Twisted skyrmions through dipolar interactions

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Abstract

The manifestations of the dipolar interaction in magnetic nano-devices can take subtle forms. In this work, we address the effects of dipolar interactions on skyrmions textures. The so-called twisted skyrmion is a natural state between the Bloch skyrmion, that arises from the bulk-like Dzyaloshinskii-Moriya (DM) interaction, and the Néel-skyrmion, that results from the interfacial form of the DM interaction. Often neglected, or approximated by local interactions, we show how, when explicitly included, dipolar interactions generate gaps in the phase diagram for certain helicities. Besides, this interaction allows the existence of skyrmions with two chiralities, which are unstable when the dipolar interaction is approximated by a shape anisotropy.

1. Introduction

Recently magnetic skyrmions [1,2] have been intensely studied from the theoretical [3,4] and experimental [5–7] perspectives. Among their characteristics, one can highlight their small size, their stability under external disturbances, and the possibility of controlling their position with ultra low current densities. Due to these properties, among others, such magnetic configurations offer potential applications in information storage and processing devices [8–12]. Within the family of skyrmions-like magnetic textures, there are two structures that have concentrated most of the attention, namely the Néel and Bloch skyrmions [13,14]. The Néel skyrmion is also known as hedgehog skyrmion, and has a radial magnetization different from zero, while the Bloch skyrmion presents a magnetic texture similar to a vortex, but instead of having only one section, it has two regions in which the magnetization points out of the plane. However, similar to what happens with magnetic vortices [15], Néel and Bloch skyrmions are two extremes of a wide continuous range of possible intermediate states.

It is well known that skyrmions can be obtained in thin magnetic systems by means of a Dzyaloshinskii-Moriya (DM) interaction [16,17]. This interaction depends on the DM vector $D_{\mathbf{R}}$, associated to the symmetry of the crystal structure. A usual crystalline symmetry group in several materials is the $D_{n}$ group, where $n$ represents the crystal symmetry. In this symmetry, the interaction energy contribution is proportional to $\mathbf{n} \cdot (\mathbf{\nabla} \times \mathbf{n})$ [13]. A few years ago, Thiaville et al. [18] showed that Bloch-like skyrmions are associated with DM vectors $\mathbf{D}_{\mathbf{R}}$ parallel to $\mathbf{n} - \mathbf{n}$, whereas Néel-like skyrmions are associated with DM vectors $\mathbf{D}_{\mathbf{R}}$ perpendicular to $\mathbf{n} - \mathbf{n}$.

In this work, we explore the stability of skyrmions characterized by a magnetic texture in between the Néel and Bloch ones. Such magnetic configurations have been recently studied using differential equations and assuming a strong uniaxial anisotropy [19]. Besides they have been addressed as skyrmion-based spin-torque nano-oscillator by Garcia-Sanchez et al. [20] and proposed for signal generation and for neuro-inspired applications. These magnetic textures occur naturally because of a competition between DM (chiral behavior) and dipolar (not chiral) interactions [21]. For our study, we consider two DM interactions. In the bulk, we assume that there exists a DM interaction favorable to Bloch-like skyrmions, and at the interface with a substrate, we consider a DM interaction that favors the Néel-like skyrmions. Such DM interactions are due to a break of the inversion symmetry in the bulk, and to the spin-orbit coupling at the interface. Besides, we include the dipolar interaction that plays a role favoring Bloch-like skyrmion. The competition between these three contributions to the energy (interfacial and bulk DM, and dipolar interactions) determines the final state of the system, which may exhibit a complex magnetic configuration. Following these ideas, we study the impact that dipolar energy has in the equilibrium magnetization in an explicit way, as opposed to considering the dipolar energy as an effective anisotropy.

2. Model

For our work we consider a cylindrical dot with radius $R$ and height $L$ ($R \gg L$), coupled to a non-magnetic substrate, as shown in Fig. 1. We assume that the interaction with the substrate occurs in a section of width $t = 0.6$ nm of the dot, as in Ref. [18].
The cylinder has a DM energy due to the symmetry of its crystalline structure defined by a DM constant $D_b$ and interacts with the substrate through a DM interaction determined by a constant $D_t$. We start by using the explicit expression for the dipolar energy and considering the reduced magnetization $\mathbf{m}$ as

$$\mathbf{m} = \sin \Theta \cos \phi \mathbf{e}_r + \sin \Theta \sin \phi \mathbf{e}_\theta + \cos \Theta \mathbf{e}_z,$$

