Chapter 18

ALLOCATION AND VALUATION OF TRAVEL-TIME SAVINGS*

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1. Introduction

Understanding travel demand is nearly like understanding life itself. The day has 24 hours, and travel time usually consumes a substantial proportion of the truly uncommitted time. In general, individuals would rather be doing something else, either at home, at work, or somewhere else, than riding a bus or driving a car. Accordingly, travelers would like to diminish the number of trips, to travel to closer destinations and to reduce travel time for a given trip. Therefore, individuals are willing to pay some amount for a travel-time reduction, which has a behavioral dimension that seems more a consequence of a general time-allocation problem than an isolated decision. On the other hand, the individual reallocation of time from travel to other activity has a value for "society" as well, either because production increases or simply because the individual is better off and that matters socially. This implies that changes in the transport system that lead to travel-time reductions generate reactions that are important to understand from a behavioral viewpoint, and increase welfare, which has to be quantified for social appraisal of projects.

In general, the reassignment of time from one activity to another that is more pleasurable indeed has a value for the individual. This subject has been explored for more than 30 years by researchers from many different perspectives, including those with an interest in either the study of the labor market, the analysis of home activities, or the understanding of travel behavior. The theories of time allocation deal with the issue of time valuation in many different ways. From these, many concepts of value of time emerge, depending on how a period of time is looked at: as a valuable resource, as something to be reassigned, or as something to be reduced. In the following section these concepts are clearly identified, and their evolution in the literature is presented.

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On the other hand, the subjective value of travel time (SVTT) is the amount the individual is willing to pay in order to reduce by one unit his or her travel time. The simplest manifestation of this is the choice between fast expensive modes and cheap slow ones. However, straight comparison of travel times and travel costs is inadequate because these are not the only characteristics that influence choice and, even if it was the case, simple observations would only provide bounds for the willingness to pay (or save), at the most. The usual procedure to measure SVTT is to estimate discrete travel-choice models and to calculate the rate of substitution between time and money from the estimated utility function. The interpretation of this ratio depends on the underlying theory that generates such utility. In Section 3 we show the relation between the SVTT and the different elements in the theory, starting with the goods–leisure framework, expanding it to all activities and to the consideration of aggregate relations between consumption and leisure. In Section 4 we provide the elements to understand and to calculate social prices of travel-time savings (SPT) to be used in (social) project appraisal. The final section includes a synthesis and conclusions.

2. Time allocation theory and the subjective value of time

When time is considered in consumer theory, there are three important aspects to be taken into account: first, its role in the utility function; second, the need to include a time constraint; and third, the need to identify the relations between time allocation and goods consumption. We will see here that each of these aspects plays an important role in the generation of money measures of activity-time reductions.

In its simplest form, consumer theory models individual behavior as if what the individual does (consumes) is governed by the search for satisfaction, which is limited by income. If one starts with a utility function that depends on consumption, and consumption means expenses, it is a natural step to consider that additional time can be assigned to work in order to increase income, but also that this process has a limit because consumption requires time. Becker (1965) took this step with a twist: he postulated the idea of "final goods" Z_i as those which directly induced satisfaction, and he focused on market goods and preparation time as necessary inputs for Z_i. His main idea was that work time was in fact total time in a period minus preparation–consumption time. Thus, consuming had a time cost; i.e., the cost of not earning money. This was the origin of a value of time equal to the individual wage rate, irrespective of the specific assignment of time to different types of activity.

In terms of the three main aspects mentioned above, in Becker's theory time entered utility as a necessary input to prepare final goods, a time constraint was introduced and then replaced in the income constraint, and the relation between market goods and time was not mentioned at all, although a unit of final good Z_i was said to require goods and time in fixed proportions. Perhaps his emphasis on the conversion of time into money through the wage rate kept somewhat hidden the implicit fixed conversion coefficients that turned goods into time, and vice versa.

Soon after Becker's paper appeared, Johnson (1966) established that the reason behind a value of time equal to the wage rate, was the absence of work time in the utility function. He showed that correcting this omission led to a value of time equal to the wage rate plus the subjective value of work (ratio between the marginal utility of work and the marginal utility of income). Johnson claimed that this was the value of leisure, which in turn was equal to the value of travel time. This, in fact, made sense, as a reduction in travel time could be assigned to either leisure, work or both, but both values should be adjusted until equality through the variation of working hours. Three years later, Oort (1969) mentioned that travel time should be included in utility as well, and a third term appeared in the SVTT notion; namely, the value of the direct perception of travel time in utility. This was also intuitively attractive, as an exogenous reduction in travel time would not only increase leisure or work, but also diminish travel time itself, which might make it even more attractive if travel was unpleasurable in itself.

