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MICROECONOMÍA DE LA DINÁMICA URBANA DE LOCALIZACIÓN RESIDENCIAL: UN ENFOQUE JERÁRQUICO ESPACIO-TEMPORAL

TESIS PARA OPTAR AL GRADO DE DOCTOR EN
SISTEMAS DE INGENIERÍA

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RESUMEN EJECUTIVO

Esta tesis consistió en el desarrollo de avances teóricos en el estado del arte en temas relevantes asociados a la modelación microeconómica del consumidor en el ámbito urbano. En particular se analiza las variaciones en las decisiones debido a la inclusión de algunos elementos del comportamiento como la memoria, el aprendizaje o formación de hábitos, las escalas temporales y espaciales definidas a través de la jerarquía que existe, tanto a nivel espacial como temporal, de las decisiones de consumo y asignación de tiempo. Específicamente, se considera la influencia de las decisiones de largo plazo (localización residencial o elección de trabajo, consumo de bienes durables) sobre las decisiones de corto plazo (asignación de tiempo y localización de actividades de ocio, consumo de bienes no durables) y el aprendizaje o fenómenos de memoria que generan todo tipo de elecciones y decisiones como aporte a la dinámica de las decisiones de largo plazo (por ejemplo, localización residencial). Esta tesis fue escrita en formato de dos artículos científicos. A continuación se describe sintéticamente los dos artículos:

En el primer artículo titulado "*Un modelo microeconómico jerárquico de asignación de tiempo a actividades*", se extiende el enfoque microeconómico desde un punto de vista teórico para explicar el comportamiento de los consumidores respecto a la elección de las actividades, bienes de consumo y la asignación de tiempo mediante la inclusión explícita de la dimensión temporal en el proceso de toma de decisiones. La premisa de este enfoque es que algunas actividades y consumo de bienes se asumen en un plazo de tiempo más largo, como empleo y educación, mientras que otras se hacen y deciden en el corto plazo, como las opciones de ocio, compras y transporte. Se diseña un modelo microeconómico con una estructura llamada de multi-escala temporal que captura el ajuste de las variables: duración, localización y consumo de bienes, y la magnitud de los recursos (tiempo y dinero) invertidos. Se formula y analiza un modelo microeconómico con dos escalas de tiempo: los niveles macro y micro, concluyendo que la observación de las preferencias a nivel micro, como la elección del modo de transporte, están fuertemente condicionados por las decisiones que prevalecen en la escala macro obteniendo una jerarquía en la toma de decisiones. Este resultado teórico explica resultados empíricos previos de valoración y asignación de tiempo de los individuos, así como en la localización de las actividades, como se ilustra en algunas simulaciones numéricas realizadas.

Para la formulación del modelo se incorpora una estructura jerárquica de decisiones donde las elecciones de largo plazo se ajustan en un punto del tiempo (punto de ajuste) y se mantienen fijas durante una ventana de tiempo de largo plazo, llamada ventana macro. Durante cada ventana de tiempo macro, los individuos toman decisiones de corto plazo que tienen una escala de duración mucho menor y están siendo frecuentemente cambiadas (por ejemplo, actividades de ocio). Estas decisiones de corto plazo dependen de los recursos en tiempo, dinero y bienes durables que transfieren las decisiones de largo plazo o fijas. Las formulaciones se basan en los modelos clásicos de uso y valoración del tiempo, en especial Jara-Díaz (2003), Evans (1972) y De Serpa, (1966).

El segundo artículo también es un análisis de tipo teórico titulado "*Modelo Microeconómico de Relocalización Residencial Incorporando Evolución de los Agentes, Aprendizaje Individual y Expectativas en el Ciclo de Vida* ". Dicha modelación surge de la necesidad de analizar el efecto del ciclo de vida de los agentes en la distribución urbana en el corto y largo plazo, teniendo en cuenta la incorporación de dinámicas del ciclo de vida de los hogares en la valoración de los bienes inmuebles tales como aprendizaje, formación de hábitos, generación de expectativas, que no han sido analizados en los modelos inter-temporales de uso de suelo ni en modelos de

segregación residencial (Grauwin, et. al., 2009) y que son procesos que han sido evidenciados en algunos modelos empíricos econométricos de elección de vivienda, procesos de relocalización o movilidad intra-urbana (Nijkamp, et al. 1993; Páez, et al. 2008; Eluru, et al. 2009; Chen, et al. 2009; Chen y Lin, 2011). Este trabajo se basa en la teoría microeconómica del uso del suelo urbano, donde se supone que cada bien inmueble es asignado al agente (hogar o firma) que tiene la mayor la disposición a pagar por dicho bien.

En primer lugar se desarrolla un modelo de elección residencial cuya novedad es incluir la experiencia obtenida de localizaciones previas asociándolo a un proceso de aprendizaje dinámico de los hogares y que cada agente obtiene de acuerdo a la utilidad asociada a cada bien. Por otro lado, se formula un segundo modelo microeconómico de localización residencial que incorpora las expectativas de cambio socioeconómico asociadas al ciclo de vida de los hogares por medio de probabilidades de transición entre categorías de hogares y la hipótesis de imitación de los agentes bajo la consideración de que se comportan racionalmente. De esta forma se obtiene una función de postura que incluye un ingreso esperado por unidad de tiempo y una utilidad consistente con el comportamiento de los agentes que son potencialmente imitables. Para los dos modelos se desarrolla una versión estocástica bajo una estructura logit multinomial, donde se supone que la disposición a pagar se determina por una parte determinista más un parte aleatoria con distribución de probabilidad iid Gumbel. Además, debido a las dinámicas del ciclo de vida analizadas se identifica la necesidad de incorporar una restricción de ingreso por unidad de tiempo sobre las posturas. Se presentan ejemplos numéricos y simulaciones usando funciones lineales de postura similares a las usadas en modelos de segregación residencial, obteniendo interesantes resultados asociados a la dinámica de dicho fenómeno urbano en el corto y largo plazo debido a la inclusión de los procesos de aprendizaje y expectativas descritos anteriormente.

Tabla de contenido

Índice de ilustraciones

Parte I:

INTRODUCCIÓN A LOS ARTÍCULOS

Capítulo 1

Introducción a los artículos

1.1. Motivación y objetivos

El estudio y análisis de la dinámica del uso del suelo y del sistema transporte de una ciudad, involucra la descripción y modelación de procesos interdependientes debido a la diversidad de agentes que interactúan y toman decisiones, de forma diferenciada en el tiempo y el espacio, en cuanto a localización y asignación de tiempo a actividades y consumo de bienes (continuos y discretos). Algunos procesos importantes en la dinámica de la toma de decisiones del sistema urbano son la memoria, el aprendizaje, la formación de hábitos, las escalas temporales y espaciales, la incertidumbre sobre la disponibilidad de recursos y la fluctuación en los cambios sociales y económicos; esto configura un sistema complejo con estructura similar a otros sistemas sociales y naturales (Gunderson y Holling, 2002). Un elemento importante es la jerarquía que existe, tanto a nivel espacial como temporal, entre las decisiones de consumo y asignación de tiempo. Por ejemplo, la influencia de las decisiones de largo plazo (localización residencial o elección de trabajo, consumo de bienes durables) sobre las decisiones de corto plazo (asignación de tiempo y localización de actividades de ocio, consumo de bienes no durables) y el aprendizaje o fenómenos de memoria que generan todo tipo de elecciones y decisiones como aporte a la dinámica de las decisiones de largo plazo (por ejemplo, localización residencial).

De acuerdo a lo descrito anteriormente y como motivación de esta investigación, sería interesante analizar e incorporar nuevas estrategias de modelación económica que expliquen algunas dinámicas no examinadas en los modelos clásicos y que son parte de la estructura de decisiones del sistema urbano; Por ejemplo, la jerarquía que se tiene en las decisiones donde las elecciones de largo plazo (localización residencial, asignación de tiempo y localización del trabajo) condicionan las decisiones de corto plazo y a la vez incorporar efectos como el aprendizaje o formación de hábitos, en decisiones como la localización residencial.

De esta forma se propone generar una estructura analítica que permita ayudar a explicar algunas de estas características de la complejidad del sistema y la dinámica asociada de la configuración de la ciudad (por ejemplo, análisis de fenómenos de segregación); esto constituye el objetivo del análisis y la formulación de las modelaciones propuestas en esta tesis.

1.1.1. Objetivos y motivación

El objetivo general de esta tesis es formular y desarrollar un modelo con jerarquías temporales en el consumo de bienes, asignación de tiempo y localización de actividades. La idea es generar resultados analíticos y experimentales que ayuden a explicar la dinámica en la toma de decisiones en el sistema urbano; Por otro lado, interesa estudiar la decisión de localización residencial incorporando fenómenos de aprendizaje o generación de expectativas del ciclo de vida de los hogares, analizando los estados de equilibrio de la distribución urbana, en especial en procesos de segregación residencial en el corto (cada corte temporal) y largo plazo.

Para lograr este objetivo, es necesario alcanzar los siguientes objetivos específicos:

1. Formular un modelo de asignación y valoración de tiempo que involucre la jerarquía y la dinámica en la toma de decisiones. Este objetivo busca delinear una base teórica microeconómica para el desarrollo de modelos propios de economía urbana. Además, se busca obtener nociones y resultados generales de la implicancia que tiene la hipótesis de jerarquía en las decisiones y comportamiento del consumidor.

2. Formular un modelo determinístico microeconómico de localización de actividades, basado en la formulación jerárquica propuesta en el objetivo anterior. Desarrollar ejemplos numéricos que expliquen la bondad del modelo.

3. Desarrollar modelos estocásticos o reglas de comportamiento de estructura Logit asociados a la decisión de localización residencial, que incorporen la heterogeneidad de los agentes y dinámicas como aprendizaje y generación de expectativas. A través de esta formulación estocástica se generarán escenarios numéricos para explicar el modelo desarrollado.

4. Analizar el efecto de los modelos propuestos sobre el equilibrio urbano. En este caso el equilibrio a analizar incluye la noción de equilibrio urbano en cada corte temporal y adicionalmente, en el largo plazo.

1.1.2. Estructura de la tesis

Esta tesis se divide en tres capítulos; aparte de la presente introducción, en el primero se desarrolla breve revisión bibliográfica que describe algunos elementos de la teoría de elecciones discretas, economía urbana y modelos microeconómicos de asignación de tiempo; estas áreas fundamentan la investigación realizada en la tesis.

Los capítulos 2 y 3 corresponden a dos documentos escritos en formato de artículo en inglés debido a que la presente tesis fue desarrollada como un compendio de dos publicaciones. En ese sentido, en el capítulo 2 se presenta la primera publicación titulada "*Un modelo microeconómico jerárquico de asignación de tiempo a actividades*"; en este desde artículo se extiende la modelación clásica microeconómica de asignación de tiempo a actividades y el consumo de bienes un punto de vista teórico. La premisa del modelo presentado es que algunas actividades y consumos de bienes se asumen en un plazo de tiempo más largo, mientras que otras se hacen y deciden en el corto plazo. Se formula y analiza un modelo microeconómico con dos escalas de tiempo: los niveles macro (largo plazo) y micro (corto plazo), concluyendo que la observación de preferencias a nivel micro, está fuertemente condicionada por las decisiones que prevalecen en la escala macro. Este resultado teórico permite explicar algunos resultados empíricos de valoración y asignación de tiempo de los individuos, y de localización de actividades; esto se ilustra en algunas simulaciones numéricas realizadas.

Finalmente, en el capítulo 3 se presenta la segunda publicación titulada "*Modelo Microeconómico de Relocalización Residencial Incorporando Evolución de los Agentes, Aprendizaje*

Individual y Expectativas en el Ciclo de Vida ". Este trabajo se basa en la teoría microeconómica del uso del suelo urbano, donde se supone que cada bien inmueble es asignado al agente (hogar o firma) que tiene la mayor disposición a pagar por dicho bien. En primer lugar se desarrolla un modelo de elección residencial cuya novedad es incluir la experiencia obtenida de localizaciones previas, asociándolo a un proceso de aprendizaje dinámico de los hogares; dicho nivel de aprendizaje se define con la utilidad obtenida. también, se formula un segundo modelo microeconómico de localización residencial, que incorpora las expectativas de cambio socioeconómico asociadas al ciclo de vida de los hogares por medio de probabilidades de transición entre categorías de hogares y una hipótesis de imitación entre agentes (bajo la consideración de que se comportan racionalmente). Para ambos modelos se desarrolla una versión estocástica con estructura Logit multinomial. Se presentan ejemplos numéricos y simulaciones usando funciones lineales de postura similares a las utilizadas en modelos de segregación residencial, obteniendo interesantes resultados asociados a la dinámica de dicho fenómeno urbano en el corto y largo plazo.

1.2. Breve revisión bibliográfica

La investigación desarrollada se fundamenta en varias áreas propias de la economía e ingeniería, dentro de las que se encuentran la teoría de elecciones discretas, economía urbana y modelos microeconómicos de asignación de tiempo. Cada uno de estos temas se describe brevemente en las siguientes secciones.

1.2.1. Teoría de elecciones discretas y teoría de la utilidad aleatoria

Los modelos de elección discreta son una de las principales herramientas usadas para predecir y analizar la elección que realiza un individuo o agente de una única alternativa entre un conjunto de posibilidades mutuamente excluyentes, finito y conocido por el tomador de decisiones (Manski, 1977). Esta teoría se basa en el supuesto de racionalidad donde siempre se opta por la opción que genera mayor utilidad. En términos matemáticos, Sea V_{hi} la utilidad que obtiene un individuo h por el bien discreto i. Dicha utilidad es completamente conocida o determinada por el tomador de decisiones. Además, V_{hi} es una utilidad indirecta condicional asociada a un problema microeconómico del consumidor. En los modelos de utilidad aleatoria (Block y Marschak, 1960) se obtiene la probabilidad de que el individuo h elija la alternativa i de la siguiente forma:

$$P_{hi} = P(V_{hi} \geq V_{hj},\ \forall j \in A_h), \tag{1.1}$$

donde A_h es el conjunto de alternativas disponibles para el individuo h. Mc Fadden (1968) hace una extensión econométrica de esta teoría considerando que una población de individuos realiza la misma elección sobre un conjunto de alternativas y determina la fracción de la población que elige una alternativa determinada; esta población puede ser dividida en H grupos socioeconómicos observables (ingreso, edad, profesión, etc.).

En estos modelos se considera que V_{hi} es conocida por el tomador de decisiones, pero no por el modelador, por lo que éste la representa como la suma de dos componentes:

$$V_{hi} = \widetilde{V}_{hi} + \xi_{hi}, \tag{1.2}$$

una determinística o sistemática $\widetilde{V}_{hi}$, conocida por el modelador y que es función del vector de atributos Z_i que definen la alternativa i , y otra componente aleatoria ξ_{hi} que representa y recoge la incapacidad del analista de apreciar todos los atributos y variaciones en los gustos o comportamiento de los individuos, los errores de medición y modelación, la falta de mejor información y la especificación dada a la función de utilidad determinística. Por lo tanto, si se asume que los errores ξ_{hi} siguen una distribución Gumbel independientes e idénticamente distribuidos (iid) con parámetro de escala (μ) entonces la probabilidad de elección tiene una forma funcional Logit multinomial (Ortúzar y Willumsen, 2001):

$$P_{hi} = \frac{\exp(\mu \widetilde{V}_{hi})}{\sum_{j \in A_h} \exp(\mu \widetilde{V}_{hj})}. \tag{1.3}$$

El parámetro de escala μ no es identificable, por lo que habitualmente se fija su valor en 1. Un supuesto interesante de (1.3) es que los individuos siguen un comportamiento compensatorio, es decir, utilizan una estrategia de decisión que establece la posibilidad de un intercambio o variación entre el valor de los atributos para mantener un nivel de utilidad. Dado que existen elecciones en diferentes contextos donde ese supuesto compensatorio no es realista pues las características de los bienes están acotadas por umbrales dados por el tomador de decisiones o regulaciones del sistema, se hace necesario reducir el número de alternativas a aquellas que cumplen dichas restricciones. En este trabajo se describirá el modelo *Logit Restringido* (*CMNL*; Martínez, et al. 2009) debido a su incorporación en la formulación de los modelos de elección residencial. Basados en los trabajos de Swait (2001) y Cascetta y Papola, (2001), Martínez et al. (2009) asumen que la utilidad se puede separar en un término compensatorio y otro no compensatorio que indica la factibilidad de esa alternativa para h:

$$V_{hi} = \widetilde{V}_{hi} + \frac{1}{\mu}\ln(\phi_{hi}(z_i)) + \xi_{hi}, \tag{1.4}$$

donde $\ln(\phi_{hi}(z_i))$ es una función *cut-off* o de penalización impuesto por el individuo h a la alternativa i y μ es el parámetro de escala. Dicha penalización mediante una función logaritmo logra que se tenga una transición suave entre el espacio factible compensatorio y el espacio infactible no compensatorio, permitiendo que las restricciones puedan ser sutilmente incumplidas por el tomador de decisiones. Suponiendo que ξ_{hi} de (1.4) distribuye (iid) Gumbel, la probabilidad de elección de i en el CMNL es:

$$P_{hi} = \frac{\phi_{hi}(z_i)\exp(\mu\widetilde{V}_{hi})}{\sum_j \phi_{hj}(z_j)\exp(\mu\widetilde{V}_{hj})}, \tag{1.5}$$

El modelo *CMNL* puede considerar tanto restricciones inferiores ϕ^L_{hik} (por ejemplo, un mínimo salario en la elección de trabajo) como superiores ϕ^U_{hik} (en elección modal, un tiempo máximo de espera asociado a la elección de modo). Estas restricciones son definidas por medio del logit binomial.

Tal como lo describe Bierlaire et al., (2010) el modelo *CMNL* puede ser visto como una heurística o aproximación práctica a la solución del problema restringido, en especial en el caso que las restricciones sean determínisticas; esto tiene como caso particular un número grande de alternativas. Por otro lado, Castro et al. (2013) estudian la estimación de los parámetros del modelo *CMNL* utilizando la función de máxima verosimilitud con datos sintéticos y reales. Los autores concluyen que el modelo *CMNL* parece ser más adecuado en algunas aplicaciones, ya que presenta mejor ajuste que el modelo Logit Multinomial bajo el supuesto de comportamiento acotado y replica las estimaciones del modelo Logit multinomial (MNL) en el caso sin restricciones. Además, encuentran diferencias significativas en el valor del tiempo y las elasticidades entre los resultados del modelo compensatorio MNL y el semi-compensatorio CMNL, mostrando que dichas diferencias aumentan a medida que los umbrales de los atributos se activan.

1.2.2. Conceptos y modelos de economía urbana

La economía urbana se basa en dos enfoques primordiales para explicar la elección de localización de los agentes urbanos. En el primero, llamado Bid (postura), se asume un mercado del tipo remate bajo la hipótesis que una vivienda es un bien cuasi-único, donde los agentes realizan ofertas por las distintas localizaciones, que se adjudican al mejor postor (Alonso, 1964). La postura depende de variables como los atributos de la localización o vivienda, variables de accesibilidad, así como del nivel de ingreso, consumo y nivel de utilidad de los agentes. El segundo enfoque, denominado Choice (elección), asume que los agentes escogen aquellas localizaciones que maximizan su nivel de utilidad (Mc Fadden, 1978; Anas, 1982) donde se desarrolla un modelo que estudia el mercado urbano como un mercado normal competitivo en que se alcanza un equilibrio estático de tipo Walras (Varian, 1978). A continuación se explican brevemente los dos enfoques en base a los trabajos de Martínez y Araya (2000) y Martínez (1992); para esto se supone el siguiente problema estático y determinístico de localización residencial usando la teoría de elecciones discretas donde el hogar tipo $h \in H$ en una ventana de tiempo t busca un bien inmueble (i) que maximice su utilidad,

$$\max_i \max_x \; U(x, Z_i^t) \;\text{ sujeto a }\; p^t x + r_i^t \leq I_h^t \tag{1.6}$$

donde Z_i^t es el conjunto de atributos del bien (i) que se pueden dividir en los atributos propios de la vivienda, atributos en el uso de suelo en la zona donde está ubicado el bien y las características de accesibilidad y atractividad (Louviere y Timmermamns, 1990); r_i^t es la renta por el bien (i); I_h^t es el ingreso del hogar h y p^t es el vector de precios asociado a un conjunto de bienes de mercado x. Dada la solución óptima del problema (1.6) en el consumo de bienes x entonces la función de utilidad indirecta es condicional a la localización en (i):

$$V_{hi}^t \equiv V_h^t(I_h^t - r_i^t, Z_i^t, p^t) \tag{1.7}$$

Dado un nivel de utilidad $\overline{U}_h$ fijo, la utilidad indirecta se puede invertir en la variable renta (*en caso de existir la función inversa*) :

$$r_{vi}^t = I_h^t - V_h^{-1}(\overline{U}_h, Z_i^t, p^t) \tag{1.8}$$

Bajo la consideración de un mercado de remate la variable renta puede ser vista como la disposición a pagar (Ellickson, 1981):

$$B_{hi}^t = I_h^t - V_h^{-1}(\overline{U}_h, Z_i^t, p^t) \tag{1.9}$$

Es fácil demostrar que si la función de utilidad directa es cuasi-lineal, es decir de la forma $U_h(x, Z_i^t) = ax_0 + f(x_1, ..., x_n, Z_i^t)$ entonces la formas funcionales de la utilidad indirecta y de la disposición a pagar son respectivamente:

$$V_{hi}^t(Z_i^t, I_h^t - r_i^t) = \lambda_h^t(I_h^t - r_i^t) + \lambda_h^t b_{hi}^t(Z_i^t),$$

$$B_{hi}^t = I_h^t + b_{hi}^t(Z_i^t) - \frac{U_h^t}{\lambda_h^t},$$

donde λ_h^t es la utilidad marginal del ingreso y $b_{h,i}^t(Z_i^t)$ es una función que mide la valoración de los atributos del bien por parte del hogar h. La expresión previa de B_{hi}^t es denotada en Martínez y Henríquez (2007) como:

$$B_{hi}^t = a_h^t + b_{hi}^t(Z_i^t) \tag{1.10}$$

donde el valor de $a_h^t = I_h^t - \frac{\overline{U}_h^t}{\lambda_h^t}$ dependen del ingreso del hogar I_h^t, la utilidad marginal del ingreso λ_h^t y un nivel de utilidad $\overline{U}_h^t$ que es obtenida por la condición de equilibrio de mercado, que es, todo agente debe localizarse.

Por otro lado, si suponemos que la disposición a pagar se puede modelar como $\widetilde{B}_{hi}^t = B_{hi}^t + \varepsilon_{hi}^t$, donde ε_{hi}^t se distribuye idéntica e independiente Gumbel con parámetro de dispersión μ y B_{hi}^t es la parte determinística o modelable dada por (1.10), entonces la probabilidad que el hogar h sea mejor postor por el bien inmueble i es:

$$Q_{h|i}^t = \frac{\exp(\mu B_{hi}^t)}{\sum_{g \in H} \exp(\mu B_{gi}^t)} \tag{1.11}$$

Usando la corrección por tamaño de cada alternativa o grupo de hogares en (1.11) propuesta por Mc Fadden (1978), la probabilidad de que el hogar que pertenece al grupo h sea el mejor postor $Q^t_{h|i}$ en la localización i , es:

$$Q^t_{h|i} = \frac{H^t_h \exp\left(\mu B^t_{hi}\right)}{\sum_{g\in H} H^t_g \exp\left(\mu B^t_{gi}\right)} \tag{1.12}$$

donde H^t_h is el tamaño del grupo (hogar tipo h) en el período t.

En el modelo RB&SM (Martínez y Henríquez, 2007) se considera la existencia de externalidades asociadas a la distribución urbana en cada zona, que afectan la disposición a pagar de los hogares, por lo tanto $B^t_{hi}(H^t_{gi}, \forall g,)$ y de esta forma la probabilidad de elección $Q^t_{h|i}$ depende de las demás probabilidades de localización $Q^t_{g|i}$ generando un sistema no lineal de ecuaciones de punto fijo en (1.12).

Adicionalmente, la renta en cada localización i se obtiene de forma endógena a través del valor esperado de la máxima postura:

$$r^t_i = \frac{1}{\mu}\left\{\ln\left(\sum_h H^t_h \exp(\mu B^t_{hi}) + \gamma\right)\right\} \tag{1.13}$$

donde γ es la constante de Euler. Por otro lado, la condición de equilibrio asegura que cada hogar será localizado, por lo tanto se tiene que

$$\sum_i Q^t_{h|i} S^t_i = H^t_h,\ \forall h \tag{1.14}$$

De (1.14) se obtiene la siguiente expresión para el valor de a^t_h:

$$a^t_h = -\frac{1}{\mu}\ln\left(\sum_i S^t_i \exp(\mu(b^t_{hi} - r^t_i))\right),\ \ \forall h. \tag{1.15}$$

Como las rentas r^t_i dependen de cada a^t_g entonces (1.15) constituye un sistema de ecuaciones nolineales de punto fijo, es decir, $a^t_h = f(a^t_g, \forall g)$, cuya solución define el nivel máximo de utilidad en equilibrio.

El modelo anterior es extendido y operacionalizado en MUSSA (Modelo de uso de suelo para Santiago de Chile[1]) donde se incorpora la necesidad de incluir una restricción de ingreso en la elección de los bienes inmuebles que garantice que las posturas de los agentes urbanos sean factibles dado los niveles de utilidad referenciales (Martínez y Donoso, 2010).

En este trabajo supondremos que la oferta S^t_i es exógena y no se expondrá con mucho detalle los diferentes enfoques que existen sobre la modelación de está. Sin embargo, es importante anotar que entre mayor claridad se tenga sobre los factores que afectan la demanda en el largo y corto plazo, más factible será tener mejores resultados en los modelos de generación de oferta de bienes inmuebles (Nijkamp, et al., 1993).

A continuación se describen algunos modelos dinámicos (basados en la demanda de los bienes) descritos en el artículo de elección de localización residencial y que a su vez sirvieron de base

[1]Recibe el nombre de MUSSA para la modelación realizada en Santiago de Chile, pero vale la pena anotar que en la actualidad, el modelo está siendo internacionalizado bajo el nombre Cube Land por la compañía internacional de software Citilabs.

y generaron el interés de incorporar la dinámica propia del ciclo de vida de los hogares tales como variación de sus características (cambio de grupo), memoria y expectativas de cambio en un modelo de demanda en la decisión de localización residencial.

Un trabajo reciente en la modelación dinámica del uso del suelo es el desarrollado por Martínez y Hurtubia (2006), donde el problema del máximo postor se escribe de la forma:

$$\max_{h \in H} B_{hi}^t(z_i^{t-1}, \overline{U}_h^t), \ \text{ sujeto a } \ B_{hi}^t \leq I_h^t \tag{1.16}$$

Es importante anotar que en este caso la postura en el periodo t depende del vector de atributos definido en el equilibrio del periodo anterior $(t-1)$, por lo que no existe equilibrio estático por externalidades y de esta forma no es necesario resolver la demanda de bienes a partir de ecuaciones de punto fijo no lineal (1.12). Este supuesto de rezago en las externalidades y características del bien i se justifica en el proceso temporal que tienen los tomadores de decisiones en la recolección de la información. Como en el caso anterior, para modelar la heterogeneidad idiosincrática en el comportamiento de los hogares de un mismo grupo se asume que la postura es una variable aleatoria $\widetilde{B}_{hi}^t = B_{hi}^t + \epsilon_{hi}$ donde el término aleatorio se distribuye Gumbel IID. De esta forma la probabilidad de que h sea el mejor postor por la vivienda i en el periodo t es

$$Q_{h|i}^t = \frac{H_h^t \phi_{hi}^t \exp\left(\mu B_{hi}^t\right)}{\sum_{g \in H} H_g^t \phi_{gi}^t \exp\left(\mu B_{gi}^t\right)} \tag{1.17}$$

donde ϕ_{hi}^t es la probabilidad de disponer con recursos para la postura, es decir $B_{hi}^t \leq I_h^t$ que se modela usando el logit restringido (CMNL) de la forma:

$$\phi_{hi}^t = \frac{1}{1 + \exp(w(B_{hi}^t - I_h^t + \theta))} \tag{1.18}$$

Este valor tiende a 1 si la postura está muy por debajo del ingreso y tiende a un valor constante θ si la postura tiende al ingreso. Usando este factor de corrección la renta r_i^t se estima de la siguiente forma:

$$r_i^t = \frac{1}{\mu}\left(\sum_g H_g^t \phi_{gi}^t \exp(\mu B_{gi}^t) + \gamma\right) \tag{1.19}$$

Además, se obtiene la probabilidad de elección de tipo *choice* $P_{i|h}^t$, es decir:

$$P_{i|h}^t = \frac{S_i^t \phi_{hi}^t \exp(\mu(b_{hi}^t - r_i^t))}{\sum_j S_j^t \phi_{hj}^t \exp(\mu(b_{hj}^t - r_j^t))}, \tag{1.20}$$

donde S_j^t es la reserva de viviendas en el período t y $b_{hi}^t - r_i^t$ es la utilidad (o excedente del consumidor) asociada al bien i con rentas exógenas r_i^t.

El sub-modelo de oferta desarrollado en este trabajo se basa en conceptos de economía dinámica y asume que los productores son homogéneos y se pueden modelar como un agente que maximiza ganancias futuras. La decisión de cuánto y dónde construir se toma en un periodo t para entregar las viviendas al mercado en $t+n$, generando un retraso temporal y produciendo un des-equilibrio entre oferta y demanda. El modelo asume que no existe demolición de viviendas ni relocalización de hogares. Para el desarrollo de simulaciones se supone que la postura es linealmente dependiente del ingreso promedio por zona en $t-1$ y de la densidad promedio de uso de suelo.

Otro modelo importante en el análisis de la dinámica del mercado inmobiliario, es el de Anas y Arnott (1991), que plantea y resuelve equilibrios para todo el vector temporal de manera simultánea y hace supuestos sobre la capacidad de predicción de los productores respecto a los estados futuros del mercado inmobiliario, conociendo los costos de construcción y los valores futuros de las rentas ("*perfect foresight*"). En particular ese supuesto de conocimiento del sistema futuro puede ser debatible o no de fácil cumplimiento, debido a las características dinámicas y cambiantes de la ciudad y de los agentes (hogares, firmas, etc), tales como el ciclo de vida, valoración, incorporación y dinámica de las redes sociales, formación de hábitos, expectativas de cambio, etc.

Otro tipo de modelación que involucra dinámica urbana es la relacionada con la segregación residencial. A continuación se exponen algunos conceptos generales de dichos modelos basados en Grauwin, et al. (2009). En este trabajo los autores proponen una solución analítica de los modelos de segregación residencial en general (Schelling, 1969; Schelling, 1971; Schelling, 1978; Zhang, 2004, entre otros) para un variedad de funciones de utilidad. Basados en teoría de juegos evolutiva, encuentran condiciones de existencia de una función potencial, lo que caracteriza la configuración global urbana, maximizando la utilidad social en el estado estacionario o de largo plazo.

Analíticamente, se define una ciudad artificial como un arreglo bidimensional $N \times N$. Cada celda corresponde a una unidad de vivienda de idénticas condiciones en calidad. Se supone que existen dos tipos de grupos de hogares que se denotan como rojos y verdes. Cada ubicación puede ser ocupada por un agente rojo, un agente verde, o puede estar vacía. Sea $N_V \geq 0$ el número de celdas vacantes, N_R el número de hogares rojos y N_G el número de hogares de tipo verde ($N_V = N^2 - N_R - N_G$). Por otro lado, un estado x de la ciudad es un vector N^2 donde cada elemento contiene un agente rojo, verde o vacío cumpliendo N_V, N_R, N_G en x. Cada estado x representa así una configuración específica de la ciudad. Se denota por X al conjunto de todas las posibles configuraciones. Un concepto importante en los modelos de segregación es la vecindad o zona, en este tipo de trabajos existen dos formas de concebir la vecindad.

- *Modelo de zonas acotadas:* La ciudad es divida en unidades geográficas. De esta forma la vecindad de cada agente se compone de los agentes presentes en la misma unidad geográfica, similar a los modelos de economía urbana descritos anteriormente.

- *Modelo de vecindades continuas:* En este caso cada celda tiene su propio conjunto de vecinos y está centrado en la percepción local de cada agente.

Además, se supone que cada tipo de agente tiene un nivel de utilidad que sólo depende de su vecindario y sus preferencias. Si se considera un agente cuyo vecindario se compone de R agentes rojos, G agentes verdes y V celdas vacías y el tamaño del vecindario es fijo (H), entonces $R + G + V = H$, por lo cual sólo se necesitan dos parámetros independientes (H y G) para describir la composición de la vecindad de un agente. Por lo tanto se puede escribir la utilidad como una función de R y G como:

$$u_R(R, G), \quad u_G(R, G) \tag{1.21}$$

En estos modelos, el nivel de utilidad es normalizado con el fin de facilitar la comparación de las funciones de utilidad; un nivel de utilidad cero denota una insatisfacción completa del agente y una utilidad de uno denota satisfacción completa. Por otro lado, el nivel de utilidad social en la ciudad es:

$$U(x) = \sum_k u_k \tag{1.22}$$

donde u_k es la utilidad del agente k y $U(x)$ denota la utilidad colectiva asociada a la configuración x.

La base fundamental de los modelos dinámicos de segregación es que los agentes tienen la oportunidad de relocalizarse o moverse para aumentar su utilidad individual. Una vez que se tiene una descripción estática de la ciudad x, se debe añadir una regla dinámica que regule dichos movimientos. De esta forma la configuración de la ciudad evoluciona en función de un proceso iterativo. En cada iteración, se elige al azar un agente y una celda vacía. El agente escogido puede decidir avanzar a esa celda vacía con una probabilidad $Pr\{move\}$ de moverse, que depende de la ganancia de utilidad Δu que lograría si ocupa ese espacio:

$$Pr\{move\} = \frac{1}{1 + \exp(-\Delta u/T)} \tag{1.23}$$

donde T es un parámetro fijo. Si el agente se mueve generará una nueva celda vacía y se iterará hasta que el proceso se estabilice (es decir, ningún agente tenga la posibilidad de moverse mejorando su utilidad). Esta regla de comportamiento es miope en el sentido que el agente no elige la mejor localización dentro de las opciones vacías.

La ecuación (1.23) representa una función de elección logit binomial. El escalar $T > 0$ en estos modelos se usa para realizar sensibilidad en las simulaciones.

