1,\tau)\dots\hat{u}(\mathbf{k}_n,\tau)\rangle_{\Lambda^4} = i\frac{\Lambda^4}{f^4}\sum_{m=1}^{\infty}\frac{(-1)^m}{(2m)!}$$
$$\int_{-\infty}^{\tau}d\tau'\left(\frac{H\tau'}{f}\right)^{2m-4} F(\tau',\tau,\mathbf{k}_1,\dots\mathbf{k}_n), \tag{4.14}$$

where we have defined the function $F(\tau',\tau,\mathbf{k}_1,\dots\mathbf{k}_n)$ as

$$F \equiv \int_z\langle 0|\left[\left(u_I(\mathbf{z},\tau')\right)^{2m},\hat{u}_I(\mathbf{k}_1,\tau)\dots\hat{u}_I(\mathbf{k}_n,\tau)\right]|0\rangle. \tag{4.15}$$

Because of invariance under spatial translations, this function is proportional to a $\delta^{(3)}(\sum_j\mathbf{k}_j)$. In addition, it is nonvanishing only for even values of n. The function F may be represented as a sum of diagrams with a vertex located in τ' with $2m$ legs, some of these connected to the n-external legs labeled by the momenta $\mathbf{k}_i$ at a time τ (see Fig. 4.1). It will be enough for us to consider the fully connected diagrams, in which every external leg is attached to the vertex. Let us refer to these contributions as F_c. It is clear that $F_c \neq 0$ only if $2m \geq n$. The number of ways in which the n external legs may be connected to the $2m$ vertex legs is $(2m)!/(2m-n)!$. In addition, F_c will have $m - n/2$ loops resulting from vertex legs that are not attached to any of the external legs. The number of different ways in which such legs can be connected into loops is given by $(2m-n)!/[2^{m-n/2}(m-n/2)!]$.

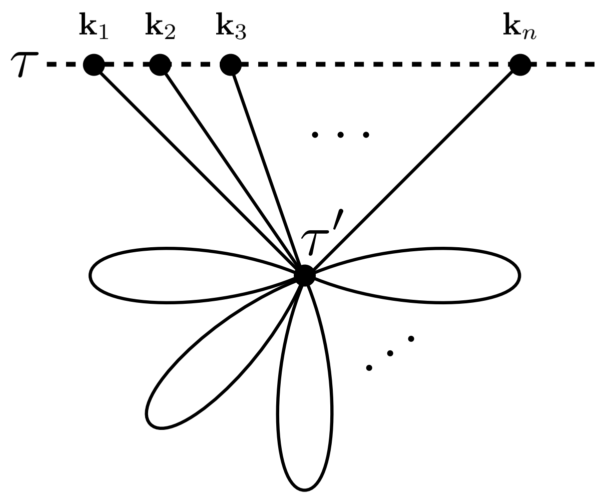

Figure 4.1: A typical connected diagram of the order of Λ^4. All the n-external legs are attached to one of the available $2m$ legs of the vertex. The remaining legs from the vertex become loops.

All of this tells us that F_c will have a combinatorial factor $(2m)!/[2^{m-n/2}(m-n/2)!]$ due to the different possible ways of contracting the various interaction fields u_I present in Eq. (4.15). More precisely, we find

$$\begin{aligned} F_c = & -i\,(2\pi)^3\delta^{(3)}\Big(\sum_j \mathbf{k}_j\Big)\frac{(2m)!}{2^{m-n/2}(m-n/2)!} \\ & \Big[\int_k \Delta(\tau',\tau',k)\Big]^{m-n/2} G_c(\tau',\tau,k_1,\ldots k_n), \end{aligned} \tag{4.16}$$

where we have defined $G_c(\tau',\tau,k_1,\ldots k_n)$ as

$$\begin{aligned} G_c(\tau',\tau,k_1,\ldots k_n) = \; & i\sum_{l=1}^{n}\Delta(\tau,\tau',k_1)\ldots\Delta(\tau,\tau',k_{l-1}) \\ & [\Delta(\tau',\tau,k_l)-\Delta(\tau,\tau',k_l)] \\ & \Delta(\tau',\tau,k_{l+1})\ldots\Delta(\tau',\tau,k_n). \end{aligned} \tag{4.17}$$

