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COSMIC INFLATION IN A LANDSCAPE OF HEAVY FIELDS

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Resumen

Un estudio sobre la influencia de campos masivos durante inflación es presentada. Las recientes observaciones, de que estos modos masivos pueden influenciar la evolución de lo modos de curvatura inflacionarios, tiene implicancias fenomenológicas de largo alcance. Dado que hay solo una dirección plana en el potencial inflacionario, los campos masivos (en este contexto, excitaciones ortogonales a la trayectoria de la solución de fondo) puede ser integrados, resultando en una teoría efectiva de campos (TEC) para los modos adiabáticos exhibiendo una velocidad del sonido reducida.

Es demostrado por construcción, que es de hecho posible tener una solución inflacionaria de fondo donde la velocidad del sonido permanece suprimida y las condiciones de *rodamiento lento* persisten por el tiempo suficiente. Una condición de adiabaticidad es derivada, bajo la cual la TEC permanece una descripción confiable para la evolución lineal de los modos de curvatura. Encontramos que esta condición consiste en demandar que la tasa de cambio de la velocidad angular del giro permanezca suprimida con respecto a las masa de los modos masivos.

Finalmente, la TEC es calculada en el caso cuando un segundo campo masivo (o que haya una tercera dirección ortogonal a la liviana inflacionaria y a la previa dirección masiva) es incluido. Es probado que la existencia de campos adicionales induce la existencia de diferentes acoplamientos que los del caso de un solo campo, implicando que una detallada caracterización de no-Gausianidades permitirá constriñir esta clase de escenarios.

Abstract

A study on the influence of heavy fields during inflation is presented. The recent observations that this heavy modes can influence the evolution of curvature inflationary ones has far reaching phenomenological implications. Provided that there is only one flat direction in the inflaton potential, heavy fields (in the present context, field excitations orthogonal to the background trajectory) can be integrated out, resulting in a low energy effective field theory (EFT) for adiabatic modes exhibiting a reduced speed of sound.

It is shown by construction that it is indeed possible to have inflationary backgrounds where the speed of sound remains suppressed and slow-roll persists for long enough. An adiabaticity condition is derived, under which the EFT remains a reliable description for the linear evolution of curvature modes. We find that this condition consists on demanding that the rate of change of the turn's angular velocity stays suppressed with respect to the masses of heavy modes.

It is finally computed the inflationary EFT when a second heavy-field (orthogonal to both the light inflationary and heavy isocurvature directions) is included. It is probed that the existence of additional fields induce the existence of different couplings from the single field case, impliying that a detailed characterization of non-Gausianities will allow us to constrain this class of scenarios.

Memory is a kind
of accomplishment,
a sort of renewal
even
an initiation, since the spaces it opens are new places
inhabited by hordes
heretofore unrealized,
of new kinds-
since their movements
are toward new objectives
(even though formerly they were abandoned).

The descent, Williams Carlos Williams

A mis padres.

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Contents

List of Tables

List of Figures

Chapter 1

Introduction

The 20th century was an epoch of great scientific achievements. In physics, quantum mechanics and special relativity were discovered, leading to a profound change in the way we understand our reality. Cosmology, in this context, is a science that drove to a deeper understanding about our universe, producing a cosmological model that allows us to describe the universe in great detail.

Today we know that the universe is flat, expanding and that in the past was dense and hotter. We know that there is a relic radiation called the cosmic microwave background (CMB) that fills all the space. This radiation comes from the very early instant when the universe became cold enough that photon decoupled from the rest of matter. This radiation is almost perfectly equal in all directions, but it has some small anisotropies that have the same estructure that those the seeds for the formation of galaxies and stars. The CMB allow us today to know the geometry and age of the universe with a lot of precission. Many of these discoveries have brought profound questions which are still unsolved. An universe starting just from an initial singularity, as in the Big Bang, is incomplete and the discovery of the acceleration of the universe has put in doubt many basic physical principles and a correct answer is still needed. This is an intense time for cosmology.

Cosmic inflation is a central part in the current picture about our universe. It was first conceived by Alan Guth [72] in 1980, as a solution to the so called flatness and horizon problems present in the standard Big Bang cosmology. Soon after being proposed, it became clear that it could also explain the initial conditions neccesary to produce the temperature anisotropies present in the cosmic microwave background radiation.

Despite its success, inflation is still not completely well understood. The necessary theory to give rise of its microphysical origin is not yet avalaible, leading to a multitude of possible inflationary scenarios, with different physical origins, and therefore with different predictions. For example, there is an intense ongoing effort to understand whether is possible to accomodate a period of inflation within fundamental theories such as string theory or supergravity.

Despite of our ignorance, inflation can be studied under a simple phenomenological model called *slow roll* inflation. First proposed by Andrei Linde [88], and by Albrecht and Steinhardt [12], this model consists in a universe filled with a single scalar field, usually called the *inflaton*, with its vacuum expectation value slowly rolling down a very flat potential. The idea is that, during inflation the inflaton kinetic energy becomes suppresed with respect to the inflationary potential energy, thus making it the main contribution to the energy density of the universe. While this effect holds, the universe becomes exponentially accelerated, solving the cosmological flatness and horizon problems of the Big Bang. Quantum field theory predicts the existence of fluctuations in this model, which because of the exponentially accelerated expansion are stretched from subhorizon scales to cosmological scales, where they become classical and freeze out, thus producing the temperature fluctuation seen in the CMB. Because the potential is required to be flat, this process happens adiabatically and the two point correlation function (or power spectrum) predicted is almost scale invariant. This is in almost exact agreement with current CMB observation, such as *WMAP* [85] and *Planck* [10], and it is improved when large scale structure observations are taken into account.

Multifield Inflation

As we mention before, there is a likely possibility that inflation could be embedded in a UV complete theory, such as string theory or supergravity. These description comes always with a plethora of different features, some of them inherited to low energy theories, such as inflation. For example, it is very common, that very high energy theories predict the existence of several scalar fields coupled together. This lead to the possibility tp consider inflation driven by more than one field, or to the case in which other fields are coupled to the inflaton field. This type of inflation theories are called multifield inflation.Moreover, multifield inflation is a natural extension to the conventional single field inflationary paradigm, and it is worth of studying.

Inflationary theories with several scalar fields produces some different predictions to the single scalar field case: primordial isocurvature perturbations and large non-Gaussianities being the two most important. Isocurvature modes can be understood, in this case, as perturbations orthogonal to the main curvature components, that can lead to very different astronomical predictions than the standard case. Gaussianity means that all the information about the perturbations is encoded in just the power spectrum. It has been shown that the primordial power spectrum is almost perfectly Gaussian, but this depends on which configuration it is considered. For single field inflation there is a lot of restrictions, and just the simple cases of non-Gaussianity can be considered, which are almost ruled out by CMB observations like *Planck*. On the other hand multifield inflation allows to consider many other forms of non-Gaussianity, which are less restricted, and while some are constrained by observations, more study is needed before giving a final word.

Non-Gaussinities and isocurvature modes predictions, if detected would discard almost any model that contains just a single field. In addition the existence of other fields predicts deviations from a scale invariant power spectrum, because when multiple fields are considered the amplitude of adiabatic perturbations generated during inflation can continue to evolve after their wavelengths have exited the horizon, affecting the final curvature power spectrum.

Thus our motivation for studying inflationary theories with various fields is twofold. On the one hand it allows to constrain inflationary models coming from UV-complete theories such as string theory, on the other hand, it stablishes a window of opportunity to study physics at energies not yet accessible.

Effective field theory

Effective field theory (EFT) is an important tool for contemporary theoretical physics. Its success is based on the fact that it allows to separate the physics valid at different energy scales. Therefore one can isolate the important degrees of freedom valid up to a certain range of energy, while the remaining is encoded in operators of the low energy theory. Their use in the context of inflation emerges just recently but its development has been promising. Cheung *et al.* [50] and Senatore and Zaldarriaga [118], developed an EFT which characterises the dynamics of the inflationary fluctuations at horizon crossing. This EFT allows to unify most of the single field models of inflation in just a few parameters. The trick is to indentify inflationary perturbations as the Goldstone bosons π parameterizing the spontaneously broken time-translation invariance. The action determining the evolution of the Goldstone boson was found to be

$$S = -M_{\rm Pl}^2 \int d^4x \dot{H} \left[\frac{1}{c_s^2} \left(\dot{\pi}^2 - c_s^2 \frac{(\partial_i \pi)^2}{a^2} \right) - \frac{1-c_s^2}{c_s^2} \left(\frac{(\partial_i \pi)^2}{a^2} + \frac{A}{c_s^2} \dot{\pi}^2 \right) \dot{\pi} + \cdots \right], \tag{1.1}$$

where c_s is the speed of sound at which Goldstone boson quanta propagate, and A is a quantity that parameterizes different models of inflation. This is a powerful tool to study inflation and it has been systematically used, since its introduction, because it allows a model independent study of inflation. An important use of this EFT is in the study of non-Gaussianity, because it is possible to derive that large non-Gaussianity, is produced when the Goldstone boson π is characterized by a low speed of sound. This effect can be due to symmetries in the theory but also because of the effects of UV-physics that is encoded in that operator.

A possible approach is to think cosmic inflation as a low energy (and long wavelength) EFT where the complete high energy theory is still unknown, but their effects could be present (through operators) in the inflationary action. This be the case, one is permitted to ask about the possible effects of a UV complete theory of inflation, as for example from a string theory or supergravity motivated inflationary theory, on the low energy EFT. Given the fact that inflation can predict observables which are within the current experimental limits, the use of EFT techniques are very compelling to test our ideas about fundamental theories which, because of their gravitational nature, are known for their lack of predictions at current experimental limits. For example, in the action (1.1) a very low c_s would determine a higher amplitude in the third order terms, this low c_s could be due to a modified theory, as for example, in the string theory motivated DBI inflation ([121]). This can eventually lead to a large non-Gaussianity signal, which is constrained by CMB observations, and thus it is possible to put limits on the speed of sound at which curvature modes propagate which at the same time constraining theories like DBI.

Multifield theories correspond tipically to the low energy version of some more fundamental theory valid at higher energies. These theories usually have a multitude of heavy fields, which produce isocurvature perturbations that can interact with curvature modes, but as they are heavy, compared to the scale of inflation H^2, these modes are suppressed once their wavelengths are larger than the horizon, and therefore not detectable. Moreover, multifield theories incorporate the idea of a scalar geometry: Scalars can be seen as coordinates of a scalar manifold, and its manifold can be curved and have a nontrivial topology. This geometry may have consequences in the dynamics of the scalar fields, in which case may not be ignored. Beyond that, massive modes are very energetic, far beyond the scale at which inflation occurs, thus they can be integrated out using the EFT formalism. In any case, it has been recently shown that in certain context, when the field trajectory is very curved, these isocurvature modes can affect the evolution of curvature modes and lead to imprints on its power spectrum. Using the EFT formalism it is possible to write an action for the curvature mode only and where the product of having integrated out these massive fields is that the effective speed of sound for the curvature perturbation modes becomes :

$$c_s^{-2} = 1 + \frac{4\dot{\theta}^2}{M_{\text{eff}}^2}, \tag{1.2}$$

where $\dot{\theta}$ is the angular velocity of the inflationary trajectory, and M_{eff}^2 the effective mass of the heavy fields. Therefore the speed of sound depends on the scale of the integrated heavy fields and on the geometry of the inflaton trajectory. With this, it is posssible to have a very low speed of sound, even in the case when $M_{\text{eff}}^2 \ll H^2$. And, as we described earlier, this low c_s would predict a large non-Gaussianity thus having a hypotethical detectable signal in the CMB experiments, which reflects the influence of physics at energies not accesible by terrestrial experiments, for example the LHC.

This result motivates further investigation, since this class of models is generic to multifield inflationary theories and as it is well known, may predict features on the curvature power spectrum and a large amount of non-Gaussianity, which may allow us to constrain this class of models using current available data.

1.1 Overview

In this thesis we further expand the understanding of this class of inflationary scenarios. We start by studying the current model of cosmology called ΛCDM model. We derive from first principles the equations necessary to account of observed data, and we explain the evolution of the universe through time. We end the first chapter by presenting two of the cosmological problems that initially motivated inflation. In the second chapter we explain inflation, and its most simple realization called *slow roll* single field inflation. We then study the generation of quantum perturbations that produces the primordial curvature perturbations measured in the CMB. Finally we explain how to connect inflation to experiments. In the third chapter we explain in some detail the theory of multifield inflationary theories. The focus of this chapter is to give a detailed account of the formalism needed to study non geodesic inflationary trajectories, which are the basis of the rest of the thesis.

The following chapters constitute the core part of this thesis. In chapter 4 we discuss and clarify the validity of effective single field theories of inflation obtained by integrating out heavy degrees of freedom in the regime where adiabatic perturbations propagate with a suppressed speed of sound. We show by construction that it is indeed possible to have inflationary backgrounds where the speed of sound remains suppressed and slow-roll persists for long enough. In this class of models, heavy fields influence the evolution of adiabatic modes in a manner that is consistent with decoupling of physical low and high energy degrees of freedom. In chapter 5 we analyse under which general conditions this EFT remains a reliable description for the linear evolution of curvature modes. We find that the main condition consists on demanding that the rate of change of the turn's angular velocity stays suppressed with respect to the masses of heavy modes. This adiabaticity condition allows the EFT to accurately describe a large variety of situations in which the multi-field trajectory is subject to sharp turns. To test this, we analyze several models with turns and show that, indeed, the power spectra obtained for both the original two-field theory and its single-field EFT are identical when the adiabaticity condition is satisfied. In particular, when turns are sharp and sudden, they are found to generate large features in the power spectrum, accurately reproduced by the EFT. We finally show that when a second heavy-field (orthogonal to both the light inflationary and heavy isocurvature directions) is included it may have a strong influence on the evolution of curvature modes. We compute the effective field theory for the low energy curvature perturbations obtained by integrating out the two heavy fields and show that the presence of the second heavy field implies the existence of additional self-interactions that are not accounted for in the case where only one field is integrated out. In particular, we show that the couplings induced by the presence of a second heavy-field are distinguishable from those appearing in the single heavy-field case, implying that a detailed characterization of non-Gaussianities will allow us to constrain this class of scenarios.

Chapter 2

The concordance model of cosmology

In this chapter we present a brief review of Standard Cosmology. We start by detailing the simplest mathematical model available of the universe: the Friedmann-Robertson-Walker (FRW) Universe. This description will permit us to study the way in which the universe, together with matter and geometry evolved until reaching its present state. This will give us the necessary theoretical background to understand the state of the art of observational cosmology. After this, we give a qualitative description of the thermal history of the Universe, and describe it's current status based on actual observations. The present model, also called ΛCDM or concordance model, still has some flaws. At the end of this chapter we show two problems that forces us to consider the possibility of a new paradigm.

We will follow the conventions used in Carroll book ([43])

2.1 Background dynamics

Based on current observations, we start by considering an isotropic (rotationally invariant spacetime) and homogeneous (translationally invariant) universe. It can be shown that a spacetime with these symmetries is described by the Friedmann-Robertson-Walker metric

$$\begin{aligned} ds^2 &= g_{\mu\nu} dx^\mu dx^\nu \\ &= -dt^2 a^2(t) \left[\frac{dr^2}{1-kr^2} + r^2 d\Omega \right], \end{aligned} \tag{2.1}$$

where $a(t)$ is the scale factor describing the relative size of a spacelike hypersurface Σ_3 at different times. The FRW geometry is characterized by the curvature of such spatial slices, where the parameter k is 0 for flat euclidean space; 1 for positive curvature spacetimes and -1 for negative ones..

To study this class of spacetime it is useful to analyze the propagation of signals emitted by a source at a given time t_1, and observed by us today at a time t_0. Because of the expansion of space, this signals become stretched or elonged while they travel through the universe. We define the redshift of light between the time of departure t_1 and arrival t_0 as

$$1 + z \equiv \frac{a(t_0)}{a(t_1)}. \tag{2.2}$$

Then if $a(t)$ is increasing with time, the frequency is shifted towards the infrared (redshift today), by a a factor $(a(t_1)/a(t_0))$, and z is different from 0.

The expansion rate is characterized by the Hubble parameter $H(t) \equiv \dot{a}/a$, which is positive for an expanding universe and negative for contracting universes. It is usual to define $a(t_0) = 1$ and $H(a(t_0)) \equiv H_0$, where H_0 the Hubble parameter observed today H_0 is called *Hubble constant*.

The comoving distance to an object at redshift z,

$$\chi(z) \equiv \int_0^z \frac{dz'}{H(z')}, \tag{2.3}$$

allow us to define two important *cosmological distance measures*:

- **Angular diameter distance:** Is defined as the ratio of an object's physical transverse size to its angular size at emission time. This is given by

$$d_A(z) = \frac{1}{1+z}\chi(z), \tag{2.4}$$

  for the case when $k = 0$.

- **Luminosity distances:** Measure the corrected distance, due to redshifting of the luminous distance emitted by an object as observed on Earth. It is given by

$$d_L(z) = (1+z)\chi(z), \tag{2.5}$$

  for $k = 0$.

The importance of these measures is that they give us the possibility to consider measurements of distances at large redshifts, say $z > 0,1$, when the effects of cosmological expansion are considerable. Measurements at large redshift, tell us whether the universe is expanding or contracting and how fast. We will apply these results in the following sections.

2.1.1 Einstein equations

To describe the dynamics of the FRW spacetime defined previously (2.1) we should solve the Einstein's equations. As a first approximation, we assume that the interaction between different matter components is negligible. Therefore the universe can be modelled as filled with a perfect fluid of pressure p and density ρ. This type of matter is consistent with the symmetries of the spacetime considered previously, and is described by the following energy-momentum tensor:

$$T^{\mu}_{\ \nu} = \begin{pmatrix} \rho(t) & & & \\ & -p(t) & & \\ & & -p(t) & \\ & & & -p(t) \end{pmatrix}, \tag{2.6}$$

Einstein equations $G_{\mu\nu} = 8\pi T_{\mu\nu}$ become

$$H^2 + \frac{k^2}{a^2} = \frac{8\pi}{3M_{\rm pl}^2}\rho, \tag{2.7}$$

$$\left(\frac{\ddot{a}}{a}\right)^2 = -\frac{4\pi}{3M_{\rm pl}^2}(\rho + 3p). \tag{2.8}$$

On the other hand conservation of energy-momentum tensor $T^{0\nu}_{\ \ ;\nu}$ gives:

$$\dot{\rho} + 3H(\rho + p) = 0. \tag{2.9}$$

Solving this set of equations will permit us to study the evolution of the scale factor $a(t)$ as a function of the matter content that fill the universe. To proceed we will consider low density fluids. This will allow us to write the pressure as a function of the density as: $p = w\rho$. Therefore eq. (2.9) can be rewritten as

$$\frac{d{\rm ln}\rho}{d{\rm lna}} = -3(1 + w), \tag{2.10}$$

thus implying that $a \sim \rho^{-3(1+w)}$, which gives us the scale factor as a function of time:

$$a(t) \propto \begin{cases} t^{\frac{2}{3(1+w)}} & \text{for } w \neq -1 \\ e^{Ht} & \text{for } w = -1 \end{cases}. \tag{2.11}$$

Therefore we see that, depending on the type of matter that we consider, we get a different evolution of the scale factor $a(t)$ and thus the Hubble parameter $H(t)$. As the universe is filled with a mixture of different matter components, then it will be useful to classify them by their contribution to the pressure. Let us examine four important cases:

- **Matter** We will use the term matter, (also called dust) to refer to any form of matter for which the pressure is much smaller than energy density $|p| \ll \rho$. This is the case for a gas of non-relativistic particles (where the energy density is dominated by mass term). This includes **cold dark matter** and **baryons** (nuclei and electrons). Setting $w = 0$ in the identity (2.11) we get:

$$\rho \propto a^{-3}. \tag{2.12}$$

This dilution of the energy density simply reflects the expansion of a volume $V \propto a^3$.

- **Radiation**
  Radiation denotes anything for which the pressure is about a third of the energy density, $p = \frac{1}{3}\rho$. This is the case for a gas of relativistic particles, for which the energy is dominated by the kinetic energy. This includes **photons**, **neutrinos** and **gravitons**. Eq. (2.11) implies

$$\rho \propto a^{-4}. \tag{2.13}$$

  The dilution now includes an extra factor a^{-1} due to the redshifting of the energy $E \propto a^{-1}$

- **Dark Energy**
  As we will see, the universe may also contain a negative pressure component characterized by an equation of state $p = -\rho$. This is unlike anything we have ever encountered in the lab, but is hypothesized as the cause of the actual expansion of the Universe. We find that the energy density is constant

$$\rho \propto a^{0}. \tag{2.14}$$

  Since in this case, the energy density doesn't dilute, energy has to be created as the universe expands.

- **Curvature**
  We can also include the effect of curvature of spacetime as a type of fluid characterized by an energy density:

$$\rho \propto a^{-2}. \tag{2.15}$$

It will be useful to rewrite the different components of the stress energy tensor in terms of the critical density for a flat universe $\rho_c \equiv 3M_{\rm pl}^2 H^2$, which evaluated today becomes,

$$\rho_{c,0} = \frac{3H_0^2}{8\pi G}. \tag{2.16}$$

The dimensionless densities evaluated today are:

$$\Omega_{i,0} \equiv \frac{\rho_{i,0}}{\rho_{c,0}}. \tag{2.17}$$

The Friedmann equation (2.7) then becomes

$$H^2 = H_0^2 \left[\Omega_{r,0} a^{-4} + \Omega_{m,0} a^{-3} + \Omega_{k,0} a^{-2} + \Omega_\Lambda\right]. \tag{2.18}$$

Where we have defined

$$\Omega_{k,0}(a) = -\frac{k}{a^2 H_0^2}, \tag{2.19}$$

as a measure of the relative curvature contribution. In the literature is usual to drop the *today* index $'0'$, and thus in the rest of this thesis we follow that convention.

This parametrization is consistent with a a spatially flat universe, and in what follows we will set $\Omega_k = 0$.

2.2 Concordance model

One of the great achievements of physics during the XX century, is the fact that we can characterize our universe with just six parameter. The concordance or ΛCDM model is based on well-tested physical principles, including general relativity that describes the dynamics of an expanding universe, the quantum mechanical laws that govern the creation of species during early times, and the Boltzmann equation which allows us to track the evolution of each of these species.

Parameter	Best fit	68% limits
$\Omega_b h^2$	0.022161	0.02214 ± 0.00024
$\Omega_c h^2$	0.11889	0.1187 ± 0.0017
$100\theta_{MC}$	1.04148	1.04147 ± 0.00056
τ	0.0952	0.092 ± 0.013
n_s	0.9611	0.9608 ± 0.0054
$\log(10^{10} A_s)$	3.0973	3.091 ± 0.025

Table 2.1: Cosmological parameters values for the *Planck* best-fit 6 parameter ΛCDM model including external data set. The parametrization includes the fraction of baryonic $\Omega_b h^2$ and cold dark matter $\Omega_c h^2$ today. The angular distance τ, the optical depth at reionization, $100\theta_{MC}$. Also are included the values of the initial conditions needed in this model, the spectral index n_s and the amplitude of the initial scalar perturbation A_s. [9]

However, most of the parameters in the concordance model contain information about physics we still have no detailed understanding. The relative fractions of baryons, dark matter and dark energy in the universe are all governed by fundamental processes that still lie outside the current Standard Model of particle physics, and may extend up to the TeV, GUT or even Planck scales. The set of variables required by the concordance cosmology is not fixed, but is dictated by the quality of the available data and our ignorance of fundamental physical parameters and interactions.

Table (2.2) details the six parameters used in the ΛCDM model and their current values according *Planck* combined with supernovae data and LSS observations. Here we can see that the current model favours a flat universe dominated by a 68.25% of dark energy, 26.8% of dark matter and a 4.9% of ordinary or baryonic matter.

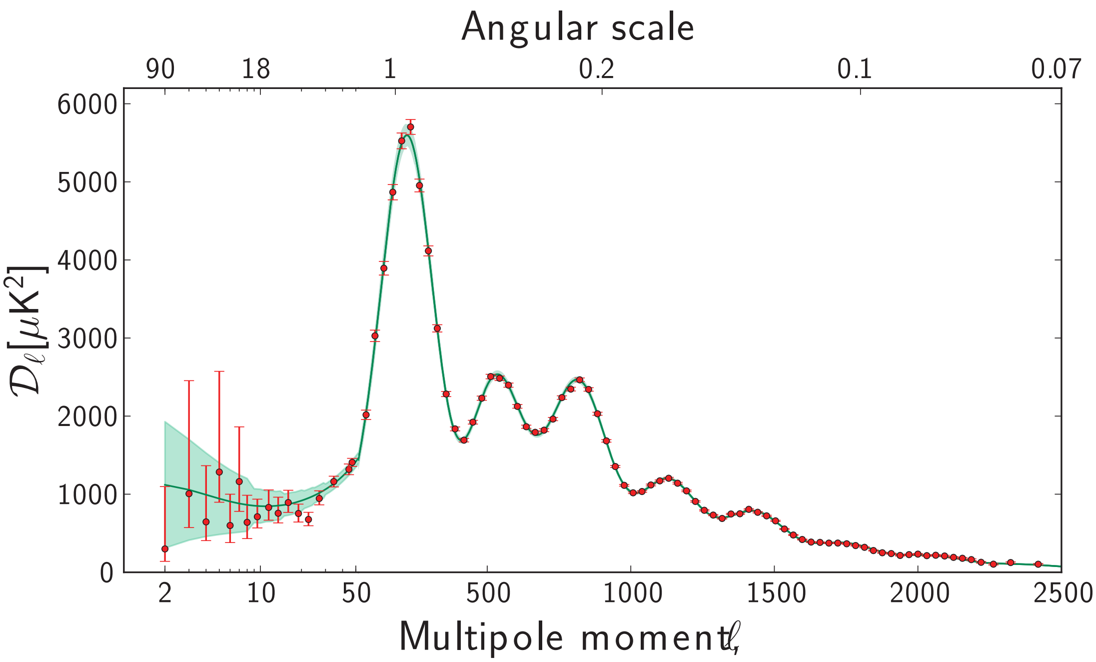

Figure 2.1: Temperature angular power spectrum of the CMB from *Planck*. The measurements is well fit by the six parameters ΛCDM model. The shaded area around the best fit curve represents cosmic variance, The error bars on individual points also include cosmic variance. The horizontal axis is logarithmic up to $l = 50$, and linear beyond. The vertical scale is $D_l = l(l+1)C_l$ where C_l is the angular power spectrum [8].

This parameterisation also includes data about the initial condition necessary to produce the CMB power spectrum. Assuming that the initial state of the universe was filled with small adiabatic curvature perturbations parametrized by a power spectrum of the form

$$P_{\mathcal{R}}(k) = A_s \left(\frac{k}{k_0}\right)^{n_s - 1 + \frac{1}{2} dn_s/s \log k \log(k/k_0)}, \tag{2.20}$$

where k_0 is chosen to be 0.05Mpc^{-1} [1]. We have that the last two parameters in Table 2.2 indicates that (2.20) forms a near scale invariant power spectrum.

2.2.1 Thermal History

In the last section we examine the composition of our universe today. The concordance models describes how the universe is now dominated by dark energy, and it is composed of a determined number of species. Since dark energy remains constant in time, at early time, matter and radiation should have dominated. We examine now, briefly, the evolution of different types of matter, that produces the universe we see nowadays.

[1] k_0 is chosen to be the middle of the scales covered by *Planck*

At the beginning our universe was hot and dense, and spacetime was expanding. This means that early epochs are characterized by high energies, at which certain broken symmetries in the laws of physics were restored, and by the fact that the expansion rate $H(t)$ plays an important role as a time scale. Interaction between particles freeze out or decouple when the interaction rate drops below the expansion rate. Table 2.2.1 summarizes the various phase transitions related to symmetry breaking events. We will give a quick summary of the most important events in the evolution of the Universe. This description will start at 100 GeV where we still have a detailed picture on the physics present.

Event	time t	redshift z	temperature T
Electroweak phase transition	10^{-10} s	10^{15}	100 GeV
QCD phase transition	10^{-9} s	10^{12}	150 MeV
Neutrino decoupling	1 s	6×10^9	1 MeV
Electron-positron annihilation	6 s	2×10^9	500 KeV
Big Bang nucleosynthesis	180 s	4×10^8	100 KeV
Matter radiation equality	60 kyr	3200	0.75 eV
Recombination	260 kyr	1100	0.26 eV
CMB decoupling	380 kyr	1200	0.23eV
Reionization	100 Myr	11	2.6 meV
Dark energy -matter equality	9 Gyr	0.4	0.33 meV
Present	13.7 Gyr	0	0.24 meV

Table 2.2: Thermal history of the Universe

Above 100 GeV the electroweak symmetry is restored and the Z and $W^{\pm}$ bosons are massless. Interactions are strong enough to keep quarks and leptons in thermal equilibrium. Below 100 GeV the symmetry between the electromagnetic and the weak forces is broken, Z and $W^{\pm}$ bosons acquire mass, and the cross-section of weak interactions decreases as the temperature of the universe drops. As a result, at 1 MeV, neutrinos decouple from the rest of the matter. Shortly after, at 1 second, the temperature drops below the electron rest mass and electrons and positrons annihilate efficiently. Only an initial matter-antimatter asymmetry of one part in a billion survives. The resulting photon-baryon fluid is in equilibrium. Around 2.2 MeV the strong interaction becomes important and protons and neutrons combine into the light elements (H, He, Li) during Big Bang nucleosynthesis. ($\sim 200s$). The successful prediction of the light elements (H, He and Li) abundances is one of the most striking consequences of the Big Bang theory. The matter and radiation densities are equal at around 1 eV (10^{11} s). Charged matter particles and photons are strongly coupled in the plasma and fluctuations in the density propagate as cosmic ‘sound waves’. Around 0.1 eV (380,000 yrs) protons and electrons combine into neutral hydrogen atoms. This epoch, at which the first atoms start to form (H) is referred to as *recombination*, despite of the fact that electrons had never before combined into atoms.

At a temperature greater than about $3000K$ the Universe consisted of an ionized plasma of mostly protons, electrons and photons, with a few helium nuclei and a tiny trace of lithium. The important characteristic of this plasma is that it was opaque, or more precisely, the mean free path of a photon was a great deal smaller than the Hubble length. As the universe cooled and expanded, the plasma ’recombines’ into neutral atoms, first the helium, then a little later the hydrogen.

After recombination, and once the gas is in a neutral state, the mean free path for a photon becomes much larger than the Hubble length. The universe is then full of a background of freely propagating photons with a black body distribution of frequencies. At the time of recombination. the background radiation has a temperature of $T = T_R = 3000$K, and as the universe expands photons redshifted, so that the temperature of the photons drops with the increase of the scale factor $T \propto a(t)^{-1}$. We can detect these photons today. Looking at the sky, this background of photons come to us evenly from all directions, with an observed temperature of $T_0 = 2.73$K. This is the cosmic microwave background. Since by looking at higher and higher redshift objects, we are looking further and further back in time, we can view the observation of CMB photons as imaging a uniform *surface of last scattering* at redshift 1100.

To the extent that recombination happens at the same time and in the same way everywhere, the CMB will be of precisely uniform temperature. While the observed CMB is highly isotropic, it is not perfectly isotropic . The largest contribution to the anisotropy of the CMB as seen from earth is simply Doppler shift due to the earth’s motion through space. As seen in fig. 2.2 CMB photons are slightly blueshifted in the direction of our motion and slighly redshifted opposite the direction of our motion. This motion shifts the temperature of the CMB so the effect has the characteristic form of a dipole temperature anisotropy. The dipole anisotropy, however, is a local phenomenon. Any intrinsic, or primordial, anisotropy of the CMB is potentially of much greater cosmological interest. To describe the anisotropy of the CMB, we remember that the surface of last scattering appears to us as a spherical surface at a redshift of $z = 1100$. Therefore the natural parameters to use to describe the anisotropy of the CMB sky is an expansion in terms of spherical harmonics $Y_{lm}(\theta, \phi)$

$$\frac{\Delta T}{T} = \sum_{l=1}^{\infty} \sum_{m=-l}^{l} a_{lm} Y_{lm}(\theta, \phi). \tag{2.21}$$

If we assume isotropy, then there is no preferred direction in the universe, and we

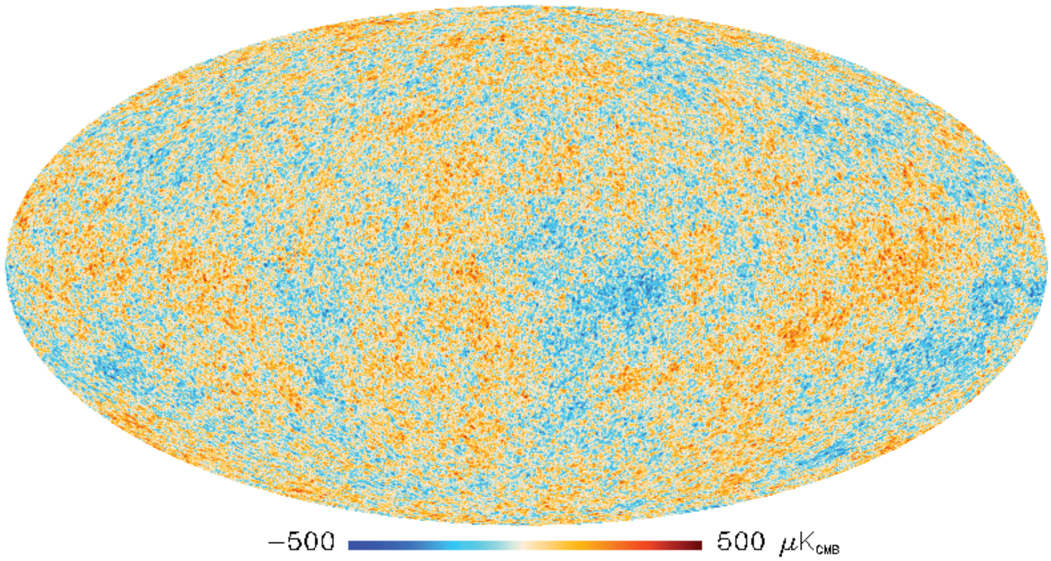

Figure 2.2: Temperature fluctuations in the CMB, as obtained by *Planck* satellite in 2013 [8]

expect the anisotropy $\Delta T/T$ to be independent of the index m. We can define

$$C_l \equiv \frac{1}{2l+1}\sum_m |a_{lm}|^2, \tag{2.22}$$

which is the rotationally invariant angular spectrum. This quantity simplifies a lot of the information contained in a CMB map pixel, and thus is used to analyze CMB power spectrums.

The $l = 1$ contribution is just the dipole anisotropy

$$\left(\frac{\Delta T}{T}\right)_{l=1} \sim 10^{-3}. \tag{2.23}$$

The dipole was first measured in the 1970's by several groups. It was until more than a decade after, that the first observation was made of anisotropy for $l > 2$, by COBE satellite. COBE observed that

$$\left(\frac{\Delta T}{T}\right)_{l>1} \approx 10^{-5}, \tag{2.24}$$

this anisotropy represents intrinsic fluctuations in the CMB itself, due to the presence of tiny primordial density fluctuations in the cosmological matter present at the time of recombination

These small density perturbations, grow via gravitational instability to form the large scale structures observed in the late universe. A competition between the background pressure and the universal attraction of gravity determines the details of the growth of structure. During radiation domination the growth is slow $\delta\rho \sim \log a$.

Clustering becomes more efficiently after matter dominates the background density, $\delta\rho \approx a$. Small scales become non-linear first $\delta\rho \gtrsim 1$, and form gravitationally bound objects that decouple from the overall expansion. This leads to a picture of hierarchical structure formation with small-scale structures (like stars and galaxies) forming and then merging in larger structures (clusters and superclusters of galaxies). Around redshift $z \approx 25$, high energy photons from the first stars begin to ionize the hydrogen in the inter-galactic medium. This process of reionization is completed at $z \approx 6$. Meanwhile, the most massive stars run out of nuclear fuel and explode as *supernovae*. In these explosions the heavy elements (C,O, etc.) necessary for the formation of life are created. At $z \approx 1$ negative pressure dark energy comes to dominate the universe. The background spacetime is accelerating and the growth of structure ceases $\delta\rho \sim const.$

2.3 Causal structure

We now move to study the causal structure of flat *FRW* spacetimes. To do so we begin by changing from *proper time* t to *conformal time* τ using the following relation

$$d\tau = \frac{dt}{a(t)}. \tag{2.25}$$

We see that the FRW metric then factorizes into a Minkowski metric $\eta_{\mu\nu}$ multiplied by a time dependent conformal factor $a(\tau)$

$$ds^2 = a^2(\tau)\left[-d\tau^2 + dr^2 + r^2 d\Omega\right] = a^2(\tau)\eta_{\mu\nu}dx^\mu dx^\nu. \tag{2.26}$$

Therefore, the radial propagation of particles is characterized by the following line element

$$ds^2 = a^2(\tau)\left[-d\tau^2 + dr^2\right]. \tag{2.27}$$

In particular the null geodesic followed by photons are represented by just straight lines at ± 45 angles in the $\tau - r$ plane.

$$r(\tau) = \pm r + const. \tag{2.28}$$

The maximal distance a photon can travel between an initial time t_i and later time $t > t_i$ is given by:

$$\Delta r = \delta\tau \equiv \tau - \tau_i = \int_{t_i}^{t} \frac{dt'}{a(t')}. \tag{2.29}$$

Thus the maximal distance travelled is equal to the amount of conformal time elapsed during the interval $\Delta = t - t_i$. The initial time is often taken to be the 'origin

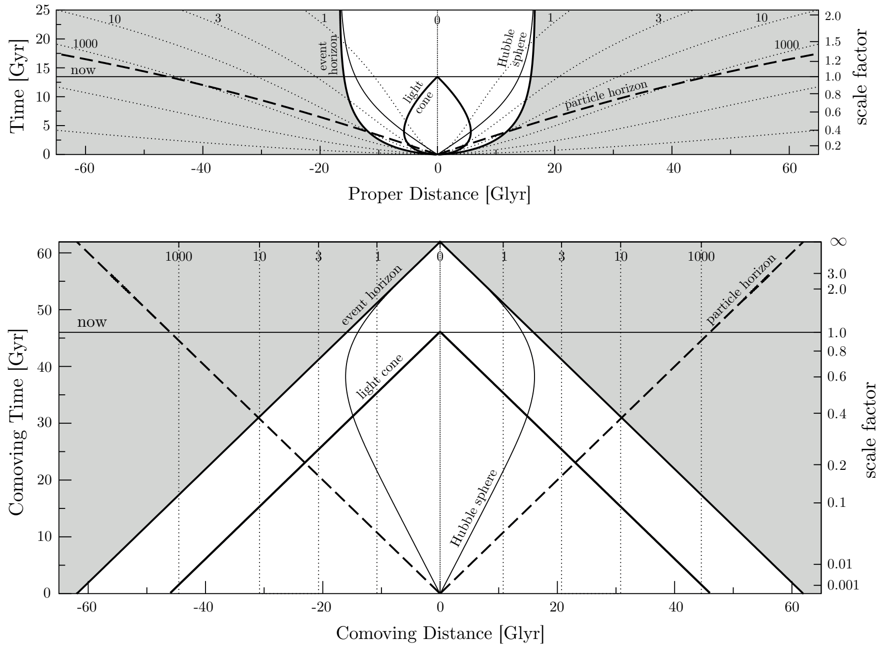

Figure 2.3: Spacetime diagrams showing the main features of the expansion of the universe, both in physical coordinates (top figure) and comoving coordinates (bottom figure). Dotted lines show the worldlines of comoving objects. We are the central vertical worldline. The current redshifts of the comoving galaxies shown appear labeled on each comoving world line. All events that we currently observe are on our past light cone. All comoving objects beyond the Hubble sphere (thin solid line) are receding faster than the speed of light.

of the Universe':$t_i \equiv 0$, defined formally by the initial singularity $a_i \equiv a(t_i) \equiv 0$. We then obtain

$$\Delta r_{\max}(t) = \int_0^t \frac{dt'}{a(t')} = \tau(t) - \tau(0), \tag{2.30}$$

which is called the *comoving particle horizon.*

If we rewrite the conformal time as

$$\tau \equiv \int \frac{dt}{a(t)} = \int \frac{d\ln a}{aH}, \tag{2.31}$$

we deduce that the elapsed conformal time depends on the evolution of the comoving Hubble radius $(aH)^{-1}$. For example, for an Universe dominated by a fluid with equation of state $\omega \equiv p/\rho$ we find that this evolves as

$$(aH)^{-1} \approx a^{\frac{1}{2}(1+3\omega)}. \tag{2.32}$$

Note the dependence of the exponent on the combination $(1+3\omega)$. All the familiar sources satisfy the strong energy condition (SEC), $1+3\omega > 0$, so it was reasonable, prior to the knowledge of the acceleration of the universe, to assume that the comoving Hubble radius increases as the universe expands. Performing the integral (2.31) gives

$$\tau \approx \frac{2}{(1+3\omega)} a^{\frac{1}{2}(1+3\omega)}. \tag{2.33}$$

For conventional matter sources the initial singularity is at $\tau_i = 0$.

$$\tau_i \approx a_i^{\frac{1}{2}(1+3\omega)}, \text{ for } \omega > -1/3. \tag{2.34}$$

And the comoving horizon is finite

$$\Delta r_{\max}(t) \approx a(t)^{\frac{1}{2}(1+3\omega)}, \text{ for } \omega > -1/3. \tag{2.35}$$

2.3.1 Flatness Problem

Let us recall that the evolution of curvature in FRW spacetime, is described by the following density parameter:

$$\Omega_k(a) = -\frac{k}{a^2 H^2(a)}. \tag{2.36}$$

Then if we assume, for simplicity, that the expansion is dominated by some form of matter with equation of state equal to w, we have $a \sim t^{\frac{2}{3(1+w)}}$ and we have the following expressions are satisfied:

$$\dot{\Omega}_k = \Omega_k H(1+3w), \tag{2.37}$$

$$\frac{\partial \Omega_k}{\partial \log a} = \Omega_k(1+3w). \tag{2.38}$$

If we further assume that $w > -1/3$, then the solution $\Omega_k = 0$ is an unstable point. Thus if $\Omega_k > 0$ at some point, Ω_k will keep growing. And viceversa, if $\Omega_k < 0$ at some point, it will keep decreasing. Of course at most $|\Omega_k| = 1$ in which case w becomes $-1/3$ if $k < 0$, or otherwise the universe collapses if $k > 0$.

The surprising fact is that Ω_k is now observed to be smaller than about 10^{-2}. Taking into account the content of matter of the universe, this mean that at earlier times it was even closer to zero. For example, at BBN epoch, it has to be $|\Omega_k| \lesssim 10^{-18}$, at the Planck scale $|\Omega_k| \lesssim 10^{-63}$. In others words, since curvature depends on redshift as a^{-2}, it tends to dominate in the future with respect to other forms of matter. So, if today curvature is not already dominating, it means that it was almost negligible in the past. The value of Ω_k at those early times represents a remarkable small number.

A possible solution could be that $k = 0$ in the initial state of the universe. While this possibility could be true, it is unknown why the universe should choose such a precise state initially. A second possibility, would be that in some epoch the universe would have been dominated by some matter content with $\omega < -1/3$.

2.3.2 Horizon Problem

Let us digress briefly to make a simple calculation. Given that the Universe seems very homogeneous at large distances, it is valid to ask ourselves, if we can trace back this to the beginning of the Universe. To do this let us compute the angle subtended by the comoving horizon at recombination. This is defined as the ratio of the comoving particle horizon at recombination and the comoving angular diameter distance from us (an observer at redshift $z = 0$) to recombination ($z \approx 1100$).

$$\theta_{\rm hor} = \frac{d_{\rm hor}}{d_{\rm A}}. \tag{2.39}$$

A fundamental quantity is the comoving distance between redshifts z_1 and z_2

$$\tau_2 - \tau_1 = \int_{z_1}^{z_2} \frac{dz}{H(z)} \equiv \mathcal{I}(z_1, z_2). \tag{2.40}$$

The comoving particle horizon at recombination is

$$d_{\rm hor} = \tau_{\rm rec} - \tau_i \approx \mathcal{I}(z_{\rm rec}, \infty). \tag{2.41}$$

In a flat Universe, the comoving angular diameter distance from us to recombination is

$$d_{\rm A} = \tau_0 - \tau_{\rm rec} = \mathcal{I}(0, z_{\rm rec}). \tag{2.42}$$

The angular scale of the horizon at recombination therefore is

$$\theta_{\rm hor} \equiv \frac{d_{\rm hor}}{d_{\rm A}} = \frac{\mathcal{I}(z_{\rm rec}, \infty)}{\mathcal{I}(0, z_{\rm rec})}. \tag{2.43}$$

Using $H(z)$ from eq.(2.18) we find that

$$\theta_{\rm hor} = 1.16^\circ. \tag{2.44}$$

Therefore in a matter or radiation dominated universe no physical influence could have smoothed out initial inhomogeneities and brought points at last scattering that are separated by more than; $\theta > \theta_c \equiv 2\theta_{\rm hor} = 2.3^\circ$ to the same temperature, in contradiction with the nearly perfect isotropy of the microwave background at large angular scales observed ever since the background radiation was discovered.

This striking fact tells us that even at the time of last scattering, the largest scales were still outside the horizon. This is called the *Horizon Problem.*

Chapter 3

Inflation

Inflation is the key idea that permeates this thesis. First presented as a solution to some cosmological problems ([72]), as the horizon and flatness problems. It is also the most successful mechanism to explain the existence of anisotropies in the CMB.

We will start this chapter by presenting inflation as a solution to the horizon and flatness problems, which are based on a logical use of the growing Hubble sphere we developed previously. We then consider the field equations needed to describe inflation, and we move to consider the quantum theory of fluctuations produced during inflation. This remarkable achievement gives us a mechanism to produce the initial conditions that gave birth to the anisotropies seen today, and thus the galaxies, stars and planets. We end this chapter by linking inflation to cosmological observables.

3.1 Background

Equation (2.31) implies that the comoving horizon τ is the logarithmic integral of the comoving Hubble radius $(aH)^{-1}$. The Hubble radius, as we mentioned previously, is the distance over which particles can travel in the course of one expansion time, *i.e.*, it is roughly the time in which the scale factor doubles. So the Hubble radius is another way of measuring whether particles are causally connected with each other: If they are separated by distances larger than the Hubble radius then they cannot currently communicate.

There is a subtle distinction between the comoving horizon τ and the comoving Hubble radius $(aH)^{-1}$. If particles are separated by distances greater than τ, they never could have communicated with one another; if they are separated by distances greater than $(aH)^{-1}$ they cannot talk to each other now. It is therefore possible that τ could have been much larger than $(aH)^{-1}$ now, so that particles cannot communicate today but were in causal contact early on. This might happen if the comoving Hubble radius early on was much larger than it is now so that the comoving horizon τ got most of its contribution from early times. This could happen, but is not possible during matter or radiation dominated epochs, because in those cases, the comoving Hubble radius increases with time, so typically we expect the largest contribution to τ to come from most ancient times.

All this suggests a solution to the horizon problem. If there exists a brief time, where the universe was not dominated by radiation or matter, and moreover the Hubble radius decreased. Then the comoving horizon τ would get most of its contributions not from recent times but rather from primordial epochs. Particles separated by many Hubble radii today, would have been in causal contact before the epoch of rapid expansion and this could explained the smoothness of the CMB observed today. This epoch of dramatically decreasing Hubble radius is called *Inflation*.

As noted earlier, a decreasing Hubble radius requires a violation of the SEC, $1 + 3w < 0$. Therefore we notice that the Big Bang singularity is now pushed to negative conformal time,

$$\tau_i \propto \frac{2}{(1+3w)} a_i^{\frac{1}{2}(1+3w)} = -\infty \text{ for } w < -\frac{1}{3}. \tag{3.1}$$

This implies that there was much more conformal time between the singularity and decoupling than we had thought. The past light cone of widely separated points in the CMB now had time to intersect before the time $\tau = 0$ which is not the initial singularity, but instead becomes, what it is called, the time of *reheating*. There is time both before and after $\tau = 0$. A decreasing comoving horizon means that large scales entering the present universe were inside horizon before inflation. Causal physics before inflation therefore had time to stablish spatial homogeneity. With a period of inflation, the uniformity of the CMB is not a mystery any more.

To continue with the discussion, let us notice that a shrinking Hubble radius $(aH)^{-1}$ corresponds to

$$\frac{d}{dt}(aH)^{-1} = \frac{d}{dt}\dot{a}^{-1} = -\frac{\ddot{a}}{\dot{a}^2}. \tag{3.2}$$

Thus,

$$\frac{d^2 a}{dt^2} > 0; \tag{3.3}$$

which in turns implies that a shrinking comoving horizon produces an accelerated expansion. Also inflation implies some constraint on the evolution of the Einstein equations. If we write,

$$\frac{d}{dt}(aH)^{-1} = -\frac{\dot{a}H + a\dot{H}}{(aH)^2} = -\frac{1}{a}(1-\varepsilon), \text{ where } \varepsilon \equiv -\frac{\dot{H}}{H^2} > 0, \tag{3.4}$$

the shrinking Hubble sphere therefore also corresponds to

$$\varepsilon = -\frac{\dot{H}}{H^2} = -\frac{d\ln H}{dN} < 1. \tag{3.5}$$

Here we have defined $dN \equiv d\ln a = H dt$, which measures the number of e-folds N of inflationary expansion. Eq. (3.5) implies that the fractional change of the Hubble parameter per e-fold is small. Moreover, to solve the horizon problem, we want inflation to last for a sufficiently long time. To achieve this requires ε to remain small for a sufficiently large number of Hubble times. This condition is measured by a second parameter,

$$\eta \equiv \frac{\dot{\varepsilon}}{H\varepsilon} = \frac{d\ln\varepsilon}{dN}. \tag{3.6}$$

Then, for $|\eta| \ll 1$ the fractional change of ε per Hubble time is small and inflation persists. We should also ask ourselves what form of stress energy source permits accelerated expansion. Assuming a perfect fluid with pressure p and density ρ, the Friedmann equations (2.7) become:

$$\dot{H} + H^2 = -\frac{1}{6M_{\rm pl}^2}(\rho + 3p) = -\frac{H^2}{2}\left(1 + 3\rho\right). \tag{3.7}$$

Then

$$\varepsilon = -\frac{\dot{H}}{H^2} = \frac{3}{2}\left(1 + \frac{p}{\rho}\right) < 1 \Longleftrightarrow w \equiv \frac{p}{\rho} < -\frac{1}{3}. \tag{3.8}$$

The question we would like to answer now is how we can implement inflation on a field theoretical context. To do this we have a large number of options. This is a big problem in inflation; up to date, it has been impossible to find an unique model that match all the predictions of inflation, but instead an enormous variety of models have appeared since inflation was first presented in the eighties. This subject partially is the bulk of this thesis. But to give a didactic introduction we give a brief description of the simplest inflationary models.