Además, existe una probabilidad de transición entre configuraciones de tipo Markoviano pues sólo depende de la iteración anterior:

$$Pr(x^t = x|x^{t-1}, \ldots, x^1, x^0) = Pr(x^t = x|x^{t-1}) \tag{1.24}$$

La cadena de Markov que describe el sistema urbano es irreducible, aperiódica y todos los estados son recurrentes. Estas tres propiedades garantizan que la probabilidad de observar cualquier estado x después de t iteraciones converge hacia un estado estacionario independiente de la situación de partida. Este tipo de consideraciones son usadas por los autores para analizar la estabilidad del proceso con diferentes funciones de utilidad, incluyendo estrategias de tarificación por moverse. En este sentido, logran obtener diferentes estrategias y resultados basados en escenarios de simulación (cambios de parámetros y funciones de utilidad) donde los niveles de segregación aumentan o disminuyen de acuerdo al tipo de preferencias de los agentes o las limitaciones impuestas por un regulador urbano.

Tal como lo menciona Grauwin, et. al. (2009), es importante anotar que tanto los modelos de segregación residencial como los modelos dinámicos de mercado inmobiliario (Martínez y Hurtubia, 2006; Anas y Arnott, 1991) no incluyen dinámicas de aprendizaje, formación de hábitos o efectos en el ciclo de vida que son propias de los consumidores y menos su efecto en el equilibrio del sistema; algunas de estas dinámicas han sido analizadas en varios trabajos asociados a modelos empíricos econométricos de elección de vivienda, procesos de relocalización o movilidad intra-urbana (Nijkamp, et al. 1993; Páez, et al. 2008; Eluru, et al. 2009; Chen, et al. 2009; Chen y Lin, 2011)

Por ejemplo, en Nijkamp et al. (1993) la dinámica de los hogares se modela utilizando la metodología llamada "*demografía multidimensional*". El artículo muestra una estructura analítica y de estimación empírica de un modelo de elección discreta desarrollado para investigar la dinámica del mercado inmobiliario, con especial atención en los cambios en la composición del hogar causados por el ciclo de vida. La relación entre el concepto de ciclo de vida y la demanda en el mercado de la vivienda se encuentra por medio de un modelo logit jerárquico. En primera instancia se obtiene la probabilidad de moverse $P(move|kzhh')$ para un hogar que pasa de un grupo h a un hogar de tipo h' en un periodo t y además está localizado en una vivienda tipo k en la zona z. Esta probabilidad se estima por medio del modelo logit binario o binomial de la forma:

$$P(move|kzhh') = \frac{exp(A_{kzhh'})}{1 + exp(A_{kzhh'})}, \tag{1.25}$$

donde

$$A_{kzhh'} = \alpha_z + \beta_{1z}\left(\frac{s_{h'} - r_k}{r_k}\right) + \beta_{2z}U_k + \beta_{3z}B_k + \beta_{4z}\log\left(\frac{s_h}{s_{h'}}\right) + \beta_{5z}X_g$$

Cada término de $A_{kzhh'}$ se explica a continuación:

- $s_{h'},\ s_h$: tamaño del hogar h ó h'.
- r_k : es el número de habitaciones en una casa tipo k.
- U_k : 1 si la casa tipo k era apartamento ó 0 en otro caso.
- B_k : 1 si es una vivienda comprada ó 0 si es en arriendo.
- X_g : edad del jefe de familia al principio del periodo.

Además, cada estimador o parámetro β depende de la zona actual de localización residencial; de esta forma se incluyen efectos de valoración asociadas a grupos socioeconómicos espaciales.

Por otro lado, si $P(move|kzhh')$ es mayor que un umbral dado, entonces se obtiene la probabilidad que ese tipo de hogar (h, k, z) elija la opción (k', z'):

$$P_{k'z'|(h,k,z)} = P_{z'|(h,k,z)} * P_{k'|z'(h,k,z)} \tag{1.26}$$

y para obtener $P_{k'z'|(h,k,z)}$ se utiliza un Logit jerárquico.

Chen y Lin (2011) proponen un modelo de elección residencial mediante la generación de un proceso dinámico asociado a la valoración de las características de los bienes, basados en una perspectiva del ciclo de vida de los hogares y postulan que los lugares donde se habita inicialmente pueden tener un efecto duradero en las preferencias de vivienda en futuras decisiones. Además se incluye la hipótesis que la influencia de la ubicación previa también está dada por la duración y el carácter reciente de la estancia anterior.

Habib y Miller (2009) estiman un modelo de elección de vivienda asociada a relocalización basados en teoría del riesgo (*Reference-Dependent Theory*) donde la función de utilidad asociada a un bien i, sabiendo que en t estaba localizado en j es:

$$U_{ijt} = \beta_0 + \sum_k \beta_{1k} gain_{ijtk} + \sum_k \gamma_k loss_{ijtk} + \beta_2 X_{lit} \tag{1.27}$$

donde

- $gain_{ijt}$: se refiere a la cantidad del atributo k en i que excede sobre en valor de k en la vivienda actual j.

- $loss_{ijt}$: es la cantidad del valor del atributo k de la vivienda i que falta para alcanzar el valor de j.
- X_{lit} : son otros atributos no comparables entre la nueva opción i y la residencia previa j.

Los autores afirman que la contribución fundamental del modelo es la incorporación de la dependencia en la localización previa, reconociendo un papel importante al status quo y capturando respuestas asimétricas hacia las ganancias y pérdidas en la toma de decisiones. Para estimar el modelo aplicaron una formulación de estructura Logit mixto y concluyen que esta formulación genera mejores ajustes a un modelo de elección sin dependencia y proporciona importantes lineamientos para el análisis de comportamiento.

Por otro lado, Páez, et al. (2008) definen la elección de vivienda por medio de la influencia social que tienen unos agentes sobre otros; esto se puede ver como un proceso de aprendizaje o imitación del comportamiento de otros agentes. Para esto la probabilidad que un individuo h seleccione la alternativa i en el tiempo t es:

$$P_{ih}^{t} = \frac{\exp(U_{ih}^{t})}{\sum_j \exp(U_{jh}^{t})} \tag{1.28}$$

donde la utilidad que se recibe por un bien tipo i es de la forma $U_{ih}^{t} = \beta_1 \frac{r_i^t}{I_h^t} + \beta_2 d_i + \beta_3 cr_i^{t-1} + \delta A_{i,h}^{t-1}$. Cada componente de la utilidad U_{ih}^{t} es:

- d_i es la distancia al centro de empleos.
- cr_i^{t-1} es la tasa de crímenes en el vecindario de i.
- $A_{i,h}^{t-1}$: es una valoración del agente h a la estructura social en $t-1$. Matemáticamente,

  $$A_{i,h}^{t-1} = \sum_{g \in H} w_{hg} y_{gi}^{t-1}$$

  donde y_{gi}^{t-1} es una variable indicadora que toma el valor 1 si el individuo g decide localizarse en i en $t-1$ y w_{hg} es la valoración social que h tiene sobre las decisiones de g (ver Leenders, 2002; Marsden y Friedkin, 2004; Páez et. al. 2008).

Los autores desarrollan simulaciones indicando que se pueden obtener parámetros sesgados al ignorar el impacto de la estructura social en las decisiones de los individuos. Una posible mejoría de este trabajo sería incorporar la dinámica del ciclo de vida y su efecto en la formación de las redes sociales de cada tipo de agente.

Eluru et. al. (2009) desarrollan un modelo econométrico que articula la elección de cambio de residencia (*movimiento*) y la razón principal para relocalizarse. Sus resultados muestran que la situación socio-económica y demográfica de los hogares tienen un impacto directo en dicha decisión.

Para obtener los resultados estiman el siguiente conjunto de utilidades:

$$U_{11} = \epsilon_{11} \tag{1.29}$$

$$U_{12} = \delta' w_{12} + \gamma' A_2 + \epsilon_{12} \tag{1.30}$$

$$U_{2i_2} = \beta_2 x_{2i_2} + \epsilon_{2i_2} \tag{1.31}$$

donde U_{11} y U_{12} son la utilidad de no moverse y la utilidad de moverse respectivamente. Además, U_{2i_2}, ($i_2 = 1, \ldots, 9$) es la utilidad asociada a la razón cualitativa principal de moverse (por ejemplo, costos asociados a la vivienda, calidad de la casa y tamaño de su casa) que activa las decisiones de relocalización residencial. La vectores w_{12} y x_{2i_2} son columnas de atributos exógenos, mientras que $A_2 \in \{0,1\}^9$ indica la razón primaria cualitativa (entre nueve opciones) para la relocalización (notar que $\sum_{\ell=1}^{9} A_{2\ell} = 1$). De acuerdo al modelo, sólo uno de los nueve motivos cualitativos constituye la categoría base asociada a la ecuación de reubicación. La intención de incluir el vector A es examinar el tipo de hogares (con cierto tipo de preferencias) que tienen mayor o menor probabilidad de relocalización. Así, por ejemplo, según los autores puede ocurrir que los hogares que son particularmente sensibles a los costos sean menos propensos a moverse, mientras que aquellos que ponen énfasis en la calidad escolar tengan mayor propensión a moverse (ya que quizás están constantemente en la búsqueda de lugares con mejor calidad escolar).

Kennan y Walker (2011) desarrollan un modelo econométrico de migración, centrándose en la variación de los ingresos esperados como la influencia económica principal de este fenómeno en la dinámica urbana. En este caso, el modelo obtiene secuencias óptimas de decisiones de localización. Además, los resultados numéricos y empíricos sugieren que la relación entre los ingresos y las decisiones de relocalización son impulsadas por las diferencias geográficas en los salarios medios y se traducen en una tendencia a desplazarse en búsqueda de una mejor ubicación cuando los ingresos asociadas a ella no son favorables; en esta formulación se asume que cada localización genera distintos ingresos, asociando el trabajo a la zona de residencia lo que constituye un supuesto discutible.

Por otro lado, se han utilizado algunas herramientas de microsimulación para entender o explicar la dinámica urbana; en estas, cada agente es modelado en forma desagregada basándose en una formulación de reglas de comportamiento. Además, en la microsimulación cada objeto o agente del sistema tiene su propia identidad, estado y comportamiento que evolucionan a través del tiempo y de esta forma son identificables en el sistema.

Por ejemplo, en el modelo de desequilibrio UrbanSim (Waddell, 2002) los agentes eligen su localización en función de probabilidades y reglas exógenas que limitan su posibilidades de elección. Recibe el nombre de desequilibrio debido a que los distintos procesos de ajuste ocurren en diferentes momentos y escalas espaciales. El modelo supone una evolución anual de las localizaciones de los hogares y del mercado laboral. Estos resultados son usados en los siguientes periodos para generar la dinámica de la ciudad. El proceso de UrbanSim se divide en 5 módulos:

- Módulo de accesibilidad: las medidas de accesibilidad influyen en los atributos de localización y es valorada según el nivel socioeconómico o ciclo de vida de los hogares.

- Módulo de transición: Este módulo se divide en dos sub-módulos: Un módulo de transición económica que obtiene la variación de la oferta laboral y otro de transición demográfica que considera los cambios en la distribución de los hogares.

- Módulo de movilidad: Se separa en los modelos de movilidad de empleo y de hogares

- Módulo de elección de localización: Dado que el módulo de movilidad determina los hogares y empleos que existen por localizar, en este caso se determina la elección de localización de los agentes que se mueven y de la nueva demanda generada.

- Módulo de desarrollo inmobiliario: se estima una probabilidad logit multinomial que considera la maximización de la utilidad de los productores.

- Módulo de precios del suelo: Los precios del suelo son exógenos al proceso de localización y son un indicador de ajuste entre oferta y demanda y los precios en el periodo previo.

Vale la pena anotar que UrbanSim no considera la existencia de estrategias de remate ni precios de equilibrio de mercado, por lo que resulta muy difícil establecer propiedades analíticas sobre estos valores. Por otro lado, el modelo PECAS (Hunt y Abraham, 2007) genera progresos en este sentido, ya que integra procesos de simulación con condiciones de equilibrio de mercado, usando matrices insumo-producto, incluyendo bienes, servicios, empleos, terrenos y vaciamiento del mercado con precios de intercambio. Se caracteriza por utilizar sub-modelos agrupados por un gran modelo logit multinomial anidado en tres etapas. Dada esta una estructura compleja de modelación no se dan pruebas de la existencia ni la estabilidad del equilibrio.

Otro modelo de microsimulación importante es el conocido como ILUTE (Salvini y Miller, 2005) que es un sistema integrado de modelación urbana que consta de cuatro componentes interrelacionados:

- Desarrollo inmobiliario

- Elección de localización

- Actividades / viajes

- Tenencia de auto

Cada uno de estos componentes se desarrolla por una serie de sub-modelos que están relacionados entre sí e incorporan la interacción de oferta/demanda. Por ejemplo, el uso del suelo inmobiliario se diseña en respuesta a las necesidades de localización de las familias y las empresas, y a su vez de generación y necesidad de nuevos puestos de trabajo. En particular, la hipótesis básica del modelo de elección de localización residencial de ILUTE es que con el tiempo, las características demográficas, sociales y económicas de la población analizada va a cambiar. Así, la lista de agentes debe ser actualizada en el modelo de modo que siga siendo representativa de la población en cada punto en el tiempo. Los procesos que afectan a la evolución de la población en el tiempo combinan factores que pueden ser endógenos (envejecimiento, nacimientos, defunciones, etc.) y exógenos como la inmigración. Este supuesto también es incluido en el modelo UrbanSim descrito anteriormente y es relevante en la generación de nuevos modelos dinámicos de economía urbana o uso de suelo como el desarrollado en esta tesis, etc. Según ILUTE el proceso de relocalización residencial puede ser descrito como:

1. *Movilidad residencial:* La primera etapa de reubicación residencial es la decisión participar en el mercado inmobiliario. Las principales motivaciones de un hogar para entrar en esta etapa del proceso de relocalización son los cambios del mercado (generación de nueva oferta), cambios del ciclo de vida propio de la familia, como matrimonios, divorcios, un nuevo miembro de la familia, una muerte en la familia, o un nuevo trabajo.

2. *Búsqueda de residencia:* Dado que el proceso anterior se activó, es decir, que una familia desea relocalizarse, entonces comienza la búsqueda de una vivienda. Se busca explicar este proceso a través de los atributos de los hogares, fuentes de información y características

de los bienes inmuebles. Una dificultad importante del análisis de la búsqueda de residencia radica en el hecho que los compradores se basan en información imperfecta sobre el mercado de la vivienda, dando resultados no necesariamente óptimos teniendo en cuenta sus necesidades y prioridades.

3. *Resultados de máxima postura*

   Cuando ya se tienen vacantes de alojamiento alternativo identificadas en el proceso de búsqueda, el consumidor (es decir, el posible comprador de vivienda) debe tomar decisiones sobre si comprar una alternativa dada y generar una postura por dicho bien (*bid*). En los mercados de bienes raíces, las posturas o disposiciones a pagar por un bien pueden ser menores o mayores al precio de venta. El oferente debe decidir si acepta la oferta o si la rechaza. En el primer caso, la unidad de vivienda cambia de manos en el precio acordado, la búsqueda del consumidor termina *con éxito*, y el consumidor vuelve a un estado pasivo. Si la oferta es rechazada, entonces el consumidor puede actualizar su información dada la experiencia adquirida durante el proceso de remate y luego puede decidir si continua la búsqueda o si le pone fin sin éxito para volver al estado pasivo sin lograr la relocalización.

Discusión

Los modelos explicados en esta sección tienen por objetivo mostrar un panorama general de las diferentes estrategias que existen en el ámbito de la modelación asociada a la elección de vivienda o localización residencial. Es importante anotar que en los modelos de equilibrio urbano o de segregación residencial no se involucra los cambios en el ciclo de vida de los hogares como un componente primordial para la toma de decisiones de relocalización residencial, aunque estos si han sido explorados por varios trabajos de tipo económetrico o tenidos en cuenta como un parámetro exógeno en algunos modelos de microsimulación. Por esta razón, la investigación teórica desarrollada en esta tesis doctoral busca analizar el efecto de dichos factores en la elección de vivienda, y además su influencia en la distribución urbana en el largo y corto plazo.

1.2.3. Modelos microeconómicos de asignación de tiempo

En las últimas décadas se han desarrollado diversos modelos microeconómicos que buscan explicar el comportamiento de las personas en cuanto a la asignación y valoración del tiempo en diversas actividades, tales como el trabajo, el hogar y el ocio, entre otras, incluyendo su relación con el consumo de bienes discretos o continuos. En esta sección se muestran algunos de los modelos y trabajos más relevantes en dicha área de la economía, que constituyen una base importante para la tesis desarrollada, en especial el modelo jerárquico de actividades. La hipótesis básica de estos modelos es que el tiempo es un recurso económico de gran importancia debido su característica de escaso y no acumulable. A su vez, es un recurso que una vez asignado a diferentes actividades genera utilidad a cada una de las personas. En el sector de ingeniería de transporte es de vital importancia poder valorar los beneficios que se generan por la disminución del tiempo de viaje de los usuarios cuando se implementan nuevos proyectos. Para esto se obtiene un valor llamado Valor Subjetivo del Tiempo de una actividad (VST), que tiene por objetivo mostrar la disposición a pagar por aumentar o disminuir el tiempo de las actividades. En economía de transporte este valor se obtiene a partir del análisis de las decisiones tomadas por los individuos cuando se enfrentan a la elección de modo de transporte o rutas entre diferentes opciones que se describen por sus características. El método generalmente aceptado y usado para la estimación de un valor subjetivo del tiempo de viaje(VST), consiste en encontrar la tasa marginal de sustitución entre el tiempo de viaje y gastos del viaje, generalmente a partir de modelos desagregados de elección discreta basados en la teoría de la utilidad aleatoria (véase, por ejemplo, Gaudry et al., 1989). En este caso particular, la interpretación de dicha tasa de sustitución es la de la disposición a pagar para reducir el tiempo de viaje en una unidad.

La literatura microeconómica al respecto parte del reconocimiento que el tiempo es un recurso esencial y escaso del que todos los individuos están dotados en la misma cantidad (24 horas, una semana, etc). Además, tal como lo menciona dicha literatura, el consumidor podrá transferir libremente su tiempo de unas actividades a otras siempre que las restricciones y regulaciones lo permitan. Un supuesto importante de estos modelos es que las distintas asignaciones de tiempo a diferentes actividades tienen distinto valor que puede ser medido en dinero. Por ejemplo, una persona podría aumentar (o disminuir) su tiempo asignado al trabajo a cambio del pago (o disminución) del salario, o elegir un modo de transporte que genere mayor tiempo de viaje ahorrando dinero, etc. Una suposición básica de la teoría es que los individuos eligen la asignación de tiempo que maximiza su utilidad, sujeta al hecho de que el recurso tiempo no puede ser almacenado.

El primer trabajo importante en este campo fue el presentado por Becker (1965) que postula y analiza una teoría de uso del tiempo donde se asume que la función de utilidad de los hogares o individuos depende de bienes compuestos o finales Z_i, que son producidos combinando bienes de mercado X_i (con precios unitarios asociados P_i) y tiempo de dedicación al consumo del bien compuesto T_i. El individuo asume una restricción de ingreso monetario, que le indica que no puede gastar en los bienes de mercado más que su ingreso. Además, dicho ingreso es compuesto por la suma de I (componente no salarial) y otra componente salarial dada por el producto de las horas de trabajo T_w y la tasa salarial w. Además, el individuo tiene una restricción de tiempo factible, donde el tiempo dedicado a las actividades, en este caso, es el tiempo dedicado a la producción de *commodities* más el tiempo asignado a trabajar T_w, no debe superar el tiempo total factible de cada individuo (τ). La relación entre los bienes (insumos) X_i y los tiempos T_i asignados a fabricar el $i-$ésimo commodity está dado por una función de producción f_i. Es decir, en este caso los individuos son simultáneamente consumidores y productores combinando el tiempo y los bienes de mercado.

El problema del consumidor según Becker (1965) se escribe de la siguiente forma:

$$\max_{X,T} U(Z(X,T)), \text{ sujeto a } \sum_i P_i X_i \leq I + wT_w, \quad \sum_i T_i + T_w = \tau \tag{1.32}$$

donde τ es el tiempo total disponible (por ejemplo, 24 horas). Es importante anotar que el tiempo dedicado al trabajo T_w no está en la función de utilidad, sino que es una decisión exógena al individuo debido a que sólo se obtiene satisfacción de la combinación de bienes y tiempo dedicado a otras actividades. Del modelo del consumidor propuesto por Becker (1.32) se puede mostrar que la disposición a pagar del individuo para aumentar en una unidad de tiempo cualquier actividad i es igual a la tasa salarial w. Es decir, la valoración del tiempo para cada actividad diferente del trabajo es la misma y se puede analizar como la pérdida de oportunidad por dejar de trabajar. Este resultado fue la primera valoración obtenida de lo que se conoce como el valor del tiempo como recurso $VT = w$.

Johnson (1966) señala que la conclusión obtenida por Becker sobre la valoración del tiempo como el costo de oportunidad de una hora de ocio (tiempo no asignado al trabajo) igual a la tasa salarial no es un estimador correcto de la valoración del ocio o de otras actividades como el tiempo asignado a los viajes. En este sentido propone un modelo donde la utilidad del individuo depende del tiempo asignado al ocio, el tiempo dedicado a trabajar, el número de viajes de placer realizados y del consumo (en este caso, es un solo bien agregado). Notese que en el modelo de Johnson, al individuo le genera utilidad (o des-utilidad) el tiempo dedicado al trabajo, de esta forma el valor de la des-utilidad marginal del trabajo más la tasa salarial es igual al valor del tiempo o valor del ocio. Este resultado es avalado por otros autores como se mostrará más adelante. Por ejemplo, Chiswick (1967) supone que la utilidad proviene de la cantidad que se consume de cada bien X_i y del tiempo asignado a trabajar W_j con $j \geq 0$, bajo el supuesto que el individuo puede tener varios trabajos. Además, cada bien i requiere un tiempo para su consumo por unidad t_i. Al igual que en Becker, en el modelo del consumidor de Chiswick (1967) se tiene una restricción de ingreso y una restricción temporal que se plantea de la siguiente forma:

$$\max U(X, W) \tag{1.33}$$

sujeto a

$$\sum_j w_j W_j - \sum_i p_i X_i = 0, \quad (\lambda) \tag{1.34}$$

$$\tau - \sum_j W_j - \sum_i t_i X_i = 0, \quad (\mu) \tag{1.35}$$

donde λ y μ son los multiplicadores de Lagrange de las restricciones y representan la utilidad marginal del ingreso y la utilidad marginal del tiempo, respectivamente. De las condiciones de primer orden, en equilibrio se cumple la siguiente condición:

$$\frac{\mu}{\lambda} = w_j + \frac{\frac{\partial U}{\partial W_j}}{\lambda} \tag{1.36}$$

El valor de $\frac{\mu}{\lambda}$ es el valor del tiempo como recurso VT (tasa marginal de sustitución entre el ingreso y el tiempo total disponible). La ecuación (1.36) indica que el valor del tiempo como recurso es igual a la tasa salarial (w_j) más la valoración del tiempo asignado al trabajo $\frac{\frac{\partial U}{\partial W_j}}{\lambda}$. Dicho resultado es equivalente al encontrado por Johnson (1966) y difiere del resultado de Becker en el caso que $\frac{\partial U}{\partial W_j} \neq 0$. Es decir, tal como lo concluye el autor, suponer que $VT = w$ puede sobre-estimar u sub-estimar la valoración del tiempo de cada individuo.

Por otro lado, Oort (1969) presenta un modelo basado en los siguientes supuestos:

- Al individuo le genera utilidad la asignación del tiempo al trabajo.
- La reducción del tiempo de viaje es equivalente a un incremento en el tiempo de ocio o trabajo.

En este caso, que el individuo maximiza una función de utilidad, que depende del ingreso (I) que a su vez representa de forma agregada el consumo de bienes, del tiempo asignado al trabajo (T_w) y del tiempo de ocio (L). De esta forma, el modelo del consumidor planteado por Oort se escribe de la forma:

$$\max U(I, L, T_w), \text{ sujeto a } I = wT_w, \; L + T_w = \tau. \tag{1.37}$$

El modelo (1.37) se puede re-escribir en términos del tiempo asignado al ocio de la forma:

$$\max U(I = w(\tau - L), L, \tau - L) \tag{1.38}$$

De las condiciones de primer orden asociadas al problema (1.38) con respecto a L se obtiene:

$$\frac{\partial U}{\partial L} = w\frac{\partial U}{\partial I} + \frac{\partial U}{\partial T_w}$$

Diviendo por $\frac{\partial U}{\partial I}$ se obtiene un resultado equivalente a (1.36):

$$\frac{\frac{\partial U}{\partial L}}{\frac{\partial U}{\partial I}} = w + \frac{\frac{\partial U}{\partial T_w}}{\frac{\partial U}{\partial I}} \tag{1.39}$$

Por otro lado, en el trabajo presentando por De Serpa (1971) se supone que el individuo o consumidor maximiza una función de utilidad que incluye a distintos bienes de consumo X_i y los tiempos T_i asignados a las diferentes actividades utilizados para el consumo de los bienes y además, el tiempo asignado al trabajo T_w. En este trabajo se supone que existen tantos bienes de consumo como actividades. Adicionalmente, el consumidor está sujeto a una restricción de ingreso, a una restricción de tiempo (similares a las presentadas por Becker, 1965) y a un conjunto de restricciones tecnológicas, llamadas restricciones de consumo de tiempo. Dichas restricciones acotan inferiormente el tiempo dedicado al consumo de un bien, condicionada por la cantidad obtenida del mismo. Dicho conjunto de restricciones generan interesantes resultados sobre al análisis del valor del tiempo y la disposición a pagar por disminuir el tiempo de algunas actividades. Matemáticamente, el modelo propuesto por De Serpa (1971) se escribe de la siguiente forma:

$$\text{máx}\, U(X_1, \ldots, X_n, T_1, \ldots, T_n, T_w) \tag{1.40}$$

sujeto a

$$\sum_i X_i P_i = I, \quad (\lambda) \tag{1.41}$$

$$\sum_i T_i + T_w = \tau, \quad (\mu) \tag{1.42}$$

$$T_i \geq \alpha_i X_i, \quad \forall i \neq w, \quad (\kappa_i) \tag{1.43}$$

donde λ, μ, κ_i son los multiplicadores de Lagrange asociados a las respectivas restricciones. El parámetro α_i de las restricciones tecnológicas (1.43) son interpretados como un tiempo mínimo de consumo por una unidad del bien X_i. De Serpa señala que el ingreso I puede aumentar con la cantidad de horas trabajadas T_w, este efecto generaría unas variaciones menores al modelo, definiendo un bien X_w llamado de trabajo, con $P_w < 0$ y de esta forma el consumo de dicho bien hace que el ingreso aumente. Teniendo que $T_w = a_w X_w$ entonces el tiempo de trabajo podría incorporarse en la restricción de ingreso. De las condiciones de primer orden del problema (1.40)-(1.43) se obtiene que:

$$\frac{\kappa_i}{\lambda} = \frac{\mu}{\lambda} - \frac{\frac{\partial U}{\partial T_i}}{\lambda} \tag{1.44}$$

A continuación se describe cada término de la ecuación anterior

- $\frac{\mu}{\lambda}$: Valor del tiempo como recurso.
- $\frac{\kappa_i}{\lambda}$:valor de ahorrar tiempo al consumo de X_i o disposición a pagar por disminuir tiempo asignado a la actividad i. Por condición del holgura complementaria ($\kappa_i(T_i - \alpha X_i) = 0$). Si $\kappa_i > 0$ entonces el individuo asigna el mínimo posible para realizar la actividad i y

en el caso que $\kappa_i = 0$ entonces el individuo asigna más tiempo a la actividad i que el mínimo posible, obteniendo que $\frac{\mu}{\lambda} = \frac{\frac{\partial U}{\partial T_i}}{\lambda}$. Dichas actividades ($\kappa_i = 0$) son definidas como actividades de ocio pues se les asigna más del mínimo necesario y todas tienen el mismo valor de utilidad marginal.

En 1972, Evans propone un modelo donde la hipótesis más importante es que la función de utilidad de cada consumidor depende solamente de la asignación de tiempo a las diferentes actividades. Evans plantea un modelo con tres tipos de restricciones, donde incorpora las relaciones tecnológicas entre tiempos de actividades. En este documento se presenta la formulación matricial donde los bienes que son utilizados por el individuo como insumos para cada una de las actividades y se supone que las columnas de una matriz llamada Q contienen los requerimientos unitarios de bienes para llevar a cabo las distintas actividades (una actividad corresponde a una columna). Existe la posibilidad que las actividades tengan un pago o costo asociado. El individuo se ve enfrentado a tres tipos de restricciones: de ingreso, de tiempo total y las relaciones entre el tiempo de actividades. En la restricción de ingreso, se supone que el gasto en los bienes requeridos para la realización de actividades está definido por los precios P. El individuo se enfrenta a una restricción de tiempo factible en un periodo de duración τ. Adicionalmente, existe una restricción donde se supone que pueden existir dependencias lineales entre los tiempos asignados a diferentes actividades. Esas relaciones se expresan de la forma $BT \leq 0$, donde la matriz B representa las relaciones (B tiene dimensión $J \times I$, donde I es el número de actividades y J el número de relaciones entre actividades).

Matemáticamente, el modelo matricial de Evans se puede escribir de la forma,

$$\max U(T), \tag{1.45}$$

sujeto a,

$$P'QT \leq 0, \quad (\lambda) \tag{1.46}$$

$$BT \leq 0, \quad (\kappa) \tag{1.47}$$

$$\mathbf{1}T = \tau, \quad (\mu) \tag{1.48}$$

Donde λ, μ son los multiplicadores de Lagrange asociados a las restricciones de ingreso y tiempo, respectivamente. Además, $\kappa = (\kappa_1, ..., \kappa_J)$ es un vector que contiene los multiplicadores de Lagrange de cada una de las J restricciones definidas por la matriz B. Las condiciones de primer orden de (1.45)-(1.48) son:

$$\frac{\partial U}{\partial T_i} - \mu + \lambda P' q_i - \sum_j \kappa_j b_{ji} \leq 0, \;\; \forall j, \;\; \mu \neq 0, \;\; \kappa_j \geq 0, \;\; \lambda \geq 0, \tag{1.49}$$

donde q_i es la columna $i-$ésima de Q y b_{ji} es el elemento de la fila j y columna i de la matriz B. La desigualdad que se presenta (1.49) es estricta si $T_i = 0$. De las condiones de primer orden se determina que la asignación de tiempo es óptima si un incremento marginal del tiempo asignado a dicha actividad, seguido de una disminución de las restantes, no le afecta en cuanto a la utilidad obtenida. En este caso podría suceder que $\lambda = 0$ implicando que el individuo tiene un ingreso neto mayor a su tasa de gasto y en ese caso la asignación óptima de tiempo depende de su utilidad y de su impacto sobre las otras actividades.

De forma paralela a Evans, De Donnea (1972) desarrolla tres modelos de asignación y valoración del tiempo, para explicar las razones de la generación, distribución y elección de modo y ruta. El objetivo de su trabajo es extender formulaciones previas para entender la naturaleza de las decisiones de viaje de los individuos. En esta revisión, solamente explicaremos el modelo que incluye el tiempo asignado al trabajo, aunque el autor propone otras formulaciones considerando el trabajo exógeno. En los modelos desarrollados, similar a Becker (1965) el autor considera que los consumidores son unidades de producción de las actividades de consumo, que requieren dos tipos de inputs: bienes (o servicios) y el tiempo. La función de producción de una actividad de consumo o bien final i se escribe como:

$$A_i = f(X_{ki}, t_i, X_{ki}^v, t_i^v),$$

donde

- A_i : Nivel de producción de la i-ésima actividad, $(i = 1, \ldots, m)$.
- X_{ki} : La cantidad del bien o servicio k usado en la producción de la actividad i, $(k = 1, \ldots, n)$.
- t_i : tiempo asignado en la producción de la actividad i.
- X_{ki}^v : cantidad del bien o servicio k usado para el viaje a la i-ésima actividad.
- t_i^v : tiempo asignado para viajar a realizar la actividad i.

De esta forma el modelo planteado por de Donnea es el siguiente:

$$\text{máx}\, U(A_i, t_i, t_i^v, T_w) \tag{1.50}$$

$$wT_w = \sum_{k=1}^{n}\sum_{i=1}^{m} P_k X_{ki} + \sum_{k=1}^{n}\sum_{i=1}^{m} P_k X_{ki}^v, \quad (\lambda) \tag{1.51}$$

$$\tau - \sum_{i}^{m} t_i - \sum_{i}^{m} t_i^v = 0, \quad (\mu) \tag{1.52}$$

De Donnea incluye el tiempo de trabajo en la función de utilidad T_w y define un ingreso endógeno en la modelación que depende de la horas trabajadas wT_w. Además, no considera el tiempo asignado al trabajo como un insumo para cualquier actividad. Por otro lado, de las condiciones de primer orden del problema (1.50)-(1.52) se denotan las siguientes utilidades marginales:

- $\frac{\partial U}{\partial t_i}$: satisfacción o insatisfacción marginal del tiempo consumido para producir A_i. Dicha valoración es el resultado de las circunstancias en las que el tiempo es gastado. De forma análoga se define $\frac{\partial U}{\partial t_i^v}$.
- $\frac{\partial U}{\partial A_i}\frac{\partial A_i}{\partial t_i}$: Utilidad marginal del tiempo como insumo para producir i.

Nótese que la utilidad marginal de cualquier actividad i está dada por la suma de las dos componentes anteriores, es decir, $(\frac{\partial U}{\partial t_i} + \frac{\partial U}{\partial A_i}\frac{\partial A_i}{\partial t_i})$.

Además, se obtiene la siguiente expresión:

$$\mu = \frac{\partial U}{\partial t_i} + \frac{\partial U}{\partial A_i}\frac{\partial A_i}{\partial t_i} = \frac{\partial U}{\partial t_i^v} + \frac{\partial U}{\partial A_i}\frac{\partial A_i}{\partial t_i^v} \tag{1.53}$$

Es decir, la conclusión primordial de la ecuación (1.53) es que la utilidad marginal del tiempo de viaje asociado a una actividad i ($\frac{\partial U}{\partial t_i^v} + \frac{\partial U}{\partial A_i}\frac{\partial A_i}{\partial t_i^v}$) es igual a la utilidad marginal del tiempo necesario para producir la respectiva actividad ($\frac{\partial U}{\partial t_i} + \frac{\partial U}{\partial A_i}\frac{\partial A_i}{\partial t_i}$).

Por otro lado, el autor hace un análisis del valor del tiempo como recurso $\frac{\mu}{\lambda}$ en términos de la valoración marginal de las actividades.