Notice that $\int_k \Delta(\tau',\tau',k)$ in Eq. (4.16) represents a closed loop. We may now plug F_c back into Eq. (4.14) to obtain an expression for the connected contribution to the n-point correlation function:

$$\begin{aligned} \langle u(\mathbf{k}_1,\tau)\ldots u(\mathbf{k}_n,\tau)\rangle_c = (-1)^{n/2}\frac{\Lambda^4}{H^4}(2\pi)^3\delta^{(3)}\Big(\sum_j \mathbf{k}_j\Big) \\ \sum_{m=n/2}^{\infty}\frac{1}{(m-n/2)!}\left[-\frac{1}{2}\left(\frac{H\tau'}{f}\right)^2\int_k \Delta(\tau',\tau',k)\right]^{m-n/2} \\ \int_{-\infty}^{\tau} d\tau'\frac{1}{(\tau')^4}\left(\frac{H\tau'}{f}\right)^n G_c(\tau',\tau,k_1,\ldots k_n). \end{aligned} \tag{4.18}$$

Now, one should notice that the m sum, which comes from the expansion of the cosine, involves only closed loop contributions. If one defines $m'=m-n/2$, this sum becomes

$$\sum_{m'}\frac{1}{m'!}\left[-\frac{1}{2}\left(\frac{H\tau'}{f}\right)^2\int_k \Delta(\tau',\tau',k)\right]^{m'} = e^{-\frac{\sigma_0^2}{2f^2}}, \tag{4.19}$$

where we have used (4.11) to identify σ_0. Thus, the resummation of all the loop diagrams leads to the following n-point correlation function

$$\begin{aligned} \langle u(\mathbf{k}_1,\tau)\ldots u(\mathbf{k}_n,\tau)\rangle_c = (-1)^{n/2}(2\pi)^3\delta^{(3)}\Big(\sum_j \mathbf{k}_j\Big) \\ \frac{\Lambda^4}{H^4}e^{-\frac{\sigma_0^2}{2f^2}}\int_{-\infty}^{\tau} d\tau'\frac{1}{(\tau')^4}\left(\frac{H\tau'}{f}\right)^n G_c(\tau',\tau,k_1,\ldots k_n). \end{aligned} \tag{4.20}$$

What remains is to solve the τ' integral. This integral is hard to solve in general, but we may find a useful expression valid in the limit where the $\mathbf{k}_i$ momenta are soft. More precisely, the integration may be divided into two regions $\tau'\in(-\infty,\tau_0)$ and $\tau'\in[\tau_0,\tau]$, with τ_0 chosen in such a way that $k_j|\tau_0|\ll 1$. The first contribution remains finite due to the oscillatory nature of G_c in the asymptotic limit $\tau' \to -\infty$. On the other hand, the second contribution has a piece that diverges as $\tau \to 0$ given by

$$\int_{\tau_0}^{\tau} d\tau' \tau^n \left(\tau'\right)^{n-4} G_c \to \frac{1}{3} \frac{k_1^3 + \cdots + k_n^3}{2^{n-1} k_1^3 \ldots k_n^3} \ln\left(\frac{\tau_0}{\tau}\right). \tag{4.21}$$

As long as the momenta satisfy $k_j |\tau_0| \ll 1$, this expression dominates the τ' integral in the limit $|\tau| \ll |\tau_0|$. This allows us to finally arrive to an expression for the n-point correlation functions:

$$\begin{aligned} \langle u(\mathbf{k}_1, \tau) \ldots u(\mathbf{k}_n, \tau) \rangle_c = (-1)^{n/2} (2\pi)^3 \delta^{(3)} \Big(\sum_j \mathbf{k}_j \Big) \\ \frac{\Lambda^4}{H^4} e^{-\frac{\sigma_0^2}{2f^2}} \left(\frac{H}{f\tau} \right)^n \frac{1}{3} \frac{k_1^3 + \cdots + k_n^3}{2^{n-1} k_1^3 \ldots k_n^3} \ln\left(\frac{\tau_0}{\tau}\right). \end{aligned} \tag{4.22}$$

This result gives us the connected contribution to the n-point correlation functions in momentum space for superhorizon fluctuations (with $|\tau_0| k_j \ll 1$). It captures the effects of the cosine potential to first order in Λ^4 but to all orders in $1/f$.

