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CHARACTERIZATION AND COMPLETATION OF THE CUSTOMER DATA FROM A RETAIL COMPANY USING GRAPH MINING TECHNIQUES

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Resumen

La industria del *retail* en Chile ha presentado un crecimiento sostenido en los últimos años. El aumento de la competencia ha obligado a hacer grandes esfuerzos de retención y fidelización. Las tecnologías de información han permitido registrar las interacciones del cliente con la empresa, sin embargo ha nacido una nueva problemática, el manejo de grandes volúmenes de datos y su procesamiento. Este trabajo contó con 400 millones de registros transaccionales provenientes de dos cadenas de supermercados.

El enfoque utilizado por los *retailers* chilenos para agrupar los clientes es según montos de compra en un determinado periodo, no en la similitud de sus canastas de compra como propone la literatura. Al aplicar técnicas tradicionales, tanto para identificar patrones de compra similares (*clustering*) como para identificar canastas de compras (*Market Basket Analysis*), los resultados son de baja calidad, haciendo muy difícil la interpretación y no identificando grupos de productos relacionados que permitan clasificar a un cliente en base a sus compras históricas.

Dado que el enfoque clásico no funcionó, se decidió buscar otra forma de abordar el problema. Se utilizó un enfoque basado en redes sociales, entendiendo que la presencia simultánea de dos productos en una misma boleta implica una relación entre ellos. En la misma línea, se aplicaron algoritmos de detección de comunidades, permitiendo obtener grupos de productos que pueden ser etiquetados, clasificados e interpretados por los analistas. Luego, es posible clasificar a un cliente de acuerdo a sus compras previas.

La detección de comunidades fue abordada utilizando algoritmos que buscan comunidades traslapadas, lo que permite la presencia simultánea de un producto en más de un grupo. Esto considerando que es habitual que las personas adquieran en ciertos casos, los mismos productos aunque presenten comportamientos de compra distintos.

La calidad del modelo propuesto se comprueba, en hechos como la estabilidad de los grupos de productos generados a lo largo de diferentes periodos temporales. Además la modularidad obtenida es más alta que la encontrada generalmente en redes sociales. Este hecho demuestra la robustez de la estructura de comunidades.

Finalmente, dentro de las conclusiones se destaca que el nuevo modelo entrega grupos de productos fuertemente relacionados, los cuales son fácilmente interpretables por los analistas. Con la existencia de grupos de productos, se puede encontrar el grado de pertenencia de un cliente a dichos grupos, esto se traduce en una caracterización del cliente y además permite completar los datos existentes del mismo. La técnica propuesta permite manejar grandes volúmenes de datos, entregando resultados en tiempos acordes a las necesidades de la empresa.

A Mi Madre, Isabel.
Quien Me Dio La Vida,
Principios y Valores
Para Ser Quien Soy.
Muchísimas Gracias. Te Amo.

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Dejé para el final la escritura de estas lineas, no porque sean sencillas de escribir o poco importantes, por el contrario, para mí son sumamente importantes. El solo hecho de sentarme a escribirlas significa que ha finalizado el trabajo de tesis que en este documento se presenta. Hoy se cierra una etapa, una que comenzó hace ya varios años cuando un día de Marzo de 2003 mis padres –*Isabel* y *Juan Carlos*– fueron a dejarme a la facultad con una mochila llena de sueños, ideas y esperanzas tanto de ellos como míos. Sueños, ideas y esperanzas de un futuro nuevo y a la vez desconocido, el cual yo con ansias quería descubrir. Y lo hice, poco a poco fui descubriendo ese camino nuevo, no sin escollos ni dificultades, pero nadie dijo que sería fácil, por el contrario, los vaticinios eran de un camino con más espinas que rosas. Hoy, mirando hacia atrás puedo decir con orgullo que el camino tenía más rosas que espinas. A mi *Familia* le digo que espero, de corazón, haber cumplido sus expectativas.

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Abstract

Retail industry in Chile has shown sustained growth in the last years. In response to the growing number of competitors, companies have made great efforts in areas such as retention and loyalty. Information technologies have enabled the registering of customer interactions with the company, although a new problem has arisen; handling big data and processing it. This project utilized 400 million transactional records belonging to two supermarket chains.

The approach used by Chilean retailers to form groups of customers is by studying the amount they spend in a certain period of time, not considering similarities in their market baskets as proposed in the literature. When applying traditional techniques, both to identify similar purchase patterns (clustering) and market baskets (Market Basket Analysis), the results are of low quality, making it very difficult to interpret and identify groups of products that would allow classifying of a client based on their historical purchases.

Since the classical approach did not work, it was decided to find another way to address the problem. An approach based on social networks was used, understanding the simultaneous presence of two products in a single ticket implies a relationship between them. In the same tone, community detection algorithms were applied; obtaining product groups that can be labeled, classified and interpreted by analysts. It is then possible to classify a customer according to their previous purchases.

Overlapping community detection approach was used, allowing the simultaneous presence of a product in more than one group. It is usual that people, with different buying behavior, had acquired products in common.

The quality of the proposed model is proven comparing groups of products generated in different time periods. These comparisons have shown stable results. Modularity obtained is higher than what is usually seen in social networks. This indicates that a robust community structure has been built.

Finally, the conclusions highlight that the new model delivers strongly related product groups, which are easily interpreted by analysts. As there are groups of products, the belonging degree of a client to such groups can be found; this translates to a characterization of the customer and also allows completing his existing data. The proposed technique allows the handling of large volumes of data, delivering results on time according to business' requirements.

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Chapter 1

Introduction

"It always seems impossible until its done."

Nelson Mandela

A general overview of the purpose of this thesis project will be presented in this chapter. Initially, the problem to be solved is introduced, then general aspects about retail industry are mentioned, followed by the general and specific objectives of this work. Finally, the methodology used and the overall structure of this thesis is depicted.

In recent years –since 2010 and thereafter– the Chilean economy has been growing about 5% per year. Retail industry[1], has been growing at an average rate of 10% per year since 2008, showing incomes above US$ 45 billion in year 2012 [22]. From these US$ 45 billion about US$ 15 billion corresponds, only, to Chilean supermarkets [34], representing nearly 5 % of the Gross Domestic Product of Chile (US$ 268 billion) [33]. This growth brings an increase both in revenue and challenges, such as a better understanding of the customer and his preferences. This thesis aims to resolve a common problem for many retailers, but in a novel way. The next section describes the problem.