3.1.1 Single field *slow-roll* inflation

The simplest discussion we can give to push forward our intuition is to consider inflation as embedded on a field theoretical context. We start by considering a scalar field ϕ, called *inflaton* with potential $V(\phi)$ which dynamics is given by the Lagrangian (we now set $M_{\rm pl}^2 = 1$)

$$\mathcal{L} = -\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi). \tag{3.9}$$

The idea of inflation is to fill a small region of the initial universe with an homogeneously distributed scalar field sitting on top of its potential $V(\phi)$. The equations of motion described by the action (3.9), using a FRW background, are

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0, \tag{3.10}$$

and

$$H^2 = \frac{1}{3}\left(\frac{1}{2}\dot{\phi}^2 + V(\phi)\right). \tag{3.11}$$

Using this equations we derive the continuity equation, wich is found to be given by

$$\dot{H} = -\frac{1}{2}\dot{\phi}^2. \tag{3.12}$$

In addition the stress energy tensor is given by

$$T_{\mu\nu} = -\frac{2}{\sqrt{-g}}\frac{\delta S_\phi}{\delta g^{\mu\nu}} = \partial_\mu\phi\partial\nu\phi - g^{\mu\nu}\left(\frac{1}{2}\partial_\rho\phi\partial^\rho\phi + V(\phi)\right). \tag{3.13}$$

This result leads to the following expressions for the energy-density and pressure

$$\rho_\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi), \tag{3.14}$$

$$p_\phi = \frac{1}{2}\dot{\phi}^2 - V(\phi). \tag{3.15}$$

Therefore the equation of state is

$$w_\phi = \frac{p_\phi}{\rho_\phi} = \frac{\frac{1}{2}\dot{\phi}^2 - V(\phi)}{\frac{1}{2}\dot{\phi}^2 + V(\phi)}. \tag{3.16}$$

From (3.16), to inflation occurs this condition is equivalent,to have an eq. of state with $w_\phi \approx -1 < \frac{1}{3}$. Thus the potential energy V, should dominate over the kinetic energy $\dot{\phi}^2$. This may be expressed using the *slow roll* parameter ε (3.5). Then

$$\varepsilon = -\frac{\dot{H}}{H^2} = \frac{1}{2}\frac{\dot{\phi}^2}{H^2} \ll 1. \tag{3.17}$$

Also notice that eq. (3.10) is the same as the one of a particle rolling down a potential. This particle is subject to friction through the term $H\dot{\phi}$. Like for particle trajectory, this means that the solution where $\dot{\phi} \approx V_\phi/(3H)$ is a slow roll attractor solution if friction is large enough. This can be written in terms of the *slow roll* parameter η (3.6) as

$$\eta = -\frac{\ddot{\phi}}{H\dot{\phi}} \ll 1. \tag{3.18}$$

Then we have found that the two slow roll parameters have to be much smaller than 1. The first parameter ε meaning that we are on a background solution where

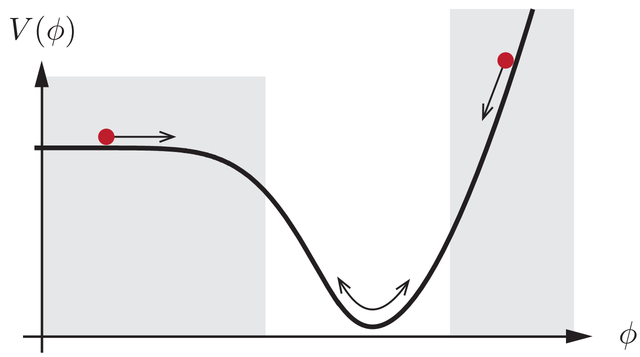

Figure 3.1: Example of a slow-roll potential. Inflation occurs in the shaded parts of the potential.

the Hubble rate changes very slowly with time. The second parameter means that we are on an attractor solution, and also that this phase of accelerated expansion ($w \approx -1$, $a \sim e^{HT}$) will last for a long time. For this condition to persists the acceleration of the scalar field has to be small. To achieve this, it is useful to define dimensionless acceleration per Hubble time

We now use the above conditions, $\varepsilon \sim |\eta| \ll 1$ to simplify the equations of motion. This is the so-called the *slow roll* approximation. First we notice that the parameters can be written in terms of the potential as,

$$\varepsilon \approx \frac{M_{\rm pl}^2}{2}\left(\frac{V'}{V}\right)^2, \tag{3.19}$$

$$\eta \approx M_{\rm pl}^2\frac{V''}{V} - \frac{M_{\rm pl}^2}{2}\left(\frac{V'}{V}\right)^2. \tag{3.20}$$

These conditions lead to the following simplification of the inflation equations (3.11), (3.10), (3.12), which now reduce to:

$$\dot{\phi} \approx \frac{V'}{3H}, \tag{3.21}$$

$$H^2 \approx \frac{V}{3M_{\rm pl}^2}, \tag{3.22}$$

$$a \sim e^{3Ht}. \tag{3.23}$$

Let us notice that inflation will end when w ceases to be ≈ -1, which in terms of the *slow roll* parameters is,

$$\varepsilon \sim \eta \sim 1. \tag{3.24}$$

In terms of the potential, this means that the field that starts on top of his potential will slowly roll down while the Hubble parameter H will decrease, providing less friction. Hence, it will be a point when the potential will become too steep to guarantee that the kinetic energy is negligible with respect to the potential energy. We will call the point in field space where this occur $\phi_{\rm end}$. At that point (immediately after inflation occurs), a period dominated by a form of energy $w > -1/3$ is expected to begin. This period is called *reheating*.

The amount of inflation required to solve the cosmological problems, is most easily measured using the number of *e-foldings*. These are defined as the logarithm of the ratio of the scale factor at the end and at the beginning of inflation. We then have

$$\begin{aligned} N(\phi) &\equiv \log\left(\frac{a_{\rm end}}{a}\right) = \int_{a_i}^{a_f} d\log a = \int_{t_i}^{t_f} H dt \\ &= \int_{\phi}^{\phi_{\rm end}} \frac{H}{\dot{\phi}} d\phi \approx \int_{\phi_{\rm end}}^{\phi} \frac{V}{V'} d\phi. \end{aligned} \tag{3.25}$$

where we have used that $a \sim e^{Ht}$ and the *slow roll* approximations. The largest scales observed in the CMB are produced some 40 to 60 *e-folds* before the end of inflation, thus

$$N_{\rm CMB} = \int_{\phi_{\rm end}}^{\phi} \frac{V}{V'} d\phi \approx 40 \; - \; 60. \tag{3.26}$$

3.1.2 Reheating

As we mentioned in the last section, just after inflation ends a new epoch called reheating has to start where the equation of state becomes $w > -1/3$. At the point $\phi_{\rm end}$, typically the inflaton begins to oscillate around the bottom of the potential. In this regime it drives the universe as if it were dominated by non-relativistic matter. The equation of motion for the inflaton indeed becomes

$$\frac{\partial \rho_\phi}{\partial t} + (3H + \Gamma)\rho_\phi = 0. \tag{3.27}$$

For $\Gamma = 0$, this is the dilution equation for non-relativistic matter. Γ represents the inflation decay rate. Indeed, in this period of time the inflaton is supposed to decay into other particles. These thermalize and, once the inflaton has decayed enough, their energy density start dominating the universe. This is the start of the standard big-bang cosmology

3.1.3 Case of study: $V(\phi) = m^{\alpha-4}\phi^{\alpha}$

As an example, let us analyze the *slow-roll* inflation by considering the following inflationary potential:

$$V(\phi) = \frac{1}{2}m^{\alpha-4}\phi^{\alpha}. \tag{3.28}$$

These models belong to a class of models called *large field inflation*. The slow roll parameters are

$$\varepsilon_v \sim M_{\text{pl}}^2\left(\frac{V_{,\phi}}{V}\right)^2 \sim \alpha^2\left(\frac{M_{\text{pl}}}{\phi}\right)^2. \tag{3.29}$$

$$|\eta_v| \sim \alpha^2\left(\frac{M_{\text{pl}}}{\phi}\right)^2. \tag{3.30}$$

To satisfy the slow roll conditions $\varepsilon_v, |\eta_v| \ll 1$, we need to consider super Planckian values of the inflaton, hence the name "large field".

$$\phi > \sqrt{2}M_{\text{pl}} \equiv \phi_{\text{end}}. \tag{3.31}$$

There is no problem with this, since we can trust general relativity and its semi-classical description of spacetime at the energies of inflation. Nevertheless in the case we would like to embed this theory on a UV complete theory of gravity such as string theory, we will need to be able to control all M_{pl} suppressed operators. Though this is possible, it is very demanding and also an active line of research. The relation between the inflaton field value and the number of e-folds before the end of inflation is

$$N(\phi) = \frac{\phi^2}{4M_{\text{pl}}^2} - \frac{1}{2}. \tag{3.32}$$

Fluctuations observed in the CMB are created at

$$\phi_{\text{CMB}} = 2\sqrt{N_{\text{CMB}}}M_{\text{pl}} \sim 15M_{\text{pl}}. \tag{3.33}$$

This kind of models are still a possibility when tested with observations. In the next section we will see how further to constrain them by using observational results .

3.2 Quantum origin of structures

Something remarkable happens when one considers quantum fluctuations of the inflation: this theory combined with quantum mechanics provides an elegant mechanism for generating the initial seeds of all structure in the universe. A very intuitive way of understanding how the quantum fluctuations of the inflaton field $\delta\phi(\mathbf{x})$ is via the time delay formalism developed by Guth and Pi. The basic idea is that ϕ controls the time at which inflation ends, because we consider ϕ to play the role of a local clock reading the amount of inflation expansion remaining. Because ϕ is a quantum mechanical object, necessarily will have some variance, then the inflaton will have spatially varying fluctuations $\delta\phi(t,\mathbf{x}) = \phi(t,\mathbf{x}) - \bar{\phi}(t)$. This small quantum fluctuations in the value of the inflaton field translate into differences in the end of inflation for different regions of space $\delta t(\mathbf{x})$. Regions acquiring a negative frozen fluctuation $\delta\phi$ remain potentially longer than regions where $\delta\phi$ is positive. Hence, fluctuations of the field ϕ lead to a local delay of the end of inflation. In quantum theory, local fluctuations in $\delta\rho(t,\mathbf{x})$ and hence ultimately in the CMB temperature anisotropy $\Delta T(\mathbf{x})$ are unavoidable and produce the small fluctuations that lead to the formation of the structure seen today.

$$\delta t = \frac{\delta\phi}{\dot{\phi}} \sim \frac{H}{\dot{\phi}}. \tag{3.34}$$

After reheating, the energy density evolves as $\rho = 3M_{\rm pl}^2 H^2$ where $H \sim t^{-1}$, so that

$$\frac{\delta\rho}{\rho} \sim 2\frac{\delta H}{H} \sim H\delta \sim H\frac{H}{\dot{\phi}}. \tag{3.35}$$

This process therefore induces tiny density variations $\delta\rho$ which via gravitationally instability grow to form the observed large-scale structure of the universe. In addition, quantum fluctuations during inflation excite tensor metric perturbations, $\delta g \sim H/M_{\rm pl}$. Future experiments hope to detect this stochastic background of gravitational waves from inflation.

This makes explicit the idea that introducing inflation makes it necessary to join quantum mechanics with general relativity. Thus producing inevitable quantum effects that in this way produces inhomogeneities to the background.

3.2.1 Classical Perturbations

To make the discussion simpler we will consider the case of single field slow-roll models. Taking the inflationary action:

$$S = \int d^4x\sqrt{-g}\left[\frac{1}{2}R_{(4)} - \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi)\right], \tag{3.36}$$

and separating the time dependent background inflationary solution from the rest, we have

$$\phi(t,\mathbf{x}) = \phi_0(t) + \delta\phi(t,\mathbf{x}), \tag{3.37}$$

$$g^{\mu\nu} = g^{\mu\nu} + \delta g^{\mu\nu}. \tag{3.38}$$

Gauge choice

A crucial subtlety in the study of cosmological perturbations is the fact that the split at eqn. (3.38) into background and perturbations is not unique, as it depend on the choice of coordinates (or choice of gauge). As it is well known, this gauge freedom could lead to non-physical perturbations. Therefore to resolve any ambiguity between real and fake perturbations in general relativity we need to consider the complete set of perturbations, *i.e.* we need both the matter field perturbations and the metric perturbations. Crucially, a gauge transformation can trade one for the other. To avoid misinterpretation of fictitious gauge modes it will also usebeful to study gauge-invariant combinations of perturbations. By definition, fluctuations of gauge-invariant quantities cannot be removed by a coordinate transformation, therefore offering a reliable way of computing observables. To start, we consider the following line element:

$$ds^2 = -(1+2\Phi)dt^2 + 2a(t)B_{,i}dx^i dt + a^2(t)[(1-2\Psi)\delta_{ij} + 2E_{,ij}]dx^i dx^j. \quad (3.39)$$

We can classify the perturbations contained in this line element according to how they transform under rotations under one axes. Formally a perturbation δ of wavenumber $\vec{k}$ has helicity λ if, under a rotation θ under the axis $\hat{k}$, it transforms as

$$\delta \longrightarrow e^{i\lambda\theta}\delta. \quad (3.40)$$

Scalars have helicity zero, vectors one, and tensors two. It can be showed that, at linear order, perturbation with different helicities do not mix. Then we can consider each element separately. We have from the action, 5 scalar degrees of freedom. One coming from the inflation scalar degree of freedom and 4 from the metric perturbation. Given that the line element is invariant under time and space gauge transformations,

$$\begin{aligned} t &\longrightarrow t + \varepsilon_0, & (3.41)\\ x_i &\longrightarrow x_i + \partial_i\varepsilon, & (3.42)\end{aligned}$$

The scalar metric perturbations transform as,

$$\begin{aligned} \Phi &\rightarrow \Phi - \dot{\varepsilon_0} & (3.43)\\ B &\rightarrow B - a^{-1}\varepsilon_0 - a\dot{\varepsilon} & (3.44)\\ E &\rightarrow E - \varepsilon & (3.45)\\ \Psi &\rightarrow \Psi + H\varepsilon_0. & (3.46)\end{aligned}$$

Therefore, because of gauge invariance, we are left with 3 physical degrees of freedom. The constraints in the action (3.36) removes two more. Then we just have a single scalar degree of fredoom.

We will choose the comoving gauge which is defined by the vanishing of the scalar momentum density. Then we have

$$\begin{aligned} \delta\phi &= 0, & (3.47) \\ \delta g_{ij} &= a^2(1-2\zeta)\delta_{ij} + a^2 h_{ij}. & (3.48) \end{aligned}$$

In this gauge ϕ is a clock that tell us when inflation ends. We have that the curvature in the 3-space is

$$R_{(3)} = \frac{4}{a^2}\nabla^2\zeta. \tag{3.49}$$

Also in the limit of long wavelength we obtain

$$\lim_{k\ll aH} \dot{\zeta} = 0. \tag{3.50}$$

Inserting the gauge choice in the action (3.36) we get

$$S = \frac{1}{2}\int dt d^3x a^3 \frac{\dot{\phi}}{H^2}\left[\dot{\zeta}^2 - \frac{1}{a}(\nabla\zeta)^2\right]. \tag{3.51}$$

Changing to conformal time and using the Mukhanov's variable instead

$$\begin{aligned} v &= z\zeta, & (3.52) \\ z^2 &= a^2\frac{\dot{\phi}^2}{H^2} = 2a^2\varepsilon. & (3.53) \end{aligned}$$

action (3.51) becomes

$$S = \frac{1}{2}\int d\tau d^3x \left[(v')^2 - (\nabla v)^2 + \frac{z''}{z}v^2\right], \tag{3.54}$$

where we change to conformal time $d\tau = \frac{dt}{a(t)}$. Let us notice that this action is similar to the Klein-Gordon action but with a time dependent mass

$$m^2(\tau) = -\frac{z''}{z}. \tag{3.55}$$

This time dependent mass accounts for the interactions on the scalar field ζ with the gravitational background. Fourier transforming the Mukhanov variable:

$$v(\tau, x) = \int \frac{d^3x}{(2\pi)^{3/2}} v_k e^{ikx}, \tag{3.56}$$

and varying this action (3.54) we get,

$$v''_{\mathbf{k}} + \left(k^2 - \frac{z''}{z}\right)v_{\mathbf{k}} = 0, \tag{3.57}$$

called *Mukhanov-Sasaki* equation. We can rewrite this equation as

$$v_k'' + \omega_k^2(\tau) v_k = 0, \tag{3.58}$$

where $\omega_k^2(\tau) = k^2 - \frac{z''}{z} = k^2 + m^2$. In the short wavelength limit $k \gg -\frac{z''}{z}$ this equation becomes

$$v_k'' + k^2 v_k = 0. \tag{3.59}$$

Therefore in this regime the equation has oscillatory solutions $v_k \propto e^{\pm ik\tau}$. This is because well inside the horizon there is no difference between a curved spacetime-geometry and Minkowski spacetime.On the other hand, in the long wavelength limit we have that

$$v_k''(\tau) \approx \frac{2}{\tau^2} v_k(\tau), \tag{3.60}$$

which implies that $v_k \propto \frac{1}{\tau}$, therefore outside the horizon the ζ_k modes freezes.

3.2.2 Quantization

Canonical quantization proceeds in the standard way; we promote the field v and its canonical conjugate momentum $\pi = v'$ to quantum operators $\hat{v}$ and $\hat{\pi}$, which satisfy the standard equal times commutation relations

$$[\hat{v}(\tau, \mathbf{x}), \hat{\pi}(\tau, \mathbf{y})] = i\delta(\mathbf{x} - \mathbf{y}), \tag{3.61}$$

and

$$[\hat{v}(\tau, \mathbf{x}), \hat{v}(\tau, \mathbf{y})] = [\hat{\pi}(\tau, \mathbf{x}), \hat{\pi}(\tau, \mathbf{y})] = 0. \tag{3.62}$$

From (3.54) it follows that the commutation relation (3.61) holds at any times if it is satisfied at a given time. The Hamiltonian is

$$\hat{H}(\tau) = \frac{1}{2} \int d^3x [\hat{\pi}^2 + (\partial\hat{v})^2 + m_{\text{eff}}^2(\tau)\hat{v}^2]. \tag{3.63}$$

The constants of integration $a_k^{\pm}$ in the mode expansion of v become operators $\hat{a}_k^{\pm}$, so that the operator $\hat{v}$ is expanded as

$$\hat{v}(\tau, \mathbf{x}) = \int \frac{d^3\mathbf{k}}{(2\pi)^{3/2}} \left[\hat{a}_{\mathbf{k}}^- v_k(\tau) e^{i\mathbf{k}x} + \hat{a}_{\mathbf{k}}^+ v_k^*(\tau) e^{-i\mathbf{k}x}\right]. \tag{3.64}$$

Substituting this equation in the canonical commutation relation (3.61) implies

$$[\hat{a}_{\mathbf{k}}^-, \hat{a}_{\mathbf{k}'}^+] = \delta(\mathbf{k} - \mathbf{k}') \text{ and } [\hat{a}_{\mathbf{k}}^-, \hat{a}_{\mathbf{k}'}^-] = [\hat{a}_{\mathbf{k}}^+, \hat{a}_{\mathbf{k}'}^+] \;\; = \;\; 0. \tag{3.65}$$

As usual, the operators $\hat{a}_{\mathbf{k}}^-$ and $\hat{a}_{\mathbf{k}}^+$ may be interpreted as creation and annihilation operators, respectively. Quantum states in the Hilbert space are constructed by defining the vacuum state $|0\rangle$ via

$$\hat{a}_{\mathbf{k}}^- |0\rangle = 0, \tag{3.66}$$

and by producing excited states by repeated application of creation operators.

3.2.3 Choice of vacuum

To proceed let us first recall that in a *time-independent* spacetime a preferable set of mode functions, and thus an unambiguous physical vacuum, can be defined by requiring that the expectation value of the Hamiltonian in the vacuum state is minimized. This gives rise to

$$v_k(\tau) = \frac{1}{\sqrt{2k}} e^{-ik\tau}. \tag{3.67}$$

This defines the preferred mode functions for fluctuations in Minkowski space. This vacuum prescription does not generalize straightforward to *time dependent* spacetimes, since the mode equations (3.57) involves time dependent frequencies $\omega_k(\tau)$ and the minimum energy vacuum depends on the time τ_0 at which it is defined. Repeating the above argument, one can nevertheless determine the vacuum which instantaneously minimize the expectation value of the Hamiltonian at some time τ_0. One finds that the initial conditions,

$$v_k(\tau_0) = \frac{1}{\sqrt{2k\omega_k(\tau_0)}} e^{-ik\omega_k(\tau_0)\tau_0}, \; v_k'(\tau_0) = -i\omega_k(\tau_0)\chi_k(\tau_0), \tag{3.68}$$

select the preferred mode functions which determine the vacuum $|0\rangle_{\tau_0}$. However, since $\omega_k(\tau)$ changes with time, the mode function satisfies the same conditions at a different time $\tau_1 \neq \tau_0$. This implies that $|0\rangle_{\tau_1} \neq |0\rangle_{\tau_0}$ and the state $|0\rangle_{\tau_0}$ is not the lowest energy state at a later time τ_1.

To solve this ambiguity we note that at sufficiently early times all modes of cosmological interest are deep inside the horizon:

$$\frac{k}{aH} \sim |k\tau| \gg 1. \tag{3.69}$$

This means that in the remote past all observable modes had time independent frequencies,

$$\omega_k^2 \sim k^2 - \frac{2}{\tau^2} \to k^2. \tag{3.70}$$

Therefore the corresponding modes are not affected by gravity and behave just like in Minkowski space. Given that all modes have time-independent frequencies at sufficiently early times, we can now avoid the ambiguity in defining the initial conditions for the mode functions that afflicts the treatment in more general time-dependent spacetimes. This means solving the Mukhanov-Sasaki equation (3.57) with the initial condition

$$\lim_{\tau \to -\infty} v_k(\tau) = \frac{1}{\sqrt{2k}} e^{-ik\tau}. \tag{3.71}$$

This defines a preferable set of mode functions and a unique physical vacuum, the *Bunch-Davies vacuum.*

3.2.4 Power spectrum

We now compute the effect of quantum zero point fluctuations. It follows that

$$\begin{aligned}\langle \hat{v}_{\mathrm{k}} \hat{v}_{\mathrm{k}'} \rangle &= \langle 0 | \hat{v}_{\mathrm{k}} \hat{v}_{\mathrm{k}'} | 0 \rangle \\ &= |v_k|^2 \delta(\mathbf{k} - \mathbf{k}') \\ &= P_v(k) \delta(\mathbf{k} - \mathbf{k}').\end{aligned}$$

Where $P_v(k)$ is the power spectrum for canonically normalized fields. For the *slow roll* inflationary case, at first order in *slow roll* parameters, equation (3.57) reduces to

$$v_k'' + \left(k^2 - \frac{\nu^2 - \frac{1}{4}}{\tau^2}\right) v_k = 0, \tag{3.72}$$

where

$$\nu \equiv \frac{3}{2} + \varepsilon + \frac{1}{2}\eta. \tag{3.73}$$

For constant ν eq.(3.72) has an exact solution in terms of the Hankel functions of the first and second kind:

$$v_k(\tau) = \sqrt{-\tau} \left[\alpha H_\nu^{(1)}(-k\tau) + \beta H_\nu^{(2)}(-k\tau)\right]. \tag{3.74}$$

To impose Bunch-Davies boundary conditions at early times, we consider the limit,

$$\lim_{k\tau \to -\infty} v_k(\tau) = \sqrt{\frac{2}{\pi}} \left[\alpha \frac{1}{\sqrt{k}} e^{-ik\tau} + \beta \frac{1}{\sqrt{k}} e^{ik\tau}\right], \tag{3.75}$$

where we used a known limit of the Hankel functions. Comparing with (3.71) we get,

$$\beta = 0 \text{ and } \alpha = \sqrt{\frac{\pi}{2}}. \tag{3.76}$$

Hence, the Bunch-Davies mode functions to first order in slow-roll are

$$v_k(\tau) = \sqrt{\frac{\pi}{2}} (-\tau)^{1/2} H_\nu^{(1)}(-k\tau). \tag{3.77}$$

To compute the power spectrum of curvature fluctuations we use that at first order in slow roll, $P_{\mathcal{R}} = z^{-2} P_v$, and therefore

$$P_{\mathcal{R}} \sim \frac{\pi}{2} (-\tau)^{2\nu} |H_\nu^{(1)}(-k\tau)|^2, \tag{3.78}$$

or in its dimensionless form

$$\Delta_s^2(k) \equiv \frac{k^3}{2\pi^2} P_{\mathcal{R}}(k) \sim k^{3-2\nu}. \tag{3.79}$$

The common way to quantify the deviations from the scale invariance is via the scalar spectral index n_s:

$$n_s - 1 \equiv \frac{d\ln\Delta_s^2}{d\ln k}. = 3 - 2\nu. \tag{3.80}$$

Or in terms of the slow roll parameters

$$n_s - 1 = -2\varepsilon - \eta. \tag{3.81}$$

This shows that the spectrum for slow roll inflation is almost scale invariant with deviations from $n_s = 1$ at a percent level.

3.2.5 Tensor Perturtions

From the line element (A.3) we have that tensor perturbations are gauge invariants, and uncoupled from the rest of the perturbations, at least at linear level. Thus we can write the perturbed metric as

$$\delta g_{ij} = a^2 \gamma_{ij}. \tag{3.82}$$

Expanding the action (3.36) up to second order, and inserting (3.82), we have

$$S = \int d^4x a^3 M_{\rm pl}^2 \left[(\dot{\gamma_{ij}})^2 - \frac{1}{a^2}(\partial_l \gamma_{ij})^2 \right] = \sum_{+,\times} \int dt d^3k a^3 M_{\rm pl}^2 \left[\dot{\gamma}_{\mathbf{k}}^s \dot{\gamma}_{-\mathbf{k}}^s - \frac{k^2}{a^2} \gamma_{\mathbf{k}}^s \gamma_{-\mathbf{k}}^s \right] \tag{3.83}$$

where $+, \ \times$ are the two possible polarization states and,

$$\gamma_{ij}^{+,\times} \equiv \gamma_s(t) e_{ij}^{(+,\times)}. \tag{3.84}$$

As the action for each polarization is the same, that we found, for curvature modes but with a different canonical normalization, quantization is straightforward. Thus the power spectrum for gravity waves is,

$$\left\langle \gamma_{\mathbf{k}}^s \gamma_{\mathbf{k}'}^{s'} \right\rangle = (2\pi)^3 \delta^3(\mathbf{k} + \mathbf{k}') \delta_{s,s'} \frac{H^2}{M_{\rm pl}^2} \frac{1}{k^3}. \tag{3.85}$$

The tilt of gravity waves is

$$n_t - 1 = 2\varepsilon. \tag{3.86}$$

Tensor perturbations are often normalized relative to their size to curvature perturbations $\Delta_{\mathcal{R}}^2 \approx \Delta_s^2 \approx 10^9$. The *tensor to scalar ratio* is

$$r \equiv \frac{\Delta_t^2(k)}{\Delta_s^2(k)} \tag{3.87}$$

Since Δ_s^2 is fixed and Δ_t^2 is proportional to the Hubble scale H, the tensor to scalar ratio is a direct measure of the energy scale of inflation.

$$H^{1/4} \sim \left(\frac{r}{0.01} \right)^{1/4} 10^{16}\text{GeV}. \tag{3.88}$$

3.3 Contact with observations

CMB Basics

We now discuss briefly how to relate inflationary quantities as $P_{\mathcal{R}}$ to observations of the CMB and large-scale structure. To do so we will need to define the transfer function: for a given perturbation $\delta X(k,\tau)$ at a given time τ and with Fourier mode k, we can define its transfer function for the quantity X at that time τ and for the Fourier mode k as,

$$\delta X(\tau)T(k,\tau,\tau_{\rm in})X(\tau_{\rm in}) \tag{3.89}$$

This must be so in the linear approximation. We can take $\tau_{\rm in}$ early enough so that the mode k is smaller than aH, in this way $\mathcal{R}_k(\tau_{\rm in})$ represents the constant value $\mathcal{R}$ took at freeze out during inflation.

For the CMB temperature, we may perform a spherical harmonics decomposition

$$\frac{\delta T}{T}(\tau_0,\hat{n}) = \sum_{l,m} a_{lm}Y_{lm}(\hat{n}). \tag{3.90}$$

and then due to statistical isotropy, the power spectra reads

$$\langle a_{lm}a_{l'm'}\rangle = C_l\delta_{ll'}\delta_{mm'}. \tag{3.91}$$

Since the temperature anisotropy is dominated by the scalar modes $\mathcal{R}$, we have:

$$a_{lm} = \int d^3k\Delta_l(k)\mathcal{R}_kY_{lm}(\hat{k}). \tag{3.92}$$

Therefore using (3.91), and the identity;

$$\sum_{m=-l}^{l} Y_{lm}(\hat{k})Y_{lm}(\hat{k}') = \frac{2l+1}{4\pi}P_l(\hat{k}\cdot\hat{k}'), \tag{3.93}$$

we find that:

$$C_l = \int dkk^2\Delta_l(k)^2P_{\mathcal{R}}(k), \tag{3.94}$$

where $\Delta_l(k)$ contains both the effect of the transfer functions and also of the projection on the sky. This is a hard calculation, and to calculate the transfer functions $\Delta_l(k)$ needs a numeric tool to be computed, which is beyond the scope of this thesis. Anyway we make some approximations to get a qualitative result.

- **Large scales** If we look at very large scales, we find modes that were still outside H^{-1} at the time of recombination. Nothing could have happened to them. Then there has been no evolution and only projection effects. In this regime the transfer function could be approximated by a Bessel function:

$$\Delta_l(k) \approx j_l(k(\tau_0 - \tau_{\rm rec})), \tag{3.95}$$

and therefore

$$C_l \approx \int dk k^2 P_{\mathcal{R}} j_l^2(k(\tau_0 - \tau_{\rm rec})). \tag{3.96}$$

We notice that $j_l^2(k(\tau_0 - \tau_{\rm rec}))$ is sharply peaked at $k(\tau_0 - \tau_{\rm rec}) \sim l$, so we can approximately perform the integral, to obtain:

$$\begin{aligned} C_l &\approx k^3 P_{\mathcal{R}}|_{k=l/(\tau_0-\tau_{\rm rec})} \times \int d\log x j_l^2(x) \\ &\sim k^3 P_{\mathcal{R}}|_{k=l/(\tau_0-\tau_{\rm rec})} \times \frac{1}{l(l+1)}. \end{aligned} \tag{3.97}$$

Therefore the quantity $l(l+1)C_l$ is nearly flat.

- **Small scales** On short scales, mode entered inside H^{-1} and begun to feel both the gravitational attraction of denser zones, but also their pressure repulsion. This leads to the following solutions for the temperature perturbation,

$$\ddot{\delta T} + c_s^2 \nabla^2 \delta T \approx F_{\rm gravity}(\zeta), \tag{3.98}$$

thus,

$$\delta T_k \approx A_k \cos(k\eta) + B_k \sin(k\eta) = \tilde{A} \cos(k\eta + \phi_k). \tag{3.99}$$

Here A_k and B_k depends on the initial conditions. In inflation we have

$$\tilde{A}_k \approx \frac{1}{k^3}, \ \phi_k = 0 \tag{3.100}$$

All the modes are in phase coherence. Notice, dynamics and wavenumber force all modes of a fixed wavenumber to have the same frequency. However, they need not have necessarily the same phase. Inflation, or super Hubble fluctuations, forces $\zeta \approx \frac{\delta T}{T} = \text{const}$ on large scales, which implies $\phi_k = 0$. This is what leads to acoustic oscillations in the CMB

$$\delta T(k, \eta) \sim \delta T_{\rm in}(k) \times \cos(k\eta). \tag{3.101}$$

Thus

$$\delta T(k, \eta_0) \sim \delta T_{\rm in}(k) \times \cos(k\eta_{\rm rec}). \tag{3.102}$$

Then

$$\langle \delta T(k, \eta_0) \rangle \sim \langle \delta T_k^2 \rangle \cos^2(k\eta_{\rm rec}), \tag{3.103}$$

which implies the acoustic oscillations.

3.3.1 Non Gaussianities

As we have seen the curvature power spectrum is very Gaussian. Nevertheless it seem plausible that given the high quality of the data available, we can put further constrain and even study the non Gaussian contribution to the correlations presents in the CMB.

Moreover, being a direct measurement of inflaton interactions, constrain on primordial non-Gaussianities will permit us to learn a lot about inflationary dynamics. To study the leading non-Gaussian effect we need to expand the action for the curvature mode $\mathcal{R}$, at least, up to third order. And calculate higher order correlation functions. This calculations can be particularly cumbersome. Here we just give a summary of the main results.

The main correlation function used to study non-Gaussianities is the bispectrum:

$$\langle \mathcal{R}_{\mathbf{k}_1}\mathcal{R}_{\mathbf{k}_2}\mathcal{R}_{\mathbf{k}_3}\rangle = (2\pi)^3\delta(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)B_{\mathcal{R}}(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3). \tag{3.104}$$

where the delta function is present because of the translational invariance of the background.

One way of parametrizing the effect of non-Gaussianities is through a non linear correction to a Gaussian perturbation $\mathcal{R}_g$,

$$\mathcal{R}(x) = \mathcal{R}_g(x) + \frac{3}{5}f_{\mathrm{NL}}^{\mathrm{local}}\left[\mathcal{R}_g(x)^2 - \langle \mathcal{R}_g(x)^2\rangle\right]. \tag{3.105}$$

This definition is local in spacetime and thus called *local non-Gaussianity*. The bispectrum of local-nongaussianity may be derived using (3.105):

$$B_{\mathcal{R}}(k_1,k_2,k_3) = \frac{6}{5}f_{\mathrm{NL}}^{\mathrm{local}} \times \left[P_{\mathcal{R}}(k_1)P_{\mathcal{R}}(k_2) + P_{\mathcal{R}}(k_2)P_{\mathcal{R}}(k_3) + P_{\mathcal{R}}(k_3)P_{\mathcal{R}}(k_1)\right] \tag{3.106}$$

For a scale invariant power spectrum $P_{\mathcal{R}}(k) \sim k^{-3}$, we have,

$$B_{\mathcal{R}}(k_1,k_2,k_3) \sim \frac{6}{5}f_{\mathrm{NL}}^{\mathrm{local}} \times \left[\frac{1}{(k_1k_2)^3} + \frac{1}{(k_2k_3)^3} + \frac{1}{(k_3k_1)^3}\right]. \tag{3.107}$$

Which is larger when one of the three momenta is very small. This limit is called the *squeezed limit*, and the bispectrum becomes (if we assume $k_3 \ll k_1 \approx k_2$):

$$\lim_{k_3\ll k_1\approx k_2} B_{\mathcal{R}}(k_1,k_2,k_3) = \frac{12}{5}f_{\mathrm{NL}}^{\mathrm{local}} \times P_{\mathcal{R}}(k_1)P_{\mathcal{R}}(k_3). \tag{3.108}$$

Because of the delta function in (3.104), the three modes form a closed triangle. Different models of inflation predicts maximal signals for some specific geometries. Therefore a possible probe of the existence of non-Gaussian correlations is to study this geometry. A common parametrization is the *shape function* defined as, [1]

$$\mathcal{S}(k_1,k_2,k_3) \equiv N(k_1,k_2,k_3)^2 B_{\mathcal{R}}(k_1,k_2,k_3), \tag{3.113}$$

where N is an appropriate normalization factor. Two commonly discussed shapes are the *local* model

$$\mathcal{S}^{\text{local}}(k_1,k_2,k_3) \propto \frac{K_3}{K_{111}}, \tag{3.114}$$

and the *equilateral* model,

$$\mathcal{S}^{\text{equilateral}}(k_1,k_2,k_3) \propto \frac{\tilde{k}_1\tilde{k}_2\tilde{k}_3}{K_1 11}. \tag{3.115}$$

As expected experimental constrain to $f_{\text{NL}}^{\text{local}}$ are very small

f_{NL}		
Local	Equilateral	Orthogonal
2.7 ± 5.8	-42 ± 75	-25 ± 39

Table 3.1: Estimates of primordial f_{NL} for local, equilateral and orthogonal shapes, as obtained by *Planck*.

[1] Here we use the following notation :

$$\begin{aligned} K_p &= \sum_i (k_i)^p \text{ with } K = K_1 && (3.109)\\ K_{pq} &= \frac{1}{\delta_{pq}} \sum_{1\neq j} (k_i)^p (k_j)^q && (3.110)\\ K_{pqr} &= \frac{1}{\delta_{pqr}} \sum_{1\neq j\neq l} (k_i)^p (k_j)^q (k_l)^q && (3.111)\\ \tilde{k}_{ip} &= K_p - 2(k_i)^p \text{ with } \tilde{k}_i = \tilde{k}_{i1}, && (3.112) \end{aligned}$$

where $\Delta_{pq} = 1 + \delta_{pq}$ and $\Delta_{pqr} = \Delta_{pq}(\Delta_{qr} + \delta_{pr})$.

Chapter 4

Multifield Inflation

In this chapter we summarize the main results coming from previous work related to the study of multi-field inflation. We will put emphasis on a geometrical description of the inflationary trajectories..

4.1 Isocurvature perturbations

There are two general classes of perturbations in a homogeneous universe filled with multiple types of fluids components: Adiabatic (or density) perturbations, in which the fluctuations of all of the components are in phase with the density perturbations, and isocurvature perturbations, in which the fluctuations of two or more components have independent phases. Single field slow roll inflation is distuinguished for predicting purely adiabatic primordial perturbations, for the simple reason that there is only one field responsible for the generations of density perturbations. The current data is, in fact, consistent with the adiabatic pertubations [10], however, current limits on isocurvature fractions are substantially weak. If isocurvature modes are detected, it would inmediatly rule out single field models of inflation. Multifield models, on the other hand, naturally contain multiple order parameters and can generate isocurvature modes corresponding to relative perturbations between the various scalar fields.

4.2 Background dynamics

For arbitrary number of scalar fields, the most general action is

$$S = \int d^4x \sqrt{\mathrm{detg}} \left[-\frac{1}{2} g^{\mu\nu} \gamma_{ab} \partial_\mu \phi^a \partial_n u \phi^b - V(\phi) \right], \tag{4.1}$$

where $V(\phi)$ is an arbitrary real potential, and $\gamma_{ab}(\phi)$ is an arbitrary real symmetric positive-definite matrix which is called *field metric*. The field equation for this action are:

$$\ddot{\phi_0}^a + \gamma^a_{bc}\dot{\phi}^b_0\dot{\phi}^c_0 + 4H\dot{\phi}^a + \gamma^{ab}\partial_b V(\phi_0) = 0, \tag{4.2}$$

where γ^{ab} is the reciprocal of the field metric γ_{ab}, and γ^a_{ab} is the affine connection associated the field space,

$$\gamma^a_{bc}(\phi_0) = \frac{1}{2}\gamma^{ad}(\phi_0)\left(\frac{\partial\gamma_{db}(\phi_0)}{\partial\phi^c_0} + \frac{\partial\gamma_{dc}(\phi_0)}{\partial\phi^b_0} - \frac{\partial\gamma_{bc}(\phi_0)}{\partial\phi^d_0}\right), \tag{4.3}$$

and H is the usual expansion rate given by $H \equiv \dot{a}/a$. We can also define the covariant derivative in field space as

$$\frac{D}{dt}v^n \equiv \frac{\partial}{\partial t}v^n + \gamma^a_{bc}v^b_0 v^c. \tag{4.4}$$

With this notation eq.(4.2) becomes

$$\frac{D}{dt}\dot{\phi}^a_0 + 3H\dot{\phi}^a_0 + V^a = 0. \tag{4.5}$$

4.2.1 Orthogonal Basis

Before consider perturbation theory, we note that, because γ^{ab} is positive definite, it can be written in terms of a set of vielbeins vectors e^a_N (with N running over as many values as a), as

$$\gamma^{ab}(\phi_0) = \sum_N e^a_N(t)e^b_N(t). \tag{4.6}$$

Which are defined to satisfy the basic relations $e^N_a e^M_b \gamma^{ab} = \delta^{MN}$ and $e^N_a e^M_b \delta_{MN} = \gamma^{ab}$. From these relations one deduces the identities

$$e^M_a \frac{D}{dt}e^a_N = -e^a_N \frac{D}{dt}e^M_a, \tag{4.7}$$

$$e^a_M \frac{D}{dt}e^M_b = -e^M_b \frac{D}{dt}e^a_M. \tag{4.8}$$

It will be useful to the rest of the thesis if we take the following approach: Let us consider trajectories on field space on such a way that can be parametrized using the vielbein previously defined. Then we can define the two first vielbeins as unit vectors T^a and N^a distinguishing tangent and normal directions to the trajectory respectively as,

$$e^a_1 \equiv T^a = \frac{\dot{\phi}^a_0}{\dot{\phi}_0} \tag{4.9}$$

$$e^a_2 \equiv N^a = s_N(t)\left(\gamma_{bc}\frac{DT^b}{dt}\frac{DT^c}{dt}\right)^{-1/2}\frac{DT^a}{dt}, \tag{4.10}$$

where $\dot{\phi}_0 = \sqrt{\gamma_{ab}\dot{\phi}^a\dot{\phi}^b}$ and $s_N(t) = \pm 1$, denotes the orientation of N^a with respect to the vector DT^a/dt. That is if $s_N(t) = 1$, then N^a is pointing in the same directions as DT^a/dt, whereas if $s_N(t) = -1$ then N^a is pointing in the opposite direction. We permit $sN(t)$ to flip sings each time DT^a/dt becomes zero, in such a way that both N^a and DT^a/dt remain a continuous function of t.

Notice that the tangent vector T^a offers an alternative way of defining the total time derivative D/dt along the trajectory followed by the scalar fields. Then:

$$\frac{D}{dt} \equiv \dot{\phi}_0 T^a \nabla_a = \dot{\phi}_0 \nabla_\phi. \tag{4.11}$$

Using field eqs. (4.5) and taking time derivative of T^a we get,

$$\frac{DT^a}{dt} = -\frac{\ddot{\phi}_0}{\dot{\phi}_0}T^a - \frac{1}{\dot{\phi}_0}\left(3H\dot{\phi}_0^a + V^a\right). \tag{4.12}$$

Projecting this equating along the two orthogonal directions T^a and N^a, we obtain the following two independent equations:

$$\ddot{\phi}_0 + 3H\dot{\phi}_0 + V_\phi = 0, \tag{4.13}$$

$$\frac{DT^a}{dt} = -\frac{V_N}{\dot{\phi}_0}N^a. \tag{4.14}$$

Where we have defined $V_\phi \equiv T^a V_a$ and $V_N \equiv N^a V_a$. In the context of inflation we define ths following parameters:

$$\varepsilon \equiv -\frac{\dot{H}}{H^a} = \frac{\dot{\phi}_0^2}{2M_{\rm pl}^2}, \tag{4.15}$$

$$\eta^a \equiv -\frac{1}{H\dot{\phi}_0}\frac{D\dot{\phi}_0^a}{dt}. \tag{4.16}$$

We can decompose η^a in terms of T^a and N^a as,

$$\eta^a = \eta_\parallel T^a + \eta_\perp N^a, \tag{4.17}$$

$$\eta_\parallel \equiv -\frac{\ddot{\phi}_0}{H\dot{\phi}}, \tag{4.18}$$

$$\eta_\perp \equiv \frac{V_N}{\dot{\phi}_0 H}, \tag{4.19}$$

where we have used the eq. (4.5) again to simplify the above expressions. It is important to observe that $\eta_\perp$ is directly related to the rate of change of the tangent unit vector T^a, since we can write using (4.14),

$$\frac{DT^a}{dt} = -H\eta_\perp N^a. \tag{4.20}$$

We can define the angle determining the orientation of T^a in time, by the relation

$$\dot{\theta} \equiv H\eta_\perp \tag{4.21}$$

An important fact is that we can relate $\eta_\perp$ to the radius of curvatures κ characterizing the bending of the trajectory followed by the scalar fields. To do so, let us recall that given a curve $\gamma(\phi_0)$ in field space, we may define the radius of curvature κ associated to that curve through the following relation:

$$\frac{1}{\kappa} = \left(\gamma_{bc} \frac{DT^b}{d\dot{\phi}_0} DT^c \dot{\phi}_0 \right). \tag{4.22}$$

Here κ stands for the radius of curvature in the scalar manifold $\mathcal{M}$ spanned by the ϕ_0^a fields, and therefore it has dimension of mass. Using (4.13) we can identify that:

$$\frac{1}{\kappa} = \frac{H|\eta_\perp|}{\dot{\phi}_0}. \tag{4.23}$$

By definition any geodesic curve $\gamma(\phi_0)$ in $\mathcal{M}$ satisfies the relation $D\dot{\phi}_0^a/dt \propto \dot{\phi}_0^a$, which corresponds to the case $\kappa^{-1} = 0$, or alternatively to the case $\eta_\perp = 0$. Thus, we see that the dimensionless parameter $\eta_\perp$ is a useful quantity that parametrises the bending of the inflation trajectory with respect to geodesics in $\mathcal{M}$

4.3 Perturbation theory

We now consider the dynamics of scalar perturbations parametrizing departures from the homogeneous and isotropic background $a(t)$ and $\phi_0^a(t)$. This may be done by defining perturbations $\delta\phi^a(t,\mathbf{x})$ as:

$$\phi^a(t,\mathbf{x}) = \phi_0^a(t) + \delta\phi^a(t,\mathbf{x}). \tag{4.24}$$

For simplicity we will study just the case of two fields. The generalization is straigthforward. Instead of directly working with $\delta\phi^a(t,\mathbf{x})$, it is more convenient to work with gauge invariant fields v^T and v^N given by[1]

$$v^T = a\,T_a\delta\phi^a + a\frac{\dot{\phi}}{H}\psi, \tag{4.25}$$

$$v^N = a\,N_a\delta\phi^a, \tag{4.26}$$

where ψ is the scalar perturbation of the spatial part of the metric (proportional to δ_{ij}) in flat gauge. It is useful to consider a second set of fields (u^X, u^Y) in addition to (v^T, v^N). Let us consider the following time dependent rotation in field space

$$\begin{pmatrix} u^X \\ u^Y \end{pmatrix} \equiv R(\tau) \begin{pmatrix} v^N \\ v^T \end{pmatrix}, \tag{4.27}$$

[1] Notice that these fields are projections of the form $v^T = aT_aQ^a$ and $v^T = aT_aQ^a$ where the Q^a fields are the usual Mukhanov-Sasaki variables $Q^a \equiv \delta\phi^a + \frac{\dot{\phi}^a}{H}\psi$ [100, 113].

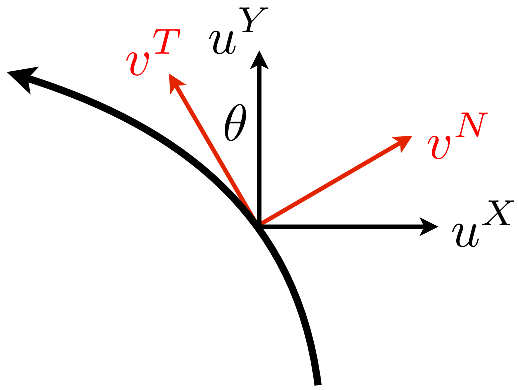

Figure 4.1: The u-fields represent fluctuations with respect to a fixed local frame, whereas the v-fields represent fluctuations with respect to the path (parallel and normal).

where the time dependent rotation matrix $R(\tau)$ is defined as

$$R(\tau) = \begin{pmatrix} \cos\theta(\tau) & -\sin\theta(\tau) \\ \sin\theta(\tau) & \cos\theta(\tau) \end{pmatrix}, \qquad \theta(\tau) = \theta_0 + \int_{-\infty}^{\tau} d\tau \, aH\eta_\perp, \tag{4.28}$$

where θ_0 is the value of $\theta(\tau)$ at $\tau \to -\infty$. The rotation angle $\theta(\tau)$ precisely accounts for the total angle covered by all the turns during the inflationary history up to time τ, and coincides with the definition introduced in eq. (4.21). Figure 4.1 illustrates the relation between the v-fields introduced earlier and the canonical u-fields. To continue, the equations of motion for the canonically normalized fields are

$$\frac{d^2u^I}{d\tau^2} - \nabla^2 u^I + \left[R(\tau)\Omega R^t(\tau)\right]^I{}_J u^J = 0, \qquad I = X, Y, \tag{4.29}$$

where R^t represents the transpose of R. In addition, Ω is the mass matrix for the v-fields, with elements given by

$$\Omega_{TT} = -a^2H^2(2 + 2\varepsilon - 3\eta_\parallel + \eta_\parallel\xi_\parallel - 4\varepsilon\eta_\parallel + 2\varepsilon^2 - \eta_\perp^2), \tag{4.30}$$

$$\Omega_{NN} = -a^2H^2(2 - \varepsilon) + a^2(V_{NN} + H^2\varepsilon\mathbb{R}), \tag{4.31}$$

$$\Omega_{TN} = a^2H^2\eta_\perp(3 + \varepsilon - 2\eta_\parallel - \xi_\perp), \tag{4.32}$$

where $\xi_\parallel = -\dot{\eta}_\parallel/(H\eta_\parallel)$ and $\xi_\perp = -\dot{\eta}_\perp/(H\eta_\perp)$. Additionally, we have defined the tree level mass V_{NN} as the second derivative of the potential projected along the perpendicular direction $V_{NN} = N^aN^b\nabla_a\nabla_b V$. To finish, expanding the original action (6.3) to quadratic order in terms of the u-fields, one finds:

$$S = \frac{1}{2}\int d\tau d^3x \left\{ \sum_I \left(\frac{du^I}{d\tau}\right)^2 - (\nabla u^I)^2 - [R(\tau)\Omega R^t(\tau)]^I{}_J u^I u^J \right\}. \tag{4.33}$$

Thus, we see that the fields $u^I = (u^X, u^Y)$ correspond to the canonically normalized fields in the usual sense. Given that these fields are canonically normalized, it is now straightforward to impose Bunch-Davies conditions on the initial state of the perturbations.

