

# Introduction to Focus Issue: Instabilities and nonequilibrium structures

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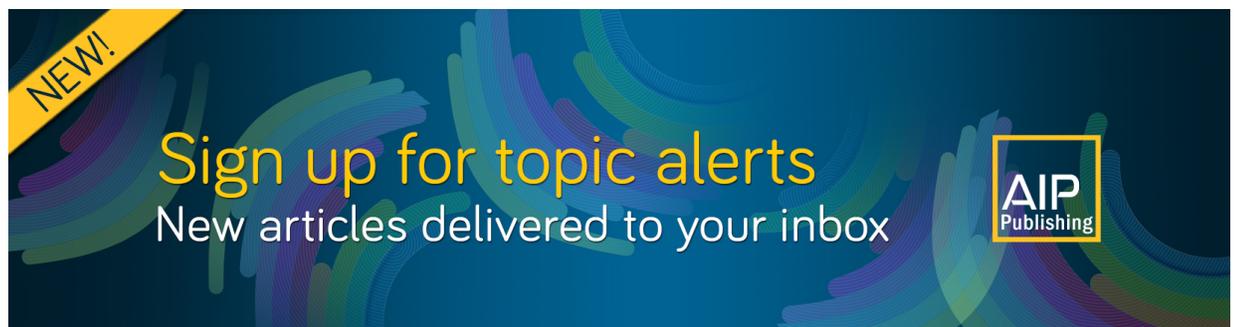
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## ABSTRACT

This Focus Issue on instabilities and nonequilibrium structures includes invited contributions from leading researchers across many different fields. The issue was inspired in part by the “VII Instabilities and Nonequilibrium Structures 2019” conference that took place at the Pontificia Universidad Católica de Valparaíso, Chile in December 2019. The conference, which is devoted to nonlinear science, is one of the oldest conferences in South America (since December 1985). This session has an exceptional character since it coincides with the 80th anniversary of Professor Enrique Tirapegui. We take this opportunity to highlight Tirapegui’s groundbreaking contributions in the field of random perturbations experienced by macroscopic systems and in the formation of spatiotemporal structures in such systems operating far from thermodynamic equilibrium. This issue addresses a cross-disciplinary area of research as can be witnessed by the diversity of systems considered from inert matter such as photonics, chemistry, and fluid dynamics, to biology.

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## I. INTRODUCTION

Since the dawn of physics and applied mathematics, the study of equilibria and their stability has been a central issue. The loss of stability of an equilibrium state gives rise to the concept of *instability*. One of the first treatises on this issue is the buckling instability of plates and elastic columns.<sup>1</sup> At the onset of this instability, there is a critical load below which the column remains in a straight stable state. This phenomenon is of fundamental relevance in the field of construction and in understanding material failure. Instabilities have continued to be vigorously studied up to the present day in all areas of natural sciences. Either isolated or in contact with a thermostat, macroscopic systems exhibit a stationary equilibrium, which is invariant of spatial, temporal, and rotational translation.<sup>2</sup>

When systems operate out of equilibrium subject to a permanent injection with dissipation of energy, particles, and momentum, equilibria are strongly affected.<sup>3–8</sup> In this case, equilibria are attractive; that is, when an equilibrium is slightly altered after a transient, the system converges back to its initial equilibrium. Likewise,

more than an exception, out-of-equilibrium systems simultaneously exhibit many equilibria, *multistability*, which in general are separated by a complex basin of attraction. When the energy injection is small compared to the energy dissipation, out-of-equilibrium systems are usually characterized by exhibiting uniform equilibria similar to those observed in thermodynamical systems. With an increase in the injection of energy, these equilibria can suffer instabilities that generate the emergence of novel uniform equilibria or by a spontaneous spatial symmetry-breaking giving rise to the appearance of patterns,<sup>4–11</sup> among others. These emergent phenomena of self-organization are called *nonequilibrium structures*. Nature is full of these spatial structures such as dunes, animal fur, fish decoration, snowflake, pigmentation on mollusk shells, fir waves, pattern of cirrus clouds, tiger bush vegetation, mountain ripples, and cloud patterns, which have attracted the attention of humanity due to their regularities or irregularities, *defects*, that generate pattern diversity.

In recent decades, much scientific effort has been devoted to the study of spatial dissipative structures or patterns arising in

diverse branches of natural sciences. Aided with new concepts, calculations, tools, experiments, and numerical simulations, various studies have been carried out for a deeper understanding of instabilities and nonequilibrium structures. This Focus Issue aims to capitalize the state-of-the-art research on nonlinear dynamics of out-of-equilibrium systems and promote the incentives for further research related to instabilities and nonequilibrium structures from experts in various scientific disciplines.

