A Multicriteria Stochastic Optimization Framework for Sustainable Forest Decision Making Under Uncertainty

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Abstract

A core process in forestry planning corresponds to the design of optimal harvesting policies along with road network layouts. In the most common setting, decision makers seek for solutions that maximize the profit of the forest while respecting operative and market constraints. Due to the long-term nature of the industry, the inherent uncertainty in both forest growth and market conditions should be taken into account. Nowadays, forest planning must target towards a sustainable management; the maximization of carbon sequestration and the minimization of land erosion are two common environmental goals.

The planning challenge addressed in this paper integrates uncertainty of future forest growth and timber prices with the need for considering three criteria; net-present value, carbon sequestration, and land erosion caused by the road construction within the forest. By using mathematical programming tools and stochastic optimization techniques, we develop a stochastic multicriteria model that enables decision makers to have not only one, but a pool of long term planning policies.

Moreover, a risk-averse variant of the framework is also considered. To the best of our knowledge, this is the first time that this type of forestry planning setting, which responds to the new challenges of the industry, is addressed.

The proposed tool is used on an eucalyptus forest located in Portugal; the obtained results show the benefit of the proposed framework for producing a pool of sustainable forest plans with efficient trade-offs among the three considered criteria.

Keywords: Forest Management, Uncertainty, Multicriteria Optimization, Stochastic Programming, Risk Management

1. Introduction and Motivation

Forest management concerns the definition of strategic, tactical and operational planning decisions whose performances are evaluated under conflicting criteria. Such planning decisions shall endorse economical, operational and regulation targets for ensuring the viability of the project. In a typical setting, decision makers need to design a forest management plan comprised by harvesting and road building decisions. The first group of decisions are embodied in a harvest scheduling plan, i.e., decision makers decide on when and how the different units of the forest must be harvested. The second group of decisions are planned through a building construction schedule that must enable (i) to access to the land units being harvested, and (ii) to transport the extracted timber to the selling points [the reader is referred to 21, 48, for general references on forest management]. Due to the nature and time span of the forest decision making context, managers must take into account the variability of market conditions (e.g., wood price) and of resource availability (e.g., timber yield), among other sources of uncertainty. Moreover, in today's forestry industry, the climate change process entails ecological concerns that incorporate additional difficulties for managers. On the one hand, there is more uncertainty regarding the forest productivity; and on the other hand, it calls for the fulfillment of new environmental regulations for guaranteeing the sustainability of the forestry industry [25, 28].

There are many papers addressing the different sources of uncertainty in forest planning [see 4, 33, 35, 50, for thorough reviews]. The underlying presence of uncertainty has been typically managed by means of scenarios [see, e.g., 1, 16, 18, 27]; nonetheless, alternative uncertainty models, such as fuzzy sets have been also used [see, e.g. 26].

As pointed out before, over the last years, managers are more and more concerned about sustainability issues associated with forest planning; as a matter of fact, multicrtieria models for sustainable forest plannings have been proposed in the last decade [see, e.g., 38, 40, and the references therein]. Moreover, multicriteria forest planning has emerged as one of the most active areas in the field of forest management. In [13] one can find an extensive review of related topics; later references on the same area can be found in [9, 44, 43]. Issues associated with environmental sustainability such as carbon sequestration, land erosion, water use efficiency, or responsible exploitation planning, have become inevitable aspects that managers must take into account in their planning process [see, e.g., 9, 38, 40, 45].

Some of the environmental requirements that managers face can be addressed by imposing spatial constraints to their harvest schedules. More precisely, environmental goals such as wildlife protection, water quality protection and scenic beauty preservation, can be accomplished by specially devised spatial constraints such as adjacency among harvest areas [see, e.g., 20, 49], or connectivity requirements [see, e.g., 10, 7]. The reader is referred to [46] for an in-deep discussion on optimization forest planning problems dealing with spatial constraints.

Based on the different works presented so far, we can conclude that effective forestry decision-aid tools shall be able to cope with several economic, technical and environmental issues. The development of such type of integrated framework corresponds to the core of our paper. From the decision making point of view, one of the main attributes of the proposed framework is that it provides not a single but a handful of solutions, each of them embodying different trade-offs among the different criteria. This means that decision makers will be enabled to include additional considerations, some of them that might have not been incorporated into the optimization model, when selecting the harvesting policy that will be implemented.

Contribution and Paper Outline Despite of the broad body of literature addressing the different aspects described above, to the best of our knowledge, no model tackling all these aspects simultaneously has been developed. In other words, our first contribution is the development of a stochastic forest management framework for deciding a harvest and road building schedule that addresses (i) economic and sustainability criteria, (ii) uncertainty from market conditions and forest growth, and (iii) spatial (adjacency) requirements.

Moreover, we formulate a risk-averse variant of the model so as to design forest management policies that respond relatively well even in adverse future events. Along with the definition of these mathematical models, we also implement an algorithmic strategy for computing pools of Pareto efficient solutions that enable decision makers to select the policies that better fit their priorities.

Using a real Eucaliptus forest located in the central region of Portugal, and scenario-based estimations of future conditions of both forest yield and market selling prices, we computationally investigate the capacity of the model for producing solutions exhibiting different trade-offs among the considered criteria. Computational results suggest that, with the appropriate setting, it is possible to compute harvest and road constructions schedules that perform well for both economical and environmental criteria. Moreover, complementary results show the ability of the risk-averse model for reducing the impact of negative future possible scenarios.

The paper is organized as follows. Mixed integer programming (MIP) models for the considered problem and the corresponding algorithmic scheme are presented in the Materials and Methods section 2. In Section 3 we report numerical results on the study case. Finally, conclusions and paths for future work are drawn in Section 4.

2. Material and Methods

2.1 Multicriteria Optimization and Stochastic Optimization in Forest Management

Already from the 60's, researches have been studying optimization schemes for addressing multicriteria decision making under uncertainty [see 11, for one of the earliest reference on this topic]. One of the most typical approaches corresponds to stochastic goal programming [see, e.g., 2, 3, 5, and the references therein]. This approach allows to handle several criteria at once allowing decision makers to define preferences among them; nonetheless, it requires decision makers to set explicit goals or target values that shall be achieved by the objective function associated with each criterion. Hence, the approach is suitable only for conditions in which decision makers have clear insights regarding these target values and their feasibility. For a relatively recent review on different multiobjective or multiocriteria stochastic programming methods, we refer the reader to [6].

The development of multicriteria stochastic optimization models in forestry applications is not new; as a matter of fact, one of the first references dates from 1997 [34] where a stochastic multiobjective framework for stand management decisions is proposed. More recent applications of similar tools on forest management optimization can be found in [17] and [24].

2.2 The Multicriteria Stochastic Forest Management Problem

We now provide a description of our multicriteria decision making model for sustainable forest management. As we will show, this model addresses uncertainty and spatially constrained harvesting decisions along with a road building schedule.

Sets The forest is divided into (*management*) *units* or *stands*, which comprise set $\mathcal{I}$. Associated with each unit $i \in \mathcal{I}$, there is a set of *adjacent* units $\delta(i) \subset \mathcal{I}$, i.e., all units sharing a common border.

Commonly, within the forest there are intermediate facilities where harvested timber is stored; such facilities, which are commonly called *origins*, are represented by set $\mathcal{O}$. For each origin $o \in \mathcal{O}$, there is a set units, $\eta(o) \subseteq \mathcal{I}$, which correspond to the units whose harvested timber will be stored in o. Likewise, for each unit $i \in \mathcal{I}$, we denote by $o_i \subseteq \mathcal{O}$ to the set of origins where the corresponding harvested timber can be stored. Along with these intermediate facilities, we also need to consider (i) a set of *exit* points $\mathcal{S}$, from where the extracted timber leave the forest, and (ii) a set of *intersection* points $\mathcal{J}$, which correspond to specifically located crossroads inside the forest.