4.3.1 Curvature and isocurvature modes

Another useful field parametrization for the perturbations is in terms of curvature and isocurvature fields $\mathcal{R}$ and $\mathcal{S}$ [68]. In terms of the v-fields, these are defined as:

$$\mathcal{R} = \frac{H}{a\dot{\phi}} v^T, \tag{4.34}$$

$$\mathcal{S} = \frac{H}{a\dot{\phi}} v^N. \tag{4.35}$$

Instead of working directly with $\mathcal{S}$, it is in fact more convenient to define:

$$\mathcal{F} = \frac{\dot{\phi}_0}{H}\mathcal{S}. \tag{4.36}$$

Then, the quadratic action for the pair $\mathcal{R}$ and $\mathcal{F}$ is found to be

$$S_{\text{tot}} = \frac{1}{2}\int d^4x\, a^3 \left[\frac{\dot{\phi}_0^2}{H^2}\dot{\mathcal{R}}^2 - \frac{\dot{\phi}_0^2}{H^2}\frac{(\nabla\mathcal{R})^2}{a^2} + \dot{\mathcal{F}}^2 - \frac{(\nabla\mathcal{F})^2}{a^2} + 4\dot{\phi}_0\eta_\perp\dot{\mathcal{R}}\mathcal{F} - M_{\text{eff}}^2\mathcal{F}^2\right], \tag{4.37}$$

where we have defined the effective mass M_{eff} of the field $\mathcal{F}$ as

$$M_{\text{eff}}^2 = V_{NN} + H^2\varepsilon\mathbb{R} - \dot{\theta}^2, \tag{4.38}$$

(recall that $\dot{\theta} = H\eta_\perp$). It may be noticed that the reason behind the appearance of the term $-\dot{\theta}^2$ in M_{eff}^2 is due to the fact that the potential receives a correction coming from the centripetal force experimented by the turn. This introduces a centrifugal barrier to the effective potential felt by the heavy modes. The equations of motion for this system of fields is then

$$\ddot{\mathcal{R}} + (3 + 2\varepsilon - 2\eta_{||})H\dot{\mathcal{R}} - \frac{\nabla^2\mathcal{R}}{a^2} = -2\frac{H^2}{\dot{\phi}_0}\eta_\perp\left[\dot{\mathcal{F}} + (3 - \eta_{||} - \xi_\perp)H\mathcal{F}\right], \tag{4.39}$$

$$\ddot{\mathcal{F}} + 3H\dot{\mathcal{F}} - \frac{\nabla^2\mathcal{F}}{a^2} + M_{\text{eff}}^2\mathcal{F} = 2\dot{\phi}_0\eta_\perp\dot{\mathcal{R}}. \tag{4.40}$$

Notice that the configuration $\mathcal{R}$ = constant and $\mathcal{F} = 0$ constitutes a non trivial solution to the system of equations. Since $\mathcal{F}$ is assumed to be heavy, the configuration $\mathcal{F} = 0$ is reached shortly after horizon exit, and the curvature mode $\mathcal{R}$ will necessarily become frozen. For this reason, in the presence of mass hierarchies, we may only concern ourselves with curvature perturbations and disregard isocurvature components after inflation.

4.3.2 Power spectrum

From the observational point of view, the main quantities of interest coming from inflation are its predicted n-point correlation functions characterizing fluctuations.

These quantities provide all the relevant information about the expected distribution of primordial inhomogeneities that seeded the observed CMB anisotropies. It is of particular interest to compute two-point correlation functions, corresponding to the variance of inhomogeneities' distribution. To deduce such quantities we have to consider the quantization of the system, and this may be achieved by expanding the canonical pair u^X and u^Y in terms of creation and annihilation operators $a^\dagger_\alpha(\mathbf{k})$ and $a_\alpha(\mathbf{k})$ respectively, as

$$u^I(\tau, x) = \int \frac{d^3k}{(2\pi)^{3/2}} \sum_\alpha \left[e^{i\mathbf{k}\cdot\mathbf{x}} u^I_\alpha(\mathbf{k},\tau) a_\alpha(\mathbf{k}) + e^{-i\mathbf{k}\cdot\mathbf{x}} u^{I*}_\alpha(\mathbf{k},\tau) a^\dagger_\alpha(\mathbf{k}) \right], \tag{4.41}$$

where $\alpha = 1, 2$ labels the two modes to be encountered by solving the second order differential equations for the fields $u^I_\alpha(\mathbf{k},\tau)$. In order to satisfy the conventional field commutation relations, the mode solutions need to satisfy the additional constraints consistent with the equations of motion:[2]

$$\sum_\alpha \left(u^I_\alpha \frac{u^{J*}_\alpha}{d\tau} - u^{I*}_\alpha \frac{u^J_\alpha}{d\tau} \right) = i\delta^{IJ}. \tag{4.42}$$

By examining the action (4.33) one sees that in the short wavelength limit $k^2/a^2 \gg \Omega$, where Ω symbolizes both eigenvalues of the matrix Ω, the equation of motion for the u-fields reduce to

$$\frac{d^2u^I}{d\tau^2} - \nabla^2 u^I = 0, \qquad I = X, Y, \tag{4.43}$$

allowing us to choose the following initial conditions for the fields

$$u^X_\alpha(\mathbf{k},\tau) = \frac{e^{-ik\tau}}{\sqrt{2k}} \delta^1_\alpha, \qquad u^Y(\mathbf{k},\tau) = \frac{e^{-ik\tau}}{\sqrt{2k}} \delta^2_\alpha. \tag{4.44}$$

Notice that here we have chosen to associate the initial state $\alpha = 1$ with the field direction X and $\alpha = 2$ with the field direction Y. This identification is in fact completely arbitrary and does not affect the computation of two-point correlation functions. In other words, we could modify the initial state (4.44) by considering an arbitrary (time independent) rotation on the right hand side, without changing the prediction of observables.

Then, given the set of solutions u^X_α and u^Y_α, one finds that the two-point correlation function associated to curvature modes $\mathcal{R}$ is given by:

$$\mathcal{P}_\mathcal{R}(k,\tau) \quad = \quad \frac{k^3}{2\pi^2} \sum_\alpha \mathcal{R}_\alpha(k,\tau)\mathcal{R}^*_\alpha(k,\tau), \tag{4.45}$$

where $\mathcal{R}_\alpha$ is related to the pair u^X_α and u^Y_α by the field redefinitions described in the previous sections. When (4.45) is evaluated at the end of inflation, for wavelengths

[2] See refs. [71] and [3] for a more detailed discussion of the quantization of these type of system.

k far away from the horizon ($k/a \ll H$), it corresponds to the power spectrum of curvature modes. For completeness, let us mention that one may also define the two-point correlation function $\mathcal{P}_{\mathcal{S}}(k,\tau)$ and the cross-correlation function $\mathcal{P}_{\mathcal{RS}}(k,\tau)$ in analogous ways.

Chapter 5

Heavy fields, reduced speeds of sound and decoupling during inflation

The recent observation that heavy fields can influence the evolution of adiabatic modes during inflation [127] has far reaching phenomenological implications [3–5, 44] that, a posteriori, require a refinement of our understanding of how high and low energy degrees of freedom decouple [42] and how one splits "heavy" and "light" modes on a time-dependent background. Provided that there is only one flat direction in the inflaton potential, heavy fields (in the present context, field excitations orthogonal to the background trajectory) can be integrated out, resulting in a low energy effective field theory (EFT) for adiabatic modes exhibiting a reduced speed of sound c_s, given by

$$c_s^{-2} = 1 + 4\dot{\theta}^2/M_{\rm eff}^2\,, \tag{5.1}$$

where $\dot{\theta}$ is the turning rate of the background trajectory in multi-field space, and $M_{\rm eff}$ is the effective mass of heavy fields, assumed to be much larger than the expansion rate H. Depending on the nature of the trajectory, (5.1) can render features in the power spectrum [3, 44] and/or observably large non-Gaussianity [5, 127].

Given that $M_{\rm eff}$ is the mass of the fields we integrate out, one might doubt the validity of the EFT in the regime where the speed of sound is suppressed [18, 61, 62, 120], as this requires $\dot{\theta}^2 \gg M_{\rm eff}^2$. In this chapter we elaborate on this issue by studying the dynamics of light and heavy degrees of freedom when $c_s^2 \ll 1$. What emerges is a crucial distinction, in time-dependent backgrounds, between isocurvature and curvature field excitations, and the true heavy and light excitations. We show that the light (curvature) mode $\mathcal{R}$ indeed stays coupled to the heavy (isocurvature) modes when strong turns take place ($\dot{\theta}^2 \gg M_{\rm eff}^2$), however, decoupling between the physical low and high energy degrees of freedom persists in such a way that the deduced EFT remains valid. This is confirmed by a simple setup in which H decreases adiabatically, allowing for a sufficiently long period of inflation. In this construction, *high energy degrees of freedom* are never excited, and yet *heavy fields* do play a role in lowering the speed of sound of adiabatic modes.

Although this is completely consistent with the principles of EFT, it seems to have escaped previous analyses due to some subtleties that we summarize in points I-IV below. Furthermore, inflationary scenarios with sustained turns and uninterrupted slow roll appear to be consistent with all the observational constraints on the primordial power spectrum of primordial perturbations, while predicting enhanced equilateral non-gaussianity. We give explicit examples at the end.

5.1 Two Field Inflation

The simplest setup that allows a quantitative analysis (see Refs. [3,5,44] for details) is a two-scalar system with action

$$S = \int \sqrt{-g} \left[\frac{1}{2} R - \frac{1}{2} g^{\mu\nu} \gamma_{ab} \partial_\mu \phi^a \partial_\nu \phi^b - V(\phi) \right] , \tag{5.2}$$

(in units $8\pi G = 1$) where R is the Ricci scalar, V is the scalar potential and γ_{ab} is the posssibly non-canonical sigma model metric of the space spanned by ϕ^a, with $a = 1, 2$. The background solution to the equations of motion is an inflationary trajectory $\phi_0^a(t)$ and a spacetime FRW metric, that solves the Friedmann equations, $ds^2 = -dt^2 + a^2(t)\delta_{ij} dx^i dx^j$, where $a(t)$ is the scale factor and $H = \dot{a}/a$ the Hubble parameter. As usual, we take unit vectors T^a and N^a tangent and normal to the trajectory [?] given by $T^a = \dot{\phi}_0^a/\dot{\phi}_0$ and $D_t T^a = -\dot{\theta} N^a$, which also defines the turning rate $\dot{\theta}$, the angular velocity described by the bends of the trajectory. $D_t = \dot{\phi}_0^a \nabla_a$ is the (covariant) time derivative along the background trajectory, and $\dot{\phi}_0^2 \equiv \gamma_{ab} \dot{\phi}_0^a \dot{\phi}_0^b$. Projecting the equations of motion on the normal direction relates the turning rate to the potential gradient as $\dot{\theta} \equiv V_N/\dot{\phi}_0$ (with $V_N \equiv N^a V_a$). The tangential projection gives the usual single-field equations $\ddot{\phi}_0 + 3H\dot{\phi}_0 + V_T = 0$ with $V_T \equiv T^a V_a$. We may define the slow-roll parameters $\varepsilon \equiv -\dot{H}/H^2$ and $\eta_{||} \equiv -\ddot{\phi}_0/\big(H\dot{\phi}_0\big)$. The conditions $\varepsilon \ll 1$ and $|\eta_{||}| \ll 1$ ensure that H evolves adiabatically for sufficiently long.

We are interested in the dynamics of scalar perturbations $\delta\phi^a(t, \boldsymbol{x}) = \phi^a(t, \boldsymbol{x}) - \phi_0^a(t)$. We work in the flat gauge and define the comoving curvature and heavy isocurvature perturbations as $\mathcal{R} \equiv -\big(H/\dot{\phi}\big) T_a \delta\phi^a$ and $\mathcal{F} \equiv N_a \delta\phi^a$, respectively. (A definition of $\mathcal{R}$ and $\mathcal{F}$ valid to all orders in perturbation theory is given in [5]). The quadratic order action for these perturbations is

$$
\begin{aligned}
S_2 \;=\; & \frac{1}{2}\int a^3\left[\frac{\dot\phi_0^2}{H^2}\dot{\mathcal{R}}^2-\frac{\dot\phi_0^2}{H^2}\frac{(\nabla\mathcal{R})^2}{a^2}+\dot{\mathcal{F}}^2-\frac{(\nabla\mathcal{F})^2}{a^2}\right. \\
& \left. -M_{\rm eff}^2\mathcal{F}^2-4\dot\theta\frac{\dot\phi_0}{H}\dot{\mathcal{R}}\mathcal{F}\right]. \qquad (5.3)
\end{aligned}
$$

Here $M_{\rm eff}$ is the effective mass of $\mathcal{F}$ given by

$$
M_{\rm eff}^2 = m^2-\dot\theta^2\,, \qquad (5.4)
$$

where $m^2 \equiv V_{NN}+\varepsilon H^2\mathbb{R}$ and $V_{NN}\equiv N^aN^a\nabla_a\nabla_b V$. $\mathbb{R}$ is the Ricci scalar of the sigma model metric γ_{ab}. Notice that $\dot\theta$ couples both fields and reduces the effective mass, suggesting a breakdown of the hierarchy that permits a single field effective description as $\dot\theta^2\sim m^2$. As we are about to see, this expectation is somewhat premature. The linear equations of motion in Fourier space are

$$
\begin{aligned}
& \ddot{\mathcal{R}}+(3+2\varepsilon-2\eta_{||})H\dot{\mathcal{R}}+\frac{k^2}{a^2}\mathcal{R} \\
& \quad = 2\dot\theta\frac{H}{\dot\phi_0}\left[\dot{\mathcal{F}}+\left(3-\eta_{||}-\varepsilon+\frac{\ddot\theta}{H\dot\theta}\right)H\mathcal{F}\right], \qquad (5.5) \\
& \ddot{\mathcal{F}}+3H\dot{\mathcal{F}}+\frac{k^2}{a^2}\mathcal{F}+M_{\rm eff}^2\mathcal{F} = -2\dot\theta\frac{\dot\phi_0}{H}\dot{\mathcal{R}}\,. \qquad (5.6)
\end{aligned}
$$

Note that $\mathcal{R}=$ constant and $\mathcal{F}=0$ are non-trivial solutions to these equations for arbitrary $\dot\theta$. Since $\mathcal{F}$ is heavy, $\mathcal{F}\to 0$ shortly after horizon exit, and $\mathcal{R}\to$ constant, as in single field inflation.

We are interested in (5.5) and (5.6) in the limit where $\dot\theta$ is constant and much greater than $M_{\rm eff}$. We first consider the short wavelength limit where we can disregard Hubble friction terms and take $\dot\phi_0/H$ as a constant. In this regime, the physical wavenumber $p\equiv k/a$ may be taken to be constant, and (5.5) and (5.6) simplify to

$$
\begin{aligned}
\ddot{\mathcal{R}}_c+p^2\mathcal{R}_c = & \;+2\dot\theta\dot{\mathcal{F}}\,, \\
\ddot{\mathcal{F}}+p^2\mathcal{F}+M_{\rm eff}^2\mathcal{F} = & \;-2\dot\theta\dot{\mathcal{R}}_c\,, \qquad (5.7)
\end{aligned}
$$

in terms of the canonically normalized $\mathcal{R}_c=\big(\dot\phi_0/H\big)\mathcal{R}$. The solutions are found to be [4]

$$
\begin{aligned}
\mathcal{R}_c = & \;\mathcal{R}_+e^{i\omega_+t}+\mathcal{R}_-e^{i\omega_-t}\,, \\
\mathcal{F} = & \;\mathcal{F}_+e^{i\omega_+t}+\mathcal{F}_-e^{i\omega_-t}\,, \qquad (5.8)
\end{aligned}
$$

where the two frequencies ω_- and ω_+ are given by

$$
\omega_\pm^2 = \frac{M_{\rm eff}^2}{2c_s^2}+p^2\pm\frac{M_{\rm eff}^2}{2c_s^2}\sqrt{1+\frac{4p^2(1-c_s^2)}{M_{\rm eff}^2c_s^{-2}}}\,, \qquad (5.9)
$$

with c_s given by (5.1). The pairs $(\mathcal{R}_-, \mathcal{F}_-)$ and $(\mathcal{R}_+, \mathcal{F}_+)$ represent the amplitudes of both low and high frequency modes respectively, and satisfy

$$\mathcal{F}_- = \frac{-2i\dot{\theta}\omega_-}{M_{\rm eff}^2 + p^2 - \omega_-^2}\mathcal{R}_- \,, \quad \mathcal{R}_+ = \frac{-2i\dot{\theta}\omega_+}{\omega_+^2 - p^2}\mathcal{F}_+ \,. \tag{5.10}$$

Thus the fields in each pair oscillate coherently. In this regime there is a double hierarchy of (inverse) timescales

$$H \ll \omega_- \ll \omega_+ \tag{5.11}$$

The first inequality, which justifies neglecting Hubble friction, defines what is meant by the short wavelength regime: $H \ll pc_s$. The second inequality ensures that the high frequency degrees of freedom do not participate in the dynamics of the adiabatic mode and defines the regime of validity of the EFT:

$$p^2 \ll M_{\rm eff}^2 c_s^{-2} \,. \tag{5.12}$$

or, equivalently, $p^2 \ll 4m^2/(3c_s^2+1)$. This shows that, contrary to the naive expectation based on $M_{\rm eff}$, the range of comoving momenta for low energy modes *actually increases as the speed of sound decreases* . Furthermore, upon quantization [4] one finds $|\mathcal{R}_-|^2 \sim c_s^2/(2\omega_-)$ and $|\mathcal{F}_+|^2 \sim 1/(2\omega_+)$, implying that high frequency modes are relatively suppressed in amplitude. Thus, we can safely consider only low frequency modes, in which case $\mathcal{F}$ is completely determined by $\mathcal{R}_c$ as $\mathcal{F} = -2\dot{\theta}\dot{\mathcal{R}}_c/\big(M_{\rm eff}^2 + p^2 - \omega_-^2\big)$. Notice that $\omega_-^2 \ll M_{\rm eff}^2 + p^2$, so ω_-^2 may be disregarded here.

As linear perturbations evolve, their physical wavenumber $p \equiv k/a$ decreases and the modes enter the long wavelength regime $p^2 c_s^2 \lesssim H^2$, where they become strongly influenced by the background and no longer have a simple oscillatory behaviour. Now the low energy contributions to $\mathcal{F}$ satisfy $\dot{\mathcal{F}} \sim H\mathcal{F}$, and because $H^2 \ll M_{\rm eff}^2$, we can simply neglect time derivatives in (5.6). On the other hand, high energy modes continue to evolve independently of the low energy modes, diluting rapidly as they redshift. Thus for the entire low energy regime (5.12), time derivatives of $\mathcal{F}$ can be ignored in (5.6) and $\mathcal{F}$ may be solved in terms of $\dot{\mathcal{R}}$ as

$$\mathcal{F} = -\frac{\dot{\phi}_0}{H}\frac{2\dot{\theta}\dot{\mathcal{R}}}{k^2/a^2 + M_{\rm eff}^2} \,. \tag{5.13}$$

Replacing (5.13) into (5.3) gives the tree level effective action for the curvature perturbation. To quadratic order [5]:

$$S_{\rm eff} = \frac{1}{2}\int a^3 \frac{\dot{\phi}_0^2}{H^2}\left[\frac{\dot{\mathcal{R}}^2}{c_s^2(k)} - \frac{k^2\mathcal{R}^2}{a^2}\right] \,, \tag{5.14}$$

where $c_s^{-2}(k) = 1+4\dot{\theta}^2/\big(k^2/a^2+M_{\rm eff}^2\big)$. This k-dependent speed of sound is consistent with the modified dispersion relation $\omega_-^2 = p^2c_s^2 + (1-c_s^2)^2p^4/(M_{\rm eff}^2c_s^{-2})$, where c_s is given by (5.1). Ref. [44] studied the validity of (5.14) in the case where turns appear suddenly. Consistent with the present analysis, it was found that this EFT is valid even when $\dot{\theta}^2 \gg M_{\rm eff}^2$, *provided the adiabaticity condition*

$$|\ddot{\theta}/\dot{\theta}| \ll M_{\rm eff}, \tag{5.15}$$

is satisfied. This condition states that the turn's angular *acceleration* must remain small in comparison to the masses of heavy modes, which otherwise would be excited. The above straightforwardly implies the more colloquial adiabaticity condition $|\dot{\omega}_+/\omega_+^2| \ll 1$. If (5.15) is violated by the background, high energy modes can be produced and the EFT does indeed break down, as confirmed by [66].

We now outline four crucial points that underpin our conclusions:

(I) The mixing between fields $\mathcal{R}$ and $\mathcal{F}$, and modes with frequencies ω_- and ω_+ is *inevitable* when the background trajectory bends. If one attempts a rotation in field space in order to uniquely associate fields with frequency modes, the rotation matrix would depend on the scale p, implying a non-local redefinition of the fields.

(II) Even in the absence of excited high frequency modes, the heavy field $\mathcal{F}$ is forced to oscillate in pace with the light field $\mathcal{R}$ at a frequency ω_-, so $\mathcal{F}$ continues to participate in the low energy dynamics of the curvature perturbations.

(III) When $\dot{\theta}^2 \gg M_{\rm eff}^2$, the high and low energy frequencies become $\omega_+^2 \simeq M_{\rm eff}^2c_s^{-2} \sim 4\dot{\theta}^2$ and $\omega_-^2 \simeq p^2(M_{\rm eff}^2+p^2)/(4\dot{\theta}^2)$. Thus the gap between low and high energy degrees of freedom is amplified, and one can consistently ignore high energy degrees of freedom in the low energy EFT.

(IV) In the low energy regime, the field $\mathcal{F}$ exchanges kinetic energy with $\mathcal{R}$ resulting in a reduction in the speed of sound c_s of $\mathcal{R}$, the magnitude of which depends on the strength of the kinetic coupling $\dot{\theta}$. This process is adiabatic and consistent with the usual notion of decoupling in the low energy regime (5.12), as implied by (5.15).

At the core of these four observations is the simple fact that in time-dependent backgrounds, the eigenmodes and eigenvalues of the mass matrix along the trajectory do not necessarily coincide with the curvature and isocurvature fluctuations and their characteristic frequencies. With this in mind, it is possible to state more clearly the refined sense in which decoupling is operative: *while the fields $\mathcal{R}$ and $\mathcal{F}$ inevitably remain coupled, high and low energy degrees of freedom effectively decouple.*

We now briefly address the evolution of modes in the ultraviolet (UV) regime $p^2 \gtrsim M_{\rm eff}^2c_s^{-2}$. Here both modes have similar amplitudes and frequencies, and so in principle could interact via relevant couplings beyond linear order (which are proportional to $\dot{\theta}$). Because these interactions must allow for the non-trivial solutions $\mathcal{R} =$

constant and $\mathcal{F} = 0$ (a consequence of the background time re-parametrization invariance), their action is very constrained [5]. Moreover, in the regime $p^2 \gg M_{\rm eff}^2 c_s^{-2}$ the coupling $\dot{\theta}$ becomes negligible when compared to p, and one necessarily recovers a very weakly coupled set of modes, whose $p \to \infty$ limit completely decouples $\mathcal{R}$ from $\mathcal{F}$. This can already be seen in (5.13), where contributions to the effective action for the adiabatic mode at large momenta from having integrated out $\mathcal{F}$, are extremely suppressed for $k^2/a^2 \gg M_{\rm eff}^2$, leading to high frequency contributions to (5.14) with $c_s = 1$.

5.2 Constant turns

We now analyze a model of uninterrupted slow-roll inflation that executes a constant turn in field space, implying an almost constant, suppressed speed of sound for the adiabatic mode. Take fields $\phi^1 = \theta$, $\phi^2 = \rho$ with a metric $\gamma_{\theta\theta} = \rho^2$, $\gamma_{\rho\rho} = 1$, $\gamma_{\rho\theta} = \gamma_{\theta\rho} = 0$ and potential

$$V(\theta, \rho) = V_0 - \alpha\theta + m^2(\rho - \rho_0)^2/2 \,. \tag{5.16}$$

This model would have a shift symmetry along the θ direction were it not broken by a non-vanishing α. This model is a simplified version of one studied in [47], where the focus instead was on the regime $M_{\rm eff} \sim m \sim H$ (see also [49] where the limit $M_{\rm eff}^2 \gg H^2 \gg \dot{\theta}^2$ is analyzed). The background equations of motion are

$$\begin{aligned} \ddot{\theta} + 3H\dot{\theta} + 2\dot{\theta}\dot{\rho}/\rho &= \alpha/\rho^2 \,, \\ \ddot{\rho} + 3H\dot{\rho} + \rho(m^2 - \dot{\theta}^2) &= m^2\rho_0 \,. \end{aligned} \tag{5.17}$$

The slow-roll attractor is such that $\dot{\rho}$, $\ddot{\rho}$ and $\ddot{\theta}$ are negligible. This means that H, ρ and $\dot{\theta}$ remain nearly constant and satisfy the following algebraic equations near $\theta = 0$

$$\begin{aligned} 3H\dot{\theta} &= \frac{\alpha}{\rho^2} \,, \quad \dot{\theta}^2 = m^2(1 - \rho_0/\rho) \,, \\ 3H^2 &= \frac{1}{2}\rho^2\dot{\theta}^2 + V_0 + \frac{1}{2}m^2(\rho - \rho_0)^2 \,. \end{aligned} \tag{5.18}$$

These equations describe circular motion with a radius of curvature ρ and angular velocity $\dot{\theta}$. Here $M_{\rm eff}^2 = m^2 - \dot{\theta}^2$, implying the strict bound $m^2 > \dot{\theta}^2$. Thus the only way to obtain a suppressed speed of sound is if $\dot{\theta}^2 \simeq m^2$. Our aim is to find the parameter ranges such that the background attractor satisfies $\varepsilon \ll 1$, $c_s^2 \ll 1$ and $H^2 \ll M_{\rm eff}^2$ simultaneously. These are given by

$$1 \gg \frac{\rho_0}{4}\left(\frac{m\sqrt{3V_0}}{\alpha}\right)^{1/2} \gg \frac{V_0}{6m^2} \gg \frac{\alpha}{4\sqrt{3V_0}m} \,. \tag{5.19}$$

If these hierarchies are satisfied, the solutions to (5.18) are well approximated by

$$\rho^2 = \frac{\alpha}{\sqrt{3V_0}m}\,, \quad \dot{\theta} = m - \frac{m\rho_0}{2}\left(\frac{m\sqrt{3V_0}}{\alpha}\right)^{1/2}, \tag{5.20}$$

and $H^2 = V_0/3$, up to fractional corrections of order ε, c_s^2 and $H^2/M_{\rm eff}^2$. We note that the first inequality in (5.19) implies $\rho \gg \rho_0$, and so the trajectory is displaced off the adiabatic minimum at ρ_0. However, the contribution to the total potential energy implied by this displacement is negligible compared to V_0. After n cycles around $\rho = 0$ one has $\Delta\theta = 2\pi n$, and the value of V_0 has to be adjusted to $V_0 \to V_0 - 2\pi n\alpha$. This modifies the expressions in (5.20) accordingly, and allows us to easily compute the adiabatic variation of certain quantities, such as $s \equiv \dot{c}_s/(c_s H) = -\varepsilon/4$, and $\eta_{||} = -\varepsilon/2$, where $\varepsilon = \sqrt{3}\alpha m^2/(2V_0^{3/2})$. These values imply a spectral index $n_{\mathcal{R}}$ for the power spectrum $\mathcal{P}_{\mathcal{R}} = H^2/(8\pi^2\varepsilon c_s)$ given by $n_{\mathcal{R}} - 1 = -4\varepsilon + 2\eta_{||} - s = -19\varepsilon/4$.

It is possible to find reasonable values of the parameters such that observational bounds are satisfied. Using (5.20) we can relate the values of V_0, α, m and ρ_0 to the measured values $\mathcal{P}_{\mathcal{R}}$ and $n_{\mathcal{R}}$, and to hypothetical values for c_s and $\beta \equiv H/M_{\rm eff}$ as

$$\begin{aligned} V_0 &= 96\pi^2(1-n_{\mathcal{R}})\mathcal{P}_{\mathcal{R}}c_s/19\,, \\ m^2 &= 8\pi^2(1-n_{\mathcal{R}})\mathcal{P}_{\mathcal{R}}/(19c_s\beta^2)\,, \\ \alpha &= 6(16/19)^2\pi^2(1-n_{\mathcal{R}})^2\mathcal{P}_{\mathcal{R}}c_s^2\beta\,, \\ \rho_0 &= 16c_s^3\beta\sqrt{2(1-n_{\mathcal{R}})/19}\,. \end{aligned} \tag{5.21}$$

Following WMAP7, we take $\mathcal{P}_{\mathcal{R}} = 2.42 \times 10^{-9}$ and $n_{\mathcal{R}} = 0.98$ [85]. Then, as an application of relations (5.21), we look for parameters such that

$$c_s^2 \simeq 0.06\,, \quad M_{\rm eff}^2 \simeq 250H^2\,, \tag{5.22}$$

(which imply $H^2 \simeq 1.4\times10^{-10}$), according to which $V_0 \simeq 5.9\times10^{-10}$, $\alpha \simeq 1.5\times10^{-13}$, $m \simeq 4.5 \times 10^{-4}$ and $\rho_0 \simeq 6.8 \times 10^{-3}$, from which we note that m, ρ_0 and $\alpha^{1/4}$ are naturally all of the same order. In terms of the mass scale m, the above implies that we have excited a *field* with mass of order $H^2/m^2 \sim 1230$, completely consistent with decoupling and the validity of the EFT. We have checked numerically that the background equations of motion are indeed well approximated by (5.20), up to fractional corrections of order c_s^2. More importantly, we obtain the same nearly scale invariant power spectrum $\mathcal{P}_{\mathcal{R}}$ using both the full two-field theory described by (5.5) and (5.6), and the single field EFT described by the action (5.14). The evolution of curvature perturbations in the EFT compared to the full two-field theory for the long wavelength modes is almost indistinguishable given the effectiveness with which (5.12) is satisfied, with a marginal difference $\Delta\mathcal{P}_{\mathcal{R}}/\mathcal{P}_{\mathcal{R}} \simeq 0.008$. This is of order $(1-c_s^2)H^2/M_{\rm eff}^2$, which is consistent with the analysis of Ref. [44]. Despite the suppressed speed of sound in this model, a fairly large tensor-to-scalar ratio of $r = 16\varepsilon c_s \simeq 0.020$ is predicted.

As expected, for $c_s^2 \ll 1$ a sizable value of $f_{\rm NL}^{(\rm eq)}$ is implied. The cubic interactions leading to this were deduced in Ref. [5] which for constant turns is given by [6]

$$f_{\rm NL}^{(\rm eq)} = \frac{125}{108}\frac{\varepsilon}{c_s^2} + \frac{5}{81}\frac{c_s^2}{2}\left(1-\frac{1}{c_s^2}\right)^2 + \frac{35}{108}\left(1-\frac{1}{c_s^2}\right). \tag{5.23}$$

This result is valid for any single-field system with constant c_s obtained by having integrated out a heavy field. We note that this prediction is cleanly distinguishable from those of other single field models (such as DBI inflation) through the different sign and magnitude for the M_3 coefficient generated in the EFT expansion of [50], as derived in [5].

Recalling that the spectral index n_T of tensor modes is $n_T = -2\varepsilon$, for $c_s \ll 1$ we find a consistency relation between three potentially observable parameters, given by $f_{\rm NL}^{(\rm eq)} = -20.74\, n_T^2/r^2$. In the specific case of the values in (5.22), we have $f_{\rm NL}^{(\rm eq)} \simeq -4.0$. This value is both large and negative, so future observations could constrain this type of scenario. Finally, one can ask if the EFT corresponding to (5.22) remains weakly coupled throughout. For this, one needs to satisfy [5] $\omega_-^4 < \Lambda_{\rm sc}^4$, where $\Lambda_{\rm sc}^4 \simeq 4\pi\varepsilon H^2 c_s^5/(1-c_s^2)$ is the strong coupling scale [28, 50]. In order to assess this, we notice that for $c_s^2 \ll 1$, values of ω_-^2 of order $M_{\rm eff}^2$ are well within the low energy regime ($M_{\rm eff}^2 \ll \omega_+^2$) and denotes a reference energy scale up to which we may trust the low energy EFT. We find $M_{\rm eff}^4/\Lambda_{\rm sc}^4 \simeq 0.18$. Furthermore, although we did not address how inflation ends, the choice (5.22) allows for at least 45 e-folds of inflation, necessary to solve the horizon and flatness problems.

We stress that various other values can be chosen in (5.22) to arrive at similar conclusions. For example, requiring 35 e-folds with $M_{\rm eff}^2 \simeq 100H^2$, $c_s^2 \simeq 0.02$, implies $V_0 \simeq 3.4\times10^{-10}$, $\alpha \simeq 8.1\times10^{-13}$, $m \simeq 3.8\times10^{-4}$, $\rho_0 \simeq 2.1\times10^{-4}$, so that the strong coupling scale becomes $M_{\rm eff}^4/\Lambda_{\rm sc}^4 \simeq 0.34$. In this case we find $f_{\rm NL}^{(\rm equil)} \simeq -14$. As an illustrative limit, we can try to saturate the strong coupling bound given a particular hierarchy between H and $M_{\rm eff}$. In doing so, we entertain the situation where our model approximates the dynamics of inflation only over the range where modes accessible to us by observations had exited the horizon. Requiring $M_{\rm eff}^2/H^2 \simeq 100$ and $\Lambda_{\rm sc} \simeq M_{\rm eff}$, we find that approximately 14 e-folds can be generated where the speed of sound is reduced to $c_s^2 \simeq 0.01$.

In summary, the active ingredients of this toy example are rather minimal and may well parametrize a generic class of inflationary models, such as axion-driven inflationary scenarios in string theory. Our results complement those of Ref. [3–5, 44, 127] and emphasize the refined sense in which EFT techniques are applicable during slow-roll inflation [28,50]. In particular, contrary to the standard perspective regarding the role of UV physics during inflation, heavy fields may influence the evolution of curvature perturbations $\mathcal{R}$ in a way consistent with decoupling between low and high energy degrees of freedom.

Chapter 6

On the importance of heavy fields during inflation

6.1 Introduction

The fact that inflation is formulated within a field theoretical framework [24, 93] makes it particularly compelling to test our ideas about fundamental theories, such as supergravity and string theory, characterized for consistently incorporating the gravitational strength among their couplings. Because these theories generically predict the existence of a large number of degrees of freedom, the need of a period of inflation at early times is found to impose strong restrictions on their interactions. In particular, if inflation happened at sufficiently high energies, curvature perturbations could have strongly interacted with other degrees of freedom, implying a variety of observable effects departing from those predicted in standard single-field slow-roll inflation [12, 88, 95], including features in the power spectrum of primordial inhomogeneities [3, 7, 16, 17, 21, 45, 54, 67, 104, 108, 110, 124, 128, 129], large primordial non-Gaussianities [13, 22, 23, 37, 46, 90, 94] and isocurvature perturbations [53, 63, 68, 70, 71, 87, 89, 109, 123]. A detection of any of these signatures would therefore represent an extremely significant step towards elucidating the fundamental nature of physics taking place during the very early universe.

Despite of its simplicity, the construction of satisfactory models of inflation within supergravity and string theory is known to constitute a notoriously hard challenge. The vacuum expectation values (v.e.v.'s) of scalar fields participating of the inflationary dynamics must evolve along flat directions of the scalar potential's landscape for a sufficiently long time. But because the interaction strength of these theories is of a gravitational nature, the scalar potential is naturally subject to changes of order 1 when the scalar fields v.e.v.'s traverse distances of the order of the Planck scale. This translates into the well known η-problem of supergravity and string theory [56, 57, 76, 97], where second derivatives of the scalar potential V'' are typically of order equal or larger than H^2 (where H is the universe's expansion rate) therefore impeding the slow-roll evolution of the fields.[1] However, this problem may be cured if the theory contains a set of shift symmetries at non-perturbative level, ensuring the existence of exactly flat directions in the potential [125]. Then, if these symmetries are mildly broken, the expected result is an inflationary scenario with a large mass hierarchy between the modular fields representing flat directions and the rest of the scalar fields, expected to have masses much larger than H [32, 55, 69, 91, 122].

Conventional wisdom dictates that UV-degrees of freedom with masses $M \gg H$ necessarily have a marginal role in the low energy dynamics of curvature modes. After these heavy degrees of freedom are integrated out, one expects a low energy effective field theory (EFT) for curvature perturbations where UV-physics is parametrized by nontrivial operators suppressed by factors of order H^2/M^2. The resulting low energy EFT is therefore expected to offer negligible departures from a truncated version of the same theory, wherein heavy fields are simply disregarded from the very beginning. However, general field theoretical arguments due allow for large sizable corrections to the low energy EFT [50, 130]. In the specific case of multi field models, there are special circumstances where the background inflationary dynamic is such that the interchange of kinetic energy between curvature perturbations and heavy degrees of freedom may be dramatically enhanced [3–5, 28, 47, 48, 61, 62, 120, 127]. For example, if the inflationary trajectory is subject to a sharp turn in such a way that the heavy scalar fields stay normal to the trajectory (see Figure 6.1 for an illustration) then unsuppressed interactions —kinematically coupling curvature perturbations with heavy fields— are unavoidably turned on. As a consequence, if the rate of turn is large compared to the rate of expansion H, the impact of heavy physics on the low energy dynamics becomes substantially amplified, introducing large non-trivial departures from a naively truncated version of the theory.

In the particular case of two field models —at linear order in the fluctuations— heavy fields are identified with isocurvature perturbations, and their role is reduced to modify the speed of sound c_s of curvature perturbations at the effective field theory level. The result is a non-trivial effective single-field theory where the time dependence of c_s is dictated by the specific shape of the two-field background trajectory, in such a way that departures from unity $c_s \neq 1$ exist whenever the trajectory is subject to a turn. More specifically, one finds that the speed of sound depends on the angular velocity $\dot{\theta}$ characterizing the turn as

$$c_s^{-2} = 1 + 4\dot{\theta}^2/M_{\text{eff}}^2, \tag{6.1}$$

where $M_{\text{eff}}^2 = M^2 - \dot{H}\,\mathbb{R} - \dot{\theta}^2$ is the effective mass of isocurvature perturbations, with

[1] Another major obstacle towards the construction of models of inflation within string theory is related to the stabilization of moduli. See for instance refs. [58, 59] for a discussion on the stabilization of moduli in supergravity and string theory.

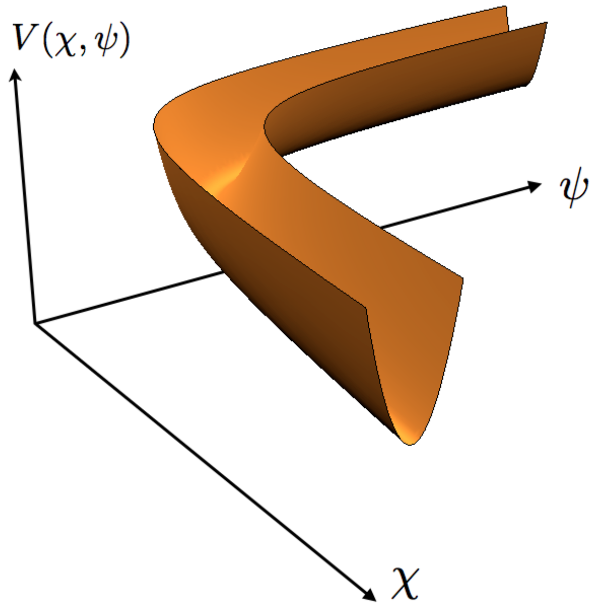

Figure 6.1: The figure illustrates a prototype example of a multi-field potential (depending on two fields χ and ψ) with a mass hierarchy in which the flat direction is subject to a turn.

M the tree-level bare mass of heavy modes, and $\mathbb{R}$ the Ricci scalar of the scalar field target space. In this way, sudden turns of the trajectory translate into sudden time variations of c_s (hence modifying the value of the sound horizon c_s/H) and therefore generating features in the power spectrum of primordial inhomogeneities [3].[2] Moreover, if the turn is such that $c_s \ll 1$, cubic interactions become unsuppressed [5], implying large levels of primordial non-Gaussianities in the distribution of curvature perturbations [46].

The purpose of this chapter is to study two-field models of inflation characterized by a large mass hierarchy.[3] We are particularly interested in assessing the general conditions under which the EFT deduced by integrating out the heavy field remains a reliable description of the inflationary dynamics. We show that the main condition simply consists on the requirement that the rate of variation of the angular velocity $\dot{\theta}$ characterizing the turn stays suppressed with respect to the effective mass $M_{\rm eff}$ of heavy modes. That is:

$$\left| \frac{d}{dt} \ln \dot{\theta} \right| \ll M_{\rm eff}. \tag{6.2}$$

We show that this *adiabaticity* condition is sufficiently mild, as it still allows for the effective field theory to describe, with great accuracy, a large variety of situations where very sharp turns take place (*i.e.* situations where $|\dot{\theta}| \gtrsim H$). We check this

[2]For a recent discussion on features in the power spectrum generated by variations of the speed of sound see ref. [104].

[3]For other interesting work regarding non-trivial effects on the dynamics of curvature perturbations coming from massive degrees of freedom, see for instance refs. [81, 82, 111].

condition by studying various models with turns and compare the power spectra of these models obtained from both, the full two-field inflationary model and the respective effective field theory. We find that turns are able to generate large features in the power spectra, with the amplitude of these features depending on how large departures of c_s from unity are.

This work is organized as follows: In Section **??** we summarize the main results concerning two-field models of inflation. Then, in Section 6.3 we offer a simple derivation of the general class of single-field EFT emerging from two-field models with mass hierarchies, and derive condition (6.2) dictating the validity of this theory in terms of background quantities. In Section 6.4 we discuss the different classes of turns and deduce the type of reactions that turns have on the background inflationary trajectory. In particular, we study the case of sudden turns, where the inflationary trajectory is subject to a single turn for a brief period of time Δt smaller (or much smaller) than an e-fold ($\Delta t \lesssim H^{-1}$). Then, in Section 6.5, we consider two toy models and compute the power spectrum for different cases of turns. There we show that, consistent with (6.2), the effective field theory remains reliable as long as $\Delta t \gg 1/M_{\rm eff}$, where $M_{\rm eff}$ is the effective mass of the heavy field. We also show that large features on the power spectrum are easily produced, with the details of the effects depending on the different parameters characterizing the type of turns. Finally, in Section 6.6 we offer our concluding remarks to this work.

6.2 Setup

Using the results from the previous chapter, the simplest setup that allows a quantitative analysis a two-scalar system with action

$$S = \int d^4x\sqrt{-g}\left[\frac{1}{2}R - \frac{1}{2}g^{\mu\nu}\gamma_{ab}\partial_\mu\phi^a\partial_\nu\phi^b - V(\phi)\right] , \tag{6.3}$$

(in units $8\pi G = 1$) where R is the Ricci scalar, V is the scalar potential and γ_{ab} is the posssibly non-canonical sigma model metric of the space spanned by ϕ^a, with $a = 1, 2$. The background solution to the equations of motion is an inflationary trajectory $\phi_0^a(t)$ and a spacetime FRW metric, that solves the Friedmann equations,

$$ds^2 = -dt^2 + a^2(t)\delta_{ij}dx^i dx^j, \tag{6.4}$$

where $a(t)$ is the scale factor and $H = \dot{a}/a$ the Hubble parameter. As we see in chapter 3, for convenience we take unit vectors T^a and N^a tangent and normal to the trajectory [68, 70, 71] given by

$$T^a = \dot{\phi}_0^a/\dot{\phi}_0 \text{ and} \tag{6.5}$$

$$D_t T^a = -\dot{\theta} N^a, \tag{6.6}$$

which also defines the turning rate $\dot{\theta}$, the angular velocity described by the bends of the trajectory. $D_t = \dot{\phi}_0^a \nabla_a$ is the (covariant) time derivative along the background trajectory, and $\dot{\phi}_0^2 \equiv \gamma_{ab}\dot{\phi}_0^a\dot{\phi}_0^b$.

Projecting the equations of motion on the normal direction relates the turning rate to the potential gradient as $\dot{\theta} \equiv V_N/\dot{\phi}_0$ (with $V_N \equiv N^a V_a$). The tangential projection gives the usual single-field equations

$$\ddot{\phi}_0 + 3H\dot{\phi}_0 + V_T = 0 \text{ with} V_T \equiv T^a V_a. \tag{6.7}$$

We may define the slow-roll parameters:

$$\varepsilon \equiv -\dot{H}/H^2 \tag{6.8}$$

$$\eta_{||} \equiv -\ddot{\phi}_0/\big(H\dot{\phi}_0\big). \tag{6.9}$$

The conditions $\varepsilon \ll 1$ and $|\eta_{||}| \ll 1$ ensure that H evolves adiabatically for sufficiently long. We can also define the parameter

$$\eta_\perp \equiv \frac{V_N}{\dot{\phi}H} \tag{6.10}$$

that is a measure of deviations from the geodesic trajectory. We are interested in the dynamics of scalar perturbations $\delta\phi^a(t,\boldsymbol{x}) = \phi^a(t,\boldsymbol{x}) - \phi_0^a(t)$. We work in the flat gauge and define the comoving curvature and heavy isocurvature perturbations as $\mathcal{R} \equiv -\big(H/\dot{\phi}\big)T_a\delta\phi^a$ and $\mathcal{F} \equiv N_a\delta\phi^a$, respectively. (A definition of $\mathcal{R}$ and $\mathcal{F}$ valid to all orders in perturbation theory is given in [5]). The quadratic order action for these perturbations is

$$S_2 = d^4x\frac{1}{2}\int a^3\left[\frac{\dot{\phi}_0^2}{H^2}\dot{\mathcal{R}}^2 - \frac{\dot{\phi}_0^2}{H^2}\frac{(\nabla\mathcal{R})^2}{a^2} + \dot{\mathcal{F}}^2 - \frac{(\nabla\mathcal{F})^2}{a^2} - M_{\text{eff}}^2\mathcal{F}^2 - 4\dot{\theta}\frac{\dot{\phi}_0}{H}\dot{\mathcal{R}}\mathcal{F}\right] \tag{6.11}$$

Here M_{eff} is the effective mass of $\mathcal{F}$ given by

$$M_{\text{eff}}^2 = m^2 - \dot{\theta}^2\,, \tag{6.12}$$

where $m^2 \equiv V_{NN} + \varepsilon H^2\mathbb{R}$ and $V_{NN} \equiv N^a N^a \nabla_a\nabla_b V$. $\mathbb{R}$ is the Ricci scalar of the sigma model metric γ_{ab}.

The linear equations of motion in Fourier space are

$$\ddot{\mathcal{R}} + (3 + 2\varepsilon - 2\eta_{||})H\dot{\mathcal{R}} + \frac{k^2}{a^2}\mathcal{R} = 2\dot{\theta}\frac{H}{\dot{\phi}_0}\left[\dot{\mathcal{F}} + \left(3 - \eta_{||} - \varepsilon + \frac{\ddot{\theta}}{H\dot{\theta}}\right)H\mathcal{F}\right]\,, \tag{6.13}$$

$$\ddot{\mathcal{F}} + 3H\dot{\mathcal{F}} + \frac{k^2}{a^2}\mathcal{F} + M_{\text{eff}}^2\mathcal{F} = -2\dot{\theta}\frac{\dot{\phi}_0}{H}\dot{\mathcal{R}}\,. \tag{6.14}$$

Note that $\mathcal{R} =$ constant and $\mathcal{F} = 0$ are non-trivial solutions to these equations for arbitrary $\dot{\theta}$. Since $\mathcal{F}$ is heavy, $\mathcal{F} \to 0$ shortly after horizon exit, and $\mathcal{R} \to$ constant, as in single field inflation.

6.3 Effective Field Theory

It is possible to deduce an effective theory for the curvature mode $\mathcal{R}$ by integrating out the heavy field $\mathcal{F}$ when $M_{\rm eff}^2 \gg H^2$, provided that certain additional conditions are met. To see this, let us first briefly analyze the expected evolution of the fields $\mathcal{R}$ and $\mathcal{F}$ when the trajectory is turning at a constant rate ($\dot{\theta} = \text{constant}$). To start with, because we are dealing with a coupled system of equations for $\mathcal{R}$ and $\mathcal{F}$, in general we expect the general solutions for $\mathcal{R}$ and $\mathcal{F}$ to be of the form [4]

$$\mathcal{R} \sim \mathcal{R}_+ e^{-i\omega_+ t} + \mathcal{R}_- e^{-i\omega_- t}, \tag{6.1}$$

$$\mathcal{F} \sim \mathcal{F}_+ e^{-i\omega_+ t} + \mathcal{F}_- e^{-i\omega_- t}, \tag{6.2}$$

where ω_+ and ω_- denote the two frequencies at which the modes oscillate. The values of ω_+ and ω_- will depend on the mode's wave number k in the following way: In the regime $k/a \gg M_{\rm eff}$, both fields are massless and therefore oscillate with frequencies of order $\sim k/a$. As the wavelength enters the intermediate regime $M_{\rm eff} \gg k/a \gg H$ the degeneracy of the modes break down and the frequencies become of order

$$\omega_- \sim k/a, \qquad \omega_+ \sim M_{\rm eff}. \tag{6.3}$$

Subsequently, when the modes enter the regime $k/a < H$ the contributions coming from ω_+ will quickly decay and the contributions coming from ω_- will freeze (since they are massless).