4.4 Probability distribution function in the long-wavelength limit

Let us recall that cosmological observables connected to inflation, via hot big-bang era initial conditions, are determined by superhorizon fluctuations. Fortunately, we can use Eq. (4.22) to compute n-point correlation functions in coordinate space as long as we focus on long-wavelength contributions. To this end, we may decompose ψ into short- and long-wavelength contributions as $\psi = \psi_{\rm S} + \psi_{\rm L}$, where $\psi_{\rm L}$ contains the contributions from momenta satisfying $k|\tau_0| \ll 1$. Then, given that (4.22) is valid only for soft momenta, we may use it to compute n-point functions $\langle \psi_{\rm L}(\mathbf{x}_1) \ldots \psi_{\rm L}(\mathbf{x}_n) \rangle_c$. In the particular case where the proper distances $L_{ij}(\tau) = |\mathbf{x}_i - \mathbf{x}_j| / H|\tau|$ are of the order of H^{-1} or smaller, at any time $\tau > \tau_0$, it makes no difference to evaluate all the coordinates $\mathbf{x}_j$ at a common value, say $x_j = 0$. Thus, we may compute $\langle \psi_L^n \rangle_c \equiv \langle \psi_{\rm L}(\mathbf{x}_1) \ldots \psi_{\rm L}(\mathbf{x}_n) \rangle_c \big|_{\mathbf{x}_j \to 0}$, which, after using (4.22), is found to be

$$\langle \psi_{\rm L}^n \rangle_c = (-1)^{n/2} n \frac{A^2}{\sigma_{\rm L}^2} e^{-\frac{\sigma_{\rm L}^2}{2f^2}} \left(\frac{\sigma_{\rm L}^2}{f} \right)^n \tag{4.23}$$

(recall that we are assuming even values of n), where we have defined

$$A^2 \equiv \frac{\Lambda^4}{3H^2} \ln\left(\frac{\tau_0}{\tau}\right) e^{-\frac{\sigma_{\rm S}^2}{2f^2}}. \tag{4.24}$$

In the previous expressions, we have introduced $\sigma_{\rm S}^2$ and $\sigma_{\rm L}^2$ as the short- and long-wavelength contributions to the variance, in such a way that $\sigma_0^2 = \sigma_{\rm S}^2 + \sigma_{\rm L}^2$. In particular,

$$\sigma_L^2 = H^2 \tau^2 \int_{k_L} u_k^I(\tau) u_k^{I*}(\tau), \tag{4.25}$$

where the label k_L tells us that we are integrating in a range such that $k \ll |\tau_0|^{-1}$. Now, we may wonder about what class of PDF gives n-point correlation functions such as those of Eq. (4.23). To be precise, there must exist a probability distribution function $\rho(\psi)$ such that

$$\langle \psi_{\rm L}^n \rangle = \int d\psi \; \psi^n \rho(\psi). \tag{4.26}$$

Notice that $\langle \psi_{\rm L}^n \rangle_c$ shown in Eq. (4.23) gives the connected part of $\langle \psi_{\rm L}^n \rangle$. This is the contribution coming from the right-hand side of (4.26) that brings the higher number of powers of $1/f$. This is because each external leg in the diagrammatic expansion of $\langle \psi(\mathbf{x}_1) \dots \psi(\mathbf{x}_n) \rangle$ performed in the previous section, carries a factor $1/f$ when it is connected to a vertex of the order of Λ^4. To find $\rho(\psi)$, we may proceed by adopting the following trick. We first rewrite Eq. (4.23) in the following way:

$$\langle \psi_{\rm L}^n \rangle_c \; = \left(1 + 2 f^2 \frac{\partial}{\partial \sigma_{\rm L}^2} \right) \left[(-1)^{n/2} \frac{A^2}{2 f^2} e^{-\frac{\sigma_{\rm L}^2}{2 f^2}} \left(\frac{\sigma_{\rm L}^2}{f} \right)^n \right]. \tag{4.27}$$