[1] considering supermarkets, department stores and home improvement

1.1 The Problem

This thesis aims to find a way to characterize customers based on previous purchases. Usually, customers are grouped only by the amount of money spent in their purchases. This is a classical measure utilized by Chilean retailers in order to build customer profiles or segments.

We observe, in figure 1.1, the money spent by three different customers in the same period of time. the customer depicted spent relatively the same amount of money, just distributed differently over the range of products available. It is clear that customers 2 (blue line) and 3 (light blue line) have a similar purchase pattern, which is different than customer 1 (red line).

A typical retail analysis would have classified these three customers in the same group, because they spent relatively the same amount of money. We would like to classify customer 1 in a group and customers 2 and 3 in another, because the products purchased by 2 and 3 and the amount of money spent on each product is very similar. In the rest of the chapters we will show our approach to this problem.

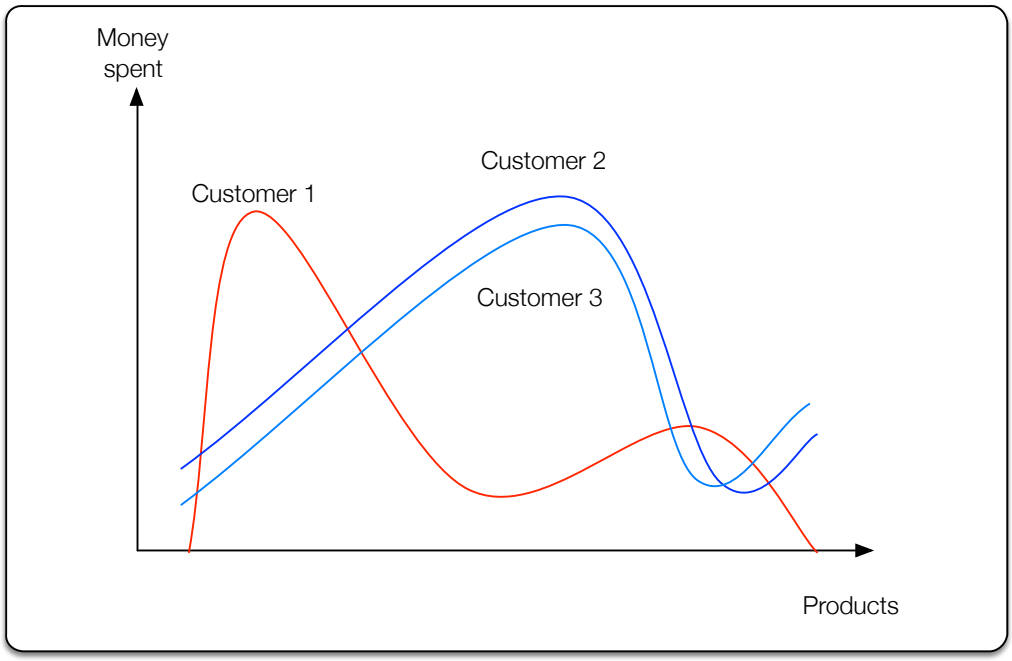


Figure 1.1: Money spent per customer on different products available.

1.2 Motivation

Companies have developed a new way to manage their relationship with customers. This new way has been widely studied and it is commonly called *Customer Relationship Management* (CRM).

Typically, marketers had been trying to acquire customers, either new ones or the competitors. Today, they are trying to retain them instead.

Reichheld [99] studied the dramatic increase in profits after small increases in customer retention rates. He showed an example where an increase in retention of 5% had an impact as high as 95% on the net present value delivered by customers. The fact exposed by Reichheld is complemented by Winer [117] who explains that long–term relationships with customers can deliver long–term profits. These facts have changed the way marketers see the world.

The customer relationship has evolved over the years, from a marketing oriented approach, to a customer oriented one [54]. Marketing approach is basically oriented to the quality of the product or service delivered; to maximize profits in the short term, i.e., seeks to attract a large number of consumers to achieve the greatest possible number of transactions. Instead, the consumer-centric approach aims to deliver value and satisfaction to the consumer, rather than maximizing the number of transactions; it seeks to attract and retain customers in order to establish long-term customer relationships.

This approach has pushed companies to apply a number of different techniques so they can respond to customer requests and provide them a highly customized experience. For example, a website can personalize their content depending on the visitor characteristics as shown in [35, 81, 93]. Verhoef [113] explains that customer loyalty programs that reward the customer have positive effects in retention rates. On the other hand sending mass emails has no effect on customer loyalty, however extending payment dates does show some benefit.

Generating a strong bond between customers and marketers requires a deep understanding of the consumer. From the large volumes of costumer data that companies today can store, great knowledge can be obtained. Processing this data in different ways may lead to improve different aspects of customer relationship; from marketing and advertising to customer service and experience.

It is important for a company to know its group of customers, because optimized promotions and recommendation strategies can be built [2, 100] on this knowledge.

Configuring layout of products at the store [28], placing related products nearby or making them accessible to customers, could be some of these actions.

In the first part of this motivation we have shown the importance of customer relationship management. Now we will explain that working with large amounts of data is a motivation in itself, because it implies significant challenges in extracting useful knowledge from it. Indeed, governments, scientific communities and other industries already use advance data mining techniques for processing large volumes of data [30].

In this case we will work with data from Chilean retailers that have big databases full of customer transactions. As with most retailers, each customer is identified thanks to loyalty programs[2] [109, 112]. These huge databases provide the necessary information to apply a methodology such as Knowledge Discovery in Databases (KDD) [38] or Cross Industry Standard Process for Data Mining (CRISP-DM) [118].

Following this approach, data quality becomes more relevant than data volume. Inaccurate, incomplete or simply absent data would produce very poor results. Despite the process with data mining techniques, the information obtained would be of low quality. The value of high quality data for companies is evident and depicted by facts such as companies are willing to give gifts and/or prizes to its customers when they update their information or submit their RUT [3] in each purchase. In this line, Wong [119] shows that when data is missing, obsolete or has scale differences (i.e., Fahrenheit to Celsius degrees) the process of finding patterns is affected and the results can be incorrect.