Notice that the amplitudes $\mathcal{R}_+$ and $\mathcal{F}_-$ necessarily arise from the couplings mixing curvature and isocurvature perturbations, and therefore they vanish in the case $\eta_\perp = \dot{\theta}/H = 0$. Additionally, on general grounds, the amplitudes $\mathcal{F}_+$ and $\mathcal{R}_+$ are expected to be parametrically suppressed by $k/M_{\rm eff}$ in the regime $M_{\rm eff} \gg k/a$, and therefore we may disregard high frequency contributions to (6.1) and (6.2). Then, in the regime $M_{\rm eff} \gg k/a$, time derivatives for $\mathcal{F}$ can be safely ignored in the equation of motion (6.14) and we may write:

$$-\frac{\nabla^2 \mathcal{F}}{a^2} + M_{\rm eff}^2 \mathcal{F} = 2\dot{\phi}_0 \eta_\perp \dot{\mathcal{R}}. \tag{6.4}$$

(Because $H \ll M_{\rm eff}$, we may also disregarded the friction term $3H\dot{\mathcal{F}}$). This leads to an algebraic relation between $\mathcal{F}$ and $\dot{\mathcal{R}}$ in Fourier space given by:

$$\mathcal{F}_{\mathcal{R}} = \frac{2\dot{\phi}_0 \eta_\perp \dot{\mathcal{R}}}{(k^2/a^2 + M_{\rm eff}^2)}, \tag{6.5}$$

which precisely tells us the dependence of low frequency contributions $\mathcal{F}_-$ in terms $\mathcal{R}_-$ defined in eqs. (6.1) and (6.2). To continue, we notice that (6.4) is equivalent to disregard the term $\dot{\mathcal{F}}^2$ of the kinetic term in the action (6.11). Keeping this in mind, we can replace (6.5) back into the action and obtain an effective action for the curvature perturbation $\mathcal{R}$, given by

$$S_{\text{eff}} = \frac{1}{2}\int d^4x\, a^3 \frac{\dot{\phi}_0^2}{H^2}\left[\frac{\dot{\mathcal{R}}^2}{c_s^2(k)} - \frac{k^2\mathcal{R}^2}{a^2}\right], \tag{6.6}$$

where c_s is the speed of sound of adiabatic perturbations, given by:

$$c_s^{-2}(k) = 1 + \frac{4H^2\eta_\perp^2}{k^2/a^2 + M_{\text{eff}}^2}. \tag{6.7}$$

In deriving this expression we have assumed that $\dot{\theta}$ remained constant. In the more general case where $\dot{\theta}$ is time dependent we expect transients that could take the system away from the simple behavior shown in eqs. (6.1) and (6.2), and the effective field theory could become invalid. The validity of the effective theory will depend on whether the kinetic terms for $\mathcal{F}$ in eq. (6.14) can be ignored, and this implies the following condition on $\mathcal{F}_{\mathcal{R}}$ of eq. (6.5):

$$|\ddot{\mathcal{F}}_{\mathcal{R}}| \ll M_{\text{eff}}^2|\mathcal{F}_{\mathcal{R}}|. \tag{6.8}$$

Now, recall that unless there are large time variations of background quantities, the frequency of $\mathcal{R}$ is of order $\omega_- \sim k/a$. Thus, any violation of condition (6.8) will be due to the evolution of background quantities, which will be posteriorly transmitted to $\mathcal{R}$. This allows us to ignore higher derivatives of $\dot{\mathcal{R}}$ in (6.8) and simply rewrite it in terms of background quantities as:

$$\left|\frac{d^2}{dt^2}\left(\frac{2\dot{\phi}_0\eta_\perp}{(k^2/a^2 + M_{\text{eff}}^2)}\right)\right| \ll M_{\text{eff}}^2\left|\frac{2\dot{\phi}_0\eta_\perp}{(k^2/a^2 + M_{\text{eff}}^2)}\right|. \tag{6.9}$$

This relation expresses the adiabaticity condition that each mode k needs to satisfy in order for the effective field theory to stay reliable. To further simplify this relation, we may take into consideration the following points: (1) When $k^2/a^2 \gg M_{\text{eff}}^2$ the two modes decouple (recall eq. (4.43)) and the turn has no influence on the evolution of curvature modes. On the other hand, in the regime $k^2/a^2 \lesssim M_{\text{eff}}^2$, contributions coming from the time variation of k^2/a^2 are always suppressed compared to M_{eff}^2 due to the fact that we are assuming $H^2 \ll M_{\text{eff}}^2$. (2) We observe that the main background quantity parametrizing the rate at which the turn happens is $\eta_\perp = \dot{\theta}/H$. Quantities such as $\dot{\phi}_0$ and H, which describe the evolution of the background along the trajectory, will therefore only be affected by the turn through the time dependence of $\eta_\perp$. This implies that time derivatives of these quantities will be less sensitive to the turn than $\eta_\perp$ itself, and therefore their contribution to (6.9) will necessarily be subsidiary.[4] (3) Similarly, the rate of change of M_{eff}^2 will necessarily

[4]Here we are implicitly assuming that *parallel* quantities such as $\dot{\phi}_0$ and H will not have variations larger than $\eta_\perp$ due to other effects, unrelated to the turns (such as steps in the potential).

be at most of the same order than $\dot{\theta}$. Then, by neglecting time derivatives coming from $\dot{\phi}_0$, H, k^2/a^2 and $M_{\rm eff}$ and focussing on the order of magnitude of the various quantitates appearing in (6.9) we can write instead a simpler expression given by:

$$\left| \frac{d^2}{dt^2} \dot{\theta} \right| \ll M_{\rm eff}^2 \left| \dot{\theta} \right| . \tag{6.10}$$

Actually, a simpler alternative expression may be obtained by conveniently reducing the number of time derivatives, and disregarding effects coming from the change in sign of $\dot{\theta}$:

$$\left| \frac{d}{dt} \ln \dot{\theta} \right| \ll M_{\rm eff}. \tag{6.11}$$

This adiabaticity condition simply states that the rate of change of the turn's angular velocity must stays suppressed with respect to the masses of heavy modes, which otherwise would become excited. Notice that, we may also choose to express this relation in terms of the variation of the speed of sound, which is a more natural quantity from the point of view of the effective field theory:

$$\left| \frac{d}{dt} \ln (c_s^{-2} - 1) \right| \ll M_{\rm eff}. \tag{6.12}$$

To finish, we can estimate the order of magnitude of departures between the full solution for $\mathcal{R}$ and the one appearing in the EFT. For this, let us write $\mathcal{F} = \mathcal{F}_{\mathcal{R}} + \delta\mathcal{F}$ where $\mathcal{F}_{\mathcal{R}}$ is the adiabatic result of (6.5) and $\delta\mathcal{F}$ denotes a departure from this value. Then $\delta\mathcal{F}$ respects the following equation of motion:

$$\delta\ddot{\mathcal{F}} + 3H\delta\dot{\mathcal{F}} + \left(\frac{k^2}{a^2} + M_{\rm eff} \right) \delta\mathcal{F} = -(\ddot{\mathcal{F}}_{\mathcal{R}} + 3H\dot{\mathcal{F}}_{\mathcal{R}}). \tag{6.13}$$

Given that we are assuming that the behavior of the system is dominated by low frequency modes, we can consistently disregard the kinetic term $\delta\ddot{\mathcal{F}} + 3H\delta\dot{\mathcal{F}}$ at the left hand side of this equation, but we cannot disregard the term $-(\ddot{\mathcal{F}}_{\mathcal{R}} + 3H\dot{\mathcal{F}}_{\mathcal{R}})$ at the right hand side, which constitutes a source for $\delta\mathcal{F}$. Then, we deduce that

$$\delta\mathcal{F} = -\frac{\ddot{\mathcal{F}}_{\mathcal{R}} + 3H\dot{\mathcal{F}}_{\mathcal{R}}}{k^2/a^2 + M_{\rm eff}}. \tag{6.14}$$

Then, we may compute the derivatives appearing in the right hand side by using the effective equation of motion (coming from the variation of the effective action (6.6)) to express $\ddot{\mathcal{R}}$ in terms of $\dot{\mathcal{R}}$ whenever it becomes necessary. We obtain

$$\mathcal{F} = \mathcal{F}_{\mathcal{R}} \left[1 + \mathcal{O}\left(\frac{(\ddot{\theta}/\dot{\theta})^2}{k^2/a^2 + M_{\rm eff}^2} \right) + \mathcal{O}\left(\frac{\delta_{||} H^2}{k^2/a^2 + M_{\rm eff}^2} \right) \right], \tag{6.15}$$

where $\delta_{||}$ represents terms of order ε and $\eta_{||}$. Finally, eq. (6.15) allows us to deduce that the EFT expressed in (6.6) is only accurate up to operators of the form:

$$S_{\rm tot} = S_{\rm eff} + \frac{1}{2}\int d^4x\, a^3\, (c_s^{-2} - 1)\, \dot{\mathcal{R}}^2 \left[\mathcal{O}\left(\frac{(\ddot{\theta}/\dot{\theta})^2}{M_{\rm eff}^2}\right) + \mathcal{O}\left(\frac{\delta_{||} H^2}{M_{\rm eff}^2}\right)\right]. \qquad (6.16)$$

This result expresses the validity of the effective field theory (6.6), and shows with eloquence the order of magnitude of the expected discrepancy with the exact two field solution for $\mathcal{R}$. In the following section we verify that indeed the adiabatic condition (6.11) constitutes a good guide to discriminate the validity of the effective theory (6.6).

6.4 Turning trajectories

We now study the consequences of turning trajectories on the dynamics of fluctuations. We start this analysis by considering the particular case of sudden turns, where the potential $V(\phi)$ and/or the sigma model metric γ_{ab} entering the action (6.3) are such that the inflationary trajectory becomes non-geodesic for a brief period of time, smaller than an e-fold. In order to characterize this class of turns it is useful to introduce the arc distance $\Delta\phi$ covered by the scalar field's v.e.v. in target space, when the turn takes place. Given the radius of curvature κ characterizing a turn, defined in eq. (4.22), we may define $\Delta\phi$ through the relation

$$\Delta\phi \equiv \kappa |\Delta\theta|, \qquad (6.1)$$

where $\Delta\theta = \int H\eta_\perp dt$ corresponds to the total angle covered during a sudden turn. It is clear that in two-field canonical models $\Delta\phi$ can be at most of order κ ($\Delta\phi \lesssim \kappa$), and that in order to have turns with $\Delta\phi \gg \kappa$ one needs a non-canonical model with a scalar geometry with a topology allowing for such situations. Figure 6.2 shows various examples of turns according to the total angle $\Delta\theta$ covered by a turn (which is determined by the relative size of $\Delta\phi$ and κ).

Notice that the arc length $\Delta\phi$ implies a timescale $T_\perp$ characterizing the overall duration of a turn. This is simply given by:

$$T_\perp \equiv \frac{\Delta\phi}{\dot{\phi}}. \qquad (6.2)$$

Then, because $\eta_\perp = \dot{\theta}/H \simeq \Delta\theta/(T_\perp H)$, we can use eq. (6.1) to derive the following estimation for the value of $\eta_\perp$ characterizing a particular turn:

$$\eta_\perp \sim \frac{1}{HT_\perp}\frac{\Delta\phi}{\kappa}. \qquad (6.3)$$

Notice that a turn requires that the inflationary trajectory departs from the flat minima of the potential (otherwise $V_N = 0$ and eq. (6.10) would require $\eta_\perp = 0$). Then, because the turn happens suddenly during a brief period of time, the various interactions present in the theory will inevitably make the background trajectory oscillate about the flat locus of the potential, with a period T_M given by

$$T_M \equiv \frac{1}{M_{\rm eff}}. \tag{6.4}$$

In other words, a turn cannot be arbitrarily sharp without waking up these background fluctuations. Recall that we are interested in models where $M_{\rm eff} \gg H$, and therefore we necessarily have $T_M \ll H^{-1}$. If present, it is clear that such oscillations will dominate the behavior of the trajectory whenever T_M is of the same order or larger than $T_\perp$ ($T_M \gtrsim T_\perp$). In fact, the adiabaticity condition (6.11) precisely translates into the following condition involving these two timescales

$$T_\perp \gg T_M, \tag{6.5}$$

which is consistent with the notion that the effective field theory will remain valid as long as heavy fluctuations are not participating of the low energy dynamics.

6.4.1 Displacement from the flat minima

Given certain turn (characterized by $\Delta\phi$ and κ) we can estimate the perpendicular displacement of the trajectory away from the flat minima of the potential while the turn takes place. By calling this quantity Δh, we can roughly relate it to other background quantities through the relation $V_N \simeq M_{\rm eff}^2 \Delta h$. Then, inserting this result back into eq. (6.10) we obtain

$$\frac{\Delta h}{\kappa} \simeq \frac{\Delta\phi^2}{\kappa^2}\frac{T_M^2}{T_\perp^2}, \tag{6.6}$$

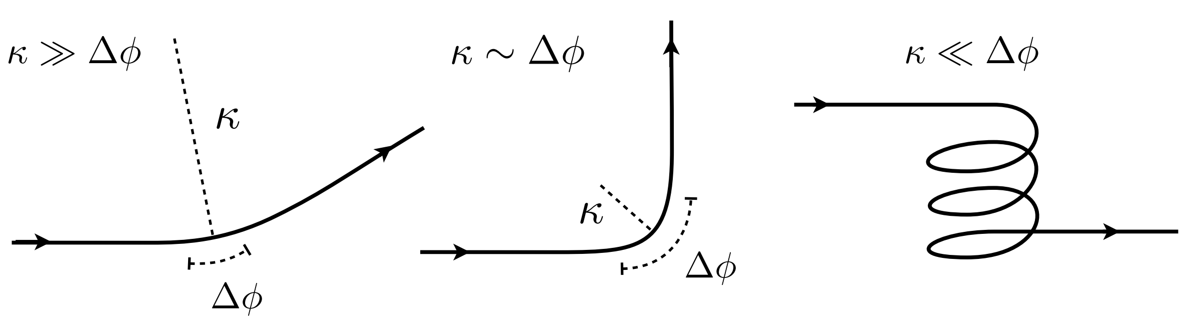

Figure 6.2: Examples of turns according to the relative size of $\Delta\phi$ and κ. In the case $\kappa \ll \Delta\phi$ the target space requires a non trivial topology.

where we made use of eqs. (6.2) and (6.4). This relation gives us the relative size of background oscillations Δh compared to the radius of curvature κ of the turn. We see that the size of the displacement depends on the total angle covered by the turn $\Delta\theta = \Delta\phi/\kappa$ and the ratio $T_M/T_\perp$ between the two relevant timescales. In what follows we briefly analyze the two relevant regimes posed by these two timescales.

6.4.2 Adiabatic turns $T_M \ll T_\perp$

If the turn is such that $T_M \ll T_\perp$, then the adiabatic condition (6.11) is satisfied and the system admits an effective field theory of the form (6.6) as deduced in the previous section. This means that we can parametrize effects in terms of a reduced speed of sound c_s, which synthesizes all the nontrivial information regarding the heavy physics. Putting together eqs. (6.7) and (6.3) we see that the speed of sound is given by

$$c_s^{-2} \simeq 1 + \frac{\Delta\phi^2}{\kappa^2}\frac{T_M^2}{T_\perp^2}. \tag{6.7}$$

Given that the effective theory requires $T_M \ll T_\perp$, the only way of having large non-trivial departures from conventional single field inflation is by having a large ratio $\Delta\phi/\kappa \gg 1$. This implies that the total angle $\Delta\theta = \Delta\phi/\kappa$ covered by the turn must extend for several cycles, and the only way of achieving this consistently is by considering models with non-trivial sigma model metrics. In particular, to produce sizable changes of the speed of sound (say of order one) we require:

$$\frac{\Delta\phi^2}{\kappa^2} \gtrsim \frac{T_\perp^2}{T_M^2}. \tag{6.8}$$

Comparing with (6.6), we see that this is equivalent to have a large displacement from the flat minima of the potential. However, because the timescale T_M is much smaller than $T_\perp$, the displacement happens adiabatically, and background fluctuations are not turned on. Correspondingly, the dimensionless parameter $\eta_\perp$ is a smooth function of time with a characteristic timescale given by $T_\perp$. Figure 6.3 illustrates this situation.

6.4.3 Non-adiabatic turns $T_M \gtrsim T_\perp$

In this case the trajectory moves away from the flat minima of the potential quickly, and the background trajectory will inevitably start oscillating about this locus. The amplitude of these oscillations will be given by Δh as in eq. (6.6). If $\Delta h \gtrsim \kappa$ then the oscillations are large and they completely dominate the behavior of the background trajectory. Needless to say, the effective field theory would offer a poor representation of the evolution of curvature perturbations, and the original two-field theory would be needed to study the system. Figure 6.4-(a) shows this

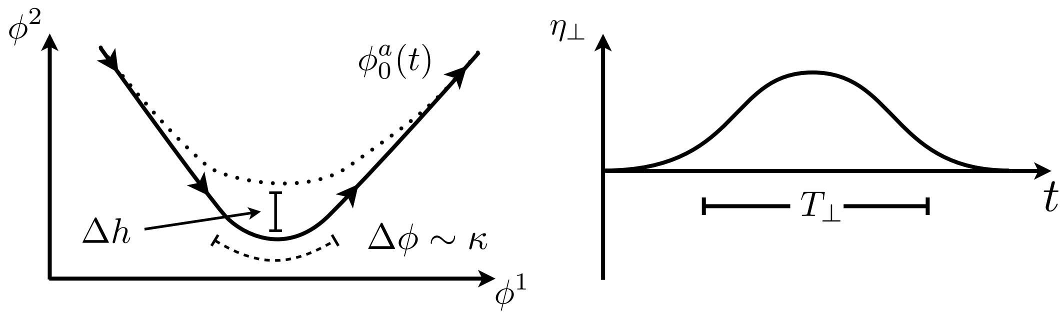

Figure 6.3: The figure illustrates the case of a turn for which the adiabatic condition $T_M \ll T_\perp$ is respected. In this case the turn happens adiabaticaly, in the sense that the timescale T_M plays no role during the turn. If $\Delta h/\kappa \sim 1$ the displacement from the flat minima is large and the speed of sound will be reduced considerably.

type of situation for the case $T_\perp \sim T_M$. There, the trajectory is subject to an initial kick that lasts T_M, and after that continues oscillating at a period given by T_M. Consequently, heavy oscillations will overtake the background trajectory, which translates into a heavily oscillating $\eta_\perp$ as a function of time. This means that instead of a single turn we end up with a succession of turns. However, after these transient turns have taken place, the overall angle $\Delta\theta = \int H\eta_\perp dt$ will correspond to the turn determined by the flat minima of the potential.

On the other hand, if $\Delta h \ll \kappa$, then the amplitude of the oscillations are small. In this case $\Delta\phi/\kappa \ll T_\perp/T_M$, implying, after considering eq. (6.3), that the angular velocity is small compared to the mass of heavy modes:

$$H|\eta_\perp| = |\dot{\theta}| \ll M_{\text{eff}}. \tag{6.9}$$

In this case the impact of background oscillations on the evolution of perturbations is negligible. Despite of this fact, since we are in the regime $T_M \gtrsim T_\perp$, the effective field theory continues to offer a poor representation of these effect, no matter how tiny they are. Figure 6.4-(b) illustrates this situation.

6.5 Examples of models with turns

We now study examples of models where turns play a relevant role on the evolution of perturbations. We will first consider the case of canonical models —in which turns are uniquely due to the shape of the potential— and later consider the case of models where the turns are due to the specific shape of the sigma model metric.

6.5.1 Model 1: Sudden turns in canonical models

Let us start by considering the case of canonical models ($\gamma_{ab} = \delta_{ab}$) in which the turn is due solely to the shape of the potential. To simplify the present analysis, we only

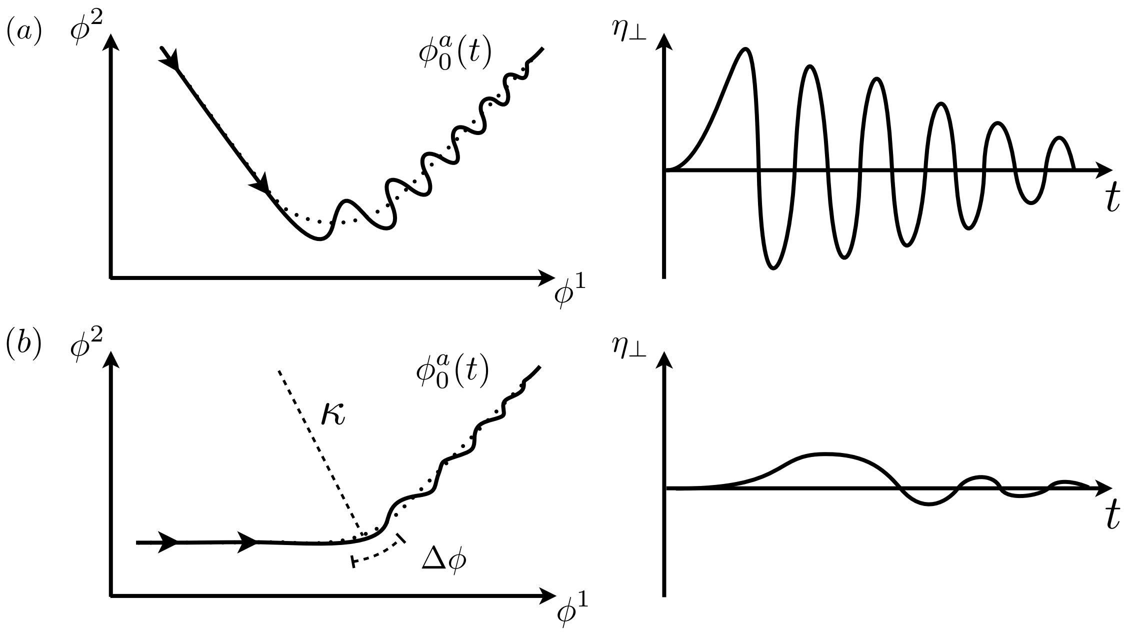

Figure 6.4: Examples of non-adiabatic turns characterized by $T_M \ll T_\perp$. Case (a) shows a typical example in which $\Delta\phi_0 \sim \kappa$, whereas case (b) shows a situation for which $\Delta\phi_0 \ll \kappa$.

consider the relevant part of the potential where the turn takes place, and disregard any details concerning the end of inflation. Our aim is to understand what are the possible consequences of a turn on the generation of primordial inhomogeneities accessible to observations today. For this reason, we assume that the turn takes place precisely when presently accessible curvature perturbations were crossing the horizon, and choose cosmological parameters accordingly. Having said this, let us adopt the notation $\phi^a = (\chi, \psi)$ and consider the following potential:

$$V(\chi, \psi) = V_0 + V_\phi(\chi - \psi) + \frac{M^2}{2}\frac{(\chi\psi - a^2)^2}{(\chi + \psi)^2} + \cdots . \tag{6.1}$$

In this expression, V_0 and V_ϕ are constants parametrizing the hight and slope of the flat direction in the potential. The third term in (6.1) has been added to ensure that a turn takes place. Notice that the third term vanishes for the locus of points defined by the equation:

$$\psi = a^2/\chi. \tag{6.2}$$

Away from this curve the slope of the potential becomes steep, with a quadratic growth characterized by the mass parameter M. Figure 6.5 shows the relevant part of the potential $V(\chi, \psi)$ containing the turn. Because we are assuming a mass hierarchy, it may be anticipated that the inflationary trajectory will stay close to the curve defined by eq. (6.2). Roughly speaking, the turn is located about the position $(\chi, \psi) = (\sqrt{a}, \sqrt{a})$ and that its radius of curvature is of order a:

$$\kappa \sim a. \tag{6.3}$$

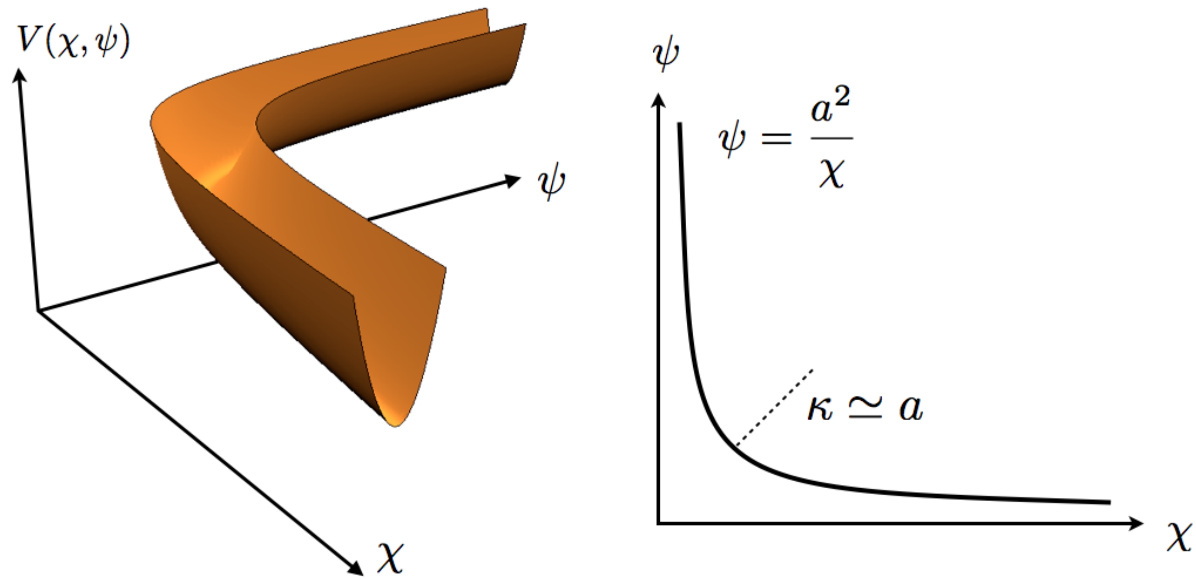

Figure 6.5: The figure shows the section of the potential $V(\chi, \psi)$ of equation (6.1) containing the turn.

The quantities V_0 and V_ϕ are chosen in such a way that the scalar fields v.e.v.'s approach the turn from the asymptotic direction $(\chi, \psi) \to (0, +\infty)$. Once the turn is left behind, the v.e.v.'s. continue evolving towards the asymptotic direction $(\chi, \psi) \to (+\infty, 0)$, until inflation ends (see Figure 6.5).

Analysis of the model

Before studying the exact evolution of the system with the help of numerics, let us estimate the behavior of the system by examining the parameters entering the potential. First of all, if the potential is flat enough, the slow roll evolution of the fields imply that *away from the location of the turn* the following relations are satisfied:

$$3H\dot{\phi}_0 \simeq V_\phi, \tag{6.4}$$

$$3H^2 \simeq V_0. \tag{6.5}$$

Notice that they imply that $\varepsilon \simeq V_\phi^2/2V_0^2$. To continue, simple examination of the potential shows that the arc length $\Delta\phi$ characterizing the turn is of the same order than the radius of curvature κ which, as stated, is of order a. Thus, we have $\kappa \simeq \Delta\phi \simeq a$. Then, putting together the previous expressions we find that time $T_\perp$ characterizing the duration of the turn, is given by

$$T_\perp \simeq \frac{a}{\sqrt{2\varepsilon}H}. \tag{6.6}$$

In terms of e-folds, the duration of the turn is given by

$$\Delta N_\perp \simeq \frac{a}{\sqrt{2\varepsilon}}. \tag{6.7}$$

Then, using (6.3) we see that the value $\eta_\perp$ characterizing the turn is roughly given by:

$$\eta_\perp \simeq \frac{\sqrt{2\varepsilon}}{a}. \tag{6.8}$$

With these relations at hand, we can now estimate the range of parameters for which the adiabatic condition (6.11) is satisfied. Using eq. (6.5) with $T_M = 1/M$, we deduce that the EFT will remain accurate as long as:

$$\alpha \equiv \frac{T_\perp^2}{T_M^2} = \frac{M^2 a^2}{2\varepsilon H^2} \gg 1. \tag{6.9}$$

Then, assuming that we are in the realms of validity of the theory, we find that the speed of sound is given by the following combination of parameters

$$c_s^{-2} \simeq 1 + \frac{8\varepsilon H^2}{a^2 M^2}, \tag{6.10}$$

which, because of eq. (6.9), implies that $c_s \sim 1$, consistent with our general analysis of Section 6.4.2.

Numerics

We now present our numerical analysis of this model. In order to study this model, we have chosen the following fixed values (in units of Planck masses) for parameters associated to the flat direction of the potential:

$$V_0 = 6.5 \times 10^{-9}, \quad V_\phi = -5.4 \times 10^{-11}. \tag{6.11}$$

Away from the turn, this choice of parameters ensure the following value for the rate of expansion $H \sim 1.4 \times 10^{-5}$. Additionally, they imply values for the slow roll parameters given by

$$\varepsilon \simeq -\eta \simeq 0.0033, \tag{6.12}$$

consistent with a power spectrum for curvature perturbations with a spectral index given by $n_s \simeq 0.98$ and an amplitude satisfying COBE's normalization. The other two parameters left are a and M. These may be expressed in terms of $\Delta N_\perp$ and α introduced in eqs. (6.7) and (6.9).

Since we are interested in turns taking place during a period of time shorter than an e-fold ($T_\perp \lesssim H^{-1}$), we are required to choose $\Delta N_\perp \lesssim 1$. Figure 6.6 shows the numerical results for the fixed value $\Delta N_\perp = 0.25$ and three choices for the mass M, given by $\alpha = 6.25$, $\alpha = 25$ and $\alpha = 625$. On the left hand side of the figure we show the numerical solution for the $\eta_\perp$ and $\eta_{||}$ as functions of e-fold N. On the right hand side, we show the resulting power spectra for the relevant modes exiting the horizon when the turn takes place. For simplicity, we have normalized the power spectrum with respect to the amplitude determined by the largest scales (smallest values of k), which have been chosen to correspond to the largest scales characterizing horizon reentry today ($k_0 = 0.002\text{Mpc}^{-1}$). The blue solid line corresponds to the complete-two dimensional theory, whereas the red dashed line corresponds to the result predicted by the effective field theory.

It may be seen that for $\alpha = 6.25$, the period of massive oscillations T_M is of the same order than $T_\perp$, and consequently the rate of turn $\eta_\perp$ is dominated by oscillations (notice that these oscillations have a period of about ~ 0.25 e-folds, in agreement with our choice of parameters). This indicates that indeed the inflationary trajectory stays oscillating about the minimum once the turn has occurred (recall case (a) of Figure 6.4). It may be also observed that $\eta_{||}$ is considerably affected by these oscillations, momentarily acquiring values as large as $\eta_{||} \sim 1$. This is however not enough to break down the overall slow-roll behavior of the background solution for two reasons: First, the value of ε does not change much during the turn (that is ε is found to be less sensitive to the turn than $\eta_{||}$). And second, after the turn takes place, the system goes back to small values of $\eta_{||}$ quickly. Given that the adiabatic condition is far from being satisfied (α is close to 1), it comes to no surprise that both power spectra differ considerably.

The second case $\alpha = 25$ shows an intermediate situation where T_M is smaller than $T_\perp$ but still able to generate small oscillations about the minimum of the potential. In this case the adiabaticity condition is mildly violated, which is reflected on the small discrepancy between both power spectra. Finally, the case $\alpha = 625$ shows a situation where T_M is much smaller than $T_\perp$. In this case heavy modes are not excited enough and the turn happens smoothly, which is reflected on the behavior of both $\eta_\perp$ and $\eta_{||}$ as functions of e-folds. In addition, we now see that both power spectra agree considerably. It may call our attention the persistence of a large feature in the power spectra even for the case $\alpha = 625$ where both, versions of the theory agree. In this case the speed of sound $c_s \simeq 1$ during the turn and we infer that the feature cannot be due to the fast variation of the speed of sound. Instead, the feature is due to the large variation of parallel background quantities, specially $\eta_{||}$, as during the turn the trajectory is forced to go up and down. In the examples of the following model we will examine a rather different situation where the features in the power spectrum are due to large variations of the speed of sound c_s.

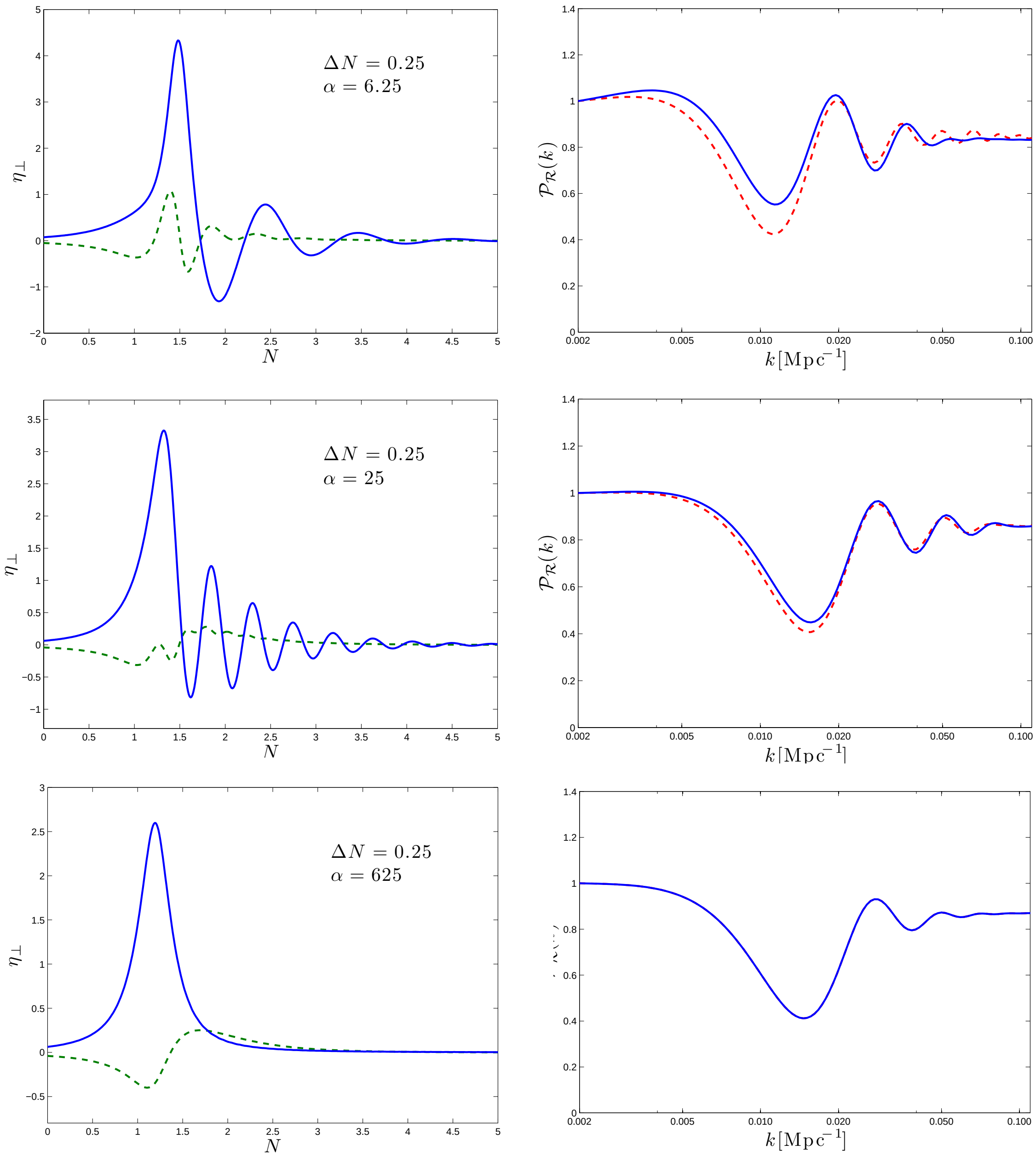

Figure 6.6: The figure shows $\eta_\perp$ and $\eta_{||}$ (left panels) and the resulting power spectra (right panels) for three choices of parameters for the potential of our model 1. From top to bottom: $\alpha = 6.25$, 25 and 625. In the case of the left panels, the blue solid line corresponds to $\eta_\perp$ whereas the green dashed line corresponds to $\eta_{||}$. In the case of right panels, the blue solid line corresponds to the power spectrum for the full two-field model, whereas the red dashed line corresponds to the power spectrum deduced using the EFT.

6.5.2 Model 2: Sudden turns induced by the metric

We now study a case in which the turn is due to the sigma model metric γ_{ab}. We will adopt the following notation $\phi^1 = \chi$ and $\phi^2 = \psi$, and consider the following separable potential

$$V(\chi, \psi) = V_0 + V_\phi \, \chi + \frac{M_\psi^2}{2} \psi^2. \tag{6.13}$$

Just like in the previous example, V_0 and V_ϕ are constants parametrizing the flat part of the potential driving inflation. As before, we omit in our analysis any detail concerning the end of inflation. For this potential, if the system were canonical ($\gamma_{ab} = \delta_{ab}$), there would be no turns and the v.e.v. of the heavy field ψ would stay sitting at its minimum $\psi_0 = 0$. To produce a single turn with the help of the metric, we consider the following model:

$$\gamma_{ab} = \begin{pmatrix} 1 & \Gamma(\chi) \\ \Gamma(\chi) & 1 + \Gamma^2(\chi) \end{pmatrix}, \tag{6.14}$$

where

$$\Gamma(\chi) = \frac{\Gamma_0}{2} \left(1 + \tanh\left[2(\chi - \chi_0)/\Delta\chi\right]\right). \tag{6.15}$$

Notice that $\Gamma(\chi)$ is a function that grows monotonically from the asymptotic value $\Gamma = 0$ at $\chi \to -\infty$ to the asymptotic value $\Gamma = \Gamma_0$ at $\chi \to +\infty$. The transition takes place at $\chi = \chi_0$ and is characterized by the width parameter $\Delta\chi$. Notice that once Γ reach the constant value Γ_0, the metric becomes canonical again (which means that one can find a new parametrization of the fields in which $\gamma_{ab} = \delta_{ab}$). We will consider values of V_0 and V_ϕ such that the field χ evolves from $\chi \to -\infty$ to $\chi \to +\infty$.

Analysis of the model

It is clear that $\Gamma(\chi)$ parametrizes departures from the canonical configuration $\gamma_{ab} = \delta_{ab}$. The potential is such that it will force the trajectory to stay on the locus of points $\psi = 0$. However, since the geometry of the target space is non-trivial, the trajectory will be subject to a turn. To estimate the effects of the turn on the relevant background quantities, notice first that the timescale associated to heavy fluctuations about the minimum $\psi = 0$ is trivially given by:

$$T_M = \frac{1}{M_\psi}. \tag{6.16}$$

Second, since the trajectory remains close to $\psi = 0$ (due to the mass hierarchy), the arc length of the turn in target space $\Delta\phi$ will be equal to the width of the function $\Gamma(\chi)$:

$$\Delta\phi = \Delta\chi. \tag{6.17}$$

Then, it immediately follows that the timescale $T_\perp$ characterizing the duration of the turn is simply given by:

$$T_\perp = \frac{\Delta\chi}{\sqrt{2\varepsilon}}, \tag{6.18}$$

where $\varepsilon = V_\phi^2/2V_0^2$. Again, we may express this period of time terms of e-folds, which is given by

$$\Delta N_\perp \simeq \frac{\Delta\chi}{\sqrt{2\varepsilon}}. \tag{6.19}$$

Then, in terms of the parameters of the model, the adiabaticity condition reads

$$\alpha \equiv \frac{T_\perp^2}{T_M^2} = \frac{M_\psi^2 \Delta\chi^2}{2\varepsilon H^2} \gg 1. \tag{6.20}$$

We may compute the radius of curvature κ of the turn by noticing that the unit vectors associated to a curve following the minimum of the potential $\psi = 0$ are given by

$$T^a = (1, 0), \qquad N^a = (\Gamma, -1). \tag{6.21}$$

Plugging these expressions back into eq. (6.6) we find that the characteristic radius of curvature while the turn is at its pick, is given by:

$$\kappa = \frac{1}{|\partial_\chi \Gamma|} \simeq \frac{\Delta\chi}{|\Gamma_0|}. \tag{6.22}$$

This implies that $\eta_\perp = \dot{\phi}/H\kappa$ characterizing the turn is given by

$$\eta_\perp \sim \frac{\sqrt{2\varepsilon}|\Gamma_0|}{\Delta\chi}, \tag{6.23}$$

whereas the speed of sound is found to be

$$c_s^{-2} \simeq 1 + \frac{8\varepsilon H^2 \Gamma_0^2}{\Delta\chi^2 M^2}. \tag{6.24}$$

Notice that the only difference between this model and the previous one (where a plays the role of $\Delta\chi$) is the appearance of Γ_0. By defining the following dimensionless parameter,

$$\beta \equiv \frac{8\varepsilon H^2 \Gamma_0^2}{\Delta\chi^2 M_\psi^2}, \tag{6.25}$$

we see that, in order to have large effects on the power spectrum due to the turn, we are required to have $\beta \sim 1$. Notice that one may satisfy this combination of parameters and still stay within the region of validity of the effective field theory, given by condition (6.20).

Numerics

We now present our numerical analysis of this model. As in the example of Section 6.5.1, we choose the following values for the potential parameters associated to the flat inflationary direction: $V_0 = 6.5 \times 10^{-9}$ and $V_\phi = -5.4 \times 10^{-11}$ which in the absence of turns, imply $H \sim 1.4 \times 10^{-5}$, $\varepsilon \simeq -\eta \simeq 0.0033$, and $n_s \simeq 0.98$. Notice that the rest of the parameters in charge of characterizing the multi-field turn are $\Delta\chi$, Γ_0 and M_ψ. These may be expressed in terms of α, β and $\Delta N_\perp$ introduced earlier as

$$M_\psi^2 = \alpha^2 \frac{2\varepsilon H^2}{\Delta\chi^2}, \tag{6.26}$$

$$\Gamma_0^2 = \beta \frac{\Delta\chi^2 M^2}{8\varepsilon H^2}, \tag{6.27}$$

$$\Delta\chi = \Delta N_\perp \sqrt{2\varepsilon}. \tag{6.28}$$

Recall that the adiabaticity condition (6.20) is equivalent to $\alpha \gg 1$, and therefore we expect a poor matching between the full two-field theory and the EFT for small values of α.

Figure 6.7 shows three examples of turns for different values of the parameters α and β, but for a fixed value $\Delta N_\perp = 0.4$. The top panels correspond to the choice $\alpha = 9$ and $\beta = 0.25$. It may be seen that given that the value of α is relatively low, the adiabaticity condition is not satisfied. Consistent with the discussions of the previous sections, the dependence on time of $\eta_\perp$ is dominated by fluctuations with a period of oscillation determined by $T_M \sim 1/M_\psi$, and the power spectrum obtained from the effective field theory (dashed red line) does not coincide with the one obtained from the complete two-field model (solid blue line). The middle panels correspond to the choice $\alpha = 36$ and $\beta = 1$. Here the adiabaticity condition is slightly improved while the speed of sound suffers a sizable change. Finally, the lower panels show the situation $\alpha = 625$ and $\beta = 0.64$. Here the adiabaticity condition is fully satisfied and the speed of sound becomes suppressed during a brief period of time. This is reflected by the excellent agreement between both power spectra, and their large features.

It may be noticed that in the first two cases $(\alpha, \beta) = (9, 0.25)$ and $(36, 1)$, there is a sizable variation of $\eta_{||}$. Just like in the case of our Model 1, this variation happens only during a brief period of time, and it is not enough to break the overall slow-roll behavior of the system. In addition, the value of ε is not affected considerably by the turn. In the last case $(\alpha, \beta) = (625, 0.64)$ the variation of $\eta_{||}$ is attenuated and the only background quantity varying considerably turns out to be $\eta_\perp$. This in turns makes c_s to have large variations, therefore producing the large features observed in the resulting power spectrum.

To finish, it is instructive to verify how does the rate of change of $\dot\theta$ evolves while

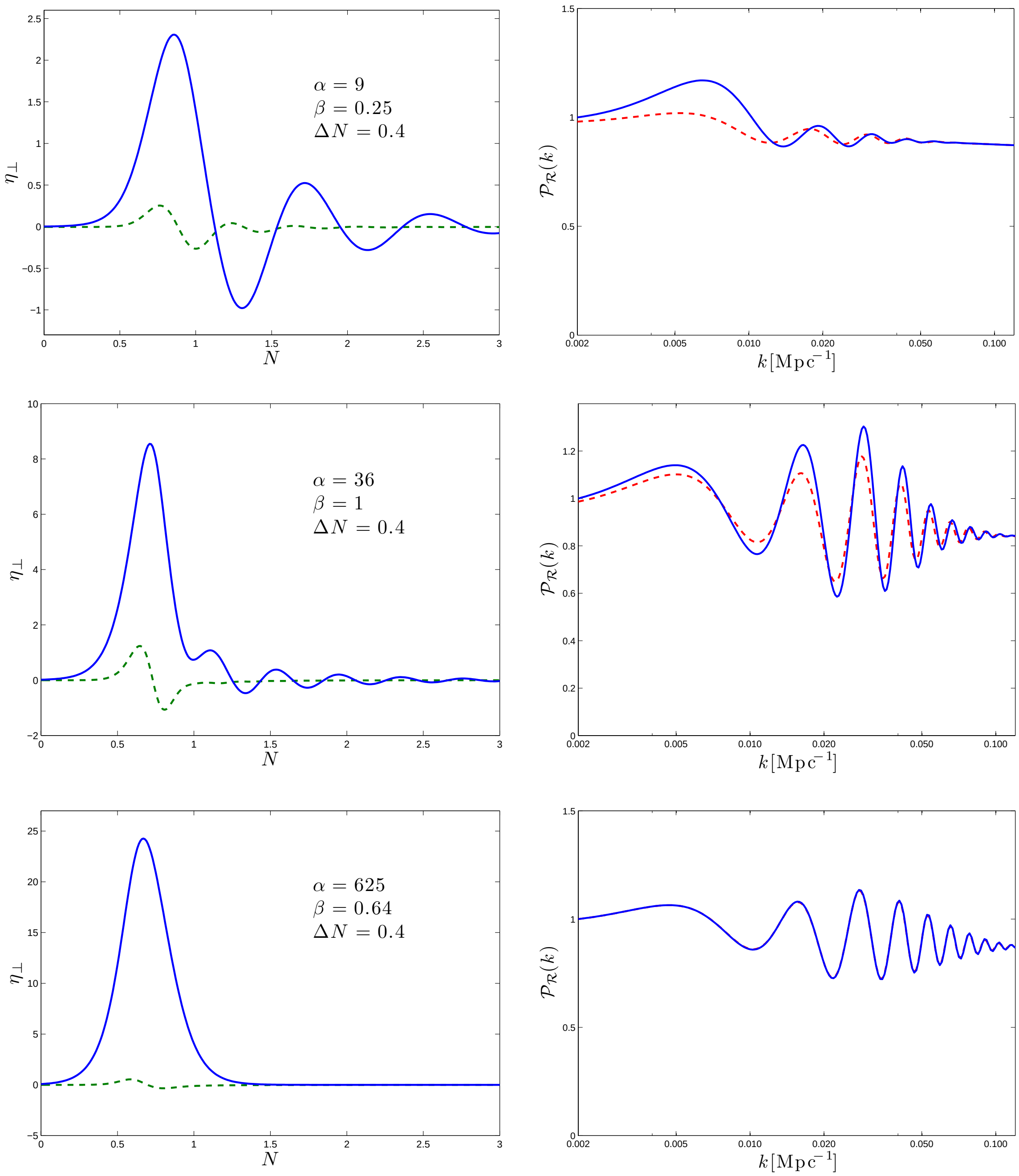

Figure 6.7: The figure shows $\eta_\perp$ and $\eta_{||}$ (left panels) and the resulting power spectra (right panels) for three choices of parameters for the potential of our model 2. From top to bottom: $(\alpha, \beta) = (9, 0.25)$, $(36, 1)$ and $(625, 0.64)$. In the case of the left panels, the blue solid line corresponds to $\eta_\perp$ whereas the green dashed line corresponds to $\eta_{||}$. In the case of right panels, the blue solid line corresponds to the power spectrum for the full two-field model, whereas the red dashed line corresponds to the power spectrum deduced using the EFT.

the turn takes place. For this, we define

$$f(t) = \frac{1}{M_{\text{eff}}^2} \frac{1}{\dot{\theta}} \frac{d^2}{dt^2} \dot{\theta}. \tag{6.29}$$

According to our discussion in Section 6.3, the adiabaticity condition will be satisfied if $|f(t)| \ll 1$. Figure 6.8 shows f as a function of e-fold N for the cases $(\alpha, \beta) = (9, 0.25)$ and $(625, 0.64)$ respectively. It may be appreciated that indeed case $(\alpha, \beta) = (9, 0.25)$ is far from satisfying the adiabaticity condition, whereas the case $(\alpha, \beta) = (625, 0.64)$ satisfies it.

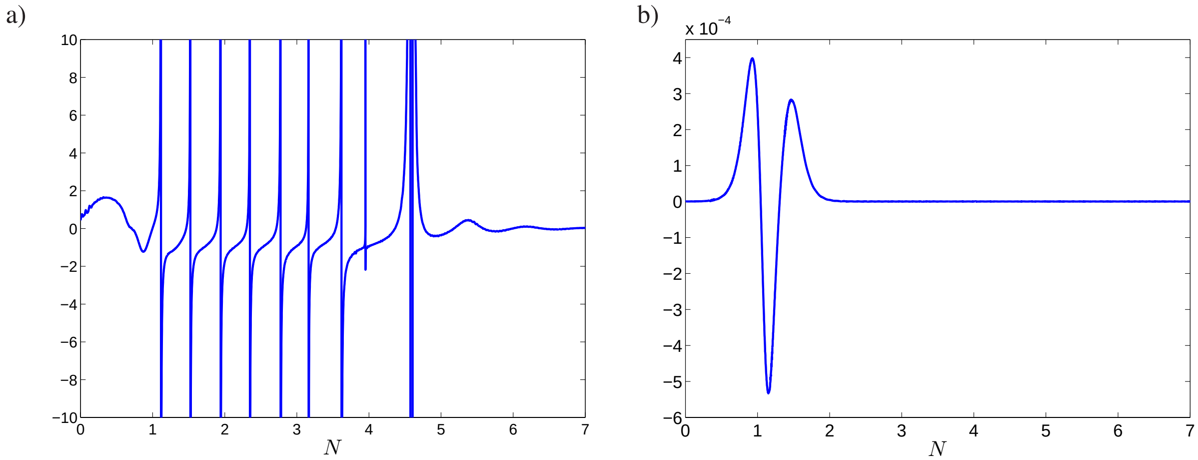

Figure 6.8: The figure shows the the quantity $f(t)$ defined in eq. (6.29) for the cases $(\alpha, \beta) = (9, 0.25)$ and $(625, 0.64)$ respectively. This function assesses whether the adiabatic condition is being satisfied during a turn. (We have chosen to plot this function in terms of e-folds N to facilitate its comparison with other quantities).

6.6 Conclusions

In this work we have analyzed the dynamics of two-field models of inflation with large mass hierarchies. We have focussed our attention on the role that turning trajectories have on the evolution of perturbations. If the mass M of the heavy field is much larger than the rate of expansion H, then the heavy field may be integrated out giving rise to a low energy effective field theory valid for curvature perturbations $\mathcal{R}$, with a quadratic order action given by eq. (6.6). At this order, all of the effects inherited from the heavy sector are reduced in the speed of sound c_s, which is given by eq. (6.7). We found that a good characterization of the validity of the low energy effective theory is given by the following adiabaticity condition:

$$\left| \frac{d}{dt} \ln \dot{\theta} \right| \ll M_{\text{eff}}, \tag{6.1}$$

where M_{eff} is the effective mass of heavy modes. Our numerical analysis is consistent with this condition, and we find that several non-trivial effects are still significant within this allowed region of parameters. For instance, in Section 6.4 we were able to provide two simple toy models for which the effective field theory remains fully trustable. In these examples large features are generated and appear superimposed on the primordial power spectrum of curvature perturbations (recall the examples of Figures 6.6 and 6.7). In addition, we verified that indeed as soon as the adiabaticity condition starts to fail, this is reflected in noticeable discrepancies in the power spectra predicted by both the complete two-dimensional model and the effective field theory (recall Figure 6.8).

Our results contradict those of recent works regarding the validity of effective field theories obtained from multi-field inflation in various respects. For instance, in ref. [120] it is claimed that the effective field theory (6.6) is only valid in the regime where turns are such that $|\dot{\theta}| \ll H$.[5] The main argument made there is that the ratio $\eta_\perp = \dot{\theta}/H$ corresponds to the coupling determining the kinetic energy transfer between the light curvature mode with the heavy fields. A large value of $\eta_\perp$ would therefore imply large transfer of energy from curvature perturbations to the heavy mode, exciting the heavy modes and rendering the effective field theory invalid. However, as we have seen, this energy interchange between both modes may happen adiabatically without implying a breakdown of the effective field theory. Indeed, the heavy mode is receiving energy from the light degree of freedom at the same rate than it is giving it back, and therefore it is possible to have turns whereby the heavy-mode's high-frequency fluctuations stay suppressed (recall our discussion of Section 6.3).