Now, instead of looking for a PDF that gives us back (4.23), we may look for a distribution $\xi(\psi)$ such that it gives us back the expression inside the square brackets:

$$\int d\psi \psi^n \xi(\psi) \Big|_c = (-1)^{n/2} \frac{A^2}{2 f^2} e^{-\frac{\sigma_{\rm L}^2}{2 f^2}} \left(\frac{\sigma_{\rm L}^2}{f} \right)^n , \tag{4.28}$$

where the subscript c denotes that we are keeping only the connected part after the integration is performed. It should be clear from (4.26)–(4.28) that the relation between ρ and ξ is given by

$$\rho(\psi) \Big|_c = \xi(\psi) \Big|_c + 2 f^2 \frac{\partial}{\partial \sigma_{\rm L}^2} \xi(\psi) \Big|_c . \tag{4.29}$$

Next, it is not difficult to see that the part of the distribution $\xi(\psi)$ that gives the desired connected contribution must be proportional to the combination $\exp(-\psi^2/2\sigma_{\rm L}^2)\cos(\psi/f)$. This implies that

$$\xi(\psi) \Big|_c = \frac{1}{\sqrt{2\pi\sigma_{\rm L}^2}} e^{-\frac{\psi^2}{2\sigma_{\rm L}^2}} \frac{A^2}{2 f^2} \cos\left(\frac{\psi}{f} \right). \tag{4.30}$$

Now, putting together Eqs. (4.29) and (4.30), and taking into account the contributions from the omitted disconnected diagrams (which, after all, come from the Gaussian free part of the theory), we finally find that the desired PDF is exactly given by

$$\rho(\psi) = \frac{e^{-\frac{\psi^2}{2\sigma_{\rm L}^2}}}{\sqrt{2\pi}\sigma_{\rm L}} \left[1 - A^2 \left(\frac{\sigma_{\rm L}^2 - \psi^2 - \sigma_{\rm L}^4/f^2}{2\sigma_{\rm L}^4} \right) \cos\left(\frac{\psi}{f} \right) \right]. \tag{4.31}$$

This is one of our main results. It describes the PDF of measuring the amplitude of ψ at a given value. Equation (4.31) is valid as long as the second term inside the square brackets remains small compared to unity. Besides this limitation, the result tells us that the probability is larger at those values that minimize the cosine potential, as it should.

4.5 Discussion

Let us do some guesswork in order to generalize Eq. (4.31) to the case in which the second term inside the square brackets is allowed to be large. If these terms come from exponentials, we are immediately led to the following possible resummation:

$$\rho(\psi) = \frac{1}{\mathcal{N}} \frac{e^{-\frac{\psi^2}{2\sigma^2(\psi)}}}{\sqrt{2\pi}\sigma(\psi)} \exp\left[\frac{A^2}{2f^2}\cos(\psi/f)\right], \tag{4.32}$$

where $\mathcal{N}$ is a normalization constant that depends on σ_L, f, and A and $\sigma(\psi)$ is a function given by

$$\sigma(\psi) \equiv \sigma_{\rm L} \exp\left[\frac{A^2}{2\sigma_{\rm L}^2}\cos(\psi/f)\right]. \tag{4.33}$$

There are good reasons to think that Eq. (4.32) is the correct PDF valid for all values of the parameter A, keeping in mind that we are interested in the description of long-wavelength modes. If this is the case, then (4.32) would be the result of taking into account those terms in Eq. (4.10) beyond the linear order in Λ^4. This conjecture is reinforced by the fact that Eq. (4.32) acquires the correct nontrivial expression in the limit $f \to +\infty$ and $\Lambda \to +\infty$, while keeping $m \equiv \Lambda^2/f$ fixed. Here, m is the mass of ψ at the stable value $\psi = 0$; that is, $m^2 = v''(\psi)\big|_{\psi=0}$.