In order to obtain valuable information retailers have been using several approaches to determine customer profiles including methods such as K-means [53], Self Organized Maps [67], Neural Networks [37, 62, 111], Decision Trees [96, 105] just to name a few. In the case of retail, these algorithms are applied on transactional data for discovering customer behavior that is not trivially observed. Transactions are segmented into groups (or clusters) with alike characteristics between them and different between groups. Each group found has characteristics that let us describe its members. As retailers have transactional information for each customer, once clusters are determined, it is easy to find the groups where the customer belongs.

Retailers worldwide have commonly used Market Basket Analysis techniques to discover frequent item sets, i.e., products that are commonly present on purchases.

[2]Customers typically register their personal information with the company and are given a unique identifier, such as a numerical ID or membership card, and use that identifier when making a purchase. [56]

[3]RUT: Rol Único Tributario in Spanish. National identification number.

The main family of algorithms are known as *Association Rules* [4, 5]. These kinds of algorithms deliver results that enable analysts to describe and interpret relationships between products. These relationships mean that when a product is purchased –with high probability– the other product will be too [66].

After customer profiles and relationships between products are obtained, we can go one step further in analyzing new techniques in order to enhance the results obtained until now. Along this line, Raeder et al. and Clauset et al. [29,97] proposed an approach where products purchased are represented as a social network. Social Network Analysis (SNA) [115] is an area of graph mining which uses graphs to represent and model people, products, organizations, etc. and its relations or interactions. Usually these graphs are called *networks*. Networks can describe various types of complex systems and permit social network analysis [102] for world wide web [36], epidemiology [83], scientific collaboration [74], to mention some of the fields where graph theory is applied.

Algorithms from SNA are applied to discovering product communities [29]. Other studies propose a utility measure for those communities [97], and others replicate this SNA approach to the purchased problem [64]. An approach for community discovering is Overlapping Community Detection; a process for finding communities in a graph. This process is similar to the classical community discovering process. The most important difference is that a node can belong to more than one community. It follows logic, because a person can belongs to several communities in a social network, or a product may belong to several communities. For example, one kind of juice may belong to a community of soft drinks and to a community of soft beverages.

As we can see, there are several ways to address this problem. We will explore these new techniques and will show our novel approach throughout this work.

1.3 Significance of the study

This study presents a new approach for solving a main issue for the retail industry. Starting from communities of products detected we characterize and complete customer data adding significant information about the belonging grades to certain communities. This methodology permits improving customer segmentation, better profiling and enhancing recommender systems amongst other.

For generating accurate groups of products it is necessary to process very large databases with million of records. Techniques used in this project makes groups easily visualized, enhancing the analysis and interpretation process.

1.4 Objectives

This section presents the objectives of this thesis project. Both, main objectives and specific objectives are explained.

Main Objective

The main objective of this thesis project is to characterize and complete retail customer data, applying graph mining techniques over millions of transactional records from a supermarket retail chain.

Specific Objectives

1. Research the state–of–the–art in clustering, market basket analysis and overlapping community detection.
2. Benchmark the results when applying current clustering and market basket analysis techniques for big data.
3. Apply graph mining techniques over transactional data and compare the results with those obtained previously.
4. Generate customer profiles characterizing and completing existing customer data.
5. Develop and build a knowledge management software that allows the handling of customer profiles and groups of products.

1.5 Expected Results / Contributions

1. Generate a report (chapter) with state–of–the–art techniques of clustering, market basket analysis and graph mining.
2. Use an efficient software implementation of the state–of–the–art techniques on large and real transactional data (big data).
3. Use an existing software or develop one for applying graph mining techniques on transactional data.
4. Measure the quality of the results, obtained with our novel method, and compare against those obtained previously.
5. Design and development of a software that encapsulates the knowledge obtained in previous stages, allowing analysts to operate both communities and customer profiles.

1.6 Thesis Structure

In order to carry out this thesis project, an understanding of market basket analysis and community detection is needed. Main literature about data sources, customer profiles, market basket analysis and the state–of–the–art in Social Network Analysis are presented in chapter 2. For fully understanding this thesis project several definitions are given. The algorithms involved in this thesis project will be presented.

Other chapters of the thesis are described next, where the methodology explains every stage completed through the application of the novel method on real data and finally conclusions and future work are presented.

Methodological Process

The methodology of this thesis is based on Cross Industry Standard Process for Data Mining (CRISP-DM) combined with Graph Mining. For achieving the main objective –customer characterization– frequent item sets from transactional data must be built first. This goal is reached going through various stages of data processing: Network Construction, Overlapping Community Detection, Community Characterization and finally, generating Customer Profiles. Chapter 3 exhibits the details.

As a summary, the process can be depicted in five general stages as we show in figure 1.2.

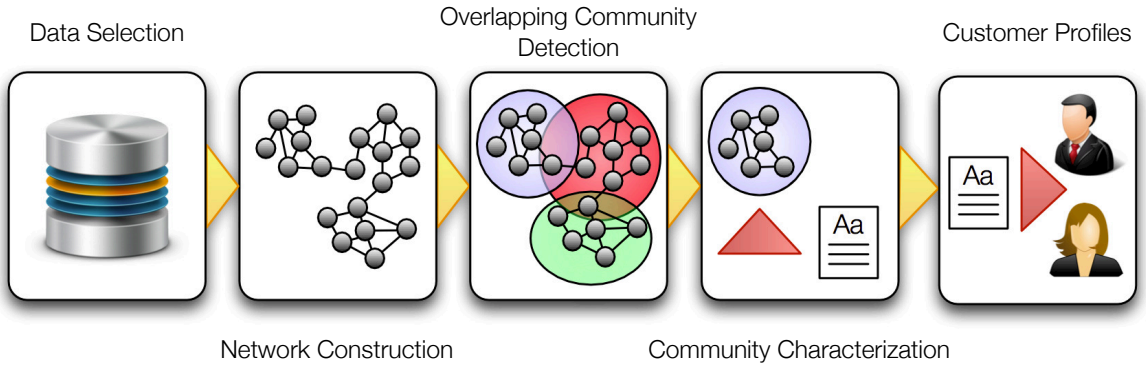


Figure 1.2: Customer Characterization Process.