One key point here is that even for large values of $\eta_\perp = \dot{\theta}/H$ the heavy modes will not become easily excited unless they receive a sufficiently strong kick. For example, even if the turn is such that

$$\left| \frac{d}{dt} \ln \dot{\theta} \right| \gtrsim H, \tag{6.2}$$

the trajectory is necessarily subject to a large angular acceleration $|\ddot{\theta}| \gtrsim H|\dot{\theta}|$, momentarily violating slow-roll [19] but without exciting heavy modes. Moreover, as we have seen, if these accelerations are brief (as in our examples) slow-roll is only interrupted for a short period of time, and the system quickly goes back to the slow-roll attractor state (within an e-fold). During these transients, $\eta_{||}$ were typically found to have sizable variations in response to the turns, whereas ε was found to stay close to its suppressed value.[6] The net effect of this process are oscillatory features in the power spectrum, with the frequency of the oscillation depending on how brief was the overall turn. While current observational constraints on features are still poor [11, 39, 75, 77–79, 86, 99, 102, 105, 126], future data will certainly put

[5]A similar claim is made in ref. [106], where it is argued that heavy fields can only be integrated out consistently if the rate of turn satisfies $|\dot{\theta}| \ll H$.

[6]This was also found numerically in ref. [3], and in ref. [120] an analytic argument was given to explain this effect.

strong constraints on primordial features, therefore improving our understanding of the role of UV-physics on the very early universe.

Another important point of departure from previous works regards the procedure employed to integrate heavy fields. In the present work we have integrated out high energy fluctuations (heavy degrees of freedom) about the exact time-dependent background trajectory, offered by the homogeneous equations of motion of the system. This contrasts with other schemes [35, 65, 131] where heavy fields are taken care of at the action level, regardless of the background dynamics, by imposing that they locally minimize the inflationary potential (with the minima depending on the inflaton v.e.v.). In our present language this is equivalent to $V_N = 0$, implying that there are no turns at all (and therefore missing all of the interesting features we have studied so far). Although such a case corresponds to a genuine limit shared by a large family of multi-field potentials with mass hierarchies [40, 64], it misses the more general situation in which the inflationary trajectory meanders away from the locus of minima offered by the potential.

We should emphasize that our results are strictly valid only at linear order in the fluctuations. For a complete analysis one should examine the relevance of higher order interaction terms which could introduce important corrections when the speed of sound is suppressed ($c_s \ll 1$). This is similar to the case of DBI inflation [13, 121] where a suppressed speed of sound makes the perturbation theory to enter the strong-coupling regime [50]. In ref. [5] the full structure of effective field theories arising from two-field models with large mass hierarchies is studied, and other relevant constraints involving higher order terms are analyzed.

Chapter 7

Cosmic inflation in a landscape of heavy-fields

Undoubtedly, canonical models of single-field slow-roll inflation[1] give us the simplest resolution to the horizon and flatness problems encountered in hot big-bang cosmology [12,72,88], and offer us an elegant explanation to the origin of primordial curvature perturbations, characterized by a nearly scale invariant power spectrum [101]. Although such predictions are fully compatible with current observations [36,77,85], there is still plenty of room for a change in paradigm in the advent of future experiments, such as large scale structure surveys [1,14] and 21cm cosmology [98]. A possible observation of scale dependence in the primordial spectra (*i.e.* in the form of features and/or running) [3, 7, 16, 17, 21, 26, 44, 45, 52, 54, 67, 74, 75, 83, 96, 104, 108, 110, 119, 124, 128, 129] and/or large non-Gaussianity [13, 22, 23, 37, 46, 47, 90, 94, 95] would force us to leave this simple picture behind, and move on to consider models of inflation where the evolution of curvature perturbations was influenced by nontrivial self-couplings and/or interactions with additional degrees of freedom.

Elucidating how future observations will guide our understanding of inflationary cosmology beyond the standard single-field paradigm has been the main focus of much effort during recent years [27, 84]. One particularly powerful and compelling framework to analyze inflation in a model independent way is the recently proposed effective field theory approach [50] (see also [41, 130]). In this scheme, the broken time translation invariance of the inflationary background is parametrized by introducing a Goldstone boson field $\pi(x,t)$, defined as the perturbation along the broken time translation symmetry. At the same time, curvature perturbations are intimately related to the Goldstone boson, whose action appears highly constrained by the symmetries of the original ultraviolet (UV)-complete action. In particular, the unknown UV-physics is parametrized by self-interactions of the

[1] By canonical models of single-field slow-roll inflation we mean models derived from an action of the form $S = S_{\rm EH} - \int \left[\frac{1}{2}(\partial\phi)^2 + V(\phi)\right]$ where $S_{\rm EH}$ is the usual Einstein-Hilbert term.

Goldstone boson that non-linearly relate field operators at different orders in perturbation theory. This framework has offered a powerful approach to analyze the large variety of infrared observables potentially predicted by inflation, including the prediction of non-trivial signals in the primordial power spectrum and bispectrum [5,6,28–31,33,34,51,60,73,92,103,114–118]. At short wavelengths, for instance, one finds that the Goldstone boson action is given by [50]

$$S = -M_{\rm Pl}^2 \int d^4x \dot{H} \left[\frac{1}{c_s^2} \left(\dot{\pi}^2 - c_s^2 \frac{(\partial_i \pi)^2}{a^2} \right) - \frac{1 - c_s^2}{c_s^2} \left(\frac{(\partial_i \pi)^2}{a^2} + \frac{A}{c_s^2} \dot{\pi}^2 \right) \dot{\pi} + \cdots \right], \tag{7.1}$$

where c_s is the speed of sound at which Goldstone boson quanta propagate, and A is a quantity that parametrizes different models of inflation (for instance, DBI inflation [13] corresponds to the particular case $A = -1$). Current available data [36] mildly constrain c_s and A, suggesting that future observations might rule out a large variety of models of inflations.

Arguably, the simplest class of theories incorporating a departure from canonical single-field slow-roll inflation is offered by models in which adiabatic modes (or equivalently, Goldstone boson modes) interact with heavy scalar fields, with masses much larger than the expansion rate during inflation [2–4,44,66,107,112,127]. Crucially, such models continue to be of the single field type [2], but come dressed with properties that differ significantly from those encountered in standard single-field models. Indeed, near horizon crossing the Goldstone boson modes do not carry enough energy to excite their high-energy counterparts implied by the heavy-fields, meaning that curvature perturbations are generated by a single low energy degree of freedom. Nevertheless, the presence of heavy-fields can induce self-couplings for adiabatic perturbations that may have a sizable impact on their evolution (for example, by modifying the dispersion relation of the Goldstone boson mode). This has been understood gradually in a series of recent articles [2, 5, 73], and for the particular case of models with a Goldstone boson mode interacting with a single heavy-field[2], our current understanding may be summarized as follows:

- There exists a background inflationary trajectory which traverses the multifield landscape determined by the scalar field potential of the theory. In general, this trajectory is expected to be non-geodesic, meaning that the flat directions of the scalar potential do not necessarily align with the family of geodesic paths defined by the scalar manifold of the theory's target space. It is possible to think of such non-geodesic trajectories as turning trajectories, characterized by an angular velocity $\dot{\theta}$ (the rate of turn of the trajectory).
- To study the perturbations of the system, it is useful to define perturbations along the trajectory and perpendicular to it. The first class defines the Gold-

[2]That is, in the particular case where the original theory consists of a two-scalar field model with a potential such that there is only one flat direction, followed by the inflationary trajectory.

stone boson field $\pi(t, x)$ and the second one corresponds to a heavy scalar field with an effective mass $M_{\rm eff}$ given by $M_{\rm eff}^2 = m^2 - \dot{\theta}^2$, where m is the standard value of the mass computed from the potential alone. The angular velocity $\dot{\theta}$ is found to have an important role on the dynamics of these two perturbations, as it implies nontrivial interactions between the Goldstone boson and the heavy-field.

- Because of these interactions, both the Goldstone boson π and the heavy-field are found to depend on a mixture of low- and high-energy modes. Crucially, the gap between these two energies increases as the strength of the turn increases, making high-energy modes more difficult to access at energy scales comparable to the horizon inverse length-scale. As a consequence, although the Goldstone boson stays coupled to the heavy-field, low- and high-energy modes decouple and evolve independently. The end result is a system where only low-energy modes play a relevant role for the generation of curvature perturbations.

Given these characteristics, one may deduce a single-field EFT governing the dynamics at low energy modes (valid at horizon crossing) by integrating out the heavy-field under question.[3] This turns out to be equivalent to truncate the high-energy modes everywhere in the theory, implying that the heavy-field takes the role of a Lagrange multiplier, to be solved in terms of the Goldstone boson field. The result is a low energy EFT for the Goldstone boson alone, with nontrivial self-interactions leading to interesting properties that differ significantly from those predicted by canonical single-field inflation. For example, a first outstanding property is that the speed of sound c_s at which Goldstone boson perturbations propagate is reduced whenever there is a turn $\dot{\theta} \neq 0$, with a value determined by the relation

$$\frac{1}{c_s^2} = 1 + \frac{4\dot{\theta}^2}{M_{\rm eff}^2}, \tag{7.2}$$

where $\dot{\theta}$ and $M_{\rm eff}$ are the quantities already introduced. As shown in [44], such an effective field theory remains valid as long as

$$|\ddot{\theta}| \ll M_{\rm eff}|\dot{\theta}|, \tag{7.3}$$

which is a necessary condition ensuring that heavy-fields will not become excited during a turn[4]. Furthermore, and consistent with the non-linear realization of the Goldstone boson self-interactions, at small speeds of sounds $c_s^2 \ll 1$ the effective

[3]For alternative approaches on effective field theories deduced by integrating heavy fields, please see refs. [18, 25, 49, 61, 62, 80–82, 120].

[4]This condition is in fact equivalent to ask the familiar adiabaticity condition $|\dot{\omega}_+/\omega_+^2| \ll 1$, where ω_+ is the frequency of the high-energy modes implied by the heavy-fields [2].

field theory contain sizable cubic self-interactions that inevitably lead to large non-Gaussianity. For instance, at long wavelengths, one find that the EFT is of the form (7.1), with A given by

$$A = -\frac{1}{2}(1 - c_s^2). \tag{7.4}$$

On the other hand, the interaction with a heavy-field may imply the appearance of a *new physics* regime, a range of energy for which the Goldstone boson dispersion relation becomes dominated by a quadratic dependence on the momentum $\omega \sim p^2$ [2,30,73]. As such, if horizon crossing happened during this regime, the prediction of observables are drastically affected by the *new physics* scale dependent operators. This class of EFT's remains weakly coupled all the way up to the cutoff scale at which heavy-fields are allowed to be integrated out [30, 73].

The previous set of findings has paved the way for a more refined understanding of how low energy effective field theories of inflation relate to the ultraviolet parent theories from which they decent. However, there is still much to be learned about the way heavy-fields affect the low energy evolution of adiabatic curvature perturbations. For instance, one may ask how would this picture change if not only one, but several massive fields interacted with the Goldstone boson parametrizing inflation.[5] The purpose of this chapter is to extend the previous body of work by deducing and analyzing the class of single field EFT's obtained in those cases where the Goldstone boson interacted with multiple heavy-fields, all of them representing fluctuations orthogonal to the trajectory. We have two main reasons to pursue this goal: First, we wish to know if the effects of heavy-fields on the low energy dynamics of curvature perturbations increase as the number of heavy-fields increases. In second place, we would like to understand in which way the new couplings, due to additional heavy-fields, would affect the Goldstone boson self-interactions.

With these two previous motivations in mind, we extend the analysis of a Goldstone boson interacting with a single heavy-field to the case in which it interacts with two heavy-fields. We compute the effective field theory obtained by integrating out the two heavy-fields and analyze the conditions for this limit to remain a fair description of the low energy dynamics of the system. We show that the existence of a third heavy-field indeed may imply larger effects on the low energy dynamics, and that its presence generally induces new self-interactions for the Goldstone boson that are not accounted for in the simpler case of a single heavy-field. Similar to the single-heavy-field case, these new couplings appear whenever the background trajectory in multi-field target space becomes non-geodesic. We find that low energy observables, such as the power spectrum and bispectrum, are sensitive to these

[5] Fundamental theories such as supergravity and string theory typically predict a large number of scalar fields, most of them expected to remain stabilized (heavy) during inflation. However, since in these theories scalar fields have a geometrical origin, it is still an open challenge to construct models of inflation where all the fields (other than the inflaton) remain stabilized [38, 56–59, 76].

couplings, and therefore future observations can be used to discern the number of heavy-fields with which the Goldstone boson interacted during inflation. In particular, we deduce that at long wavelengths, the effective action describing this class of models is of the form (7.1), with A generically constrained to be:

$$A \leqslant -\frac{1}{2}(1 - c_s^2). \tag{7.5}$$

This result implies that, under the assumption that during horizon crossing modes are parametrized by (7.1), future observations might rule out the existence of interactions between curvature perturbations and a large number of heavy fields.[6]

We have organized this chapter as follows: In Section 7.1 we present the basic setup to be studied and introduce the notation that will be used throughout our work to handle inflationary trajectories traversing a landscape of heavy-fields. In Section 7.2, we analyze the specific case in which the fields orthogonal to the inflationary trajectory are heavy enough that they can be integrated out. We analyze the full multi-field dynamics of this regime and deduce the effective field theory governing the low energy dynamics of the Goldstone boson fluctuations. Then, in Section 7.3 we discuss our results by analyzing the observational consequences of the resulting effective field theory for the Goldstone boson. Finally, in Section 7.4 we provide our concluding remarks.

7.1 Inflation in a heavy-field landscape

We commence by presenting the basic inflationary setup to be analyzed in the rest of this work. We are interested in studying inflationary systems with three scalar fields $\phi^a(t, x)$ (with $a = 1, 2, 3$) described by a generic action of the form

$$S_{\rm tot} = \frac{M_{\rm Pl}^2}{2} \int d^4x \sqrt{-g} R + S_{\rm scalar}, \tag{7.1}$$

where $M_{\rm Pl}$ stands for the Planck mass, R is the Ricci scalar constructed out of the metric $g_{\mu\nu}$ with a $(-, +, +, +)$ signature, and $S_{\rm scalar}$ represents the action for the scalar sector of the theory, given by

$$S_{\rm scalar} = -\frac{1}{2} \int d^4x \sqrt{-g} \left[g^{\mu\nu} \partial_\mu \phi^a \partial_\nu \phi_a + 2V(\phi) \right], \tag{7.2}$$

where $V(\phi)$ is the scalar field potential. Given that we are interested in a general model-independent analysis, we will not specify the dependance of the potential

[6] Another possibility is that modes crossed the horizon during the new physics regime, in which the Goldstone boson is described in terms of a modified dispersion relation $\omega \sim p^2$. In such case, one is forced to parametrize the period of horizon crossing with a different EFT incorporating operators with nontrivial scalings [30, 73].

$V(\phi)$ on the scalar fields ϕ^a. Instead, we shall only specify local properties of the potential along the background trajectory, consistent with the existence of heavy-fields interacting with the inflaton.

7.1.1 Background dynamics

We assume that the potential V is such that there exist homogeneous time-dependent solutions of the system in which the universe inflates. This, in turn, means that there exists a background scalar field trajectory in the 3-field target space, parametrized by t, hereby denoted by $\phi_0^a(t)$. Then, assuming a flat Friedmann-Robertson-Walker background metric of the form $ds^2 = -dt^2 + a^2 d\mathbf{x}^2$, the background equations of motion determining the trajectory $\phi_0^a(t)$ for the scalar fields are given by

$$\ddot{\phi}_0^a + 3H\dot{\phi}_0^a + V^a = 0, \qquad a = 1, 2, 3, \tag{7.3}$$

where $H \equiv \dot{a}/a$ is the usual Hubble expansion rate. These three equations need to be supplemented with Friedmann's equation which, in the present context, is found to be given by

$$3H^3 = \frac{1}{M_{\rm Pl}^2}\left(\frac{1}{2}\dot{\phi}_0^2 + V\right), \tag{7.4}$$

where $\dot{\phi}_0^2 \equiv \delta_{ab}\dot{\phi}_0^a\dot{\phi}_0^a$. Putting these two equations together, one deduces an additional equation relating the change of the expansion rate with the rapidity $\dot{\phi}_0$ of the scalar field along the trajectory:

$$\dot{H} = -\frac{\dot{\phi}_0^2}{2M_{\rm Pl}^2}. \tag{7.5}$$

To study the nontrivial aspects implied by a given path traversing the landscape, it is convenient to define a triad of unit vectors moving along with the trajectory, parametrized by t. We choose to work with a standard basis consisting of a tangent vector T^a, a normal vector N^a and a binormal vector B^a, all of them defined as

$$T^a = \dot{\phi}_0^a/\dot{\phi}_0, \tag{7.6}$$

$$N^a \propto \dot{T}^a, \tag{7.7}$$

$$B^a \propto (\delta_b^a - T^a T_b)\dot{N}^b, \tag{7.8}$$

with positive proportionality coefficients, such that vectors are normalised as $N^a N_a = B^a B_a = T^a T_a = 1$ (we rise and lower indices with δ^{ab} and δ_{ab} respectively). These vectors remain mutually orthogonal, and their time evolution may be parametrized by two angular velocities $\dot{\theta}$ and $\dot{\varphi}$, defined as:

$$\dot{T}^a = -\dot{\theta}N^a, \tag{7.9}$$

$$\dot{N}^a = \dot{\theta}T^a - \dot{\varphi}B^a, \tag{7.10}$$

$$\dot{B}^a = \dot{\varphi}N^a. \tag{7.11}$$

It may be seen that $\dot{\theta}$ is the rate of change of T^a along the direction $-N^a$, whereas $\dot{\varphi}$ is the rate of change of B^a along the direction $+N^a$. In other words, $\dot{\theta}$ is the angular velocity of the turning trajectory, whereas $\dot{\varphi}$ parametrizes how this turn spirals (see Figure 7.1). Having introduced this set of vectors [68, 70, 71], the background

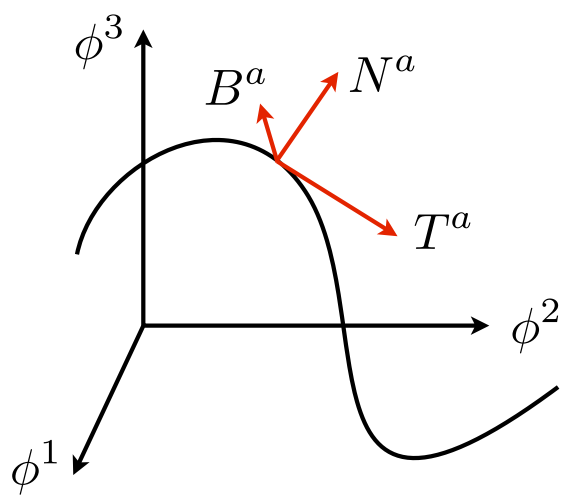

Figure 7.1: A schematic plot of the triad of vectors $\{T^a, N^a, B^a\}$ defined with respect to the background trajectory $\phi_0^a(t)$.

equations of motion (7.3) may be rewritten by projecting them along the three available directions. One obtains

$$\ddot{\phi}_0 + 3H\dot{\phi}_0 + V_T = 0, \tag{7.12}$$

$$\dot{\theta} = \frac{V_N}{\dot{\phi}_0}, \tag{7.13}$$

$$B^a V_a = 0, \tag{7.14}$$

where we have defined the projections $V_T \equiv T^a V_a$ and $V_N \equiv N^a V_a$. The first equation (7.12) is nothing but the usual equation of motion for a single-field background, with the inflaton rolling down a potential of slope V_T. Using (7.12) and (7.4) we may now characterise the inflationary dynamics in terms of slow roll parameters as usual. That is, by defining the following dimensionless slow roll parameters

$$\varepsilon = -\frac{\dot{H}}{H^2}, \qquad \eta = -\frac{\ddot{\phi}_0}{H\dot{\phi}_0}, \qquad \xi = -\frac{\dddot{\phi}_0}{H\ddot{\phi}_0}, \tag{7.15}$$

one deduces from (7.4) and (7.12) the following relations among these quantities,

$$\varepsilon = \frac{M_{\rm Pl}^2}{2}\left(\frac{V_T}{V}\right)^2\left(\frac{3-\varepsilon}{3-\eta}\right)^2, \tag{7.16}$$

$$3(\varepsilon + \eta) = M_{\rm Pl}^2 \frac{V_{TT}}{V}(3-\varepsilon) + \xi\eta, \tag{7.17}$$

where $V_{TT} \equiv T^a \nabla_a V_T \equiv T^a \nabla_a (T^a \nabla_a V)$. Slow roll inflation will persist as long as $\varepsilon \ll 1$, $\eta \ll 1$ and $\xi \ll 1$ hold. With (7.16) and (7.17) these slow-roll conditions are seen to be equivalent to[7]

$$\varepsilon = \frac{M_{\rm Pl}^2}{2} \left(\frac{V_T}{V} \right)^2, \qquad \varepsilon + \eta = M_{\rm Pl}^2 \frac{V_{TT}}{V}, \tag{7.18}$$

which further translates into restrictions on the shape of the potential along the trajectory. At this point, it is very important to emphasise that these slow roll conditions only imply restrictions on background quantities along the trajectory, but tell us nothing about the turns of the trajectory. As discussed in full detail in refs. [44] and [2], in the case of two-field models of inflation, it is perfectly possible to have sudden turns with $\dot{\theta} \gg H$ without implying a violation of the aforementioned slow-roll conditions. The same arguments can be used to state that, in the case of three-field models of inflation, one can have $\dot{\theta} \gg H$ and $\dot{\varphi} \gg H$ simultaneously, without necessarily violating slow-roll whatsoever.

7.1.2 Perturbation dynamics

We now move on to consider perturbations about an arbitrary inflationary trajectory. A convenient way of studying scalar fluctuations without specifying the inflationary model, is by introducing the Goldstone boson π as the fluctuation along the direction of broken time translation symmetry [5]. In the present context, this is precisely equivalent to define the Goldstone boson as the fluctuation along the inflationary trajectory.[8] In addition to the Goldstone boson, there are two other scalar field fluctuations, hereby called $\mathcal{F}_1$ and $\mathcal{F}_2$, which denote fluctuations away from the trajectory, along the two available directions N^a and B^a. The definition of these three scalar fluctuations may be summarized by writing the complete set of scalar fields $\phi^a(t, x)$ in terms of the background fields $\phi_0^a(t)$, and the vectors $N^a(t)$ and $B^a(t)$ as:

$$\phi^a(t, x) \equiv \phi_0^a(t + \pi) + N^a(t + \pi)\mathcal{F}_1 + B^a(t + \pi)\mathcal{F}_2. \tag{7.19}$$

Notice that $\pi(t, x)$ appears through the replacement $t \to t + \pi(t, x)$ in the argument of background quantities. To deal with the gravitational sector, we may adopt the Arnowitt-Deser-Misner (ADM) formalism [15] to parametrize space-time, requiring that we write the metric as

$$ds^2 = -N^2 dt^2 + \gamma_{ij}(N^i dt + dx^i)(N^j dt + dx^j), \tag{7.20}$$

[7]Notice that with definition (7.15), the usual η_V-parameter defined in terms of the second derivative of the potential is given by $\eta_V = \varepsilon + \eta$.

[8]This is simply because the inflationary trajectory consists of a path parametrized by t.

where N and N^i are the lapse and shift functions (here playing the role of Lagrange multipliers) and γ_{ij} is the induced metric describing the 3-D spatial foliations parametrized by t. In terms of these quantities, the components of the metric $g_{\mu\nu}$ and its inverse $g^{\mu\nu}$ are given by

$$\begin{aligned} g_{00} &= -N^2 + \gamma_{ij}N^iN^j, \quad g_{0i} = \gamma_{ij}N^j, \quad g_{ij} = \gamma_{ij}, \\ g^{00} &= -\frac{1}{N^2}, \quad g^{0i} = \frac{N^i}{N^2} \quad g^{ij} = \gamma^{ij} - \frac{N^iN^j}{N^2}, \end{aligned} \tag{7.21}$$

where γ^{ij} is the inverse of γ_{ij}. Moreover, we adopt the flat gauge, in which the spatial metric γ_{ij} takes the form:

$$\gamma_{ij} = a^2\delta_{ij}. \tag{7.22}$$

To obtain the action for the perturbations, we may now introduce the parametrization (7.19) for $\phi^a(t,x)$ back into the action (7.1). The result is given by the following full action, including background fields and fluctuations:

$$\begin{aligned} S &= \int d^4x \frac{Na^3}{2}\Big\{ -\frac{6M_{\rm Pl}^2H^2}{N^2} + \frac{4M_{\rm Pl}^2H}{N^2}N^i{}_{,i} + \frac{M_{\rm Pl}^2}{2N^2}\left(N^i{}_{,j}N^j{}_{,i} + \delta_{ij}N^{i,k}N^j{}_{,k} - 2N^i{}_{,i}N^j{}_{,j}\right) \\ &+\frac{1}{N^2}\left[(\dot\phi_0 + \dot\theta\mathcal{F}_1)^2 + \dot\varphi^2(\mathcal{F}_1^2+\mathcal{F}_2^2)\right]\left[\left(1+\dot\pi - N^i\pi_{,i}\right)^2 - \frac{N^2}{a^2}\left(\nabla\pi\right)^2\right] \\ &+\frac{2\dot\varphi}{N^2}\left(1+\dot\pi - N^i\pi_{,i}\right)\left[\mathcal{F}_2(\dot{\mathcal{F}}_1 - N^i\mathcal{F}_{1,i}) - \mathcal{F}_1(\dot{\mathcal{F}}_2 - N^i\mathcal{F}_{2,i})\right] \\ &-\frac{2\dot\varphi}{a^2}\nabla\pi\left[\mathcal{F}_2\nabla\mathcal{F}_1 - \mathcal{F}_1\nabla\mathcal{F}_2\right] + \frac{1}{N^2}\left(\dot{\mathcal{F}}_1 - N^i\mathcal{F}_{1,i}\right)^2 + \frac{1}{N^2}\left(\dot{\mathcal{F}}_2 - N^i\mathcal{F}_{2,i}\right)^2 \\ &-\frac{(\nabla\mathcal{F}_1)^2}{a^2} - \frac{(\nabla\mathcal{F}_2)^2}{a^2} - 2V\left(\phi_0^a + N^a\mathcal{F}_1 + B^a\mathcal{F}_2\right)\Big\}. \end{aligned} \tag{7.23}$$

To deal with this action, we need to solve the constraint equations for N and N^i. To simplify this, we set ourselves to obtain the action for π, $\mathcal{F}_1$ and $\mathcal{F}_2$ only up to cubic order in the fields. This implies that it is only necessary to solve the constraint equations up to linear order in $N-1$ and N^i. Then, by writing $N = 1+\delta N$ and $N^i = \partial^i\psi + v^i$, with $\partial_i v^i = 0$, we find the solutions

$$v^i = 0, \tag{7.24}$$

$$\delta N = \varepsilon H\pi, \tag{7.25}$$

$$\frac{\Delta}{a^2}\psi = -\varepsilon H(\dot\pi - \varepsilon H\pi) - \frac{\dot\theta\dot\phi_0}{HM_{\rm Pl}^2}\mathcal{F}_1. \tag{7.26}$$

Replacing these expressions back into (7.23) we obtain the full action for the fluctuations up to cubic order. However, because we are interested in studying inflation in the slow roll retime, where $\varepsilon \ll 1$, we are allowed to consider the decoupling limit, where the gravitational effects implied by δN and N^i on the evolution of the Goldstone boson become negligible. More specifically, in the regime where the Goldstone boson fluctuations carry energies $\omega \gg \Lambda_{\rm dec} \sim \varepsilon H$ one may drop the couplings coming from the constraint solutions (7.25) and (7.26), which otherwise imply terms of order ε. This step leads us to consider the following action valid at the decoupling limit $\omega \gg \Lambda_{\rm dec}$

$$
\begin{aligned}
S_{\rm dec} \;=\; & \frac{1}{2}\int d^4x a^3 \Big\{ (\dot\phi_0 + \dot\theta \mathcal{F}_1)^2 \left[\dot\pi^2 - \frac{1}{a^2}(\nabla\pi)^2 \right] + 2\dot\theta \left(2\dot\phi_0 + \dot\theta\mathcal{F}_1 \right) \mathcal{F}_1 \dot\pi \\
& + \dot\varphi^2 (\mathcal{F}_1^2 + \mathcal{F}_2^2) \left[2\dot\pi + \dot\pi^2 - \frac{1}{a^2}(\nabla\pi)^2 \right] + 2\dot\varphi\,(1+\dot\pi) \left[\mathcal{F}_2 \dot{\mathcal{F}}_1 - \mathcal{F}_1 \dot{\mathcal{F}}_2 \right] \\
& - \frac{2\dot\varphi}{a^2} \nabla\pi \left[\mathcal{F}_2 \nabla\mathcal{F}_1 - \mathcal{F}_1 \nabla\mathcal{F}_2 \right] + \dot{\mathcal{F}}_1^2 + \dot{\mathcal{F}}_2^2 - \frac{(\nabla\mathcal{F}_1)^2}{a^2} - \frac{(\nabla\mathcal{F}_2)^2}{a^2} \\
& - \sum_{ij} \mathcal{M}_{ij} \mathcal{F}_i \mathcal{F}_j - \sum_{ij} \mathcal{C}_{ijk} \mathcal{F}_i \mathcal{F}_j \mathcal{F}_k \Big\} ,
\end{aligned}
\tag{7.27}
$$

where the mass matrix $\mathcal{M}^2_{ij}$ is found to have elements given by:

$$
\mathcal{M}^2 = \begin{pmatrix} V_{NN} - \dot\theta^2 - \dot\varphi^2 & V_{NB} \\ V_{NB} & V_{BB} - \dot\varphi^2 \end{pmatrix} . \tag{7.28}
$$

In this expression, $V_{NB} \equiv N^a B^b V_{ab} \equiv B^a N^b V_{ab}$. In addition, the cubic term proportional to $\mathcal{C}_{ijk}$ appears from third derivatives of the potential V away from the inflationary trajectory. It is worth noting that at quadratic order the Goldstone boson only interacts with the isocurvature field $\mathcal{F}_1$, which is precisely due to the parametrisation of the inflationary trajectory in terms of the triad (7.6)-(7.8). Because this triad is aligned with respect to the trajectory (and not with respect to the mass matrix of the fields $\mathcal{M}^2$) in general we expect the existence of non-vanishing off-diagonal terms $\mathcal{M}^2_{12} = \mathcal{M}^2_{21} \neq 0$.

For completeness, we write down the equations of motion for the fluctuations deduced by varying the action (7.27) with respect to the three fields, π, $\mathcal{F}_1$ and $\mathcal{F}_2$. First, the equation of motion for the Goldstone boson π is found to be:

$$
\begin{aligned}
& \frac{1}{a^3}\frac{d}{dt}\left[a^3 \left((\dot\phi_0 + \dot\theta\mathcal{F}_1)^2 + \dot\varphi^2 (\mathcal{F}_1^2 + \mathcal{F}_2^2) \right) \dot\pi \right] - \frac{1}{a^2}\nabla \left[\left((\dot\phi_0 + \dot\theta\mathcal{F}_1)^2 + \dot\varphi^2(\mathcal{F}_1^2 + \mathcal{F}_2^2) \right) \nabla\pi \right] \\
& = -\frac{d}{dt}\left[\dot\theta \left(2\dot\phi_0 + \dot\theta\mathcal{F}_1 \right) \mathcal{F}_1 + \dot\varphi^2 (\mathcal{F}_1^2 + \mathcal{F}_2^2) + \dot\varphi \left(\mathcal{F}_2 \dot{\mathcal{F}}_1 - \mathcal{F}_1 \dot{\mathcal{F}}_2 \right) \right] \\
& - \frac{\dot\varphi}{a^2} \nabla \left[\mathcal{F}_2 \nabla\mathcal{F}_1 - \mathcal{F}_1 \nabla\mathcal{F}_2 \right] .
\end{aligned}
\tag{7.29}
$$

The equation of motion for the heavy-field $\mathcal{F}_1$ is found to be:

$$\ddot{\mathcal{F}}_1 + 3H\dot{\mathcal{F}}_1 - \frac{\nabla^2}{a^2}\mathcal{F}_1 + \mathcal{M}_{11}^2\mathcal{F}_1 - (\dot{\varphi}^2 + \dot{\theta}^2)\left[2\dot{\pi} + \dot{\pi}^2 - \frac{1}{a^2}\left(\nabla\pi\right)^2\right]\mathcal{F}_1$$
$$= \dot{\theta}\dot{\phi}_0\left[2\dot{\pi} + \dot{\pi}^2 - \frac{1}{a^2}\left(\nabla\pi\right)^2\right] - \mathcal{M}_{12}^2\mathcal{F}_2 - 2\dot{\varphi}(1+\dot{\pi})\dot{\mathcal{F}}_2$$
$$-3H\dot{\varphi}\mathcal{F}_2 - \ddot{\varphi}(1+\dot{\pi})\mathcal{F}_2 + 2\frac{\dot{\varphi}}{a^2}\nabla\pi\nabla\mathcal{F}_2 - \dot{\varphi}\left[\ddot{\pi} + 3H\dot{\pi} - \frac{1}{a^2}\nabla^2\pi\right]\mathcal{F}_2. \quad (7.30)$$

And finally, the equation of motion for the heavy-field $\mathcal{F}_2$ is found to be:

$$\ddot{\mathcal{F}}_2 + 3H\dot{\mathcal{F}}_2 - \frac{\nabla^2}{a^2}\mathcal{F}_2 + \mathcal{M}_{22}^2\mathcal{F}_2 - \dot{\varphi}^2\left[2\dot{\pi} + \dot{\pi}^2 - \frac{1}{a^2}\left(\nabla\pi\right)^2\right]\mathcal{F}_2 = -\mathcal{M}_{21}^2\mathcal{F}_1 + 2\dot{\varphi}(1+\dot{\pi})\dot{\mathcal{F}}_1$$
$$+3H\dot{\varphi}\mathcal{F}_1 + \ddot{\varphi}(1+\dot{\pi})\mathcal{F}_1 - 2\frac{\dot{\varphi}}{a^2}\nabla\pi\nabla\mathcal{F}_1 + \dot{\varphi}\left[\ddot{\pi} + 3H\dot{\pi} - \frac{1}{a^2}\nabla^2\pi\right]\mathcal{F}_1. \quad (7.31)$$

In agreement with the analysis of ref. [5], the previous equations are consistent with the particular solution π =constant, and $\mathcal{F}_1 = \mathcal{F}_2 = 0$, which is reached shortly after horizon crossing.

7.1.3 The linear regime

We now examine the evolution of fluctuations in the linear regime, paying special attention to their dynamics on sub-horizon scales (*i.e.* when the the wavelength of perturbations is shorter than the de Sitter radius H^{-1}). Keeping linear terms in eqs. (7.29)-(7.31), and expressing them in Fourier space, we obtain

$$\ddot{\pi} + 3H\dot{\pi} - \frac{k^2}{a^2}\pi = -\frac{2}{\dot{\phi}_0}\left[\dot{\theta}\dot{\mathcal{F}}_1 + \ddot{\theta}\mathcal{F}_1\right], \quad (7.32)$$
$$\ddot{\mathcal{F}}_1 + 3H\dot{\mathcal{F}}_1 - \frac{k^2}{a^2}\mathcal{F}_1 + \mathcal{M}_{11}^2\mathcal{F}_1 = 2\dot{\theta}\dot{\phi}_0\dot{\pi} - \mathcal{M}_{12}^2\mathcal{F}_2 - 2\dot{\varphi}\dot{\mathcal{F}}_2 - 3H\dot{\varphi}\mathcal{F}_2 - \ddot{\varphi}\mathcal{F}_2, \quad (7.33)$$
$$\ddot{\mathcal{F}}_2 + 3H\dot{\mathcal{F}}_2 - \frac{k^2}{a^2}\mathcal{F}_2 + \mathcal{M}_{22}^2\mathcal{F}_2 = -\mathcal{M}_{21}^2\mathcal{F}_1 + 2\dot{\varphi}\dot{\mathcal{F}}_1 + 3H\dot{\varphi}\mathcal{F}_1 + \ddot{\varphi}\mathcal{F}_1, \quad (7.34)$$

where we have also dropped terms suppressed by the slow roll parameters, to stay consistent with the decoupling limit. Recall that the triad $\{T^a, N^a, B^a\}$ has been chosen so that it remains aligned with the inflationary trajectory, as in eqs. (7.6)-(7.8). As a consequence, at linear order the Goldstone boson π remains coupled only to the isocurvature field $\mathcal{F}_1$, with the strength of the coupling determined by the value of $\dot{\theta}$. On the other hand, the coupling between the isocurvature mode $\mathcal{F}_1$ and the binormal mode $\mathcal{F}_2$ is determined by the combination $\mathcal{M}_{12}^2 \pm 2\dot{\varphi}\partial_t$ (with the sign depending on the field ∂_t acts upon). The mass matrix (7.28) is fixed by the choice of this basis, and any attempt to diagonalize it will change this interaction structure by coupling $\mathcal{F}_2$ with π. Thus, in general, we expect a non-vanishing value of $\mathcal{M}_{12}^2$ even in the absence of spiraling turns ($\dot{\varphi} = 0$).

To learn more about the kinematical structure of the system, we disregard time derivatives of $\dot{\theta}$ and $\dot{\varphi}$ and focus our attention on sub-horizon modes, with $p \equiv k/a \gg H$. Then, the previous equations simplify to

$$\ddot{\pi}_c + p^2 \pi_c = -2\dot{\theta}\dot{\mathcal{F}}_1, \tag{7.35}$$

$$\ddot{\mathcal{F}}_1 + p^2 \mathcal{F}_1 + \mathcal{M}_{11}^2 \mathcal{F}_1 = 2\dot{\theta}\dot{\pi}_c - \mathcal{M}_{12}^2 \mathcal{F}_2 - 2\dot{\varphi}\dot{\mathcal{F}}_2, \tag{7.36}$$

$$\ddot{\mathcal{F}}_2 + p^2 \mathcal{F}_2 + \mathcal{M}_{22}^2 \mathcal{F}_2 = -\mathcal{M}_{21}^2 \mathcal{F}_1 + 2\dot{\varphi}\dot{\mathcal{F}}_1, \tag{7.37}$$

where $\pi_c = \dot{\phi}_0 \pi$ is the canonically normalised Goldstone boson. Notice that since $p \equiv k/a \gg H$, one has $|\dot{p}|/p^2 \ll 1$, implying that we may consider the adiabatic approximation whereby p is treated as a constant. Then, by assuming the ansatz $\pi, \mathcal{F}_1, \mathcal{F}_2 \propto e^{-i\omega}$, the previous eqs. (7.35)-(7.37) take the form

$$\Omega \begin{pmatrix} \pi_c \\ \mathcal{F}_1 \\ \mathcal{F}_2 \end{pmatrix} = 0. \tag{7.38}$$

where the frequency matrix Ω is given by:

$$\Omega \equiv \begin{pmatrix} -\omega^2 + p^2 & -2i\dot{\theta}\omega & 0 \\ 2i\dot{\theta}\omega & -\omega^2 + p^2 + \mathcal{M}_{11}^2 & \mathcal{M}_{12}^2 - 2i\dot{\varphi}\omega \\ 0 & \mathcal{M}_{21}^2 + 2i\dot{\varphi}\omega & -\omega^2 + p^2 + \mathcal{M}_{22}^2 \end{pmatrix}. \tag{7.39}$$

To solve these equations, we must demand $\det \Omega = 0$, which determines the following cubic algebraic equation for ω:

$$(p^2 - \omega^2)(\mathcal{M}_{12}^4 + 4\dot{\varphi}^2\omega^2) - (\mathcal{M}_{22}^2 + p^2 - \omega^2)\left(p^2\mathcal{M}_{11}^2 + p^4 - (\mathcal{M}_{11}^2 + 2p^2 + 4\dot{\theta}^2)\omega^2 + \omega^4\right) = 0. \tag{7.40}$$

For $p = 0$ one of the solutions corresponds to the case $\omega = 0$. This is consistent with the fact that $\pi =$ constant and $\mathcal{F}_1 = \mathcal{F}_2 = 0$ is a solution of the system, and implies that there is a massless mode (to be identified as the Goldstone boson mode). Then, expressing the three frequencies about $p = 0$, we find

$$\omega_{\text{light}}^2 = c_s^2 p^2 + \frac{(1 - c_s^2)^2}{\det \mathcal{M}^2 c_s^{-2}} \left[\mathcal{M}_{22}^2 + \frac{\mathcal{M}_{12}^4}{\mathcal{M}_{22}^2} - \frac{4c_s^2\dot{\varphi}^2}{1 - c_s^2}\right] p^4 + \mathcal{O}(p^6), \tag{7.41}$$

$$\omega_{\text{I}}^2 = \frac{\text{tr}\mathcal{M}^2 + 4(\dot{\theta}^2 + \dot{\varphi}^2)}{2} - \frac{1}{2}\sqrt{\left[\text{tr}\mathcal{M}^2 + 4(\dot{\theta}^2 + \dot{\varphi}^2)\right]^2 - 4\det\mathcal{M}^2\, c_s^{-2}}, \tag{7.42}$$

$$\omega_{\text{II}}^2 = \frac{\text{tr}\mathcal{M}^2 + 4(\dot{\theta}^2 + \dot{\varphi}^2)}{2} + \frac{1}{2}\sqrt{\left[\text{tr}\mathcal{M}^2 + 4(\dot{\theta}^2 + \dot{\varphi}^2)\right]^2 - 4\det\mathcal{M}^2\, c_s^{-2}}, \tag{7.43}$$

where we have defined the speed of sound c_s via the relation:

$$\frac{1}{c_s^2} = 1 + \frac{4\dot{\theta}^2\mathcal{M}_{22}^2}{\det \mathcal{M}^2}. \tag{7.44}$$

Notice that we have dropped the p-dependence of $\omega_{\rm I}$ and $\omega_{\rm II}$ for simplicity. In addition, notice that $\omega_{\rm I} \leqslant \omega_{\rm II}$ by definition. A direct check of these relations shows that in the limit $\mathcal{M}_{12}^2 = \dot{\varphi}^2 = 0$ we recover the case in which only one massive field interacts with the Goldstone boson [2]:

$$\omega_{\rm light}^2 = c_s^2 p^2 + \left(1 - c_s^2\right)^2 \frac{p^4}{\mathcal{M}_{11}^2 c_s^{-2}} + \mathcal{O}(p^6) \tag{7.45}$$

$$\omega_{\rm I}^2 = \mathcal{M}_{11}^2 + 4\dot{\theta}^2 = \mathcal{M}_{11}^2 c_s^{-2} \tag{7.46}$$

$$\omega_{\rm II}^2 = \mathcal{M}_{22}^2, \tag{7.47}$$

where we have assumed $\mathcal{M}_{11}^2 + 4\dot{\theta}^2 < \mathcal{M}_{22}^2$ for definiteness (otherwise we would have obtained the inverted relations $\omega_{\rm I}^2 = \mathcal{M}_{22}^2$ and $\omega_{\rm II}^2 = \mathcal{M}_{11}^2 + 4\dot{\theta}^2$).

In general, we see that the coupled system of equations (7.38) imply that the fields π, $\mathcal{F}_1$ and $\mathcal{F}_2$ are linear combinations of modes with frequencies $\omega_{\rm light}$, $\omega_{\rm I}$ and $\omega_{\rm II}$ in the following form

$$\pi = \pi_{\rm light} e^{-i\omega_{\rm light} t} + \pi_{\rm I} e^{-i\omega_{\rm I} t} + \pi_{\rm II} e^{-i\omega_{\rm II} t}, \tag{7.48}$$

$$\mathcal{F}_1 = \mathcal{F}_{\text{1-light}} e^{-i\omega_{\rm light} t} + \mathcal{F}_{\rm 1I} e^{-i\omega_{\rm I} t} + \mathcal{F}_{\rm 1II} e^{-i\omega_{\rm II} t}, \tag{7.49}$$

$$\mathcal{F}_2 = \mathcal{F}_{\text{2-light}} e^{-i\omega_{\rm light} t} + \mathcal{F}_{\rm 2I} e^{-i\omega_{\rm I} t} + \mathcal{F}_{\rm 2II} e^{-i\omega_{\rm II} t}. \tag{7.50}$$

The amplitudes $\pi_{\rm light}$, $\pi_{\rm I}$, $\pi_{\rm II}$, $\mathcal{F}_{\text{1-light}}$, $\mathcal{F}_{\rm 1I}$, $\mathcal{F}_{\rm 1II}$, $\mathcal{F}_{\text{2-light}}$, $\mathcal{F}_{\rm 2I}$ and $\mathcal{F}_{\rm 2II}$ are all functions of p, and determined trivially by (7.38) except three normalization coefficients, that may be fixed by quantizing the theory. In the particular case where the inflationary trajectory is not subject to turns (*i.e.* $\dot{\theta} = \dot{\varphi} = 0$), the matrix of eq. (7.38) becomes diagonal, and only $\pi_{\rm light}$, $\mathcal{F}_{\rm 1I}$, and $\mathcal{F}_{\rm 2II}$ remain non-vanishing. In such a case, assuming that $\mathcal{M}_{11}^2 \leqslant \mathcal{M}_{22}^2$, the frequencies reduce to

$$\omega_{\rm light}^2 = p^2, \qquad \omega_{\rm I}^2 = \mathcal{M}_{11}^2, \qquad \omega_{\rm II}^2 = \mathcal{M}_{22}^2, \tag{7.51}$$

and there is a one to one correspondence between frequencies and fields. However, it is important to emphasize that in the presence of turns (*i.e.* $\dot{\theta} \neq 0$ and $\dot{\varphi} \neq 0$) there will always be a mixing between fields and modes, implying non-trivial consequences for the dynamics of the low energy Goldstone boson, as we shall verify in the following section.

7.2 Effective field theory

In the previous section we analysed the dynamics of inflationary systems with three scalar fields, which may be understood in terms of a Goldstone boson interacting with two massive scalar fields. We now move on to consider the case in which these two massive fields remain heavy, and therefore contribute with heavy degrees of freedom to the particle content of the theory. Such a regime exists only for wavelengths such that the frequency of the light mode is found to be much smaller than the frequencies of the two heavy degrees of freedom:

$$\omega^2_{\rm light} \ll \omega^2_{\rm I} \leqslant \omega^2_{\rm II}. \tag{7.1}$$

As long as this condition is satisfied, the creation of high-energy quanta of energies $\omega_{\rm I}$ and $\omega_{\rm II}$ will remain kinematically precluded to processes involving low-energy degrees of freedom characterized by $\omega_{\rm light}$. Thus $\omega_{\rm I}$ constitutes the cut-off energy scale defining the validity of the effective field theory for low energy modes of frequency $\omega_{\rm light}$. However, because $\omega_{\rm I}$ and $\omega_{\rm II}$ depend on time-dependent background quantities, eq. (7.1) needs to be complemented with the additional adiabaticity conditions [2]

$$\frac{|\dot\omega_{\rm I}|}{\omega^2_{\rm I}} \ll 1, \qquad \frac{|\dot\omega_{\rm II}|}{\omega^2_{\rm II}} \ll 1, \tag{7.2}$$

ensuring that high-frequency quanta will not be excited by strong sudden turns of the background inflationary trajectory.

7.2.1 Preliminaries

In what follows we deduce the effective field theory describing the dynamics of the low energy modes characterized by the frequency $\omega_{\rm light}$, subject to the hierarchy (7.1). First, because there is a large number of parameters involved in the definition of both $\omega^2_{\rm I}$ and $\omega^2_{\rm II}$, we need to make some simplifying assumptions about their values. To start with, we assume that both $\omega^2_{\rm I}$ and $\omega^2_{\rm II}$ are of the same order. By inspecting eqs. (7.42) and (7.43) we see that this condition implies that the cutoff scale is of order

$$\Big[{\rm tr}\mathcal{M}^2 + 4(\dot\theta^2 + \dot\varphi^2)\Big]^2 \sim 4\,{\rm det}\mathcal{M}^2\, c_s^{-2}. \tag{7.3}$$

in order to avoid a hierarchy between $\omega^2_{\rm I}$ and $\omega^2_{\rm II}$. In second place, we only consider inflationary trajectories where $\dot\varphi$ is at most of order $\dot\theta$. Putting together these two assumptions, one finds that both $\mathcal{M}^2_{22}$ and $\mathcal{M}^2_{11} + 4\dot\theta^2$ are of the same order as the cutoff scale $\Lambda^2_{\rm UV} = \omega^2_{\rm I}$ of the effective field theory:

$$\mathcal{M}^2_{22} \sim \mathcal{M}^2_{11} + 4\dot\theta^2 \sim \Lambda^2_{\rm UV}. \tag{7.4}$$

It is important to realize that under the present assumptions, $\mathcal{M}^2_{11}$ and $\dot\theta^2$ are not necessarily of the same order, and a hierarchy among their values is perfectly possible [44].

Next, we may anticipate the range of validity of the low energy EFT in terms of the momentum carried by the fluctuations. For this, we see that the EFT will remain valid as long as $\omega^2_{\rm light} \ll \Lambda^2_{\rm UV}$. Then, noticing from (7.45) and (7.4) that the dispersion relation for the light mode is of the general form

$$\omega^2_{\rm light} \sim c_s^2 p^2 + \frac{(1-c_s^2)^2}{\Lambda^2_{\rm UV}} p^4 + \mathcal{O}(p^6/\Lambda^4_{\rm UV}), \tag{7.5}$$

we see that, independently of the value of c_s, the effective field theory is valid as long as the wavelength

$$p^2 \ll \Lambda^2_{\rm UV}. \tag{7.6}$$

Finally, we argue that the term proportional to $\dot{\varphi}^2$ appearing in the light mode dispersion relation (7.41), is always subleading when compared to any other term in the expression. Indeed, from eq. (7.5) we see that the contribution quartic in p dominates only if $c_s^2 \ll 1$, in which case the contribution due to $\dot{\varphi}^2$ will be suppressed by a factor c_s^2 against the remaining term $\mathcal{M}^2_{22} + \mathcal{M}^4_{12}/\mathcal{M}^2_{22}$ (recall that we are taking $\dot{\varphi}^2$ at most of order $\sim \dot{\theta}^2$). Thus, we are allowed to take

$$\omega^2_{\rm light} = c_s^2 p^2 + \frac{(1-c_s^2)^2}{\det \mathcal{M}^2 c_s^{-2}} \left[\mathcal{M}^2_{22} + \frac{\mathcal{M}^4_{12}}{\mathcal{M}^2_{22}} \right] p^4 + \mathcal{O}(p^6/\Lambda^4_{\rm UV}), \tag{7.7}$$

as the dispersion relation for the light mode, with terms of order $\mathcal{O}(p^6)$ always subleading [73].