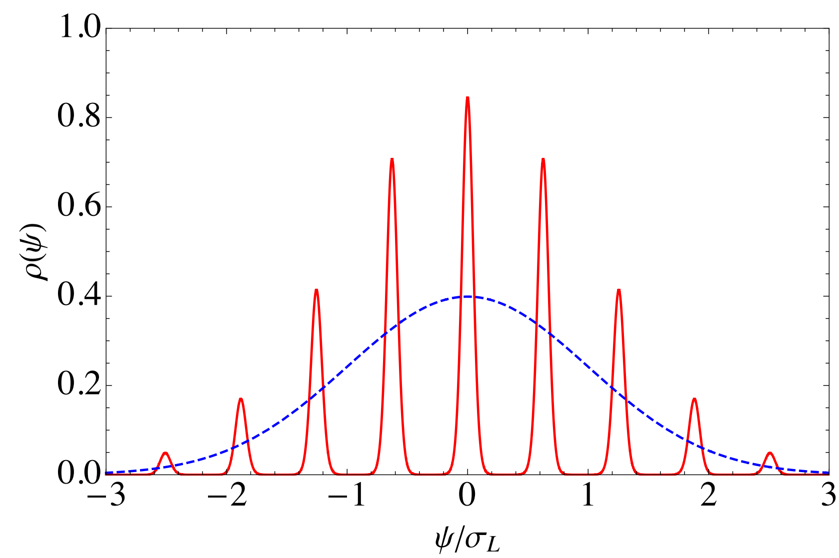

Figure 4.2: The figure shows an example of the PDF of Eq. (4.32) for the choice of parameters $f/\sigma_L = 10^{-1}$ and $A^2/\sigma_L^2 = 10^{-1}$ (red solid curve). For comparison, we have plotted a Gaussian distribution of variance σ_L (blue dashed curve). The distributions are not normalized.

The PDF of Eq. (4.32) is plotted in Fig. 4.2 for certain values of A and f. The function consists of a multimodal distribution where $\Delta\psi = 2\pi f$ determines the distance between consecutive peaks. On the other hand, A controls the magnitude with which the probability of measuring a value of ψ laying in the vicinity of a minimum or maximum of $v(\psi)$ is enhanced or suppressed respectively. Notice that the non-Gaussian effects due to the periodicity of the potential are relevant only if $f < \sigma_L$. Given that σ_L is of the order of H, our result calls for sub-Planckian values of f, which in fact are favored in string theory [117, 118] and quantum gravity [119]. In what follows, we discuss three possible consequences of our result that might be worth exploring.

4.5.1 Isocurvature fluctuations after inflation

It is common lore that scalar fields with large masses during inflation lead to the production of suppressed levels of isocurvature modes in the CMB. Our results show that this is not necessarily the case: If fields are able to tunnel the potential barriers separating their VEV from other minima, then the PDFs describing them at the end of inflation can be highly non-Gaussian, regardless of how massive they are. In the case that we have studied, the mass of the axion about any local minima is given by $m = \Lambda^2/f$, and a large value (as compared to H) does not preclude fluctuations from leaking from one minimum to another. Thus, a landscape with a rich structure may have strong quantum effects even with massive extra scalar fields. It would be interesting to study how these effects could affect current studies of inflation with many fields such as those of Refs. [114–116], and CMB observables.

Some recent results appeared in the context of axionic fields treated as isocurvature modes. In [129], we can see that the non-Gaussian statistics generated by the mechanism in this chapter are transferred to the statistics of the comoving curvature perturbations $\mathcal{R}$. Recent extensions to any kind of potential for the isocurvature modes were developed in [130], giving hints of how a similar mechanism could give information about the shape of the inflationary landscape.

4.5.2 Role of inflation to determine SM properties

Our results also reinforce the intriguing possibility that the SM of particle physics could be just one realization among many others, taking place in a confined region of our Universe [120]. This would be the case if the field ψ determines the value of couplings of other fields that appear in the SM, such as the Higgs field. The PDF of Eq. (4.32) tells us that different patches of the Universe could emerge out of inflation with a long-wavelength value of ψ corresponding to a minimum of $v(\psi)$ different from $\psi = 0$, which, by assumption, was the vacuum expectation value of the field. Our results should, in principle, allow one to compute the probability with which these patches emerge.

4.5.3 Dark matter

Another interesting possibility is that ψ corresponds to dark matter (DM) [121–123]. If this is the case, our result predicts that, for certain values of the parameters, it could be possible that within our observable Universe the DM distribution started with nontrivial initial conditions, resulting from the multimodal PDF (4.32). We foresee that, if the initial conditions for DM come from (4.32), then there should be certain observational signatures that could serve as a test. It is a pending challenge to deduce them.

Conclusions

During this thesis, we have studied the effect of extra degrees of freedom on the primordial statistics of the universe during the epoch of Inflation. This period is characterized by an accelerated expansion which could be driven by the evolution of one or more scalar fields. Along with their evolution, the fields can interact with different kind of sources that can leave an imprint in the primordial statistics of the curvature perturbations.