## II. SUMMARY OF AREAS COVERED

This issue is intended to provide an overview by experts from various scientific disciplines working on instabilities, nonequilibrium structures, and self-organization, involving experimentalists, theoreticians, mathematicians, and engineers. This issue contains 30 contributions addressing comprehensive overviews of current hot topics, state of the art, recent progress, new ideas, novel techniques, and challenges in instabilities and nonequilibrium structures. They may be broken into three categories of active research: Dynamics of out-of-equilibrium systems, localized structures, nonlinear waves, self-organization, and emergent phenomena.

### A. Dynamics of out-of-equilibrium systems

One of the distinctive characteristics of out-of-equilibrium systems is the possibility of presenting permanent attractive dynamics such as oscillatory, chaotic, turbulent, spatiotemporal chaos, nonlinear waves, among others that Prigogine denominated dissipative structures.<sup>4–7,9,10</sup> Mancini and co-workers<sup>12</sup> studied the periodicity of regular regions embedded in chaotic states for the case of an anisotropic magnetic particle. They computed two-dimensional phase diagrams in the parameter space for the Lyapunov exponents and isospikes. In addition, they observed multiple transitions among periodic states, revealing complex topological structures in the parameter space typical of such dynamical systems. Likewise, out-of-equilibrium systems can present attractive homogeneous stationary equilibria, which are not governed by thermodynamic laws.

Out-of-equilibrium systems tend to form spatial structures which the thermal fluctuations tend to destroy. Wio *et al.*<sup>13</sup> studied the variational approach of a prototype model of surface evolution with inherent fluctuations. Using the stochastic Kardar–Parisi–Zhang equation allows one to readily obtain fluctuation theorems as well as information about the large deviation function for the entropy production. It is expected that such an approach could offer alternative information about critical exponents for the surface model. Equally, Chembo *et al.*<sup>14</sup> investigated the effects of environmental stochastic fluctuations on Kerr optical frequency combs. Many applications require the frequency combs to operate in an ultra-low-noise regime, where they feature the highest temporal coherence. They have provided a generic methodology to compute statistical estimators and have successfully compared the analytical results with numerical simulations. Malomed and co-workers<sup>15</sup> analyzed an efficient transformation of trapped nonlinear-wave modes of effectively one-dimensional Bose–Einstein condensates. They

concluded that the self-attractive nonlinearity in Bose–Einstein condensates can help them to shorten the minimal time. This result may have fundamental implications for the consideration of the quantum speed limit and in the third law of thermodynamics in quantum systems. Gaveau and Moreau<sup>16</sup> studied the coarse-grained distribution of a Hamiltonian system on the space partition determined by the initially measured inaccuracies. They considered the non-stationary mesoscopic process induced by the Hamiltonian evolution from an inhomogeneous initial distribution. In general, this process has infinite memory, but they have shown that its memory fades out with time. They have obtained an ordinary Markov system by introducing a time coarse-graining on “*n*” successive elementary time steps. Therefore, in a generic case, the system eventually tends to equilibrium for any initial mesoscopic distribution. Tirapegui and co-workers<sup>17</sup> have shown the explicit form of the nonlinear partial differential equations for transport coefficients when the thermodynamic system is subject to two thermodynamic forces. Based on the general arguments considered, they proposed equations as a good candidate for describing transport in thermodynamic systems also far from equilibrium. The preliminary test was carried out by analyzing a concrete example where Onsager’s relationships manifestly disagree with experiments: magnetically confined tokamak-plasmas.

The characterization of equilibria and their instabilities is of fundamental importance in dynamical systems. Experimentally, the characterization of transitions is complex due to time scales’ separation, the effect of thermal fluctuations, and inherent experimental imperfections. Morel *et al.*<sup>18</sup> have investigated and determined the Fréedericksz transition using hue measurements of the transmitted light in thin nematic liquid crystal cells. Based on birefringent retardation experienced by transmitted light due to molecular reorientation, they characterized the color adjustment of the nematic liquid crystal cells under white light illumination. By monitoring the hue of the transmitted light, the bifurcation diagram is determined and characterized.