7.2.2 Computation of the effective field theory

We are now ready to compute the desired effective field theory. We will do this by expressing both heavy-fields, $\mathcal{F}_1$ and $\mathcal{F}_2$ in terms of the light Goldstone boson π, with the help of the equations of motion (7.30) and (7.31). The following two considerations will help in this task:

- We first notice that the absence of $\dot{\varphi}$ in the dispersion relation (7.7) allows us to drop any term containing $\dot{\varphi}$ in (7.30) and (7.31), as long as it is linear in the fields. However, we must keep $\dot{\varphi}$ in those terms which are of higher order in the fields.
- In addition, the modified dispersion relation (7.7) is consistent with $\omega^2 \ll p^2 + \mathcal{M}^2_{11}$ and $\omega^2 \ll p^2 + \mathcal{M}^2_{22}$ for all values of p up to the cutoff scale $\Lambda_{\rm UV}$. This means that we can drop the second time derivatives $\ddot{\mathcal{F}}_1$ and $\ddot{\mathcal{F}}_2$ in the equations of motion (7.30) and (7.31) respectively.

To appreciate the relevance of these two points more clearly, we may analyze their effects when applied to the linear equations of motion (7.38) valid at sub-horizon scales. In this case, the matrix Ω is found to be:

$$\Omega \equiv \begin{pmatrix} -\omega^2 + p^2 & -2i\dot{\theta}\omega & 0 \\ 2i\dot{\theta}\omega & p^2 + \mathcal{M}^2_{11} & \mathcal{M}^2_{12} \\ 0 & \mathcal{M}^2_{21} & p^2 + \mathcal{M}^2_{22} \end{pmatrix}, \tag{7.8}$$

from where it is straightforward to deduce the following dispersion relation for the light mode:

$$\omega^2_{\text{light}} = c_s^2 p^2 + \frac{(1-c_s^2)}{\det \mathcal{M}^2 c_s^{-2}} \left[\mathcal{M}_{22}^2 + \frac{\mathcal{M}_{12}^4}{\mathcal{M}_{22}^2} \right] p^4 + \mathcal{O}(p^6). \tag{7.9}$$

The only difference between this expression and that found in (7.7) is a missing extra factor $(1-c_s^2)$ in front of the quartic term of (7.9). This comes from having neglected the second time derivatives of the heavy-fields (see ref. [73] for a detailed explanation of this in the case of a single heavy-field). However, this difference is marginal, as the quartic term is only relevant if the speed of sound is suppressed ($c_s^2 \ll 1$).

Next, we write the equations of motion at most linear in the heavy-fields $\mathcal{F}_1$ and $\mathcal{F}_2$, but to quadratic order in π (this will allow us to consistently deduce an EFT action for π valid to cubic order in π):

$$\left[-\frac{\nabla^2}{a^2} + \mathcal{M}_{11}^2 - 2\dot{\pi}(\dot{\varphi}^2 + \dot{\theta}^2) \right] \mathcal{F}_1 = \dot{\theta}\dot{\phi}_0 \left[2\dot{\pi} + \dot{\pi}^2 - \frac{1}{a^2} (\nabla\pi)^2 \right] - \mathcal{M}_{12}^2 \mathcal{F}_2, \tag{7.10}$$

$$\left[-\frac{\nabla^2}{a^2} + \mathcal{M}_{22}^2 - 2\dot{\pi}\dot{\varphi}^2 \right] \mathcal{F}_2 = -\mathcal{M}_{21}^2 \mathcal{F}_1. \tag{7.11}$$

Since we are neglecting second order time derivatives, these equations may be interpreted as constraint equations for the Lagrange multipliers $\mathcal{F}_1$ and $\mathcal{F}_2$. As such, they automatically provide the low energy evolution of the heavy-fields $\mathcal{F}_1$ and $\mathcal{F}_2$ as sourced by the Goldstone boson π. The solution to these equations are given by

$$\mathcal{F}_1 = \Omega_2 \frac{\dot{\theta}\dot{\phi}_0}{\Omega_1\Omega_2 - \mathcal{M}_{21}^4} \left[2\dot{\pi} + \dot{\pi}^2 - \frac{1}{a^2} (\nabla\pi)^2 \right], \tag{7.12}$$

$$\mathcal{F}_2 = -\dot{\theta}\dot{\phi}_0 \frac{\mathcal{M}_{21}^2}{\Omega_1\Omega_2 - \mathcal{M}_{21}^4} \left[2\dot{\pi} + \dot{\pi}^2 - \frac{1}{a^2} (\nabla\pi)^2 \right], \tag{7.13}$$

where the operators Ω_1 and Ω_2 are defined as:

$$\Omega_1 \equiv \left[-\frac{\nabla^2}{a^2} + \mathcal{M}_{11}^2 - 2\dot{\pi}(\dot{\varphi}^2 + \dot{\theta}^2) \right], \tag{7.14}$$

$$\Omega_2 \equiv \left[-\frac{\nabla^2}{a^2} + \mathcal{M}_{22}^2 - 2\dot{\pi}\dot{\varphi}^2 \right]. \tag{7.15}$$

Replacing the solutions (7.12) and (7.13) back into the full action (7.27), and consistently dropping those terms in the action the led to disregarded terms in the equations of motion, we are led to the single-field Goldstone-boson action in the decoupling limit:

$$
\begin{aligned}
S_{\rm EFT} &= \frac{1}{2}\int d^4x a^3 \dot\phi_0^2 \Bigg\{ \left[\dot\pi^2 - \frac{1}{a^2}(\nabla\pi)^2\right] + 4\dot\theta^2 \dot\pi \frac{\mathcal{M}_{22}^2 - \nabla^2/a^2}{(\mathcal{M}_{11}^2 - \nabla^2/a^2)(\mathcal{M}_{22}^2 - \nabla^2/a^2) - \mathcal{M}_{21}^4}\dot\pi \\
&+8\dot\theta^2\dot\varphi^2 \left[\dot\pi \frac{\mathcal{M}_{12}^2}{(\mathcal{M}_{11}^2 - \nabla^2/a^2)(\mathcal{M}_{22}^2 - \nabla^2/a^2) - \mathcal{M}_{21}^4}\right]^2 \dot\pi \\
&+8\dot\theta^2(\dot\theta^2 + \dot\varphi^2) \left[\dot\pi \frac{(\mathcal{M}_{22}^2 - \nabla^2/a^2)}{(\mathcal{M}_{11}^2 - \nabla^2/a^2)(\mathcal{M}_{22}^2 - \nabla^2/a^2) - \mathcal{M}_{21}^4}\right]^2 \dot\pi \\
&+2\dot\theta^2 \left[\dot\pi^2 - \frac{1}{a^2}(\nabla\pi)^2\right] \frac{\mathcal{M}_{22}^2 - \nabla^2/a^2}{(\mathcal{M}_{11}^2 - \nabla^2/a^2)(\mathcal{M}_{22}^2 - \nabla^2/a^2) - \mathcal{M}_{21}^4}\dot\pi \\
&+2\dot\theta^2 \dot\pi \frac{\mathcal{M}_{22}^2 - \nabla^2/a^2}{(\mathcal{M}_{11}^2 - \nabla^2/a^2)(\mathcal{M}_{22}^2 - \nabla^2/a^2) - \mathcal{M}_{21}^4} \left[\dot\pi^2 - \frac{1}{a^2}(\nabla\pi)^2\right] \Bigg\}.
\end{aligned} \tag{7.16}
$$

This action may be further simplified by recalling that our formalism only allows us to integrate heavy fields at wavelengths such that (7.6) is respected. This allows us to write:

$$
\begin{aligned}
S_{\rm EFT} &= \frac{1}{2}\int d^4x a^3 \dot\phi_0^2 \Bigg\{ \left[\dot\pi^2 - \frac{1}{a^2}(\nabla\pi)^2\right] + \dot\pi \frac{4\dot\theta^2}{\det \mathcal{M}^2/\mathcal{M}_{22}^2 - \nabla^2/a^2}\dot\pi \\
&+\frac{1}{2}\left(1 + \frac{\dot\varphi^2}{\dot\theta^2}\frac{\mathcal{M}_{22}^4 + \mathcal{M}_{12}^4}{\mathcal{M}_{22}^4}\right) \left[\dot\pi \frac{4\dot\theta^2}{\det \mathcal{M}^2/\mathcal{M}_{22}^2 - \nabla^2/a^2}\right]^2 \dot\pi \\
&+\frac{1}{2}\left[\dot\pi^2 - \frac{1}{a^2}(\nabla\pi)^2\right] \frac{4\dot\theta^2}{\det \mathcal{M}^2/\mathcal{M}_{22}^2 - \nabla^2/a^2}\dot\pi \\
&+\frac{1}{2}\dot\pi \frac{4\dot\theta^2}{\det \mathcal{M}^2/\mathcal{M}_{22}^2 - \nabla^2/a^2} \left[\dot\pi^2 - \frac{1}{a^2}(\nabla\pi)^2\right] \Bigg\}.
\end{aligned} \tag{7.17}
$$

This action constitutes one of our main results. It summarises the effect of two heavy-fields on the evolution of a single adiabatic mode, parametrized by the Goldstone boson mode π. The dispersion relation for the Goldstone boson mode may be read from the quadratic part of the action, and is found to be given by:

$$
\omega^2 = \frac{(\mathcal{M}_{11}^2 + p^2)\mathcal{M}_{22}^2 - \mathcal{M}_{21}^4}{(\mathcal{M}_{11}^2 + p^2)\mathcal{M}_{22}^2 - \mathcal{M}_{21}^4 + 4\dot\theta^2\mathcal{M}_{22}^2} p^2. \tag{7.18}
$$

Expanding this expression in powers of p^2, we obtain back the dispersion relation (7.9). It may be seen that at energies larger than

$$
\Lambda_{\rm new}^2 \sim \Lambda_{\rm UV}^2 c_s, \tag{7.19}
$$

the dispersion relation changes from a linear dependence on the momentum $\omega \propto p$ to a quadratic dependence $\omega \propto p^2$. This regime has been dubbed new physics regime [28], and it signals the regime where the non-trivial contributions due to the Laplacian ∇^2 become important in (7.16).

7.3 Discussion

We now wish to highlight and discuss some of the main characteristics emerging from the effective field theory deduced in the previous section. First of all, the form of action (7.17) coincides with that studied in ref. [73], where general arguments about the effects of heavy fields on curvature perturbations where given. There, the non trivial effects coming from heavy physics was parametrized by a single mass scale M, representing the mass of a single heavy field modifying the kinematics of the low energy Goldstone boson. Direct comparison between both approaches allows us to identify M in terms of the entries of the mass matrix $\mathcal{M}^2$ as:

$$M^2 = \det \mathcal{M}^2/\mathcal{M}^2_{22}. \tag{7.1}$$

In addition, motivated by the EFT parametrization of ref. [50], in ref. [73] the Goldstone boson self couplings were parametrized with the help of a set of the couplings M_n^4, where n denoted the order of expansion of the EFT in terms of the Newtonian potential $g^{00}+1$. In the present case, it is direct to read that the relation between M_3^4 and M_2^4 is given by

$$\frac{M_3^4}{M_2^4} = -\frac{3}{4}(c_s^{-2}-1)\left(1+\frac{\dot{\varphi}^2}{\dot{\theta}^2}\frac{\mathcal{M}^4_{22}+\mathcal{M}^4_{12}}{\mathcal{M}^4_{22}}\right), \tag{7.2}$$

where one sees that all the nontrivial effects due to the presence of the second field are due to the ratio $\dot{\varphi}^2/\dot{\theta}^2$.

There are two relevant limits of the deduced effective field theory, depending on the value of $\Lambda^2_{\rm new} = \Lambda^2_{\rm UV}c_s$ relative to H^2. If $H^2 \ll \Lambda^2_{\rm new}$, the dispersion relation takes the simple form $\omega = c_s p$ during horizon crossing, and the relevant EFT action parametrizing this process becomes:

$$\begin{aligned} S_{\rm EFT} &= \frac{1}{2}\int d^4x a^3 \dot{\phi}_0^2 \Bigg\{ \left[\frac{1}{c_s^2}\dot{\pi}^2 - \frac{1}{a^2}(\nabla\pi)^2\right] + \left(\frac{1}{c_s^2}-1\right)\dot{\pi}\left[\dot{\pi}^2 - \frac{1}{a^2}(\nabla\pi)^2\right] \\ &\quad + \frac{1}{2}\left(\frac{1}{c_s^2}-1\right)^2\left(1+\frac{\dot{\varphi}^2}{\dot{\theta}^2}\frac{\mathcal{M}^4_{22}+\mathcal{M}^4_{12}}{\mathcal{M}^4_{22}}\right)\dot{\pi}^3\Bigg\}. \end{aligned} \tag{7.3}$$

This form of the action may be compared with the one found in the original EFT analysis of ref. [50]. In terms of the parametrization offered by eq. (7.1) one deduces that:

$$A = -\frac{1}{2}(1-c_s^2)\left(1+\frac{\dot{\varphi}^2}{\dot{\theta}^2}\frac{\mathcal{M}^4_{22}+\mathcal{M}^4_{12}}{\mathcal{M}^4_{22}}\right). \tag{7.4}$$

Thus we see that a second heavy-field enlarges the EFT parameter encountered in the single field case. Crucially, this happens only in one direction, and we are able to conclude that the generic effect implied by a second field is to allow the inequality:

$$A \leqslant -\frac{1}{2}(1-c_s^2). \tag{7.5}$$

This inequality may be tested, by means of the relations [114]:

$$f_{\rm NL}^{\rm eq} = \frac{1-c_s^2}{c_s^2}(-0.276+0.0785A), \tag{7.6}$$

$$f_{\rm NL}^{\rm orth} = \frac{1-c_s^2}{c_s^2}(0.0157-0.0163A), \tag{7.7}$$

as long as large non-Gaussian signatures are observed.

Next, we may consider the limit in which horizon crossing happens during the new physics regime. Here the dispersion relation is dominated by a quadratic term ($\omega \sim p^2$), and one is forced to consider the full action (7.17). This form of the action was studied in detail in ref. [30, 73] where it was noticed that the nontrivial scale dependence implied by the insertions $4\dot{\theta}^2/(M^2-\nabla^2/a^2)$ (with $M^2 = \det \mathcal{M}^2/\mathcal{M}^2_{22}$), would modify drastically the computation of observables in terms of background inflationary quantities. For instance, the quadratic part of the action (7.17) in the new physics regime reads

$$S_{\rm EFT}^{(2)} = \frac{1}{2}\int d^4x a^3 \dot{\phi}_0^2 \left\{ \left[\dot{\pi}^2 - \frac{1}{a^2}\left(\nabla\pi\right)^2\right] - \dot{\pi}\frac{4\dot{\theta}^2}{\nabla^2/a^2}\dot{\pi}\right\}, \tag{7.8}$$

from where one deduces that the power spectrum $\mathcal{P}_\zeta$, and the tensor to scalar ratio r, are given by:

$$\mathcal{P}_\zeta \simeq \frac{5.4}{100}\frac{H^2}{M_{\rm Pl}^2\varepsilon}\sqrt{\frac{\dot{\theta}}{H}}, \qquad r \simeq 3.8\varepsilon\sqrt{\frac{H}{\dot{\theta}}}. \tag{7.9}$$

A detailed characterization of the shape of non-Gaussianity in the new physics regime is still missing, but it is possible to infer that the size of equilateral non-Gaussianity is of order

$$f_{\rm NL} \sim \frac{\dot{\theta}}{H}, \tag{7.10}$$

with its actual value determined by M_3^4 given by (7.2).

7.4 Conclussions

We have deduced the effective field theory describing the evolution of curvature perturbations during inflation, in the specific case where the Goldstone boson mode interacted with two heavy fields. Our main result is summarized by eq. (7.17), which provides the explicit effective field action for the Goldstone boson field π. Crucially, the couplings induced by the presence of a second heavy-field are distinguishable from those appearing in the single heavy-field case, implying that a detailed characterization of non-Gaussianities will allow us constrain this class of scenarios. In particular, the presence of a second field implies the following general inequality involving the parameters c_s and A, appearing in the EFT of eq. (7.1)

$$A \leqslant -\frac{1}{2}(1 - c_s^2), \tag{7.1}$$

which is saturated in the single-field case. In terms of the angular velocities $\dot{\theta}$ and $\dot{\varphi}$ parametrizing the multi-field inflationary trajectory, such an inequality becomes stronger as $\dot{\varphi}$ becomes of the same order than $\dot{\theta}$ (which may be as large as the cutoff scale $\Lambda_{\rm UV}$).

Our results represent a significant step towards a better understanding of the collective effects that many heavy fields may have on the evolution of adiabatic perturbations, and highlight the importance of using effective field theory techniques to interpret future observations. In particular, our results show, in an eloquent manner, how different values of c_s and A correspond to different, and potentially distinguishable, UV realizations of inflation. Several outstanding questions remain to be answered: For instance, in the present analysis, we have assumed that the parameter space is such that the two high frequencies $\omega_{\rm I}$ and $\omega_{\rm II}$ are of the same order, simplifying the derivation of the desired effective field theory. Thus, it would be desirable to study the system in other limits allowed by the parameters, such as $\omega_{\rm light}^2 \ll \omega_{\rm I}^2 \ll \omega_{\rm II}^2$, and/or $\dot{\varphi}^2 \gg \dot{\theta}^2$. Also, given that one generically expects several heavy fields to have interacted with curvature perturbations during inflation, it would be important to know whether the inclusion of additional heavy fields would modify inequality (7.1). Last but not least, it would be interesting to study the way in which a second heavy field would generate features in the primordial spectra due to possible sudden turns of the inflationary trajectory in the landscape.

Conclussions

During this thesis, we have systematically studied the effects of heavy physics in inflationary theories. This fields are called "heavy" because their masses are heavier when compared to the scale of inflation H. Thus, it is expectable that their degrees of freedom will not become excited at the energies when inflation happens. Nevertheless, it can be the case that this heavy fields produce deviations from the geodesic trajectories followed by the inflaton. When this happens heavy modes mixes with the low energy modes, in such a way that the high energy degrees of freedom do not become excited. The influence of the heavy fields, thus can be encoded, in operators of an effective field theory for the curvature modes. By this mechanism, heavy fields can produce a number of different predictions in the inflationary observables, such as reduced speed of sound for the curvature perturbations and features in the curvature power spectrum.

The use of EFT, allows a model independent way of studying this effects and opens the possibility of constrain some models using recent observations. In this work, we study in detail the regime of validity of the effective field theory for this case. First we analyze the decoupling between high and low energy modes that allows an effective theory description in the case when heavy fields are coupled to the curvature modes. We show that this holds, when the heavy field is very massive compared to the scale of inflation H. We also found an adiabaticity condition for the case when the inflationary trajectory is curved due to the effects of heavy physics. This condition tells that while the logarithmic derivative stays suppressed with respect to the mass of the heavy fields, high energy degrees of freedom will not become excited and the EFT will stay reliable. We also analyze the case when additional heavy fields are considered . In those cases, EFT can also be constructed, by the extension of the previous conditions, but since new parameters arise, newer features can be used to distinguish between the existence of one or several fields.

The results of *Planck*, although highly in agreement with models of single field inflation, opens many more question for the inflationary paradigm. Because a detailed study of its microphysics is still unknown, studying possible variations of the single field case, allows for a better understanding of the entire paradigm. Furthermore, a reconstructions of the power spectrum using CMB data still shows features that has to be explained. Also, there is the possibility that observations can constrain the shape of the features and their origin, thus restricting the possibility of the mechanism presents, in this type of models.

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Appendix A

Cosmological Peturbation Theory

In this appendix we summarize the main results of cosmological perturbation theory.

A.1 Perturbed Universe

We consider perturbations to the homegeneous spacetime given by the FRW metric:

$$ds^2 = -dt^2 - a^2 d\mathbf{x}^2, \tag{A.1}$$

and the stress-energy of the universe:

$$T^{\mu}_{\ \nu} = \begin{pmatrix} \rho(t) & & & \\ & -p(t) & & \\ & & -p(t) & \\ & & & -p(t) \end{pmatrix}. \tag{A.2}$$

A.1.1 Metric perturbations

At first order, the most generall perturbationas to (A.1) is given by

$$ds^2 = -(1+2\Phi)dt^2 + 2a(t)B_{,i}dx^i dt + a^2(t)[(1-2\Psi)\delta_{ij} + 2E_{ij}]dx^i dx^j, \tag{A.3}$$

where Φ is a 3-scalar called *lapse*, B_i is a 3-vector called the $shift$, Ψ a a 3-scalar called the spatial *curvature perturbation*, and E_{ij} is a spatial *shear* 3-tensor which is symmetric and traceless, $E^i_i = 0$. 3-surfaces of constant time t are called *slices* and curves of constant spatial coordinates x^i but variiying time t are called *threads*.

A.1.2 Matter perturbation

In general stress-energy tensors may be described by a density ρ, a pressure p, a 4-velocity u^μ of the frame in which the 3-momentum density vanishes, and an anisotropic stress $\Sigma^{\mu\nu}$.

Density and pressure perturbationas are defined as:

$$\begin{aligned}\delta\rho(t,x^i) &\equiv \rho(t,x^i)-\bar{\rho}(t),\\ \delta p(t,x^i) &\equiv p(t,x^i)-\bar{p}(t).\end{aligned} \tag{A.4}$$

Where we denote unperturberd quantitiaes by a overber. The 4-velocity has only three independent components since it has to satisfy the constraint $g_{\mu\nu}u^\mu u^\nu$=-1. In the perturbed metric (A.3) the perturbed 4-velocity is:

$$u_\mu \equiv (-1-\Phi, v_i) \text{ or} \tag{A.5}$$

$$u^\mu \equiv (1-\Phi, v^i+B^i). \tag{A.6}$$

Anisotropic stress vanishes in the unperturbed FRW universe, so $\Sigma^{\mu\nu}$ is a first order perturbation. Furthermore, $\Sigma^{\mu\nu}$ is constrained by

$$\Sigma^{\mu\nu}u_\nu = \Sigma^\mu_\mu = 0. \tag{A.7}$$

The orthogonality with u_μ implies $\Sigma^{00}=\Sigma^{0j}=0$, *i.e.* only the spatial components Σ^{ij} are non-zero. The trace condition then implies $\Sigma^i_i=0$. Anisotropic stress is therefore a traceless symmetric 3-tensor.

Finally, with these definitions the perturbed stress-tensor is

$$T^0_0 = -(\bar{\rho}+\delta\rho), \tag{A.8}$$

$$T^0_i = (\bar{\rho}+\bar{p})v^i, \tag{A.9}$$

$$T^i_0 = -(\bar{\rho}+\bar{p})(v^i+B^i), \tag{A.10}$$

$$T^i_j = \delta^i_j(\bar{p}+\delta p)+\Sigma^i_j. \tag{A.11}$$

If there are several contributions to the stress-energy tensor, they are added: $T_{\mu\nu}=\sum_I T^I_{\mu\nu}$. This implies

$$\delta\rho = \sum_i \delta\rho_I, \tag{A.12}$$

$$\delta p = \sum_i \delta p_I, \tag{A.13}$$

$$(\bar{p}+\bar{\rho})v^i = \sum_i (\bar{p}_I+\bar{\rho}_I)v^i_I, \tag{A.14}$$

$$\Sigma^{ij} = \sum_I \Sigma^{ij}_I. \tag{A.15}$$

As velocities do not simply add, for convenience we define the 3-momentum density as

$$\delta q^i \equiv (\bar{p}+\bar{\rho})v^i, \tag{A.16}$$

such that

$$\delta q^i = \sum_I \delta q^i_I. \tag{A.17}$$

A.2 Scalars, vectors and tensors

A.2.1 Real Space SVT-decomposition

By Hemholtz theorem we can decompose the perturbation in scalar, vector an tensors. A 3-scalar is obviously a helicity scalar

$$\alpha = \alpha^S. \tag{A.18}$$

Consider a 3-vector β_i. We argue that it can be decomposed as

$$\beta_i = \beta_i^S + \beta_i^V, \tag{A.19}$$

where

$$\beta_i^S = \nabla_i b\hat{e}ta, \quad \nabla^i \beta_i^V = 0, \tag{A.20}$$

where we have defined $\beta \equiv k^i \beta_i$.

Similarly, a traceless, symmetric 3-tensor can be written as

$$\gamma_{ij} = \gamma_{ij}^S + \gamma_{ij}^V + \gamma_{ij}^T, \tag{A.21}$$

where

$$\gamma_{ij}^S = \left(\nabla_i \nabla_j - \frac{1}{3}\delta_{ij}\nabla^2\right)\hat{\gamma} \tag{A.22}$$

$$\gamma_{ij}^V = \frac{1}{2}(\nabla_i \hat{\gamma}_j + \nabla_j \hat{\gamma}_i), \ \nabla_i \hat{\gamma}_i = 0 \tag{A.23}$$

$$\nabla_i \gamma_{ij}^T = 0. \tag{A.24}$$

A.3 Scalars

A.3.1 Metric Perturbations

For scalar metric perturbations Φ, B, i, $\Psi\delta_{ij}$ and $E_{,ij}$ may be constructed from 3-scalars, their derivatives and the background spatial metric, *i.e.*

$$ds^2 = -(1+2\Phi)dt^2 + 2a(t)B_{,i}dx^i dt + a^2(t)[(1-2\Psi)\delta_{ij} + 2E_{,ij}]dx^i dx^j, \tag{A.25}$$

where we wrote the $\nabla^2 E\delta_{ij}$ part of the helicity scalar E_{ij}^S in $\Psi\delta_{ij}$.

The intrinsic Ricci scalar curvature of constant time hypersurfaces is

$$R_{(3)} = \frac{4}{a^2}\nabla^2\Psi. \tag{A.26}$$

This explains why Ψ is often referred to as the curvature perturbations.

There are two scalar gauge transformations

$$\begin{aligned} t &\longrightarrow t+\alpha, && \text{(A.27)}\\ x_i &\longrightarrow x_i+\delta^{ij}\beta,j. && \text{(A.28)} \end{aligned}$$

Under these coordinate transformations the scalar metric perturbations transform as,

$$\begin{aligned} \Phi &\to \Phi-\dot{\alpha} && \text{(A.29)}\\ B &\to B-a^{-1}\alpha-a\dot{\beta} && \text{(A.30)}\\ E &\to E-\beta && \text{(A.31)}\\ \Psi &\to \Psi+H\alpha. && \text{(A.32)} \end{aligned}$$

Note that the combination $\dot{E}-B/a$ is independent of the spatial gauge and only depends on the the temporal gauge. It is called the scalar potential for the anisotropic shear of world lines orthogonal to constant time hypersurfaces. To extract physical results it is useful to define gauge-invariant combinatiosn of the scalar metric perturbations. Two important gauge-invarian quantities are [20]:

$$\begin{aligned} \Phi_B &\equiv \Phi-\frac{d}{dt}[a^2(\dot{E}-B/a)] && \text{(A.33)}\\ \Psi_B &\equiv \Psi+a^2H(\dot{E}-B/a). && \text{(A.34)} \end{aligned}$$

A.3.2 Matter Perturbations

Matter perturbations are also gauge-dependents, *e.g.* density and pressure perturbations transform as follows under temporal gauge transformations

$$\begin{aligned} \delta\rho &\longrightarrow \delta\rho-\dot{\bar{\rho}}\alpha, && \text{(A.35)}\\ \delta p &\longrightarrow \delta p-\dot{\bar{p}}\alpha. && \text{(A.36)} \end{aligned}$$

Adiabatic pressure perturbations are defined as

$$\delta p_{ad}\equiv\frac{\bar{p}}{\bar{\rho}}\delta\rho. \qquad \text{(A.37)}$$

The non-adiabatic, or entropic, part of the pressure perturbations is then gauge invariant,

$$\delta p_{en}\equiv\delta p-\frac{\bar{p}}{\bar{\rho}}\delta\rho. \qquad \text{(A.38)}$$

The scalar part of the 3-momentum density δq transform as

$$\delta q\leftrightarrow\delta q+(\bar{\rho}+\bar{p})\alpha. \qquad \text{(A.39)}$$

We may define the gauge-invariant comoving density perturbation

$$\delta\rho_m \equiv \delta\rho - 3H\delta q. \tag{A.40}$$

Finally, two important gauge-invariant quantities are formed from combinations of matter and metric perturbations. The *curvature perturbation on uniform density hypersurfaces* is

$$-\mathcal{R} \equiv \psi + \frac{H}{\delta\bar{\rho}}\delta\rho. \tag{A.41}$$

The *comoving curvature perturbation* is

$$\zeta \equiv= \psi - \frac{H}{\bar{\rho}+\bar{p}}\delta q. \tag{A.42}$$

We will show that ζ and $\mathcal{R}$ are equal on superhorizon scales, where they become time-independent. The computation of the inflationary perturbation spectrum is most clearly phrased in terms of ζ and $\mathcal{R}$

A.3.3 Einstein Equations

To relate the metric and stress-energy perturbations, we consider the perturbed Einstein equations

$$G_{\mu\nu} = 8\pi G\delta T_{\mu\nu} \tag{A.43}$$

We work at linear order. This leads to the *energy and momentum constraint equations*

$$3H(\dot{\Psi} + H\Phi) + \frac{k^2}{a^2}\left[\psi + H(a^2\dot{E} - aB)\right] = -4\pi G\delta\rho \tag{A.44}$$

$$\dot{\Psi} + H\Phi = -4\pi G\delta q. \tag{A.45}$$

These can be combined into the gauge invariant *Poisson equation*

$$\frac{k^2}{a^2}\Psi_B = -4\pi G\delta\rho_m. \tag{A.46}$$

The Einstein equations also yield two *evolution equations*

$$\ddot{\Psi} + 3H\dot{\Psi} + H\dot{\Phi} + (3H^2 + 2\dot{H})\Psi = 4\pi G\left(\delta p - \frac{2}{3}k^2\delta\Sigma\right) \tag{A.47}$$

$$(\partial_t + 3H)(\dot{E} - B/a) + \frac{\Psi - \Phi}{a^2} = 8\pi G\delta\Sigma. \tag{A.48}$$

The last equation may be written as

$$\Psi_{\rm B} - \Phi_{\rm B} = 8\pi Ga^2\delta\Sigma. \tag{A.49}$$

In the absence of anisotropic stress this implies, $\Psi_{\rm B} = \Phi_{\rm B}$.

Energy - momentum conservation, $\nabla_\mu T^{\mu\nu} = 0$, gives the *continuity equation* and the *Euler equation*:

$$\delta\dot\rho + 3H(\delta\rho + \delta p) = \frac{k^2}{a^2}\delta q + (\bar\rho + \bar p)\left[3\dot\Psi + k^2(\dot E + (B/a))\right], \tag{A.50}$$

$$\delta\dot q + 3H\delta q = -\delta p + \frac{2}{3}k^2\delta\Sigma - (\bar\rho + \bar p)\Phi. \tag{A.51}$$

Expressed in terms of the curvature perturbation on uniform-density hypersurfaces, ζ, reads

$$\dot\zeta = -H\frac{\delta p_{en}}{\bar\rho + \bar p} - \Pi, \tag{A.52}$$

where Π is the secalar shear along comoving worldlines

$$\frac{\Pi}{H} \equiv -\frac{k^2}{3H}\left[\dot E - B/A + \frac{\delta q}{a^2(\bar\rho + \bar p)}\right] \tag{A.53}$$

$$= -\frac{k^2}{3a^2H^2}\left[\zeta - \Psi_{\rm B}\left(1 - \frac{2\bar\rho}{9(\bar\rho + \bar p)}\frac{k^2}{a^2H^2}\right)\right]. \tag{A.54}$$

For adiabatic perturbations, $\delta p_{en} = 0$ on superhorizon scales, $k/(aH) \ll 1$ (*i.e* $\Pi/H \to 0$ for finite ζ and $\Psi_{\rm B}$), the curvature perturbations ζ is constant. This is a crucial result for our computation of the inflationary spectrum of ζ. It justifies computing ζ at horizon exit and ignoring superhorizon evolution.

A.3.4 Popular gauges

For reference we now give the Einstein equations and the conservations equations in various popular gauges

- **Synchronous gauge**
  A popular gauge, specially for numerical implementations of the perturbation equations is synchronous gauge. It is defined by

$$3H\dot\psi + \frac{k^2}{a^2}\left[\Psi + Ha^2\dot E\right] = -4\pi G\delta\rho, \tag{A.55}$$

$$\dot\Psi = -4\pi G\delta q, \tag{A.56}$$

$$\ddot\Psi + 3H\dot\Psi = 4\pi G\left(\delta p - \frac{2}{3}k^2\delta\Sigma\right), \tag{A.57}$$

$$(\partial_t + 3H)\dot E + \frac{\Psi}{a^2} = 8\pi G\delta\Sigma. \tag{A.58}$$

  The conservation equation are

$$\delta\dot\rho + 3H(\delta\rho + \delta p) = \frac{k^2}{a^2}\delta q + (\bar\rho + \bar p)[3\dot\Psi + k^2\dot E], \tag{A.59}$$

$$\delta\dot q + 3H\delta q = -\delta p + \frac{2}{3}k^2\delta\Sigma. \tag{A.60}$$

- **Uniform density gauge** The uniform density gauge is useful for describing the evolution of perturbations on superhorizon scales. As its name suggests it is defined by

$$\delta\rho = 0. \tag{A.61}$$

In addition, it is convenient to take

$$E = 0, \quad -\Psi \equiv \mathcal{R}. \tag{A.62}$$

The Einstein equations are

$$\begin{aligned}
3H(\dot{\mathcal{R}} + H\Psi) - \frac{k^2}{a^2}[\mathcal{R} + aHB] &= 0, && \text{(A.63)}\\
-\dot{\mathcal{R}} + H\Psi &= -4pG\delta q, && \text{(A.64)}\\
-\ddot{\mathcal{R}} - 3H\dot{\mathcal{R}} + H\dot{\Psi} + (3H^2 + 2\dot{H})\Psi &= 4\pi G\left(\delta p - \frac{2}{3}k^2\delta\Sigma\right), && \text{(A.65)}\\
(\partial_t + 3H)B/a + \frac{\mathcal{R} + \Phi}{a^2} &= -8\pi G\delta\Sigma. && \text{(A.66)}
\end{aligned}$$

The continuity equations are

$$\begin{aligned}
3H\delta p &= \frac{k^2}{a^2}\delta q + (\bar{\rho} + \bar{p})[-3\dot{\mathcal{R}} + k^2 B/a], && \text{(A.67)}\\
\delta\dot{q} + 3H\delta q &= -\delta p + \frac{2}{3}k^2\delta\Sigma - (\bar{\rho} + \bar{p})\Phi. && \text{(A.68)}
\end{aligned}$$

- **Comoving gauge**
  Comoving gauge is defined by the vanishing of the scalar momentum density,

$$\delta q = 0, \; E = 0. \tag{A.69}$$

It is also convenient to set $-\Psi = \zeta$ in this gauge. The Einstein equations are

$$\begin{aligned}
3H(-\dot{\zeta} + H\Phi) + \frac{k^2}{a^2}[-\zeta - aHB] &= -4\pi G\delta\rho, && \text{(A.70)}\\
-\dot{\zeta} + H\Phi &= 0, && \text{(A.71)}\\
-\ddot{\zeta} - 3H\dot{\zeta} + H\dot{\Phi} + (3H^2 + 2\dot{H})\Phi &= 4\pi G\left(\delta p - \frac{2}{3}k^2\delta\Sigma\right), && \text{(A.72)}\\
(\partial_t + H)B/a + \frac{\zeta + \Phi}{a^2} &= -8\pi G\delta\Sigma. && \text{(A.73)}
\end{aligned}$$

The continuity equations are

$$\begin{aligned}
\delta\dot{\rho} + 3H(\delta\rho + \delta p) &= (\bar{\rho} + \bar{p})[-3\dot{\zeta} + k^2 B/a], && \text{(A.74)}\\
0 &= -\delta p + \frac{2}{3}k^2\delta\Sigma - (\bar{\rho} + \bar{p})\Phi. && \text{(A.75)}
\end{aligned}$$

Appendix B

Articles Published

B.1 On the importance of heavy fields during inflation

We study the dynamics of two-field models of inflation characterized by a hierarchy of masses between curvature and isocurvature modes. When the hierarchy is large, a low energy effective field theory (EFT) exists in which only curvature modes participate in the dynamics of perturbations. In this EFT heavy fields continue to have a significant role in the low energy dynamics, as their interaction with curvature modes reduces their speed of sound whenever the multi-field trajectory is subject to a *sharp turn* in target space. Here we analyze under which general conditions this EFT remains a reliable description for the linear evolution of curvature modes. We find that the main condition consists on demanding that the rate of change of the turn's angular velocity stays suppressed with respect to the masses of heavy modes. This *adiabaticity* condition allows the EFT to accurately describe a large variety of situations in which the multi-field trajectory is subject to *sharp turns*. To test this, we analyze several models with turns and show that, indeed, the power spectra obtained for both the original two-field theory and its single-field EFT are identical when the adiabaticity condition is satisfied. In particular, when turns are sharp and sudden, they are found to generate large features in the power spectrum, accurately reproduced by the EFT.

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On the importance of heavy fields during inflation

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Abstract. We study the dynamics of two-field models of inflation characterized by a hierarchy of masses between curvature and isocurvature modes. When the hierarchy is large, a low energy effective field theory (EFT) exists in which only curvature modes participate in the dynamics of perturbations. In this EFT heavy fields continue to have a significant role in the low energy dynamics, as their interaction with curvature modes reduces their speed of sound whenever the multi-field trajectory is subject to a *sharp turn* in target space. Here we analyze under which general conditions this EFT remains a reliable description for the linear evolution of curvature modes. We find that the main condition consists on demanding that the rate of change of the turn's angular velocity stays suppressed with respect to the masses of heavy modes. This *adiabaticity* condition allows the EFT to accurately describe a large variety of situations in which the multi-field trajectory is subject to *sharp turns*. To test this, we analyze several models with turns and show that, indeed, the power spectra obtained for both the original two-field theory and its single-field EFT are identical when the adiabaticity condition is satisfied. In particular, when turns are sharp and sudden, they are found to generate large features in the power spectrum, accurately reproduced by the EFT.

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Contents

1 Introduction

Cosmic inflation [1] persists as the undisputed mechanism explaining the origin of primordial curvature perturbations [2] necessary to account for the Cosmic Microwave Background (CMB) anisotropies [3–5] and the large scale structure of our universe [6–9]. The fact that inflation is formulated within a field theoretical framework [10, 11] makes it particularly compelling to test our ideas about fundamental theories, such as supergravity and string theory, characterized for consistently incorporating the gravitational strength among their couplings. Because these theories generically predict the existence of a large number of degrees of freedom, the need of a period of inflation at early times is found to impose strong restrictions on their interactions. In particular, if inflation happened at sufficiently high energies, curvature perturbations could have strongly interacted with other degrees of freedom, implying a variety of observable effects departing from those predicted in standard single-field slow-roll inflation [12–14], including features in the power spectrum of primordial inhomogeneities [15–28], large primordial non-Gaussianities [29–35] and isocurvature perturbations [36–44]. A detection of any of these signatures would therefore represent an extremely significant step towards elucidating the fundamental nature of physics taking place during the very early universe.

Despite of its simplicity, the construction of satisfactory models of inflation within supergravity and string theory is known to constitute a notoriously hard challenge. The vacuum expectation values (v.e.v.'s) of scalar fields participating of the inflationary dynamics must evolve along flat directions of the scalar potential's landscape for a sufficiently long time. But because the interaction strength of these theories is of a gravitational nature, the scalar potential is naturally subject to changes of order 1 when the scalar fields v.e.v.'s traverse distances of the order of the Planck scale. This translates into the well known η-problem of supergravity and string theory [45–48], where second derivatives of the scalar potential V'' are typically of order equal or larger than H^2 (where H is the universe's expansion rate) therefore impeding the slow-roll evolution of the fields.[1] However, this problem may be cured if the theory contains a set of shift symmetries at non-perturbative level, ensuring the existence of exactly flat directions in the potential [49]. Then, if these symmetries are mildly broken, the expected result is an inflationary scenario with a large mass hierarchy between the modular fields representing flat directions and the rest of the scalar fields, expected to have masses much larger than H [52–56].

Conventional wisdom dictates that UV-degrees of freedom with masses $M \gg H$ necessarily have a marginal role in the low energy dynamics of curvature modes. After these heavy degrees of freedom are integrated out, one expects a low energy effective field theory (EFT) for curvature perturbations where UV-physics is parametrized by nontrivial operators suppressed by factors of order H^2/M^2. The resulting low energy EFT is therefore expected to offer negligible departures from a truncated version of the same theory, wherein heavy fields are simply disregarded from the very beginning. However, general field theoretical arguments due allow for large sizable corrections to the low energy EFT [57, 58]. In the specific case of multi field models, there are special circumstances where the background inflationary dynamic is such that the interchange of kinetic energy between curvature perturbations and heavy degrees of freedom may be dramatically enhanced [26, 62–70]. For example, if the inflationary trajectory is subject to a sharp turn in such a way that the heavy scalar fields stay normal to the trajectory (see figure 1 for an illustration) then unsuppressed interactions — kinematically coupling curvature perturbations with heavy fields — are unavoidably turned on. As a consequence, if the rate of turn is large compared to the rate of expansion H, the impact of heavy physics on the low energy dynamics becomes substantially amplified, introducing large non-trivial departures from a naively truncated version of the theory.

In the particular case of two field models — at linear order in the fluctuations — heavy fields are identified with isocurvature perturbations, and their role is reduced to modify the speed of sound c_s of curvature perturbations at the effective field theory level. The result is a non-trivial effective single-field theory where the time dependence of c_s is dictated by the specific shape of the two-field background trajectory, in such a way that departures from unity $c_s \neq 1$ exist whenever the trajectory is subject to a turn. More specifically, one finds that the speed of sound depends on the angular velocity $\dot{\theta}$ characterizing the turn as

$$c_s^{-2} = 1 + 4\dot{\theta}^2/M_{\rm eff}^2\,, \tag{1.1}$$

where $M_{\rm eff}^2 = M^2 - \dot{H}\,\mathbb{R} - \dot{\theta}^2$ is the effective mass of isocurvature perturbations, with M the tree-level mass of heavy modes, and $\mathbb{R}$ the Ricci scalar of the scalar field target space. In this way, sudden turns of the trajectory translate into sudden time variations of c_s (hence

[1] Another major obstacle towards the construction of models of inflation within string theory is related to the stabilization of moduli. See for instance refs. [50, 51] for a discussion on the stabilization of moduli in supergravity and string theory.

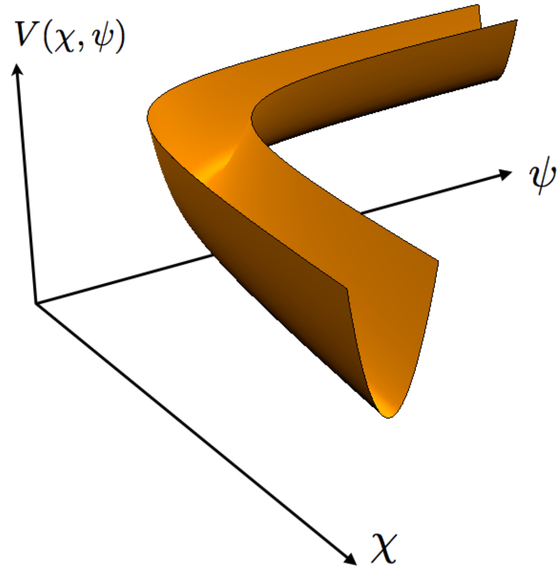

Figure 1. The figure illustrates a prototype example of a multi-field potential (depending on two fields χ and ψ) with a mass hierarchy in which the flat direction is subject to a turn.

modifying the value of the sound horizon c_s/H) and therefore generating features in the power spectrum of primordial inhomogeneities [26].[2] Moreover, if the turn is such that $c_s \ll 1$, cubic interactions become unsuppressed, implying large levels of primordial non-Gaussianities in the distribution of curvature perturbations [35].

The purpose of this article is to study two-field models of inflation characterized by a large mass hierarchy.[3] We are particularly interested in assessing the general conditions under which the EFT deduced by integrating out the heavy field remains a reliable description of the inflationary dynamics. We show that the main condition simply consists on the requirement that the rate of variation of the angular velocity $\dot{\theta}$ characterizing the turn stays suppressed with respect to the effective mass $M_{\rm eff}$ of heavy modes. That is:

$$\left| \frac{d}{dt} \ln \dot{\theta} \right| \ll M_{\rm eff} \, . \tag{1.2}$$

We show that this *adiabaticity* condition is sufficiently mild, as it still allows for the effective field theory to describe, with great accuracy, a large variety of situations where very sharp turns take place (i.e. situations where $|\dot{\theta}| \gtrsim H$). We check this condition by studying various models with turns and compare the power spectra of these models obtained from both, the full two-field inflationary model and the respective effective field theory. We find that turns are able to generate large features in the power spectra, with the amplitude of these features depending on how large departures of c_s from unity are.

This work is organized as follows: in section 2 we provide a self contained review on two-field models of inflation and summarize the main known results concerning the existence of

[2]For a recent discussion on features in the power spectrum generated by variations of the speed of sound see ref. [28].

[3]For other interesting work regarding non-trivial effects on the dynamics of curvature perturbations coming from massive degrees of freedom, see for instance refs. [59–61].

mass hierarchies. Then, in section 3 we offer a simple derivation of the general class of single-field EFT emerging from two-field models with mass hierarchies, and derive condition (1.2) dictating the validity of this theory in terms of background quantities. In section 4 we discuss the different classes of turns and deduce the type of reactions that turns have on the background inflationary trajectory. In particular, we study the case of sudden turns, where the inflationary trajectory is subject to a single turn for a brief period of time Δt smaller (or much smaller) than an e-fold ($\Delta t \lesssim H^{-1}$). Then, in section 5, we consider two toy models and compute the power spectrum for different cases of turns. There we show that, consistent with (1.2), the effective field theory remains reliable as long as $\Delta t \gg 1/M_{\rm eff}$, where $M_{\rm eff}$ is the effective mass of the heavy field. We also show that large features on the power spectrum are easily produced, with the details of the effects depending on the different parameters characterizing the type of turns. Finally, in section 6 we offer our concluding remarks to this work.

2 Two-field inflation

In this section we summarize the main results coming from previous work related to the study of multi-field inflation [26, 40, 41].[4] We shall specialize these results to the particular case of two-field models, to be studied in detail throughout this work. To start with, let us consider a non-canonical scalar field system with an action given by

$$S = \int d^4x\sqrt{-g}\left[\frac{1}{2}R - \frac{1}{2}g^{\mu\nu}\gamma_{ab}\partial_\mu\phi^a\partial_\nu\phi^b - V(\phi)\right], \tag{2.1}$$

where R is the Ricci scalar constructed out of the spacetime metric $g_{\mu\nu}$ (notice that we are working in units where the Planck mass is set to unity $M_{\rm Pl} = 1$). Additionally, $V(\phi)$ is the scalar field potential and γ_{ab} with $a = 1, 2$ is the sigma model metric describing the abstract geometry of the scalar space spanned by the pair of fields ϕ^1 and ϕ^2. It is extremely useful to adopt a covariant notation with respect to the geometrical space offered by the scalar fields. This will allow us to deduce general results without making any reference to particular models in which γ_{ab} and $V(\phi)$ acquire specific dependences on the fields. We therefore define a set of Christoffel symbols given by

$$\Gamma^a_{bc} = \frac{1}{2}\gamma^{ad}(\partial_b\gamma_{dc} + \partial_c\gamma_{bd} - \partial_d\gamma_{bc}), \tag{2.2}$$

where γ^{ab} is the inverse sigma model metric. Then, the equations of motion for the scalar fields are found to be

$$\Box\phi^a + \Gamma^a_{bc}\partial_\mu\phi^b\partial^\mu\phi^c - V^a = 0, \tag{2.3}$$

where $V^a \equiv \gamma^{ab}V_b$ with $V_b = \partial_b V$. We will also encounter the need of introducing the Ricci scalar, defined as $\mathbb{R} = \gamma^{ab}\mathbb{R}^c{}_{acb}$, where $\mathbb{R}^a{}_{bcd}$ is the Riemann tensor given by:

$$\mathbb{R}^a{}_{bcd} = \partial_c\Gamma^a_{bd} - \partial_d\Gamma^a_{bc} + \Gamma^a_{ce}\Gamma^e_{db} - \Gamma^a_{de}\Gamma^e_{cb}. \tag{2.4}$$

Because we are specializing our analysis to two-field models, the Riemann tensor depends on a single degree of freedom, and therefore may be expressed in terms of the Ricci scalar $\mathbb{R}$ as:

$$\mathbb{R}_{abcd} = \frac{1}{2}\mathbb{R}(\gamma_{ac}\gamma_{bd} - \gamma_{ad}\gamma_{cb}). \tag{2.5}$$

In what follows we proceed to study the dynamics of this system by considering separately the homogeneous and isotropic background and the perturbations of the system.

[4]For other general approaches to multi-field models of inflation, see refs. [71–74].

2.1 Homogeneous and isotropic backgrounds

Let us first devote our attention to homogeneous and isotropic cosmological backgrounds characterized by a scalar field solution $\phi^a = \phi_0^a(t)$ only dependent on time. For this we consider a flat Friedmann-Robertson-Walker metric of the form

$$ds^2 = -dt^2 + a^2(t)\delta_{ij}dx^i dx^j, \tag{2.6}$$

where $a(t)$ is the scale factor describing the expansion of flat spatial foliations. Then, the equations of motion determining the evolution of the system of fields $a(t)$, $\phi_0^1(t)$ and $\phi_0^2(t)$ are given by

$$\frac{D}{dt}\dot{\phi}_0^a + 3H\dot{\phi}_0^a + V^a = 0\,, \tag{2.7}$$

$$3H^2 = \frac{1}{2}\dot{\phi}_0^2 + V, \tag{2.8}$$

where $H = \dot{a}/a$ is the rate of expansion. Equation (2.7) corresponds to the equation of motion derived by varying the action with respect to ϕ^a. There, we have introduced a covariant time derivative D_t defined to satisfy

$$\frac{D}{dt}X^a = \dot{X}^a + \Gamma^a_{bc}\dot{\phi}_0^b X^c, \tag{2.9}$$

where $X^a = X^a(t)$ is an arbitrary vector field with the property of transforming like $X^{a'} = \frac{\partial \phi^{a'}}{\partial \phi^b}X^b$ under a general field reparametrizations $\phi^{a'} = \phi^{a'}(\phi^1, \phi^2)$. On the other hand, eq. (2.8) (Friedmann's equation) determines the expansion rate in terms of the energy density of the system $\rho = \frac{1}{2}\dot{\phi}_0^2 + V$. There, we are using the following notation to define the scalar field velocity $\dot{\phi}_0$:

$$\dot{\phi}_0^2 \equiv \gamma_{ab}\dot{\phi}_0^a\dot{\phi}_0^b\,. \tag{2.10}$$

By combining (2.7) and (2.8) we may derive the following useful relation:

$$\dot{H} = -\frac{\dot{\phi}_0^2}{2}\,. \tag{2.11}$$

Given a set of initial conditions for the scalar fields, there will exist a unique solution $\phi_0^a(t) = (\phi_0^1(t), \phi_0^2(t))$ defining a curve in field space parametrized by cosmic time t. To characterize this curve, it is convenient to construct a set of orthogonal unit vectors T^a and N^a in such a way that, at a given time t, $T^a(t)$ is tangent to the path, and $N^a(t)$ is normal to it. We may define this set of vectors as:[5]

$$T^a = \dot{\phi}_0^a/\dot{\phi}_0\,, \tag{2.12}$$

$$N_a = (\det\gamma)^{1/2}\epsilon_{ab}T^b \tag{2.13}$$

where ϵ_{ab} is the two dimensional Levi-Civita symbol with $\epsilon_{11} = \epsilon_{22} = 0$ and $\epsilon_{12} = -\epsilon_{21} = 1$. These definitions ensure that $T_aT^a = N_aN^a = 1$ and $T^aN_a = 0$. Notice that N_a has a fixed orientation with respect to the path, as shown in figure 2. These two unit vectors may be used

[5]We use the metric γ_{ab} and its inverse γ^{ab} to lower and rise indices whenever it is required. In particular, we have $T_a = \gamma_{ab}T^b$ and $N^a = \gamma^{ab}N_b$.