One interesting effect appeared when we parametrized the deviations from a de-Sitter background with a time dependent function. This induced a self-interaction term for both the curvature perturbations and the tensor modes in the Einstein-Hilbert Lagrangian. The in-in formalism turned out to be a very useful tool to compute primordial correlators for both scalar and tensor degrees of freedom coming from the perturbed metric. We found that the self-interactions due to the inclusion of the time-dependent background functions allow us to relate the deviations from scale invariance, or features, from the scalar power spectrum with the features in the power spectrum for tensor modes.

In particular, we have studied the possible appearance of scale-dependent features in the power spectrum of primordial tensor perturbations due to non-trivial inflationary dynamics in a model-independent way. Our main result is eq. (3.35) – or eqs. (3.38), (3.39) in the more general case of EFT's with a sound speed – which consist of relations linking features in the tensor power spectrum to those appearing in the scalar power spectrum, allowing us to estimate the amplitude and shape of the former given the latter. In general, we find that the tensor spectrum is expected to be featureless: Indeed, eq. (3.35) shows that any feature appearing in the tensor spectrum is generically suppressed with respect to those appearing in the scalar one for two reasons: firstly due to slow-roll [77], and more importantly, due to the fact that features should, in general, be sharp enough in order to leave an imprint in the CMB.

One may wonder about other mechanisms producing features in the tensor sector of the theory. For instance, in principle, we could consider a Lagrangian describing the dynamics of tensor modes with a sound speed c_t experiencing rapid variations producing features in the tensor spectrum. However, in [96] it was shown that under a disformal transformation, models with a non-trivial tensor sound speed (and canonical scalar sector) map into models with a non-trivial scalar sound speed (and canonical tensor sector). Since the spectra are invariant under such a transformation, our formalism to relate features in the tensor spectrum to those appearing in the scalar spectrum would continue to be valid. Moreover, in the special case where only c_t varies, the disformal transformation would lead to an equivalent system where both c_s and H vary, but in such a way that the scalar spectrum remains featureless [78]. Given that we are interested in understanding the consequences of features in the scalar spectrum on the tensor one, this class of situations is out of our scope.

Current CMB observations show the existence of departures from scale invariance in the power spectrum of primordial curvature perturbations in the multipole range $\ell \sim 20$. If we interpret this behavior as the result of the dynamics of inflation, we are led to conclude that the tensor power spectrum will not show any consequential departure from scale invariance in this region. The importance of this conclusion may be appreciated more clearly by inverting the statement: If tensor modes are observed to have strong departures from scale invariance in the aforementioned multipole range, then we will have good reasons to suspect that the departures appearing in the scalar spectrum are not of primordial origin.

As we mentioned, the quantum fluctuations for both scalar and tensor degrees of freedom allow us to relate features in the power spectra coming from its respective self-interactions. The extra self-interacting terms turned out to be hidden in the description of the background quantities from a quasi de-Sitter universe. In the same way that this interaction was hidden, different extra degrees of freedom could be concealed due to the energy scale in which the theory is developed. This was the motivation for the next topic this thesis. Axion-like fields appear in the different context of inflationary models and a successful description of their properties could tell us something about the primordial universe.

The second result appeared during the analysis of the inflationary landscape, in this case, generated by the inclusion of an extra scalar field called axion. The axion potential is described by a periodic function which introduces many vacua in the landscape. We studied the effect of quantum fluctuations of the axion-like field evolving during the primordial expansion given by inflation. Since the axion-like field is evolving through the landscape, we wanted to describe its statistical behavior as it evolves along with the inflaton field. For this, we computed an n-point correlation function, in the connected limit, of the axion field using the in-in formalism. Starting from this correlation, we focused on the long-wavelength limit for the scales and derived a probability distribution function (PDF). This result shows how these spectator fields are able to tunnel between the various vacua of the axionlike potential [1]. We concluded that using the PDF, the field had non-perturbative non-Gaussian statistics for its quantum fluctuations. This non-Gaussian statistic could be transferred to the curvature perturbations and, in principle, induce a different type of non-Gaussianity (tomographic non-Gaussianity) for the primordial curvature perturbations during inflation.

[1] It would be interesting to understand the relation between our results and other well-known tunneling solutions in expanding backgrounds, such as the Hawking-Moss solution [124].

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