The sets described before, whose elements correspond to points within the forest, are part of the network of potential roads that shall be built for the transportation of the timber. Such network is denoted by set $\mathcal{A}$, which is comprised by roads among origins ($\subset \mathcal{O} \times \mathcal{O}$), roads among origins and intersections ($\subset \mathcal{O} \times \mathcal{J}$), roads among intersections ($\subset \mathcal{J} \times \mathcal{J}$), roads among origins and exit points ($\subset \mathcal{O} \times \mathcal{S}$), roads and among intersections and exit points ($\subset \mathcal{J} \times \mathcal{S}$). We consider that the elements encompassing set $\mathcal{A}$ can be divided into two sets. Set $\mathcal{A}_1 \subset \mathcal{A}$ is regarded as the set of *weak* roads, i.e., roads whose construction associates higher levels of land erosion; and set $\mathcal{A}_2 \subset \mathcal{A}$, which are roads whose construction does not entail particular damage on the land' surface (note that $\mathcal{A}_2 \subset \mathcal{A}$, $\mathcal{A}_1 \cup \mathcal{A}_2 = \mathcal{A}$ and $\mathcal{A}_1 \cap \mathcal{A}_2 = \emptyset$). For ease of exposition, we identify the set of roads associated with the different points, i.e., $\gamma^-(o) \subset \mathcal{A}$ roads entering to origin point $o \in \mathcal{O}$, $\gamma^+(o) \subset \mathcal{A}$ roads leaving from origin point $o \in \mathcal{O}$, $\gamma^-(j) \subset \mathcal{A}$ roads entering to intersection point $j \in \mathcal{J}$, $\gamma^+(j) \subset \mathcal{A}$ roads leaving from intersection point $j \in \mathcal{J}$, and $\gamma^-(s) \subset \mathcal{A}$ roads entering to exist point $s \in \mathcal{S}$.

Besides these sets that characterize the topological structure of the forest, the planning horizon will be encoded by a set of 1-year time periods $T = \{1, \ldots, t_{max}\}$, the set of growth and yield scenarios is given by Ω, and future price realizations are represented by set Φ. Since sets Ω and Φ respond to different sources of uncertainty, and it is realistic to assume that they are independent [see 47], we define set Π as the set of *all* possible scenarios $\Pi = \Omega \times \Phi$, and we associate probability $\rho^\pi \geq 0$ to each of them (clearly, $\sum_{\pi \in \Pi} \rho^\pi = 1$).

Parameters The state of the forest during the planning horizon, for the different scenarios, is expressed by the following parameters. Let Vh_{it}^π be volume of wood that can be harvested from unit $i \in \mathcal{I}$ in period $t \in T$ if scenario π occurs [m^3]; likewise, let Vf_{it}^π be the standing volume of wood, at the end of the planning horizon, if unit $i \in \mathcal{I}$ is harvested in period $t \in T$ if scenario π occurs [m^3]. From a sustainability point of view, let C_{it}^π be the average mass of carbon that is captured during the planning horizon in case unit $i \in \mathcal{I}$ is harvested in period $t \in T$ if scenario π occurs [Ton].

The future market conditions are characterized by parameter p_t^π, which corresponds to the discounted (at some fixed and known discount rate) profit obtained by harvesting a cubic meter of timber in period t if scenario π is realized [\$/m^3]; this value depends on the sale price (subject to uncertainty), the harvesting cost and the production cost. Along with this market indicator, we also consider $\overline{D}_t$ and $\underline{D}_t$ which correspond to an upper and lower bound, respectively, on total demand of wood in period $t \in T$ [m^3].

Two important requirements in forest management are characterized by the following two parameters; V_f is the required volume of standing wood at the end of the planning horizon [m^3], and α is a harvesting balance factor which ensures a maximum variability of the harvested volume between consecutive years.

Roads are also characterized by several parameters. From a planning point of view, let c_{klt} be the cost of building road $(kl) \in \mathcal{A}$ in period $t \in T$ [\$]; and let d_{klt} be the length of road $(kl) \in \mathcal{A}$ [km]. Complementary, and from a operative point of view, let u_{klt} be the capacity of road $(kl) \in \mathcal{A}$ in period $t \in T$ [m^3].

Variables In the proposed model, we consider three types of variables; one associated with harvesting decisions, one associated with road building decisions, and a third one associated with timber transportation decisions. The first set of variables is given by vector $\mathbf{x} \in \{0,1\}^{|\mathcal{I}|\times|T|\times|\Pi|}$, so that

$$x_{it}^{\pi} = \begin{cases} 1, & \text{if unit } i \in \mathcal{I} \text{ is harvested in period } t \in T \text{ if scenario } \pi \in \Pi \text{ occurs} \\ 0, & \text{otherwise} \end{cases}.$$

The second, is given by vector $\mathbf{y} \in \{0,1\}^{|\mathcal{A}|\times|T|\times|\Pi|}$, such that

$$y_{klt}^{\pi} = \begin{cases} 1, & \text{if road } (kl) \in \mathcal{A} \text{ is built in period } t \in T \text{ if scenario } \pi \in \Pi \text{ occurs} \\ 0, & \text{otherwise} \end{cases}.$$

The third set of variables, given by $\mathbf{f} \in \mathbb{R}_{\geq 0}^{|\mathcal{A}|\times|T|\times|\Pi|}$, is such that f_{klt}^{π} corresponds to the amount of wood transported through road $(kl) \in \mathcal{A}$ in period $t \in T$ if scenario $\pi \in \Pi$ occurs [m^3].

Constraints As said in the introduction, one of the issues addressed by our model corresponds to harvest adjacency requirements; such requirements impose that if a given unit $i \in \mathcal{I}$ is harvested at a given period, say t, then none of the adjacent units (i.e., those in $\delta(i)$) can be harvested at the same time. This constraint is ensured by the following inequality,

$$\sum_{a \in \delta(i)} x_{at}^{\pi} \leq 1 - x_{it}^{\pi}, \ \forall i \in \mathcal{I}, \ \forall t \in T, \ \forall \pi \in \Pi. \tag{ADJ}$$

Complementary, we assume that units can be harvested only once during the planning horizon, which means that constraint

$$\sum_{t \in T} x_{it}^{\pi} \leq 1, \ \forall i \in \mathcal{I}, \ \forall \pi \in \Pi \tag{H}$$

must always hold.

At each period, harvested timber shall be transported from the inner areas of the forest towards the exit points $\mathcal{S}$ through the available road network. From the units, harvested wood first flows to origin points; this is expressed by constraint

$$\sum_{i \in \eta(o)} x_{it}^{\pi} V h_{it}^{\pi} + \sum_{(ko) \in \gamma^{-}(o)} f_{kot}^{\pi} = \sum_{(ol) \in \gamma^{+}(o)} f_{olt}^{\pi}, \ \forall o \in \mathcal{O}, \ \forall t \in T, \ \forall \pi \in \Pi, \tag{F.1}$$

which ensures that the wood that comes from both, harvested units and other incoming roads, shall exit the origin point. As well as for origin points, intersection points shall also verify flow conservation, meaning that constraint

$$\sum_{(kj)\in\gamma^-(j)} f^{\pi}_{kjt} = \sum_{(jl)\in\gamma^+(j)} f^{\pi}_{jlt},\ \forall j \in \mathcal{J},\ \forall t \in T,\ \forall \pi \in \Pi, \tag{F.2}$$

shall always hold. Timber that flows towards the exist points should satisfy the demand requirements, given by $\overline{D}_t$ and $\underline{D}_t$, at each period; this is expressed by

$$\underline{D}_t \le \sum_{s\in\mathcal{S}} \sum_{(ks)\in\gamma^-(s)} f_{kst} \le \overline{D}_t,\ \forall t \in T,\ \forall \pi \in \Pi, \tag{F.3}$$

which basically forces that all the timber volume that flows towards the exit points shall be greater or equal (resp. less or equal) than $\underline{D}_t$ (resp. $\overline{D}_t$). Complementary, we assume that no stock is maintained between periods, meaning that all harvested timber should leave the forest. Such requirement is modeled by

$$\sum_{i\in\mathcal{I}} Vh^{\pi}_{it}x^{\pi}_{it} = \sum_{s\in\mathcal{S}} \sum_{(ks)\in\gamma^-(s)} f^{\pi}_{kst},\ \forall t \in T,\ \forall \pi \in \Pi. \tag{F.4}$$

The constraints presented so far only concern the flow balance of timber within the road network. For ensuring the feasibility of such flow, we have to force that if there is flow on road $(kl) \in \mathcal{A}$, then the corresponding road was built, and the volume of such flow does not exceed the capacity of the road, i.e.,

$$f^{\pi}_{klt} \le U_{klt} \sum_{\theta\le t} y^{\pi}_{kl\theta},\ \forall (kl) \in \mathcal{A},\ \forall t \in T,\ \forall \pi \in \Pi. \tag{F.5}$$