El modelo propuesto por Train y McFadden (1978) es formulado como un problema típico de decisión microeconómico en el ámbito de las decisiones discretas donde el individuo elige un modo de transporte y además tiempo de ocio (L) y bienes de consumo (G):

$$\max_{j,G,L} U(G,L) \text{ sujeto a } G + c_j = wT_w + E, \quad T_w + L + t_j = \tau \tag{1.54}$$

donde E es un ingreso externo, w es la tasa salarial, c_j y t_j son el costo y tiempo de viaje del modo de transporte j, τ es el tiempo total disponible en el periodo de análisis y T_w es el tiempo de trabajo (horas trabajadas). Es importante anotar que j pertenece a un conjunto finito M de posibilidades de modos factibles.

Basado este modelo, según lo muestra Jara-Díaz (2000) se puede resolver este problema usando los conceptos clásicos de los problemas de elecciones discretas por medio de la optimización en dos etapas:

$$\max_{j\in M}\left\{\max_{T_w} U\left(T_w w + E - c_j, \tau - T_w - t_j\right)\right\} \tag{1.55}$$

La solución óptima de T_w que se notará T_w^* es condicional a los valores de c_j y t_j. Obteniendo una función de utilidad indirecta que depende del modo de transporte elegido:

$$V_j \equiv U(T_w^*(c_j, t_j)) \tag{1.56}$$

Luego el modo elegido será el que entregue la máxima utilidad V_j.

Usando las diferentes formas funcionales de la utilidad se obtienen diferentes soluciones del problema. Si se supone una función Cobb-Douglas ($U = KG^{1-\alpha}L^{\alpha}$), entonces (Jara-Díaz, 2007):

$$V_j = -K(1-\beta)^{1-\beta}\beta^{\beta}(w^{-\beta}c_j + w^{1-\beta}t_j) \tag{1.57}$$

Así, en este modelo la tasa marginal de sustitución entre bienes y ocio, que es equivalente al valor de tiempo, es igual a w. Sin embargo, en la versión estocástica de este modelo, como Jara-Díaz (1991) señala, los valores de tiempo difieren de la tasa salarial, porque la especificación de la función de utilidad que se suele utilizar es la siguiente:

$$V_j = \alpha_j + \beta\left(\frac{c_i}{w}\right) + \gamma t_i. \tag{1.58}$$

El resultado (1.58) expresa el valor subjetivo del tiempo como una proporción de la tasa salarial.

Posteriormente, existen otros desarrollos teóricos tal como el modelo de Small (1982) que añade consideraciones de programación de actividades (scheduling) para la función de utilidad y las restricciones. Supongamos que existe un bien numerario x y tres tipos de actividades: el ocio, el trabajo h, y el tiempo de consumo t. La actividad de consumo, implica un costo c y debe llevarse a cabo en un momento determinado del día, s. Por ejemplo, para los viajes de trabajo, s puede ser el momento de comenzar el viaje. Las consideraciones de scheduling se representan en tres puntos.

- En primer lugar, para permitir que las preferencias de la asignación de tiempo a actividades incluyan la programación de las mismas, de esta forma s se incluye en la función de utilidad.
- En segundo lugar, el costo $c(s)$ y el tiempo de consumo $t(s)$ dependen funcionalmente de s, teniendo en cuenta fenómenos de congestión y cambios de precios.
- En tercer lugar, para dar cuenta de las limitaciones impuestas por el marco institucional o laboral dentro del cual se encuentran oportunidades de empleo, se añade una restricción sobre s y h. Las limitaciones que figuran en la restricción pueden ser flexibles o inflexibles.

Alternativamente, el salario puede depender de la hora de llegada, así como las horas de trabajo:

$$w = w(s, h)$$

Si las sanciones por el retraso son muy fuertes, la restricción podría ser simplemente $s \leq s_o$. Según el autor, la inclusión de s en la función de utilidad no es esencial, ya que es posible presentar casi cualquier preferencia de scheduling, como algún tipo de restricción. Sin embargo, en muchos casos parece más natural: por ejemplo, un trabajador puede simplemente disfrutar hacer actividades de ocio en la tarde.

El modelo matemático es el siguiente:

$$\max U(x, l, h, s) \tag{1.59}$$

sujeto a

$$x + c(s) = Y + wh \tag{1.60}$$

$$l + h + t(s) = \tau \tag{1.61}$$

$$F(s, h; w) = 0 \tag{1.62}$$

donde $F(s, h; w) = 0$ es una función que relaciona s con las horas asignadas a trabajar. Dado este modelo microeconómico, Small (1982) muestra la dependencia del valor del ocio de fenómenos de congestión marginal y programación de actividades.

En una línea similar de trabajo, Winston (1987) considera una modelo microeconómico de asignación de tiempo en un entorno dinámico y continuo. Es decir, las decisiones de consumo están en función del instante de tiempo. De forma más precisa, el modelo descrito en Winston (1987) hace las siguientes consideraciones:

$$U(a(t), z(t)) \text{ con } \frac{\partial U}{\partial z} > 0, \; \frac{\partial^2 U}{\partial z^2} < 0 \tag{1.63}$$

$$z(t) = z_a(x(t), t) \text{ con } \frac{\partial U}{\partial x} > 0, \; \frac{\partial^2 U}{\partial x^2} < 0 \tag{1.64}$$

donde $a(t)$ es una función que asocia a cada instante de tiempo una actividad y $z(t)$ es una función que modela la intensidad de la actividad y que está definida por una función de producción descrita en la ecuación (1.64). El autor se refiere a la intensidad de cada actividad como una tasa de realización de la misma que se puede entender de forma análoga a la tasa de flujo de producción instantánea en una firma. Por otro lado, dado que se supone total separabilidad entre actividades entonces se obtiene con dicho modelo un patrón de actividades y sus respectivas valoraciones del tiempo. Aunque los autores obtienen resultados teóricos interesantes de dicho modelo, El autor omite la inclusión de restricciones tecnológicas entre actividades o entre actividades y consumo de bienes, descritos previamente y que son necesarias para entender y analizar la asignación y valoración del tiempo.

En una interesante revisión de la teoría de la modelación microeconómica, Juster (1990) describe los flujos de utilidad de los diferentes individuos dependientes de las diferentes actividades que realiza y los bienes usados para su realización. El postula que la función de utilidad de cada individuo está caracterizada de la siguiente forma:

$$U = u(x_i,\, Z_i,\, t_i, K_0, K_1), \tag{1.65}$$

donde x_i son los bienes y servicios del mercado, Z_i son los bienes producidos por el hogar, t_i es el tiempo asignado a las actividades y K_0 es la reserva de capital inicial y K_1 es el capital final. Este tipo de bienes de capital, Juster los define como bienes que trascienden en el tiempo y se podrían representar como medidas de *calidad de vida*, por ejemplo, estado de los bienes durables, inversiones, seguridad financiera, etc.

Luego de definir y analizar la función de utilidad (1.65), Juster (1990) afirma que tanto los bienes como el uso de tiempo generan un beneficio del proceso (PB_i). Además, supone que los bienes de capital durante un periodo de tiempo K_0^* generan utilidad directa y dicho consumo es independiente de cualquier uso de tiempo, de esta forma se podría plantear el modelo de la siguiente forma:

$$\text{máx}\, U = u(PB_i, K_0^*, K_1) \tag{1.66}$$

sujeto a

$$K_0 - K_0^* = K_1 \tag{1.67}$$

$$\sum_i t_i = T \tag{1.68}$$

La ecuación (1.67) no aparece de esta forma en el documento original, el propósito de presentarla así es detallar los conceptos que se incluyen en la modelación, en especial la transcendencia intertemporal de los bienes de capital. Es interesante la propuesta de modelación y la descripción de Juster (1990), donde los bienes durables generan utilidad directa. Sin embargo, la formulación no permite obtener resultados analíticos sobre valoración del tiempo como recurso y su dependencia de los bienes de capital de cada hogar.

Jara Díaz (1994) plantea y analiza un modelo microeconómico que integra elecciones discretas y asignación de tiempo a actividades, permitiendo el estudio de elementos importantes de la ingeniería de transporte como la partición modal y la generación y distribución de viajes bajo el supuesto que otras variables son fijas o se mantienen constantes. Un primer modelo de Jara-Díaz (1994) tiene la siguiente formulación:

$$\underset{T,W_v,m,B,X}{\text{máx}}\, U(T, W_v, W_f, X) \tag{1.69}$$

sujeto a

$$\sum_i T_i + W_v + W_f + \sum_{j=1}^{B} \sum_{m\in M_j} \delta_{mj} t_{mj} = \tau \tag{1.70}$$

$$F(X, T, W_f, W_v, t) \geq 0 \tag{1.71}$$

$$\sum_i \sum_d P_{id} X_{id} + \sum_{j=1}^{B} \sum_{m \in M_j} \delta_{ij} c_{ij} = I_f + w W_v \tag{1.72}$$

$$B = B(X) \tag{1.73}$$

donde $F(.)$ es la función de transformación entre bienes y tiempo y viceversa, y además incluye las interrelaciones entre el tiempo asignado a las actividades. B es el número de viajes en un período de análisis τ y a su vez B es condicional a los bienes consumidos X, t_{mj} y c_{mj} el tiempo y costo de viaje en el modo m para el viaje j, X_{id} es cantidad de bien i que comprado en la zona d a un precio P_{id} y M_j son los modos disponibles para el viaje j. Además, I_f el ingreso fijo, T el vector de duración de actividades sin incluir el trabajo ni el tiempo de viaje, W_f es la duración del trabajo fijo, W_v la duración del trabajo variable y w es la tasa salarial. Resolviendo el problema condicional en el modo resulta posible obtener valores óptimos para el tiempo asignado a las actividades, y la cantidad de trabajo variable en función de los parámetros del modelo. Es importante anotar que las variables fijas o parámetros son W_f, I_f, t_{mj}, c_{mj}, P_{id} y w, mientras que las variables de decisión son T_i, δ_{mj}, X_{id}, W_v y B. La solución para B es el modelo de la generación, la solución para X es el modelo de distribución, y la solución de δ es el modelo de elección del modo. En ese mismo trabajo, el autor hace una simplificación del modelo (1.69)-(1.73) eliminando la dimensionalidad de B y de la distribución zonal de los bienes d. Un aspecto interesante de la modelación previa es reconocer distintos tipos de relaciones tecnológicas entre bienes y tiempo, que en un trabajo posterior del mismo autor (Jara-Díaz, 2003) serían analizadas de manera más detallada basándose en las propiedades de las funciones de multi-producción. Además incorpora el uso de suelo como un elemento importante para la realización de actividades. Un enfoque similar es utilizado por Jara-Díaz y Martínez (1999) para proveer una base teórica del análisis de la decisión de localización residencial, obteniendo un término asociado a la accesibilidad para cada lugar, donde se incorpora la utilidad obtenida de desarrollar actividades en diferentes lugares y el costo generalizado de viajar a realizar dichas actividades.

A continuación se describe con más detalle los modelos de Jara-Díaz (2003) y Jara-Díaz y Martínez (1999). En Jara-Díaz (2003), el autor presenta un sistema completo de las relaciones tecnológicas entre el consumo de bienes X y tiempo asignado a las actividades T durante un mismo periodo de tiempo, que se describen a continuación.

- Dado un vector de las actividades fijos T^0, es necesario saber los bienes que son necesarios para realizarlas. Hay algunas combinaciones de productos que permiten la asignación dada por T^0.

- Otro tipo de relaciones factibles con el consumo de bienes que está permitido por la asignación de tiempo a un vector de actividades T^0. Es decir, dado T^0 hay algunas combinaciones y cantidades de bienes X que pueden ser consumidos y otros que no.

- Un conjunto determinado de bienes X^0 se puede consumir durante ciertas combinaciones factibles de tiempo asignados a las actividades. Esto significa que hay estructuras de actividades T que no son compatibles con el consumo de X^0.

- Además, existe una relación que asocia una determinada cantidad de bienes con la duración posible de un conjunto de actividades. En otras palabras, dado una cantidad y la combinación de bienes X^0, existen algunas combinaciones de actividades factibles a realizar, y algunos que no son factibles debido a la falta de bienes disponibles.

Las cuatro representaciones presentadas anteriormente se resumen en dos tipos de relaciones entre bienes de consumo y tiempo asignado a actividades. El hecho de que las actividades requieren bienes, hace definir una función de posibilidad de actividades A que abarca dos tipos de relaciones, a saber:

$$A(X,T) \geq 0 \tag{1.74}$$

tal que $A(X,T^0) \geq 0$ representa la combinación en X que son necesarias para T^0 y $A(X^0,T) \geq 0$ representa la combinación en T que son permitidos con X^0.

Otro tipo de las relaciones tecnológicas da la combinación de actividades que permiten una estructura de consumo dado por X^0, y también la combinación en consumo X permitido por una determinada estructura T^0. De esta forma se define una función de posibilidad de consumo G

$$G(X,T) \geq 0 \tag{1.75}$$

tal que $G(X^0,T) \geq 0$ da la combinación de actividades que son necesarias para X^0, y $G(X,T^0) \geq 0$ describe el consumo que es permitido por T^0.

El autor muestra un ejemplo de las relaciones (1.75) y (1.74) con el siguiente problema del consumidor:

$$\text{máx}\, U(X_1,\cdots,X_n,T_1,\cdots,T_m) \tag{1.76}$$

sujeto a

$$wT_w - \sum_{i=1}^{n} P_i X_i \geq 0, \quad (\lambda) \tag{1.77}$$

$$\tau - \sum_{j=1}^{m} T_j = 0, \quad (\mu) \tag{1.78}$$

$$T_j - f_j(X) \geq 0, \quad \forall j, \quad (\kappa_j) \tag{1.79}$$

$$X_i - g_i(T) \geq 0, \quad \forall i, \quad (\psi_i) \tag{1.80}$$

Las desigualdades (1.80) afirman que sobre el consumo de bienes se imponen unos niveles mínimos basados en el tiempo asignado a las diferentes actividades. Dichas restricciones son propuestas como una interpretación que hace el autor a la matriz de requerimiento unitario de bienes Q de la restricción (1.46) en el modelo de Evans (1972). Las desigualdades (1.79) son una extensión multi-consumo a las desarrolladas por De Serpa (1971) indicando que el tiempo dedicado a las actividades está acotado inferiormente por unos niveles mínimos dado por la cantidad de los bienes de consumo. Tal como lo indica Jara-Díaz (2003) la incorporación de ambos tipos de restricciones (tanto en la modelación como en los desarrollos econométricos) pueden ser necesarias para hacer un análisis más completo del uso y valoración del tiempo. Este análisis es importante debido a ser un sustento teórico necesario para el modelo de actividades desarrollado en esta tesis.

De las condiciones de primer orden de (1.76)-(1.80) se obtienen los siguientes resultados:

- El valor del tiempo o disposición a pagar para actividades diferentes al trabajo:

$$Si \quad \kappa_j \neq 0 \Rightarrow \quad \frac{\kappa_j}{\lambda} = \frac{\mu}{\lambda} - \frac{1}{\lambda}\frac{\partial U}{\partial T_j} + \frac{1}{\lambda}\sum_{i=1}^{n} \psi_i \frac{\partial g_i}{\partial T_j} \tag{1.81}$$

La ecuación (1.81) se puede analizar con tres efectos:

1. $\frac{\mu}{\lambda}$: Valor del tiempo.
2. $\frac{1}{\lambda}\frac{\partial U}{\partial T_j}$: Valoración de asignar tiempo a la actividad j.
3. $\frac{1}{\lambda}\sum_{i=1}^{n}\psi_i\frac{\partial g_i}{\partial T_j}$: efecto de la variación en el consumo.

$$Si \quad \kappa_j = 0 \Rightarrow \quad \frac{\mu}{\lambda} = \frac{1}{\lambda}\frac{\partial U}{\partial T_j} - \frac{1}{\lambda}\sum_{i=1}^{n}\psi_i\frac{\partial g_i}{\partial T_j} \tag{1.82}$$

El resultado de la ecuación (1.82) sobre la valoración del tiempo cambia el resultado sobre lo mostrado en De Serpa (1971), ya que se incluye el efecto de variación en el consumo.

- El valor del tiempo asignado al trabajo se puede analizar de la forma siguiente:

$$\frac{\mu}{\lambda} = w + \frac{1}{\lambda}\frac{\partial U}{\partial T_w} - \frac{1}{\lambda}\sum_{i=1}^{n}\psi_i\frac{\partial g_i}{\partial T_w} \tag{1.83}$$

Usando las ecuaciones (1.83) y (1.81) el autor concluye que la expresión para calcular la disposición a pagar por disminuir el tiempo de una actividad diferente al trabajo se ve afectada por varios efectos:

$$\frac{\kappa_j}{\lambda} = w + \frac{1}{\lambda}\frac{\partial U}{\partial T_w} - \frac{1}{\lambda}\frac{\partial U}{\partial T_j} + \frac{1}{\lambda}\sum_{i=1}^{n}\psi_i\left\{\frac{\partial g_i}{\partial T_j} - \frac{\partial g_i}{\partial T_w}\right\} \tag{1.84}$$

La disposición a pagar (1.84) depende de la tasa salarial, más el valor de la utilidad marginal del trabajo, más el valor de la reducción marginal de la actividad, más el valor del patrón de cambio en el consumo. Notando además que (1.84) muestra la relación entre la asignación y valoración del tiempo de actividades no laborales con la asignación y valoración del tiempo al trabajo. De forma análoga es posible encontrar la disposición a pagar por disminuir el patrón de consumo $\frac{\psi_i}{\lambda}$.

Una de las limitaciones de los modelos de tipo teórico como el descrito anteriormente es su difícil aplicabilidad a datos reales. En este sentido, en Jara-Díaz y Guevara (2003) se propone una simplificación de dicha formulación para estimar econométricamente el tiempo asignado al trabajo y la valoración del tiempo de forma conjunta a la elección de modo de transporte.

En primera medida, los autores analizan el siguiente modelo microeconómico:

$$\text{máx}\, U(X,T) \tag{1.85}$$

$$I_f + wT_{w_v} - \sum_{j=1}^{m} p_jX_j - c_r \geq 0, \quad (\lambda) \tag{1.86}$$

$$\tau - \sum_{i=1}^{n} T_i - T_r = 0, \quad (\mu) \tag{1.87}$$

$$T_i - h_i(X) \geq 0, \quad (\kappa_i), \quad \forall i \neq r,\, w_f \tag{1.88}$$

$$T_k - T_k^{min} \geq 0, \quad (\kappa_k), \quad k = r,\, w_f \tag{1.89}$$

donde w_f corresponde al trabajo fijo, w_v corresponde al trabajo variable, w es la tasa salarial, c_r y t_r es el costo y tiempo de viaje, r es el sub-índice asociado al viaje y T_k^{min} es el mínimo tiempo que se puede asignar a la actividad k. λ, μ, κ_i, κ_k son los respectivos multiplicadores de Lagrange. Basados en dicha formulación, los autores analizan y describen conceptos clásicos de la literatura como el valor del tiempo como recurso $\frac{\mu}{\lambda}$, el valor de ahorrar tiempo en una actividad $\frac{\kappa_i}{\lambda}$ y la valoración marginal de asignar tiempo a una actividad $\frac{\frac{\partial U}{\partial T_i}}{\lambda}$.

Además, consideran el caso particular del tiempo de viaje, donde explican como el valor de $\frac{\kappa_r}{\lambda}$ es igual a la tasa marginal del sustitución entre tiempo y costo de viaje, que es estimado por medio de la utilidad modal en modelos de elecciones discretas. Dicha afirmación también está sustentada por resultados de trabajos previos, como Bates (1987), Troung y Hensher (1985), entre otros.

Basados en la propiedad descrita anteriormente, Jara-Díaz y Guevara (2003) presentan un modelo microeconómico simplificando las condiciones de (1.85)-(1.89) para ser usado en estimaciones econométricas. La formulación es la siguiente:

$$\text{máx}\, U(X,T) = \Omega T_w^{\theta_w} T_t^{\theta_t} \prod_i T_i^{\theta_i} \prod_j X_j^{\varphi_j} \tag{1.90}$$

$$I_f + wT_w - \sum_{j=1}^{m} p_j X_j - c_t \geq 0, \quad (\lambda) \tag{1.91}$$

$$\tau - T_t - T_w - \sum_{i=1}^{n} T_i = 0, \quad (\mu) \tag{1.92}$$

$$T_t - T_t^{min} \geq 0, \quad (\kappa_t) \tag{1.93}$$

donde θ_i y φ_j son los parámetros en la función de utilidad Cobb-Douglas, Ω es una utilidad constante y T_t^{min} es el tiempo mínimo de viaje al trabajo. Por otro lado, si la utilidad modal V es aproximada de forma lineal:

$$V_t^j \approx \gamma^j - \gamma^t t_j - \gamma^c c_j \tag{1.94}$$

donde c_j y t_j es el costo y tiempo de viaje para el modo j, entonces se tendrá la siguiente relación para el valor subjetivo del tiempo de viaje ($VSTV$):

$$\frac{\gamma^t}{\gamma^c} = \frac{\kappa_t}{\lambda} = VSTV \tag{1.95}$$

Note que dicha relación implica que $\kappa_t > 0$ y por lo tanto $T_t = T_t^{min}$.

En este mismo trabajo, usando las condiciones de primer orden, los autores expresan el tiempo asignado al trabajo en términos del costo y tiempo de viaje, y algunos parámetros que caracterizan las preferencias de los individuos.

$$T_w = \beta(\tau - T_t) + \alpha\frac{c_t}{w} + \sqrt{\left[\beta(\tau - T_t) + \alpha\frac{c_t}{w}\right]^2 - \left[2(\alpha+\beta) - 1\right]\frac{c_t}{w}(\tau - T_t)} \tag{1.96}$$

donde β y α son parámetros a ser estimados. Además, dada la formulación de la función de utilidad se obtiene la siguiente expresión para el valor del tiempo como recurso:

$$\frac{\mu}{\lambda} = \left(\frac{1-2\beta}{1-2\alpha}\right)\left(\frac{wT_w - c_t}{\tau - T_w - T_t}\right) \tag{1.97}$$

Soportados en las ecuaciones (1.97) y (1.95) los autores obtienen la valoración del tiempo de viaje $\frac{\frac{\partial U}{\partial T_t}}{\lambda}$ y la valoración del tiempo asignado al trabajo $\frac{\frac{\partial U}{\partial T_w}}{\lambda}$ usando las siguientes expresiones basados en las condiciones de primer orden del modelo microeconómico:

$$\frac{\kappa_t}{\lambda} = \frac{\mu}{\lambda} - \frac{\frac{\partial U}{\partial T_t}}{\lambda} = w + \frac{\frac{\partial U}{\partial T_w}}{\lambda} - \frac{\frac{\partial U_t}{\partial T_t}}{\lambda} \tag{1.98}$$

Las relaciones descritas anteriormente fueron establecidas orginalmente por Oort (1969).

Por otro lado, en Jara-Díaz y Guerra (2003) se propone una extensión del modelo de Jara-Díaz y Guevara (2003) para hacer nuevas estimaciones econométricas del valor del ocio y de la asignación del tiempo a diferentes actividades, en especial el tiempo asignado a trabajar. Para explicar dicho modelo se usará la descripción realizada en Jara-Díaz et al. (2008), donde los autores parten de una formulación microeconómica sin elección de modo:

$$\text{máx}\, U(X,T) = \Omega T_w^{\theta_w} \prod_i T_i^{\theta_i} \prod_j X_j^{\varphi_j} \tag{1.99}$$

sujeto a

$$I + wT_w - \sum_j P_j X_j \geq 0, \quad (\lambda) \tag{1.100}$$

$$\tau - T_w - \sum_i T_i = 0, \quad (\mu) \tag{1.101}$$

$$T_i - T_i^{min} \geq 0, \quad (\kappa_i), \ \forall i \tag{1.102}$$

$$X_j - X_j^{min} \geq 0, \quad (\eta_j), \ \forall j \tag{1.103}$$

donde T_i es el tiempo asignado a la actividad i, X_j es el consumo del bien j durante un período de tiempo τ, T_w es el tiempo asignado al trabajo con una tasa salarial w y se tiene un ingreso exógeno I. Además, λ, μ, κ_i, η_j son los multiplicadores de Lagrange de las distintas restricciones. Vale la pena anotar que las restricciones tecnológicas en este problema (1.102) y (1.103) definen una cota inferior exógena al consumo de bienes y asignación de tiempo a actividades. Es decir, no se consideran restricciones que relacionen el consumo de bienes y el tiempo asignado a las diferentes actividades.

De las condiciones de primer orden de este problema se obtiene un sistema de ecuaciones que determinan el tiempo óptimo de trabajo y los tiempos óptimos de las actividades irrestrictas (Ver Jara-Díaz et al. 2008, páginas 947 a 950). Para obtener dichos valores se definen los siguientes parámetros:

- *Gasto comprometido* E_c: Este valor se define como la suma de los gastos fijos que según este modelo es el gasto asociado a los bienes que se les asigna el mínimo necesario. Si se define G^r al conjunto de dichos bienes se tiene que:

$$E_c = \sum_{j \in G^r} X_j^{min} P_j - I$$

- *Tiempo comprometido* T_c: Es la suma de todos los tiempos de las actividades a que se les asigna el mínimo necesario. Si se define A^r al conjunto de dichas actividades se tiene que:

$$T_c = \sum_{i \in A^r} T_i^{min}$$

De esta forma, en Jara-Díaz et al.(2008) se obtienen las siguientes expresiones para la asignación del tiempo al trabajo, asignación del tiempo a actividades libres y asignación al consumo de bienes:

$$T_w = \beta(\tau - T_c) + \alpha\frac{E_c}{w} + \sqrt{\left(\beta(\tau - T_c) + \alpha\frac{E_c}{w}\right)^2 + (2\alpha + 2\beta - 1)(\tau - T_c)\frac{E_c}{w}} \tag{1.104}$$

Además,

$$T_i = \frac{\eta_i}{(1-2\beta)}(\tau - T_w - T_c), \ \ \forall i \notin A^r \tag{1.105}$$

$$X_j = \frac{\varsigma_j}{P_j(1-2\alpha)}(wT_w - E_c), \ \ \forall j \notin G^r \tag{1.106}$$

donde α, β, η_i y ς_j son parámetros a ser estimados.

En este caso, el valor del tiempo como recurso tiene la siguiente expresión

$$\frac{\mu}{\lambda} = \left(\frac{1-2\beta}{1-2\alpha}\right)\left(\frac{wT_w - E_c}{\tau - T_w - T_c}\right) \tag{1.107}$$

Al igual que en el modelo de Jara-Díaz y Guevara (2003), es posible obtener la valoración de cada actividad restringida ($T_i = T_i^{min}$) y la valoración del tiempo de trabajo usando expresiones similares a (1.98). En particular, se tiene que:

$$\frac{\frac{\partial U}{\partial T_w}}{\lambda} = \left(\frac{2(\alpha+\beta)-1}{1-2\alpha}\right)\left(\frac{wT_w - E_c}{T_w}\right) \tag{1.108}$$

En Munizaga et al. (2008) hacen una extensión microeconómica y econométrica de este modelo incluyendo la elección de modo de transporte.

Los resultados obtenidos en Jara-Díaz y Guevara (2003) y Jara-Díaz y Guerra (2003), surgen de una formulación microeconómica basada en restricciones tecnológicas exógenas (sin interacción bienes y tiempo). En este sentido, sería interesante proporcionar una estructura más general con restricciones como las descritas en Jara-Díaz (2003).

Un trabajo que integra asignación de tiempo a actividades y uso de suelo es el desarrollado por Jara-Díaz y Martínez (1999) donde bajo una perspectiva estática se analizan los factores de la realización y asignación de tiempo a actividades que influyen en la decisión de localización residencial, y a su vez estas decisiones de largo plazo, generan posibilidades de realización de actividades en términos de tiempo, frecuencia y localización espacial, basados en la distribución de bienes y servicios y los costos y tiempos de transporte. En dicho modelo se obtiene la disposición a pagar por un bien inmueble usando teoría de elecciones discretas. La formulación es la siguiente:

$$\max_s \max_{x,f,T} U(x, z, f, T, tv_s) \tag{1.109}$$

sujeto a

$$T_{FW} + \sum_{k \neq FW} f_k T_k + tv_s = TT \tag{1.110}$$

$$\sum_i p_i x_i + \sum_\ell B_{s\ell}(f,\delta) c_{s\ell} + p(z_s) = I + \sum_{k \neq FW} w_k f_k T_k \tag{1.111}$$

$$x_i = x_i(f,T), \ \ \forall i \tag{1.112}$$

$$f_k \geq f_k^{min}, \ \ \forall k \tag{1.113}$$

$$T_k \geq T_k^{min}, \ \ \forall k \tag{1.114}$$

$$x_i \geq 0$$

con la siguiente notación y explicación de términos:

- k, s, ℓ : índices para actividades, localización del hogar y otras localizaciones, respectivamente.
- f_k, T_k frecuencia y tiempo asignado a la actividad k.
- tv, T_{FW}, TT, (*variables de tiempo*:) tiempo de viaje, tiempo fijo de trabajo, y tiempo total factible en el periodo modelado.
- I, w_k : Ingreso fijo y tasa salarial por actividad k.
- x_i, p_i : consumo del bien i y su respectivo precio por unidad. Además, $x_i(f,T)$ es una función de transformación de tiempo asignado a actividades en los bienes requeridos para realizarlas.
- $p(z_s)$: renta por el bien inmueble s como función de los atributos z_s.
- $B_{s\ell}$, $c_{s\ell}$, $tv_{s\ell}$, (*variables de transporte*:) número de viajes entre las localizaciones s y ℓ y sus respectivos costos y tiempos de viaje.
- δ : matriz de destinos de viajes condicional a s, donde los elementos $\delta_{ks\ell}$ toman el valor de 1 si la actividad k se puede realizar en ℓ viviendo en s y 0 en otro caso.
- Además, el tiempo agregado de viaje desde s es

$$tv_s \equiv \sum_\ell tv_{s\ell} \equiv B_{s\ell}(f,\delta) t_{s\ell}$$

La formulación (1.109)-(1.114) extiende los trabajos descritos anteriormente pues incluye la frecuencia de las actividades, así como las posibilidades factibles de elección dada una localización residencial. Además, identifican las variables de tiempo de viaje y número de viajes en términos espaciales y temporales. Aunque en dicho artículo no se obtiene una expresión explícita para el valor del tiempo como recurso, claramente se ve la influencia de las elecciones espaciales sobre dicho valor.

Por otro lado, los autores obtienen una expresión para la disposición a pagar por el bien s, que depende de los beneficios de realizar actividades en s y la accesibilidad a otras opciones ℓ que combina el beneficio de visitar actividades en diferentes localizaciones dado un costo generalizado de transporte.

Por el objetivo de este trabajo no se presentan más desarrollos de modelos microeconómicos de asignación y valoración del tiempo, sin embargo, existen excelentes documentos que resumen y extienden varios de los trabajos desarrollados en dicha área, tales como Jara-Díaz (2007) y González (1997).

Otros trabajos y modelos que buscan analizar el comportamiento de los individuos, en especial en la asignación de tiempo, es el enfoque llamado *Basado en Actividades* (*Activity Based*). El objetivo primordial de este enfoque es entender el contexto de la decisión de viajar, reconociendo que la estructura de actividades, ya sea de un individuo o de un hogar, está distribuida en el espacio y el tiempo, generando los diferentes tipos de viajes. Existen varias perspectivas para estudiar los modelos basados en actividades, que son: ecuaciones de tiempo, ecuaciones estructurales y generación de programas de actividades (Ver Astroza, 2012). Por la similitud con las formulaciones microeconómicas descritas anteriormente, sólo se mostraran algunos detalles de las ecuaciones de tiempo, que tienen su fundamentación en el modelo de Kitamura (1984), donde el autor plantea un modelo microeconómico de elección discreta de participación de actividades (una actividad puede o no realizarse dentro del mismo día) y continuo de asignación de tiempo que se escribe de la forma:

$$\text{máx}\, U(T_1, \ldots, T_n, q_1, \ldots, q_n) = \sum_{i=1}^{n} U_i(T_i, q_i), \quad \text{sujeto a} \quad \sum_{i=1}^{n} T_i = \tau \tag{1.115}$$

donde T_i es el tiempo asignado a la actividad i, q_i es un vector de características de dicha actividad. Además, se supone que la utilidad total U es la suma de la utilidad obtenida de cada actividad $U_i(T_i, q_i)$. En dicho trabajo, se considera la siguiente función de utilidad para el caso en el que se les asigna tiempo positivo a cada actividad:

$$U_i = \epsilon_i \gamma_i f_i(q_i) \ln(T_i) \tag{1.116}$$

donde $f(q_i)$ es una función que indica la valoración de cada individuo de las características de los actividades, ϵ_i es un término de error positivo y γ_i es un parámetro positivo. Kitamura (1984) admite el hecho que cuando a las actividades discrecionales no se les asigna tiempo entonces el individuo no obtendrá utilidad por las mismas. De esta forma se tendría la siguiente función para U_i

$$U_i = \begin{cases} \epsilon_i \gamma_i f_i(q_i) \ln(T_i), & si \;\; T_i > 0 \\ 0, & si \;\; T_i = 0 \end{cases} \tag{1.117}$$

El autor muestra que esta asignación de tiempo discreta/continua se puede expresar como un modelo Tobit para la estimación de los parámetros.

Existen diferentes avances en la formulación de la función de utilidad de estos modelos, tales como Munshi (1993), Kitamura et al. (1996), Kim et al. (2002), Bhat (2005), Bhat (2008), entre otros, que han utilizado una base similar a la de (1.115) de asignación de recursos para determinar la participación individual o por hogares en una actividad y la duración de la misma, pero vale la pena mencionar que dichos modelos de ecuaciones de tiempo no permiten obtener información sobre el valor del tiempo como recurso de los individuos dado que sólo se considera la restricción temporal. De esta forma, hasta el momento han sido más aporte en la estimación econométrica que en la modelación microeconómica de la asignación del tiempo. Sin embargo, una trabajo reciente de Castro et al. (2012) avanza en la idea de la modelación microeconómica, incorporando varios tipos de restricciones. Para ser más exactos, los autores consideran el siguiente problema genérico de elección discreta-continua:

$$\max U = \sum_{k=1}^{K} \frac{\gamma_k}{\alpha_k} \psi_k(q^k) \left(\left(\frac{x_k}{\gamma_k} + 1 \right)^{\alpha_k} - 1 \right) \tag{1.118}$$

$$\sum_{k=1}^{K} p_k x_k = E, \quad (\lambda) \tag{1.119}$$

$$\sum_{k=1}^{K} g_k x_k = T, \quad (\mu) \tag{1.120}$$

donde

- $\psi_k > 0$ es la utilidad marginal cuando el consumo de x_k es cero. Además, ψ_k es función del vector de atributos q^k de la actividad k
- $0 < \alpha_k \leq 1$ este valor captura los efectos de saciedad. Cuando $\alpha_k = 1$ para todo k, se representa el caso de la ausencia de efectos de saciedad o, de manera equivalente, utilidad marginal constante (elección es de tipo discreto). Cuando α_k decrece entonces el efecto de saciedad para el bien k aumenta.
- $\gamma_k > 0$ un parámetro positivo para cada bien k.
- En la restricción presupuestaria (1.119), E es el gasto total en todos los bienes y $p_k > 0$ es el precio por unidad de bien k
- En la restricción temporal (1.120), T es el tiempo disponible para el consumo de todos los bienes y $g_k > 0$ es la tasa de tiempo a la que se consume una unidad del bien k, es decir, $t_k = g_k x_k$.
- λ y μ son los multiplicadores de Lagrange de las restricciones de ingreso y tiempo respectivamente.