Transactional data is selected according to different time window criteria. Then, networks of products –a graph representation using products as nodes and presence in a buying opportunity as edges– are built.

For discovering communities of products, overlapping community algorithms are applied. Communities obtained are characterized according to the products that it contains.

Finally, customer profiles are generated by analyzing previous purchases.

Application on Real Data

Our methodology will be tested with real data from two Chilean retail companies.

The application over real transactional data from two Chilean retail companies will be presented in chapter 4. Thresholds and filters developed to reduce spurious edges are explained. For handling millions of records, a parallel processing will be shown.

Later, the results obtained from the application of the described methodology are compared to a classical approach on both analytical and expert criteria points of view. At the end of the chapter the system developed for handling communities and customer knowledge is presented. Its construction, interfaces and purpose are explained.

Conclusions and Future Work

Finally, in chapter 5, main conclusions are described, including relevant findings and contributions from this thesis project. Also, future work and following lines of research are shown. Experts found results very promising. The method provides much better results than classical techniques of clustering and market basket analysis.

Chapter 2

State of the Art

"I have not failed. I've just found 10,000 ways that won't work."

Thomas A. Edison

In this chapter, the literature will be reviewed. It is intended to give readers a brief introduction to the methods and algorithms required to understand this thesis. In the first place, a description is provided on big data, sources of data, market basket analysis and the main algorithms used to discover frequent item sets. Afterwards, is presented a general review about graph mining and social network analysis. Finally, the current approaches to mix market basket analysis and graph mining are described.

2.1 Big Data

It is common to hear that *"We live in the information age"*. However, we believe that we live in the data age. To illustrate this, here are some facts: every day 2.5 quintillion bytes of data are created [55]. In fact, 90% of the data in the world today has been

created in the last two years alone. This data came from different sources such as, sensors, climate information, posts to social media sites, digital pictures and videos, purchase transaction records, and cell phone GPS signals, to name a few.

In [59] these facts are shown in numbers: Every minute in the world, 48 hours of video are uploaded to Youtube, email users send over 200 million messages, Google receives over 2 million search queries, Facebook shares more than six–hundred thousand pieces of content, consumers spend almost three–hundred thousand dollars on web shopping. Apple receives about forty–seven thousand app downloads, Flickr users add 3,125 new photos, Foursquare users perform around two thousand *check-ins*[1]. These numbers show the importance of not only storing information, but being able to process it. We will describe the sources of information and how it can be processed in order to obtain valuable information.

2.2 Data Sources

The customer information file is the building block for a customer–focused company, because every decision should be based on the analysis of customer data.

Customer information may include everything about customers from their names, address, phone numbers, transaction histories and any information from customer contacts. It is a good idea to prioritize different data elements to be included in the customer information file because there is a trade–off between costs and benefits of collecting data. Storing and processing information has an economical cost. The benefit is that getting detailed information leads to better decisions.

There are no standard ways to classify the types of data elements included in a customer information file. In fact, the types of data elements are different according to the industry or company where they belong. In [16] a classification in six different groups is presented: (1) customer identification data, (2) demographic data, (3) psychographic or lifestyle data, (4) transaction data, (5) marketing action data and (6) other types of data. Each type is described in the following subsections:

[1]Check-in: the process whereby a person announces their arrival at a hotel, airport, etc. in several mobile applications.

2.2.1 Customer Identification Data

It is the most basic customer information, covering various classification data, the customer's contact addresses and other useful identification data. The typical information from a customer is their name, a unique id, home and business addresses, home, business and mobile telephone numbers, email addresses, date of birth. In the case of business customer data, names of contacts, departments, fax numbers, etc. can also be included.

Customer identification information is rarely used in building response models partially because it is nominally scaled. However, they are critical in maintaining the customer relationship with the company, because they provide the means to contact the customer. It is impossible to contact a customer if the email address or home address is incorrect.

Is important to mention one customer identification field, the customer ID, that is uniquely assigned to each customer upon her first contact with the company. This customer ID is used as a key field to link to other databases. This ID is used in every interaction of the customer with the company, for example this ID helps to keep track of every customer purchase.

2.2.2 Demographic Data

Demographic data is data obtained typically from a census carried out every 10 years by the government. This data includes the age of the head of household, family income, family size, occupation of head of household, marital status, presence of children, length of residence, education level, own or rent, type of dwelling, car ownership, gender, race, and so on.

Demographic information is particularly useful for targeting prospects. Marketing researchers have found that the best predictors for a customer's future purchase behaviors are found in their historical purchase/transaction information. Nevertheless, transactional data is only available for current customers. To be able to carry out a targeting of prospects without any transactions, the demographic (or psychological) characteristics are used. For example, one could use a predictive model to identify what types of demographic characteristics a high–value customer has. Then we target prospects whose demographic profiles are similar to those of current high–value customers. Then, once prospects become customers, transactional data is collected and used to adjust the model.

2.2.3 Psychographic or Lifestyle Data

Lifestyle is a way of life or style of living that reflects the attitudes and values of a consumer while psychographics are psychological characteristics of consumers such as attitudes, values, lifestyles and opinions. According to [94], to measure lifestyles there are specific questions that consist of three groups: *activities* at work, shopping, entertainment, hobbies, etc.; *interests* in fashion, recreation, job, family, food, media, etc.; *opinions* on business, politics, economics, future, products, etc.

This kind of information could be useful for a company to infer customer attitudes and behavior from their product usage. For example, a technological company would like to know if their customers tend to hike, because they might be able to buy a GPS[2] device.

2.2.4 Transaction Data

In general, each record in a transactional database captures a transaction, such as a customer's purchase, a flight booking, or a user click on a web page. A transaction typically includes a unique transaction identity number (*transaction_ID*) and a list of the items making up the transaction, such as the items purchased in the transaction, purchase date, salesperson ID, discount, sales tax, return code, sizes, quantity, prices, purchase amount, among others. A transactional database may contain additional tables with information related to the transactions, such as item description, information about the salesperson or the branch.