So far, the analytics can be synthesized as follows, where goods and timeout of work or travel collapse into G and L respectively (see the notation section):

$$\text{Max } \mathrm{U}(G, L, W, t), \tag{1}$$

subject to

$$wW - G \geq 0 \qquad (\lambda) \tag{2}$$

and

$$\tau - (L + W + t) = 0 \qquad (\mu). \tag{3}$$

Eqs. (1) to (3) constitute a simple Oort-like model, having Johnson's and Becker's as particular cases (without t in U in the former, without t and W in U in the latter). First-order conditions are

$$\frac{\partial \mathrm{U}}{\partial G} - \lambda = 0, \tag{4}$$

$$\frac{\partial \mathrm{U}}{\partial L} - \mu = 0, \tag{5}$$

and

$$\frac{\partial U}{\partial W} + \lambda w - \mu = 0, \tag{6}$$

from which one can obtain the following results:

$$\frac{\mu}{\lambda} = \frac{\partial U/\partial L}{\partial U/\partial G} = w + \frac{\partial U/\partial W}{\partial U/\partial G} \tag{7}$$

and

$$-\frac{dU/dt}{\lambda} = w + \frac{\partial U/\partial W}{\partial U/\partial G} - \frac{\partial U/\partial t}{\partial U/\partial G}, \tag{8}$$

where dU/dt is the total effect on utility of an exogenous change in travel time.

Eq. (7) shows that the (money) value of leisure equals the wage rate plus the (money) value of work (the marginal utility of spending time at work converted into money terms). Eq. (8), originally given by Oort (1969) in a footnote, says that the value of a reduction in the minimum necessary travel time is equal to the value of leisure minus the money value of travel time in U (note that no minimum travel-time constraint was included in problem 1–3, and Oort had to deal with this qualitatively). The main corollary is evident: the value of a reduction in travel time would be equal to the wage rate only if both work and travel do not affect utility directly. Thus, the Johnson and Becker results on the value of time are particular cases of eqs. (7) and (8).

In spite of his notation, which actually obscured his results, DeSerpa (1971) made a relevant contribution to the discussion of the value of time by introducing explicitly a set of technical constraints relating time and goods. He postulated a utility function dependent on all goods and all time periods (which he soon called "activities"), including work and travel. The technical constraints established that consumption of a given good required a minimum assignment of time (which facilitates the derivation of eq. (8) above). Within this framework, DeSerpa defined three different concepts of time value. The first is the *value of time as a resource*, which is the value of extending the time period, equivalent to the ratio between the marginal utility of (total) time and the marginal utility of income, or μ/λ in the previous model. The second is the value of time allocated to a certain activity (*value of time as a commodity*), given by the rate of substitution between that activity and money in U, which is equal to μ/λ only if the individual assigns more time to an activity than the minimum required; for the case of travel this corresponds to $(\partial U/\partial t)/(\partial U/\partial G)$ applied to eq. (1). The third concept is the *value of saving time in activity i*, defined as the ratio K_i/λ, where K_i is the multiplier of the corresponding new constraint. He showed that this ratio is equal to the algebraic difference between the value of time assigned to an alternative use (the resource value) and the value of time as a commodity, which is exactly what is shown in equation (8), because the multiplier is the variation in utility after a unit relaxation of the constraint.

We can see that each of DeSerpa's definitions in fact corresponds to different concepts that had previously appeared in the literature. One of his most interesting comments is related with "leisure", which he defined as the sum of all activities that are assigned more time than is strictly necessary according to the new set of constraints. For these activities, the value of saving time is zero, and the value of time allocated to the activity (his "value of time as a commodity") is equal for all such activities and equal to μ/λ, the resource value of time or, what is now evident, to the value of leisure time.