Para estimar el modelo suponen la siguiente forma funcional de ψ_k :

$$\psi_k = \exp(\beta' q^k + \epsilon_k) \tag{1.121}$$

donde q^k es un conjunto de atributos que caracterizan la alternativa k y el valor de ϵ_k captura las características idiosincrásicas (no observada) que afectan a la utilidad del bien k. Esta parametrización garantiza la positividad de la función de utilidad.

Usando las condiciones de primer orden y técnicas econométricas, los autores estiman los diferentes parámetros para obtener adicionalmente el valor del tiempo como recurso $\frac{\mu}{\lambda}$. Lo interesante de este trabajo reciente, es que logra delinear posibles comparaciones entre los resultados tanto teóricos como prácticos obtenidos por los modelos microeconómicos y los modelos basados en actividades. Además, las distintas formulaciones de las funciones de utilidad usadas en los modelos basados en actividades podrían motivar nuevos desarrollos en el área de la modelación microeconómica.

A manera de conclusión, se tiene que la modelación microeconómica de asignación de tiempo a actividades y consumo de bienes tiene varios aspectos importantes a considerar que son la función de utilidad, las restricciones del problema y los parámetros exógenos del modelo. En esta línea, la hipótesis o justificación para el desarrollo del modelo jerárquico de actividades y consumo de bienes presentado en esta tesis es que los modelos microeconómicos estáticos de asignación y valoración del tiempo (por estáticos nos referimos a que representan un solo cohorte temporal) suponen la existencia de decisiones exógenas que generan diferencias en el valor del tiempo como recurso de cada individuo (por ejemplo, localización residencial o de trabajo, modo de transporte y en algunos casos un tiempo de trabajo fijo, etc.); o a su vez, dichas diferencias en la valoración del tiempo también pueden ser generadas por los parámetros tanto en la función de utilidad o de las restricciones de ingreso y tiempo (por ejemplo, ingreso exógeno, tasa salarial que depende de decisiones previas, gastos y tiempo comprometido, costo y tiempo de viaje, etc). Además, los parámetros de las restricciones tecnológicas para el consumo de bienes y asignación de tiempo pueden ser un elemento diferenciador del valor del tiempo entre individuos (por ejemplo, tiempo mínimo de viaje).

Basándose en estos supuestos clásicos de los modelos estáticos de asignación de tiempo se busca explorar algunos efectos inter-temporales que pueden afectar la valoración del tiempo definidos por una jerarquía asociada a los plazos de validez de las distintas decisiones. Es más, un aspecto importante de la modelación inter-temporal y jerárquica, es poder explicar varios de los parámetros exógenos o fijos en un determinado período, como decisiones tomadas en periodos previos.

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Parte II:

ARTÍCULOS O PUBLICACIONES

Capítulo 2

Primer articulo: A time-hierarchical microeconomic model of activities

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ABSTRACT

The microeconomic approach to explain consumers behavior regarding the choice of activities, consumption of goods and use of time is extended in this paper by explicitly including the temporal dimension in the choice-making process. Recognizing that some activities, such as a job and education, involve a long-term commitment and that other activities, such as leisure and shopping, are conducted and modified in the short term, we make these differences explicit in a microeconomic framework. Thus, a hierarchical temporal structure defines the time window or frequency of adjusting the variables of activities (duration, location and consumption of goods) and the magnitude of the resources (time and money) spent. We specify and analyze a stylized microeconomic model with two time scales, the macro and micro level, concluding that preference observations at the micro level, such as transport mode choice, are strongly conditioned by the prevailing choices at the macro scale. This result has strong implications for the current theory of the value and allocation of time, as well as on the location of activities, as illustrated by numerical simulations.
Keywords: value of time, hierarchical decisions, temporal scale, long-term and short-term activities.

2.1. Introduction

In the last decades several microeconomic models have been proposed to explain people's behavior, including the consumption of discrete and continuous goods, the choice of feasible activities within the urban system and the use of time for them. Analogous to the proposal of Lancaster (1966) in the theory of consumption, Becker (1965) proposes a basic theory of time use assuming that the utility function of households and individuals is dependent on the set of nonworking activities, which are produced by combining consumption goods and the time allocated. The trade-off between working and leisure times yields the author concluding that the value of time equals the consumer's wage rate. De Serpa (1971) adds the working time as an argument in the utility function, and a set of technological relationships between goods consumption and duration time for each activity, while consumption of any good requires a minimum amount of time. This model contributes to differentiate time values allocated to each activity. Evans (1972) modifies fundamentally this approach by assuming that time spent in activities is the only argument of the utility function, arguing that goods play a role limited to make activities feasible. This argument questions the definition of activities and what yields utility (goods or time) and introduces a new set of constraints on the time allocated to different activities. By explicitly modeling the choice of transport mode as a component of the activities, Train and McFadden (1978) derived the specification of the indirect utility function for the mode choice, whose parameters yield values of time derived in this case from the consumer's choice between the alternative time and cost options offered by different transport modes. In addition, Small (1982) presents a static microeconomic model that considers the scheduling of the individual's activities, assuming duration as well as starting time, finding and expression to compute the leisure time value depending, among other things, on both the marginal contribution of traffic congestion and the starting time of activities; the model is applied to characterize work trips. In the same line, Winston (1987) developed a model of continuous scheduling of activities, showing that the value of time is a dynamic measurement for each individual that can vary according to each individual's environment. Juster (1990) showed empirically as well as descriptively the dependence between time allocation of activities and durables (goods that last between periods). Later Jara-Díaz and Guevara (2003) and Jara-Díaz and Guerra (2003) integrated the choice of activities and transport mode in the calculus of value of time. In both approaches individuals choose their working hours and time spent in all other activities to attain static equilibrium.

In sum, since De Serpa, researchers conclude that for a given individual the value of time differs for each activity performed. More precisely, individuals' value of time is composed by an individual's specific component that values the general scarcity of time, given a time budget, and another component that depends on whether the activity performed is work (generates income) or leisure (consumes income and time). Subsequently, empirical evidence supports this argument. For example, Jara-Diaz et al. (2008) and Munizaga et al. (2008) developed an econometric approach to calculate and estimate the value of working time and the value of leisure time based on a utility maximization model, including goods and time allocated to activities. The econometric model was applied to data from three different cities: Santiago (Chile), Karlsruhe (Germany) and Thurgau (Switzerland). Their analysis shows that the average value of leisure time is less than the average wage rates in Santiago, unlike the other European cities; on average, in Karlsruhe the valuation of working time is positive, unlike in Santiago and Thurgau. Olguín (2008) used the 2001 origin-destination survey of Santiago to report different values of time: by gender, age and residential location. Another interesting result is that the value of leisure time of residents in the high-income area is 3 to 4 times this value in other areas. Additionally, the overall value of working time reported is negative, but it is unexpected positive for individuals under 25 years and for women between 25 and 64. Greeven (2006) also obtained different time values associated to differentials in households' expenditure in long term commitments, like housing, domestic services, medical services, education, communications, consumer durables, etc.

The causes of the differences in time allocation to different activities were empirically studied by Levinson (1999), concluding that the demography, socioeconomics, season and scheduling affect the distribution of time to various activities. His analysis also shows that the time allocated to an activity is positively correlated with the duration of the trip. He also concludes that time allocated to household activities, shopping, etc., is correlated with the individual's gender, age, location and residential density, income, seasonality, marital status, age of children, and frequency of activities. Bullard and Feigenbaum (2007) proposed a general equilibrium model for the cycle of life, where household's utility include consumption and leisure in each period. The calibration shows that there are significant changes in consumption over the cycle of life. The model identifies the income, asset holdings, hours worked, in different temporal cohorts, for individuals with age greater than 25 years old. They estimate that the average age of maximum consumption of nondurable goods is close to 45 years old. On the other hand, in terms of hours spent at work, the peak is between 50 to 55 years on average, but after this period is likely to decay. Similarly, Guang-Zhen and Yew-Kwang (2009), studied life cycles using a microeconomic model to show the age-dependent dynamics of the value of time. In their model long-term choices are represented by the structure of risk or investment assumed by the individual a priori. In addition, they analyzed the changes obtained in the value of time, after the retirement of individual and the influence of mortality and interest rates on the valuation of time.

The static microeconomic theoretical models of time allocation (static refers to a single and representative temporal cohort) assumes that the activity/consumption choices are optimized simultaneously (fixed consumption/activities are assumed as exogenous parameters). Additionally, both decisions and exogenous elements of the model generate differences in the value of time as resource of each individual (for example, residential or work location, age, mode, transport cost, travel time, and in some cases the time allocated to work, education level, etc.). These exogenous decisions can be incorporated in the parameters of both, the utility function (change in taste) and the budget constraints (for example, exogenous income or wage rate that depends on previous decisions, expenses and time commitments in budget constraints defined in Jara-Díaz et al., 2008).

In addition, the value of time as resource can be different across individuals due to the variety of parameters associated with technological constraints (see DeSerpa, 1971; Evans, 1972, Jara-Díaz, 2003; Jara-Diaz, 2007), for example, in the case that each activity has an individuals' specific minimum time allocation and, similarly, each consumption good has a lower exogenous bound, as proposed by Jara-Diaz et al. 2008.

However, this theoretical literature cannot explain explicitly the empirical findings that show differences in the value of time between individuals with similar resources (for example, similar income or job type) but with differences such as age, housing type or location, gender, marital status, car ownership or number and age of children (see for example, Jara-Díaz et al., 2012).

In this paper, we explore the implications of relaxing the assumption of a simultaneous choice-making process, introducing time scales in a dynamic (inter-temporal) choice of activities in the context of an otherwise similar utilitarian approach as that commonly used in the literature. This dynamic assumption can explain what in static models is represented by some exogenous parameters, represented in our approach as decision variables in previous periods, for example, the residential and job location decisions generate a value of the minimum travel time feasible, this parameter is usually used in static models of allocation of time (see Jara-Díaz and Guevara, 2003). A basic argument is that activities performed by an individual can be clearly differentiated by duration, i.e., the time window during which the choice lasts, and we assume this difference explains a structural dependency among activities and time values. We also propose that time windows are defined by the proportion of available resources (time and income) consumed or produced by the activity, in contrast to an arbitrary taxonomy. For example, we can easily observe that work and study involve time windows of years of commitment to a given choice and a major allocation of daily time and monetary resources, whereas other activities that involve less daily time, such as shopping or leisure activities, can be modified within a day or week and involve a comparatively small amount of time and income.

Such time-scale differences is a common feature in many physical and biological systems, which provide relevant knowledge of their dynamic structures and has been recently applied to study social and economic systems (see Gunderson and Holling, 2002). These systems dynamic are modeled assuming a hierarchy in the processes involved, with different time scales for each sub-system, which is the approach borrowed in this paper to model human activities. There are other features common to these systems, like stochastic shocks and memory effects, as well as conditions on the decisions sequence, all representing a potentially strong complexity in the dynamic process. However, despite their relevance to the individuals behavior, to keep this paper focused and simple they are not be included in our model; indeed, we seek to isolate the effect of introducing explicitly the temporal scale in the decision process.

Our model considers a hierarchical structure in the activity decision-making process based on the time windows in which choices are made, representing both the speed of decision changes and the amount of resources consumed. For simplicity, we concentrate on two temporal scales, named the macro and micro scale, which are considered sufficient to understand the basic effects that can later be extended to a multi-scale structure. Using this approach, we seek to analyze the dynamics of an individual's behavior and, particularly, to study the effects that long-term choices may have on a set of short-term decisions and on the interpretation of the value of time derived from choices observed at the micro scale. An important assumption of this model is the timing or synchrony of decision making at the macro and micro levels. We assume that at the time of the beginning of each macro scale, called the adjusting point, all short-term and long-term decisions are adjustable. We also consider that individuals make decisions based on the current economic information in each given time window, i.e., myopic decisions, although the hierarchical approach can be extended using future expectations without fundamentally modifying our main conclusions.

With the microeconomic approach presented here, we indicate that the time value associated with micro-scale activities is naturally dependent on the set of choices made at the macro scale, including time and money expenditures in durables, as observed in the above-mentioned empirical studies. The macro-micro bidirectional dependency among activities is not explicit in previous microeconomic models, where all activities belong to the same hierarchy level. Thus, the time-hierarchical model provides a more comprehensive choice-making framework, where long and short-term choices, or all choices in life, become theoretically integrated in a dynamic approach. A practical conclusion for transport studies follows: the value of time estimated from the standard econometric calibration of transport mode choice models is bound to be different according to the individuals choice of durables (housing, car(s), education level, job) and the location of macro-scale activities, such as work and study; the implication is that there is a potential bias in the estimation of time values in the applied research. We also conclude on how to specify short-term utility functions based on the conditionality of durables and long-term assignments of time and money budgets.

The essential aim of the paper is to improve the understanding of individuals behavior in the urban context, where individuals face a highly complex set of interconnected decisions. Considering this complexity, we question the assumptions of the standard optimization paradigm adopted from the microeconomic literature and introduce a fundamental structure of the decision process, the temporal structure. In this paper, we particularly concentrate on the effect of this approach on understanding time values. Our aim is to develop a better model of urban agents. We also derive and discuss the practical implications of our model for applications .

2.2. Microeconomic hierarchical model of consumer

The proposed model considers the hierarchy of activities depending on the frequency of changes in consumption of both continuous and discrete goods over a specific horizon of time. To analyze this hierarchy, the feasible space of activity choices is divided in two scales, called macro scale (long-term activities) and micro scale (short-term activities). The macro scale choices are exemplified by work and study, which after defining their basic contract conditions (salary or fees, duration and location) they remain essentially fixed during the macro period of time. Thus, the model proposed in this paper extends the modeling of Jara-Díaz et al. (2008) where the authors assume that long-term decisions are those that agents set to an exogenous minimum value, using a static microeconomic model. The micro scale includes decisions that can be adjusted frequently (e.g. daily), exemplified by leisure and shopping. Macro decisions generate expenses or income and committed in time for activities, these parameters affect micro decisions. The individual's choice on the location of activities are assumed to be decided at the corresponding scale of activity, but decisions regarding transport mode is taken in the micro scale, irrespective of the activity to which the consumer is travelling. Activities are defined as self-contained, meaning that activities do not share any action, goods or time, and are interdependent, meaning that activities may need resources acquired on other activities to be feasible. At each temporal scale consumers make choices assuming that all exogenous variables, like prices, are constant during the time window; hence, the longer the time window the stronger the price prediction the consumer has to make.

The model also assumes that some goods obtained at the macro scale are durable in nature, noting that such goods in economic theory are defined as those goods (or services) that once acquired can be used many times over time, also known as reusable assets (Sullivan and Stevens, 2003). In addition, their characteristics may affect the level of consumption at the micro scale. The set of decisions taken by the agent in short and long terms are based on his/her information at the beginning of the respective time window. The model developed in this paper is deterministic and continuous, but can easily be extended to discrete choices, such as residential location of activities, see Pérez et al. (2003) or transport mode choice (Train and McFadden, 1978), as shown in the simulation example below. In the proposed model, we assume that the decisions are based on the individual's current preferences only, ignoring -for simplicity- other effects as memory and learning in consumption, or the temporal dependence of decisions across different time windows and scales.

2.2.1. Notation

Relevant sets and indices:

- Δh: Duration of a micro time window; one day by default. All activities, macro or micro, and their consumption goods and durations, are defined in a common "time unit"Δh, irrespective of the temporal scale. The one day time window is not arbitrary because it is the biological cycle where activities are scheduled. A week is also considered a plausible time window, although it is also organized as a set of days with different activities.

- $V\Delta h$: Duration of a macro time window, e.g. one to five years. V is the number of micro time windows in the macro window.
- J : Sets of time scales. In the two scales model $J = \{j_1 = m, j_2 = M\}$, with m denoting micro and M denoting macro scales. In addition, Ω^j and Λ^j are the sets of feasible activities and consumption goods, respectively, associated with scale j. $\Omega^M \cap \Omega^m = \emptyset$; $\Lambda^M \cap \Lambda^m = \emptyset$.
- i, k: Indices for activities and consumption goods, respectively.
- n, v : Time windows indices denoting the v^{th} micro time window in the n^{th} macro time window; $v = 1, \dots, V$; $n = 1, \dots, N$. Note that a double index for time is needed to differentiate between macro and micro time windows.

Parameters:

- $r_i^{n,v}$: Income ($r_i^{n,v} \geq 0$) or cost ($r_i^{n,v} < 0$), per time unit obtained in *micro* activity i in the micro time window (n, v); $i \in \Omega^m$, vector $r^{n,v} = (r_i^{n,v}, i \in \Omega^m)$.
- $R_i^{n,v}$: Income ($R_i^{n,v} \geq 0$) or cost ($R_i^{n,v} < 0$), per time unit obtained in *macro* activity i in the micro time window (n, v); $i \in \Omega^M$, vector $R^{n,v} = (R_i^{n,v}, i \in \Omega^M)$.
- $p_k^{n,v}$, $P_k^{n,v}$: Unitary price of good k, micro and macro level, respectively, in n, v; $k \in \Lambda^m \cup \Lambda^M$, vectors $p^{n,v} = (p_k^{n,v}, k \in \Lambda^m)$; $P^{n,v} = (P_k^{n,v}, k \in \Lambda^M)$
- $I^{n,v}$: Exogenous income obtained in (n, v) from real estate rents, capital investments in previous periods, bank loans, inheritance, etc.
- $C^{n,v}$: Adjustment costs associated with transactions of durable goods.

Decision Variables:
The classical decision variables are the allocation of time and the consumption of goods in each activity. The new variables in this model are the time and wealth surpluses, which are transferred from the macro scale to the micro scale; these are called dynamic variables.
Decision variables in the direct utility specification:

- $X_k^{n,v}$, $x_k^{n,v}$ Consumption of good k (macro and micro, respectively) -per unit of time Δh- that belongs to the micro time window (n, v); .
- $T_i^{n,v}$, $t_i^{n,v}$: Time allocated to activity i (macro or micro, respectively) in (n, v); $i \in \Omega^M \cup \Omega^m$, vectors $t^{n,v} \equiv (t_i^{n,v}, i \in \Omega^m)$, $T^{n,v} \equiv (T_i^{n,v}, i \in \Omega^M)$.

Dynamic Variables (macro-micro transference):

- $S^{n,v}$: Surplus of wealth -per time unit-, which is transferred from the macro scale in time period (n, v) to the micro time window v; ($S^{n,v} \in \mathbb{R}$) with $S^{n,v}$ a debt (negative) or a saving (positive).
- τ^n Time saved from macro window n to all micro scale activities.

Note that the model parameters, such as prices, and variables have a double time superscript (n, v), which is important to denote the macro-scale window and the specific micro-scale window within the macro scale, assuming that the micro-scale superscript v is reset at the beginning of each macro-time window n.

2.2.2. Minimum or maximum time constraints

Within each time window, following Jara-Díaz (2003), we assume that the decision of time assigned to certain activity depends on the level of consumption of goods. In the macro scale, the decision depends on the consumption goods at that scale only, but at the micro scale it depends on the choices of consumption at the macro as well as the micro scale. This assumption reflects that some micro scale activities require inputs decided in the long term (infrastructure, durables). For example, a car is a durable that is an input to choose this mode on every activity; housing is another durable whose type (floor space, land lot size and building quality) affects micro scale choices like leisure time. Let us define the following technological constraints associated with the macro scale:

$$g_i^n(T_i^{n,v}, X^{n,v}) \geq 0,\ i \in \Omega^M$$

The times and goods allocated to each macro activity within a time window are mutually dependent; for example, the time allocated to work is limited by the consumption of durables, such as car ownership, residential and job location, etc. We consider particularly the two different forms proposed by De Serpa (1971) and extended by Jara-Díaz (2003):
$\underline{D}_i^n(X^{n,v}) \leq T_i^{n,v} \leq \overline{D}_i^n(X^{n,v}),\ i \in \Omega^M$: The maximum/minimum feasible time for the realization of an activity is constrained by the quantity of consumption goods purchased on the same macro scale time window n. Note that, the macro technological constraints are fixed for the entire time window n.

With regard to the micro scale, we identify the following types of constraints,

$$g_i^{n,v}(t_i^{n,v}, x^{n,v}; X^{n,v}) \geq 0,\ i \in \Omega^m$$

; these constraints relate the time assigned to a short-term activity at the micro time scale, with both micro and macro consumption decisions. As described for the macro case, this constraint can be, for example:

$$\underline{d}_i^{n,v}(x^{n,v}, X^{n,v}) \leq t_i^{n,v} \leq \overline{d}_i^{n,v}(x^{n,v}, X^{n,v})$$

That is, in both spaces, of micro and macro decisions, the time allocated to any activity may be jointly bounded from above and from below by the resources obtained or by endogenous and exogenous constraints. For instance, the technology and physical equipment at home generate different bounds on the time allocation for home activities. The specific features of the job and home location generate more or less options to assign time to leisure as well as availability of transport modes. Notice that v is an indicator of the deterioration of the durable goods X, and therefore, such technological constraints at the micro level varies over time.

2.2.3. Optimization model

In this section we propose the two levels microeconomic model of consumers' behavior defined on the same time window (n, v), for macro or long-term and micro or short-term activities. Long term decisions are made at the beginning of each macro time window (n), which is micro time window $(n, 1)$, along with all micro decisions associated with this micro time window. In subsequent micro time windows, the consumer adjusts his/her short term decisions conditional on macro choices (durables).The set of optimal choices made at the macro level in $(n,1)$ is yield by the following:

$$\max_{T,X} U_M^{n,v}(X,T), \text{ Subject to: } F_M^{n,v} \equiv \left\{ \begin{array}{c} R^{n,v}T^{n,v} - P^{n,v}X^{n,v} - C^{n,v} + I^{n,v} - S^{n,v} = 0 \\ \sum_{i\in\Omega^M} T_i^{n,v} = \Delta h - \tau^n \\ \underline{D}^n(X^{n,v}) \leq T^{n,v} \leq \overline{D}^n(X^{n,v}) \end{array} \right\} \tag{2.1}$$

In this case $v=1$; however, we use a general notation because $F_M^{n,v}$ in (2.1) is used later for any v.

Similarly, the set of optimal short-term choices at any micro time window (n,v) is the solution of:

$$\max_{t,x} U_m^{n,v}(x,t), \text{ Subject to: } F_m^{n,v} \equiv \left\{ \begin{array}{c} r^{n,v}t^{n,v} - p^{n,v}x^{n,v} + S^{n,v} = 0 \\ \sum_{i\in\Omega^m} t_i^{n,v} = \tau^n \\ \underline{d}^{n,v}(x^{n,v}, X^{n,v}) \leq t^{n,v} \leq \overline{d}^{n,v}(x^{n,v}, X^{n,v}) \end{array} \right\} \tag{2.2}$$

Following De Serpa (1971) and Jara-Díaz (2003), problems (2.1) and (2.2) assume that utility depends on goods and time, and the feasible set is defined by income and time budgets, in addition to technological constraints. These problems are interdependent by the set of savings in money ($S^{n,v}$) and time (τ^n) at the macro level problem (2.1), which defines budget constraints in the micro level problem (2.2). Additionally, macro decisions imply consumption in durables that modifies the set of feasible micro scale options. The macro income constraint contains a fixed adjustment cost associated with the transaction of durables, denoted as cost $C^{n,v}$. Note that even though the adjustment costs only affect the consumption of durables variation, the technological relation between goods and time, makes that these costs have an indirect effect on time allocation in long-term activities. At the micro scale problem (2.2) we assume that income is exhausted at every period v, i.e micro intertemporal savings are endogenous decisions, as is assumed in Juster (1990). This assumption simplifies the consumers behavior model in order to focus on the hierarchical structure of the decision process. However, an extension introducing intertemporal savings to this model, would yield a dynamic model. The microeconomic problems (2.1) and (2.2) define two different dynamics in the choice making process.

- *Vertical dynamic (hierarchical)*: this dynamic is analyzed through the time (τ_n) and monetary ($S_{n,v}$) budget transferences between the different decision scales, and trough the effects of durables in micro scale choices.

- *Horizontal dynamic (temporal)*: this dynamic is explained by the temporal variation of the parameters associated with the economy over time, together with the variation in the perception of the durables $X^{n,v}$. An example is the deterioration of the durables that may be modeled by means of $U(X^{n,v}) = \sigma U(X^{n,v-1})$ where $\sigma < 1$; a different example are changes in specific features of the durables, such as residential location externalities. Some of these dynamics described above are analyzed conceptually in Juster (1990).

2.2.4. Hierarchical choice

In this section, we show the process of making decisions based on a hierarchical structure starting with the macro scale choices. At every time window $(n,1)$ the individual adjusts her/his long-term decisions; we call this the adjusting point. At this point the individual is assumed to solve both problems simultaneously, macro and micro together, including their aversion to risk here encapsulated in parameter α. Note that the simultaneous macro-micro choice allows the influence of the micro scale on the macro scale because at the adjusting point all choices are made simultaneously, thus, at this point in time, short term choices directly affect long term ones, and vice versa.

Then, at every adjusting point $(n,1)$ the individual solves the following:

$$\max_{X,T,x,t} \alpha U_M^{n,1}(X^{n,1},T^{n,1}) + (1-\alpha)U_m^{n,1}(x^{n,1},t^{n,1}), \text{ subject to : } F_M^{n,1} \cup F_m^{n,1} \tag{2.3}$$

The set $F_M^{n,1} \cup F_m^{n,1}$ is the simultaneous macro-micro feasible set at $(n,1)$. In problem (2.3) micro choices affect macro decisions. For example, the residential location is a macro choice made based on the environmental amenities, but also with regards to accessibility to micro choice activities (shopping, social activities, interaction with other agents, etc.). In the set $F_M^{n,1} \cup F_m^{n,1}$ the income and time constraints are added, and therefore, the transferences of budget ($S^{n,v}$) as well as time (τ^n) make no sense at the adjusting point because the hierarchical structure is lost; analytically we have the budget constraints $R^{n,1}T^{n,1} - P^{n,1}X^{n,1} - C^{n,1} + I^{n,1} + r^{n,1}t^{n,1} - p^{n,1}x^{n,1} = 0$ and $\sum_{i\in\Omega^M} T_i^{n,1} + \sum_{i\in\Omega^m} t_i^{n,1} = \Delta h$. In addition, the future-present valuation of utilities generates a trade-off parametrically controlled by the discount rate $\alpha \in [0,1]$, such that if $\alpha \longrightarrow 1$ represents a long-term planner individual and $\alpha \longrightarrow 0$ represents preference for short-term benefits only. In addition, we also consider that the individual makes myopic decisions based on the current economic information at each given time window; this assumption does not affect our main conclusions. Solving the simultaneous problem (2.3) the individual gets the following demands for durable goods and long-term time allocated to activities, conditional on the parameters of the optimization problem, including Δh:

$$X^{n,1*}(\xi_M^{n,1}, \xi_m^{n,1}, \Delta h), \quad T^{n,1*}(\xi_M^{n,1}, \xi_m^{n,1}, \Delta h), \tag{2.4}$$

where $\xi_M^{n,v} \equiv (R^{n,v}, P^{n,v}, I_C^{n,v}, \theta_M^{n,v}, \alpha)$, $\xi_m^{n,v} \equiv (r^{n,v}, p^{n,v}, \theta_M^{n,v})$, $I_C^{n,v} = I^{n,v} - C^{n,v}$ and $\theta_M^{n,v}$ are the parameters of technological constraints in (n,v).

The set of long-term choices defined in (2.4) at $(n,1)$ remain fixed for all (n,v), i.e. $X^{n,v} = X^{n,1*}$, $T^{n,v} = T^{n,1*}$, $\forall v$ denoted as X^n and T^n and macro-scale decisions depend on the intertemporal parameter α. However, it is worth noting that the utility obtained from this set changes along the macro time window because exogenous parameters of the economy (prices, income, technological constraints) change, as well as the individual's perceptions. In addition, the deterioration of durables generates variations in the utility perceived by the agents at each micro-time period.

Long-term commitments (durable goods, fixed-time activities) define savings, e.g. money transfer (cost / revenue committed) and time transfer for short-term decisions, as follows:

$$S^{n,v} = R^{n,v}T^{n,1*} - P^{n,v}X^{n,1*} + I_C^{n,v} \tag{2.5}$$

and

$$\tau^n = \Delta h - \sum_{i\in\Omega^M} T_i^{n,1*} \tag{2.6}$$

When $v > 1$, then $I_C^{n,v} = I^{n,v}$ because the adjustment costs are zero, and therefore they are not considered in the computation of $S^{n,v}$. Notice that while τ^n is constant for all time windows v

in (n,v), the money saving for every v also changes at every time window v following variations in the exogenous parameters of the economy. Formally,

$$S^{n,v}(\xi_M^{n,v}|\xi_M^{n,1},\xi_m^{n,1},\Delta h) \tag{2.7}$$

On the other hand, we have the following decision process at the micro level. After solving the optimization macro problem (2.3) in the adjusting point $(n,1)$, the individual makes short-term decisions in every time window (n,v) for $v \neq 1$. This is modeled solving micro problem (2.2), assuming that the macro decisions are fixed, so that the solution to this problem generates the following optimal choice set:

$$x^{n,v*}(\xi_m^{n,v},S^{n,v},\tau^n,X^n),\;\; t^{n,v*}(\xi_m^{n,v},S^{n,v},\tau^n,X^n) \tag{2.8}$$

From equation (2.8) it is clear that micro activities change along the duration of the macro time window due to changes in the economy, and those decisions such as location of work, home, time allocated to work and consumption of durables, defines the dynamics of short-term decisions. In addition, changes in income, real estate rents and other long-term monetary assets also generate differences in short-term decisions. Additionally, in (2.8) the intertemporal parameter α implicitly affects the micro-scale consumption and time through the macro consumption X^n. Note that at the micro level, for all $i \in \Omega^m$ we have that $t_i^{n,v}(\xi_m^{n,v},S^{n,v},\tau^n,X^n)$, respectively for all $k \in \Lambda^m$ we have that $x_k^{n,v}(\xi_m^{n,v},S^{n,v},\tau^n,X^n)$, are increasing functions with decreasing rates in $S^{n,v}$ and τ^n:

$$\frac{\partial t_i^{n,v}}{\partial S^{n,v}} \geq 0,\;\; \frac{\partial^2 t_i^{n,v}}{\partial S^{n,v^2}} \leq 0,\;\; \frac{\partial t_i^{n,v}}{\partial \tau^n} \geq 0,\;\; \frac{\partial^2 t_i^{n,v}}{\partial \tau^{n^2}} \leq 0,\;\; \forall i \in \Omega^m \tag{2.9}$$

$$\frac{\partial x_k^{n,v}}{\partial S^{n,v}} \geq 0,\;\; \frac{\partial^2 x_k^{n,v}}{\partial S^{n,v^2}} \leq 0,\;\; \frac{\partial x_k^{n,v}}{\partial \tau^n} \geq 0,\;\; \frac{\partial^2 x_k^{n,v}}{\partial \tau^{n^2}} \leq 0,\;\; \forall k \in \Lambda^m \tag{2.10}$$

which follows directly from observing that $S^{n,v}$ and τ^n are constraints in problem (2.2).

This two levels (macro-micro) hierarchical process can be extended to a multilevel process. At the adjusting point of each level, the individual optimizes choices of all lower levels, i.e. all levels with shorter time windows, and defines the initial time and money savings for these lower levels. Notice that at the adjusting point, changes on macro scale activities represents shocks in the micro scale choice process.

2.2.5. Conditional indirect utility functions

From problem (2.1) we now define the macro indirect utility function evaluated at any time window (n,v). This utility is yield by the optimal consumption set on goods and time decided at $(n,1)$, denoted (X^*,T^*), and given by $V_M^{n,v} = U_M^{n,v}(X_k^{n,1*},T_i^{n,1*})$; replacing optimal goods and time given at equation (2.4) yields $V_M^{n,v} \equiv V_M^{n,v}(\xi_M^{n,1},\xi_m^{n,1},\Delta h)$.
Observe that, although the optimal set (X^*,T^*) remains fixed for all v in macro period n, the macro scale utility may change along this period if the consumer's perceptions change; hence $V_M^{n,1}$ is not necessarily equal to $V_M^{n,v},\; v>1$. Secondly, observe that, except from variable perceptions, the macro scale utility remains fixed depending only on the economy at the adjusting point $(n,1)$.

Similarly, we define the micro indirect utility function for the micro time window (n,v) as $V_m^{n,v} = U_m^{n,v}(x_k^{n,v*},t_i^{n,v*})$ which becomes $V_m^{n,v} \equiv V_m^{n,v}(\xi_m^{n,v},S^{n,v},\tau^n,X^n)$. Note that $V_m^{n,v}$ makes

clear how the utility of micro decisions is conditional on the decisions taken at the macro level, by way of the availability of durables, fixed-time to long-term activities and monetary savings. Additionally, the indirect utility is conditional on the parameters of the current economy at (n, v).
The conditional indirect micro-level utility function increases monotonically with time and money budgets. Thus, we have the following important properties for $v > 1$ of the dynamics of the following key optimization parameters (see proof in Jara-Díaz and Gschwender, 2008):

$$\lambda^{n,v}(S^{n,v}) = \frac{\partial V_m^{n,v}}{\partial S^{n,v}} \geq 0, \ \ \frac{\partial \lambda^{n,v}}{\partial S^{n,v}} = \frac{\partial^2 V_m^{n,v}}{\partial S^{n,v^2}} \leq 0 \tag{2.11}$$

$$\mu^{n,v}(\tau^n) = \frac{\partial V_m^{n,v}}{\partial \tau^n} \geq 0, \ \ \frac{\partial \mu^{n,v}}{\partial \tau^n} = \frac{\partial^2 V_m^{n,v}}{\partial \tau^{n^2}} \leq 0 \tag{2.12}$$

where $\lambda^{n,v}$, $\mu^{n,v}$ are the Lagrange multipliers associated with the income and time constraints at period (n, v). The conditions (2.11) and (2.12) indicate that if micro scale income $(S^{n,v})$ or the micro scale time (τ^n) budgets grow, then the marginal utility -of income or time- decreases. This result becomes important below because it is a support to analyze the sub-optimality in the transfer of macro-micro resources and the sub-optimality of long-term and short time decisions.