An example of the records inside a transactional database are depicted in 2.1:

Transaction ID	Date	SKU	Customer ID	Quantity	Price	Total Price
925	05-07-2009	P1	10021	1	350	350
925	05-07-2009	P2	10021	3	500	1500
925	05-07-2009	P4	10021	2	500	1000
926	05-07-2009	P3	-1	4	600	2400
926	05-07-2009	P4	-1	9	500	4500
927	05-07-2009	P1	1308	4	350	1400
927	05-07-2009	P3	1308	7	600	4200

Table 2.1: Example of a transaction set.

[2]GPS: Global Positioning System

This kind of data is the most powerful data for predicting future customer purchase behavior. In fact, transactional data researchers for the last two decades have developed various models predicting consumer purchase behavior of packaged goods based on transaction histories [27, 48, 82].

Because of data storage and maintenance costs, some companies only save part of transaction data or its summary. For example, a telecommunication company has millions of customers and each customer makes several calls a day. Also, their mobile devices are constantly *pinging*[3] the cell tower. The size of calling and logs data easily becomes terabytes in weeks. Instead of saving all calling data and logs, the transaction data is recorded in summarized quantities with only successful calls made each day; the most frequently called numbers, etc.

2.2.5 Marketing Action Data

Transaction data and marketing action data may be the most important types of data for efficient marketing strategies and tactics. Marketing action information covers all marketing efforts directed at customers and prospects, whereas transaction information is their responses to the marketing effects. For example, if we send out a catalog and the customer responds, the mailing of a catalog is a marketing action data and the response or no response is transaction data.

2.2.6 Other Types of Data

For instance, financial data is especially important for financial institutions and service providers who require regular payments over time. They use external data providers that supply a customer–level financial data, such as a credit score[4], credit card history, etc. As an example, in Chile, several credit scores exist from different companies, but only one *Boletín Comercial*, which is, a centralized credit history for an individual.

In this thesis project, we will focus mainly on transactional data and how useful information is extracted. In the following sections 2.4 and 2.6, we will describe the principal methods to apply over transactional data.

[3]Ping: Utility used to test the reachability of a host in a network.
[4]Credit Score: The probability that credit users will pay their bills.

2.3 Characterization and Completion of Customer Profile

This section explains what is understood by a customer profile for this thesis project. A customer profile, according to [3] has two parts: factual and behavioral. The factual part is based on information like, name, gender, date of birth, etc. Factual part can contain information obtained from transactional data, such as "the favorite cookie of the customer is 'Fantastic Cookies' ". On the other hand, the behavioral part models the customer's actions and is usually derived from transactional data. For example, "On late Fridays when beer is purchased, the customer usually bought diapers".

First of all, we will explain the process in obtaining a customer profile. The process begins when customers purchase a set of products in a retail store generating transactional data. This data is store in a database. The information is then passed through a clustering process in order to obtain products that are related. In other words, these are products that are usually purchased together. This process is depicted in figure 2.1. There are several methods of obtaining groups of related products. This fact allows an analyst to choose the best method best fits its needs in terms of the quality and usefulness. In section 2.4 we will present the most popular method to perform a cluster analysis.

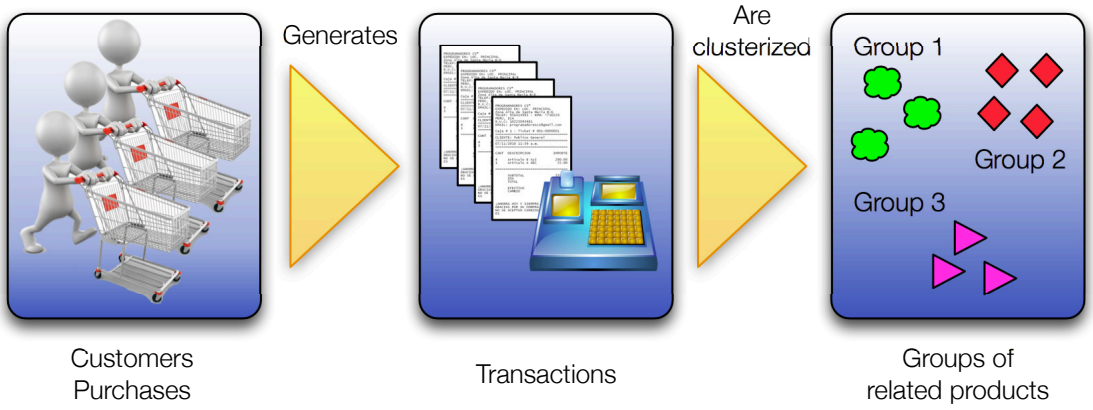


Figure 2.1: Step 1 of the customer profile generation.

Once the groups of related products are found, it is time to characterize groups by a retail analyst. Characterization is the process of labeling the groups of products

found. For example, a group that contains cookies, chocolate bars and peanut could be characterized as *Snacks*. Then the transactional data of a particular customer will be processed to obtain the degree of belonging with each of the groups found in step 1 (In figure 2.2 the situation is described, also are represented the belonging factors a, b and c).

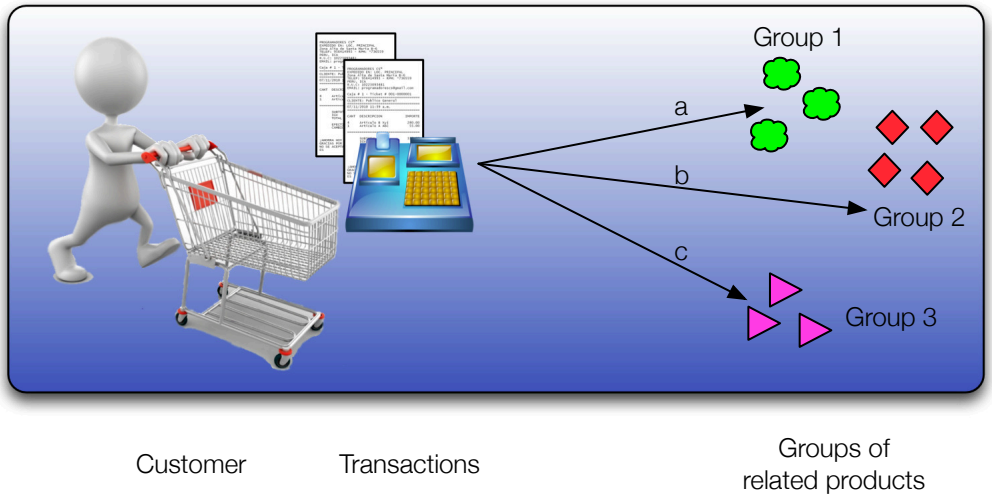


Figure 2.2: Step 2 of the customer profile generation.