We have praised elsewhere the pioneering work by Evans (1972), who was the first to formulate a model for consumer behavior in which utility depended only on time assigned to activities. Regarding value of time, Evans made some particularly sharp remarks (he did not seem to be aware of DeSerpa's). First, he criticized Johnson (1966) because of the confusion between value of time and value of leisure, extending the critique to Oort (1969), who had compared a reduction in travel time with an extension of the day. Secondly, and due to the explicit introduction of a family of constraints dealing with the interrelation among activities, Evans ended up finding the possibility of a zero value for the marginal utility of income for individuals that earn money faster than their capability to spend it; thus, their time constraint is binding and the income constraint is not, which means an infinite value of time as a resource and an infinite value of saving time (but, of course, a finite value for the time allocated to an activity, which does not depend on the constraints).

Two other models are worth mentioning for their contribution to value of time analysis. One is the review on home economics by Gronau (1986), who in fact extended Becker by including work time in utility. His value of time as a resource ends up being the marginal wage rate plus the value of work minus the value of work inputs. Gronau's approach does not extend to the value of saving time in an activity, but the introduction of input goods value is indeed a contribution because reassigning time in fact induces a marginal change in the structure of consumption (for a formal treatment see Guevara (1999)). It should be stressed that Gronau focuses on work at home. The second approach that introduces novel concepts is that of Small (1982), who includes departure time as a variable, which influences utility, travel time and travel cost. The introduction of an institutional constraint that links departure time, working hours, and the wage rate generates a resource value of time that depends on the work schedule. This is an important point because a travel-time reduction exogenously induced might favor a pleasurable rescheduling of activities. There are other time-related microeconomic models that deal with the discussion of the value of time, such as De Donnea (1971), Pollack and Watcher (1975), Michael and Becker (1973), Biddle and Hamermesh (1990), or Dalvi (1978). In Table 1 we summarize analytically what we consider the main contributions to the analysis of the value of time.

Table 1
Value of time from the main time-allocation approaches

Author	Model	Value of time
Becker (1965)	$\text{Max U} = \text{U}(Z_1, \ldots, Z_n)$ $\sum_{i=1}^{n} P_i X_i = wW + I_f \rightarrow \lambda$ $\sum_{i=1}^{n} T_i = \tau - W \rightarrow \mu$ $Z_i = f_i(X_i, T_i) \quad i = 1, \ldots, n$	$\frac{\mu}{\lambda} = w$
Johnson (1966)	$\text{Max U} = \text{U}(L, W, G)$ $G = wW \rightarrow \lambda$ $\tau = L + W \rightarrow \mu$	$\frac{\mu}{\lambda} = w + \frac{\partial U/\partial W}{\lambda} = \frac{\partial U/\partial L}{\lambda}$
Oort (1969)	$\text{Max U} = \text{U}(L, W, t, G)$ $\tau = L + W + t \rightarrow \mu$ $G + c = wW \rightarrow \lambda$	$-\frac{\text{dU/d}t}{\lambda} = w + \frac{\partial \text{U}/\partial W}{\lambda} - \frac{\partial \text{U}/\partial t}{\lambda}$ $\frac{\mu}{\lambda} = \frac{\partial \text{U}/\partial L}{\lambda}$
DeSerpa (1971)	$\text{Max U} = \text{U}(X_1, \ldots, X_n, T_1, \ldots, T_n)$ $\sum_{i=1}^{n} P_i X_i = I_f \rightarrow \lambda$ $\sum_{i=1}^{n} T_i = \tau \rightarrow \mu$ $T_i \geq a_i X_i \quad \rightarrow K_i \quad i = 1, \ldots, n$	$\frac{\mu}{\lambda} = \frac{\partial \text{U}/\partial L}{\lambda}$ $\frac{K_i}{\lambda} = \frac{\mu}{\lambda} - \frac{\partial \text{U}/\partial T_i}{\lambda}$
Evans (1972)	$\text{Max U} = \text{U}(T_1, \ldots, T_n)$ $\sum_{i=1}^{n} w_i T_i \geq 0 \rightarrow \lambda$ $\tau - \sum_{i=1}^{n} T_i = 0 \rightarrow \mu$ $T_i - \sum_{\forall j \neq i} b_{ij} T \geq 0 \rightarrow K_i \quad i = 1, \ldots, n$	$\frac{K_i}{\lambda} = \frac{\mu}{\lambda} - \frac{\partial \text{U}/\partial T_i}{\lambda} - w_i$ $\frac{\mu}{\lambda} = \frac{\partial \text{U}/\partial L}{\lambda} + w_\text{L}$
Small (1982)	$\text{Max U} = \text{U}(G, L, W, s)$ $G + c(s) = I_f + wW \rightarrow \lambda$ $L + t(s) = \tau - W \rightarrow \mu$ $\text{F}(s, W; w) = 0 \rightarrow \nu$	$\frac{\mu}{\lambda} = w + \frac{\partial \text{U}/\partial W}{\lambda} - \nu \frac{\partial \text{F}/\partial W}{\lambda}$