The concepts, ideas, and tools for studying instabilities and non-equilibrium structures have been mainly developed in natural sciences such as physics, chemistry, hydrodynamics, and biology and exact sciences such as mathematics. These concepts, ideas, and tools are equally applied to other areas of knowledge, such as social sciences. Rica and co-workers<sup>19</sup> analyze the 2019 Chilean social unrest episode, consisting of a sequence of events, through the lens of an epidemic-like model that considers global contagious dynamics. They adjust the parameters to the Chilean social unrest using aggregated public data available from the Undersecretary of Human Rights. They observed that the number of violent events follows a well-defined pattern already observed in various other public disorder episodes in other countries since the 1960s. Valdivia and co-workers<sup>20</sup> have studied the impact of deserting a pre-established path, determined by navigation software, on the overall city traffic. To do so, they considered a cellular automaton model for vehicular traffic where the cars travel between two randomly assigned points in the city following three different navigation strategies based on the minimization of the individual paths or travel times. They found that above a critical car density, the transport improves in all strategies if we decrease the time that the vehicles persist in trying to follow a particular strategy.

## B. Localized structures

Fields describe the elementary scale physics. The elementary particles, quarks, and leptons correspond to localized states of fields. Particularly, localized states are characterized by fields having their dominant values in a localized region of space. On the other hand, macroscopic systems are also described by fields. These fields, which arise as a consequence of the injection and dissipation of energy, also exhibit localized solutions and particle-type solutions.<sup>21–32</sup> In recent decades, localized structures have been observed in different fields, such as magnetic materials, chemical reactions, vertically driven Newtonian fluid, granular media, liquid crystals, liquid crystal light valve, colloidal fluids, electrical discharges, thermal convection, and nonlinear optics, to mention a few (see reviews<sup>21,23–32</sup> and references therein). Localized structures exhibit a series of characteristics that are often attributed to corpuscles, such as the size, position, and speed—besides, they present laws of interactions between them.

A ring resonator made of a silica-based optical fiber is a paradigmatic system for the generation of dissipative localized structures or dissipative solitons. Coulibaly and co-workers<sup>33</sup> investigated analytically the formation of pulse solutions by including the non-instantaneous nonlinear temporal response of a cavity fiber, the Raman response. Alternatively, Descalzi and Brand<sup>34</sup> approximated the Raman response using nonlinear gradient terms. They established the existence of stable dissipative solitons exclusively due to the nonlinear gradient terms. Besides, they showed the existence of a feedback loop, leading to dissipative solitons by analyzing a mechanical analog as a function of the magnitude of the amplitude.

An injected semiconductor laser with coherent injection and optical feedback is described by the time-delayed Adler equation. Javaloyes and co-workers<sup>35</sup> explored the mechanisms by which topological localized structures emerge. They have derived the effective equations governing the motion of distant topological localized structures. As a result of the lack of parity in time-delayed systems, which leads to exotic, non-reciprocal interactions between topological localized states. Cisternas *et al.*<sup>36</sup> studied the propagation of light pulses in dual-core nonlinear optical fibers. They have analyzed the dynamics of single standing and walking solitons, as well as walking soliton trains. For the characterization of different scenarios, they used an ensemble-averaged square displacement of the soliton trajectories and a time-averaged power spectrum of the background waves. Power law spectra, indicative of turbulence, were found to be associated with random walks. Gurevich and co-workers<sup>37</sup> have analyzed the dynamics and formation mechanisms of bound states of light bullets in the output of a laser coupled to a distant saturable absorber. Using an effective theory allows them to perform a multiparameter bifurcation study and to identify different mechanisms responsible for the instability of bound states. Rosenau and Pikovsky<sup>38</sup> have investigated phase waves of self-sustained oscillators with a nearest-neighbor dispersive coupling on an infinite lattice. Using an iterative procedure on the original lattice equations, they have determined the shapes of solitary waves, kinks, and flat-like solitons, which we shall refer to as flatons. Direct numerical experiments reveal that the interaction of solitons and flatons on the lattice is notably clean.

Another possibility is by considering coupled oscillators that can allow for the observation of particle-like solutions. Barbay

and co-workers<sup>39</sup> studied theoretically the propagation of nonlinear pulses in a 1D array of evanescently coupled excitable semiconductor lasers. They have shown that the propagation of pulses is characterized by hopping dynamics. Besides, the average pulse speed and the bifurcation diagram are described as a function of the coupling strength between the lasers. The pulse propagation modes evidenced are specific to the discrete nature of the 1D array of excitable lasers. Seidel *et al.*<sup>40</sup> have analyzed the formation and dynamical properties of discrete light bullets in an array of passively mode-locked lasers coupled via evanescent fields in a ring geometry. Using a generic model based upon a system of nearest-neighbor coupled master equations, they have shown numerically the existence of discrete light bullets for different coupling strengths. They have shown the existence of multistable branches of discrete localized states corresponding to different numbers of active elements in the array. Mechanisms are revealed by which the snaking branches can be created and destroyed as the line-width enhancement factor or the coupling strength is varied.