Customer will belong to each of the different groups to a factor that can vary between $[0, 1]$. These factors do not necessarily have to add up to 1. i.e., if we call a_j the belonging factor to group j, we have that:

$$\sum_{j \in groups} a_j \neq 1. \tag{2.1}$$

With all this information we will obtain a profile composed of factual information as the one shown in 2.2.1, and a behavioral information as shown previously. Figure 2.3 shows the customer profile obtained.

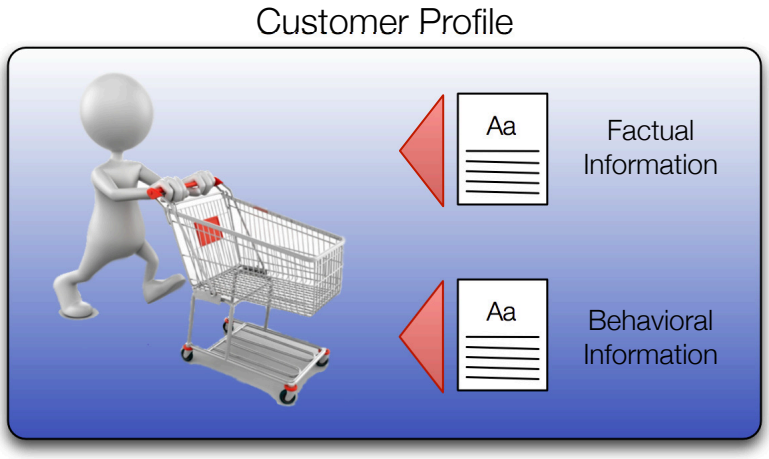


Figure 2.3: Customer profile obtained after step 1 and 2.

An important fact to note is that for each customer we are obtaining a set of belonging factors to each of the groups, ending the process of profile completion. This stage of the process is independent from the method used to obtain the groups of products.

2.4 Clustering Analysis

It is one of the vast groups of data mining techniques. The main idea is to segment a customer database so that customers within segments are similar, and different from customers in other segments. Similarity is in terms of the "clustering variables", which may be psychographics, demographics, or transaction measures such as recency, frequency or monetary value. The clusters allows rich interpretation with strong implications for which customers should be targeted with a particular offer or marketed to in a certain way.

Unlike the classification process, where a class label is given for each customer, then based on this label elements are classified. In this case, the class label of each customer is unknown. Through clustering analysis, these groupings are discovered. Clustering is the process of partitioning a set of data objects into subsets. Each subset is a cluster, and objects in a cluster are similar to one another, yet dissimilar to objects in other clusters. In this context, different clustering methods may generate different clusterings on the

same data set. The partitioning is not performed by people, but by the clustering algorithm. Consequently, clustering is useful in that it can lead to the discovery of previously unknown groups within the data.

Cluster analysis has been widely used in many applications such as image pattern recognition [7], business intelligence [1, 114], information retrieval [77], biology [14] and security [95]. In business intelligence, clustering can be used to organize a large number of customers into groups, where customers within a group share strong, similar characteristics.

In image recognition, clustering can be used to discover clusters in handwriting character recognition systems. Clustering is found in many applications in web search. For example, a keyword search may return a large number of pages that are relevant to the search. Clustering can be used to organize the search results into groups and present the results in a concise and easily accessible way. Clustering techniques also have been developed to cluster documents into topics, which are commonly used in information retrieval.

Because a cluster is a collection of data objects that are similar to one another within the cluster and dissimilar to objects in other clusters, a cluster of data objects can be treated as an implicit class. In this sense, clustering is sometimes called automatic classification. Also, one important use of clustering is *outlier detection*, where outliers[5] may be more relevant to be studied than common cases, for example an application that detects credit card fraud [25, 26].

2.5 Clustering Process

To conduct a cluster analysis, several steps are necessary :

- Select variables on which to cluster.
- Select a similarity measure and scale the variables.
- Select a clustering method.
- Determine the number of clusters.

[5]outliers : values that are far away from any cluster.

- Conduct the cluster analysis, interpret the results, and apply them.

A wide variety of methods exist that can be used for each of these steps, especially involving similarity measures and clustering methods. Choices of these methods are often subjective.

2.5.1 Selecting Clustering Variables

One of the first questions that an analyst faces is what variables should form the basis for the clustering. That is, on which set of variables do we want to form similar groups of customers? In typical applications, there are several variables available. These include:

- *Psychographics*: Attitudes and behaviors relevant to a particular product category. For example, if the product category is consumer electronics, customer self-report as innovator, opinion leader, etc. are psychographic characteristics.
- *Demographics*: Characteristics like age, income, region, etc. are included in these variables.
- *Geo-demographics*: These include characteristics inferred by where the customer lives. For example, from a census, a company can infer the average income of a customer.
- *Behavior*: This includes recency, frequency and monetary value (RFM) behaviors measured from company's customer file [20]. Recency refers to how recently did the customer make a purchase. Frequency refers to how often they make purchases. Finally, Monetary value, refers to how much they spend on each purchase.
- *Competitive measures*: These include *share–of–wallet* –which is the percentage ("share") of a customer's expenses ("of wallet") for a product that goes to the firm selling the product– [75]. This includes also, competitor preferences, etc.
- *Customer Value*: These may include responsiveness to marketing, lifetime value, customer lifetime duration, etc.

The choice of variables will depend on the application. For instance, for a cross–selling application, clustering on channel usage or RFM for various departments might be useful.

2.5.2 Similarity Measures

The main idea of clustering is to group customers into several homogeneous clusters. Customers in the same cluster are supposed to be similar while subjects across different clusters are dissimilar. The process of clustering begins with selecting a similarity measure and a set of variables. The similarity measure is applied over these chosen variables.

Two kinds of similarity measures exist: distance type and matching type, which are described next.