In order to estimate econometrically the utility functions $V_M^{n,v}$ and $V_m^{n,v}$, a similar approach to what is proposed in Bhat (2000) can be envisaged, where micro-scale utilities are conditioned on macro-scale decisions (i.e., on the set of macro decisions), thus making estimates of $V_m^{n,v}$ and the values of time dependent on the set of macro-scale decisions (e.g., car ownership, residential location, education).

On the other hand, the dynamic nature of our model includes savings on money, time available and variations in economy, which affects the short term utility maximization process. Then, the required data for utility functions' estimation in several periods of time should also consider longitudinal information of a selected group of people observing how the aforementioned factors change during certain time period, and also taking records of how the selected individuals reallocate time for his(her) short as well as long term activities. We also conclude that the clustering process used in current econometric methods is supported by our multi-scales theory, and, moreover, it helps identify the parameters that control the dynamic process.

Note that the formulation and calculation of $S^{n,v}$ as well as τ^n propose an extension to the definition of resources commitment (expense $E_c^{n,v}$ and time $T_c^{n,v}$ initially defined in Jara-Diaz and Guerra, 2003, and Jara-Díaz 2007, later used in other econometric works. The committed expenses $(E_c^{n,v})$ and time $(T_c^{n,v})$ are obtained of a particular microeconomic model with exogenous technological constraints of the form $X_k^{n,v} \geq X_{k,min}^{n,v}$, $T_i^{n,v} \geq T_{i,min}^{n,v}$, $x_k^{n,v} \geq x_{k,min}^{n,v}$ and $t_i^{n,v} \geq t_{i,min}^{n,v}$. In particular, $E_c^{n,v}$ and $T_c^{n,v}$ are defined as the resources -in money and time- generated by the decisions that agents sets to an exogenous minimum value. Compared to our model, such committed resources may be equal to $S^{n,v}$ and τ^n only in some cases, but from an analytical standpoint they are different. First, while $E_c^{n,v}$ and $T_c^{n,v}$ are based on very specific technological constraints that only consider exogenous minimum time and consumption for activities and goods, respectively, in our hierarchical model the values $S^{n,v}$ and τ^n are obtained endogenously from a more general formulation of macro scale decisions, including possible interactions between constraints on time, consumption and technology. In our model, τ^n can include activities whose time is not set to a minimum ($T_{i,min}^{n,v}$ or $t_{i,min}^{n,v}$). Additionally, even in activities where it is generally assumed that individuals allocate minimum time, such as home repairs, in our model the requirements associated with these activities vary continuously in time, therefore,

$t_{i,min}^{n,v}$ also varies between micro time windows. Finally, given the simultaneity of the optimization consumer problem used to find $E_c^{n,v}$ and $T_c^{n,v}$ they naturally depend on the economy at the time (n, v). Conversely, according to equations (2.5) and (2.6), the values of $S^{n,v}$ and τ^n depend on the parameters of the economy and technological constraints at the adjusting point $(n, 1)$, and on some other parameters (e.g. prices) at every time window.

2.3. The value of time

We now analyze the expression for the value of time, following Jara-Díaz (2003) and De Serpa (1971) to study the dynamics of such measure and the influence of the macro decisions on micro activities. We also study the willingness to pay for allocating either extra or less time to specific activities.

The subjective value of time (VT), as a resource, by definition represents the variation of utility at the margin caused by an increment in time compared with the same variation caused by an increment in income. This marginal behavior be identified at the micro scale and calculated directly from (2.11) and (2.12) as:

$$VT^{n,v} = \frac{\lambda^{n,v}}{\mu^{n,v}} = \frac{\frac{\partial V_m^{n,v}}{\partial \tau^n}}{\frac{\partial V_m^{n,v}}{\partial S^{n,v}}}, \quad \forall v \tag{2.13}$$

We assume that the value of $\lambda^{n,v}$ is nonzero, i.e. the individual exhausts the money budget for all v. Additionally, the value of $VT^{n,v}$ may vary along micro time intervals, because the economy and perceptions change; in particular the money transference may change due to price changes modifying the money budget along time. The VT calculated at any time window $v > 1$ is a micro scale value conditional on macro scale decisions, which implies that equal individuals that make different long term choices have different VT. This further implies that a proper definition of clusters of agents in studies of consumers' behavior in short-term activities should include, in addition to classical socioeconomic variables, the most important long term choices, such as location of residence and work, and car ownership.

We now consider the values of time at specific activities subject to technological constraints. Let us define $\kappa_{i(-)}^{n,v}$, $\kappa_{i(+)}^{n,v}$ as the Lagrange multipliers associated with constraints of minimum or maximum time allocated to activities, all with respect to micro activity i in time window (n, v).

We can calculate the value of time at each point in time as a dynamic process; however, it is sufficient to analyze two characteristic points: any time point $v > 1$ (the general case) and the adjusting point $v = 1$.

2.3.1. General case ($v > 1$)

Assume that the value of $\lambda^{n,v}$ is nonzero, i.e. the individual exhausts the money budget. The first order conditions for the micro scale consumption of good $k \in \Lambda^m$ in the time window (n, v) are:

$$\frac{\frac{\partial U_m^{n,v}}{\partial x_k^{n,v}}}{\lambda^{n,v}} - p_k^{n,v} + \sum_{i \in \Omega^m} \left\{ \frac{\kappa_{i(+)}^{n,v}}{\lambda^{n,v}} \frac{\partial \overline{d}_i^{n,v}(x^{n,v}, X^n)}{\partial x_k^{n,v}} - \frac{\kappa_{i(-)}^{n,v}}{\lambda^{n,v}} \frac{\partial \underline{d}_i^{n,v}(x^{n,v}, X^n)}{\partial x_k^{n,v}} \right\} = 0, \ \ \forall k \in \Lambda^m \tag{2.14}$$

This equation (2.14) makes evident that that the optimal consumption of goods at the micro level is conditional on the consumption decisions at the macro level. Additionally, we derive micro scale value of time as:

$$\frac{\mu^{n,v}}{\lambda^{n,v}} = \frac{\frac{\partial U_m^{n,v}}{\partial t_i^{n,v}}}{\lambda^{n,v}} + r_i^{n,v} + \frac{\kappa_{i(-)}^{n,v}}{\lambda^{n,v}} - \frac{\kappa_{i(+)}^{n,v}}{\lambda^{n,v}}; i \in \Omega^m;\ v = 2, \cdots, V \tag{2.15}$$

which depends on the value of the time allocated to activity i, the net income of the activity, and a factor that depends on the Lagrange multipliers of technological -upper and lower- constraints.

The calculation of the value of time implies the solutions of the system of equations (2.14) and (2.15) plus the constraints set $F_m^{n,v}$, which implies that the value of time implicitly depends on an individuals intertemporal valuation α (embedded in X^n, τ^n, and $S^{n,v}$, in (2.14) and (2.2)).

Note that, because we assume that technological constraints depends on the specific agent consumption of durables, then it follows that $\kappa_{i(-)}^{n,v}, \kappa_{i(+)}^{n,v}$ may vary across agents. For example, the macro scale decisions of residential and job location defines the minimum travel time. Thus, we conclude that it is likely to find individuals with identical conditions, i.e. the same wage rate r_i^n and the same valuation of the activity $\frac{1}{\lambda^{n,v}} \frac{\partial U_m^{n,v}}{\partial t_i^{n,v}}$, but with different value of time as a short-term resource.

Note also that from the complementary slackness conditions on the technological constraints we also have $\forall i \in \Omega^m$:

$$\begin{aligned} \kappa_{i(-)}^{n,v} \times (t_i^{n,v} - \underline{d}_i^{n,v}(x^{n,v}, X^n)) &= 0, \\ \kappa_{i(+)}^{n,v} \times (\overline{d}_i^{n,v}(x^{n,v}, X^n) - t_i^{n,v}) &= 0, \\ \kappa_{i(+)}^{n,v} \times \kappa_{i(-)}^{n,v} &= 0, \end{aligned} \tag{2.16}$$

The set of conditions of (2.16) yields two interesting cases.

1. *Lower bounded time*: $t_i^{n,v} - \underline{d}_i^{n,v}(x^{n,v}, X^n)) = 0,\ \kappa_{i(-)}^{n,v} > 0,\ \kappa_{i(+)}^{n,v} = 0$

   That is, the lower bound constraint for the time allocated to an activity is saturated in the micro time window (n, v) indicating that the individual assigns the minimum time possible. In this way, we obtain expression (2.17) for the individual's marginal willingness to pay for a marginal reduction the time spent in activity i:

$$\frac{\kappa_{i(-)}^{n,v}}{\lambda^{n,v}} = \frac{\mu^{n,v}}{\lambda^{n,v}} - \frac{\partial U_m^{n,v}/\partial t_i^{n,v}}{\lambda^{n,v}} - r_i^{n,v}; \tag{2.17}$$

   Note that in the literature $\kappa_{i(-)}^{n,v} > 0$, is defined as representing an unpleasant or mandatory activity, or are the activities of lower marginal utility, because the time assigned is the minimum amount.

2. *Upper bounded time:* $\overline{d}_i^{n,v}(x^{n,v}, X^n) - t_i^{n,v} = 0,\ \kappa_{i(+)}^{n,v} > 0,\ \kappa_{i(-)}^{n,v} = 0.$

   In this case the upper bound constraint ($\overline{d}_i^{n,v}(x^{n,v}, X^n) \geq t_i^{n,v}$) is active, then

$$\frac{\kappa_{i(+)}^{n,v}}{\lambda^{n,v}} = \frac{\partial U_m^{n,v}/\partial t_i^{n,v}}{\lambda^{n,v}} + r_i^{n,v} - \frac{\mu^{n,v}}{\lambda^{n,v}}, \tag{2.18}$$

   which represents the individual's willingness to pay for a marginal increase in the time spent in the activity above the maximum capacity.

In the literature, leisure is defined as an unbounded activity ($\kappa_{i(-)}^{n,v} = 0$). If we further assume that $\kappa_{i(+)}^{n,v} \geq 0$ then

$$\frac{\partial U_m^{n,v}/\partial t_i^{n,v}}{\lambda^{n,v}} \geq \frac{\mu^{n,v}}{\lambda^{n,v}} - r_i^{n,v}$$

which states that the marginal valuation of the individual's time allocated to a leisure activity can be higher than the cost per unit time ($r_i^{n,v} < 0$) plus the time value as a resource for micro window (n, v) (see for example Jara-Díaz, 2003).

The results (2.15) to (2.19) are consistent with those found by De Serpa (1971) and followers. The contribution of the hierarchical model is that values of time at the micro scale are dependent on the long-term choices made by the individual at $(n, 1)$, including the location and the time allocated to home, work or study and the investments in durables such as a car. This result is for two main reasons: first, by the formulation of S^n and τ^n (see equation); and second, by the technological relationship between short-term and long-term decisions (see equation). This result is supported econometrically and theoretically from previous works such as Jara-Díaz et al. (2008), Juster (1990) and others.

In this way, some results, descriptively introduced by Juster (1990), are formalized here with the hierarchical model that considers the influence of these durable and capital goods in the allocation and time valuation. Moreover, the micro-scale value of time changes within a macro-time window due to a change in money transfers caused by changes in exogenous prices and changes in the valuation of durables. This shows that the value of time is a dynamic and continuous measure, dependent on each individual's environment and can also vary according to the time point where it is computed, supporting the results of other works as Winston (1987). Thus, hierarchical model could define a temporal dynamic for the expense committed defined in Jara-Díaz et al. (2008) at a particular static microeconomic model.

2.3.2. Case of the adjusting point

At the adjusting point $(n, 1)$ the macro and micro activities are optimized simultaneously. In this case the first order conditions for the optimal micro scale consumption $k \in \Lambda^m$ are as follows:

$$(1-\alpha)\frac{\frac{\partial U_m^{n,1}}{\partial x_k^{n,1}}}{\lambda^{n,1}} - p_k^{n,1} + \sum_{i\in\Omega^m}\left\{\frac{\kappa_{i(+)}^{n,1}}{\lambda^{n,1}}\frac{\partial \overline{d}_i^{n,1}(x^{n,1}, X^n)}{\partial x_k^{n,1}} - \frac{\kappa_{i(-)}^{n,1}}{\lambda^{n,1}}\frac{\partial \underline{d}_i^{n,1}(x^{n,1}, X^n)}{\partial x_k^{n,1}}\right\} = 0, \ \ \forall k \in \Lambda^m, \tag{2.19}$$

where $\lambda^{n,1} = \alpha\dfrac{\partial V_M^{n,1}}{\partial I_C^{n,1}} + (1-\alpha)\dfrac{\partial V_m^{n,1}}{\partial I_C^{n,1}}$ and $I_C^{n,1} = I^{n,1} - C^{n,1}$.

And the first order conditions for the optimal macro scale consumption $k \in \Lambda^M$ are as follows:

$$\alpha\frac{\frac{\partial U_M^{n,1}}{\partial X_k^n}}{\lambda^{n,1}} - P_k^{n,1} + \sum_{i\in\Omega^m}\left\{\frac{\kappa_{i(+)}^{n,1}}{\lambda^{n,1}}\frac{\partial \overline{d}_i^{n,1}(x^{n,1}, X^n)}{\partial X_k^n} - \frac{\kappa_{i(-)}^{n,1}}{\lambda^{n,1}}\frac{\underline{d}_i^{n,1}(x^{n,1}, X^n)}{\partial X_k^n}\right\} +$$
$$\sum_{i\in\Omega^M}\left\{\frac{K_{i(+)}^{n,1}}{\lambda^{n,1}}\frac{\partial \overline{D}_i^n(x^{n,1}, X^n)}{\partial X_k^n} - \frac{K_{i(-)}^{n,1}}{\lambda^{n,1}}\frac{\partial \underline{D}_i^n(x^{n,1}, X^n)}{\partial X_k^n}\right\} = 0, \ \ \forall k \in \Lambda^M, \tag{2.20}$$

where $K^{n,1}_{i(-)}$, $K^{n,1}_{i(+)}$ are the Lagrange multipliers associated with minimum and maximum time allocated to macro activity i in time window $(n,1)$. Equation (2.20) shows the marginal effect of macro consumption decisions on short and long term choices.

The first order conditions associated with time allocated to micro activity $i \in \Omega^m$ are

$$\frac{\mu^{n,1}}{\lambda^{n,1}} = (1-\alpha)\frac{\frac{\partial U^{n,1}_m}{\partial t^{n,1}_i}}{\lambda^{n,1}} + r^{n,1}_i + \frac{\kappa^{n,1}_{i(-)}}{\lambda^{n,1}} - \frac{\kappa^{n,1}_{i(+)}}{\lambda^{n,1}};\ i \in \Omega^m. \tag{2.21}$$

Analogously, the first order conditions associated with time allocated to micro activity $i \in \Omega^M$ are

$$\frac{\mu^{n,1}}{\lambda^{n,1}} = \alpha\frac{\frac{\partial U^{n,1}_M}{\partial T^{n,1}_i}}{\lambda^{n,1}} + R^{n,1}_i + \frac{K^{n,1}_{i(-)}}{\lambda^{n,1}} - \frac{K^{n,1}_{i(+)}}{\lambda^{n,1}};\ i \in \Omega^M. \tag{2.22}$$

The equations (2.21) and (2.22) are equivalent because at the adjusting point macro and micro activities are simultaneously optimized, then money and time budgets are common, implying that VT are equal for both time scales in this case. Hence, we define the long-term VT as one not constrained by longer-term choices, which is revealed by the consumer's allocation of time at the adjusting point, i.e. when the optimal allocation of time of macro and micro activities is made. Moreover, we conclude that the short-term VT differs from the long-term value. This statement is valid in the context of two time scales, micro and macro; it can be extended for more time scales obtaining values of time for each scale. We also note that the long-term value of time (2.22) remains constant along $v > 1$ while the short term values of time change at every v.

Given that α is a measure of the risk assumed by the individuals in long-term decisions, then equation (2.21) can be different for two agents making equivalent decisions associated with short-term time allocation. The difference comes from the term $(1-\alpha)\ \frac{\frac{\partial U^{n,v}_m}{\partial t^{n,v}_i}}{\lambda^{n,v}}$, which implies that if, for example, women are more risk averse than men, this will be reflected in different values of time, which is in fact reported by econometric experiments.

The willingness to pay (WP) for reducing the time allocated to a macro activity is observed at the adjusting point $(n,1)$ and given by:

$$WP^{n,1}_{i(-)} = \frac{K^{n,1}_{i(-)}}{\lambda^{n,1}} = \frac{\mu^{n,1}}{\lambda^{n,1}} - \alpha\frac{\frac{\partial U^{n,1}_M}{\partial T^{n,1}_i}(T^{1*}, X^{1*})}{\lambda^{n,1}} - R^{n,1}_i \tag{2.23}$$

Note that although for all v, the allocations of time and consumption durables for long-term activities are fixed (at values T^{1*}, X^{1*}), this willingness to pay $WP^{n,1}_{i(-)}$ can vary for $v > 1$, according to (23) which can be calculated as:

$$WP^{n,v}_{i(-)} = \frac{\mu^{n,v}}{\lambda^{n,v}} - \alpha\frac{\frac{\partial U^{n,v}_M}{\partial T^{n,v}_i}(T^{1*}, X^{1*})}{\lambda^{n,v}} - R^{n,v}_i \tag{2.24}$$

Observe that may be $WP^{n,v}_{i(-)} \neq WP^{n,1}_{i(-)}$ because the value of time $\frac{\mu^{n,v}}{\lambda^{n,v}}$, marginal utility yield by time and goods (T^{1*}, X^{1*}) and income $(R^{n,v}_i)$ may change along the macro time period.

Similarly, the willingness to pay for increasing the time of a macro *leisure* activity (such as child care at home) in the micro time window $(n, v\ )$, $v > 1$, is:

$$WP^{n,v}_{i(+)} = \alpha\frac{\frac{\partial U^{n,v}_M}{\partial T^{n,v}_i}(T^{1*}, X^{1*})}{\lambda^{n,v}} - \frac{\mu^{n,v}}{\lambda^{n,v}} + R^{n,v}_i \tag{2.25}$$

Hence, the contribution of the hierarchical model to the VT theory is twofold: one contribution is the calculation of the time value, which, in contrast to the static model, depends on long-term choices. Our theoretical construct explains the origin of the differences observed in time values by characteristics such as gender, residence, age, which, according to our model, are caused by long-term decisions that condition the shortterm choices where time values are observed. The second contribution is the calculation of the willingness to pay for specific activities, differentiating the activities by the time scale of the decisions and making the willingness to pay for short-term activities dependent on the long-term choices.
So far, we have assumed that the adjusting point is exogenous for the individual, i.e., the macro-time window is fixed. A different plausible assumption is that agents adjust macro decisions following changes in environmental and economic conditions, which could be introduced by an endogenous adjusting point based on a learning process with regards to the evolution of macro and micro utilities. For example, travel times and travel costs change over the macro-time window and can modify the work duration and the job location. Analogously to the literature on economic investments, immediately after the adjusting point, the endogenous model could assume that individuals incur costs associated with a change in durables (e.g., contracts); over time, these costs may increase or decrease, through the deterioration of durables or the capitalization of real estate, for example.

2.4. Hierarchical vs simultaneous decisions

We call simultaneous microeconomic problem, or "*sim*", the classic static consumer problem where all macro and micro variables are decided simultaneously without the associated adjustment costs. This choice process defines long-term optimal values for goods consumption and values of time at any micro period. Conversely, the hierarchical process only reveals long-term optimal choices and values at the adjusting point ($v = 1$), while at other time periods choices defines sub-optimal or short-term consumptions and time values. In this section, we analyze the difference between the hierarchical (*HP*) and simultaneous (SP) consumer's problems assuming exogenous adjusting point, particularly with regards to the sub-optimality associated with decisions made in the short-term conditional on long-term choices. We also ignore adjustments costs.

The formulation of the simultaneous problem at any time period (n, v) is:

$$\max_{X,T,x,t} \alpha U_M^{n,v}(X^{n,v}, T^{n,v}) + (1-\alpha) U_m^{n,v}(x^{n,v}, t^{n,v}), \text{ subject to : } F_M^{n,v} \cup F_m^{n,v} \tag{2.26}$$

And the correspondely indirect utility is $V_{sim}^{n,v} \equiv V_{sim}^{n,v}(\xi^{n,v}, \Delta h)$, where

$$\xi^{n,v} \equiv (R^{n,v}, r^{n,v}, P^{n,v}, p^{n,v}, I^{n,v}, \theta^{n,v}, \alpha),$$

which is the complete set of macro and micro parameters without adjustment costs. By definition the following condition holds for the simultaneous and hierarchical optimization problems with exogenous adjusting point:

$$V_{sim}^{n,v}(\xi^{n,v}, \Delta h) \geq \alpha V_M^{n,v}(\xi_M^{n,1}, \xi_m^{n,1}, \Delta h) + (1-\alpha) V_m^{n,v}(\xi_m^{n,v}, S^{n,v}, \tau^n, X^n, \Delta h), \tag{2.27}$$

where equality necessarily holds at the adjusting point ($v = 1$), while the inequality is expected to hold (though not necessarily) for $v > 1$.

The optimal expenditures of money and time in long-term activities in the simultaneous problem with no adjustment costs are:

$$S^{n,v}_{sim} = R^{n,v}T^{n,v*}_{sim} - P^{n,v}X^{n,v*}_{sim} + I^{n,v} \tag{2.28}$$

$$\tau^{n,v}_{sim} = \Delta h - \sum_{i\in\Omega^M} T^{n,v*}_{sim,i} \tag{2.29}$$

where, $T^{n,v*}_{sim}$ and $X^{n,v*}_{sim}$ are the solutions of the macro variables in the simultaneous problem. Now we can define a gap functions between optimal (simultaneous) and sub-optimal (hierarchical) micro scale expenditures or transfers:

$$GS^{n,v} = S^{n,v}_{sim}(\xi^{n,v}_M, \xi^{n,v}_m, \Delta h) - S^{n,v}(\xi^{n,1}_M, \xi^{n,1}_m, \Delta h), \tag{2.30}$$

$$G\tau^{n,v} = \tau^{n,v}_{sim}(\xi^{n,v}_M, \xi^{n,v}_m, \Delta h) - \tau^{n}(\xi^{n,1}_M, \xi^{n,1}_m, \Delta h), \tag{2.31}$$

A suboptimal choice set is characterized by not null GS or $G\tau$.

These gaps allow us to identify differences between the long-term valuation of time and the value of time for short term activities. These differences arise from comparing the results of the hierarchical model -where the time budget and technological constraints are given for $v > 1$ and are conditional on the long-term decisions-, with the results of the simultaneous model -where these constraints are global. We discuss two cases of income variability to show that the simultaneous problem may over or under estimate values of time. We consider only one macro time window so we drop the index n.

- **Case 1: Fixed income and overestimates:**

  The individual assumes long-term commitments X and faces a fixed income defined as $I \equiv Y * T^{min}_{work}$, where Y is the wage rate and T^{min}_{work} is the fixed working time assumed equal to minimum time. Additionally, all short-term decisions generate net costs ($r^v_i < 0$, for all $i \in \Omega^m$). Then:

  $$r^v t^v - p^v x^v < 0.$$

  In the hierarchical model $S^v = I^v - P^v X^v$ and $\tau^v = \Delta h - T^{min}_{work}$.

  Now we introduce shocks in macro scale prices P^v, e.g. land rents inducing higher expenditure in macro scale commitments, assuming they increase compared to the adjusting point (P^1), while micro scale prices remain fixed. Then, consumption in macro scale goods decreases in the simultaneous problem while remains fixed in the hierarchical problem: $X^v_{sim} \leq X^1$ and $P^v X^v_{sim} \leq P^v X^1$. Therefore, $S^v_{sim} \geq S^v$, while in this case $T^v_{sim,work} = T^v_{work}$ and $\tau^v_{sim} = \tau^1$. That is, the individual does not have the optimum transfer of money for micro activities in periods $v > 1$, then optimal micro consumption changes at every period. Additionally, marginal utilities of the micro problem $\lambda^v(S)$ and $\tau^v(\mu)$ are decreasing functions, (see equations (2.11) and (2.12)), which yields $\lambda^v(S^v_{sim}) \leq \lambda^v(S^v)$ and $\mu^v(\tau^v_{sim}) \geq \mu^v(\tau^1)$. This implies that

  $$VT^v_{sim} = \frac{\mu^v(\tau^v_{sim})}{\lambda^v(S^v_{sim})} \geq \frac{\mu^v(\tau^1)}{\lambda^v(S^v)} = VT^v$$

  Thus, we conclude that, in this case, the individual's long-term willing to pay for extra time in micro activities is larger than the value observed in the short-term and calculated by the hierarchical problem. However, keep in mind that behavior actually follows the micro VT; the VT^v_{sim} is a theoretical construct.

- **Case 2: Variable income and underestimates:**

  Individuals make long-term contracts on working time T^v while the wage rate varies for $v > 1$. If $\partial U/\partial T^v < 0$ and, for example, the wage rate increases, then the long-term optimal working time is reduced along with wage rates, which is captured by the simultaneous problem, compared to the hierarchical or short-term one (if $T^v > T^{min}$). In this case, $S^v = Y^v T^v$ and $\tau^v = \Delta h - T^v$.

  An increases of I^v with respect to (I^1) induces $T^v_{sim} \leq T^v$, then $S^v_{sim} \leq S^v$ and $\tau^v_{sim} \geq \tau^1$. That is, in the HP model the individual does not decide the optimum transfer of money, because he/she is working longer than the SP optimal. Then,

$$VT^v_{sim} = \frac{\mu^v_{sim}(\tau^v_{sim})}{\lambda^v_{sim}(S^v_{sim})} \leq \frac{\mu^v(\tau^1)}{\lambda^v(S^v)} = VT^v$$

  In this case we conclude that long-term value of time is overestimated by observing short-term decisions (HP)■.

To analyze these cases we define the sub-optimality gap in the value of time by the instantaneous long versus short term VT values at period v, calculated as:

$$\Delta VT^{n,v} = \left| \frac{\mu^{n,v}_{sim}}{\lambda^{n,v}_{sim}} - \frac{\mu^{n,v}}{\lambda^{n,v}} \right|, \tag{2.32}$$

Note that again, at each adjusting point the gaps are equal to zero, because the simultaneous and hierarchical solutions are equal.

2.5. Numerical experiments

The objective is to quantify numerically the effects of the hierarchical approach in modeling individuals' decisions, considering an extension to the proposed model by adding choices also on discrete goods. This case is of interest because it introduces realistic discontinuities in the choice process. We use simulation tests built on a hypothetical rational agent that maximizes utility in a deterministic choice process. The agent decides macro scale activities, including discrete choice on residential (l_{home}) and job (l_{work}) locations, and continuous decisions on housing size (q_{home}) and time assigned to work (t_{work}). At the micro scale he/she decides leisure activities, including free time at home (t_{home}), time assigned to shopping activities (t_{shop}), and transport modes. For each activity the agent also decides the consumption of goods (x). The simulation considers discrete sets of location choices for activities, transport mode options and housing sizes.
For simplicity we assume one spatial dimension, which is a straight line of 25 km long, where the individual decides locations for activities. To consider the dynamic behavior of decisions on the long term, we use 40 temporal snapshots. Analytically, the following microeconomic consumer's problem for a specific snapshot v (omitting v for simplicity) extends previous static models on discrete mode choice and time allocation (Train and McFadden, 1978; Jara-Díaz, 2007), by including location choices:

$$\begin{aligned} \text{máx}\, U(t,x,q,l,m) = \alpha * [\ln(t_{work}+1) + \ln(q_{home}+1)] + \\ (1-\alpha)[\ln(t_{home}+1) + \ln(t_{shop}+1) + \ln(x_{shop}+1) + \ln(x_{home}+1) \\ - \ln(tt_m(l_{home}, l_{work})+1) - \ln(tt_m(l_{home}, l_{shop})+1)] \end{aligned} \tag{2.33}$$

subject to

$$y_{work}t_{work} - px - r(l_{home}) * q_{home} - tc_m(l_{home}, l_{work}) - tc_m(l_{home}, l_{shop}) = 0, \quad (2.34)$$

$$t_{work} + t_{home} + t_{shop} + tt_m(l_{home}, l_{work}) + tt_m(l_{shop}, l_{home}) = 24, \quad (2.35)$$

and technological constraints

$$t_{work} \geq t_{work}^{min}, \quad (2.36)$$

$$t_{home} \geq x_{home}, \quad (2.37)$$

$$t_{shop} \geq x_{shop}, \quad (2.38)$$

$$q_{home} \geq q_{min} \quad (2.39)$$

Here, a Cobb-Douglas utility, with exponent α for the macro-micro substitution effect, is maximized. The location terms l_{home}, l_{work} and l_{shop} represent exogenous amenities of locations. The solution finds the optimal bundle of: the activities' duration times (t), goods (x), housing size (q), locations (l) and travel times by mode m (tt_m). Utility variables (t, x, q, l, tt) are deviated in one unit to avoid taking logarithm of zero. The income budget (2.34) balances out working income and expenditure on goods, residential rents and travel costs; housing prices, denoted r, are per size unit and depend on the location. Additionally, there is a set of technological constraints. Equation (2.36) indicates a minimum time for labor supply, either due to legal restrictions or representing features inherent to the specific type of job. Constraints (2.37) and (2.38) limit the minimum time allocated to home and shopping regarding the level of goods consumption. Finally, there is a survival constraint on the minimum amount of residential housing (2.39).

This setting defines discrete-continuous optimization problem, and the decision variables are defined per time snapshot (*e.g. per day*). Regarding the discrete variables, the long-term choices are home and job locations, while short-term choices are transport mode and the shopping location. We note that for simplicity, and in order to reduce the computational effort, we evaluate specific discrete options for the locations choices of activities ($l_{home}, l_{shop}, l_{work}$) along the linear city. With regard to the continuous decision variables, in the long-term they are: t_{work} and q_{home}; in the short term they are: $t_{home}, t_{shop}, x_{shop}$ and x_{home}. In addition, travel times (tt) and travel costs (tc) associated are dependent on the location choices where the optional transport modes are: Bus ($m = 1$) and Car ($m = 2$). Next, we use this experiment design to show how exogenous shocks in the economy and macro scale choices modify the computed values of time and time allocation.

Example 1: Wage rate shock

In this first example, for simplicity we assume a value of $\alpha = 0{,}5$, and job and residential location are fixed. In addition, we assume that only the wage rate varies along time according Figure 2.1 and described by the following function:

$$y_{work}^{v} = \begin{cases} 7 + 0{,}3\sqrt{v}; & 1 \leq v \leq 10 \\ 12 - \sqrt{v}; & 11 \leq v \leq 40 \end{cases}$$

At time snapshot 10 there is a sudden economic shock, changing wages dynamics from increasing to decreasing along time. We solve three different problems: two hierarchical problems (HP), one with adjusting points at $v = 9$ and $v = 18$ (denoted $HP - 9$) and the other one at

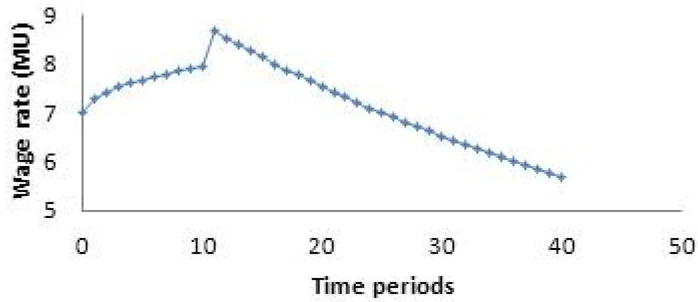

Figura 2.1: Changes on wage rate over time (in monetary units MU)

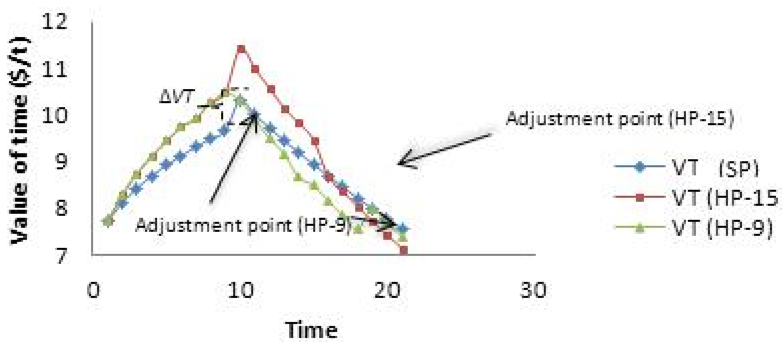

Figura 2.2: Value of time for Example 1: simultaneous versus hierarchical models

$v = 15$ $(HP - 15)$, we also solve the simultaneous problem (SP). In Figure 2.2 we show the computed variation of the value of time for the three cases.

The dynamics of the value of time (VT) in the simultaneous problem (SP) follows the changes assumed for the wage rate, because this value is the only factor changing over time in the model. While the wage rate increases in the first period before $v = 9$, the VT obtained from hierarchical models overestimate the long-term VT given by the SP model; this is because in the HP models the individual maintains longer working hours (according to low wage rates in $v = 1$ when working time was decided), while in the SP model working hours are reduced as income increases, then the time constrain is relaxed making VT to reduce. In the second period, between $v = 9$ and $v = 15$, the wage rate decreases and we see that the $HP - 9$ model adjusts VT underestimating the SP values; conversely, the $HP - 15$ model keep overestimating VT values until its adjusting point at $v = 15$.

In addition, in the Figure 2.2 we show the simultaneous versus hierarchical gap (VT), indicating sub-optimality in VT of the hierarchical problems. It worth mentioning that VT obtained by the SP problem represents the value associated to the hypothetical behavior where the agents adjusts long-term activities at every point in time, while the HP model calculates VT consistent with the slow motion of long-term decisions.

In Figure 2.3, we show the allocation of time to work obtained from these models. As the marginal valuation of labor is assumed negative, and because the wage rate increases during the first macro time window, the individual decides to work less time -in SP compared to HP-, adjusting income to expenditure and time to increase leisure. Conversely, if the wage rate is reduced, then he/she works more time.