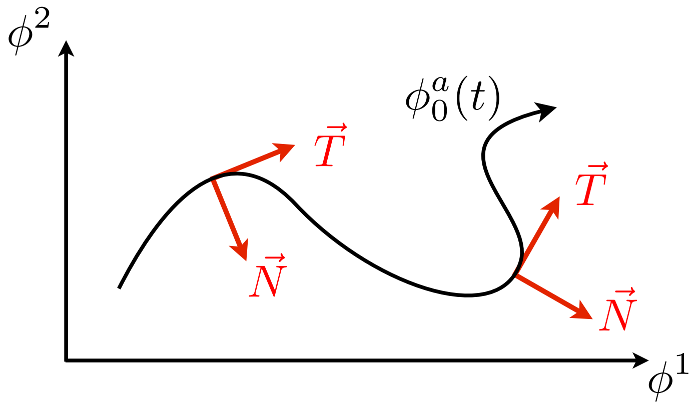

Figure 2. Relative orientation of the vector fields T^a and N^a defined with respect to the background solution $\phi_0^a(t)$.

to project the scalar field equations of motion in (2.7) along the two orthogonal directions. Projecting along T^a, one finds:

$$\ddot{\phi}_0 + 3H\dot{\phi}_0 + V_\phi = 0\,, \tag{2.14}$$

where $V_\phi \equiv \nabla_\phi V$, with $\nabla_\phi \equiv T^a \partial_a$. On the other hand, projecting along N^a, one obtains the relation

$$\frac{DT^a}{dt} = -\frac{V_N}{\dot{\phi}_0} N^a, \tag{2.15}$$

where $V_N = N^a \partial_a V$. It is customary to define dimensionless parameters accounting for the time variation of various background quantities. These are the so called slow-roll parameters ϵ, η^a and we define them as:

$$\epsilon \equiv -\frac{\dot{H}}{H^2}\,, \qquad \eta^a \equiv -\frac{1}{H\dot{\phi}_0}\frac{D\dot{\phi}_0^a}{dt}\,. \tag{2.16}$$

Notice that η^a is a two dimensional vector field telling us how fast $\dot{\phi}_0^a$ is changing in time. We may decompose η^a along the normal and tangent directions by introducing two independent parameters $\eta_{||}$ and $\eta_\perp$ as

$$\eta^a = \eta_{||} T^a + \eta_\perp N^a. \tag{2.17}$$

Then, one finds that

$$\eta_{||} = -\frac{\ddot{\phi}_0}{H\dot{\phi}_0}\,, \tag{2.18}$$

$$\eta_\perp = \frac{V_N}{\dot{\phi}_0 H}\,. \tag{2.19}$$

Notice that $\eta_{||}$ may be recognized as the usual η slow-roll parameter in single field inflation. On the other hand $\eta_\perp$ tells us how fast T^a rotates in time, and therefore it parametrizes the rate of turn of the trajectory followed by the scalar field dynamics. This may be seen more clearly by using (2.15) together with (2.19) to deduce the following relations:

$$\frac{DT^a}{dt} = -H\eta_\perp N^a, \tag{2.20}$$

$$\frac{DN^a}{dt} = +H\eta_\perp T^a. \tag{2.21}$$

Thus, if $\eta_\perp = 0$, the vectors T^a and N^a remain constant along the path. On the other hand, if $\eta_\perp > 0$, the path turns to the left, whereas if $\eta_\perp < 0$ the turn is towards the right. The value of $\eta_\perp$ is therefore telling us how quickly the angle determining the orientation of T^a is varying in time. By calling this angle θ we may therefore do the identification

$$\dot{\theta} \equiv H\eta_\perp\,. \tag{2.22}$$

With the help of this definition, one deduces that the ratio of curvature κ characterizing the turning trajectory, is given by

$$\kappa^{-1} \equiv |\dot{\theta}|/\dot{\phi}_0\,. \tag{2.23}$$

As in conventional single-field inflation, the background dynamics may be understood in terms of the values of the dimensionless parameters ϵ, $\eta_{||}$ and $\eta_\perp$. For instance, slow roll inflation will happen as long as:

$$\epsilon \ll 1\,, \qquad |\eta_{||}| \ll 1\,. \tag{2.24}$$

These two conditions ensure that both H and $\dot{\phi}_0$ evolve slowly during the period of interest. On the other hand, it is interesting to notice that a large variation of $\eta_\perp$ does not necessarily imply a violation of the slow-roll regime (2.24). We will analyze this statement in full detail in section 4 where we study the effect of sharp sudden turns on the dynamics of this class of system.

2.2 Perturbation theory

We now consider the dynamics of scalar perturbations parametrizing departures from the homogeneous and isotropic background $a(t)$ and $\phi_0^a(t)$. This may be done by defining perturbations $\delta\phi^a(t,\mathbf{x})$ as:

$$\phi^a(t,\mathbf{x}) = \phi_0^a(t) + \delta\phi^a(t,\mathbf{x})\,. \tag{2.25}$$

Instead of directly working with $\delta\phi^a(t,\mathbf{x})$, it is more convenient to work with gauge invariant fields v^T and v^N given by[6]

$$v^T = a\,T_a\delta\phi^a + a\frac{\dot{\phi}}{H}\psi\,, \tag{2.26}$$

$$v^N = a\,N_a\delta\phi^a, \tag{2.27}$$

where ψ is the scalar perturbation of the spatial part of the metric (proportional to δ_{ij}) in flat gauge. It is useful to consider a second set of fields (u^X, u^Y) in addition to (v^T, v^N). Let us consider the following time dependent rotation in field space

$$\begin{pmatrix} u^X \\ u^Y \end{pmatrix} \equiv R(\tau)\begin{pmatrix} v^N \\ v^T \end{pmatrix}, \tag{2.28}$$

[6]Notice that these fields are projections of the form $v^T = aT_aQ^a$ and $v^T = aT_aQ^a$ where the Q^a fields are the usual Mukhanov-Sasaki variables $Q^a \equiv \delta\phi^a + \frac{\dot{\phi}^a}{H}\psi$ [75, 76].

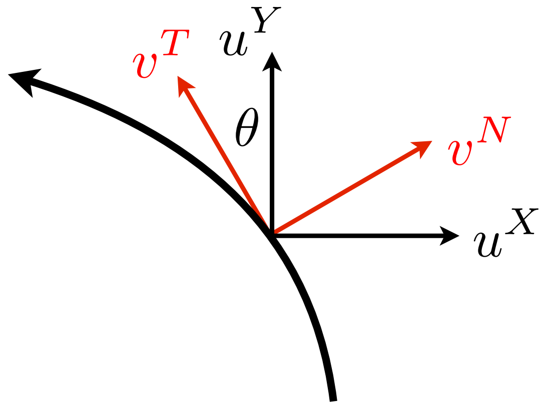

Figure 3. The u-fields represent fluctuations with respect to a fixed local frame, whereas the v-fields represent fluctuations with respect to the path (parallel and normal).

where the time dependent rotation matrix $R(\tau)$ is defined as

$$R(\tau) = \begin{pmatrix} \cos\theta(\tau) & -\sin\theta(\tau) \\ \sin\theta(\tau) & \cos\theta(\tau) \end{pmatrix}, \qquad \theta(\tau) = \theta_0 + \int_{-\infty}^{\tau} d\tau \, aH\eta_\perp \,, \tag{2.29}$$

where θ_0 is the value of $\theta(\tau)$ at $\tau \to -\infty$. The rotation angle $\theta(\tau)$ precisely accounts for the total angle covered by all the turns during the inflationary history up to time τ, and coincides with the definition introduced in eq. (2.22). Figure 3 illustrates the relation between the v-fields introduced earlier and the canonical u-fields. To continue, the equations of motion for the canonically normalized fields are

$$\frac{d^2 u^I}{d\tau^2} - \nabla^2 u^I + \left[R(\tau)\Omega R^t(\tau)\right]^I{}_J u^J = 0\,, \qquad I = X, Y, \tag{2.30}$$

where R^t represents the transpose of R. In addition, Ω is the mass matrix for the v-fields, with elements given by

$$\Omega_{TT} = -a^2H^2(2 + 2\epsilon - 3\eta_{||} + \eta_{||}\xi_{||} - 4\epsilon\eta_{||} + 2\epsilon^2 - \eta_\perp^2)\,, \tag{2.31}$$

$$\Omega_{NN} = -a^2H^2(2-\epsilon) + a^2(V_{NN} + H^2\epsilon\mathbb{R})\,, \tag{2.32}$$

$$\Omega_{TN} = a^2H^2\eta_\perp(3 + \epsilon - 2\eta_{||} - \xi_\perp)\,, \tag{2.33}$$

where $\xi_{||} = -\dot\eta_{||}/(H\eta_{||})$ and $\xi_\perp = -\dot\eta_\perp/(H\eta_\perp)$. Additionally, we have defined the tree level mass V_{NN} as the second derivative of the potential projected along the perpendicular direction $V_{NN} = N^aN^b\nabla_a\nabla_b V$. To finish, expanding the original action (2.1) to quadratic order in terms of the u-fields, one finds:

$$S = \frac{1}{2}\int d\tau d^3x \left\{ \sum_I \left(\frac{du^I}{d\tau}\right)^2 - (\nabla u^I)^2 - \left[R(\tau)\Omega R^t(\tau)\right]^I{}_J u^I u^J \right\}. \tag{2.34}$$

Thus, we see that the fields $u^I = (u^X, u^Y)$ correspond to the canonically normalized fields in the usual sense. Given that these fields are canonically normalized, it is now straightforward to impose Bunch-Davies conditions on the initial state of the perturbations.

2.3 Curvature and isocurvature modes

Another useful field parametrization for the perturbations is in terms of curvature and isocurvature fields $\mathcal{R}$ and $\mathcal{S}$ [39]. In terms of the v-fields, these are defined as:

$$\mathcal{R} = \frac{H}{a\dot{\phi}} v^T, \tag{2.35}$$

$$\mathcal{S} = \frac{H}{a\dot{\phi}} v^N. \tag{2.36}$$

Instead of working directly with $\mathcal{S}$, it is in fact more convenient to define:

$$\mathcal{F} = \frac{\dot{\phi}_0}{H}\mathcal{S}\,. \tag{2.37}$$

Then, the quadratic action for the pair $\mathcal{R}$ and $\mathcal{F}$ is found to be

$$S_{\rm tot} = \frac{1}{2}\int d^4x\, a^3 \left[\frac{\dot{\phi}_0^2}{H^2}\dot{\mathcal{R}}^2 - \frac{\dot{\phi}_0^2}{H^2}\frac{(\nabla\mathcal{R})^2}{a^2} + \dot{\mathcal{F}}^2 - \frac{(\nabla\mathcal{F})^2}{a^2} + 4\dot{\phi}_0\eta_\perp\dot{\mathcal{R}}\mathcal{F} - M_{\rm eff}^2\mathcal{F}^2\right], \tag{2.38}$$

where we have defined the effective mass $M_{\rm eff}$ of the heavy field $\mathcal{F}$ as

$$M_{\rm eff}^2 = V_{NN} + H^2\epsilon\mathbb{R} - \dot{\theta}^2, \tag{2.39}$$

(recall that $\dot{\theta} = H\eta_\perp$). It may be noticed that the reason behind the appearance of the term $-\dot{\theta}^2$ in $M_{\rm eff}^2$ is due to the fact that the potential receives a correction coming from the centripetal force experimented by the turn. This introduces a centrifugal barrier to the effective potential felt by the heavy modes. The equations of motion for this system of fields is then

$$\ddot{\mathcal{R}} + (3 + 2\epsilon - 2\eta_{||})H\dot{\mathcal{R}} - \frac{\nabla^2\mathcal{R}}{a^2} = -2\frac{H^2}{\dot{\phi}_0}\eta_\perp\left[\dot{\mathcal{F}} + (3 - \eta_{||} - \xi_\perp)H\mathcal{F}\right], \tag{2.40}$$

$$\ddot{\mathcal{F}} + 3H\dot{\mathcal{F}} - \frac{\nabla^2\mathcal{F}}{a^2} + M_{\rm eff}^2\mathcal{F} = 2\dot{\phi}_0\eta_\perp\dot{\mathcal{R}}\,. \tag{2.41}$$

Notice that the configuration $\mathcal{R} =$ constant and $\mathcal{F} = 0$ constitutes a non trivial solution to the system of equations. Since $\mathcal{F}$ is assumed to be heavy, the configuration $\mathcal{F} = 0$ is reached shortly after horizon exit, and the curvature mode $\mathcal{R}$ will necessarily become frozen. For this reason, in the presence of mass hierarchies, we may only concern ourselves with curvature perturbations and disregard isocurvature components after inflation.

2.4 Power spectrum

From the observational point of view, the main quantities of interest coming from inflation are its predicted n-point correlation functions characterizing fluctuations. These quantities provide all the relevant information about the expected distribution of primordial inhomogeneities that seeded the observed CMB anisotropies. It is of particular interest to compute two-point correlation functions, corresponding to the variance of inhomogeneities' distribution. To deduce such quantities we have to consider the quantization of the system, and this may be achieved by expanding the canonical pair u^X and u^Y in terms of creation and annihilation operators $a_\alpha^\dagger(\mathbf{k})$ and $a_\alpha(\mathbf{k})$ respectively, as

$$u^I(\tau, x) = \int \frac{d^3k}{(2\pi)^{3/2}} \sum_\alpha \left[e^{i\mathbf{k}\cdot\mathbf{x}} u_\alpha^I(\mathbf{k},\tau) a_\alpha(\mathbf{k}) + e^{-i\mathbf{k}\cdot\mathbf{x}} u_\alpha^{I*}(\mathbf{k},\tau) a_\alpha^\dagger(\mathbf{k})\right], \tag{2.42}$$

where $\alpha = 1, 2$ labels the two modes to be encountered by solving the second order differential equations for the fields $u^I_\alpha(\mathbf{k}, \tau)$. In order to satisfy the conventional field commutation relations, the mode solutions need to satisfy the additional constraints consistent with the equations of motion:[7]

$$\sum_\alpha \left(u^I_\alpha \frac{u^{J*}_\alpha}{d\tau} - u^{I*}_\alpha \frac{u^J_\alpha}{d\tau} \right) = i\delta^{IJ}. \tag{2.43}$$

By examining the action (2.34) one sees that in the short wavelength limit $k^2/a^2 \gg \Omega$, where Ω symbolizes both eigenvalues of the matrix Ω, the equation of motion for the u-fields reduce to

$$\frac{d^2 u^I}{d\tau^2} - \nabla^2 u^I = 0\,, \qquad I = X, Y, \tag{2.44}$$

allowing us to choose the following initial conditions for the fields

$$u^X_\alpha(\mathbf{k}, \tau) = \frac{e^{-ik\tau}}{\sqrt{2k}} \delta^1_\alpha\,, \qquad u^Y(\mathbf{k}, \tau) = \frac{e^{-ik\tau}}{\sqrt{2k}} \delta^2_\alpha\,. \tag{2.45}$$

Notice that here we have chosen to associate the initial state $\alpha = 1$ with the field direction X and $\alpha = 2$ with the field direction Y. This identification is in fact completely arbitrary and does not affect the computation of two-point correlation functions. In other words, we could modify the initial state (2.45) by considering an arbitrary (time independent) rotation on the right hand side, without changing the prediction of observables.

Then, given the set of solutions u^X_α and u^Y_α, one finds that the two-point correlation function associated to curvature modes $\mathcal{R}$ is given by:

$$\mathcal{P}_\mathcal{R}(k, \tau) = \frac{k^3}{2\pi^2} \sum_\alpha \mathcal{R}_\alpha(k, \tau) \mathcal{R}^*_\alpha(k, \tau)\,, \tag{2.46}$$

where $\mathcal{R}_\alpha$ is related to the pair u^X_α and u^Y_α by the field redefinitions described in the previous sections. When (2.46) is evaluated at the end of inflation, for wavelengths k far away from the horizon ($k/a \ll H$), it corresponds to the power spectrum of curvature modes. For completeness, let us mention that one may also define the two-point correlation function $\mathcal{P}_\mathcal{S}(k, \tau)$ and the cross-correlation function $\mathcal{P}_{\mathcal{R}\mathcal{S}}(k, \tau)$ in analogous ways. But, as previously stated, because of the assumed mass hierarchy these contributions may be completely disregarded.

3 Effective field theory

It is possible to deduce an effective theory for the curvature mode $\mathcal{R}$ by integrating out the heavy field $\mathcal{F}$ when $M^2_{\text{eff}} \gg H^2$, provided that certain additional conditions are met. To see this, let us first briefly analyze the expected evolution of the fields $\mathcal{R}$ and $\mathcal{F}$ when the trajectory is turning at a constant rate ($\dot\theta$ = constant). To start with, because we are dealing with a coupled system of equations for $\mathcal{R}$ and $\mathcal{F}$, in general we expect the general solutions for $\mathcal{R}$ and $\mathcal{F}$ to be of the form [65]

$$\mathcal{R} \sim \mathcal{R}_+ e^{-i\omega_+ t} + \mathcal{R}_- e^{-i\omega_- t}, \tag{3.1}$$

$$\mathcal{F} \sim \mathcal{F}_+ e^{-i\omega_+ t} + \mathcal{F}_- e^{-i\omega_- t}, \tag{3.2}$$

[7] See refs. [41] and [26] for a more detailed discussion of the quantization of these type of system.

where ω_+ and ω_- denote the two frequencies at which the modes oscillate. The values of ω_+ and ω_- will depend on the mode's wave number k in the following way: in the regime $k/a \gg M_{\rm eff}$, both fields are massless and therefore oscillate with frequencies of order $\sim k/a$. As the wavelength enters the intermediate regime $M_{\rm eff} \gg k/a \gg H$ the degeneracy of the modes break down and the frequencies become of order

$$\omega_- \sim k/a \,, \qquad \omega_+ \sim M_{\rm eff} \,. \tag{3.3}$$

Subsequently, when the modes enter the regime $k/a < H$ the contributions coming from ω_+ will quickly decay and the contributions coming from ω_- will freeze (since they are massless).

Notice that the amplitudes $\mathcal{R}_+$ and $\mathcal{F}_-$ necessarily arise from the couplings mixing curvature and isocurvature perturbations, and therefore they vanish in the case $\eta_\perp = \dot\theta/H = 0$. Additionally, on general grounds, the amplitudes $\mathcal{F}_+$ and $\mathcal{R}_+$ are expected to be parametrically suppressed by $k/M_{\rm eff}$ in the regime $M_{\rm eff} \gg k/a$, and therefore we may disregard high frequency contributions to (3.1) and (3.2). Then, in the regime $M_{\rm eff} \gg k/a$, time derivatives for $\mathcal{F}$ can be safely ignored in the equation of motion (2.41) and we may write

$$-\frac{\nabla^2 \mathcal{F}}{a^2} + M_{\rm eff}^2 \mathcal{F} = 2\dot\phi_0 \eta_\perp \dot{\mathcal{R}} \,. \tag{3.4}$$

(Because $H \ll M_{\rm eff}$, we may also disregarded the friction term $3H\dot{\mathcal{F}}$). This leads to an algebraic relation between $\mathcal{F}$ and $\dot{\mathcal{R}}$ in Fourier space given by:

$$\mathcal{F}_{\mathcal{R}} = \frac{2\dot\phi_0 \eta_\perp \dot{\mathcal{R}}}{(k^2/a^2 + M_{\rm eff}^2)} \,, \tag{3.5}$$

which precisely tells us the dependence of low frequency contributions $\mathcal{F}_-$ in terms $\mathcal{R}_-$ defined in eqs. (3.1) and (3.2). To continue, we notice that (3.4) is equivalent to disregard the term $\dot{\mathcal{F}}^2$ of the kinetic term in the action (2.38). Keeping this in mind, we can replace (3.5) back into the action and obtain an effective action for the curvature perturbation $\mathcal{R}$, given by[8]

$$S_{\rm tot} = \frac{1}{2}\int d^4x \, a^3 \frac{\dot\phi_0^2}{H^2} \left[\frac{\dot{\mathcal{R}}^2}{c_s^2(k)} - \frac{k^2 \mathcal{R}^2}{a^2} \right] , \tag{3.6}$$

where c_s is the speed of sound of adiabatic perturbations, given by:

$$c_s^{-2}(k) = 1 + \frac{4H^2\eta_\perp^2}{k^2/a^2 + M_{\rm eff}^2} \,. \tag{3.7}$$

In deriving this expression we have assumed that $\dot\theta$ remained constant. In the more general case where $\dot\theta$ is time dependent we expect transients that could take the system away from the simple behavior shown in eqs. (3.1) and (3.2), and the effective field theory could become invalid. The validity of the effective theory will depend on whether the kinetic terms for $\mathcal{F}$ in eq. (2.41) can be ignored, and this implies the following condition on $\mathcal{F}_{\mathcal{R}}$ of eq. (3.5):

$$|\ddot{\mathcal{F}}_{\mathcal{R}}| \ll M_{\rm eff}^2 |\mathcal{F}_{\mathcal{R}}| \,. \tag{3.8}$$

[8]This way of integrating heavy modes has also been employed to deduce an effective theory for the linear propagation of gravitons in bigravity theories [77], where a massive graviton kinetically interacts with the massless one. When the massive graviton is integrated out, one is left with a massless graviton with a non-trivial speed of propagation that depends on the background.

Now, recall that unless there are large time variations of background quantities, the frequency of $\mathcal{R}$ is of order $\omega_- \sim k/a$. Thus, any violation of condition (3.8) will be due to the evolution of background quantities, which will be posteriorly transmitted to $\mathcal{R}$. This allows us to ignore higher derivatives of $\dot{\mathcal{R}}$ in (3.8) and simply rewrite it in terms of background quantities as:

$$\left| \frac{d^2}{dt^2} \left(\frac{2\dot{\phi}_0 \eta_\perp}{(k^2/a^2 + M_{\rm eff}^2)} \right) \right| \ll M_{\rm eff}^2 \left| \frac{2\dot{\phi}_0 \eta_\perp}{(k^2/a^2 + M_{\rm eff}^2)} \right| . \tag{3.9}$$

This relation expresses the adiabaticity condition that each mode k needs to satisfy in order for the effective field theory to stay reliable. To further simplify this relation, we may take into consideration the following points: (1) when $k^2/a^2 \gg M_{\rm eff}^2$ the two modes decouple (recall eq. (2.44)) and the turn has no influence on the evolution of curvature modes. On the other hand, in the regime $k^2/a^2 \lesssim M_{\rm eff}^2$, contributions coming from the time variation of k^2/a^2 are always suppressed compared to $M_{\rm eff}^2$ due to the fact that we are assuming $H^2 \ll M_{\rm eff}^2$. (2) We observe that the main background quantity parametrizing the rate at which the turn happens is $\eta_\perp = \dot{\theta}/H$. Quantities such as $\dot{\phi}_0$ and H, which describe the evolution of the background along the trajectory, will therefore only be affected by the turn through the time dependence of $\eta_\perp$. This implies that time derivatives of these quantities will be less sensitive to the turn than $\eta_\perp$ itself, and therefore their contribution to (3.9) will necessarily be subsidiary.[9] (3) Similarly, the rate of change of $M_{\rm eff}^2$ will necessarily be at most of the same order than $\dot{\theta}$. Then, by neglecting time derivatives coming from $\dot{\phi}_0$, H, k^2/a^2 and $M_{\rm eff}$ and focussing on the order of magnitude of the various quantitates appearing in (3.9) we can write instead a simpler expression given by:

$$\left| \frac{d^2}{dt^2} \dot{\theta} \right| \ll M_{\rm eff}^2 |\dot{\theta}| . \tag{3.10}$$

Actually, a simpler alternative expression may be obtained by conveniently reducing the number of time derivatives, and disregarding effects coming from the change in sign of $\dot{\theta}$:

$$\left| \frac{d}{dt} \ln \dot{\theta} \right| \ll M_{\rm eff} . \tag{3.11}$$

This adiabaticity condition simply states that the rate of change of the turn's angular velocity must stays suppressed with respect to the masses of heavy modes, which otherwise would become excited. Notice that, we may also choose to express this relation in terms of the variation of the speed of sound, which is a more natural quantity from the point of view of the effective field theory:

$$\left| \frac{d}{dt} \ln(c_s^{-2} - 1) \right| \ll M_{\rm eff} . \tag{3.12}$$

To finish, we can estimate the order of magnitude of departures between the full solution for $\mathcal{R}$ and the one appearing in the EFT. For this, let us write $\mathcal{F} = \mathcal{F}_\mathcal{R} + \delta\mathcal{F}$ where $\mathcal{F}_\mathcal{R}$ is the adiabatic result of (3.5) and $\delta\mathcal{F}$ denotes a departure from this value. Then $\delta\mathcal{F}$ respects the following equation of motion:

$$\delta\ddot{\mathcal{F}} + 3H\delta\dot{\mathcal{F}} + \left(\frac{k^2}{a^2} + M_{\rm eff} \right) \delta\mathcal{F} = -(\ddot{\mathcal{F}}_\mathcal{R} + 3H\dot{\mathcal{F}}_\mathcal{R}) . \tag{3.13}$$

[9]Here we are implicitly assuming that *parallel* quantities such as $\dot{\phi}_0$ and H will not have variations larger than $\eta_\perp$ due to other effects, unrelated to the turns (such as steps in the potential).

Given that we are assuming that the behavior of the system is dominated by low frequency modes, we can consistently disregard the kinetic term $\delta\ddot{\mathcal{F}} + 3H\delta\dot{\mathcal{F}}$ at the left hand side of this equation, but we cannot disregard the term $-(\ddot{\mathcal{F}}_{\mathcal{R}} + 3H\dot{\mathcal{F}}_{\mathcal{R}})$ at the right hand side, which constitutes a source for $\delta\mathcal{F}$. Then, we deduce that

$$\delta\mathcal{F} = -\frac{\ddot{\mathcal{F}}_{\mathcal{R}} + 3H\dot{\mathcal{F}}_{\mathcal{R}}}{k^2/a^2 + M_{\text{eff}}} \,. \tag{3.14}$$

Then, we may compute the derivatives appearing in the right hand side by using the effective equation of motion (coming from the variation of the effective action (3.6)) to express $\ddot{\mathcal{R}}$ in terms of $\dot{\mathcal{R}}$ whenever it becomes necessary. We obtain

$$\mathcal{F} = \mathcal{F}_{\mathcal{R}}\left[1 + \mathcal{O}\left(\frac{(\ddot{\theta}/\dot{\theta})^2}{k^2/a^2 + M_{\text{eff}}^2}\right) + \mathcal{O}\left(\frac{\delta_{||}H^2}{k^2/a^2 + M_{\text{eff}}^2}\right)\right], \tag{3.15}$$

where $\delta_{||}$ represents terms of order ϵ and $\eta_{||}$. Finally, eq. (3.15) allows us to deduce that the EFT expressed in (3.6) is only accurate up to operators of the form:

$$S_{\text{tot}} = S_{\text{eff}} + \frac{1}{2}\int d^4x\, a^3\, (c_s^{-2} - 1)\, \dot{\mathcal{R}}^2 \left[\mathcal{O}\left(\frac{(\ddot{\theta}/\dot{\theta})^2}{M_{\text{eff}}^2}\right) + \mathcal{O}\left(\frac{\delta_{||}H^2}{M_{\text{eff}}^2}\right)\right]. \tag{3.16}$$

This result expresses the validity of the effective field theory (3.6), and shows with eloquence the order of magnitude of the expected discrepancy with the exact two field solution for $\mathcal{R}$. In the following section we verify that indeed the adiabatic condition (3.11) constitutes a good guide to discriminate the validity of the effective theory (3.6).

4 Turning trajectories

We now study the consequences of turning trajectories on the dynamics of fluctuations. We start this analysis by considering the particular case of sudden turns, where the potential $V(\phi)$ and/or the sigma model metric γ_{ab} entering the action (2.1) are such that the inflationary trajectory becomes non-geodesic for a brief period of time, smaller than an e-fold. In order to characterize this class of turns it is useful to introduce the arc distance $\Delta\phi$ covered by the scalar field's v.e.v. in target space, when the turn takes place. Given the radius of curvature κ characterizing a turn, defined in eq. (2.23), we may define $\Delta\phi$ through the relation

$$\Delta\phi \equiv \kappa|\Delta\theta|\,, \tag{4.1}$$

where $\Delta\theta = \int H\eta_\perp dt$ corresponds to the total angle covered during a sudden turn. It is clear that in two-field canonical models $\Delta\phi$ can be at most of order κ ($\Delta\phi \lesssim \kappa$), and that in order to have turns with $\Delta\phi \gg \kappa$ one needs a non-canonical model with a scalar geometry with a topology allowing for such situations. Figure 4 shows various examples of turns according to the total angle $\Delta\theta$ covered by a turn (which is determined by the relative size of $\Delta\phi$ and κ).

Notice that the arc length $\Delta\phi$ implies a timescale $T_\perp$ characterizing the overall duration of a turn. This is simply given by:

$$T_\perp \equiv \frac{\Delta\phi}{\dot{\phi}}\,. \tag{4.2}$$

Then, because $\eta_\perp = \dot{\theta}/H \simeq \Delta\theta/(T_\perp H)$, we can use eq. (4.1) to derive the following estimation for the value of $\eta_\perp$ characterizing a particular turn:

$$\eta_\perp \sim \frac{1}{HT_\perp}\frac{\Delta\phi}{\kappa}\,. \tag{4.3}$$

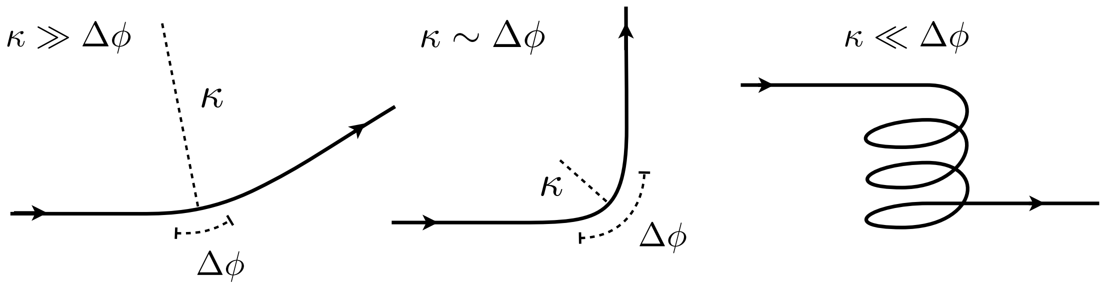

Figure 4. Examples of turns according to the relative size of $\Delta\phi$ and κ. In the case $\kappa \ll \Delta\phi$ the target space requires a non trivial topology.

Notice that a turn requires that the inflationary trajectory departs from the flat minima of the potential (otherwise $V_N = 0$ and eq. (2.19) would require $\eta_\perp = 0$). Then, because the turn happens suddenly during a brief period of time, the various interactions present in the theory will inevitably make the background trajectory oscillate about the flat locus of the potential, with a period T_M given by

$$T_M \equiv \frac{1}{M_{\rm eff}} \, . \tag{4.4}$$

In other words, a turn cannot be arbitrarily sharp without waking up these background fluctuations. Recall that we are interested in models where $M_{\rm eff} \gg H$, and therefore we necessarily have $T_M \ll H^{-1}$. If present, it is clear that such oscillations will dominate the behavior of the trajectory whenever T_M is of the same order or larger than $T_\perp$ ($T_M \gtrsim T_\perp$). In fact, the adiabaticity condition (3.11) precisely translates into the following condition involving these two timescales

$$T_\perp \gg T_M \, , \tag{4.5}$$

which is consistent with the notion that the effective field theory will remain valid as long as heavy fluctuations are not participating of the low energy dynamics.

4.1 Displacement from the flat minima

Given a certain turn (characterized by $\Delta\phi$ and κ) we can estimate the perpendicular displacement of the trajectory away from the flat minima of the potential while the turn takes place. By calling this quantity Δh, we can roughly relate it to other background quantities through the relation $V_N \simeq M_{\rm eff}^2 \Delta h$. Then, inserting this result back into eq. (2.19) we obtain

$$\frac{\Delta h}{\kappa} \simeq \frac{\Delta\phi^2}{\kappa^2} \frac{T_M^2}{T_\perp^2} \, , \tag{4.6}$$

where we made use of eqs. (4.2) and (4.4). This relation gives us the relative size of background oscillations Δh compared to the radius of curvature κ of the turn. We see that the size of the displacement depends on the total angle covered by the turn $\Delta\theta = \Delta\phi/\kappa$ and the ratio $T_M/T_\perp$ between the two relevant timescales. In what follows we briefly analyze the two relevant regimes posed by these two timescales.

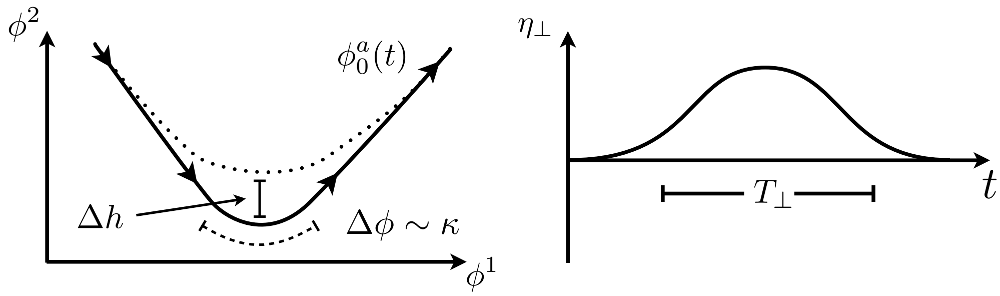

Figure 5. The figure illustrates the case of a turn for which the adiabatic condition $T_M \ll T_\perp$ is respected. In this case the turn happens adiabaticaly, in the sense that the timescale T_M plays no role during the turn. If $\Delta h/\kappa \sim 1$ the displacement from the flat minima is large and the speed of sound will be reduced considerably.

4.2 Adiabatic turns $T_M \ll T_\perp$

If the turn is such that $T_M \ll T_\perp$, then the adiabatic condition (3.11) is satisfied and the system admits an effective field theory of the form (3.6) as deduced in the previous section. This means that we can parametrize effects in terms of a reduced speed of sound c_s, which synthesizes all the nontrivial information regarding the heavy physics. Putting together eqs. (3.7) and (4.3) we see that the speed of sound is given by

$$c_s^{-2} \simeq 1 + \frac{\Delta\phi^2}{\kappa^2}\frac{T_M^2}{T_\perp^2}\,. \tag{4.7}$$

Given that the effective theory requires $T_M \ll T_\perp$, the only way of having large non-trivial departures from conventional single field inflation is by having a large ratio $\Delta\phi/\kappa \gg 1$. This implies that the total angle $\Delta\theta = \Delta\phi/\kappa$ covered by the turn must extend for several cycles, and the only way of achieving this consistently is by considering models with non-trivial sigma model metrics. In particular, to produce sizable changes of the speed of sound (say of order one) we require:

$$\frac{\Delta\phi^2}{\kappa^2} \gtrsim \frac{T_\perp^2}{T_M^2}\,. \tag{4.8}$$

Comparing with (4.6), we see that this is equivalent to have a large displacement from the flat minima of the potential. However, because the timescale T_M is much smaller than $T_\perp$, the displacement happens adiabatically, and background fluctuations are not turned on. Correspondingly, the dimensionless parameter $\eta_\perp$ is a smooth function of time with a characteristic timescale given by $T_\perp$. Figure 5 illustrates this situation.

4.3 Non-adiabatic turns $T_M \gtrsim T_\perp$

In this case the trajectory moves away from the flat minima of the potential quickly, and the background trajectory will inevitably start oscillating about this locus. The amplitude of these oscillations will be given by Δh as in eq. (4.6). If $\Delta h \gtrsim \kappa$ then the oscillations are large and they completely dominate the behavior of the background trajectory. Needless to say, the effective field theory would offer a poor representation of the evolution of curvature perturbations, and the original two-field theory would be needed to study the system. Figure 6(a) shows this type of situation for the case $T_\perp \sim T_M$. There, the trajectory is subject

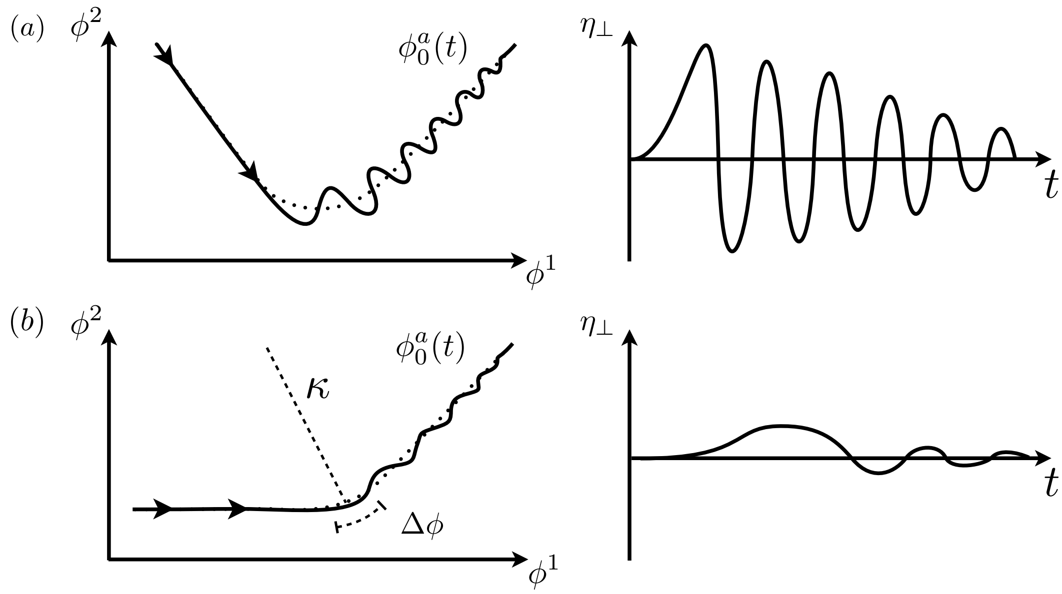

Figure 6. Examples of non-adiabatic turns characterized by $T_M \ll T_\perp$. Case (a) shows a typical example in which $\Delta\phi_0 \sim \kappa$, whereas case (b) shows a situation for which $\Delta\phi_0 \ll \kappa$.

to an initial kick that lasts T_M, and after that continues oscillating at a period given by T_M. Consequently, heavy oscillations will overtake the background trajectory, which translates into a heavily oscillating $\eta_\perp$ as a function of time. This means that instead of a single turn we end up with a succession of turns. However, after these transient turns have taken place, the overall angle $\Delta\theta = \int H\eta_\perp dt$ will correspond to the turn determined by the flat minima of the potential.

On the other hand, if $\Delta h \ll \kappa$, then the amplitude of the oscillations are small. In this case $\Delta\phi/\kappa \ll T_\perp/T_M$, implying, after considering eq. (4.3), that the angular velocity is small compared to the mass of heavy modes:

$$H|\eta_\perp| = |\dot{\theta}| \ll M_{\rm eff} \, . \tag{4.9}$$

In this case the impact of background oscillations on the evolution of perturbations is negligible. Despite of this fact, since we are in the regime $T_M \gtrsim T_\perp$, the effective field theory continues to offer a poor representation of these effect, no matter how tiny they are. Figure 6(b) illustrates this situation.

5 Examples of models with turns

We now study examples of models where turns play a relevant role on the evolution of perturbations. We will first consider the case of canonical models — in which turns are uniquely due to the shape of the potential — and later consider the case of models where the turns are due to the specific shape of the sigma model metric.

5.1 Model 1: sudden turns in canonical models

Let us start by considering the case of canonical models ($\gamma_{ab} = \delta_{ab}$) in which the turn is due solely to the shape of the potential. To simplify the present analysis, we only consider the

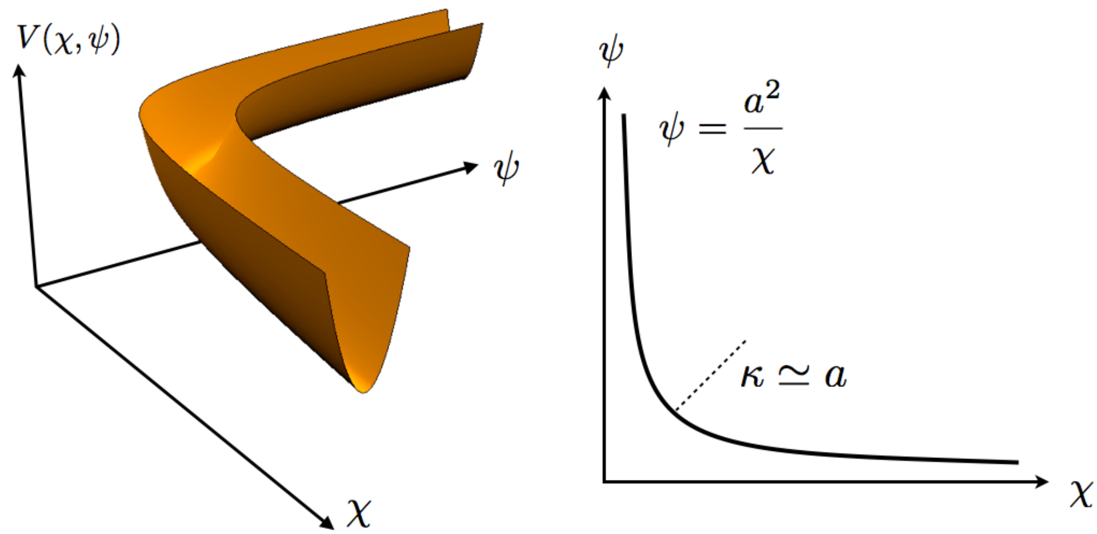

Figure 7. The figure shows the section of the potential $V(\chi, \psi)$ of equation (5.1) containing the turn.

relevant part of the potential where the turn takes place, and disregard any details concerning the end of inflation. Our aim is to understand what are the possible consequences of a turn on the generation of primordial inhomogeneities accessible to observations today. For this reason, we assume that the turn takes place precisely when presently accessible curvature perturbations were crossing the horizon, and choose cosmological parameters accordingly. Having said this, let us adopt the notation $\phi^a = (\chi, \psi)$ and consider the following potential:

$$V(\chi, \psi) = V_0 + V_\phi(\chi - \psi) + \frac{M^2}{2}\frac{(\chi\psi - a^2)^2}{(\chi + \psi)^2} + \cdots . \tag{5.1}$$

In this expression, V_0 and V_ϕ are constants parametrizing the hight and slope of the flat direction in the potential. The third term in (5.1) has been added to ensure that a turn takes place. Notice that the third term vanishes for the locus of points defined by the equation:

$$\psi = a^2/\chi . \tag{5.2}$$

Away from this curve the slope of the potential becomes steep, with a quadratic growth characterized by the mass parameter M. Figure 7 shows the relevant part of the potential $V(\chi, \psi)$ containing the turn. Because we are assuming a mass hierarchy, it may be anticipated that the inflationary trajectory will stay close to the curve defined by eq. (5.2). Roughly speaking, the turn is located about the position $(\chi, \psi) = (\sqrt{a}, \sqrt{a})$ and its radius of curvature is of order a:

$$\kappa \sim a . \tag{5.3}$$

The quantities V_0 and V_ϕ are chosen in such a way that the scalar fields v.e.v.'s approach the turn from the asymptotic direction $(\chi, \psi) \to (0, +\infty)$. Once the turn is left behind, the v.e.v.'s. continue evolving towards the asymptotic direction $(\chi, \psi) \to (+\infty, 0)$, until inflation ends (see figure 7).

5.1.1 Analysis of the model

Before studying the exact evolution of the system with the help of numerics, let us estimate the behavior of the system by examining the parameters entering the potential. First of all, if the potential is flat enough, the slow roll evolution of the fields imply that *away from the location of the turn* the following relations are satisfied:

$$3H\dot{\phi}_0 \simeq V_\phi \,, \tag{5.4}$$

$$3H^2 \simeq V_0 \,. \tag{5.5}$$

Notice that they imply that $\epsilon \simeq V_\phi^2/2V_0^2$. To continue, simple examination of the potential shows that the arc length $\Delta\phi$ characterizing the turn is of the same order than the radius of curvature κ which, as stated, is of order a. Thus, we have $\kappa \simeq \Delta\phi \simeq a$. Then, putting together the previous expressions we find that time $T_\perp$ characterizing the duration of the turn, is given by

$$T_\perp \simeq \frac{a}{\sqrt{2\epsilon}H} \,. \tag{5.6}$$

In terms of e-folds, the duration of the turn is given by

$$\Delta N_\perp \simeq \frac{a}{\sqrt{2\epsilon}} \,. \tag{5.7}$$

Then, using (4.3) we see that the value $\eta_\perp$ characterizing the turn is roughly given by:

$$\eta_\perp \simeq \frac{\sqrt{2\epsilon}}{a} \,. \tag{5.8}$$

With these relations at hand, we can now estimate the range of parameters for which the adiabatic condition (3.11) is satisfied. Using eq. (4.5) with $T_M = 1/M$, we deduce that the EFT will remain accurate as long as:

$$\alpha \equiv \frac{T_\perp^2}{T_M^2} = \frac{M^2 a^2}{2\epsilon H^2} \gg 1 \,. \tag{5.9}$$

Then, assuming that we are in the realms of validity of the theory, we find that the speed of sound is given by the following combination of parameters

$$c_s^{-2} \simeq 1 + \frac{8\epsilon H^2}{a^2 M^2} \,, \tag{5.10}$$

which, because of eq. (5.9), implies that $c_s \sim 1$, consistent with our general analysis of section 4.2.

5.1.2 Numerics

We now present our numerical analysis of this model. In order to study this model, we have chosen the following fixed values (in units of Planck masses) for parameters associated to the flat direction of the potential:

$$V_0 = 6.5 \times 10^{-9}, \qquad V_\phi = -5.4 \times 10^{-11}. \tag{5.11}$$

Away from the turn, this choice of parameters ensure the following value for the rate of expansion $H \sim 1.4 \times 10^{-5}$. Additionally, they imply values for the slow roll parameters given by

$$\epsilon \simeq -\eta \simeq 0.0033 \,, \tag{5.12}$$

consistent with a power spectrum for curvature perturbations with a spectral index given by $n_s \simeq 0.98$ and an amplitude satisfying COBE's normalization. The other two parameters left are a and M. These may be expressed in terms of $\Delta N_\perp$ and α introduced in eqs. (5.7) and (5.9).

Since we are interested in turns taking place during a period of time shorter than an e-fold ($T_\perp \lesssim H^{-1}$), we are required to choose $\Delta N_\perp \lesssim 1$. Figure 8 shows the numerical results for the fixed value $\Delta N_\perp = 0.25$ and three choices for the mass M, given by $\alpha = 6.25$, $\alpha = 25$ and $\alpha = 625$. On the left hand side of the figure we show the numerical solution for the $\eta_\perp$ and $\eta_{||}$ as functions of e-fold N. On the right hand side, we show the resulting power spectra for the relevant modes exiting the horizon when the turn takes place. For simplicity, we have normalized the power spectrum with respect to the amplitude determined by the largest scales (smallest values of k), which have been chosen to correspond to the largest scales characterizing horizon reentry today ($k_0 = 0.002\,\mathrm{Mpc}^{-1}$). The blue solid line corresponds to the complete-two dimensional theory, whereas the red dashed line corresponds to the result predicted by the effective field theory.

It may be seen that for $\alpha = 6.25$, the period of massive oscillations T_M is of the same order than $T_\perp$, and consequently the rate of turn $\eta_\perp$ is dominated by oscillations (notice that these oscillations have a period of about ~ 0.25 e-folds, in agreement with our choice of parameters). This indicates that indeed the inflationary trajectory stays oscillating about the minimum once the turn has occurred (recall case (a) of figure 6). It may be also observed that $\eta_{||}$ is considerably affected by these oscillations, momentarily acquiring values as large as $\eta_{||} \sim 1$. This is however not enough to break down the overall slow-roll behavior of the background solution for two reasons: first, the value of ϵ does not change much during the turn (that is ϵ is found to be less sensitive to the turn than $\eta_{||}$). And second, after the turn takes place, the system goes back to small values of $\eta_{||}$ quickly. Given that the adiabatic condition is far from being satisfied (α is close to 1), it comes to no surprise that both power spectra differ considerably.

The second case $\alpha = 25$ shows an intermediate situation where T_M is smaller than $T_\perp$ but still able to generate small oscillations about the minimum of the potential. In this case the adiabaticity condition is mildly violated, which is reflected on the small discrepancy between both power spectra. Finally, the case $\alpha = 625$ shows a situation where T_M is much smaller than $T_\perp$. In this case heavy modes are not excited enough and the turn happens smoothly, which is reflected on the behavior of both $\eta_\perp$ and $\eta_{||}$ as functions of e-folds. In addition, we now see that both power spectra agree considerably. It may call our attention the persistence of a large feature in the power spectra even for the case $\alpha = 625$ where both, versions of the theory agree. In this case the speed of sound $c_s \simeq 1$ during the turn and we infer that the feature cannot be due to the fast variation of the speed of sound. Instead, the feature is due to the large variation of parallel background quantities, specially $\eta_{||}$, as during the turn the trajectory is forced to go up and down. In the examples of the following model we will examine a rather different situation where the features in the power spectrum are due to large variations of the speed of sound c_s.