Finally, any road building schedule should be such that roads are built only once; this is accomplished by imposing constraint

$$\sum_{t\in T} y^{\pi}_{klt} \le 1,\ \forall (kl) \in \mathcal{A},\ \forall \pi \in \Pi. \tag{F.6}$$

One requirement that contributes to the sustainability of the forest as an economic activity, is to ensure that timber production between consecutive years does not change abruptly; this is guarantee by the following so-called *production balance* constraints,

$$\sum_{i\in\mathcal{I}} Vh^{\pi}_{it}x^{\pi}_{it} \le (1+\alpha) \sum_{i\in\mathcal{I}} Vh^{\pi}_{it-1}x^{\pi}_{it-1},\ \forall t \in T\backslash\{1\},\ \forall \pi \in \Pi \tag{P.1}$$

$$\sum_{i\in\mathcal{I}} Vh^{\pi}_{it}x^{\pi}_{it} \ge (1-\alpha) \sum_{i\in\mathcal{I}} Vh^{\pi}_{it-1}x^{\pi}_{it-1},\ \forall t \in T\backslash\{1\},\ \forall \pi \in \Pi, \tag{P.2}$$

that force that the total volume harvested at period t can be at least (resp. at most) $(1-\alpha)$ (resp.$(1+\alpha)$) times what was harvested in $t-1$. Along with this production balance requirements, decision makers are typically expected to ensure that a final volume

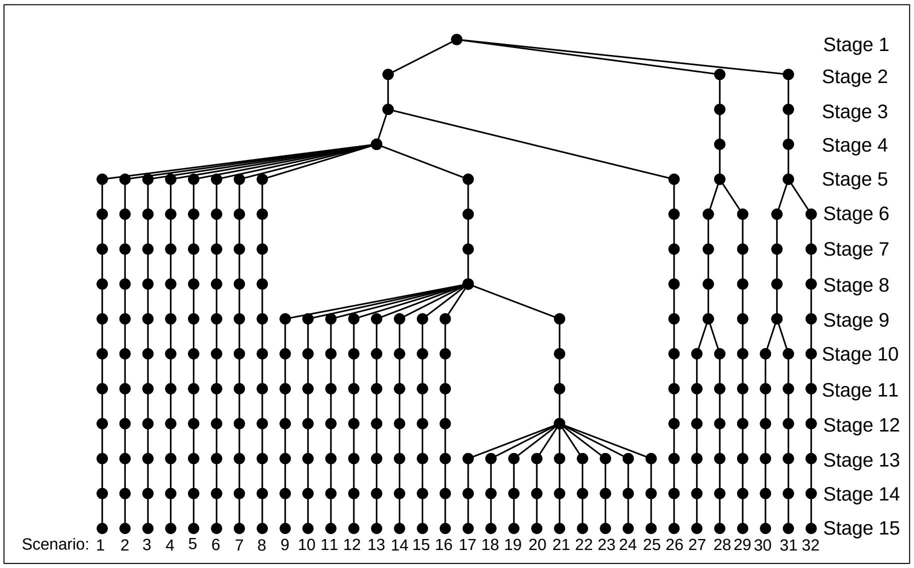


Figure 1: An example of (temporal) dependencies among a set of 32 scenarios across 15 stages.

of standing forest; in other words, the constraint

$$\sum_{t \in T} \sum_{i \in \mathcal{I}} x_{it}^{\pi} V f_{it}^{\pi} \geq V_f, \; \forall \pi \in \Pi \tag{P.3}$$

must be satisfied in order to contribute to the economical and ecological sustainability of the forest.

Since we are dealing with a stochastic program, any feasible solution embodied by a triplet $(\mathbf{x}, \mathbf{y}, \mathbf{f})$ should also satisfy the so-called non-anticipativity constraints, i.e.,

$$\mathbf{x}_t^{\pi} = \mathbf{x}_t^{\pi'}, \; \forall t \in T \backslash \{t_{max}\}, \; \forall \pi, \pi' \in \Pi^g, \; \forall g \in \mathcal{G}^t \tag{NA.1}$$

$$\mathbf{y}_t^{\pi} = \mathbf{y}_t^{\pi'}, \; \forall t \in T \backslash \{t_{max}\}, \; \forall \pi, \pi' \in \Pi^g, \; \forall g \in \mathcal{G}^t \tag{NA.2}$$

$$\mathbf{f}_t^{\pi} = \mathbf{f}_t^{\pi'}, \; \forall t \in T \backslash \{t_{max}\}, \; \forall \pi, \pi' \in \Pi^g, \; \forall g \in \mathcal{G}^t, \tag{NA.3}$$

where $\mathcal{G}^t$ corresponds to the set of scenarios that are indistinguishable up to period t. Complementary, one can also state that these constraints ensure that at each period t, the harvesting, road building, and timber flow decisions should depend only on information available at the time of the decision, i.e., on an observed realization of the economical and ecological parameters up to period t, and not on future observations. The consideration of these constraints is independent of the particular stochastic behavior of the uncertain parameters; hence, even if scenarios are equiprobable, they must be included as long as some scenarios share common branches along the *scenario tree* [37]. In Figure 1 we show an example of the typical structure of a scenario tree and how this encodes their temporal interdependence. In this example, there are 32 scenarios that evolve across 15 stages or periods. We can see that, for instance, scenarios 1 and 32 coincide only in the first stage, while scenarios 17 and 25 are the same up to stage 12. Without the non-anticipativity constraints, such temporal relations would be neglected, and the problem would decompose into $|\Pi|$ (32 in the case of the example) independent deterministic problems, one for each scenario; however, the obtained solutions are likely to lack of practical interpretation.

Although all the previously presented inequalities are enough for characterizing the set of feasible solutions of the stochastic program, it is possible to strengthen the formulation by adding the following two inequalities. First, we observe that a unit i can be harvested, in a period t, if and only if there is at least one (previously) built road *leaving* one of the origin points to which i is associated with; in other words,

$$x^{\pi}_{it} \leq \sum_{\theta \leq t} \sum_{(o_i k) \in \gamma^{+}(o_i)} y_{o_i k \theta}, \ \forall i \in \mathcal{I}, \ \forall t \in T, \ \forall \pi \in \Pi. \tag{S.1}$$

Second, and following similar arguments, it is clear that at intersection points, a road $(kl) \in \mathcal{A}$ is built if and only if another road incident to $k \in \mathcal{J}$ is (previously) built, i.e., constraint

$$y^{\pi}_{klt} \leq \sum_{\theta \leq t} \sum_{(rk) \in \gamma^{-}(k)} y^{\pi}_{rk\theta}, \ \forall (kl) \in \mathcal{A} \mid_{k \in \mathcal{J}}, \ \forall t \in T, \ \forall \pi \in \Pi, \tag{S.2}$$

ensures that no isolated road is built.

For ease of exposition, we define set $R(\Pi)$ as the set of all triplets $(\mathbf{x}, \mathbf{y}, \mathbf{f})$ simultaneously satisfying constraints (ADJ), (H), (F.1)-(F.6), (P.1)-(P.2), (NA.1)-(NA.2), and (S.1)-(S.2). A given triplet $(\mathbf{x}, \mathbf{y}, \mathbf{f}) \in R(\Pi)$ will be referred to as a (feasible) forest management *policy*.

Problem Formulation As said before, our model addresses economic and environmental sustainability criteria. Concretely, the considered criteria are (i) the net-present value (NPV), (ii) the carbon sequestration, and (iii) the land erosion; they are incorporated into the model by means of three objective functions.

The economic criterion corresponds to the value of the forest expressed by its net present value. For a given $(\mathbf{x}, \mathbf{y}, \mathbf{f}) \in R(\Pi)$, the corresponding *expected* net present value, is expressed by

$$NPV(\mathbf{x}, \mathbf{y}, \mathbf{f}) = \sum_{\pi \in \Pi} \rho^{\pi} \left(\sum_{t \in T} \sum_{i \in \mathcal{I}} p^{\pi}_{t} V h^{\pi}_{it} x^{\pi}_{it} - \sum_{t \in T} \sum_{(kl) \in \mathcal{A}} c_{klt} y^{\pi}_{klt} \right),$$

i.e., the expected value of the difference between the profit obtained from the harvested wood and the costs incurred by building the required roads. Note that this NPV criterion is probably the most common one when defining forest management policies [see, e.g., 29].