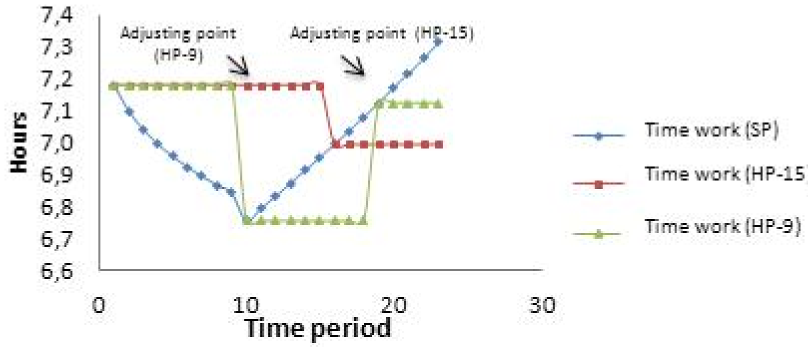

Figura 2.3: Time allocated to work for Example 1: simultaneous and hierarchical problems.

Finally, if $\alpha < 0{,}5$, then the time assigned to work is reduced not only in the simultaneous problem, but also in the hierarchical models, which is reflected in more time available for the micro activities. From this result, t_home and t_shop increase although the income decreases at each temporal snapshot; in other words, the individual has more time available but less budget for short-term consumption, which means that the value of time is reduced in all the time windows, obtaining parallel curves below the time valuations in Figure 2.2. In case of $\alpha > 0{,}5$, the valuation of time is larger with respect to Figure 2.2 as the income increases and so the work time, which is reflected in less time available for micro activities.

Example 2: Residential location, value of time and transport mode choice

In this case, we consider an individual whose job is located at point l=0, while assuming that the long-term home location choice varies in the range [0,25]. We assume that the bus travel cost is fixed and car travel cost varies with distance, as follows:

$$\begin{aligned} tc_{bus}(l_{home}, l_{work}) &= tc_{bus}(l_{home}, l_{shop}) = 10, \\ tc_{car}(l_{home}, l_{work}) &= 2 * |l_{home} - l_{work}|, \\ tc_{car}(l_{home}, l_{shop}) &= 2 * |l_{home} - l_{shop}|, \end{aligned}$$

travel times are also proportional to distance as follows:

$$tt_m(l_{home}, l_{work}) = \frac{1}{a} |l_{home} - l_{work}|, \; tc_{car}(l_{home}, l_{shop}) = \frac{1}{a} |l_{home} - l_{shop}|,$$

where,

$$a = \begin{cases} 2, & if \;\; m = BUS \\ 4, & if \;\; m = CAR \end{cases}$$

In this example for any individual who chooses a residential location between 0 and 17 the mode CAR is more convenient; beyond that limit the high travel cost by car makes BUS the

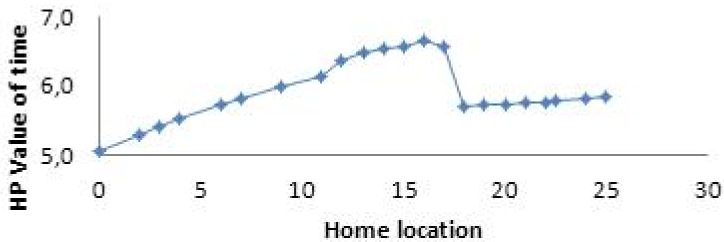

Figura 2.4: Residential location vs. value of time and transport mode.

optimal choice. The big drop observed in the hierarchical value of time shown in Figure 2.4 at $l_{home} = 17$ is the result of switching the transport mode, which is compensated by increasing the housing size and rents, hence reducing the money transfer to micro scale activities. Then, the observed micro scale behavior of those living beyond $l_{home} = 17$ reflects the lower disposable income implying a lower value of time. The most important result of this example is to clearly visualize the influence of the location of long-term activities on the values of time.

2.6. Conclusions

The paper presents an extension of the classical theory of time allocation, introducing a feature observed in several dynamic systems: the temporal hierarchy of choice processes. The model developed considers two levels, the macro- and micro-temporal levels, to describe the process of the individuals allocation of time and wealth to long- and short-term activities. No other dynamic feature was included; except for this extension, the model setting is identical, and thus comparable with the well-known one-level-simultaneous approach. We do not identify relevant shortcomings caused from choosing only two levels; in fact, we claim that the simple macro-micro framework is the nucleus of more complex hierarchies because it can represent the most-relevant effects of moving from one to multiple levels.

Despite the simplicity of the model, our analysis yields some relevant conclusions that contribute to the theory of the value and allocation of time. First, the simultaneous and the new approaches are consistent because both collapse to the same model once we assume that all choices are made and adjusted simultaneously; that is, there is no hierarchy in the process of deciding on activities and in this sense, the hierarchical model is a generalization of the former one. Second, the hierarchical model explains why estimated values of time from real data differ significantly according to socioeconomic characteristics, such as gender, residence and job locations, availability of durables (e.g., car, house), and education level. For all these conditions, our model provides a unique answer: the value of time observed is yielded by short-term activities but determined by different long-term choices. This finding implies that two identical persons, clones in everything except for having made a different choice in at least one long-term choice, are expected to have different time values. Third, we conclude that observed values of time change over time for the same individual and that usually, these values represent short-term values (conditional on long-term choices), except for at the adjusting point, where long-term preferences are revealed.

Therefore, an important concept in the hierarchical model is the adjusting point, when the macro-time window elapses and long-term choices are adjusted to initiate the next macro-time window. The adjusting point is considered exogenous to the individual's decisions, an assumption which may be extended in future work to make this point endogenous.

One major feature worth mentioning regarding the proposed formulation, is that the hierarchical model considers deterministic parameters together with a myopic behavior premise, i.e. decisions are made only on the current state of the economy in each time window (n, v). However, the results associated with the dependencies between long and short term decisions are extensible to other contexts in decision-making. For example, if at the adjusting point (n, 1) individuals make long-term decisions with parameters associated with their rational expectations ("*perfect foresight*") then, both long-term variables as well $S^{n,v}$ and τ^n will be conditional the individual's expectation on economic, all the same, the results of conditionality of long-term decisions (macro) over short-term decisions (micro), remain the same.

The proposed hierarchical model also illuminates an arising question regarding rigorous economic assessments of transport projects. Indeed, we have concluded that there is no unique value of time for a given person. In fact, there are two (long and short-term) values in the simple macro-micro model (and multiple in a multi-time-scales model), and both of them have a dynamic behavior, changing along time according to external economic conditions. Then, the question arises on how to evaluate time saving yield by transport projects lasting for a long time horizon. Of course, if the dynamics of external economic conditions are known, we can simulate short-term and long-term activities of each individual and assess the associated values of time over time. Assuming we can accurately make such calculations, should we use the short-term or long-term dynamics of time values along the time horizon? We know, from our examples above, that short-term values can either over- or underestimate the long-term values, depending on whether the individual reduces or increases income along the macro-time window. We argue that the right value of time to use for project appraisals is the short-term value of time, regardless of the gap with longterm values. This argument is supported by the fact that individuals perceive short-term values; the long-term value of time is a theoretical construct affected by the slow motion of long-term decisions, whose associated benefit are potential but not realized due to unavoidable long-term commitments and adjustment costs.

Thus, the most general conclusion is that the theory of time allocation is enriched by the hierarchical structure of activities; conversely, the assumption of one temporal level obscures important features that explain consumers behavior with relevant implications by modeling travel demand and time allocation. The main contribution of the hierarchical approach is that it makes clear how the behavior at the micro level is conditional on the decisions made at the macro level. This finding has an impact on the methodologies to estimate demand and on the understanding of the formation of the time value. Some shortcomings of the traditional one-level model can be partially overcome by clustering the population with regards to certain long-term decisions, such as car ownership and income level, based on intuitive arguments. Our model provides theoretical support to such intuitions; however, it also provides a theoretical framework to identify a complete set of variables to define the population clusters. More importantly, our results show that clustering can be avoided, at least partially, by specifying the indirect utility function at the micro level explicitly dependent on the macro-level choices. The benefit of this approach is the reduction of parameters in the estimation of the demand model, which reduces the number of observations required to obtain a given precision on the parameter estimates. Practical studies of aggregate levels of population and their choices only require average time values; in that case the contribution of the hierarchical model may be considered irrelevant. However, the more disaggregated the study of individuals' behavior, the more relevant the understanding of the variability of these values across individuals. Additionally, average values are composed of individuals values and the population distribution; hence the evolution of these values over time is better understood (thus better predicted) if the diversity of values and the evolution of the population are explicit in the model. The theory of the value and allocation of time is complemented by the hierarchical model. This model explains how macro-level activities and their value of time depends on the expected effect of other decisions at the same macro level and at the micro level (what we call the adjusting point), while decisions at the micro level are dependent on the resources allocated to activities at the macro level. Thus, the estimate of the value of time inferred from consumers behavior depends on the type of activity and whether the activity is decided at a macro or micro level.

Acknowledgements

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Capítulo 3

Segundo articulo: Microeconomic model of residential relocation incorporating agents evolution, individual learning and life cycle expectations

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ABSTRACT

This paper is focused on the dynamics of the residential location choices based on the microeconomic theory of urban land use in which it is assumed that each property is assigned to the agent (household, firm) that has the highest bid or willingness to pay. A model of residential location choice is developed, where process experience matters in a dynamic learning process and each agent evaluates these locations decisions according to the utility obtained. Additionally, household expectations in relocation decisions is incorporated in a microeconomic formulation by mean of transition probabilities among household clusters in the life cycle and the imitation's hypothesis of such agents under the consideration that they behave rationally. A imitation multi-objective bid function is obtained which includes an expected income per unit of time and a utility consistent with the behavior of the agents that are potentially imitables.

In the learning stochastic model a behavior LOGIT is assumed where willingness to pay are determined by a deterministic part based on the dwelling characteristics, the externalities associated with the zonal distribution in supply, household income, and the urban configuration in the previous period with the inclusion of learning phenomena plus a random part with Gumbel probability distribution. Similarly, we develop a stochastic model with the imitation effect.

Numerical examples and simulations are presented using linear bid functions. Additionally, concepts of urban dynamics are used in the long term assuming quasi-equilibrium in the bids, in the externalities of urban configuration, and comparing the results and the differences obtained with a static outcome (equilibrium model of land use). The comparison is done through indices of urban segregation and short and long term configurations. These studied dynamics permit to

conclude that static modeling does not explain all the features that configuration of the urban system (in short and long term).

Keywords: Residential location, learning, imitation, household life cycle, Residential Segregation, multinomial logit, Bid functions.

3.1. Introduction

The study of land use dynamics in an urban system, particularly in the residential relocation process, involves the description and modeling of the interaction of a variety of agents that change their characteristics and preferences over time and make different decisions in time and space. Moreover, this dynamic is, among others, the result of issues such as the joint location decision of various household which in turn affects the urban system configuration, causing variations in real estate market behavior. On the other hand, there are some processes associated with making location decisions of households that are not explicitly considered in the classical models of short and long term urban land use equilibrium such as memory, learning, habit formation, the generation of expectations, uncertainty about resources availability and about the fluctuation and disruption in social and economic changes that establish a complex system with a structure similar to other social and natural systems (Gunderson y Holling, 2002).

Even though households have an internal dynamics such as changes in life cycles, changes in the structure (new children, divorce, job changes, level of education, etc.) which affects the consumption patterns and residential location, and on the other hand, there are variations in the urban land use due to generation of new real estate projects of private and public investment in different areas of a city, it is known that previous experiences are an important element in decision-making (Flórez, 1998) noticing that households tend to remain in the neighborhood where they have lived or in similar housing types (apartments, houses, etc.). In turn it is possible that households tend to remain in a status quo in residential location decision-making over time, because of the knowledge generated by learning the area and the apprehension to change despite of changes in family structure and the environment. In turn, this type of household's life-cycle expected dynamics or transition among possible states or household types are part of a set of possibilities that can be anticipated by like themselves and used as a basis for decision making in urban areas in the short and long term, such as residential relocation and intra-urban mobility (Li and Tu, 2011). Such relocation forces are what Huff and Clark (1977) have called cumulative inertia (resistance to movement) and residential stress given by the possible dissatisfaction with certain attributes of the current household and their surroundings, this dissatisfaction can be generated by changes in the household life cycle.

As this work, there are other analytical and empirical studies that explain the residential relocation dynamics through the previous experiences, or the expectations of change associated with possible future states. For example, in Chen et. al. (2009) the effect of past location decisions is studied by means of the spatial correlation between new and previous decisions showing that there are preferences formed over time and interact with the life cycle; this is modeled through different extreme value distributions. Habib and Miller (2010) propose a model of residential choice founded on the discrete choice theory where individuals decide to relocate based on a reference point, this selection is given by the former location. For this, they postulate a utility function, conditional on the amount of gain or loss for each attribute or feature of the property, with respect to the former location, using concepts of the risk decision theory (Tversky and

Kahneman, 1991; Sugden, 2003). Páez, et. al (2008) present a discrete choice model in which agents are modeled through the random utility theory, where each good's utility depends on the decisions made in the past for themselves and a socially related group of agents. For this they use a formulation of the social distance grounded in the social network theory. The problem with this formulation is that the decision made by the agents is the only one analyzed, but not the valuation of each one of the location options or the utility perceived by them. Chen and Lin (2011) develop a life cycle econometric model for residential location and show that the marginal valuation of each property attributes is generated by a learning process that depends on past experiences, showing that this assessment is formed throughout time, using the bounded rationality concepts described in Camerer, (1998). In addition, they analyze the influence of life cycle changes (having children, age, marriage decisions, etc.) in location decisions. In other type of studies, the relocation decision is analyzed as a two-stage process, first as a relocation probability and then as a search for a new option. For example, Nijkamp, et. al (1993) obtain each household's probability of moving (logit binomial) from its current location associated to age changes, changes in the characteristics of the goods and if the agent is owner or not, and given the information of this probability, a new property demand function is obtained. Altogether, the probability to move into a new zone is obtained and conditional to it the probability of switching to a type of housing within the area. Other several econometric works that based on logistic regression or logit binomial find probabilities to move from the current location by means of the utility thresholds obtained using longitudinal data are Clark et. al., (2003) and Sommers and Rowell, (1992). Lee and Waddell (2010) extend this type of works including sampling in the second stage of the model that corresponds to the selection of the new housing, creating a set of bounded decision alternatives given the relocation probability.

Eluru, et al, (2009) present an econometric formulation using multinomial type choices and a system of equations that defines the reason for moving and the new choice duration. They used a retrospective survey to estimate the model parameters.

Additionally, there are several empirical studies that explain the residential relocation dynamics through effects like the expected salary in the future or the importance given by the agents to the utility drawn by others with the consumption of various goods in urban areas. One way to make these making decision changes is by assessing the anticipation of them, knowing the cluster or household type expectations or future change probabilities, for example, future expected revenues can be used as an estimate of the payment capacity for a household (Kennan and Walker, 2011). In addition, other influential causes in the relocation decision making are the possible changes in the long-term activities in any of the household members in terms of work or education (Hooimeijer, 1996; Clark, and Withers, 1999; Li and Tu, 2011), changes in the household structure by departure or arrival of new members, etc.

An important concept found in some of the relocation models are transaction costs, that contain not only monetary costs but also costs associated with distances; social or psychological losses which are explained with the disutilities of change by Miller and Haroun (2000) that according to Russ (1994) and Kennan and Walker (2011) are costs that increase with the agents' age.

Moreover, some studies in psychology and sociology show that habit formation and learning, as well as socioeconomic changes in the life cycle are important factors when a household decides to make decisions about intra-urban mobility (Rossi, 1955; Ritchey, 1976; Anderson and Milson, 1989; Aarts et. al, 1998).

In addition, there are dynamic economic models that explain and analyze the consumption of continuous and discrete goods of the agents through learning, habit formation, or memory models, analyzing long-term effects and their consequences in the demand thereof (see Milani, 2004). On other hand, in the context of game theory there are theoretical studies of imitation and learning dynamics associated with strategic decision making (Alós-Ferrer and Schlag, 2009). An interesting example in this area is the work developed by Berg (2008) where the firms make the location decisions using information of the profit obtained by other firms previously located. Finally, there are some micro simulation models that attempt to model these dynamics and their influence on the household choice. For instance, Miller and Haroun (2000) have developed a model called ILUTE which simulates the behavior of individual agents in time and space. The overall purpose of ILUTE is to simulate the evolution of an entire urban region for an extended period of time analyzing the effect of changes in the transport system, in the real estate market, and other urban policies.

The objective of this work is to formulate analytically an urban equilibrium problem of residential location that captures the dynamic behavior described above and their effects in the short and long term urban configurations.

In the second section, the theoretical background and initial considerations necessary for the model development with learning or history appreciation are described. In the third section, a discrete choice deterministic microeconomic model associated with residential location is developed incorporating the utility gained by past experiences and the transition between periods; likewise the households' willingness to pay is found when this type of phenomena is considered. In the fourth section, the use of household change's expectations are incorporated in a residential location discrete choice microeconomic formulation the by mean of transition probabilities among household clusters in the life cycle. In the fifth section, numerical examples are developed based on the dynamic models of residential segregation theory. Finally, there is a section on conclusions and final discussion.

3.2. Theoretical background

Real estate units are characterized by attributes or characteristics called "location externalities"given by the relationship between the distinctive variables that establishes the location of the agents (households, firms, schools, etc.) in a city. As one of these agents is located, changes occur in the physical and / or socio-economic setting in the chosen place; resulting in changes in the perception or the utility of the rest of the agents (Martínez and Araya, 2000). Urban economics is based on two main approaches to explain the urban agents' location choice both founded on the assumption that the housing is a quasi-unique good. In the first, called Bid, an auction-type market is assumed where agents bid for the different locations, which are awarded to the highest bidder (Alonso, 1964). The bid depends on variables such as location or housing attributes, accessibility variables, as well as the income level, consumption and utility level of the agents. The second approach, called Choice, assumes that the agents choose those locations that maximize their utility level (McFadden, 1978; Anas, 1982). Immediately after, the two approaches are briefly explained, making some necessary assumptions for the development of the learning model. Based on Martínez and Araya (2000) and Martínez (1992) the following static and deterministic problem of residential location is assumed using the discrete choice theory (3.1), where the household type $h \in H$ in a period t selects a real estate $i \in D$ that maximizes its utility[1],

$$\max_i \max_x \ U_h^t(x, Z_i^t) \text{ subject to } p^t x + r_i^t \leq I_h^t \tag{3.1}$$

where Z_i^t is the set of attributes of the property indexed by (i). Attributes can be divided into the proper housing attributes and neighborhood quality attributes, and accessibility and attractiveness features (Louviere and Timmermamns, 1990). r_i^t is the rent for the property (i), I_h^t is the exogenous income of the household h, and p^t is the price vector associated with a set of market goods x. Given the optimal solution of the problem (3.1) the indirect utility function is obtained, associated to the real estate (i)

$$V_{hi}^t \equiv V_h^t(I_h^t - r_i^t, Z_i^t, p^t) \tag{3.2}$$

Given a utility level $\overline{U}_h$, if the inverse function of V_h^t exist with respect to the rent variable then

$$r_i^t = I_h^t - V_h^{-1}(\overline{U}_h, Z_i^t, p^t) \tag{3.3}$$

Under the consideration of an auction market and the assumption that each property (i) is quasi-unique consumption good, the rent variable can be seen as the willingness to pay (Ellickson, 1981)

$$B_{hi}^t = I_h^t - V_h^{-1}(\overline{U}_h, Z_i^t, p^t) \tag{3.4}$$

It is easy to demonstrate that if the utility function is quasi-linear, i.e. $U_h(x, Z_i) = ax_0 + f(x_{-0}, Z_i)$ then the functional form of indirect utility function and the willingness to pay are, respectively:

$$V_{hi}^t(Z_i^t, I_h^t - r_i^t) = \lambda_h^t(I_h^t - r_i^t) + \lambda_h^t b_{hi}^t(Z_i^t)$$

$$B_{hi}^t = I_h^t + b_{hi}^t(Z_i^t) - \frac{\overline{U}_h^t}{\lambda_h^t}$$

where λ_h^t is the income marginal utility and $b_{h,i}^t(Z_i^t)$ is a function that measures the property attributes valuation by household side h. The previous expression of B_{hi}^t is denoted in (Martínez and Henríquez, 2007) as:

$$B_{hi}^t = a_h^t + b_{hi}^t(Z_i^t) \tag{3.5}$$

where the utility level reached, embedded in $a_h^t = I_h^t - \frac{\overline{U}_h^t}{\lambda_h^t}$, is obtained by a market equilibrium condition that every agent should be located. Additionally, if it is assumed that the willingness to pay can be modeled as $\widetilde{B}_{hi}^t = B_{hi}^t + \varepsilon_{hi}^t$, where ε_{hi}^t is identically and independently distributed Gumbel with dispersion parameter μ, and B_{hi}^t is the deterministic part, then the probability that household h be the highest bidder for the property i in the period t is:

$$Q_{h|i}^t = \frac{\exp(\mu B_{hi}^t)}{\sum_{g \in H} \exp(\mu B_{gi}^t)} \tag{3.6}$$

Using the size correction for each alternative or households cluster in (3.6) proposed by Mc Fadden (1978), the probability that household h be the highest bidder for the property i in the period t , $Q_{h|i}^t$ is:

$$Q_{h|i}^t = \frac{H_h^t \exp(B_{hi}^t)}{\sum_{g \in H} H_g^t \exp(B_{gi}^t)} \tag{3.7}$$

[1]Although the model can be defined as dis-aggregated at the level of each agent and location, in practice aggregated versions are used such as the one presented here. Consumers are classified into socio-economically homogeneous categories (index h) and the supply described by location clusters (index i).

where H_h^t is the cluster size of household type h in the period t.

This result was proposed by Ellickson (1981) and extended by Martínez (1992) demonstrating that this bid maximization approach is equivalent to the utility maximization approach of McFadden, (1978) and Anas, (1982)The latter in its stochastic formulation represents the probability that a household h chooses an option ($i \in D$) given by:

$$P_{i|h}^t = \frac{S_i^t \exp(\mu V_{hi}^t)}{\sum_{i' \in I} S_{i'}^t \exp(\mu V_{hi'}^t)}$$

where S_i^t is a deterministic and exogenous supply in each period t of real estate i. In this way, the following household distribution is obtained in a period t given the probability (3.7) as $H_{hi}^t = S_i^t Q_{h|i}^t$. In the $RB\&SM$ static model (Martínez and Henríquez, 2007), included externalities associated with the urban distribution in each zone that affects the households' willingness to pay, therefore $B_{hi}^t(H_{gj}^t, \forall g, j)$, so that $Q_{h|i}^t$ depends on $Q_{g|j}^t$, generating a fixed point system of equations. On the other hand, in the inter-temporal or dynamic extension of such a model of Martínez and Hurtubia, (2006), considered $B_{hi}^t(H_{gj}^{t-1}, \forall g, j)$, under the hypothesis that each agent's bid in a period t is a function of the urban distribution in $t-1$, preventing the fixed point calculation, and the urban dynamics are discussed given this decision making process, where the valuation is assumed with time-lag, based on an information knowing process. In any case the rent at each location is obtained endogenously by the expected value of the maximum bid, given by the log-sum function:

$$r_i^t = \frac{1}{\mu}\left\{\ln(\sum_h H_h^t \exp(\mu B_{hi}^t) + \gamma)\right\} \tag{3.8}$$

γ is Euler's constant. Finally, the equilibrium condition ensures that every home locates itself, given by $\sum_i Q_{h|i}^t S_i^t = H_h^t$, obtaining

$$a_h^t = -\frac{1}{\mu}\ln\left(\sum_i S_i^t \exp(\mu(b_{hi}^t - r_i^t))\right), \ \ \forall h. \tag{3.9}$$

Since the rents r_i^t depend on each a_g^t then (3.9) constitutes a fixed-point system of equations described in the form $a_h^t = f(a_g^t, \forall g)$ defining the maximum utility levels possible to get at equilibrium. Note that if the direct utility is quasi-linear, then $V_{hi}^t(Z_i^t, I_g^t - r_i^t) = \lambda_h^t(I_h^t - r_i^t) + \lambda_h^t b_{hi}^t(Z_i^t)$. Seeing that this utility indicates the satisfaction degree of an individual for a real estate (i) it may not affected by the agent's income, then we obtain truncated utility function:

$$V_{hi}^t = \lambda_h^t(b_{hi}(Z_i^t) - r_i^t) \tag{3.10}$$

3.3. Memory effect and learning

3.3.1. Deterministic process

In this section, a simple dynamics of endogenous learning for each household will be assumed in the discrete residential location decision-making in period t, given the convex linear combination of the current and past valuation, using the following discrete choice problem:

$$\max_i \max_x \alpha_h U_h^t(x, Z_i^t) + (1-\alpha_h) m_{hi}^{t-1}, \ \text{subject to} \ p^t x + r_i^t \leq I_h^t \tag{3.11}$$

where m_{hi}^{t-1} is the memory or learning accumulation factor from past experiences (before the time t) in the real estate (i), α_h is the valuation of the property features in the decision period t or the risk aversion or the uncertainty associated with location changes. This modeling assumes that the utility on a real estate is given by a continuous learning process formed from the accumulation of information over a long period of time or where the agents make decisions using information from the past as an extension of the present and future information. For example, the set of activities performed in previous periods generates a learning process on attractiveness and accessibility measures. These measures are described ine Martínez, (1995); Jara-Díaz and Martínez, (1998) and others. The proposed formulation entails a continuous assessment on the part of households about their residential location where factors such as experience may generate slow processes of relocation. In addition, for each period t an endogenous household dynamics related to the housing valuation is defined, but it is worth noting that this problem remains static. It is important to note that it is not intended to seek long-term equilibria ("perfect foresight") as developed in Anas and Arnott (1991), who do consider a dynamic interaction (from a temporal point of view) between urban agents, forcing to suppose a complete knowledge of future real estate market equilibria by the households and firms (mainly from real estate developers). In this way, this work's contribution is based on the static microeconomic formulation, using as a numeric solution base other models' demand algorithms such as *RB&SM* (Martínez and Henríquez, 2007) and its dynamic extension (Martínez and Hurtubia, 2006).

The following expressions or memory measures are nominated to analyze the microeconomic formulation (3.11):

- **Formulation 1: Myopic memory or learning**

  In the myopic memory assumption, the agent only bases the residential location decision on the real estate characteristics in the current and immediately prior periods, in case of relocating in the same area or in the same housing type of previous period. Mathematically, the memory expression depends on the utility gained in the period $t-1$, given by

$$mm_{hi}^{t-1} = \mathbf{1}_{h,i}^{t-1} V_{hi}^{t-1}(Z_i^{t-1}, r_i^{t-1}) = \mathbf{1}_{hi}^{t-1} \lambda_h^{t-1}(b_{hi}(Z_i^{t-1}) - r_i^{t-1}) \tag{3.12}$$

  where $\mathbf{1}_{hi}^{t-1} = \begin{cases} 1 & \text{if in } t-1 \text{ the agent } h \text{ was located in } i \\ 0 & \text{otherwise} \end{cases}$

  So that in period t, previous experience increases utility if the experiences are positive or decreases if they are negative in such a period. Although in the formulation (3.12) seems that the agent makes decisions based only on the present and immediate past, there is a clearly interrelation between periods in, since in the previous period his decision was based on two preceding periods and so on.

- **Formulation 2: memory with discount factor or average**

  This case is an extension of the formulation 1, it is assumed that the household or agent who makes decisions values the previous periods experiences much more, so the following formulation is proposed:

$$m_{hi}^{t-1} = \sum_{k \leq t-1} (1-\alpha_h)^{t-k-1} mm_{hi}^{k} = \sum_{k \leq t-1} (1-\alpha_h)^{t-k-1} \mathbf{1}_{hi}^{k} V_{hi}^{k}(Z_i^k, r_i^k) \tag{3.13}$$

where $\sum_{k\leq t-1}(1-\alpha_h)^{t-k-1}mm_{hi}^k$ is a weighted average to discount rate $(1-\alpha_h)$ over past experiences. Another formulation is the use of the utilities' average associated with previous experiences:

$$m_{hi}^{t-1} = \frac{1}{t-1}\sum_{k\leq t-1} mm_{hi}^k = \frac{1}{t-1}\sum_{k\leq t-1} \mathbf{1}_{hi}^k V_{hi}^k(Z_i^k, r_i^k) \tag{3.14}$$

Expression (3.14) loses its meaning or relevance when t is too large due to changes in urban land use such as generating real estate supply, construction of public and private property, changes in transportation systems and infrastructure, as well as the aging of goods, generate considerable variation in the previously known attributes. In this way, each period utility may not be assessed with the same weighting. On the other hand, expressions (3.13) and (3.14) allow, in the face of several options for location and a frequent relocations, to grow the utility level t in a for the set of choice options with good previous experiences ($mm_{hi}^k > 0$) and it would decrease in case of negative experiences ($mm_{hi}^k < 0$).

NOTE: The expression $\alpha_h U_h^t(x, Z_i^t) + (1-\alpha_h)m_{hi}^{t-1}$ in each memory formulation case can be viewed as an average utility where $1-\alpha_h$ represents the probability of not seeking a new location for a household h and α_h would represent the probability of seeking relocation, including the current location. Using probabilistic notation and considering an expected equilibrium utility U_h^*, this is

$$\alpha_h = P(m_{hi}^{t-1} \leq U_h^*)$$

In case of obtaining a declining history valuation because of agents or system changes, such as $m_{hi}^k < m_{hi}^{k+1}$, there will be a greater probability to seek new options due to the low utility gained in previous period.■

Assuming that U_h^t is quasi-linear then the functional form of indirect utility function of (3.11) is

$$V_{hi}^t = \alpha_h\{\lambda_h^t(I_h^t - r_i^t) + \lambda_h^t b_{hi}^t(Z_i^t)\} + (1-\alpha_h)m_{hi}^t.$$

Note that as m_{hi}^{t-1} is a constant in the consumer problem at t then the agent's bid h with associated memory to the good (i) described by m_{hi}^{t-1} is

$$\overline{B}_{hi}^t = B_{hi}^t + \frac{1-\alpha_h}{\alpha_h}\frac{m_{hi}^{t-1}}{\lambda_h^t} \tag{3.15}$$

where $\frac{m_{hi}^{t-1}}{\lambda_h^t}$ is the property valuation (i) in $t-1$ divided by the income level obtained in t. That is, each household updates prior experiences assessments to the current incomes increasing or decreasing the willingness to pay for each property. Also $B_{hi}^t = a_h^t + b_{hi}^t(Z_i^t)$, with $a_h^t = I_h^t - U_h/(\alpha_h\lambda_h^t)$, at equilibrium U_h it will be conditional to the memory level with regard to the chosen property, that is to say $U_h(m_{hi}^{t-1})$ and hence each agent h will reach a different utility depending on past experiences, creating a differentiating factor between them in addition to the proper characteristics of the household in t. Moreover, formulation (3.15) of the willingness to pay increases the amount of household differences because it includes within attributes of the cluster all the locations background, generating a great variety of attributes. Then, each household will be described by their socioeconomic characteristics in the period t and the characteristics in previous periods that define the perceived utilities in their previous locations. Thus, the proposed formulation provides a useful tool in micro-simulation models, where each agent is analyzed in detail. To avoid such a large dimensionality of classification variables, in the subsequent analyzes we will only assume a myopic learning for the equilibrium study in each period, allowing obtaining further change or relocation probabilities.

On the other hand, the microeconomic modeling of residential location with memory is an interesting contribution in the social assessment of private or public projects, because two agents of an urban system with identical conditions (residential location and household type) in a period t can perceive the utility differently at to their current residential location because of the past experiences' reference. In this sense, a social planner should include within the analysis not only the perceived utility in the period t, but also how the property characteristics, where every household is located, improve or worsen with respect to previous periods.

3.3.2. Dynamic transition of agents

In formulations (3.12), (3.13) and (3.14) of memory or learning it is assumed that the agents do not change cluster between periods, only residential relocation attributes, but an important factor associated with the intra-urban mobility resposible changes in household characteristics, such as changing jobs or income, change in family structure, for this we assume the following dynamics of decisions.

Consider a household that changes cluster between periods $t-1$ and t from g in $t-1$ to h in t noted (h,g) in the period t, then under a memory formulation, the consumer problem and willingness to pay for a good (i) can be written as:

$$\max_i \max_x \alpha_h U_h^t(x, Z_i^t) + (1-\alpha_h) m_{gi}^{t-1}, \text{ subject to } p^t x + r_i^t \leq I_h^t$$

$$B_{hg(i)}^t = a_{hg}^t + b_{h,i}^t(Z_i^t) + \frac{1-\alpha_h}{\alpha_h}\frac{m_{gi}^{t-1}}{\lambda_h^t}, \tag{3.16}$$

where $a_{hg}^t = I_h^t - U_{hg}/(\alpha_h \lambda_h^t)$. That is, as mentioned above, two agents having the same socioeconomic characteristics and located in equivalent real estates in a period t, can have different levels of utility by the generation of the experience or by the evolution of socio-economic characteristics in previous periods. More widely, the households' classification is defined by the following vector of features in the period $t:\ (h,g,j)$. The information that defines the triplet above describes a cluster to which each agent belongs to in t which is h and in the period $t-1$ corresponds to cluster g and the residential location j. For such households the bid function with myopic memory for a real estate (i) in a period t is:

$$B_{(h,g,j)i}^t = a_{h,g,j}^t + b_{h,i}^t(Z_i^t) + \mathbf{1}_{(i=j)}\frac{1-\alpha_h}{\alpha_h}\frac{\lambda_g^{t-1}}{\lambda_h^t}(b_{gi}^{t-1} - r_i^{t-1}), \tag{3.17}$$

where

$$\mathbf{1}_{(i=j)} = \begin{cases} 1, & if\ i = j \\ 0, & if\ i \neq j \end{cases} \tag{3.18}$$

Where $\frac{\lambda_g^{t-1}}{\lambda_h^t}$ is an update of the willingness to pay in periods prior to the income in the current period. Note that if $\lambda_g^{t-1} \leq \lambda_h^t$ then $I_g^{t-1} > I_h^t$ so the assessment of the past will be reduced. In turn, if $\lambda_g^{t-1} > \lambda_h^t$ then $I_g^{t-1} < I_h^t$, implying that the valuation of the past will be greater.