One of the most intriguing particle-like solutions is those that connect coherent and incoherent domains, usually referred to as chimera states. Omel'chenko<sup>41</sup> has considered a large ring of non-locally coupled phase oscillators and showed that apart from stationary chimera states, such a system also supports non-stationary coherence-incoherence patterns. He has shown for the case of identical oscillators wherein the observed patterns behave as breathing chimera states and are found in a relatively small parameter region only. It turns out that the stability region of these states enlarges dramatically when a certain amount of spatially uniform heterogeneity is introduced in the system. Moreover, he has revealed a complex bifurcation scenario for these patterns. Anti-coordination and chimera states occur in a two-layer model of learning and coordination dynamics in fully connected networks. Learning occurs in the intra-layer networks, while a coordination game is played in the inter-layer network. San Miguel and co-workers<sup>42</sup> studied the robustness of these states against local effects introduced by the local connectivity of random networks. They have identified threshold values for the mean degree of the networks such that below these values, local effects prevent the existence of anti-coordination and chimera states found in the fully connected setting. Likewise, they have studied the effect of local connectivity on the problem of equilibrium selection in the asymmetric coordination game, showing that local effects favor the selection of the Pareto-dominant equilibrium in situations in which the risk-dominant equilibrium is selected in the fully connected network.

## C. Nonlinear waves

One of the distinguishing characteristics of macroscopic systems is that they are nonlinear in nature. This allows them to exhibit multistability; that is, for fixed parameters, the system under study can present different equilibria. Solutions connecting these equilibria propagate with a well-defined speed, which depends on the relative stability of the connected states. These attractive nonlinear waves are usually referred to as fronts or wavefronts.<sup>7,11</sup> Alfaro-Bittner *et al.*<sup>43</sup> have explored fronts' propagation in a spatially forced system with a high-wavenumber. Based on a model that describes a liquid crystal light valve with spatially modulated optical forcing,

they studied equilibria and fronts as a function of forcing parameters. The forcing induces patterns coexisting with the uniform state in regions where the system without forcing is monostable. The front evolution is characterized theoretically and numerically. Experimental results verify these phenomena and the law describing bistability. Thiele and co-workers<sup>44</sup> have employed path continuation to determine moving fronts' bifurcation diagram in a simple gradient model for a non-conserved order parameter field. They have used the same methodology to systematically analyze fronts for more involved AC-type models. Gaspard<sup>45</sup> has developed a theory of multistate template-directed reversible copolymerization by extending the method based on iterated function systems to matrices. He has taken into account the possibility of multiple activation states instead of a single one for the growth process. In this extended theory, the mean growth velocity is obtained with an iterated matrix function system, and the probabilities of copolymer sequences are given by matrix products defined along with the template. The theory allows one to understand the effects of template heterogeneity.

Around a given equilibrium, disturbances can present themselves as waves, which, due to nonlinearity, can exhibit complex dynamics. Such interactions can generate highly pulsating waves known as rogue waves. Panajotov *et al.*<sup>46</sup> have investigated and reviewed the formation of two-dimensional dissipative rogue waves in cavity nonlinear optics with transverse effects. They considered two spatially extended systems: the driven Kerr optical cavities subjected to optical injection and the broad-area surface-emitting lasers with a saturable absorber. They have shown that rogue waves are controllable by means of a time-delayed feedback and optical injection. When the strength of the delayed feedback is increased, all the systems generate giant two-dimensional pulses that appear with low probability, which suddenly appear and disappear.

#### D. Self-organization and emergent phenomena

The macroscopic systems, as a result of the injection and dissipation of energy, are self-organized,<sup>4,5</sup> generating complex behaviors, which can be summarized by the expression *the whole is more than its parts*, an idea associated with emerging phenomena. This concept has its origins in the field of philosophy, notably by Aristotle.<sup>47</sup> Speckle is a wave interference phenomenon; these patterns contain spectral information of the interfering waves and of the scattering medium that generates the pattern. Yacomotti and co-workers<sup>48</sup> have studied experimentally the speckle patterns generated by light emitted by two types of semiconductor lasers. In both cases, they have analyzed the intensity statistics of the respective speckle patterns to inspect the degree of coherence of light. They have shown that the speckle analysis provides a non-spectral way to assess the coherence of semiconductor laser light.