2.5.2.1 Distance–type Similarity Measures

Distance types of similarity measures are more appropriate when variables are measured on a common metric so that the similarity between two customers can be measured on a common metric and that the similarity between two customers can be measured as their distance in a metric space.

For example, three customers are plotted in figure 2.4. Each customer is represented as a point in two–dimensional space. These attributes could represent, for example, income and age. Here, we are implicitly using the distance between points or subjects as the similarity measure.

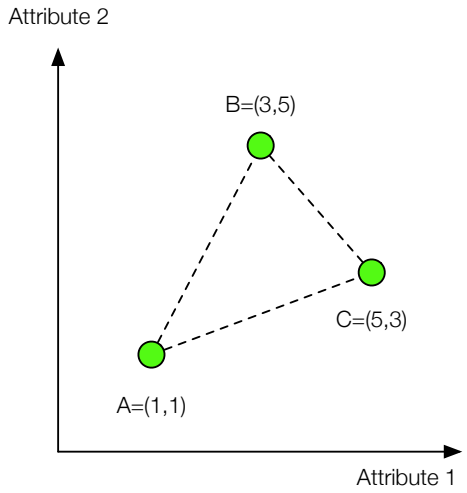


Figure 2.4: Customers and their attributes.

The most popular distance similarity measure is the Euclidean distance between two points [51]. The Euclidean distance between two subjects of dimension p , i.e., $X = [x_1, x_2, \ldots, x_p]$ and $Y = [y_1, y_2, \ldots, y_p]$ is defined as

$$d(x, y) = [(x_1 - y_1)^2 + \cdots + (x_p - y_p)^2]^{\frac{1}{2}} = [(x - y)'(x - y)]^{\frac{1}{2}} \tag{2.2}$$

Each subject is represented in two–dimensional space ($p = 2$) in figure 2.4. For example, the Euclidean distance between customers A and B is $(1-3)^2+(1-5)^2]^{\frac{1}{2}} = \sqrt{20}$ and the distance between B and C is $\sqrt{8}$. The distance between B and C is the shortest, so A and B are the most similar.

The Minkowski distance is a metric that generalizes the concept of the Euclidean distance. The Minkowski distance between two p-dimensional subjects is given by

$$d(x, y) = \left(\sum_{i=1}^{p} |x_i - y_i|^m \right)^{\frac{1}{m}} \tag{2.3}$$

It is clear that the Minkowski distances become the Euclidean distance when $m = 2$.

2.5.2.2 Matching-Type Similarity Measure

In the case of categorical variables, geometric distance is not a meaningful one. Instead, we can use the degree of matching to measure the similarity between customers when they are represented by a set of categorical characteristics. For example, two customers are treated as similar if both of them are students.

We measure the similarity between two customers as the degree of matching, specifically, the ratio of the number of matching attributes to the total number of attributes considered. For example, suppose that two customers are represented by the presence or absence (coded as 1 or 0 respectively) of five attributes. If customer A's attributes can be represented as $x = [0, 0, 1, 1, 1]$ and customer B's as $y = [1, 0, 1, 1, 0]$, then there are three matches out of five attribute comparisons and, therefore the similarity between customers A and B is 0.6.

The are some variants of matching–type similarity measures such as assigning differential weighting on matching from mismatching cases. For example, when there is a large number of zeros in the data, we assign a large weight on the matches of ones.

2.5.3 Scaling and weighting

Metric variables are frequently measured in different units, and this can distort the cluster analysis results. For instance, if we multiply one variable by a large number, then the similarity measure will be dominated by the value of the variable. The idea is to rescale the variables such that a percentage change of one variable is not more significant than the same percentage change of another variable in similarity calculation.

Several approaches exist to rescale the variables. One is to rescale all the variables to range from zero to one. If X_i is the original variable and X_i^* is the scaled variable, $X_i^* = (X_i - X_{min})/(X_{max} - X_{min})$ where X_{min} and X_{max} are the minimum and the maximum observed values for the original variable respectively. Another approach is to rescale variable $Z_i = (X_i - \bar{X})/\sigma$, where $\bar{X}$ and σ represents the mean and the standard deviation of the original variable respectively.

Another way to deal with the scaling issue is to consider weighting each of the clustering variables. For example, if we believe that the income variable is much more important than the age, it is reasonable to assign a large weight to the income variable by multiplying it by a large number (at least larger than one).

There are no correct answers to the scaling and weighting problem, so it is necessary to try several methods in order to find the best result.

2.5.4 Clustering methods

The goal of clustering is to group subjects into an arbitrary number of segments such that customers in the same segment are similar in their characteristics while customers across different segments are dissimilar. However, it is not a simple task to find the optimal clustering solution. For instance, millions of ways exist to cluster only 20 customers. While there is only one way to form a cluster with 20 customers, there are $524,287$ ways to group 20 customers into two clusters. In fact, the number of ways for sorting n subjects into k nonempty groups can be computed by a Stirling number of the second kind that is given by [45]:

$$\left\{ {n \atop k} \right\} = \frac{1}{k!} \sum_{j=0}^{k} (-1)^{k-j} \binom{j}{k} j^n \tag{2.4}$$

There are several algorithms available for clustering. That are broadly classified into four groups: partitioning, hierarchical, density–based and grid–based methods [16, 52].

Partitioning Methods The simplest version of cluster analysis is partitioning, which organizes the objects of a set into several exclusive groups or clusters. We assume that the number of clusters is given. This parameter is the starting point for partitioning methods.

Formally, given a data set, D, of n objects, and k, the number of clusters to form, a partitioning algorithm organizes the objects into k partitions ($k \leq n$), where each partition represents a cluster. The clusters are formed to optimize an objective partitioning criterion, such as a similarity function based on distance, so that the objects within a cluster are *similar* to one another and *dissimilar* to objects in other clusters in terms of the data set attributes.

Hierarchical Methods These methods are based on the creation of a hierarchical decomposition of the given set of data objects. A hierarchical method can be classified as being either *agglomerative* or *divisive*, based on how the hierarchical decomposition is formed.