As for environmental criteria, in this paper we consider carbon sequestration and land erosion [see, e.g., 38, 40, 41, and the references therein]. For a feasible forest management policy $(\mathbf{x}, \mathbf{y}, \mathbf{f}) \in R(\Pi)$, the expected total mass of sequestrated carbon (in tons), across the whole planning horizon, is given by

$$CS(\mathbf{x}, \mathbf{y}, \mathbf{f}) = \sum_{\pi \in \Pi} \rho^{\pi} \left(\sum_{t \in T} \sum_{i \in \mathcal{I}} C^{\pi}_{it} x^{\pi}_{it} \right).$$

In our setting, land erosion is measured by the total length (in kilometers) of roads built on soil which is more vulnerable to erosion; hence, those roads associated to set $\mathcal{A}_1$. This means that the expected total land erosion (in km) is measured by the function

$$LE(\mathbf{x},\mathbf{y},\mathbf{f}) = \sum_{\pi\in\Pi} \rho^{\pi} \left(\sum_{t\in T} \sum_{(kl)\in\mathcal{A}_1} d_{kl} y_{klt}^{\pi} \right).$$

Given these three functions, the goal of the optimization task is to find a forest management policy $(\mathbf{x},\mathbf{y},\mathbf{f}) \in R(\Pi)$, so that $NPV(\mathbf{x},\mathbf{y},\mathbf{f})$ is *maximized*, $CS(\mathbf{x},\mathbf{y},\mathbf{f})$ is *maximized*, and $LE(\mathbf{x},\mathbf{y},\mathbf{f})$ is *minimized*, simultaneously. Hence, the multicriteria stochastic forest management problem (MSFMP), corresponds to the following (3-objective) stochastic MIP problem

$$(\mathbf{x}^*,\mathbf{y}^*,\mathbf{f}^*) = \arg\ \max(NPV(\mathbf{x},\mathbf{y},\mathbf{f}), CS(\mathbf{x},\mathbf{y},\mathbf{f}), -LE(\mathbf{x},\mathbf{y},\mathbf{f})) \tag{MSFMP.1}$$

s.t.

$$\text{(ADJ), (H), (F.1)-(F.6), (P.1)-(P.2), (NA.1)-(NA.2), (S.1)-(S.2)} \tag{MSFMP.2}$$

$$\mathbf{x} \in \{0,1\}^{|\mathcal{I}|\times|T|\times|\Pi|},\ \mathbf{y} \in \{0,1\}^{|\mathcal{A}|\times|T|\times|\Pi|} \text{ and } \mathbf{f} \in \mathbb{R}_{\geq 0}^{|\mathcal{A}|\times|T|\times|\Pi|}. \tag{MSFMP.3}$$

In order to solve problem (MSFMP.1)-(MSFMP.3), we adopt the well-known ϵ-constrained method [see, e.g., 30]. This method, whose details will be outlined in Section 2.4, allows to approximate the whole set of solutions comprising the Pareto frontier, i.e., *efficient* solutions that offer an optimal trade-off among the different objectives.

2.3 Risk Averse Forest Management: MSFMP combined with CVaR

The stochastic model encoded by (MSFMP.1)-(MSFMP.3) is a *risk-neutral* approach, meaning that criteria performances are measured only with respect to the corresponding expected values. However, and as we will report in Section 3, such approach leads to forest management policies that, although performing well in average, verify (very) poor performances for some scenarios. This behavior is particularly critical when referring to the NPV criterion, since the eventual occurrence of averse economic outcomes can risk the overall viability of the project.

Note that reducing the magnitude of averse outcomes corresponds to reducing the *thickness* of the tail of the NPV distribution. Such reduction can be achieved by incorporating a risk-averse measure into the model [see, e.g., Part II, Section 2.9 in 8]. In this paper we consider the well-known Conditional Value-at-Risk (CVaR) measure for reducing the magnitude of net-present values attained for adverse scenarios; below we describe the details of how this measure is incorporated into our multicriteria stochastic framework.

Let us assume that the decision maker defines a *desired* net-present value threshold, say q. For a given triplet $(\mathbf{x},\mathbf{y},\mathbf{f}) \in R(\Pi)$ and a given scenario $\pi \in \Pi$, the *shortfall* with respect to q is

$$\gamma(\mathbf{x}^{\pi},\mathbf{y}^{\pi},\mathbf{f}^{\pi}) = \left(\sum_{t\in T} \sum_{i\in\mathcal{I}} p_t^{\pi} V h_{it}^{\pi} x_{it}^{\pi} - \sum_{t\in T} \sum_{(kl)\in\mathcal{A}} c_{klt} y_{klt}^{\pi} \right) - q.$$

Now, let us assume that the decision maker defines a *shortfall* threshold α, and a security level $\beta \in [0,1]$. An additional goal of the decision maker, is to find a risk-*averse* forest management policy such that the β-conditional expectation of the shortfalls $\gamma\left(\mathbf{x}^{\omega}, \mathbf{y}^{\omega}, \mathbf{f}^{\omega}\right)$ greater than α is minimum. For example, if $\alpha = 500,000$ [EUR] and $\beta = 0.95$, it means that the decision maker seeks a harvesting policy that ensures that the average of the *worst* 5% of the shortfalls greater than 500,000 [EUR], with respect to q, is as small as possible.

For a given harvesting policy $(\mathbf{x}, \mathbf{y}, \mathbf{f}) \in R(\Pi)$, the (β, α)-Conditional Value-at-Risk ((β, α)-CVaR), defined as the β-conditional expectation of the shortfalls greater than α, is given by

$$\Gamma(\mathbf{x}, \alpha, \beta) = \alpha + \frac{1}{1-\beta} \frac{1}{|\Pi|} \sum_{\pi \in \Pi} \left[\gamma(\mathbf{x}^{\pi}) - \alpha\right]^{+}, \tag{CVaR}$$

where

$$[r]^{+} = \begin{cases} r, \text{ if } & r > 0, \\ 0, \text{ if } & r \leq 0. \end{cases}$$

CVaR was proposed in the seminal paper by Rockafellar and Uryasev [36]. In that paper, CVaR corresponds to the objective of a mathematical optimization problem; such representation enabled the authors to prove that CVaR is tractable under general circumstances. Moreover, in case of discrete finite distributions (as our case), CVaR optimization problems admit linear programming formulations. As shown in [36], one can incorporate function $\Gamma(\mathbf{x}, \alpha, \beta)$ into the model by defining a weighted function combining $NPV(\mathbf{x}, \mathbf{y}, \mathbf{f})$ and $\Gamma(\mathbf{x}, \alpha, \beta)$, i.e.,

$$NPVCVaR(\mathbf{x}, \mathbf{y}, \mathbf{f}) = \lambda NPV(\mathbf{x}, \mathbf{y}, \mathbf{f}) - (1-\lambda)\Gamma(\mathbf{x}, \alpha, \beta),$$

where $\lambda \in [0,1]$ is a user defined parameter for balancing between the expected net-present value and the risk-aversion function. Therefore, the CVaR-MSFMP is the problem obtained when replacing the first objective function of formulation of MSFMP by $NPVCVaR(\mathbf{x}, \mathbf{y}, \mathbf{f})$.

Note that for incorporating function $\Gamma(\mathbf{x}, \alpha, \beta)$ into the model, it is necessary to linearize it; this can be done by introducing a vector of auxiliary variables $\mathbf{u} \in \mathbb{R}^{|\Pi|}_{\geq 0}$ and rewriting (CVaR) as

$$\Gamma(\mathbf{x}, \alpha, \beta) = \alpha + \frac{1}{1-\beta} \frac{1}{|\Pi|} \sum_{\pi \in \Pi} u_{\pi},$$

with

$$-\gamma(\mathbf{x}^{\pi}) + \alpha + u_{\pi} \geq 0, \ \forall \pi \in \Pi \tag{Γ.1}$$

$$u_{\pi} \geq 0, \ \forall \pi \in \Pi. \tag{Γ.2}$$

As we will show in Section 3, incorporating this risk-aversion measure yields to policies with a better performance in adverse scenarios, although at expenses of decreasing the expected net-present value. In the following section, we will also show how the CVaR-

MSFMP is solved.