Table 1
Continued

Author	Model	Value of time
Gronau (1986)	Max $U = U(Z_1, \ldots, Z_n, Z_W)$ $\sum_{i=1}^{n} P_i X_i + P_W X_W = I(Z_W) + I_f \rightarrow \lambda$ $\sum_{i=1}^{n} T_i + W = \tau \rightarrow \mu$ $Z_i = f_i(X_i, T_i) \quad i = 1, \ldots, n$ $Z_W = f_W(X_W, W)$	$\frac{\mu}{\lambda} = w + \frac{\partial U/\partial W}{\lambda} - P_W \frac{\partial X_W}{\partial W}$ with $Z_W = W$ and $I(Z_W) = wW$

3. Discrete travel choice and the value of time

Disaggregate discrete choice models are the most popular type of travel demand models (see Chapter 5 for more details). The most important element is the (alternative–specific) utility level, usually represented through a linear combination of cost and characteristics of each alternative, and socio-economic variables for each group of individuals. Under this approach the analyst is assumed to know, for each individual type, which variables determine the level of non-random utility associated to each discrete alternative. This poses many questions regarding model specification: the structure of decisions, the distribution of the unknown portion of utility, the functional form of the observable part, the type and form of variables that should be used, and the criteria to decide which group of individuals will be regarded as "alike".

In travel choice, the utility of an alternative is usually written in a linear form as

$$\bar{U}_i = \alpha_i + \beta c_i + \gamma t_i + \ldots, \tag{9}$$

where c_i and t_i are travel cost and travel time of alternative i, respectively (we are including a single dimension of travel time for simplicity). Using appropriate data regarding travel choices and individual characteristics, functions like eq. (9) can be estimated from different groups of individuals. From these, one can obtain the amount of money the individual is willing to pay to reduce travel time by one unit. This SVTT is calculated as $(\partial\bar{U}_i/\partial t_i)/(\partial\bar{U}_i/\partial c_i)$; i.e., the rate of substitution between time and cost for constant utility. For eq. (9), this yields an SVTT equal to γ/β. Note that a non-linear utility does not change the concept, but the SVTT would not be constant for a given group across alternatives.

The choice of the word "utility" to describe the equation that represents the level of satisfaction associated to each alternative, is not casual. It is borrowed from the terminology in microeconomics, a discipline that provides a theoretical framework to understand and specify choice models. It is important to stress that what is called "utility" to describe an alternative in discrete-choice models is in fact a conditional indirect utility function (CIUF). The discrete-choice paradigm rests on the deduction of such a CIUF which, unlike direct utility, represents the optimum in all variables but travel and, therefore, includes the constraints in its formulation. The most popular framework is the one presented by Train and McFadden (1978), for the choice of mode in a journey to work, which can be synthesized in its simplest form as

$$\text{Max U}(G, L), \tag{10}$$

subject to

$$G + c_i = wW, \tag{11}$$

$$L + W + t_i = \tau, \tag{12}$$

and

$$i \in M, \tag{13}$$

where M is the set of available modes. The relation between this model and eq. (9) is the following: once G and L are expressed in terms of W, travel cost and travel time from eqs. (11) and (12), and replaced in function (10), U can be maximized with respect to W conditional on mode choice; the optimal value of W is then a function of travel cost and time which, after being plugged back into U, yields the CIUF represented by eq. (9), which should be maximized over $i \in M$. Formally,

$$V_i(c_i, t_i) = \text{U}\{[wW^*(c_i, t_i) - c_i], [\tau - t_i - W^*(c_i, t_i)]\}. \tag{14}$$

In this original goods–leisure model, a reduction in travel time can be reassigned to either leisure or work, and the only reward from work is to increase income for consumption; thus, W will be adjusted until the value of leisure equals the value of work w, which is also the SVTT. This can be shown analytically from eq. (14), by calculating $(\partial V_i/\partial t_i)/(\partial V_i/\partial c_i)$, taking into account the first-order conditions from eqs. (10)–(12). The Train and McFadden (1978) framework is nothing but the discrete counterpart of Becker (1965).