where $\mathbf{e}_r$, $\mathbf{e}_\theta$, and $\mathbf{e}_z$ are the standard basis in cylindrical coordinates, $\Theta = \Theta(\rho)$ is a function that determines the profile of the $z$ component of the magnetization, $m_z$, of the skyrmion and $\phi$ is the helicity [22], as shown in Fig. 2. This parametrization ensures that $|\mathbf{m}| = 1$ everywhere inside the magnetic dot. If we take $\phi_{\phi} = v \pi$ with $v$ integer, we have a hedgehog or Néel-like skyrmion. On the contrary, we obtain a Bloch-like skyrmion for $\phi_{\phi} = v \pi + \pi/2$. Therefore we labeled the states as $\phi_{\phi}$-twisted-skyrmion, due to the magnetic spiral formed in the cylinder. Concerning the $\phi$ dependence on $z$, we used the thin film approximation, a widely used approximation that allows considering the magnetization profile independent of $z$ in sufficiently thin geometries [23].

Assuming exchange, anisotropy, dipolar and DM interactions, the total energy can be written as

$$E = E_{ex} + E_{an} + E_{dip} + E_{DM}^{(1)} + E_{DM}^{(2)},$$

$$E = A \int_0^R \left( \frac{d\Theta}{d\rho} \right)^2 d\rho - k_a \int_0^R m_z^2 d\rho + t_D \int_0^R \left( \nabla \cdot \mathbf{m} \right) d\rho + t_D \int_0^R \left( \nabla \times \mathbf{m} \right)^2 d\rho,$$

with $A$ and $K_a$ the stiffness and anisotropy constants, respectively, and $D_t$ and $D_b$ the DM constants in the bulk and at the interface. In this equation $M_r$ represents the saturation magnetization, $U$ corresponds to the magnetostatic potential and the easy axis is along the dot axis. The first two terms of the total energy do not depend on $\phi_{\phi}$, but the last three contributions to the total energy depend on $\phi_{\phi}$. After calculating each energy contribution, we obtain

$$E = 2\pi L A \int_0^R \left[ \frac{d\Theta}{d\rho} \right]^2 d\rho - 2\pi L K_a \int_0^R \cos^2 \Theta \rho d\rho + E_{dip}^{(1)} + E_{dip}^{(2)},$$

where $E_{dip}^{(1)}$ denotes the contribution to this energy that does not depend on $\phi_{\phi}$, and

$$E_{dip}^{(2)} = \mu_0 M_s \int_0^\infty (L q + e^{-q^2} - 1) dq \left[ \int_0^R \sin \Theta(\rho) J_1(q\rho) \rho d\rho \right]^2,$$

and

$$E_{DM} = 2\pi \int_0^R \frac{d\Theta}{d\rho} + \frac{\sin \Theta(\rho) \cos \Theta(\rho)}{\rho} \rho d\rho,$$

depend on the skyrmion profile.

In our calculations, we will consider two expressions for the dipolar energy. In the first case, the dipolar energy is explicitly calculated and then added to the total energy. In the second case, it is considered as a shape anisotropy term and included into the anisotropy constant as $K = K_a + K_{dip}$, with $K_{dip}$ the dipolar anisotropy.

### 2.1. Skyrmion structure taking full account of the dipolar energy

In this section we consider the detailed calculation of the dipolar energy. Collecting together all contributions to the total energy we can write it as $E = E_{ex} + E_{an} + E_{DM} + E_{dip}$, with $E_{DM}$ containing the $\phi_{\phi}$-dependence of the energy, obtained by adding the DM interactions and the $\phi_{\phi}$-dependent part of the dipolar interaction. Therefore,

$$E_{DM} = -2\pi \int_0^R \frac{d\Theta}{d\rho} + \frac{\sin \Theta(\rho) \cos \Theta(\rho)}{\rho} \rho d\rho,$$

and

$$E_{DM}^{(2)} = \mu_0 M_s \int_0^\infty (L q + e^{-q^2} - 1) dq \left[ \int_0^R \sin \Theta(\rho) J_1(q\rho) \rho d\rho \right]^2.$$

By evaluating $\phi_{\phi}$ minimizing energy in Eq. (3), we notice that Bloch and Néel-skyrmions ($\phi_{\phi} = \pi/2$ or 0, respectively) are formed only when $D_t$ and/or $D_b$ are zero, respectively. If both $D_t$ and $D_b$ are different from zero, a solution with intermediate $\phi_{\phi}$ is reached.