2.4 An Algorithmic Scheme for Solving the (CVaR-)MSFMP

There are different alternatives for solving a multicriteria (stochastic) optimization problems as the MSFMP [see 14, for a fundamental textbook on these issues]. The most common approach corresponds to the so-called *weighted sum-method*; in this method we *aggregate* the set of objectives into a single objective by multiplying each objective with a user defined weight or score. Such weights shall represent the preferences of the decision maker regarding the different objectives. As pointed out in [30], the main drawback of this method is that either decision makers need to have a clear idea of such preferences or shall try several possible configurations, which might still not be enough for computing the whole Pareto frontier. One alternative to tackle this drawback is the so-called ϵ*-method*. Instead of aggregating criteria, this is an iterative scheme where a single objective counterpart of the problem is solved such that the remaining objectives are handled via constraints whose corresponding right-hand-side values guarantee a good approximation of Pareto-efficiency for the attained solutions. These right-hand-values correspond to the ϵ-values which are updated in order to approximate the Pareto frontier.

Let us assume that for the carbon sequestration criterion we know, beforehand, a minimum quota, say Q_{CS}, of (expected) carbon mass to be captured. In other words, $CS(\mathbf{x}, \mathbf{y}, \mathbf{f})$ shall not be *less* than Q_{CS}. Likewise, for the land erosion criterion, let Q_{LE} a maximum value for the corresponding objective, i.e., $LE(\mathbf{x}, \mathbf{y}, \mathbf{f})$ cannot be *greater* than Q_{LE}. These values correspond to the initial ϵ-values, i.e., $\epsilon^0_{CS} = Q_{CS}$ and $\epsilon^0_{LE} = Q_{LE}$. The first iteration of the method corresponds to find the *best* value of the NPV criterion, $NPV(\mathbf{x}, \mathbf{y}, \mathbf{f})$ (or of the risk-averse NPN criterion, $CVaRNPV(\mathbf{x}, \mathbf{y}, \mathbf{f})$). Such value is found by solving

$$
\begin{aligned}
NPV^0 \text{ or } CVaRNPV^0 = \max\ & NPV(\mathbf{x}, \mathbf{y}, \mathbf{f}) \text{ or } CVaRNPV(\mathbf{x}, \mathbf{y}, \mathbf{f}) \\
\text{s.t.}\ & \\
& CS(\mathbf{x}, \mathbf{y}, \mathbf{f}) \geq \epsilon^0_{CS} \\
& LE(\mathbf{x}, \mathbf{y}, \mathbf{f}) \leq \epsilon^0_{LE} \\
& \text{(MSFMP.2) and (MSFMP.3),}
\end{aligned}
$$

which is a single-objective stochastic MIP problem. Once this problem is solved, the ϵ-values are updated as follows: the ϵ-value associated to the carbon sequestration is increased by Δ^0_{CS}, yielding $\epsilon^1_{CS} = \epsilon^0_{CS} + \Delta^0_{CS}$; and the ϵ-value associated to land erosion is decreased by Δ^0_{LE}, yielding $\epsilon^1_{LE} = \epsilon^0_{LE} - \Delta^0_{LE}$. Note that $\Delta^0_{CS} \geq 0$ and $\Delta^0_{LE} \geq 0$ correspond to the *step-length*, and the ϵ-values are updated consecutively (first ϵ_{CS} and then ϵ_{LE}), so as to appropriately explore the Pareto frontier induced by the corresponding expected values. In general, the values of Δ^ℓ_{CS} and Δ^ℓ_{LE}, of the ℓ-th iteration, shall be small enough in order to enable a good approximation of the Pareto frontier, large enough in order to avoid a computationally expensive exploration, and shall guarantee that the obtained solution differs from the one obtained in the previous iteration. Intuitively speaking, the following trade-off relations can be establish among NPV (or $CVaRNPV$), CS and LE: (i) the better the economic performance of the project ($NPV \uparrow$, $CVaRNPV \uparrow$), the more timber must be harvested ($CS \downarrow$), and the larger road network that must be built ($LE \uparrow$); (ii) the more carbon we expect to be retained by the forest ($CS \uparrow$), the less we can harvest ($NPV \downarrow$, $CVaRNPV \downarrow$), and the less roads we are likely to build on vulnerable soil ($LE \downarrow$); and, complementary, (iii) the less land erosion we are willing to accept ($LE \downarrow$), the larger road network we have to build on regular soil ($NPV \downarrow$, $CVaRNPV \downarrow$) and the more timber we have to harvest to compensate this extra cost ($CS \downarrow$). Clearly, after updating the corresponding right-hand-side values we get that $NPV^1 \leq NPV^0$ (resp. $CVaRNPV^1 \leq CVaRNPV^0$). The process repeats until a combination of ϵ-values, say ϵ_{CS}^{L+1} and ϵ_{LE}^{L+1}, yields an infeasible problem (at iteration, say, $L+1$). The collections of objective function values $(NPV^0, NPV^1, \ldots, NPV^L)$, $(\epsilon_{CS}^0, \epsilon_{CS}^1, \ldots, \epsilon_{CS}^L)$, and $(\epsilon_{LE}^0, \epsilon_{LE}^1, \ldots, \epsilon_{LE}^L)$ approximate the Pareto frontier of the expected values.

The efficacy of the described method strongly relies on how burdensome is the single-objective stochastic programming problem that must be solved at each iteration. The computational difficulty of such problem mainly depends on how large is the number of scenarios; as we will show in the following section, we incorporate a scenario reduction method so as to scale down the computational difficulty and enable a broader exploration of the solution space.

3. Computational Results for (CVaR-)MSFMP: An Application in Portugal

In this section we report the results obtained when solving both the MSFMP and CVaR-MSFMP, as described in Section 2.4, on a real eucalyptus forest located in central Portugal. At each iteration of the ϵ-method, the resulting (stochastic) MIP problems were solved using IBM CPLEX 12.6 on a Linux-based server with a 64 GB RAM and an Intel Xeon Ivy E5-2660 processor. For the considered data, the running time required for solving each of these MIP problems was below 60 seconds.

3.1 Case Study: a Forest in Portugal

The proposed methodology is applied to a eucalyptus forest located in central Portugal. The forest is comprised by 1000 management units, and extends over 12435 hectars (ha); the forest is shown in Figure 2. As can be seen in the figure, there is a whole transportation network comprised by origin points, intersection points, and potential roads.

Problem Parameter Values In the case study, there are 157 origins (set $\mathcal{O}$), 145 intersections (set $\mathcal{J}$), and 9 exit points (set $\mathcal{S}$). Additionally, there are 1040 potential roads (set $\mathcal{A}$), extending over 800 kilometers.

Based on interviews with forest managers and records in forest databases, we obtained the following parameter values. The planning horizon is 15 years; the road construction cost is uniform and corresponds to 2.6 [$/m]; the production balance factor α, see constraints (P.1)-(P.2), is set to 0.15; and, the interest rate, for the NPV calculation, is 3%.

For defining the demand levels, stakeholders suggested to follow the next strategy. Let us assume that we already have a good approximation of set Π (this is explained later).

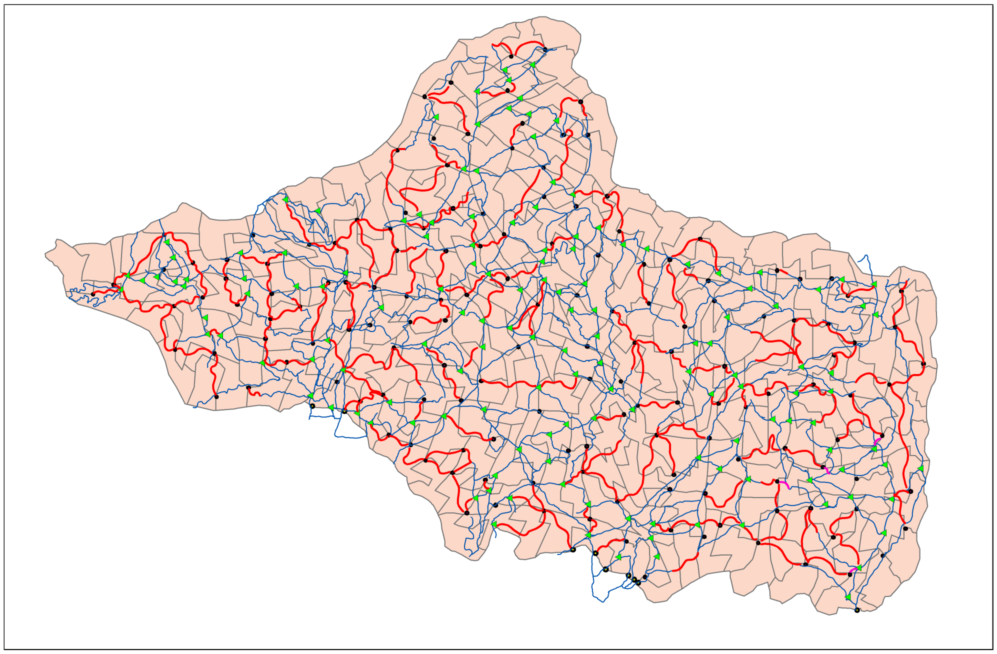


Figure 2: Representation of the forest under evaluation. Management units are delimited by grey lines, weak roads are shown as red lines, normal roads are shown as blue lines, origins are shown as green triangles, intersection points are denoted by black circles, and nine exit points are considered in the southern part of the forest.