This framework has two limitations: work and travel are assumed to be neutral regarding utility, and there is no technical relation between consumption and leisure. Thus, if all activities are potential sources of utility (or disutility), the general model for discrete choices should include W and t in U besides G and L. On the other hand, if consumption is assumed (for simplicity) to occur during leisure, L should be large enough to accommodate G. The general model in its simplest form would be (Jara-Díaz, 1997)

$$\text{Max } U(G, L, W, t), \tag{15}$$

subject to

$$\begin{aligned} &G + c_i = wW, \\ &L + W + t_i = \tau, \\ &L \geq \alpha G, \\ &i \in M, \end{aligned} \tag{16}$$

where α is consumption time per unit G. Replacing the equality constraints in eqs. (15) and (16), we get the new conditional maximization problem

$$\underset{W}{\text{Max }} U[(wW - c_i), (\tau - W - t_i), W, t_i], \tag{17}$$

subject to

$$\tau - W - t_i - \alpha(wW - c_i) \geq 0. \tag{18}$$

From this we can obtain the SVTT which happens to be given by (see the appendix to this chapter)

$$\text{SVTT} = \frac{\partial V_i / \partial t_i}{\partial V_i / \partial c_i} = w + \frac{\partial U / \partial W}{\partial U / \partial G - \alpha\theta} - \frac{\partial U / \partial t_i}{\partial U / \partial G - \alpha\theta}. \tag{19}$$

This result is quite interesting from many viewpoints. First we have to note, though, that the expression $\partial U/\partial G - \alpha\theta$ is the marginal utility of income λ; the proof can be seen in Jara-Díaz (1997) or checked by looking at eq (A.8) in the appendix, recalling that $\lambda = \partial V_i/\partial c_i$ by virtue of a property of all discrete-choice models. Then, eq. (19) shows that the ratio between $\partial V_i/\partial t_i$ and $\partial V_i/\partial c_i$ indeed captures what DeSerpa (1971) had called the value of saving time in the travel activity, previously presented by Oort (1969). It is worth noting that Bates (1987) building on Truong and Hensher (1985) showed that SVTT in discrete-choice models yields K_i/λ, although using a linear approximation of the indirect utility and a fixed income; besides, his formulation did not include a relation stating that consumption might be limited by leisure. Eq. (19) indicates that the rate of substitution between travel cost and travel time calculated from the so-called modal utility gives the difference between the value of leisure (or value of time as a resource) and the value of travel time in direct utility (or value of travel time as a commodity). As a corollary, if people like working and dislike traveling, the SVTT is unambiguously larger than the corresponding wage rate.

The effect of explicitly recognizing the need to accommodate consumption within a leisure period is very interesting. If this constraint is binding, i.e., if consumption is in fact limited by available time, the different values of time are magnified because the marginal utility of income diminishes. To see this intuitively, we can look at the non-binding case, which makes the marginal utility of income equal to the value of the marginal utility of consumption $\partial U/\partial G$. If consumption is limited by time, the effect of one additional money unit on utility should be smaller (or, if preferred, the marginal utility of goods consumption is larger than the marginal utility of income). Under these circumstances, the marginal utility of leisure is smaller than the total value of work given by the wage rate plus work itself (see eq. (A.1) in the appendix). In addition, the direct values of work and travel time ($(\partial U/\partial W)/\lambda$ and $(\partial U/\partial t_i)/\lambda$) are no longer rates of substitution between the activity and goods in the direct utility.

We have shown that the role of the "leisure enough for consumption" constraint has great importance. This is very much in accordance with Evans's observation regarding a possibly null marginal utility of income: "It must be possible for the consumer's income to accrue at a rate faster than he can spend it. ... At low levels of income the budget constraint will be effective. It is possible that at high income levels only the time constraint is effective" (Evans, 1972, p. 16). We should note that this type of phenomenon is present even if one considers that individuals can spend their money in durable goods they do not use afterwards (for which one has to assign time and money for maintenance). Also, the value of simply "having money" should enter the picture. In both cases (durables and savings), the single period-type analysis is not enough. This is a subject that should be studied further within the context of time values (some elements for discussion are included in Juster (1990)).