Furthermore, $\lambda_g^{t-1}(b_{gi}^{t-1} - r_i^{t-1})$ is the utility obtained in i in period $t-1$. The utility a_{hgj}^t in equilibrium may be different for each agent type(hgj).
In the case of having new agents in the system type h (immigrants) at time t, which will be noted as $\tilde{H}_h^t$ they will have as bid function:

$$B_{hi}^t = a_h^t + b_{h,i}^t(Z_i^t) \tag{3.19}$$

Namely, it is assumed that the new agents in the urban system have zero memory and the value of a_h^t may be different. Clearly, formulations (3.17) to (3.19) pose differentiating elements associated with the household evolution and the learning of the system given by the equilibrium level of utility and the former location.
Furthermore, a agent h will relocate in a period t in a real estate i being located in j in $t-1$ so $j \neq i$ if $\alpha_h V_{hj}^t + (1-\alpha_h) m_{hj}^{t-1} < \alpha_h V_{hi}^t$

$$V_{hi}^t - V_{hj}^t > \frac{1-\alpha_h}{\alpha_h} m_{hj}^{t-1} \equiv ac_{hj} \tag{3.20}$$

Thus, value ac_{hj} can be interpreted as a adjustment cost or pull factor, given as a lower bound associated with the utility valuation in the past experiences. It also defines a resistance factor to change or transaction cost, which provides a formal interpretation of Miller and Haroun, (2000) where the adjustment costs are defined as monetary, psychological and social losses. Conversely, $\frac{1-\alpha_h}{\alpha_h}$ shows the valuation of the past. If $\frac{1-\alpha_h}{\alpha_h} \geq 1$ then $\alpha_h \leq 0{,}5$, obtaining that the past experiences assessment is greater in regard than present utility, generated by the uncertainty of knowing the system or the agent's risk aversion h. In addition, in the case that $r_i^t > b_{hi}^t$, we have $m_{hi}^{t-1} < 0$ due to changes in land used that decrease the utility of the property, for example, location of new unwanted agents (prisons, landfills, etc.) and the dynamic effects that these locations are generating over time.

3.3.3. Stochastic modeling

To define the stochastic model it will be assumed that there is an exogenous transition matrix of all agents in the system $P(h^t|g^{t-1})$ that defines the aggregate percentage of change among type of households and it is given by life cycle changes like income, education, children, job change, macro decision-making, exogenous economic shocks to the household, etc. This transition matrix is exogenous in this model. Thus, the number of agents belonging to the cluster defined by (h,g,j) in the period t is

$$H_{hgj}^t = P(h^t|g^{t-1}) H_{gj}^{t-1}, \tag{3.21}$$

where H_{gj}^{t-1} is the exogenous urban distribution of equilibrium in the period $t-1$. Assuming that the willingness to pay $B_{(h,g,j)i}^t$ is a random variable, where the modeling error is assumed with Gumbel distribution with scale parameter μ. Additionally, at each modeling period the current income must be greater than the bid for the utility levels in equilibrium:

$$\bar{B}_{(h,g,j)(i)}^t \leq I_h^t, \quad \forall (h,g,j) \tag{3.22}$$

If the constraint (3.22) is not met, the agent (h,g,j) won't enter to the set of potential bidders for the household (i) during the period t.

For new agents in the system we have:

$$\bar{B}_{(h)(i)}^t \leq I_h^t, \quad \forall h \in \tilde{H}_h^t \tag{3.23}$$

Constraints (3.22) and (3.23) are important insomuch as in cases where the memory and present assessment is very high in some propoerty goods, the willingness to pay cannot exceed the current income, thus, if an income decrease between periods is faced, there is no bid for goods that had past experiences where the income was higher, that could generate relocation processes associated with economic shocks such as job loss of one or more members of the household or increased spending on consumer durables (car, education, etc). These restrictions are may be included in the modeling through the approximation obtained by the *Constrained Logit* (Martínez, et al 2009) with "*cut-off*"functions defined as the probability that a household of a cluster h accomplishes in each i, denoted as $\phi^t_{(h,g,j)(i)}$, given by

$$\phi^t_{(h,g,j)(i)} = \frac{1}{(1+\exp(w(\bar{B}^t_{(h,g,j)(i)} - I^t_h + \zeta)))} = \begin{cases} 1, & if \ (\bar{B}^t_{(h,g,j)(i)} - I^t_h \to -\infty) \\ \eta, & if \ (\bar{B}^t_{(h,g,j)(i)} - I^t_h \to 0) \end{cases} \tag{3.24}$$

The value of w determines how fast the probability value approach extreme one or zero values. The parameter ζ represents the tolerance of violating the income restriction, if the bid is approaching to the income level then the factor $\phi^t_{(h,g,j)(i)}$ tends to η. To achieve this we use the parameter ζ, defined by:

$$\zeta = \frac{1}{w}\ln((1-\eta)/(\eta)) \tag{3.25}$$

Analogously the probability of reaching the new agents' constraint in the system is defined as:

$$\phi^t_{h(i)} = \frac{1}{(1+exp(w(\bar{B}^t_{h(i)} - I^t_h + \zeta)))} = \begin{cases} 1, & if \ (\bar{B}^t_{h(i)} - I^t_h \to -\infty) \\ \eta, & if \ (\bar{B}^t_{h(i)} - I^t_h \to 0) \end{cases} \tag{3.26}$$

Given the heterogeneity in each cluster size of consumers and the variety of them generated by the inclusion of the memory effect as part of the household' description, the set of consumers looking for location and meeting the feasibility condition is $P(h^t|g^{t-1})H^{t-1}_{gj}\phi^t_{(h,g,j)(i)}$ for agents type (h,g,j) and there will be $\tilde{H}^t_h\phi^t_{h(i)}$ new (*immigrants*) household type h that might be located in (i) in the period t.

McFadden's argument is used to introduce these heterogeneous household sizes (McFadden, 1978), obtaining a corrected probability that the household (hgj) be the highest bidder in (i) is

$$Q_{(h,g,j)|(i)} = \frac{P(h|g)H^{t-1}_{gj}\phi^t_{(h,g,j)(i)}\exp(\mu\bar{B}^t_{(h,g,j)(i)})}{A^t_i}, \tag{3.27}$$

where

$$A^t_i = \sum_{h',g',j'} P(h'|g')H^{t-1}_{g',j'}\phi^t_{(h',g',j')(i)}\exp(\mu\bar{B}^t_{(h',g',j')(i)}) + \sum_{h'}\tilde{H}^t_{h'}\phi^t_{h'(i)}\exp(\mu\bar{B}_{h'(i)})$$

And additionally the probability that a new agent h^t be the highest bidder is

$$Q_{(h,0)|(i)} = \frac{\tilde{H}^t_h\phi^t_{h(i)}\exp(\mu\bar{B}^t_{h(i)})}{A^t_i} \tag{3.28}$$

Thus, the aggregate probability that a household type h be the highest bidder and locates on the property (i) is

$$Q^t_{h|i} = Q^t_{h,0|(i)} + \sum_{g,j} Q^t_{h,g,j|(i)} \tag{3.29}$$

The formulation of $Q^t_{h|i}$ differs mathematically from the one stated at the $RB\&SM$, model due to memory effect. The probability of relocation for (h, g, j) in the period t is:

$$P(move|hgj) = 1 - Q_{(h,g,j)|j} \tag{3.30}$$

Note that the probability of relocation captures urban system changes and household lifecycle variations. Thus, $P(move|hgj)$ integrates dissatisfaction with current property good and the search for a new home. Therefore, we extend the results of microsimulation or econometrics models where the probability of relocation depends on a binomial logit model associated with the level of the utility of the current location.

Additionally, the aggregate population distribution in the period t is:

$$H^t_{hi} = Q^t_{h|i} * S^t_i \tag{3.31}$$

In this modeling proposal, probabilities (3.27) or (3.28) can be evaluated in time period equilibrium, with atributes of bids obtained by means of fixed-point algorithms. In the dynamic model it is assumed that each b^t_{hi} depends on $H^{t-1}_{h'i'}$ under the lag assumption, then calculation of the fixed-point associated with the demand or urban distribution in each time period will be avoided. That is, an agent will make decisions with a myopic memory in a period t according to the urban distribution in $t-1$ and $t-2$ (memory effect).

Equivalent to (3.8), the rent in (i) is obtained as follows:

$$r^t_i = \frac{1}{\mu} \ln(A^t_i + \gamma) \tag{3.32}$$

where γ is Euler' constant. Note that r^t_i depends on r^{t-1}_i when $P(move|hgi) < 1$ and $\alpha_h < 1$, generating intertemporal interaction between rents, whenever there are low probabilities of relocation.

Moreover, given the following equilibrium equation for households with memory

$$\sum_i Q_{h,g,j|i} * S_i = P(h|g) H^{t-1}_{gj}, \ \ \forall \ (h, g, j) \tag{3.33}$$

and the equilibrium equation for new agents (immigrants) in the urban system:

$$\sum_i Q_{h,0|i} * S_i = \tilde{H}^t_h, \ \ \forall \ h \tag{3.34}$$

Then the utility level reached in the period t by an agent (hgj) is obtained with from following equation:

$$a^t_{hgj} = -\frac{1}{\mu} \ln \left(\sum_i S^t_i * \phi_{hgj(i)} \exp \left(\mu \left\{ b^t_{hi} + \frac{1-\alpha_h}{\alpha_h} \frac{m^{t-1}_{g,j=i}}{\lambda^t_h} - r^t_i \right\} \right) \right), \ \ \forall (hgj) \tag{3.35}$$

and equivalently for new households we have

$$a^t_h = -\frac{1}{\mu} \ln \left(\sum_i S^t_i * \phi_{h(i)} \exp \left(\mu \left\{ b^t_{hi} - r^t_i \right\} \right) \right), \ \ \forall (h) \in \tilde{H}^t_h \tag{3.36}$$

As $\phi^t_{hgj(i)}$ depend of a^t_{hgj} ($\phi^t_{h(i)}$ depend of a^t_h) and the rent r^t_i depends on all levels of $a^t_{h',g',j'}$ of the agents with memory and the utility level of all the new agents $a^t_{h'}$ then (3.35) and (3.36) define a nonlinear fixed point system of equations in each period t.

To conclude, the deterministic and stochastic formulation of learning model, captures important elements associated with the relocation process as a dependence on past decisions valuations. These past and present assessments of property goods generate analytical results on the intertemporal interaction between rents, population distributions, equilibrium utility level and relocation probabilities.

3.4. Expectation effect in residential location: Imitation model

In this section the expectation or imitation effect on residential location will be analyzed. It is important to note that there are several factors that explain the reason the decisions of some economic agents (firms, schools, households, people) are similar to the decisions made by other different agents or with other characteristics. For example, the households that belong to a socioeconomic cluster type incorporate, within their valuations or preferences for location, preferences that other socially related households (Páez et. al. 2008) may have or other agents make decisions based on future income, employment, education, and others expectations (Kennan and Walker, 2011). In the context of the housing choice or residential relocation, all these behaviors will be called imitation, similar to the process studied in game theory and strategic behavior (Alós-Ferrer and Sclag, 2009). To understand this effect on residential location choices and the urban equilibrium in the short and long term, we propose to incorporate the expectations associated with the life cycle dynamics to this type of choices, which can be obtained through an endogenous decision (changing jobs, having children, etc.) or an exogenous schoks (layoffs, economic shocks, etc.) of the households.
Based on the experience of the econometric works as Kennan and Walker, (2011), it is assumed that an agent makes a decision on a period t of location and goods consumption, according to the goods valuation, given their current state, that is the socioeconomic cluster h in the period t. And additionally, it incorporates the possible changes in life cycle that means belonging to another cluster in the next period. For this model, we modify the utility function in problem (1) of the consumer's residential location choices as follows:

$$\begin{aligned} \max_i \max_x \theta^t_h U^t_h(x, Z^t_i) + \theta^{t+1}_h \sum_{f \in H} P(f^{t+1}|h^t) V^{t+1}_f(E(Z^{t+1}_i), E(I^{t+1}_f - r^{t+1}_i)) \\ \text{subject to} \qquad p^t x + r^t_i \le I^t_h, \end{aligned} \tag{3.37}$$

where $\theta^t_h + \theta^{t+1}_h = 1$.

In problem (3.37) it is assumed that an individual belonging to the cluster h in the period t has a set of change expectations in the life cycle for the period $t+1$ with probabilities $P(f^{t+1}|h^t)$ that represent changes in the household structure (job changes, income, education, having or not car, etc.) will make them transit to cluster f. Note that for simplicity, it is assumed that the individual anticipates transition possibilities only between consecutive periods, but this assumption can be easily generalized to a long-term problem. Additionally, a household f expects a utility in $t+1$ associated with real estate (i) given by $V_{fi}(E(Z^{t+1}_i), E(I^{t+1}_f - r^{t+1}_i))$, where V_f is the obtained valuation by the conditional indirect utility that households type f have, assuming as parameters the expected values associated with the residential location (i) in the period $t+1$ noted as $E(Z_i^{t+1})$ and from the expected income and rents $E(I_f^{t+1} - r_i^{t+1})$. To estimate the expectations valuation in $t+1$ a rationality argument in decision making will be used, given by the following imitation process:

$$E(Z_i^{t+1}) \approx Z_i^t \tag{3.38}$$

$$E(I_f^{t+1} - r_i^{t+1}) \approx I_f^t - r_i^t \tag{3.39}$$

that is, the agent h estimates the expectation parameters of period $t+1$ using known information of the parameters in t of the agent f, under the assumption that is rational agent. On the other hand, I_f^t can be seen as an approximation to the present value of the future income. Moreover, if we assume that $P(f^{t+1}|h^t)$ is a homogeneous transition matrix, that is $P(f^{t+1}|h^t) = P(f^t|h^{t-1})$, then, the consumer's problem would be formulated as follows:

$$\max_i \max_x \theta_h^t U_h^t(x, Z_i^t) + \theta_h^{t+1} \sum_{f \in H} P(f^t|h^{t-1}) V_f^t(Z_i^t, I_f^t - r_i^t) \text{ subject to } p^t x + r_i^t \leq I_h^t, \tag{3.40}$$

The problem of the consumer (3.40) is static in t and it supposes that the agents belonging to the cluster h knows the utility parameters ($tastes$) of the other potentially imitable agents with an associated valuation parameter θ_h^{t+1} . Besides for the household in cluster h, the value of $\theta_h^{t+1} \sum_{f \in H} P(f^t|h^{t-1}) V_f^t(Z_i^t, I_f^t - r_i^t)$ is a constant in the continuous optimization problem in x, then the optimal allocation of goods consumption is conditional only to the parameters of h and r_i^t.

$$x_h^t(p^t, I_h^t - r_i^t, Z_i^t, \theta_h^t)$$

Therefore, for a household type h, the indirect utility conditional on the real estate i is:

$$V_{hi} \equiv \theta_h^t V_h^t(I_h^t - r_i^t, Z_i^t) + \theta_h^{t+1} \sum_{f \in H} P(f^t|h^{t-1}) V_f^t(Z_i^t, I_f^t - r_i^t) \tag{3.41}$$

To obtain the willingness to pay for the imitation problem we will build on an illustrative example. It will be assumed, without loss of generality, that there are two households (h, f) such that the agent h has the following indirect utility function associated with the real estate i,

$$V_{hi} \equiv \theta_h^t V_h^t(I_h^t - r_i^t, Z_i^t) + \theta_h^{t+1} V_f^t(Z_i^t, I_f^t - r_i^t), \tag{3.42}$$

and for the agent f, the conditional utility on the real estate i is given by:

$$V_{fi} \equiv V_f^t(Z_i^t, I_f^t - r_i^t), \tag{3.43}$$

That is, the cluster change expectations or imitation level are zero for f.

On the other hand, given a fixed utility level for household h, this can be represented as $U_{ohf} \equiv \theta_h^t U_{oh} + \theta_h^{t+1} U_{of}$, since it must be consistent with the behavior of f considering that the utility of f is U_{of}. Then for h we have

$$\theta_h^t V_h(I_h^t - r_i^t, Z_i^t) + \theta_h^t V_f^t(Z_i^t, I_f^t - r_i^t) = \theta_h^t U_{oh} + \theta_h^{t+1} U_{of} \tag{3.44}$$

Under the assumption of quasi-linear direct utility for each of the agents we have

$$\theta_h^t \left(\lambda_h^t (I_h^t - r_i^t) + \lambda_h^t b_{hi}(Z_i^t) \right) + \theta_h^{t+1} \left(\lambda_f^t (I_f^t - r_i^t) + \lambda_f^t b_{fi}(Z_i^t) \right) = \theta_h^t U_{oh} + \theta_h^{t+1} U_{of} \tag{3.45}$$

Solving the rent variable, then the willingness to pay is:

$$B_{hf(i)}^t = \theta_h^t \left(\frac{\lambda_h^t I_h^t + \lambda_h^t b_{hi}^t(Z_i^t) - U_{oh}}{\theta_h^t \lambda_h^t + \theta_h^{t+1} \lambda_f^t} \right) + \theta_h^{t+1} \left(\frac{\lambda_f^t I_f^t + \lambda_f^t b_{fi}^t(Z_i^t) - U_{of}}{\theta_h^t \lambda_h^t + \theta_h^{t+1} \lambda_f^t} \right) \tag{3.46}$$

Or equivalently

$$\begin{aligned} B_{hf(i)}^t &= \frac{\theta_h^t \lambda_h^t}{\theta_h^t \lambda_h^t + \theta_h^{t+1} \lambda_f^t} \left(I_h^t + b_{hi}^t(Z_i^t) - \frac{U_{oh}}{\lambda_h^t} \right) + \frac{\theta_h^{t+1} \lambda_f^t}{\theta_h^t \lambda_h^t + \theta_h^{t+1} \lambda_f^t} \left(I_f^t + b_{fi}^t(Z_i^t) - \frac{U_{of}}{\lambda_h^f} \right) \\ &= \frac{\theta_h^t \lambda_h^t}{\theta_h^t \lambda_h^t + \theta_h^{t+1} \lambda_f^t} \left(a_h^t + b_{hi}^t(Z_i^t) \right) + \frac{\theta_h^{t+1} \lambda_f^t}{\theta_h^t \lambda_h^t + \theta_h^{t+1} \lambda_f^t} \left(a_f^t + b_{fi}^t(Z_i^t) \right) \\ &= \frac{\theta_h^t \lambda_h^t}{\theta_h^t \lambda_h^t + \theta_h^{t+1} \lambda_f^t} B_{hi}^t + \frac{\theta_h^{t+1} \lambda_f^t}{\theta_h^t \lambda_h^t + \theta_h^{t+1} \lambda_f^t} B_{fi}^t \end{aligned} \tag{3.47}$$

The result (3.47) can be extended to any agent h described by the consumer's problem (3.37), concluding that the willingness to pay with expectations $\widetilde{B}_{hi}^t$ is of the form:

$$\widetilde{B}_{hi}^t = \theta_h^t \frac{\lambda_h^t}{\bar{\lambda}_h^t} B_{hi}^t + \theta_h^{t+1} \sum_{f \in H} P(f^{t+1}|h^t) \frac{\lambda_f^t}{\bar{\lambda}_h^t} B_{fi}^t, \tag{3.48}$$

where $\bar{\lambda}_h^t = \theta_h^t \lambda_h^t + \theta_h^{t+1} \sum_{f \in H} P(f^{t+1}|h^t) \lambda_f^t$ is interpreted as an average of the income marginal utility between periods t and $t+1$. In the case that $P(f^{t+1}|h^t)$ represents long-term probabilities, $\bar{\lambda}_h^t$ would represent an expected marginal utility per period in the long term.

Making the following change in notation:

$$\psi_{hh} = \theta_h^t \lambda_h^t + \theta_h^{t+1} P(h^{t+1}|h^t) \lambda_h^t; \qquad \psi_{hf} = \theta_h^{t+1} P(f^{t+1}|h^t) \lambda_f^t, \quad \forall f \neq h \tag{3.49}$$

with $\bar{\lambda}_h = \sum_{f \in H} \psi_{hf}$, we obtain that the bid function can be written as:

$$\widetilde{B}_{hi}^t = \sum_{f \in H} \frac{\psi_{hf}}{\bar{\lambda}_h} B_{fi}^t = \sum_{f \in H} \frac{\psi_{hf}}{\bar{\lambda}_h} (a_f^t + b_{fi}^t(Z_i^t)) = \widetilde{a}_h^t + \widetilde{b}_{hi}^t(Z_i^t) \tag{3.50}$$

Whereby $\frac{\psi_{hf}}{\bar{\lambda}_h}$ is the percentage or valuation level on the willingness to pay of h that has the expectation of being an agent f in the future. Furthermore, the bid with imitation (3.50) has a new important component given by the inclusion of an expected utility level and the expected income to be noted as $\widetilde{a}_h^t$. This allows the analysis within the formulation of possible effects of future income change, before making decisions associated with residential location, with

$$\widetilde{a}_h^t = \frac{1}{\sum_f \psi_{hf}} \sum_f \psi_{hf} a_f^t = \frac{1}{\sum_f \psi_{hf}} \sum_f \psi_{hf} \left(I_f^t - \frac{U_f^t}{\lambda_f^t} \right) \tag{3.51}$$

The formulation (3.40) to (3.48) can be extended to other contexts independent of the life cycle dynamics, for example if we change $\theta_h^{t+1} P(f^{t+1}|h^t)$ by a weight w_{hf} that measures a social relationship among this type of households (Páez et. al, 2008), then $\frac{\psi_{hf}}{\bar{\lambda}_h}$ would indicate the valuation in the willingness to pay of the agent h of the social influence that has the agent

f. Thus, the formulation of the utility (3.41) and the willingness to pay (3.50) would be an extension of the classic social network studies and their impact on decision making; insomuch as the individual incorporates within his/her preferences valuations of other socially related agents, that in the urban economy context is defined as a location externality. It is necessary to use the imitable or related agents utility as a differentiator in the property valuation, since equivalent households can exist (belonging to the same socio-economic cluster) with different choices of residential location. Furthermore, the previously proposed modeling strategy can be used when it is unclear to which specific cluster a household belongs, called fuzzy clustering, (see Valente de Oliveira and Pedrycz, 2007), and thus to generate more representative willingness to pay of such agents. In that sense, in microsimulation problems, where it is assumed that each agent has a value belonging to each distinctive cluster, a more representative formulation of the utility as well as the bid can be obtained of each household for each property good.

In general, these imitation phenomena are analyzed and understood as a collective learning form (Alós-Ferrer and Schad, 2009) which uses information from other agents for decision-making, for example, in firms's location the profit of other previously located ones can be used (e.g. imitation location).

Analogous to what was developed in the learning model, in the stochastic version of expectation model, it will be assumed that the willingness to pay $\widetilde{B}^t_{h(i)}$ is a random variable, where the modeling error is assumed with Gumbel distribution with scale parameter μ. It is important to note that the bid with expectations includes an expected income within the formulation, as noted in (3.51); however, the bid at time t should not be higher than the revenue in this period, independent of the expectation levels, therefore if holds that:

$$\widetilde{B}^t_{h(i)} \leq I^t_h \tag{3.52}$$

If the restriction (3.52) is not met for the agent h with an equilibrium utility level then will not enter to the set of potential bidders for the property (i) in the period t.

Constraint (3.52) is important because in cases where the valuation of expectations on agents who are being imitating each other is very high in some property goods, their willingness to pay cannot exceed the current income (this income includes wage rates, loans, savings, spendings on durable goods, etc). Analogous to (3.22) and (3.23), these constraints are modeled for each agent and are included in the modeling through the *Constrained Logit* (Martínez et al 2009) with "*cut-off*"functions defined as the probability that a household in a cluster h meets it in every i, denoted as $\widetilde{\phi}^t_{hi}$, given by

$$\widetilde{\phi}^t_{h(i)} = \frac{1}{(1+\exp(w(\widetilde{B}^t_{h(i)} - I^t_h + \zeta)))} = \begin{cases} 1, & if\ (\widetilde{B}^t_{h(i)} - I^t_h \to -\infty) \\ \eta, & if\ (\widetilde{B}^t_{h(i)} - I^t_h \to 0) \end{cases} \tag{3.53}$$

Thus, the population distribution H^t_{hi} at a period t is expressed as:

$$H^t_{hi} = S_i * Q_{h|i} = S_i * \frac{H^t_h * \widetilde{\phi}^t_{h(i)} * \exp(\mu \widetilde{B}^t_{hi})}{\sum_{g \in H} H^t_g * \widetilde{\phi}^t_{g(i)} * \exp(\mu \widetilde{B}^t_{gi})} \tag{3.54}$$

Equivalent to what was shown in the static model $RB\&SM$ (Martínez and Henríquez, 2007) it is considered to exist externalities associated with the population distribution in each zone that affects the willingness to pay of households; therefore $\widetilde{B}^t_{hi}(H^t_{gj}, \forall g, j)$; thus $Q^t_{h|i}$ depends on $Q^t_{g|j}, \forall g, j$ obtaining a fixed-point system of equations.

Furthermore, the rent at each location is obtained endogenously by the expectation of the maximum bid, given by the logsum function:

$$r_i^t = \frac{1}{\mu} \ln \left(\sum_{g \in H} H_g^t * \widetilde{\phi}_{g(i)}^t * \exp(\mu \widetilde{B}_{gi}^t) + \gamma \right) \tag{3.55}$$

γ is Euler's constant.

Finally, the equilibrium condition ensures that every household is allocated somewhere, given by $\sum_i Q_{h|i}^t S_i^t = H_h^t$, obtaining

$$a_h^t = -\frac{\sum_g \psi_{hg}}{\psi_{hh}\mu} \ln \left(\sum_i S_i^t * \widetilde{\phi}_{h(i)}^t * \exp \left(\mu \left\{ \frac{\psi_{hh}}{\sum_g \psi_{hg}} b_{hi}^t - r_i^t + \sum_{f \neq h} \frac{\psi_{hf}}{\sum_g \psi_{hg}} * B_{fi}^t \right\} \right) \right), \quad \forall h \tag{3.56}$$

Analogous to (3.35) and (3.36), as rents r_i^t depend on each a_g^t, $\widetilde{\phi}_{hi}^t$ depend of $\widetilde{B}_{hi}^t$ and B_{fi}^t depends on a_f^t and on the utility levels of the imitable individuals by f, and then (3.56) constitutes a non-linear fixed-point system of equations described as $a_h^t = f(a_g^t, \forall g)$ defining the maximum levels of utility which are possible to obtain in equilibrium, with $\widetilde{a}_h^t = \frac{1}{\sum_f \psi_{hf}} \sum_f \psi_{hf} a_f^t$. This value is conditioned to the other agents' utility levels (both the equilibrium and the imitation effect) as well as the expected incomes $\widetilde{I}_h = \frac{1}{\sum_f \psi_{hf}} \sum_f \psi_{hf} I_f^t$ (see equation (3.51)). I.e. current rents and household utility levels (in t) depend on future incomes. This interesting result could explain one of the causes that generates speculation or uncertainty in the property market, for example, overvaluation or inflation of some real estates rents, associated with household expectations and state of the economy.

To analyze the imitation effect on urban distribution suppose that there are two clusters f, h with an expectation or imitation normalized factor denoted as $1 > \psi_{hf} > 0$. In addition, if given a location (i) such that $B_{fi} \geq B_{hi}$ then:

$$B_{hi} \leq (1 - \psi_{hf})B_{hi} + \psi_{hf} B_{fi} = \widetilde{B}_{hi} \tag{3.57}$$

and under the assumption that $\widetilde{\phi}_{hi}^t = \widetilde{\phi}_{fi}^t = 1$ it is obtained (income constraint is fulfilled), then

$$\frac{H_h^t * \exp(\mu B_{hi}^t)}{H_h^t * \exp(\mu B_{hi}^t) + \sum_{g \neq h} H_g^t * \widetilde{\phi}_{g(i)}^t * \exp(\mu \widetilde{B}_{gi}^t)} \leq \frac{H_h^t * \exp(\mu \widetilde{B}_{hi}^t)}{H_h^t * \exp(\mu \widetilde{B}_{hi}^t) + \sum_{g \neq h} H_g^t * \widetilde{\phi}_{gi}^t * \exp(\mu \widetilde{B}_{gi}^t)} \tag{3.58}$$

Thus:

$$H_{hi} \leq H_{hi}^{imi} \tag{3.59}$$

Where H_{hi} is the urban distribution with no imitation ($\psi_{hg} = 0$) and H_{hi}^{imi} is the urban distribution with imitation or expectation in i ($\psi_{hg} > 0$). Similarly, if in a real estate good (i) such that $B_{fi} \leq B_{hi}$ then $B_{hi} \geq (1 - \phi_{hf})B_{hi} + \psi_{hf} B_{fi} = \widetilde{B}_{hi}$ and $H_{hi} \geq H_{hi}^{imi}$. Note that if $\widetilde{\phi}_{hi}^t < 1$ and $\phi_{hi}^t = 1$, it means, if the inclusion of expectation effect makes the income constraint not met strictly, the result (3.58) and (3.59) are not necessarily true, because the current income level does not fully allow to meet the change expectations in the period t.

On the other hand, the dynamic modeling of residential relocation allows to include bid generation strategies (or utilities) that integrate interaction between different temporal and spatial information. For example, Habib and Miller (2010) and Martínez and Hurtubia (2006) use former periods information for the construction of valuations for each real estate type in the current period. One way to formulate bids with dynamic interaction is assuming that every bid function B^t_{hi} depends on urban distribution in the period $t-1$. That is, $B^t_{hi}(H^{t-1}_{gj}, \forall g, j)$. It means, the fixed-point calculation associated to urban demand is avoided. But not in terms of the utility $\bar{a}^t_h$ at equilibrium, which is obtained in each period using equation (3.56). It is important to see that in an intertemporal context the change on expectations are being renovated and updated, hence it is necessary to do the analysis period by period on the number of agents that actually change the household type, and also the new cluster sizes that generate variations of urban distribution. Note that even in a city with slow relocation processes, if the households change their socioeconomic characteristics over time, then the city will have different population distributions, given the agents' endogenous dynamics.

3.5. Numerical example: analysis of a dynamic model of residential segregation

To develop a numerical example in order to explain the proposed modeling of learning or imitation and its effects on urban configuration in the short and long term, bid functions will be used based on the theory of dynamic models of residential segregation (see Zhang, 2004; Grauwin, et. al., 2009). In this type of work it is sought to analyze policies that reduce the segregation levels in an urban system through simple numerical examples where the agents' behaviors are defined by a utility level which is associated with their set of neighbors in a given period and with relocation rules, given that utility. In O'Sullivan (2009), triangular bid functions (asymmetric preferences for integration) are used to analyze the dynamic models of segregation from a deterministic and disaggregated perspective.

Secondly, the purpose of this section is to analyze the urban system dynamics (long-term and short-term configuration) with the inclusion of imitation or learning processes through stochastic modeling (Logit structure) of the bid functions.

The question is: If the agents change of cluster over time (life cycle evolution), but learn from the location experiences or imitate the behavior of other agents; is it possible to reduce the segregation levels in the short and long term?

3.5.1. Numerical examples for memory model

Linear Bids and Lagged Extenalities

For the numerical exemplification, it is assumed that there are two equally sized zones and the only differentiating factor among the dwellings is given by those who inhabit each zone. For this case, every agent valuates each property in an area i according to the proportion of each household type in the zone. There are two agent types with the same income and which we assume WP fulfills income-constraint, so for this simulation it is not necessary to use the cutoff probabilities. In a first step, a bid function with the form $B^t_{hi} = a^t_h + b^t_{hi}(H^{t-1}_{gi}, \forall g)$ will be used, where H^{t-1}_{gi} is the lagged externality in the zone i (population dsitribution in $t-1$).

The bid function with myopic memory in the period t for an agent (h^t, g^{t-1}) which was located in the area j in the period $t-1$ is:

$$If \;\; i = j \implies B^t_{hgj(i)} = a^t_{hgj} + b_{hi}(H^{t-1}_{fi}, \forall f) + \frac{1-\alpha_h}{\alpha_h}\frac{\lambda^{t-1}_g}{\lambda^t_h}\left(b_{gi}(H^{t-2}_{fi}, \forall f) - r^{t-1}_i\right) \tag{3.60}$$

$$If \;\; i \neq j \implies B^t_{hgj(i)} = a^t_{hgj} + b_{hi}(H^{t-1}_{fi}, \forall f) \tag{3.61}$$

In this example, the following distribution linear bid function will be used:

$$b^t_{hi} = \sum_f \beta_{hf} \frac{H^{t-1}_{fi}}{\sum_{h'} H^{t-1}_{h'i}}, \tag{3.62}$$

where β_{hf} is the valuation that has an agent type h of the percentage of agents f in the zone i in $t-1$. In this case the bids in a period t $(B^t_{(h,g,j)i})$ depend on the problem solution in $t-1$ associated with the lagged valuation, and on $t-2$ corresponding to the myopic memory term.

Initially, the following values for β_{hf} are assumed to simulate the attraction among peers:

$$\beta_{hf} = \begin{cases} 20, & if \;\; h = f \\ 0, & if \;\; h \neq f \end{cases} \tag{3.63}$$

The first result we compute the static equilibrium solution with no memory effect, obtaining:

$H^*_{hi} =$ (3.64)

	Zone 1	*Zone 2*
Household 1	100	0
Household 2	0	100

where the population size population is $H = [100, 100]$. Note that peer attraction yield perfect segregation.

The variation of the solution with memory (3.60) and (3.61) will be analyzed with respect to the equilibrium solution (3.64) in each stage or modeling period. For this, the two next initial configurations are defined, which are necessary for calculating the urban configuration in the period 2:

$H^0_{hi} =$

	Zone 1	*Zone 2*
Household 1	50	50
Household 2	50	50

; $H^1_{hi} =$

	Zone 1	*Zone 2*
Household 1	51	49
Household 2	49	51

(3.65)

In addition, initially in each period it is assumed the following transition matrix through agents:

$P =$ (3.66)

	Household 1	*Household 2*
Household 1	0.6	0.4
Household 2	0.4	0.6

In this case, long term decisions will be obtained if distributions are numerically equal at least in three consecutive periods, this is achieved in this example by the transition matrix symmetry for agents, having the same number of agents per cluster. But if not so, it makes no sense to talk about a long-term solution because different household cluster sizes are constantly gotten. Finally, the following dissimilarity measure in each period t as an index of segregation:

$$dis^t = \frac{\sum_{h,i} \left| H^t_{hi} - H^*_{hi} \right|}{\sum_h H^t_h}, \tag{3.67}$$

with $dis^t \in [0,1]$ which in the case of being close to zero the system is more segregated and if it is close to 1 it corresponds to fully integrated or even distributions. In this way we obtain the Figure 3.1 representing the dynamics of dis^t (% of integration) at several decision periods for different memory levels assuming initial full integration H^0_{hi}.