First, we solve the following single-objective counterpart of the MSFMP;

$$
\begin{aligned}
V^*(\mathbf{x}^*, \mathbf{y}^*, \mathbf{f}^*) = \max & \sum_{\pi\in\Pi}\sum_{t\in T}\sum_{i\in\mathcal{I}} Vh_{it}^{\pi} x_{it}^{\pi} \\
\text{s.t.} & \\
& \text{(ADJ), (H), (F.1)-(F.2), (F.4)-(F.6), (P.1)-(P.2), (NA.1)-(NA.2), (S.1)-(S.2)} \\
& \mathbf{x} \in \{0,1\}^{|\mathcal{I}|\times|T|\times|\Pi|},\ \mathbf{y} \in \{0,1\}^{|\mathcal{A}|\times|T|\times|\Pi|} \text{ and } \mathbf{f} \in \mathbb{R}_{\geq 0}^{|\mathcal{A}|\times|T|\times|\Pi|},
\end{aligned}
$$

which aims at maximizing the volume of the harvested wood from the forest without any demand constraints. Once this problem is solved (using the same machine described above), we compute, per each period $t \in T$, the average harvested volume, i.e., $V^t = \sum_{i\in\mathcal{I}} Vh_{it}^{\pi} x_{it}^{\pi\,*}$. Finally, and as suggested by decision makers, the demands levels were set to $\overline{D}_t = 0.75V^t$ and $\underline{D}_t = 0.25V^t$.

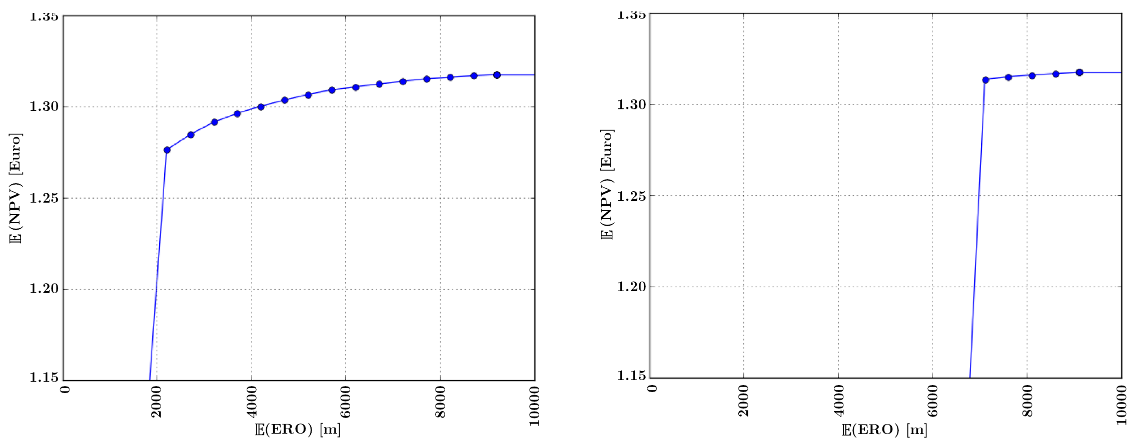
For the carbon sequestration and the land erosion criteria, the corresponding $\boldsymbol{\epsilon}_{CS}$ and $\boldsymbol{\epsilon}_{LE}$ values were defined following a similar scheme as for demand levels. For each of these criteria, we solved single objective counterparts of the MSFMP aiming at producing lower and upper bounds that guarantee, on the one hand, to be associated with feasible harvesting and road building plans, and on the other hand an efficient use of the resources. By applying this process we obtained $\boldsymbol{\epsilon}_{CS} = \{21100, 21300, 21800, 22300, 22800, 23300, 23800\}$ and $\boldsymbol{\epsilon}_{LE} = \{0, 500, 1000, 1500, 2000, \ldots, 9500, 10000\}$.

Scenario Generation As explained in §2.2, we have two sources of uncertainty; future growth and yield, and future prices. Such uncertainty is translated into scenarios, Ω and Φ, respectively; which, and due to the assumption of independence, are then combined into set Π.

Forest growth scenarios (Ω) determine both the volume of timber available for harvesting, and the carbon captured in each management unit. Each growth scenario is induced by a series of weather data over the planning horizon. Such data includes temperature, radiation, precipitation, number of frost days, number of rain days and relative humidity. We used two sources for this data. The first source corresponds to the climate dataset gathered by the ENSEMBLES project [15], which was developed by [22] using the emission scenarios developed by the Intergovernmental Panel on Climate Change (IPCC) [see IPCC Special Report on Emission Scenarios available in 31]. According to [39], the scenarios of the ENSEMBLES project are considered the most appropriate for Portuguese conditions. The second source corresponds to data collected by 8 weather stations located within the case study area. With these two sources of data as input, we used the SADfLOR system [12] as a black-box tool for generating 100 growth scenarios ($|\Omega| = 100$). SADfLOR is a decision support tool that uses a projection scheme that incorporates the process-based model designed in [42], which simulates the physiological processes involved in forest growth (e.g., photosynthesis). Due to the functionality of SADfLOR, the resulting 100 scenarios are such that they cover a wide range of possible climates for the case study area, i.e., from an extremely dry and hot climate to a cool one with more rain. It is known that extremely negative scenarios (also known as *black swans*) and extremely positive scenarios are less likely to occur than those that are concentrated around the average. To capture this pattern, and because we assigned equal weight (i.e., equal probability) to each scenario, we used a higher number of scenarios around the average expected climate, while we considered fewer scenarios with (negative and positive) extreme weather.

As for the set Φ, the price scenarios were generated based on the eucalyptus wood historical prices. On the one hand, we used information from the Portuguese market, which was reported in the Silviculture national market annual reports from 2009 to 2014 [see 23]. And on the other hand, we gathered information from international wood trading records from 2009 and up to 2016 [see 19]. We combined these sources of data to create, using a standard econometric model, a series of yearly prices (in \$/Ton), and then we used a Brownian motion scheme to create 10,000 scenarios of future wood prices for the following 15 years [see 47, for further details]. Afterwards, the scenario reduction scheme proposed in [32], which is based on scenario *clustering*, was applied to reduce these 10,000 scenarios. Preliminary results showed that 10 (*cluster*) scenarios were enough to embody the full range of variability of the original 10,000 scenarios, i.e., to cover a wide range of possible future outcomes of wood price, aiming at embodying the stochastic nature of future prices.

In total, we therefore have $100 \times 10 = 1000$ scenarios comprising set Π. In the case of both sets, Ω and Φ, the corresponding scenarios are characterized by temporal interdependencies (such as those depicted in the example shown in Figure 1); therefore, the same extends for the scenarios contained in Π. This means that scenarios comprising set Π yield non-anticipativity constraints of the type (NA.1)-(NA.2). Having 1000 scenarios yields a quite burdensome optimization model, we have followed an additional preprocessing strategy to



(a) Pareto frontier for $\epsilon_{CS} = 22300$

(b) Pareto frontier for $\epsilon_{CS} = 23300$

Figure 3: Pareto frontiers of $NPV(\mathbf{x}, \mathbf{y}, \mathbf{f})$ (values are given in *euros*, €) attained for two values of ϵ_{CS} (denoted as "Carbon" and whose values are given in tons) and several levels of ϵ_{LE} (denoted as "ERO" and whose values are given in meters)

reduce the number of scenarios. The strategy is as follows; we solve the auxiliary problem

$$NPV^1 = \max \{NPV(\mathbf{x}, \mathbf{y}, \mathbf{f}) \mid \text{(MSFMP.2)and (MSFMP.3)}\},$$

for a scenario set Π comprised by a single scenario (i.e., $|Pi| = 1$), and let NPV^1 be the corresponding solution value. Now, solve the same auxiliary problem but for $|Pi| = 2$, and, likewise, let NPV^2 be the corresponding solution value. We repeat the process until finding a solution, say the i-th solution obtained when having i scenarios, is such that the difference of NPV^i with respect to the previous corresponding value, NPV^{i-1}, is less than 10^-3. In other words, we want to find the minimum subset of Π, say Π', so that including an additional scenario does not bring much difference with respect to the policies to be defined. The order in which scenarios are selected as candidates to be included in Π is done in such way that it ensures an appropriate representation of the different outcomes, i.e., the resulting scenario tree Π' is balanced in the same manner as Π. In our experiments, this convergence was reached in $\Pi' = 92$, so we decided to use 100 scenarios in our computations. Note that these auxiliary problems were solved using the same resources described before.

