

Contents

1	Introduction	1
1.1	Cross-diffusion models	3
1.1.1	Local interaction	3
1.1.2	Main question and answer	8
1.1.3	Our contribution	8
1.2	Non-local self-diffusion models	10
1.2.1	Non-local interaction	11
1.2.2	Main question and answer	13
1.2.3	Our contribution	14
1.3	Genealogical constructions of branching processes	16
1.3.1	Branching processes and their genealogies	16
1.3.2	Main question and answer	19
1.3.3	Our contribution	20
2	Stability of a cross-diffusion system and approximation by repulsive random walks: a duality approach	23
2.1	Introduction and notation	24
2.1.1	Objectives and state of the art	25
2.1.2	Notation	28
2.2	Main objects and results	29
2.2.1	Repulsive random walks and scaling	29
2.2.2	The intermediate (semi-discrete) system	31
2.2.3	Formal insight	31
2.2.4	Statements	32
2.3	A general and rough estimate	35
2.4	Duality estimates	39
2.4.1	The continuous setting	39
2.4.2	Reconstruction operators	41
2.4.3	Prerequisites on the discrete Laplacian matrix	43
2.4.4	The discrete duality lemma	47
2.5	Quantitative estimates and proof of Theorem 2.2	52
A	Appendix: discrete–continuous dictionary	58
3	Quantitative large-population asymptotics for mean-field interacting branching diffusions via optimal transport	60
3.1	Introduction	60
3.2	Model, notations and main result	62

3.3	Preliminaries	66
3.4	Strategy of the proof	69
3.5	Pathwise constructions and coupling through optimal transport	71
3.6	Proof of Theorem 3.2: the pure binary branching case	77
3.7	Proof of Theorem 3.2: the general case	83
B	Appendix	95
4	Large population approximation of the genealogy of branching processes in varying environments	97
4.1	Introduction	97
4.2	Models and main result	99
4.2.1	Lookdown constructions	99
4.2.2	The reduced tree	101
4.3	Preliminaries on trees	103
4.4	Construction of the trees and coupling	105
4.4.1	Level evolution and threshold	106
4.4.2	Trees construction	108
4.4.3	Coupling: proof of Theorem 4.1	111
C	Appendix	116
C.1	Markov mapping theorem	116
C.2	Inhomogeneous lookdown construction	117
Bibliography		121