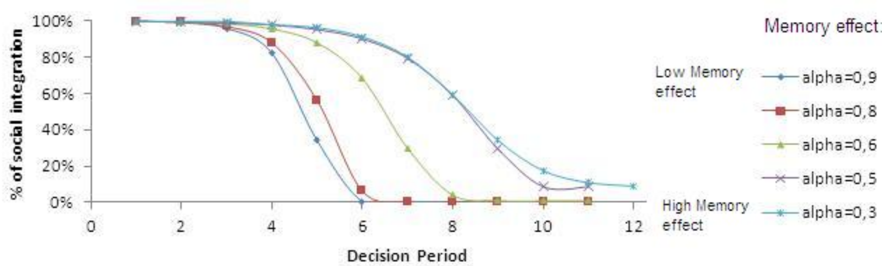

Figura 3.1: Percentage of integration in each period for different values of α_h

From Figure 3.1, we conclude that for $\alpha_h < 0{,}5$, the system eventually converge to full segregation. However, for $\alpha_h < 0{,}5$, which is a high memory level; it is achieved in the long term an integration level greater than 2 % in each time period. It means that in the long term, the equilibrium solution at each time period is different than the static solution without memory effect (segregated solution). Furthermore, the dynamic process of segregation is slower when agents value memory (in this case starting from the integrated solution as the initial configuration). For example, with a memory valuation of 0,1 in 6 periods the segregated solution is obtained. On the other hand, when having a valuation of 0,5, the segregated configuration is obtained after 9 periods. When $\alpha_h \geq 0{,}5$ the segregating process is more relevant in the long term than the agents' transition between both clusters and the memory valuation.
In addition, there is no evidence that the starting point (initial configuration) generates changes in the long-term solution. For example, given a memory value $\alpha = 0{,}2$, the long-term configuration with small level of integration is

$$H^{\ell}_{(\alpha_h=0{,}2)} = \begin{array}{c|c|c} & \textit{Zone 1} & \textit{Zone 2} \\ \hline \textit{Household 1} & 86 & 14 \\ \hline \textit{Household 2} & 14 & 86 \end{array} \tag{3.68}$$

We explore the result (3.68) using different starting points:

$$1).\ H^0 = \begin{bmatrix} 99{,}9 & 0{,}1 \\ 0{,}1 & 99{,}9 \end{bmatrix},\quad 2).\ H^0 = \begin{bmatrix} 75 & 25 \\ 25 & 75 \end{bmatrix},\quad 3).\ H^0 = \begin{bmatrix} 50 & 50 \\ 50 & 50 \end{bmatrix},\quad 4).\ H^0 = \begin{bmatrix} 60 & 40 \\ 40 & 60 \end{bmatrix}.$$

The following indicator is defined by period: $dis^{t\ell} = \frac{\sum_{h,i} \left| H^t_{hi} - H^{*\ell}_{hi} \right|}{\sum_h H^t_h}$, which measures the percentage dissimilarity between the long-term solution (3.68) H^{ℓ} and the solution found in the period

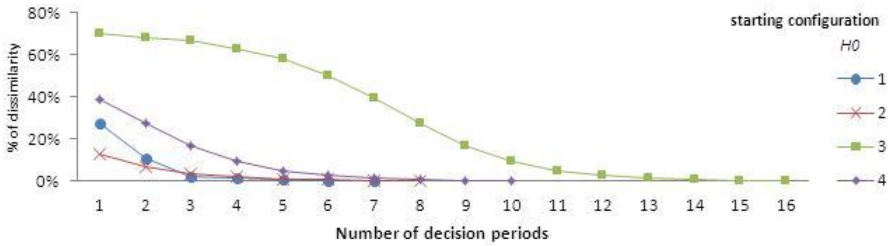

Figura 3.2: Number of convergence periods and percentage dissimilarity with different starting points

t with different starting points. Getting the following sequence per period (Figure 3.2). In all cases the long-term solution is reached, but with a different number of periods which depend on the initial dissimilarity level dis^0.

The results in Figures 3.1 and 3.2 were obtained with a fixed structure of the matrix P. Now, the sensitivity with respect to the transition matrix among agents will a symmetric matrix of the form:

$$P = \begin{array}{c|c|c} & \textit{Household 1} & \textit{Household 2} \\ \hline \textit{Household 1} & 1-p & p \\ \hline \textit{Household 2} & p & 1-p \end{array} \tag{3.69}$$

Clearly, if there are strong memory effects and transition of agents, the segregation level obtai-

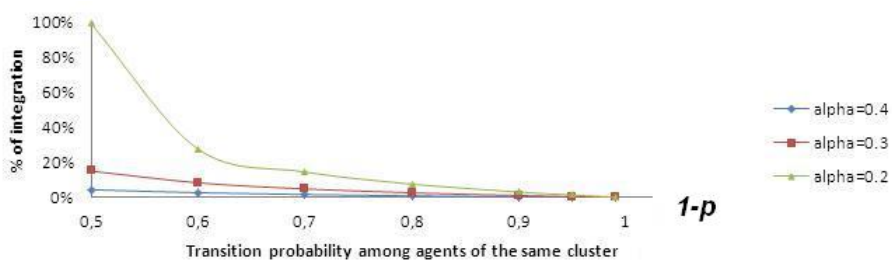

Figura 3.3: Level of social integration in the long term for different transition levels $(1-p)$

ned in the static model is diminished (Figure 3.3). When $p = 0$, which is no transition of agents, in all cases the segregated solution is reached in the long term.

In the case of $p = 0{,}1$ a level of integration of 8 % is achieved for values of $\alpha = 0{,}2$. That is, the valuation effect of externalities is quite strong independently from the memory valuation level. If probability between cluster (p) is higher then integration rates is higher too, but always less than 20 or 30 % except in the case that $\alpha = 0{,}2$ and $p = 0{,}5$ obtaining total integration.

In the following case $\alpha = 0{,}5$ will be set and the number of periods needed to reach the equilibrium solution will be analyzed, which in this case is the segregated solution by varying the transition probability value p. According to Figure 3.4, in all cases the segregated solution is reached but in a different number periods, so that when $1-p$ increases (low transition among clusters), the segregation rate is greater due to the lack of memory associated with the valuation of belonging to the other cluster.

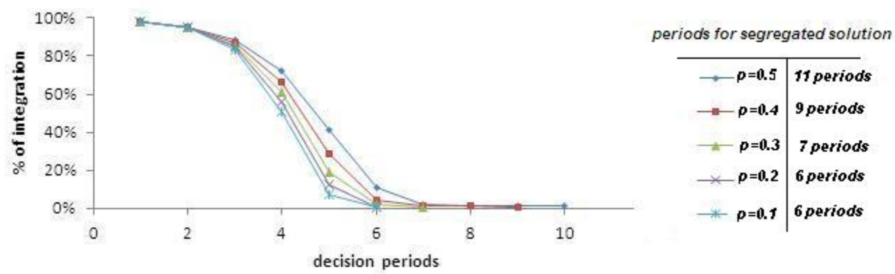

Figura 3.4: Level of integration in several periods if there are variations in p

Within the initial simulations it is assumed a scale parameter logit $\mu = 0{,}5$, this parameter is inverse to the variance of the population. It is going to be analyzed the result of long-term memory problem (3.60) and (3.61) with a lag in the urban distribution varying the scale parameter between 0.1 and 0.9 with a memory factor $1 - \alpha_h$ and assuming a transition matrix among households (3.66).
In addition, the usal segregation indicator (3.67) that compares the long-term solution $H^{\ell}_{hi}(\alpha_h, \mu)$ with the segregated solution (3.64) H^*_{hi} will be used as a measure of analysis, where $H^{\ell}_{hi}(\alpha_h, \mu)$ is the problem solution in the long term, conditional to the memory (α_h) and the scale parameter (μ).

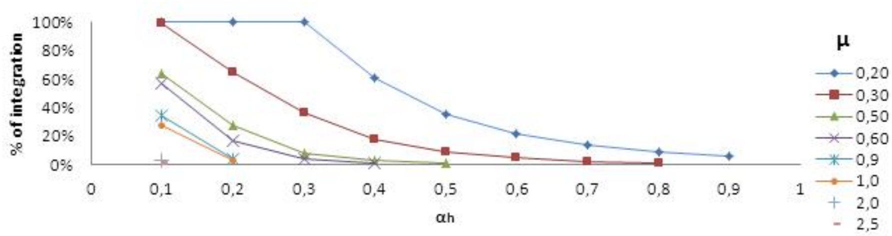

Figura 3.5: Level of integration in the long term if there are variations in α and μ

Note that from the results of Figure 3.5, for the scale factor $\mu = 0{,}2$ the long-term solution is always different than the segregated model. It means, independently of the memory value, the scale is so small that generates a large variance within the population. When $\alpha_h \geq 0{,}9$, and $\mu \geq 0{,}3$ an integration factor greater than 1 % is not achieved. To achieve integration factors greater than 50 %, with a memory of 0,9 ($\alpha = 0{,}1$) there should be a scale parameter less than 0,71. Analogously, given a memory of 0.7 ($\alpha = 0{,}1$), then the scale parameter must be less than or equal to 0.26 in order to achieve high levels of integration. Under this scenario it is necessary that besides the interaction among agents, there will be a variance of the high bid distribution to generate lower results of segregation.

In summary, the numerical examples with lagged distribution externalities show the possibility of reducing the segregation levels in the short and long term, due to the inclusion of high memory valuation ($\alpha_h < 0{,}5$), starting with no segregated solutions (integrated urban distribution). In this way, we achieve the conclusion that agents can learn from the experiences, since had the opportunity to interact with other types of household, making some levels of integration, dependent of the internal dynamics of households $1 - p > 0$ and positive utility levels of previous experience $m_i^{t-1} > 0$. This conclusion could generate possible guidelines and strategies for explain the reasons to have less segregated cities as result of endogenous agents individual learning.

Linear Bids and no Lagged Externalities

In this case the memory effect with externalities will be included in the static RB& SM model. That is, we'll analyze the solution obtained in each period of a static equilibrium model with memory. The bid function with myopic memory in a period t for an agent (hg) located in the zone j in the period $t-1$ is:

$$\text{If } \ i = j \implies B_{hgj(i)}^t = a_{hgj}^t + b_{hi}(H_{fi}^t, \forall f) + \frac{1-\alpha_h}{\alpha_h}\frac{\lambda_g^{t-1}}{\lambda_h^t}\left(b_{gi}(H_{fi}^{t-1}, \forall f) - r_i^{t-1}\right) \tag{3.70}$$

$$\text{If } \ i \neq j \implies B_{hgj(i)}^t = a_{hgj}^t + b_{hi}(H_{fi}^t, \forall f) \tag{3.71}$$

To generate a model analysis the following bid function will be used for an agent h by a zone i in a period t

$$b_{hi}^t = \sum_f \beta_{hf} \frac{H_{fi}^t}{\sum_{h'} H_{h'i}^t}. \tag{3.72}$$

That is to say, it is necessary to find the fixed point solution associated with externalities $Q_{h|i}^t$. First, we analyze the obtained difference in the period 1 with integrated memory (see (3.65) for H^0), with the segregated solution (3.64) by means of the following indicator:

$$dis^1 = \frac{\sum_{h,i} \left|H_{hi}^1 - H_{hi}^*\right|}{\sum_h H_h} \tag{3.73}$$

Where H_{hi}^1 is the static solution with memory in the period 1 assuming an integrated distribution in the former period (see, (3.65) for H^0). In a first step, it is assumed that the transition matrix among agents is as follows:

$$P = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

The results obtained for the first period are described in Figure 3.6. Note that, if having sig-

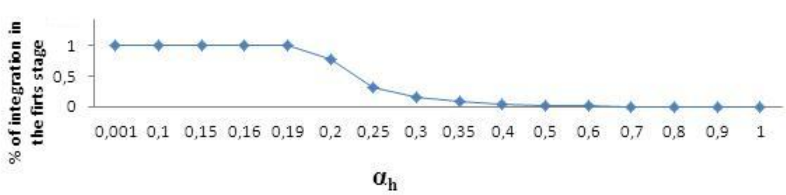

Figura 3.6: Level of integration for externalities model solution with memory $1-\alpha$

nificant memory effects $\alpha_h < 0{,}5$ lower levels of segregation are achieved in the static model with memory associated to integrated urban distribution. That is, if a high past valuation (integration), it is feasible that a complete segregation model soften its results. According with the Figure 3.6, $\alpha = 0{,}2$ it is a key value for the memory analysis, noticing nonlinear effects associated with the outcome valuation given by the probabilistic structure related to the logit formulation of the problem.

If we generate a long-run dynamics described by the recursive equations (3.70) and (3.71) all configurations converge to the segregated solution in the long-term, but with different rates or convergence periods. The table 1 shows the number of periods needed to reach a segregated configuration of 99 %. On other hand, if sensitivity is done in the transition matrix among households

α	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
periods	6	4	3	3	2	1	1	1	1

Cuadro 3.1: Number of required periods to reach a segregated solution

P with the same transition structure of (3.69), $p \in \{0{,}1; 0{,}2; 0{,}3; 0{,}4\}$ and $\alpha \in \{0{,}1; 0{,}2; 0{,}3; 0{,}4\}$, then the integration factor or dissimilarity (dis) between the segregated solution and the obtained solution in the long term (assuming a minimum of 2 % of integration) is presented in the Figure 3.7. It is visible that in the long term, given the existence of transition probabilities

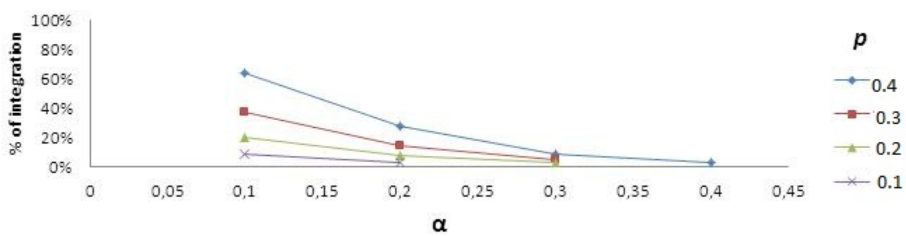

Figura 3.7: Level of integration in the long term by varying α and p

among agents in each one of the periods, it is feasible that segregated configurations are not obtained. For example if $p = 0{,}3$ and the memory factor is 0,1 ($\alpha = 0{,}9$) a 37 % level of integration is achieved. When $p = 0{,}1$, an integration factor of maximum 8.3 % is achieved when $\alpha = 0{,}1$. On the other hand, when $p = 0{,}4$ integration values greater than 10 % are achieved when there is a value of $\alpha < 0{,}3$.

As a general conclusion about the developed memory simulation models, it is clear that in the short and long terms non-segregated solutions are obtained when memory effect exists in the agents' decision-making process. It is worth to say that with other bid functions, like the triangular functions, the same effects with lower memory valuations and no transition of agents are achieved, which allows to define the effect of myopic learning as an integrating element. Furthermore, we have replicated different segregation effects through the dynamics of urban agents, that other authors have obtained assuming triangular bid functions (for example, see O'Sullivan, 2009).

3.5.2. Numerical examples for imitation model

To analyze the imitation effect in the urban distribution, we will use the bid functions of previous examples.

Example 1: Imitation and Lagged Externalities

First, we suppose that the bid of an agent h by de zone i has the following lagged externality effect:

$$\widetilde{B}_{hi}^{t} = \sum_{f \in H} \psi'_{hf}(a_f^t + b_{fi}(H_{gi}^{t-1}, \forall g)) \tag{3.74}$$

Where $\psi'_{hf} = \frac{\psi_{hf}}{\sum_f \psi_{hf}}$ is the percentage of the willingness to pay of h, that has the expectation of being an agent f in the future. This value is an aggregate and normalized weight ($\sum_f \psi'_{hf} = 1,\ \forall h$) which includes lifecycle transition probabilities $P(f|h)$, the valuation of h by expectations (θ_h^{t+1}) and the marginal utilities of income $\lambda_g^t, \forall g : P(g|h) > 0$ (see equations (3.49) and (3.50)).

In this first example, we consider a symmetrical effect in the imitation among agents to ensure the same cluster size throughout the planning horizon, as follows:

$$\psi'_{hf} = \begin{pmatrix} \psi' & 1-\psi\prime \\ 1-\psi' & \psi' \end{pmatrix}, \quad with \quad \psi' \in \{0{,}6; 0{,}7; 0{,}8; 0{,}9; 1\}, \tag{3.75}$$

further,

$$b_{fi} = \sum_g \beta_{fg} \frac{H_{gi}^{t-1}}{\sum_{h'} H_{h'i}}, \quad \beta = 15 * \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix},$$

then the static problem solution (long term) without imitation ($\psi' = 1$ and $\mu = 0{,}5$) is:

$$H_{11} = H_{22} = 99{,}95; \quad H_{21} = H_{12} = 0{,}05.$$

Figure 3.8 shows the percentage of integration under varying ψ' with different periods in the planning horizon. In the case where $\psi' = 0{,}7$, the steady state is obtained in 36 iterations,

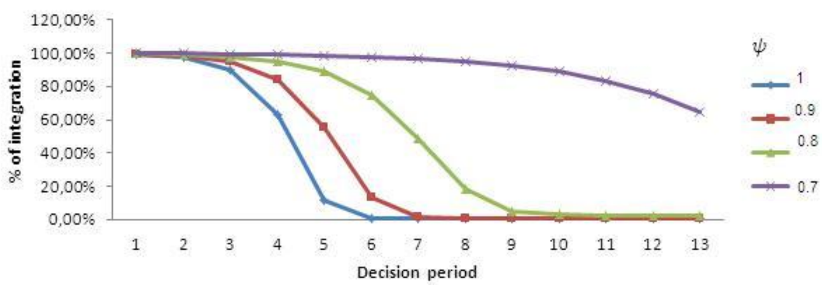

Figura 3.8: Level of integration in each period by varying ψ'.

obtaining the following urban configuration:

$H_{hi}^t =$

	Zone 1	Zone 2
Household 1	93,2	6,8
Household 2	6,8	93,2

$, \quad t \geq 36. \quad (3.76)$

For $\psi' = 0{,}6$, independent of the starting point it will be converged to the total integration state ($H_{hi} = 50,\ \forall h, i$). The above results have sense, because given the bid formulation, the higher the imitation level the more homogeneous the bids are, insomuch as the agents have the same income level. The symmetrical effect on the agents' valuations allow to obtain this type of numerical evidence; then the non-symmetrical effects in other bid functions will be tried to analyze.

Example 2: Imitation with no Lagged Externalities

Assuming now that for a static problem solution there are 3 agents and each ones valuation will be as follows:

$$B_{hi}^{t} = \sum_{f} \psi'_{hf}(a_{f}^{t} + b_{fi}^{t}(Q_{gi}^{t}; \forall g)); \text{ with } b_{fi}^{t} = \sum_{g} \beta_{fg} \frac{H_{gi}^{t}}{\sum_{h'} H_{h'i}^{t}}, \text{ and } \beta = 20 * \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \tag{3.77}$$

Moreover, there are 3 zones and 3 agent types, where the stochastic static solution without imitation has the form ($\mu = 0{,}5$):

$$H_{hi}^{*} = \begin{cases} 99{,}2, & if \;\; h = i \\ 0{,}4, & if \;\; h \neq i \end{cases} \tag{3.78}$$

Strictly speaking (3.78) can be seen as a totally segregated solution. Various scenarios of imitation will be analyzed, where on agent will be always the imitated one and the other two the imitators.

SCENARIO 1 (A leader and two followers): Agents 1 and 2 imitate agent 3 with a valuation of their bid function of the form:

$$\text{Imitation normalized matriz: } \left\{ \frac{\psi_{hf}}{\sum_{g} \psi_{hg}} \right\}_{h,f \in H} = \begin{bmatrix} \psi & 0 & 1-\psi \\ 0 & \psi & 1-\psi \\ 0 & 0 & 1 \end{bmatrix} \tag{3.79}$$

In a first step, we consider ψ between 0,6 and 0,9 will be analyzed, using the same social integration indicator achieved in equilibrium, given by

$$dis^{imi} = \frac{\sum_{h,i} \left| H_{hi}^{imi}(\psi) - H_{hi}^{*} \right|}{\sum_{h,i} H_{h}} \tag{3.80}$$

Where $H_{hi}^{imi}(\psi)$ is the long-term solution with an imitation factor given by $1 - \psi$ for agents 1 and 2. The results of dis^{imi} are shown in the Figure 3.9. A global social integration factor of

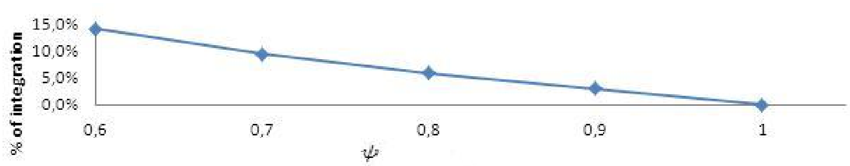

Figura 3.9: Level of long-term integration (scenario 1) by varying of ψ.

14 % is achieved when $\psi = 0{,}6$ but the interesting thing is that at the area where the agent 3 (imitable agent) is mostly located (zone 3), less integration factors are achieved. For example, when $\psi = 0{,}6$ we have the following population distribution

$H_{hi}^{imi}(\psi = 0{,}6) =$ (3.81)

	Zone 1	*Zone 2*	*Zone 3*
Household 1	92	5,4	2,6
Household 2	5,4	92	2,6
Household 3	2,6	2,6	94,8

That is, although given the imitation effect of households 1 and 2 that values strongly the preferences of 3, the equilibrium solution with such behavior results in less segregation between these two types of households. One of the reasons for this result is that there is a lower valuation of segregated living, and additionally, 3 does not change its initial preference; for this reason this agent's integration level with respect to the others is less. Note that in this case the rents have the behavior of Figure 3.10.

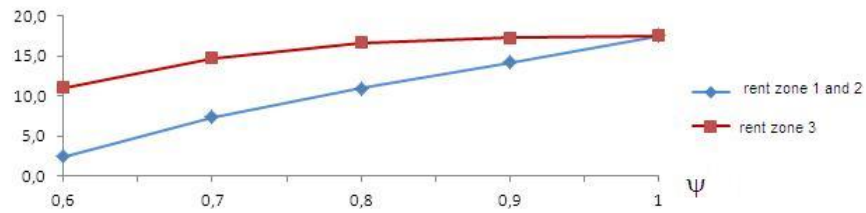

Figura 3.10: Rents in each zone (scenario 1) by varying ψ.

The results of the Figure 3.10 show that the willingness to pay for agents 1 and 2 decreases in the zones where they are mostly located. This is for two reasons, the expectancy associated with the idea of integrating with an agent 3, which cannot be achieved given the preferences of such household type, and additionally being in the same zone with the same kind of agents is less valued. In addition, integration with non-imitable agents generates less valuation to goods.
SCENARIO 2 (A leader and social interaction between followers): Agents 1 and 2 imitate each other and imitate agent 3, with a valuation of their bid function of the form:
Imitation normalized matriz:

$$\left\{\frac{\psi_{hf}}{\sum_g \psi_{hg}}\right\}_{h,f\in H} = \begin{bmatrix} \psi & \frac{1}{2}\{1-\psi\} & \frac{1}{2}\{1-\psi\} \\ \frac{1}{2}\{1-\psi\} & \psi & \frac{1}{2}\{1-\psi\} \\ 0 & 0 & 1 \end{bmatrix} \tag{3.82}$$

In this case, the integration factors increase with respect to scenario 1. The Figure 3.11 below shows the results: A social integration factor of 18 % is achieved when $\psi = 0{,}6$; in the zone

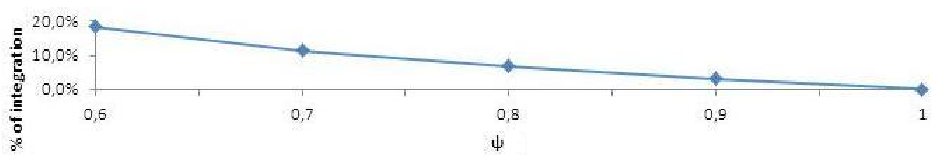

Figura 3.11: Level of long-term integration (scenario 2) by varying of ψ.

where the agent 3 (imitable agent) is mostly located lower integration factors are obtained with respect to the last scenario, because the imitation preferences of agents 1 and 2 are less than scenario 1. For example, when $\psi = 0{,}6$ we have the following urban distribution

$H_{hi}^{imi}(\psi = 0{,}6) =$ (3.83)

	Zone 1	*Zone 2*	*Zone 3*
Household 1	87,6	10,9	2,6
Household 2	10,9	87,6	1,5
Household 3	1,5	1,5	97,1

The rents follow a similar results behavior of scenario 1.
SCENARIO 3 (Each group follows the valuation of the next household cluster up in socio economics): In this last case, the following valuation matrix of each one of the agents' bids is assumed
Imitation normalized matriz:

$$\left\{\frac{\psi_{hf}}{\sum_g \psi_{hg}}\right\}_{h,f\in H} = \begin{bmatrix} \psi & \frac{1}{2}\{1-\psi\} & \frac{1}{2}\{1-\psi\} \\ 0 & \psi & 1-\psi \\ 0 & 0 & 1 \end{bmatrix} \tag{3.84}$$

In this case, the agent 1 imitates to the agents 2 and 3 with the same valuation. On other hand, the agent 2 imitates to agent 3. The results shows that integration factors increase with respect to scenario 1 and decrease with respect to scenario 2. In addition, a higher segregation level is obtained in the imitable agents (2 and 3) zones than in the zone where the agent 1 is mostly located.
Moreover, the rents in each zone have the behavior of Figure 3.12. This rents' result indicates

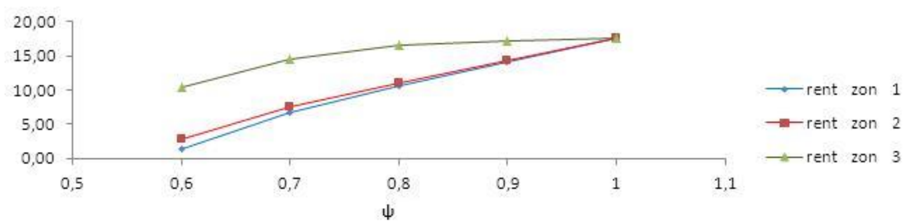

Figura 3.12: Rents in each zone (Scenario 3) by varying of ψ.

that the higher the agents valuation by imitating others' valuation and given that the others valuation is not coexisting with imitator agents, and the higher integration level, the less the willingness to pay for each property good.
The results in scenarios 1, 2 and 3 show that in face of imitation effects or change expectation, independent of other agents' valuations, it is possible to decrease the segregation levels. In these particular cases where there was a heavily imitated agent, such social integration was achieved because this effect decreases the imitator agents' valuation to live in a segregated space (only with peers), getting that in many cases its bid's decrease in that factor that pursues living with the imitable agent generates greater interaction with other imitator agents.

Example 2.1: An Indifferent Agent and Externalities

In the following case, it will be assumed that there is an agent indifferent to the neighborhood (agent 1) and an imitator agent (household type 2) and finally an imitated agent (type 3). Thus, the valuation without imitation is of the form:

$$B^t_{hi} = \sum_f \psi'_{hf}(a^t_f + b^t_{fi}(Q^t_{gi};\forall g)); \text{ with } b^t_{fi} = \sum_g \beta_{fg}\frac{H^t_{gi}}{\sum_{h'} H^t_{h'i}}, \text{ and } \beta = 20 * \begin{pmatrix} 1/3 & 1/3 & 1/3 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \tag{3.85}$$

Additionally, the imitation matrix is:

$$\left\{ \frac{\psi_{hf}}{\sum_g \psi_{hg}} \right\}_{h,f \in H} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \psi & 1-\psi \\ 0 & 0 & 1 \end{bmatrix} \tag{3.86}$$

Figure 3.13 shows the population size in each zone varying the imitation factor ψ. For example, the first figure in 3.13 show the type 1 household distribution in each zone. When $\psi = 1$ an

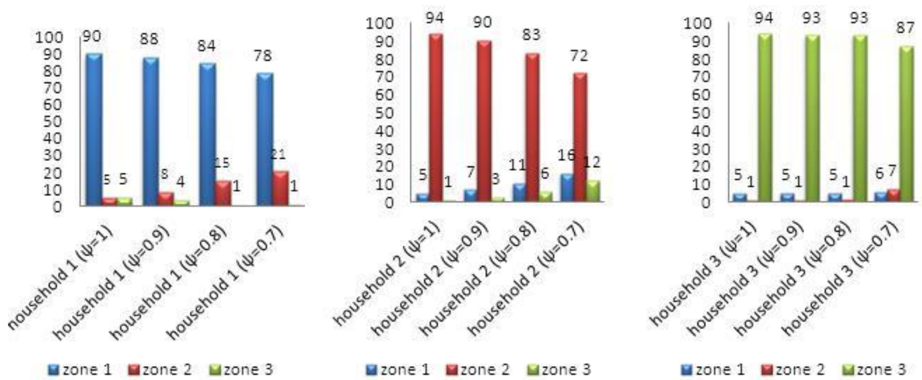

Figura 3.13: Long-term population distribution varying of ψ.

integration level of 15.2 % is accomplished given the preferences of agent 1, which is indifferent to urban distribution. In the case that $\psi = 0{,}7$ an integration level of 42 % is achieved; however in zone 3 a similar effect to the above scenarios is observed, because only 13 % of households type 3 is located outside the area where they are concentrated (zone 3). Moreover, the amount of households type 1 located in zone 3 decreases (going from 5 to 1) obtaining more type 2 households (imitators) that will be located with the agents type 3 (going from 1 to 12 type 2 households in the zone 3). That is, the area 3 has an integration of 13 % with respect to the segregated solution in that area (the solution without imitation has a zonal integration of 6 %). On the other hand, the segregation decreases more rapidly in zone 2 with the decreasing of ψ, mainly because the agents that are mostly located in this region (agents 2) imitate the valuations of agent 3 decreasing their own intentions to coexist with their own kind of households and creating the possibility of integration with households type 1 and 3. Indifferent agents (type 1) generate an important role since they reduce segregation in all cases and also allow agent 2 to achieve a greater integration with the agent 3 because of its indifferent nature in the valuation. Anyway, a greater integration would be achieved between 2 and 3 if the agent 3 had a lower willingness to pay to live with its peers.

3.6. Conclusions and final discussion

This paper presents two microeconomic formulations of a household choice model of residential location that incorporates the effects of past experiences and expected future change in the household life cycle.

In a first formulation, a static microeconomic model with endogenous learning is formulated where households give higher utility to real estate with previous positive experiences and less valuation to property in which they had negative experiences. For this, the memory of a real estate is analyzed by the gained utility in former periods. Given the process of utility maximization decision-making with endogenous learning, the willingness to pay formulation when an agent changes category between periods is obtained, generating an additional differentiating element among households based on their history. It is proposed a stochastic formulation assuming logit distributions of behavior to obtain urban distributions based on a myopic learning. Finally, some numerical simulations are presented based on classical models of residential segregation that seek to explain the presented modeling and its effects in making location decisions. For this, two scenarios are built, firstly it is assumed that the willingness to pay are evaluated with a lag, and secondly incorporating the externality effect in the same period. In all cases it was showed that the memory and the variation of agents are a differentiating factor within the static solution without learning, achieving solutions different than the static one without memory in both the short and long term.

In particular, the numerical example developed analyzes the possibility of reducing the levels or percentage of segregation in the short and long term, due to the inclusion of learning or memory processes starting with integrates solutions. In this way, we achieve the conclusion that agents can learn from the experiences since, they had the opportunity to interact with other types of household, making some levels of integration, dependent of the internal dynamics of households and positive utility levels of previous experience. This conclusion could generate possible guidelines and strategies for explain the reasons for having less segregated cities as result of endogenous agents decision.

On the other hand, the microeconomic modeling of residential location with memory is an interesting contribution in the social assessment of private or public projects, because two agents of an urban system with identical conditions (residential location and household type) in a period t can perceive the utility differently to their current residential location because of the past experiences' reference. In this sense, a social planner should include within the analysis not only the perceived utility in the period t, but also how the property characteristics, where every household is located, improve or worsen with respect to previous periods. Moreover, the formulation increases the amount of household categories because it includes within the characteristics or descriptor attributes of the cluster all the locations background, generating a great variety of attributes within the agent's classification. Then, each household will be described by their socioeconomic characteristics in the period t and the characteristics in previous periods that define the perceived utilities in their anterior locations. Thus, the proposed formulation provides a useful tool in micro-simulation models, where each agent is analyzed in detail.

As a general conclusion, this work includes a theoretical analysis of the effects of agents learning in the aggregated urban configuration and in the individual choice of residential relocation, which have been studied and evidenced in previous econometric studies.

Furthermore, the use of household lifecycle expectations in relocation decisions is incorporated in the discrete choice microeconomic formulation by means of transition probabilities among household clusters in the life cycle and the imitation's hypothesis of such agents under the consideration that they behave rationally. This postulation considers that imitable or expected agents' tastes are known. Based on a microeconomic consumer formulation, a multi-objective bid function is obtained which includes an expected income per unit of time and a utility consistent with the behavior of the agents that are potentially imitables. For both the expectations and learning model, it is necessary to include an income restriction per period, since such valuations can generate infeasible willingness to pay because they are greater than the net income of each household in that period. In the developed simulations, it is shown that the incorporation of these dynamic effects of expectations in the agents' behavior as an endogenous valuation decreases the segregation levels when there are possibilities for interaction with other agents. That is, if it is tried to partly imitate a given agent's behavior, but this one values strongly coexisting among its peers; it is very difficult to increase drastically the integration with these types of households. The microeconomic formulation and the willingness to pay of the model with imitation are extensible to other contexts, such as social networks or fuzzy clustering due to the interaction among different agents and the effect of being able to incorporate their likes (valuations) in their own valuation.

As a final conclusion of the theory developed in this paper, it can be analyzed the relocation process by means of endogenous forces or the households' own generating attraction to relocation or location in already known goods (memory effect) by the agents given by past experiences, assuming that the obtained utility level was positive or change forces given by expectations in the life cycle, such as the possibility of a better income or changes in long-term activities (work, education). In that sense, each household's bid can be seen as a valuation given by the agents to their own experiences and other ones' valuations related or imitable under the assumption that they make rational decisions.

In this way, it is proposed a consumer's problem associated with the choice by the real estate i in a period t by an agent (h, g, j) where h is the cluster in the period t, g is the cluster in the period $t-1$ and j is the location in $t-1$ and with a set of expectations described by the probability $P(f^{t+1}|h^t)$ described as:

$$\max_i \max_x \theta_h^t U_h^t(x, Z_i^t) + \theta_h^{t+1} \sum_f P(f^t|h^{t-1}) V_f(Z_i^t, I_f^t - r_i^t) + \theta_h^{t-1} m_{gj}^{t-1}$$

For simplicity it might be assumed that the agent h analyzes the current utility that an agent f receives for the good i without including the learning or experience valuation of that agent because of two reasons: the first is that it is very difficult to know the valuation given by other agents to past experiences, and on the other hand, given the formulation of the learning model, it would be difficult to include all these behaviors in the valuation of a single agent, because it would become a high dimensionality problem, since it would not only include its own history but the others' one. However, since the history process helps to generate or form valuations of every agent and under the assumption that the transition probability matrix among clusters is a sparse matrix (slow processes in the life cycle change), it might be assumed that under the imitation process the history valuations of the imitable agents are captured.

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