3.2 Results: Efficiency Analysis, Trade-offs and the Effect of Uncertainty

The different algorithmic and preprocessing strategies presented before were applied to solve the MSFMP on the Portuguese data set described in §3.1. In Figure 3, we show the Pareto frontiers obtained when solving the MSFMP for two values of ϵ_{CS} and several values of ϵ_{LE}. The frontier shown in Figure 3(a) reports on the y-axis the $NPV(\mathbf{x}, \mathbf{y}, \mathbf{f})$ values attained for solutions ensuring a total extension of roads built on weak land limited by the values shown in the x-axis, and a carbon sequestration of at least 22300 tons (ϵ_{CS} =22300). The results show that is not economically attractive to implement a forest management strategy without building at least 2000 meters of roads on weak land (ϵ_{LE}=2000) when requiring, at the same time, a quota of carbon sequestration of at least 22300 tons. Requiring

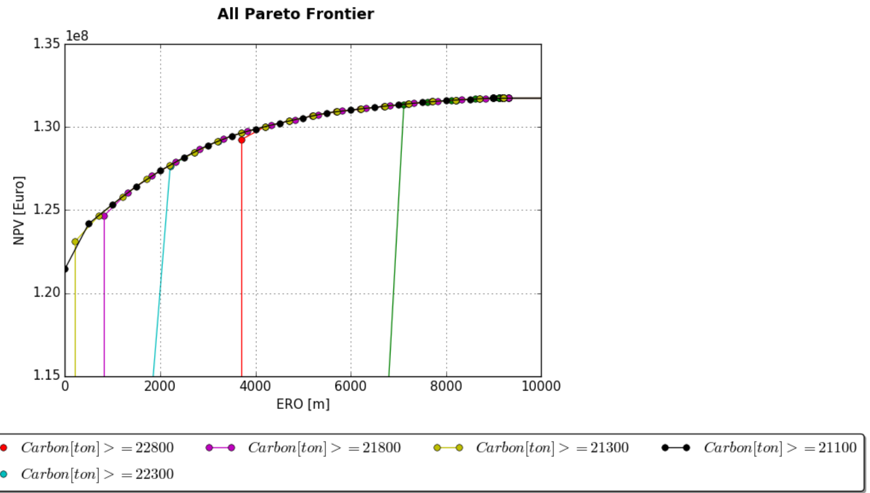
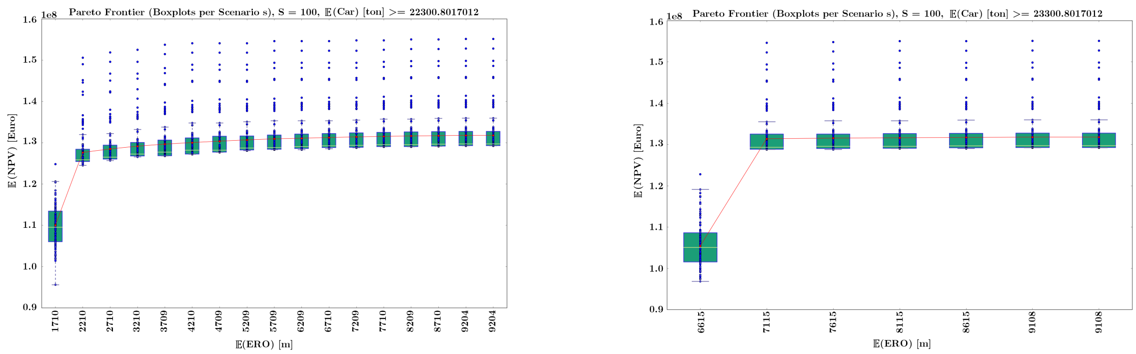


Figure 4: Pareto frontiers obtained for different combinations of ϵ_{CS} and ϵ_{LE} when solving the MSFMP

less than 2 kilometers of roads on weak land would require to build a very extensive, and thereafter too expensive, network of roads on normal soil. However, if the decision maker is willing to build roads on weak soil extending over at least 2500 meters, then it is possible to reach NPV values above 125 millions of euros. Moreover, the NPV can reach values above 130 millions if more than 10 kilometers of weak soil roads are allowed to be built, while still capturing more than 22 tons of carbon. In Figure 3(b) we report the behavior of the solutions obtained when the minimum value on the expected mass of sequestrated carbon increased from 22300 tons to 23300 tons (a 5% approx. increase). The impact on the land erosion criterion is clear; it is no longer possible to find economically attractive solutions without building at least 7 kilometers of roads on weak land (this is 300% worse when compared with the 2 kilometers threshold for the case shown in Figure 3). Such pattern might be explained as follows. Capturing more carbon requires to keep, in average, greater volumes of standing forest. Accomplishing such goal, while still exploiting the economic value of the forest and satisfying the demand, requires to harvest, each year, trees comprising a more complex mixture of ages. This calls for the construction of a larger transportation network within the forest. If at the same time we forbid constructing roads on weak land, we are basically forcing, as said before, the construction of a large number of detours, i.e., a much more expensive road network, for the access and transportation from the harvesting areas towards the exit point.

From the Pareto frontiers analyzed before, one can observe that aiming at a better performance of the carbon sequestration criterion, basically implies that attractive NPV levels can be reached only if the decision maker is willing to accept a trade-off with respect to the land erosion criterion. A better picture on this is shown in Figure 4, where the Pareto frontiers obtained for the different levels of carbon sequestration are reported. From the displayed plots we can observe that imposing greater quotas of carbon sequestration does not really affect the economic potential to be obtained from the forest; the Pareto frontiers are all overlapped and the (attractive) NPV values go from 122 millions (approx.) up to 132 millions (approx.) of euros; what changes between one frontier and the other is the



(a) Boxplots associated with $\epsilon_{CS} = 22300$

(b) Boxplots associated with $\epsilon_{CS} = 23300$

Figure 5: Boxplots of the NPV values associated with the Pareto frontiers corresponding to $\epsilon_{CS} = 22300$ and $\epsilon_{CS} = 23300$

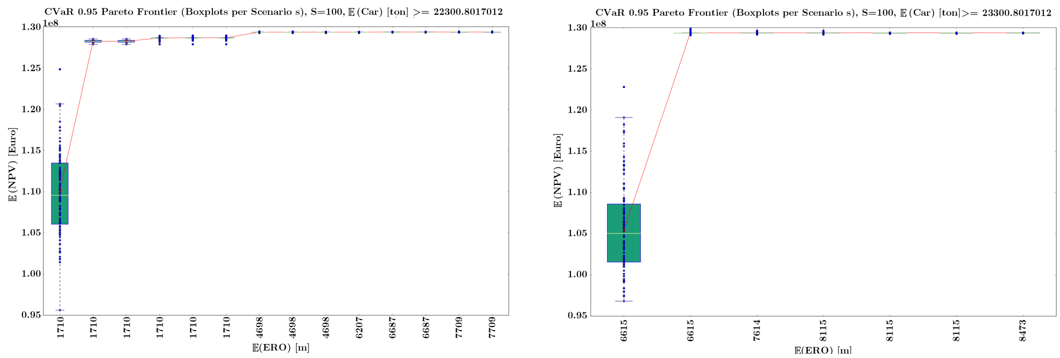
threshold of the land erosion criterion value for reaching such NPV values.