If we consider the dipolar interaction explicitly, and $D_b = 0$, the equilibrium configuration is a twisted-skyrmion when

$$\frac{\pi_{DM}}{\pi_{dip}} < \frac{2}{D_b},$$

and a Néel-skyrmion otherwise. When $D_t < D_t^*$ ($D_t^*$ is a lower value such as exist skyrmions), the ferromagnetic configuration pointing out-of-plane is an equilibrium configuration. Our study is only in the region $D_t > D_t^*$.

Now we propose an ansatz for the explicit form of the magnetization profile $\Theta(\rho)$. This ansatz has been previously used in several works such as in Refs. [4,24]. We start from the exact solution for a soliton in the non-linear sigma model [25] that gives $m_z(\rho) = (\rho^2 - 1)/(\rho^2 + 1)$. Since this solution is exact on an infinite plane, which is not our case, we introduce a finite size by replacing $\rho$ by $e^{-\sqrt{1-\rho^2}}$ in $m_z(\rho)$, with $\lambda$ and $\rho_0$ as adjustable parameters that are obtained from energy minimization. We propose

$$\Theta(\rho) = -\arccos \left[ \tanh \left( \frac{\lambda \rho_0}{\lambda} \right) \right] + \arccos \left[ \tanh \left( \frac{\lambda(\rho_0 - \rho)}{\lambda} \right) \right],$$

where the first term in this equation is included to avoid energy divergences.

We compare results from our ansatz with results obtained from similar calculations in OOMMF [26], for a cylindrical particle with $t = 0.6$ nm and $R = 50$ nm described by $M_s = 1100$ kA/m, $A = 16$ pJ/m, $K = 0.75$ MJ/m$^3$ and $L = 3t$. Our ansatz shows a very good agreement with OOMMF results, as illustrated in Fig. 3. In addition, both profiles
are similar to the one obtained by N. Romming et al. in Ref. [27] with the parameters \( w = 16.9 \) nm and nm.

Using OOMMF simulations we obtained \( \lambda \) and \( \rho_0 \) that minimize the energy. Our results for \( D_b = 0 \) are shown in Fig. 4. For simplicity we assume \( \lambda \) and \( \rho_0 \) independent of \( D_b \). This figure also defines a minimum value of \( D_b, D_i^* = 1.5 \) mJ/m\(^2\), that allows skyrmion nucleation.

**Fig. 5.** Normalized energy for a magnetic cylinder with \( L = 3\tau \) and \( D_i = 2.0 \) mJ/m\(^2\). The solid line illustrates results for \( D_b = D_i \), the dashed line represents results for \( D_b = 0.1D_i \), and the dotted line corresponds to \( D_i = 10D_b \).
sign as $D_i$ is tuned across $D_i^*$. This fact is behind the difference in the behavior of the optimal helicity below and above the threshold. Within this approximation, the optimal angle $\phi_0$ can be readily shown to scale as $\pm D_i D_i$. This simple scaling law is indeed displayed by the result of the exact calculation in the vicinity of $D_i^*$.

To clarify the differences between both models, Fig. 7 illustrates the phase diagram for different $D_b$ and $D_i$ values. For $D_b = 0$ and $D_i < D_i^*$ we obtain both negative and positive values of the helicity with a full consideration of the dipolar energy. For $D_i \neq 0$ helicities reach only values higher than zero. In addition, when considering shape anisotropy, $\phi_0$ for different $D_i$ can take any positive helicity dependent of the $D_i \neq 0$.

4. Conclusions

In conclusion, in this paper, we have shown that a proper consideration of dipolar interaction leads to skyrmions with a significantly modified internal structure as those obtained with a shape anisotropy contribution. Dipolar interaction favors the generation of twisted skyrmions and, when $D_b = 0$, allows the existence of skyrmions with two chiralities, a very important fact that, otherwise, cannot be observed. The dipolar interaction generates gaps in the phase space, restricting the helicities that can be found twisted skyrmions. In addition, for $D_b = 0$ our results show the existence of a critical $D_i$ value that allows considering the dipolar interaction as local anisotropy. Our results show the importance of a paper consideration of this interaction, which for simplicity is usually approximated by a shape anisotropy.

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