4. Towards social values

We have examined what is behind the value that the individual is willing to pay to reduce travel time by one unit. This SVTT, however, is not necessarily equal to what society is willing to pay, which is a relevant question when moving into the area of the appraisal of projects that are financed with social money, i.e., with money collected through taxes.

From the point of view of a society as a whole, reductions in individual travel time can be looked at positively for various reasons. One is the potential increase in real product if such reductions translate into more work. Other is the increase in social welfare, as this includes individual utility directly, which increases indeed as travel conditions improve. Under the approach that regards time as a productive resource only, the SPT would be the value of the individual's marginal product of labor, if travel-time reductions induce an equivalent amount of additional work. On the other hand, if working time is unaltered by travel-time changes, the social price would be nil; this would be the case in pleasure trips or trips made during the leisure period (i.e., out of the (fixed) work schedule). The social price of time would not be nil under the approach that views time as an element that influences individual utility, as all gains should be accounted for, because they mean an increase in social welfare irrespective of changes in physical product.

In a perfectly competitive labour market, the wage rate would represent the value of the marginal productivity of labour. On the other hand, in the original version of the goods–leisure model (in which neither work time nor travel time enter direct utility) the SVTT is exactly given by the wage rate. Thus, if this rate truly represents marginal productivity, and if neither work nor travel induce satisfaction *per se,* then the subjective value of travel time would be equal to the social price and both would be equal to w, under the resource approach. Under the welfare approach, however, this would be different.

Following Pearce and Nash (1981), a social utility or welfare function can be used to represent the implicit preferences in the domain of public decisions. Such a function W_s has the individual utility levels as arguments, and therefore it represents the way in which "society" takes into account individual (or group) welfare. Then,

$$W_s = W_s(U_1, \ldots, U_q, \ldots, U_n). \tag{20}$$

If dB_q is the money equivalent of variation in utility (consumer's surplus) due to a project of individual q, then social welfare would change by

$$dW_s = \frac{dW_s}{dU_q}\frac{\partial U_q}{\partial I} dB_q. \tag{21}$$

On the other hand, as shown in Jara-Díaz (1990), a consumer's surplus variation after a travel time reduction Δt_q is approximately given by

$$dB_q = \mathrm{SVTT}_q\, \Delta t_q. \tag{22}$$

As $\partial U_q/\partial I$ is the marginal utility of income λ_q, then

$$dW_s = \Omega_q \lambda_q \mathrm{SVTT}_q\, \Delta t_q, \tag{23}$$

where Ω_q is the "social weight" $\partial W_s/\partial U_q$. A factor λ_s is needed to convert dW_s into money. Gálvez and Jara-Díaz (1998) point out that the tax system provides a socially accepted equivalence between the total welfare loss of those who pay taxes and the total bill collected. They show that, for non-discriminating social weights Ω_q, a social utility of money can be calculated as a weighted average of individual marginal utilities of income, using tax proportions as weights. Irrespective of which social factor λ_s we use to convert W into money, the term that multiplies Δt_q modified by λ_s is the SPT of individual or group q under the welfare approach. In general, then

$$\mathrm{SPT}_q = \Omega_q \frac{\lambda_q}{\lambda_s} \mathrm{SVTT}_q. \tag{24}$$

Thus, even if $\mathrm{SVTT}_q = w_q$, the SPT_q would not be given by the wage rate within this framework. Note that for SPT_q to be equal to SVTT_q the social weight attached to group q should be inversely related with λ_q or directly related with income. This reveals the highly regressive assumptions behind the acceptance of the subjective value as the social price of time.

As the subjective value of time is always equal to the marginal utility of travel time $\partial V_i/\partial t_i$ over the marginal utility of cost, and this latter is identically equal to minus the marginal utility of income in discrete-choice models, we get the most synthetic form for the social price of time under the welfare approach, which is

$$\mathrm{SPT}_q = \Omega_q \frac{|\partial V_i/\partial t_i|_q}{\lambda_s}. \tag{25}$$

Even if we accept equal social weights (which are difficult to challenge), this result shows how relevant are the elements that determine the marginal utility of travel time as discussed earlier; i.e., the perception of goods, leisure, work, and travel time as arguments in direct utility. Recall that the most general result for $\partial V_i/\partial t_i$ is shown in eq. (19). Expression (25) also shows that social prices of time can vary across individuals. However, this is not because of the (usually different) SVTT, but because of potential differences in the perception of travel time itself.