5.2 Model 2: sudden turns induced by the metric

We now study a case in which the turn is due to the sigma model metric γ_{ab}. We will adopt the following notation $\phi^1 = \chi$ and $\phi^2 = \psi$, and consider the following separable potential

$$V(\chi, \psi) = V_0 + V_\phi\, \chi + \frac{M_\psi^2}{2} \psi^2. \tag{5.13}$$

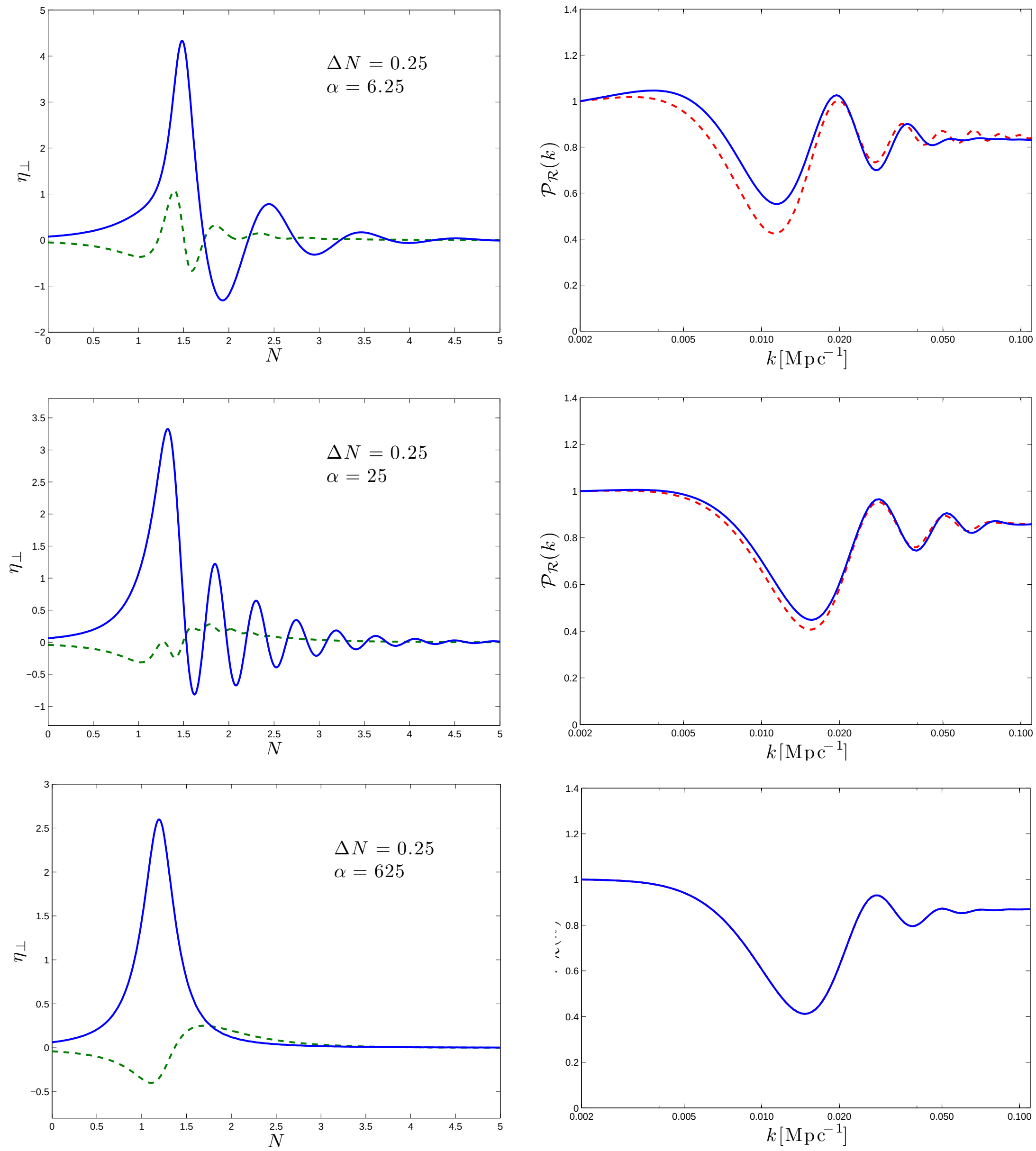

Figure 8. The figure shows $\eta_\perp$ and $\eta_{||}$ (left panels) and the resulting power spectra (right panels) for three choices of parameters for the potential of our model 1. From top to bottom: $\alpha = 6.25$, 25 and 625. In the case of the left panels, the blue solid line corresponds to $\eta_\perp$ whereas the green dashed line corresponds to $\eta_{||}$. In the case of right panels, the blue solid line corresponds to the power spectrum for the full two-field model, whereas the red dashed line corresponds to the power spectrum deduced using the EFT.

Just like in the previous example, V_0 and V_ϕ are constants parametrizing the flat part of the potential driving inflation. As before, we omit in our analysis any detail concerning the end of inflation. For this potential, if the system were canonical ($\gamma_{ab} = \delta_{ab}$), there would be no turns and the v.e.v. of the heavy field ψ would stay sitting at its minimum $\psi_0 = 0$. To produce a single turn with the help of the metric, we consider the following model:

$$\gamma_{ab} = \begin{pmatrix} 1 & \Gamma(\chi) \\ \Gamma(\chi) & 1 + \Gamma^2(\chi) \end{pmatrix}, \tag{5.14}$$

where

$$\Gamma(\chi) = \frac{\Gamma_0}{2}\left(1 + \tanh[2(\chi - \chi_0)/\Delta\chi]\right). \tag{5.15}$$

Notice that $\Gamma(\chi)$ is a function that grows monotonically from the asymptotic value $\Gamma = 0$ at $\chi \to -\infty$ to the asymptotic value $\Gamma = \Gamma_0$ at $\chi \to +\infty$. The transition takes place at $\chi = \chi_0$ and is characterized by the width parameter $\Delta\chi$. Notice that once Γ reach the constant value Γ_0, the metric becomes canonical again (which means that one can find a new parametrization of the fields in which $\gamma_{ab} = \delta_{ab}$). We will consider values of V_0 and V_ϕ such that the field χ evolves from $\chi \to -\infty$ to $\chi \to +\infty$.

5.2.1 Analysis of the model

It is clear that $\Gamma(\chi)$ parametrizes departures from the canonical configuration $\gamma_{ab} = \delta_{ab}$. The potential is such that it will force the trajectory to stay on the locus of points $\psi = 0$. However, since the geometry of the target space is non-trivial, the trajectory will be subject to a turn. To estimate the effects of the turn on the relevant background quantities, notice first that the timescale associated to heavy fluctuations about the minimum $\psi = 0$ is trivially given by:

$$T_M = \frac{1}{M_\psi}. \tag{5.16}$$

Second, since the trajectory remains close to $\psi = 0$ (due to the mass hierarchy), the arc length of the turn in target space $\Delta\phi$ will be equal to the width of the function $\Gamma(\chi)$:

$$\Delta\phi = \Delta\chi. \tag{5.17}$$

Then, it immediately follows that the timescale $T_\perp$ characterizing the duration of the turn is simply given by:

$$T_\perp = \frac{\Delta\chi}{\sqrt{2\epsilon}}, \tag{5.18}$$

where $\epsilon = V_\phi^2/2V_0^2$. Again, we may express this period of time terms of e-folds, which is given by

$$\Delta N_\perp \simeq \frac{\Delta\chi}{\sqrt{2\epsilon}}. \tag{5.19}$$

Then, in terms of the parameters of the model, the adiabaticity condition reads

$$\alpha \equiv \frac{T_\perp^2}{T_M^2} = \frac{M_\psi^2 \Delta\chi^2}{2\epsilon H^2} \gg 1. \tag{5.20}$$

We may compute the radius of curvature κ of the turn by noticing that the unit vectors associated to a curve following the minimum of the potential $\psi = 0$ are given by

$$T^a = (1, 0), \qquad N^a = (\Gamma, -1). \tag{5.21}$$

Plugging these expressions back into eq. (2.15) we find that the characteristic radius of curvature while the turn is at its pick, is given by:

$$\kappa = \frac{1}{|\partial_\chi \Gamma|} \simeq \frac{\Delta\chi}{|\Gamma_0|} \,. \tag{5.22}$$

This implies that $\eta_\perp = \dot{\phi}/H\kappa$ characterizing the turn is given by

$$\eta_\perp \sim \frac{\sqrt{2\epsilon}|\Gamma_0|}{\Delta\chi} \,, \tag{5.23}$$

whereas the speed of sound is found to be

$$c_s^{-2} \simeq 1 + \frac{8\epsilon H^2 \Gamma_0^2}{\Delta\chi^2 M^2} \,. \tag{5.24}$$

Notice that the only difference between this model and the previous one (where a plays the role of $\Delta\chi$) is the appearance of Γ_0. By defining the following dimensionless parameter,

$$\beta \equiv \frac{8\epsilon H^2 \Gamma_0^2}{\Delta\chi^2 M_\psi^2} \,, \tag{5.25}$$

we see that, in order to have large effects on the power spectrum due to the turn, we are required to have $\beta \sim 1$. Notice that one may satisfy this combination of parameters and still stay within the region of validity of the effective field theory, given by condition (5.20).

5.2.2 Numerics

We now present our numerical analysis of this model. As in the example of section 5.1, we choose the following values for the potential parameters associated to the flat inflationary direction: $V_0 = 6.5 \times 10^{-9}$ and $V_\phi = -5.4 \times 10^{-11}$ which in the absence of turns, imply $H \sim 1.4 \times 10^{-5}$, $\epsilon \simeq -\eta \simeq 0.0033$, and $n_s \simeq 0.98$. Notice that the rest of the parameters in charge of characterizing the multi-field turn are $\Delta\chi$, Γ_0 and M_ψ. These may be expressed in terms of α, β and $\Delta N_\perp$ introduced earlier as

$$M_\psi^2 = \alpha^2 \frac{2\epsilon H^2}{\Delta\chi^2} \,, \tag{5.26}$$

$$\Gamma_0^2 = \beta \frac{\Delta\chi^2 M^2}{8\epsilon H^2} \,, \tag{5.27}$$

$$\Delta\chi = \Delta N_\perp \sqrt{2\epsilon} \,. \tag{5.28}$$

Recall that the adiabaticity condition (5.20) is equivalent to $\alpha \gg 1$, and therefore we expect a poor matching between the full two-field theory and the EFT for small values of α. Figure 9 shows three examples of turns for different values of the parameters α and β, but for a fixed value $\Delta N = 0.4$. The top panels correspond to the choice $\alpha = 9$ and $\beta = 0.25$. It may be seen that given that the value of α is relatively low, the adiabaticity condition is not satisfied. Consistent with the discussions of the previous sections, the dependence on time of $\eta_\perp$ is dominated by fluctuations with a period of oscillation determined by $T_M \sim 1/M_\psi$, and the power spectrum obtained from the effective field theory (dashed red line) does not coincide with the one obtained from the complete two-field model (solid blue line). The

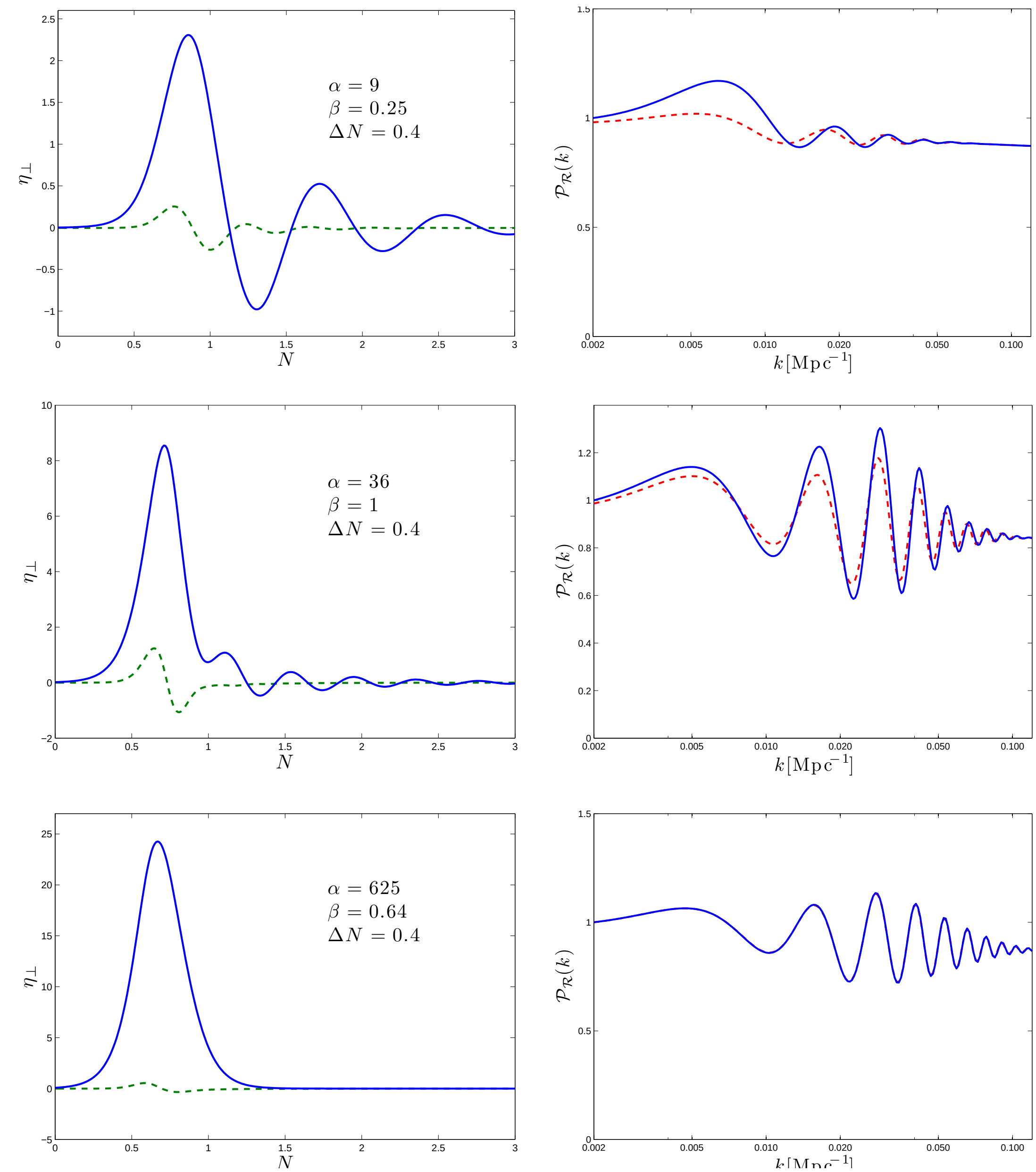

Figure 9. The figure shows $\eta_\perp$ and $\eta_{||}$ (left panels) and the resulting power spectra (right panels) for three choices of parameters for the potential of our model 2. From top to bottom: $(\alpha, \beta) = (9, 0.25)$, $(36, 1)$ and $(625, 0.64)$. In the case of the left panels, the blue solid line corresponds to $\eta_\perp$ whereas the green dashed line corresponds to $\eta_{||}$. In the case of right panels, the blue solid line corresponds to the power spectrum for the full two-field model, whereas the red dashed line corresponds to the power spectrum deduced using the EFT.

middle panels correspond to the choice $\alpha = 36$ and $\beta = 1$. Here the adiabaticity condition is slightly improved while the speed of sound suffers a sizable change. Finally, the lower panels show the situation $\alpha = 625$ and $\beta = 0.64$. Here the adiabaticity condition is fully satisfied

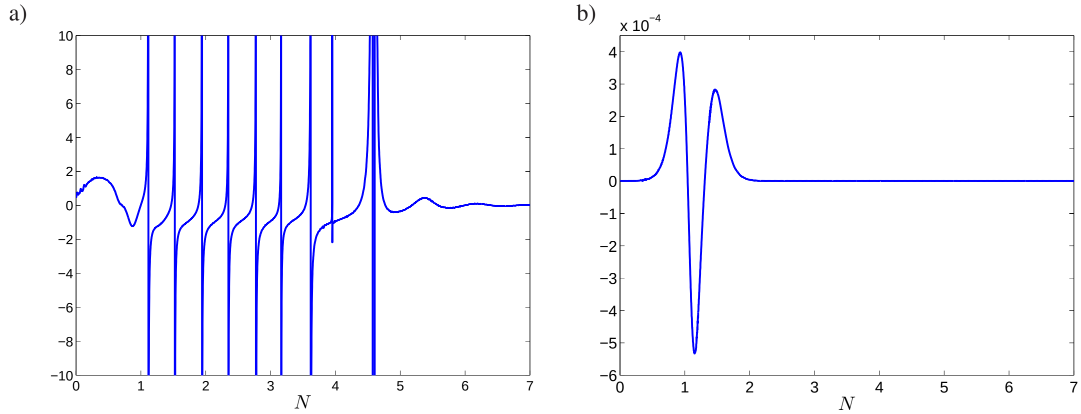

Figure 10. The figure shows the the quantity $f(t)$ defined in eq. (5.29) for the cases $(\alpha, \beta) = (9, 0.25)$ and $(625, 0.64)$ respectively. This function assesses whether the adiabatic condition is being satisfied during a turn. (We have chosen to plot this function in terms of e-folds N to facilitate its comparison with other quantities).

and the speed of sound becomes suppressed during a brief period of time. This is reflected by the excellent agreement between both power spectra, and their large features.

It may be noticed that in the first two cases $(\alpha, \beta) = (9, 0.25)$ and $(36, 1)$, there is a sizable variation of $\eta_{||}$. Just like in the case of our Model 1, this variation happens only during a brief period of time, and it is not enough to break the overall slow-roll behavior of the system. In addition, the value of ϵ is not affected considerably by the turn. In the last case $(\alpha, \beta) = (625, 0.64)$ the variation of $\eta_{||}$ is attenuated and the only background quantity varying considerably turns out to be $\eta_{\perp}$. This in turns makes c_s to have large variations, therefore producing the large features observed in the resulting power spectrum.

To finish, it is instructive to verify how does the rate of change of $\dot{\theta}$ evolves while the turn takes place. For this, we define

$$f(t) = \frac{1}{M_{\rm eff}^2} \frac{1}{\dot{\theta}} \frac{d^2}{dt^2} \dot{\theta} \, . \tag{5.29}$$

According to our discussion in section 3, the adiabaticity condition will be satisfied if $|f(t)| \ll 1$. Figure 10 shows f as a function of e-fold N for the cases $(\alpha, \beta) = (9, 0.25)$ and $(625, 0.64)$ respectively. It may be appreciated that indeed case $(\alpha, \beta) = (9, 0.25)$ is far from satisfying the adiabaticity condition, whereas the case $(\alpha, \beta) = (625, 0.64)$ satisfies it.

6 Conclusions

In this work we have analyzed the dynamics of two-field models of inflation with large mass hierarchies. We have focussed our attention on the role that turning trajectories have on the evolution of perturbations. If the mass M of the heavy field is much larger than the rate of expansion H, then the heavy field may be integrated out giving rise to a low energy effective field theory valid for curvature perturbations $\mathcal{R}$, with a quadratic order action given by eq. (3.6). At this order, all of the effects inherited from the heavy sector are reduced in the speed of sound c_s, which is given by eq. (3.7). We found that a good characterization of the validity of the low energy effective theory is given by the following adiabaticity condition:

$$\left| \frac{d}{dt} \ln \dot{\theta} \right| \ll M_{\text{eff}} \, , \tag{6.1}$$

where M_{eff} is the effective mass of heavy modes. Our numerical analysis is consistent with this condition, and we find that several non-trivial effects are still significant within this allowed region of parameters. For instance, in section 4 we were able to provide two simple toy models for which the effective field theory remains fully trustable. In these examples large features are generated and appear superimposed on the primordial power spectrum of curvature perturbations (recall the examples of figures 8 and 9). In addition, we verified that indeed as soon as the adiabaticity condition starts to fail, this is reflected in noticeable discrepancies in the power spectra predicted by both the complete two-dimensional model and the effective field theory (recall figure 10).

Our results contradict those of recent works regarding the validity of effective field theories obtained from multi-field inflation in various respects. For instance, in ref. [69] it is claimed that the effective field theory (3.6) is only valid in the regime where turns are such that $|\dot{\theta}| \ll H$.[10] The main argument made there is that the ratio $\eta_\perp = \dot{\theta}/H$ corresponds to the coupling determining the kinetic energy transfer between the light curvature mode with the heavy fields. A large value of $\eta_\perp$ would therefore imply large transfer of energy from curvature perturbations to the heavy mode, exciting the heavy modes and rendering the effective field theory invalid. However, as we have seen, this energy interchange between both modes may happen adiabatically without implying a breakdown of the effective field theory. Indeed, the heavy mode is receiving energy from the light degree of freedom at the same rate than it is giving it back, and therefore it is possible to have turns whereby the heavy-mode's high-frequency fluctuations stay suppressed (recall our discussion of section 3).

One key point here is that even for large values of $\eta_\perp = \dot{\theta}/H$ the heavy modes will not become easily excited unless they receive a sufficiently strong kick. For example, if the turn is such that

$$\left| \frac{d}{dt} \ln \dot{\theta} \right| \gtrsim H \, , \tag{6.2}$$

the trajectory is necessarily subject to a large angular acceleration $|\ddot{\theta}| \gtrsim H|\dot{\theta}|$, momentarily violating slow-roll [78] but without exciting heavy modes. Moreover, as we have seen, if these accelerations are brief (as in our examples) slow-roll is only interrupted for a short period of time, and the system quickly goes back to the slow-roll attractor state (within an e-fold). During these transients, $\eta_{||}$ were typically found to have sizable variations in response to the turns, whereas ϵ was found to stay close to its suppressed value.[11] The net effect of this process are oscillatory features in the power spectrum, with the frequency of the oscillation depending on how brief was the overall turn. While current observational constraints on features are still poor [5, 80–89], future data will certainly put strong constraints on primordial features, therefore improving our understanding of the role of UV-physics on the very early universe.

Another important point of departure from previous works regards the procedure employed to integrate heavy fields. In the present work we have integrated out high energy

[10] A similar claim is made in ref. [79], where it is argued that heavy fields can only be integrated out consistently if the rate of turn satisfies $|\dot{\theta}| \ll H$.

[11] This was also found numerically in ref. [26], and in ref. [69] an analytic argument was given to explain this effect.

fluctuations (heavy degrees of freedom) about the exact time-dependent background trajectory, offered by the homogeneous equations of motion of the system. This contrasts with other schemes [90–92] where heavy fields are taken care of at the action level, regardless of the background dynamics, by imposing that they locally minimize the inflationary potential (with the minima depending on the inflaton v.e.v.). In our present language this is equivalent to $V_N = 0$, implying that there are no turns at all (and therefore missing all of the interesting features we have studied so far). Although such a case corresponds to a genuine limit shared by a large family of multi-field potentials with mass hierarchies [93, 94], it misses the more general situation in which the inflationary trajectory meanders away from the locus of minima offered by the potential.

We should emphasize that our results are strictly valid only at linear order in the fluctuations. For a complete analysis one should examine the relevance of higher order interaction terms which could introduce important corrections when the speed of sound is suppressed ($c_s \ll 1$). This is similar to the case of DBI inflation [33, 95] where a suppressed speed of sound makes the perturbation theory to enter the strong-coupling regime [57]. In ref. [70] the full structure of effective field theories arising from two-field models with large mass hierarchies is studied, and other relevant constraints involving higher order terms are analyzed.

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B.2 Heavy fields, reduced speed of sound, and decoupling during inflation

We discuss and clarify the validity of effective single-field theories of inflation obtained by integrating out heavy degrees of freedom in the regime where adiabatic perturbations propagate with a suppressed speed of sound. We show by construction that it is indeed possible to have inflationary backgrounds where the speed of sound remains suppressed and uninterrupted slow-roll persists for long enough. In this class of models, heavy fields influence the evolution of adiabatic modes in a manner that is consistent with decoupling of physical low- and high-energy degrees of freedom. We emphasize the distinction between the effective masses of the isocurvature modes and the eigenfrequencies of the propagating high-energy modes. Crucially, we find that the mass gap that defines the high-frequency modes increases with the strength of the turn, even as the naively heavy (isocurvature) and light (curvature) modes become more strongly coupled. Adiabaticity is preserved throughout, and the derived effective field theory remains in the weakly coupled regime, satisfying all current observational constraints on the resulting primordial power spectrum. In addition, these models allow for an observably large equilateral non-Gaussianity.

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Heavy fields, reduced speeds of sound, and decoupling during inflation

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We discuss and clarify the validity of effective single-field theories of inflation obtained by integrating out heavy degrees of freedom in the regime where adiabatic perturbations propagate with a suppressed speed of sound. We show by construction that it is indeed possible to have inflationary backgrounds where the speed of sound remains suppressed and uninterrupted slow-roll persists for long enough. In this class of models, heavy fields influence the evolution of adiabatic modes in a manner that is consistent with decoupling of physical low- and high-energy degrees of freedom. We emphasize the distinction between the effective masses of the isocurvature modes and the eigenfrequencies of the propagating high-energy modes. Crucially, we find that the mass gap that defines the high-frequency modes increases with the strength of the turn, even as the naively heavy (isocurvature) and light (curvature) modes become more strongly coupled. Adiabaticity is preserved throughout, and the derived effective field theory remains in the weakly coupled regime, satisfying all current observational constraints on the resulting primordial power spectrum. In addition, these models allow for an observably large equilateral non-Gaussianity.

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The recent observation that heavy fields can influence the evolution of adiabatic modes during inflation [1] has far-reaching phenomenological implications [2–5] that, *a posteriori*, require a refinement of our understanding of how high- and low-energy degrees of freedom decouple [6] and how one splits "heavy" and "light" modes on a time-dependent background. Provided that there is only one flat direction in the inflaton potential, heavy fields (in the present context, field excitations orthogonal to the background trajectory) can be integrated out. This results in a low-energy effective field theory (EFT) for adiabatic modes exhibiting a reduced speed of sound c_s, given by

$$c_s^{-2} = 1 + 4\dot{\theta}^2/M_{\rm eff}^2, \tag{1}$$

where $\dot{\theta}$ is the turning rate of the background trajectory in multifield space, and $M_{\rm eff}$ is the effective mass of heavy fields, assumed to be much larger than the expansion rate H. Depending on the nature of the trajectory, (1) can render features in the power spectrum [3,4] and/or observably large non-Gaussianity [1,5].

Given that $M_{\rm eff}$ is the mass of the fields we integrate, one might doubt the validity of the EFT in the regime where the speed of sound is suppressed [7], as this requires $\dot{\theta}^2 \gg M_{\rm eff}^2$. In this article we elaborate on this issue by studying the dynamics of light and heavy degrees of freedom when $c_s^2 \ll 1$. What emerges is a crucial distinction, in time-dependent backgrounds, between isocurvature and curvature field excitations, and the true heavy and light excitations. We show that the light (curvature) mode $\mathcal{R}$ indeed stays coupled to the heavy (isocurvature) modes when strong turns take place ($\dot{\theta}^2 \gg M_{\rm eff}^2$); however, decoupling between the physical low- and high-energy degrees of freedom persists in such a way that the deduced EFT remains valid. This is confirmed by a simple setup in which H decreases adiabatically, allowing for a sufficiently long period of inflation. In this construction, *high-energy degrees of freedom* are never excited, and yet *heavy fields* do play a role in lowering the speed of sound of adiabatic modes.

Although this is completely consistent with the principles of EFT, it seems to have escaped previous analyses due to some subtleties that we summarize in points i–iv below. Furthermore, inflationary scenarios with sustained turns and uninterrupted slow roll appear to be consistent with all the observational constraints on the primordial power spectrum of primordial perturbations, while predicting enhanced equilateral non-Gaussianity, and we give explicit examples at the end.

The simplest setup that allows a quantitative analysis (see Refs. [3–5] for details) is a two-scalar system with an action

$$S = \int \sqrt{-g}\left[\frac{1}{2}R - \frac{1}{2}g^{\mu\nu}\gamma_{ab}\partial_\mu\phi^a\partial_\nu\phi^b - V(\phi)\right], \tag{2}$$

(in units $8\pi G = 1$) where R is the Ricci scalar, V is the scalar potential and γ_{ab} is the possibly noncanonical sigma-model metric of the space spanned by ϕ^a, with $a = 1, 2$. The background solution to the equations of motion is an inflationary trajectory $\phi_0^a(t)$ and a Friedman-Robertson-Walker metric $ds^2 = -dt^2 + a^2(t)\delta_{ij}dx^i dx^j$, where $a(t)$ is the scale factor and $H = \dot{a}/a$ the Hubble parameter. As usual, we take unit vectors T^a and N^a tangent and normal to the trajectory [8] given by $T^a = \dot{\phi}_0^a/\dot{\phi}_0$ and $D_t T^a = -\dot{\theta} N^a$, which also defines the turning rate $\dot{\theta}$, the angular velocity described by the bends of the trajectory. Here $D_t = \dot{\phi}_0^a \nabla_a$ is the covariant time derivative along the background trajectory, and $\dot{\phi}_0^2 \equiv \gamma_{ab}\dot{\phi}_0^a\dot{\phi}_0^b$ [9]. Finally, we define slow-roll parameters $\epsilon \equiv -\dot{H}/H^2 = \dot{\phi}_0^2/2$ and $\eta_{\|} \equiv -\ddot{\phi}_0/(H\dot{\phi}_0)$, whose smallness ensures that H evolves adiabatically for a sufficiently long time.

We are interested in the dynamics of scalar perturbations $\delta\phi^a(t,\boldsymbol{x}) = \phi^a(t,\boldsymbol{x}) - \phi_0^a(t)$. We work in the flat gauge and define the comoving curvature and heavy isocurvature perturbations as $\mathcal{R} \equiv -(H/\dot{\phi})T_a\delta\phi^a$ and $\mathcal{F} \equiv N_a\delta\phi^a$, respectively. (A definition of $\mathcal{R}$ and $\mathcal{F}$ valid to all orders in perturbation theory is given in Ref. [5]). The quadratic order action for these perturbations is

$$S_2 = \frac{1}{2}\int a^3\bigg[\frac{\dot{\phi}_0^2}{H^2}\dot{\mathcal{R}}^2 - \frac{\dot{\phi}_0^2}{H^2}\frac{(\nabla\mathcal{R})^2}{a^2} + \dot{\mathcal{F}}^2 - \frac{(\nabla\mathcal{F})^2}{a^2} - M_{\rm eff}^2\mathcal{F}^2 - 4\dot{\theta}\frac{\dot{\phi}_0}{H}\dot{\mathcal{R}}\mathcal{F}\bigg]. \tag{3}$$

Here $M_{\rm eff}$ is the effective mass of $\mathcal{F}$ given by

$$M_{\rm eff}^2 = m^2 - \dot{\theta}^2, \tag{4}$$

where $m^2 \equiv V_{NN} + \epsilon H^2\mathbb{R}$ and $V_{NN} \equiv N^a N^a \nabla_a\nabla_b V$. $\mathbb{R}$ is the Ricci scalar of the sigma-model metric γ_{ab}. Notice that $\dot{\theta}$ couples both fields and reduces the effective mass, suggesting a breakdown of the hierarchy that permits a single-field effective description as $\dot{\theta}^2 \sim m^2$. As we are about to see, this expectation is somewhat premature. The linear equations of motion in Fourier space are

$$\ddot{\mathcal{R}} + (3 + 2\epsilon - 2\eta_{\|})H\dot{\mathcal{R}} + \frac{k^2}{a^2}\mathcal{R} = 2\dot{\theta}\frac{H}{\dot{\phi}_0}\bigg[\dot{\mathcal{F}} + \bigg(3 - \eta_{\|} - \epsilon + \frac{\ddot{\theta}}{H\dot{\theta}}\bigg)H\mathcal{F}\bigg], \tag{5}$$

$$\ddot{\mathcal{F}} + 3H\dot{\mathcal{F}} + \frac{k^2}{a^2}\mathcal{F} + M_{\rm eff}^2\mathcal{F} = -2\dot{\theta}\frac{\dot{\phi}_0}{H}\dot{\mathcal{R}}. \tag{6}$$

Note that $\mathcal{R}$ = constant and $\mathcal{F} = 0$ are nontrivial solutions to these equations for arbitrary $\dot{\theta}$. Since $\mathcal{F}$ is heavy, $\mathcal{F} \to 0$ shortly after horizon exit, and $\mathcal{R} \to$ constant, as in single-field inflation.

We are interested in (5) and (6) in the limit where $\dot{\theta}$ is constant and much greater than $M_{\rm eff}$. We first consider the short-wavelength limit where we can disregard Hubble friction terms and take $\dot{\phi}_0/H$ as a constant. In this regime, the physical wave number $p \equiv k/a$ may be taken to be constant, and (5) and (6) simplify to

$$\begin{aligned}\ddot{\mathcal{R}}_c + p^2\mathcal{R}_c &= +2\dot{\theta}\,\dot{\mathcal{F}},\\ \ddot{\mathcal{F}} + p^2\mathcal{F} + M_{\rm eff}^2\mathcal{F} &= -2\dot{\theta}\dot{\mathcal{R}}_c,\end{aligned} \tag{7}$$

in terms of the canonically normalized $\mathcal{R}_c = (\dot{\phi}_0/H)\mathcal{R}$. The solutions are found to be [2]

$$\begin{aligned}\mathcal{R}_c &= \mathcal{R}_+e^{i\omega_+t} + \mathcal{R}_-e^{i\omega_-t},\\ \mathcal{F} &= \mathcal{F}_+e^{i\omega_+t} + \mathcal{F}_-e^{i\omega_-t},\end{aligned} \tag{8}$$

where the two frequencies ω_- and ω_+ are given by

$$\omega_\pm^2 = \frac{M_{\rm eff}^2}{2c_s^2} + p^2 \pm \frac{M_{\rm eff}^2}{2c_s^2}\sqrt{1 + \frac{4p^2(1-c_s^2)}{M_{\rm eff}^2c_s^{-2}}}, \tag{9}$$

with c_s given by (1). The pairs $(\mathcal{R}_-, \mathcal{F}_-)$ and $(\mathcal{R}_+, \mathcal{F}_+)$ represent the amplitudes of both low- and high-frequency modes, respectively, and satisfy

$$\mathcal{F}_- = \frac{-2i\dot{\theta}\omega_-}{M_{\rm eff}^2 + p^2 - \omega_-^2}\mathcal{R}_-, \qquad \mathcal{R}_+ = \frac{-2i\dot{\theta}\omega_+}{\omega_+^2 - p^2}\mathcal{F}_+. \tag{10}$$

Thus the fields in each pair oscillate coherently. Integrating out the heavy mode consists in ensuring that the high-frequency degrees of freedom do not participate in the dynamics of the adiabatic modes. This requires a hierarchy of the form $\omega_-^2 \ll \omega_+^2$, which from (9) requires

$$p^2 \ll M_{\rm eff}^2c_s^{-2}. \tag{11}$$

This defines the regime of validity of the EFT, for which one has $\omega_+^2 \simeq M_{\rm eff}^2c_s^{-2} = m^2 + 3\dot{\theta}^2$ and the low-energy dispersion relation

$$\omega_-^2(p) \simeq p^2c_s^2 + (1-c_s^2)^2\frac{p^4}{M_{\rm eff}^2c_s^{-2}}. \tag{12}$$

These expressions for ω_+ and ω_- imply a clear distinction between low- and high-energy degrees of freedom. In terms of the low-energy frequency, condition (11) may be rewritten as $\omega_-^2 \ll M_{\rm eff}^2c_s^{-2}$, in light of which the scale $\omega_+^2 \simeq M_{\rm eff}^2c_s^{-2}$ evidently cuts off the low-energy regime. Thus, we can safely consider only low-frequency modes, in which case $\mathcal{F}$ is completely determined by $\mathcal{R}_c$ as $\mathcal{F} = -2\dot{\theta}\dot{\mathcal{R}}_c/(M_{\rm eff}^2 + p^2 - \omega_-^2)$. Notice that $\omega_-^2 \ll M_{\rm eff}^2 + p^2$, so ω_-^2 may be disregarded here.

As linear perturbations evolve, their physical wave number $p \equiv k/a$ decreases and the modes enter the long-wavelength regime $p^2c_s^2 \lesssim H^2$, where they become strongly influenced by the background and no longer have a simple oscillatory behavior. Now the low-energy contributions to $\mathcal{F}$ satisfy $\dot{\mathcal{F}} \sim H\mathcal{F}$, and because $H^2 \ll M_{\rm eff}^2$, we can simply neglect time derivatives in (6). On the other hand, high-energy modes continue to evolve independently of the low-energy modes, diluting rapidly as they redshift. Thus for the entire low-energy regime (11), time derivatives of $\mathcal{F}$ can be ignored in (6) and $\mathcal{F}$ may be solved in terms of $\dot{\mathcal{R}}$ as

$$\mathcal{F} = -\frac{\dot{\phi}_0}{H}\frac{2\dot{\theta}\dot{\mathcal{R}}}{k^2/a^2 + M_{\rm eff}^2}. \tag{13}$$

Inserting (13) into (3) gives the tree-level effective action for the curvature perturbation. To quadratic order [5],

$$S_{\rm eff} = \frac{1}{2}\int a^3 \frac{\dot{\phi}_0^2}{H^2}\Big[\frac{\dot{\mathcal{R}}^2}{c_s^2(k)} - \frac{k^2\mathcal{R}^2}{a^2}\Big], \tag{14}$$

where $c_s^{-2}(k) = 1 + 4\dot{\theta}^2/(k^2/a^2 + M_{\rm eff}^2)$. This k-dependent speed of sound is consistent with the modified dispersion relation (12), where c_s is given by (1). Reference [4] studied the validity of (14) in the case where turns appear suddenly. Consistent with the present analysis, it was found that this EFT is valid even when $\dot{\theta}^2 \gg M_{\rm eff}^2$, *provided the adiabaticity condition*

$$|\ddot{\theta}/\dot{\theta}| \ll M_{\rm eff} \tag{15}$$

is satisfied. This condition states that the turn's angular *acceleration* must remain small in comparison to the masses of heavy modes, which otherwise would be excited. The above straightforwardly implies the more colloquial adiabaticity condition $|\dot{\omega}_+/\omega_+^2| \ll 1$. If (15) is violated by the background, high-energy modes can be produced and the EFT does indeed break down, as confirmed by [10].

We now outline four crucial points that underpin our conclusions:

(i) The mixing between fields $\mathcal{R}$ and $\mathcal{F}$, and modes with frequencies ω_- and ω_+ is *inevitable* when the background trajectory bends. If one attempts a rotation in field space in order to uniquely associate fields with frequency modes, the rotation matrix would depend on the scale p, implying a nonlocal redefinition of the fields.

(ii) Even in the absence of excited high-frequency modes, the heavy field $\mathcal{F}$ is forced to oscillate in pace with the light field $\mathcal{R}$ at a frequency ω_-, so $\mathcal{F}$ continues to participate in the low-energy dynamics of the curvature perturbations.

(iii) When $\dot{\theta}^2 \gg M_{\rm eff}^2$, the high- and low-energy frequencies become $\omega_+^2 \simeq M_{\rm eff}^2 c_s^{-2} \sim 4\dot{\theta}^2$ and $\omega_-^2 \simeq p^2(M_{\rm eff}^2 + p^2)/(4\dot{\theta}^2)$. Thus the gap between low- and high-energy degrees of freedom is amplified, and one can consistently ignore high-energy degrees of freedom in the low-energy EFT.

(iv) In the low-energy regime, the field $\mathcal{F}$ exchanges kinetic energy with $\mathcal{R}$ resulting in a reduction in the speed of sound c_s of $\mathcal{R}$, the magnitude of which depends on the strength of the kinetic coupling $\dot{\theta}$. This process is adiabatic and consistent with the usual notion of decoupling in the low-energy regime (11), as implied by (15).

At the core of these four observations is the simple fact that in time-dependent backgrounds, the eigenmodes and eigenvalues of the mass matrix along the trajectory do not necessarily coincide with the curvature and isocurvature fluctuations and their characteristic frequencies. With this in mind, it is possible to state more clearly the refined sense in which decoupling is operative: *while the fields $\mathcal{R}$ and $\mathcal{F}$ inevitably remain coupled, high- and low-energy degrees of freedom effectively decouple*.

We now briefly address the evolution of modes in the ultraviolet regime $p^2 \gtrsim M_{\rm eff}^2 c_s^{-2}$. Here both modes have similar amplitudes and frequencies, and so in principle could interact via relevant couplings beyond linear order (which are proportional to $\dot{\theta}$). Because these interactions must allow for the nontrivial solutions $\mathcal{R}$ = constant and $\mathcal{F} = 0$ (a consequence of the background time-reparametrization invariance), their action is very constrained [5]. Moreover, in the regime $p^2 \gg M_{\rm eff}^2 c_s^{-2}$ the coupling $\dot{\theta}$ becomes negligible when compared to p, and one necessarily recovers a very weakly coupled set of modes, whose $p \to \infty$ limit completely decouples $\mathcal{R}$ from $\mathcal{F}$. This can already be seen in (13), where contributions to the effective action for the adiabatic mode at large momenta from having integrated out $\mathcal{F}$, are extremely suppressed for $k^2/a^2 \gg M_{\rm eff}^2$, leading to high-frequency contributions to (14) with $c_s = 1$.

We now analyze a model of uninterrupted slow-roll inflation that executes a constant turn in field space, implying an almost constant, suppressed speed of sound for the adiabatic mode. Take fields $\phi^1 = \theta$, $\phi^2 = \rho$ with a metric $\gamma_{\theta\theta} = \rho^2$, $\gamma_{\rho\rho} = 1$, $\gamma_{\rho\theta} = \gamma_{\theta\rho} = 0$ and potential

$$V(\theta, \rho) = V_0 - \alpha\theta + m^2(\rho - \rho_0)^2/2. \tag{16}$$

This model would have a shift symmetry along the θ direction were it not broken by a nonvanishing α. This model is a simplified version of one studied in Ref. [11], where the focus instead was on the regime $M_{\rm eff} \sim m \sim H$ (see also Ref. [12] where the limit $M_{\rm eff}^2 \gg H^2 \gg \dot{\theta}^2$ is analyzed). The background equations of motion are

$$\begin{aligned} \ddot{\theta} + 3H\dot{\theta} + 2\dot{\theta}\dot{\rho}/\rho &= \alpha/\rho^2, \\ \ddot{\rho} + 3H\dot{\rho} + \rho(m^2 - \dot{\theta}^2) &= m^2\rho_0. \end{aligned} \tag{17}$$

The slow-roll attractor is such that $\dot{\rho}$, $\ddot{\rho}$ and $\ddot{\theta}$ are negligible. This means that H, ρ and $\dot{\theta}$ remain nearly constant and satisfy the following algebraic equations near $\theta = 0$:

$$\begin{aligned} 3H\dot{\theta} &= \frac{\alpha}{\rho^2}, \qquad \dot{\theta}^2 = m^2(1 - \rho_0/\rho), \\ 3H^2 &= \frac{1}{2}\rho^2\dot{\theta}^2 + V_0 + \frac{1}{2}m^2(\rho - \rho_0)^2. \end{aligned} \tag{18}$$

These equations describe a circular motion with a radius of curvature ρ and angular velocity $\dot{\theta}$. Here $M_{\rm eff}^2 = m^2 - \dot{\theta}^2$, implying the strict bound $m^2 > \dot{\theta}^2$. Thus the only way to obtain a suppressed speed of sound is if $\dot{\theta}^2 \simeq m^2$. Our aim is to find the parameter ranges such that the background attractor satisfies $\epsilon \ll 1$, $c_s^2 \ll 1$ and $H^2 \ll M_{\rm eff}^2$ simultaneously. These are given by

$$1 \gg \frac{\rho_0}{4}\left(\frac{m\sqrt{3V_0}}{\alpha}\right)^{1/2} \gg \frac{V_0}{6m^2} \gg \frac{\alpha}{4\sqrt{3V_0}m}. \quad (19)$$

If these hierarchies are satisfied, the solutions to (18) are well approximated by

$$\rho^2 = \frac{\alpha}{\sqrt{3V_0}m}, \qquad \dot{\theta} = m - \frac{m\rho_0}{2}\left(\frac{m\sqrt{3V_0}}{\alpha}\right)^{1/2}, \quad (20)$$

and $H^2 = V_0/3$, up to fractional corrections of order ϵ, c_s^2 and $H^2/M_{\rm eff}^2$. We note that the first inequality in (19) implies $\rho \gg \rho_0$, and so the trajectory is displaced off the adiabatic minimum at ρ_0. However, the contribution to the total potential energy implied by this displacement is negligible compared to V_0. After n cycles around $\rho = 0$ one has $\Delta\theta = 2\pi n$, and the value of V_0 has to be adjusted to $V_0 \to V_0 - 2\pi n\alpha$. This modifies the expressions in (20) accordingly, and allows us to easily compute the adiabatic variation of certain quantities, such as $s \equiv \dot{c}_s/(c_s H) = -\epsilon/4$, and $\eta_{||} = -\epsilon/2$, where $\epsilon = \sqrt{3}\alpha m^2/(2V_0^{3/2})$. These values imply a spectral index $n_{\mathcal{R}}$ for the power spectrum $\mathcal{P}_{\mathcal{R}} = H^2/(8\pi^2\epsilon c_s)$ given by $n_{\mathcal{R}} - 1 = -4\epsilon + 2\eta_{||} - s = -19\epsilon/4$.

It is possible to find reasonable values of the parameters such that observational bounds are satisfied. Using (20) we can relate the values of V_0, α, m and ρ_0 to the measured values $\mathcal{P}_{\mathcal{R}}$ and $n_{\mathcal{R}}$, and to hypothetical values for c_s and $\beta \equiv H/M_{\rm eff}$ as

$$\begin{aligned} V_0 &= 96\pi^2(1-n_{\mathcal{R}})\mathcal{P}_{\mathcal{R}}c_s/19, \\ m^2 &= 8\pi^2(1-n_{\mathcal{R}})\mathcal{P}_{\mathcal{R}}/(19c_s\beta^2), \\ \alpha &= 6(16/19)^2\pi^2(1-n_{\mathcal{R}})^2\mathcal{P}_{\mathcal{R}}c_s^2\beta, \\ \rho_0 &= 16c_s^3\beta\sqrt{2(1-n_{\mathcal{R}})/19}. \end{aligned} \quad (21)$$

Following WMAP7, we take $\mathcal{P}_{\mathcal{R}} = 2.42 \times 10^{-9}$ and $n_{\mathcal{R}} = 0.98$ [13]. Then, as an application of relations (21), we look for parameters such that

$$c_s^2 \simeq 0.06, \qquad M_{\rm eff}^2 \simeq 250H^2, \quad (22)$$

(which imply $H^2 \simeq 1.4 \times 10^{-10}$) according to which $V_0 \simeq 5.9 \times 10^{-10}$, $\alpha \simeq 1.5 \times 10^{-13}$, $m \simeq 4.5 \times 10^{-4}$ and $\rho_0 \simeq 6.8 \times 10^{-3}$, from which we note that m, ρ_0 and $\alpha^{1/4}$ are naturally all of the same order. In terms of the mass scale m, the above implies that we have excited a *field* with mass of order $H^2/m^2 \sim 1/1450$, completely consistent with decoupling and the validity of the EFT. We have checked numerically that the background equations of motion are indeed well approximated by (20), up to fractional corrections of order c_s^2. More importantly, we obtain the same nearly scale-invariant power spectrum $\mathcal{P}_{\mathcal{R}}$ using both the full two-field theory described by (5) and (6), and the single-field EFT described by the action (14). The evolution of curvature perturbations in the EFT compared to the full two-field theory for the long-wavelength modes is almost indistinguishable given the effectiveness with which (11) is satisfied, with a marginal difference $\Delta\mathcal{P}_{\mathcal{R}}/\mathcal{P}_{\mathcal{R}} \simeq 0.008$. This is of order $(1-c_s^2)H^2/M_{\rm eff}^2$, which is consistent with the analysis of Ref. [4]. Despite the suppressed speed of sound in this model, a fairly large tensor-to-scalar ratio of $r = 16\epsilon c_s \simeq 0.02$ is predicted.

As expected, for $c_s^2 \ll 1$ a sizable value of $f_{\rm NL}^{(\rm eq)}$ is implied. The cubic interactions leading to this were deduced in Ref. [5], which for constant turns is given by Ref. [14]

$$f_{\rm NL}^{(\rm eq)} = \frac{125}{108}\frac{\epsilon}{c_s^2} + \frac{5}{81}\frac{c_s^2}{2}\left(1 - \frac{1}{c_s^2}\right)^2 + \frac{35}{108}\left(1 - \frac{1}{c_s^2}\right). \quad (23)$$

This result is valid for any single-field system with constant c_s obtained by having integrated out a heavy field. We note that this prediction is cleanly distinguishable from those of other single-field models (such as Dirac-Born-Infeld inflation) through the different sign and magnitude for the M_3 coefficient generated in the EFT expansion of Ref. [15], as derived in Ref. [5].

Recalling that the spectral index n_T of tensor modes is $n_T = -2\epsilon$, for $c_s \ll 1$ we find a consistency relation between three potentially observable parameters, given by $f_{\rm NL}^{(\rm eq)} = -20.74n_T^2/r^2$. In the specific case of the values in (22), we have $f_{\rm NL}^{(\rm eq)} \simeq -4.0$. This value is both large and negative, so future observations could constrain this type of scenario. Finally, one can ask if the EFT corresponding to (22) remains weakly coupled throughout. For this, one needs to satisfy [5] $\omega_- < \Lambda_{\rm sc}$, where $\Lambda_{\rm sc}$ is the scale at which our low-energy EFT becomes strongly coupled. For the standard case in which $\omega_-^2 = c_s^2 p^2$ this scale is found to be given by $\Lambda_{\rm sc}^4 \simeq 4\pi\epsilon H^2 c_s^5/(1-c_s^2)$ [15]. Nevertheless, for small values of c_s the scaling properties offered by the quartic piece in the modified dispersion relation (12) necessarily pushes the value of $\Lambda_{\rm sc}$ to a larger value [16]. For the present case, this effect implies a strong coupling scale given by $\Lambda_{\rm sc} \simeq (8\pi c_s^2)^{2/5} \times [2\epsilon H^2/(M_{\rm eff}^2 c_s^{-4})]^{2/5} M_{\rm eff} c_s^{-1}$ [17]. For instance, for the values (22) we find that $\Lambda_{\rm sc}/M_{\rm eff}c_s^{-1} \simeq 2$, implying that the EFT obtained by integrating a heavy field remains weakly coupled all the way up to its cutoff scale $M_{\rm eff}c_s^{-1}$. Furthermore, although we did not address how inflation ends, the choice (22) allows for at least 45 e-folds of inflation, which is necessary to solve the horizon and flatness problems. We stress that various other values can be chosen in (22) to arrive at similar conclusions. For example, requiring 35 e-folds with $M_{\rm eff}^2 \simeq 100H^2$, $c_s^2 \simeq 0.02$, implies $V_0 \simeq 3.4 \times 10^{-10}$, $\alpha \simeq 8.1 \times 10^{-13}$, $m \simeq 3.8 \times 10^{-4}$, $\rho_0 \simeq 2.1 \times 10^{-4}$. In this case we find $f_{\rm NL}^{(\rm eq)} \simeq -14$.

In summary, the active ingredients of this toy example are rather minimal and may well parametrize a generic class of inflationary models, such as axion-driven inflationary scenarios in string theory. Our results complement those of Refs. [1–5] and emphasize the refined sense in which EFT techniques are applicable during slow-roll inflation [15,18]. In particular, contrary to the standard perspective regarding the role of ultraviolet physics during inflation, heavy fields may influence the evolution of curvature perturbations $\mathcal{R}$ in a way consistent with decoupling between low- and high-energy degrees of freedom.

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