In the analysis presented so far, we have basically focused on analyzing the performance of the different criteria through the expected values of the corresponding outcomes. However, an adequate analysis shall also pay attention on how the obtained solutions behaved across the different scenarios. In Figure 5 we report, by means of boxplots, the dispersion of the NPV values attained for the different scenarios. The mean values, marked as red circles, coincide with those encompassing the Pareto frontiers shown in Figure 3. As we can see from these plots, there is a clear variability of the NPV values obtained from different scenarios; this is what we refer to as the *effect of uncertainty*. For some scenarios the performance is better than the expected one, while for some others this performance is worse. This latter case is problematic, since decision makers might receive less profit than expected. As a matter of fact, in both Figures, we can see that for the lowest levels of land erosion (1710 m and 6615 m, respectively), there are scenarios for which the NPV value is notoriously worse than the expected value; these scenarios are likely to be *black swans* that associate abnormally adverse climate realizations. Although not shown, this situation occurs also for the values of ϵ_{CS}. It is precisely for hedging against this behavior that we proposed using a risk-averse counterpart of the MSFMP; the results obtained when solving the CVaR-MSFMP are reported in the following subsection.

3.3 Results: Reducing Worst-Case Shortfalls via CVaR Approach

As shown before, the policies present a variable performance, in particular for the NPV criterion, for the different scenarios. In order to mitigate some of the more adverse scenarios, we propose the use of CVaR as a risk-averse strategy (see §2.3). For this, we have adopted the following setting: (i) for each pair ϵ_{CS}, ϵ_{LE}, we set the net-present value threshold q to the corresponding $NPV(\mathbf{x}, \mathbf{y}, \mathbf{f})$ value; (ii) the shortfall threshold, α, was set to the difference between $NPV(\mathbf{x}, \mathbf{y}, \mathbf{f})$ and the value corresponding to the 3rd-quartile of the associated boxplot; and (iii) the conditional expectation was bounded by $\beta = 0.95$.

In Figure 6 we report the resulting boxplots when solving CVaR-MSFMP, with the setting described above, on the problems resulting when having $\epsilon_{CS} = 22300$ (Figure 6(a)) and $\epsilon_{CS} = 23300$ (Figure 6(b)), and the corresponding ϵ_{LE} values. When comparing the



(a) Boxplots associated with $\epsilon_{CS} = 22300$

(b) Boxplots associated with $\epsilon_{CS} = 23300$

Figure 6: Boxplots of the NPV values associated with the Pareto frontiers (corresponding to $\epsilon_{CS} = 22300$ and $\epsilon_{CS} = 23300$ for Figures 6(a) and 6(b), respectively) obtained when solving CVaR-MSFMP

reported boxplots with those obtained for the risk-neutral MSFMP (reported in Figure 5) three observations can be highlight. First, it is possible to reduce the dispersion of the attained NPV values (for almost all pairs ϵ_{CS}, ϵ_{LE}). Second, the reduction of the impact of adverse scenarios is achieved at expenses of reducing the $NPV(\mathbf{x}, \mathbf{y}, \mathbf{f})$ values, i.e., the expected values across all scenarios. And third, the impact of the land erosion criterion is more or less similar as the one for the risk-neutral model. These conclusions rely on the fact that the shortfall threshold, α, and the security level, β, were set with the aim of forcing the risk-averse NPV values to be as close to the 3rd-quartile as possible. As a matter of fact, when comparing the curves in Figure 5 with those in Figure 6, we can see that the expected risk-averse NPV values in Figures 6(a) and 6(b) are all more or less concentrated relatively close to the value associated with the upper part of the boxes displayed in Figures 5(a) and 5(b), respectively. The three observations presented above lead to the following conclusion: constraining the range of the NPV values, by a risk-averse approach, burdens the capacity of the model to exploit the trade-offs among the criteria in order to produce more diverse solutions.

Compared to the frontiers shown in Figure 4, the risk-averse harvesting and road building policies seem more sensitive with respect to the different carbon sequestration levels ϵ_{CS} (Figure 7). Despite of this variability, when allowing the construction of a road network with more than 6 kilometers of weak land roads ($\epsilon_{LE} \geq 6000$), the economical value of the policies converges towards a value near 129 millions of euros, regardless of the value of ϵ_{CS}. For the risk-neutral case, when having $\epsilon_{LE} \geq 6000$ implies that the economical value converges towards 132 millions; this 3 million euros difference is the *cost* of having risk-averse policies.

3.4 Discussion

The results obtained for the risk-neutral and risk-averse MSFMP demonstrate the capability of the framework for providing a pool of alternative policies for sustainable forest management, and a strategy for comparing them with respect to the different optimization criteria. From a managerial perspective, such feature stands as one of the main advantages

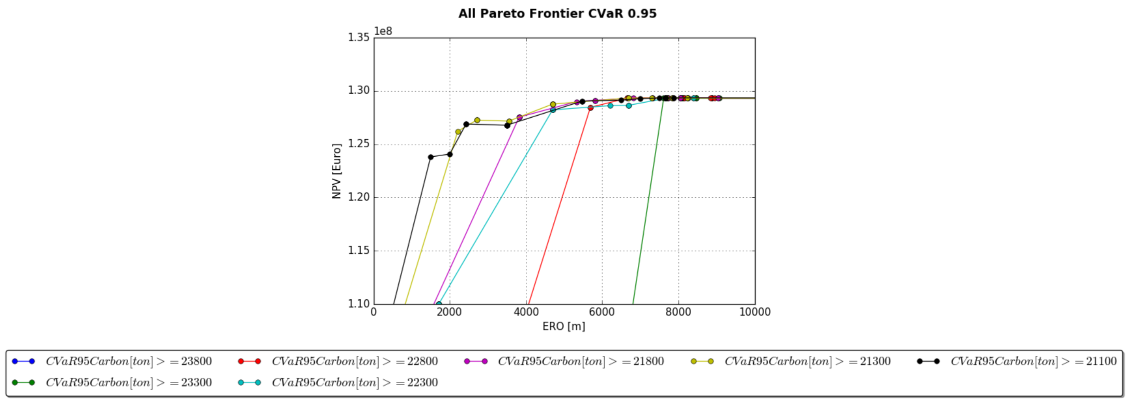


Figure 7: Pareto frontiers obtained for different combinations of ϵ_{CS} and ϵ_{LE} when solving the CVaR-MSFMP

of the designed tool. Furthermore, and in contrast to the existing literature on multicriteria stochastic approaches for forestry management [see, 17, 24], another advantage of our approach is the incorporation of road building and transportation decisions; this boosts the practical interpretation and application of the attained forest policies. However, and despite of these advantages, the main drawback of the proposed methodology is that, compared to more traditional forest management approaches, there is quite large number of input parameters involved in the resulting optimization problems. Hence, decision makers must ensure an accurate manipulation of the input data as well as to perform extensive sensitivity analyses in order to limit the impact of an eventually biased or imprecise data manipulation.

4. Conclusions

In this paper we have designed an optimization framework for assisting sustainable forestry planning when uncertainty and multiple criteria must be taken into account. The framework relies on solving an optimization problem that has been referred to as multi-criteria stochastic forest management problem (MSFMP). The resolution of the problem is performed by the so-called ϵ-method, which consists of transforming the problem into a single-objective one, while managing the remaining criteria as constraints.

By using data corresponding to an eucalyptus forest located in central Portugal, we solved the MSFMP and a risk-averse counterpart. With respect to the considered data, the obtained results allow us to draw three main conclusions: (i) the economical attractiveness of the forest is severely affected if decision makers aim at road building decisions encompassing a limited distance on weak land (i.e., implying reduced levels of land erosion), (ii) different performances of the carbon sequestration criteria (i.e., different masses of sequestrated carbon) do not seem to be accompanied by different performances of the other criteria, and (iii) incorporating road building and transportation decisions, in the planning stage, boosts the practical interpretation and application of the attained forest policies. From a policy making perspective, the reported results show the capacity of the proposed model to offer a diverse pool of solutions to the different actors involved in the exploitation of forestry resources, specially when sustainability aspects (e.g., carbon sequestration or land erosion) must be considered in the planning stage. Likewise, the results obtained when solving the risk-averse version of the problem show that incorporating Conditional-Value-at-Risk (CVaR) is an effective strategy for reducing the impact of adverse scenarios. The advantages of the proposed tool shall motivate forest managers, as well as regulatory institutions, to incorporate it into their decision making process when designing both investment and regulation standards for sustainable forest exploitation.

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