To summarize, if we follow the resource approach (time as a factor of production), emphasis will be made on quantifying the net effect of travel-time reductions on work. As observed previously, this extreme view would assign a nil value to the SPT for those individuals with fixed working schedule, because time substitution could only be made against leisure. If $\tau - W$ is looked at as time out of work, it is evident that travel-time reductions could be assigned to (unpaid) homework, to recreation, or to basic activities in a more relaxed way. In all such cases there will be an increase either in real product (although difficult to measure) or in quality of life, which the resource approach tends to diminish or ignore. In the social utility approach, all elements are implicitly considered, as the formation of a SVTT is influenced by objective quantities such as the wage rate, income, or time at work, and by the subjective perceptions of goods, leisure, work, and travel.

5. Synthesis and conclusion

We have presented the different concepts of value of time that flow from the different theories on time allocation. Coincidences and differences have been highlighted showing that there has been an evolution towards a much better understanding of the elements that determine money equivalencies for the variation in time assigned to activities. From a time value equal to the wage rate for all activities, we have jumped to values that are activity specific due to the introduction of new important elements in the underlying model for consumer behavior, affecting the arguments of utility and the set of constraints. Regarding utility, all activities are potential sources of (dis)satisfaction. Regarding constraints, there are activities that would be assigned less time if possible. For this latter case, the value of saving time in constrained activities has been shown to have at least three components: the wage rate, the value of work, and the unconstrained value of the activity itself. We have also shown that two other components should be incorporated: the value of the marginal change in the consumption pattern, and the value of rescheduling activities.

The preceding discussion applies as well to transport as an activity. Discrete-choice theory applied to mode-choice models facilitates the calculation of the value of saving travel time (as defined above), which can be obtained as the marginal rate of substitution between travel cost and travel time (for different types of individuals and circumstances) directly from the estimated modal utility, which is in fact a conditional indirect utility function. This rate is also called the SVTT. The microeconomic foundations of this type of model reveal that this subjective value reflects the three elements identified in the preceding paragraph. We propose the use of the general result represented by eq. (19) to make interpretations of estimated subjective values of travel times, taking into account potential departures towards the two facets mentioned above (change in the consumption structure and re-scheduling). This result shows that particular attention should be paid to consumption when it is limited by leisure (not by income), because in this case the marginal utility of income diminishes and the subjective values of time get larger. This is a most important dimension to understand the SVTT and behaviour in general in social environments characterized by a large income relative to leisure time.

Under very specific circumstances (i.e., individually decided working time, no effect of either W or t_i on direct utility), the subjective value of travel time would be exactly equal to the individual wage rate (exogenous), and different from it in all other cases. On the other hand, the "time as a productive resource" approach to the SPT makes this price equal to the product gain given by the value of the marginal utility of labour, which is equal to the wage rate under competitive conditions in the labour market. Thus, only for these particular set of conditions it would hold that SVTT = SPT = w. Alternatively, the social price of time can be looked at as the money equivalent of the increase in social welfare as a result of individual gains in utility due to travel-time reductions; in this case, both the induced leisure (rest, recreation or other) and the induced work would have a social value, which is determined by the relative social weight on the individual utility, the marginal utility of travel time and the social utility of money (eq. (25)).

There are indeed many sources of improvement in the modeling and understanding of the value of time, both individual and social. One is the elements that come from the theory of home economics. There, we can find research dealing with the value of (unpaid) domestic work and also research related with consumption in other areas that reveals trade-offs between time and money, which can help in revealing the specific role of the different sources of utility. In fact, accounting for the value of domestic work and the impact of increased leisure on related markets (e.g., recreational) should diminish the differences between the resource and welfare approaches to the social valuation of travel-time savings. A second source of improvement is the explicit introduction of technical constraints relating goods and activities (duration and frequency), a somewhat forgotten element in the literature (Jara-Díaz and Calderón, 2000), which should improve the interpretation of the SVTT and its relation with the consumption structure. A third aspect to incorporate is the understanding of activity scheduling and its relation with utility; i.e., accounting for the fact that some sequences of activities might be preferred to others, and that this could be affected by travel-time reductions. We see the contributions by Small (1982) and Winston (1987) as starting points in this direction. Finally, understanding and measuring the (hidden) components behind the SVTT (i.e. the subjective values of work, leisure, and activities in general) is a task that has to be dealt with if we are to capture in depth the links between travel and activity patterns.