Following the idea that dissipation in turbulence at a high Reynolds number is dominated by singular events in space-time and are described by the solutions of the inviscid Euler equations. Pomeau and co-workers<sup>49</sup> drew the conclusion that in such flows, scaling laws should depend only on quantities appearing in the Euler equations. They have focused on the drag law deduced by Newton for a projectile moving quickly in a fluid at rest. They have proposed

an explicit relationship between the Reynolds stress in the turbulent wake and the quantities depending on the velocity field. Laroze and co-workers<sup>50</sup> have studied the control effects on the convection dynamics in a viscoelastic fluid-saturated porous medium heated from below and cooled from above. A truncated Galerkin expansion was applied to balance equations to obtain a four-dimensional generalized Lorenz system. The dynamical behavior is mainly characterized by the Lyapunov exponents, bifurcation, and isospike diagrams. They have shown that within a range of moderate and high Rayleigh numbers, proportional controller gain is found to enhance the stabilization and destabilization effects on the thermal convection. Escaff and Delpiano<sup>51</sup> have presented a Kuramoto-type approach to address flocking phenomena. Analyzing a simple generalization of the Kuramoto model for interacting active particles, they have shown the flocking transition. In the case of global coupling, the proposed model reduces to the Kuramoto model for phase synchronization of identical motionless noisy oscillators. The nature of this non-equilibrium phase transition depends on the range of interaction between the particles. In addition, for larger interaction ranges, the system exhibits the same features as in the case of all-to-all interactions. They have computed the phase diagram of the model, where they distinguish the three phases as a function of the range of interaction and the effective coupling strength.

Spatially extended oscillatory systems can be entrained by pacemakers. Entrainment happens through waves originating at a pacemaker. Biological and chemical media can contain multiple pacemaker regions, which compete with each other. Gelens and co-workers<sup>52</sup> have performed a numerical analysis on the wave propagation and synchronization of the medium depending on the properties of these pacemakers. In particular, they have discussed the influence of the size and intrinsic frequency of pacemakers on the synchronization properties. Vocal production in songbirds is a key topic regarding the motor control of complex learning behavior. Birdsong is the result of an interaction between the activity of an intricate set of neural nuclei specifically dedicated to song production and learning. Amador and collaborators<sup>53</sup> have reported neural additive models embedded in an architecture compatible with the song system to provide a tool to reduce the dimensionality of the problem by considering the global activity of the units in each neural nucleus. This model is capable of generating outputs compatible with measurements of air sac pressure during song production in canaries (*Serinus canaria*). They have shown that the activity in a telencephalic nucleus required by the model to reproduce the observed respiratory gestures is compatible with electrophysiological recordings of a single-neuron activity in freely behaving animals. The dynamics of localized states of light in propagative geometries is investigated by Garbin *et al.*<sup>54</sup> The authors show theoretically and experimentally that this dynamics can be controlled by an adequate parameter modulation and the system can be described as the phase locking of oscillators. They also show that the interactions lead to the convective motion of defects and to an unlocking as a collective emerging phenomenon. Finally, the last contribution by Messaoudi *et al.*<sup>55</sup> discusses the formation of dissipative structures in an arid ecological system. These large-scale structures are generically called vegetation patterns. This contribution explains the interplay between short-range and long-range interactions governing plant communities. Weakly nonlinear analysis is performed as

well as the formation of vegetation patches under the presence of facilitative and competitive interactions between individual plants and seed dispersion.

### III. ANNOUNCEMENT

During the process of editing this issue, Professor Enrique Tirapegui (E.T.) passed away unexpectedly on March 11, 2020. It has been with great sadness that we have to complete this issue without our late friend and collaborator, Enrique. An author of many seminal papers, E.T. completed his Ph.D. in Mathematics and Theoretical Physics at the University of Paris. In 1991, he was honored with the Chilean National Prize of Distinction for his contributions on “the effects of random perturbations experienced by macroscopic systems, and in the formation of spatiotemporal structures in such systems, which are far from thermodynamic equilibrium.” In his distinguished career, E.T. published more than 120 scientific articles in journals in mainstream physics and has trained more than 20 scientists, founding the nonlinear school in Chile. His main scientific efforts were concentrated on the theory of normal forms,<sup>56</sup> description of stochastic processes using path integrals,<sup>57</sup> instabilities in fluids,<sup>58</sup> vortex interactions,<sup>59</sup> and robust effects of noise.<sup>60</sup> The *Instabilities and Nonequilibrium Structures* workshop has been a regular meeting that takes place in Viña del Mar and Valparaíso, Chile, since December 1985, led by E.T. This conference is the oldest conference in South America, which is strictly devoted to the study of Nonlinear Science and which has been an essential element in South America for the development of research in this field.

### DEDICATION

We dedicate this Focus Issue to the memory of Professor Enrique Tirapegui, 1940–2020.

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