Appendix

Notation

b_{ij}	minimum time requirement of activity i per unit of activity j
c	travel cost
c_i	travel cost (mode i)
F	function that accounts for the limitations imposed by the institutional setting within which employment opportunities are encountered
f_i	production function of commodity i
G	aggregate consumption in money units
I_f	individual's fixed income
K_i	Lagrange multiplier of minimum time requirement of activity i
L	time assigned to leisure
P_i	price of good i
P_W	price of goods associated with the work activity (nursery, travel, etc.)
s	schedule time (a specific time of the day)
t	exogenous travel time
T_i	time assigned to activity i

t_i	time assigned to travel (mode i)
U	utility function
W	time assigned to work
w	wage rate (work)
w_i	money reward of activity i
w_L	money reward of leisure
X_i	consumption of good i
X_W	consumption of goods associated with work activity
Z_i	final good i
λ	Lagrange multiplier of income restriction
μ	Lagrange multiplier of time restriction
ν	Lagrange multiplier of schedule restriction
τ	total time available

Derivation of the SVTT from the U(G, L, W, t) model

First-order conditions of eqs. (16)–(17) are

$$\frac{\partial \mathrm{U}}{\partial G}w-\frac{\partial \mathrm{U}}{\partial L}+\frac{\partial \cancel{E}^{\mathrm{U}}}{\partial W}-\theta(1+\alpha w)=0 \tag{A.1}$$

and

$$\theta[\tau-t_i+\alpha c_i-W(1+\alpha w)]=0, \qquad \theta\geq 0, \tag{A.2}$$

where θ is the multiplier of constraint (17). As $\theta > 0$ is the most general case, we can solve for W^* in eq. (A.2), which yields

$$W^*(c_i,t_i)=\frac{\tau-t_i+\alpha c_i}{1+\alpha w}. \tag{A.3}$$

Substituting W^* in U (eq. (16)) we get the conditional indirect utility function, which happens to be

$$V_i\equiv \mathrm{U}\left\{\left[\frac{w(\tau-t_i)-c_i}{1+\alpha w}\right],\left[\frac{\alpha[w(\tau-t_i)-c_i]}{1+\alpha w}\right],\left(\frac{\tau-t_i+\alpha c_i}{1+\alpha w}\right),t_i\right\}. \tag{A.4}$$

From this, we can obtain

$$\frac{\partial V_i}{\partial t_i}=-\frac{\partial \mathrm{U}}{\partial G}\left(\frac{w}{1+\alpha w}\right)-\frac{\partial \mathrm{U}}{\partial L}\left(\frac{\alpha w}{1+\alpha w}\right)-\frac{\partial \mathrm{U}}{\partial W}\left(\frac{1}{1+\alpha w}\right)+\frac{\partial \mathrm{U}}{\partial t_i} \tag{A.5}$$

and

$$\frac{\partial V_i}{\partial c_i}=-\frac{\partial \mathrm{U}}{\partial G}\left(\frac{1}{1+\alpha w}\right)-\frac{\partial \mathrm{U}}{\partial L}\left(\frac{\alpha}{1+\alpha w}\right)-\frac{\partial \mathrm{U}}{\partial W}\left(\frac{\alpha}{1+\alpha w}\right). \tag{A.6}$$

Substituting $\partial U/\partial L$ from first-order condition (A.1), the marginal utilities reduce analytically to

$$\frac{\mu}{\lambda} = w + \frac{\partial U/\partial W}{\lambda} = \frac{\partial U/\partial L}{\lambda} \tag{A.7}$$

and

$$\frac{\partial V_i}{\partial c_i} = -\frac{\partial U}{\partial G} + \alpha\theta, \tag{A.8}$$

from which we finally get

$$\text{SVTT} = \frac{\partial V_i/\partial t}{\partial V_i/\partial c_{i\,i}} = w + \frac{\partial U/\partial W}{\partial U/\partial G - \alpha\theta} - \frac{\partial U/\partial t_i}{\partial U/\partial G - \alpha\theta}. \tag{A.9}$$

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