The *agglomerative* approach, starts with each object forming a separate group. It successively merges the objects or groups close to one another, until all the groups are merged into one, or a termination condition is reached. The *divisive* approach, starts with all the objects in the same cluster. In each successive iteration, a cluster is split into smaller clusters, until eventually each object is in one cluster, or a termination condition holds.

Density–based Methods The majority of the methods cluster objects based on the distance between objects. These methods can find only spherical–shaped clusters and encounter difficulty in discovering clusters of arbitrary shapes. Other clustering methods have been developed based on the *density* notion. The main idea is to continue growing a given cluster as long as the density –understood as the number of objects– in the vicinity exceeds a particular threshold. For instance, for each data point within a given cluster, the vicinity of a given radius has to contain at least a minimum number of points. These kinds of methods can be used to filter out noise or outliers and discovery clusters of arbitrary shape.

Grid–based Methods These methods quantize the object space into a finite number of cells that form a grid structure. All the clustering operations are performed on the grid structure. The main advantage of this approach is its fast processing time, which is typically independent of the number of data objects and dependent only on the number of cells in each dimension in the quantized space.

2.5.5 Algorithms for clustering

A wide range of algorithms exist for clustering a set of elements. As depicted previously, there are four families of methods to cluster. In this section we will explain two algorithms that are widely used, K–means and Self Organizing Feature Maps.

2.5.5.1 K–Means

The K–means [53] may be the most popular clustering method among data miners. This algorithm is a centroid–based technique and belongs to the family of partitioning methods. Let us explain how this algorithm works:

Suppose a data set D contains n objects in Euclidean space. Partitioning methods distribute the objects in D into k clusters, $C_1, \ldots, C_k$, that is, $C_i \subset D$ and $C_i \cap C_j = \varnothing$ for $(1 \leq i, j \leq k)$. An objective function is used to assess the partitioning quality so that objects within a cluster are similar to one another, but dissimilar to objects in other clusters.

A centroid–based partitioning technique uses the *centroid* of a cluster, C_i, to represent that cluster. Conceptually, the centroid of a cluster is its center point. The centroid can be defined in various ways such as by the mean or medoid of the objects (or points) assigned to the cluster. The difference between an object $p \in C_i$ and c_i, the representative of the cluster, is measured by $dist(p, c_i)$, where $dist(x, y)$ is the Euclidean distance between two points x and y. The quality of the cluster C_i can be measured by the within–cluster variation, which is the sum of squared error between all objects in C_i and the centroid c_i defined as:

$$E = \sum_{i=1}^{k} \sum_{p \in C_i} dist(p, c_i)^2 \tag{2.5}$$

where E is the sum of the squared error for all objects in the data set; p is the point in space representing a given object; and c_i is the centroid of cluster C_i. This objective function tries to make the resulting k clusters as compact and as separate as possible.

Optimizing the within–cluster variation is computationally challenging. In the worst case, we would have to enumerate a number of possible partitions that are exponential to the number of clusters, and check the within–cluster variation values. It has been

shown that the problem is NP–hard in general Euclidean space even for two clusters. Moreover, the problem is NP–hard for a general number of clusters k even in the 2–D Euclidean space. If the number of clusters k and the dimensionality of the space d are fixed, the problem can be solved in time $O(n^{dk+1}\ log\ n)$ where n is the number of objects.

This algorithm defines the centroid of a cluster as the mean value of the points within the cluster. It proceeds as follows. First, it randomly selects k of the objects in D, each of which initially represents a cluster mean or center. For each of the remaining objects, an object is assigned to the cluster to which it is most similar, based on the Euclidean distance between the object and the cluster mean. The k–means algorithm then iteratively improves the within–cluster variation. For each cluster, it computes the new mean using the objects assigned to the cluster in the previous iteration. All the objects are then reassigned using the updated means as the new cluster centers. The iterations continue until the assignment is stable, that is, the clusters formed in the current round are the same size as those formed in the previous round. The k–means algorithm is described in algorithm 1.

Algorithm 1: The k–means algorithm.

Input:

k: the number of clusters

D: a data set containing n objects

Output:

A set of k clusters.

Method:

1 Arbitrarily choose k objects from D as the initial cluster centers
2 **repeat**
3 Assign each object to the cluster to which the object is the most similar, based on the mean value of the objects in the cluster
4 Update the cluster means, that is, calculate the mean value of the objects for each cluster
5 **until** *no change*
6 **return**

Figure 2.5 depicts the process with three different types of objects. The algorithm starts creating three randomly centroids or cluster centers that we call seeds. The seeds then begin as the center of the groups by computing the mean among the data vectors.

We will try to cluster products from transactional data as described in figure 2.5,

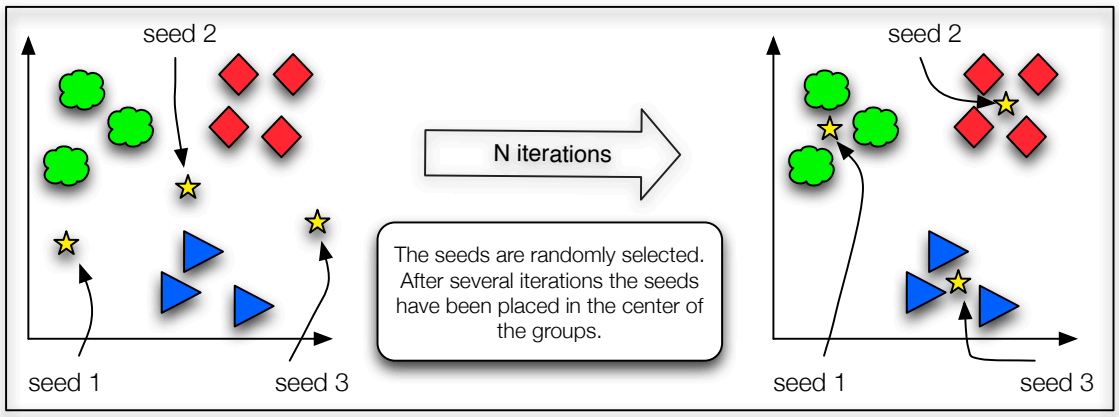


Figure 2.5: K–means algorithm.

then determine a belonging factor to each cluster based on previous transactions as depicted in figure 2.6, where a, b, c are the belonging factors to each cluster, as shown in 2.3.

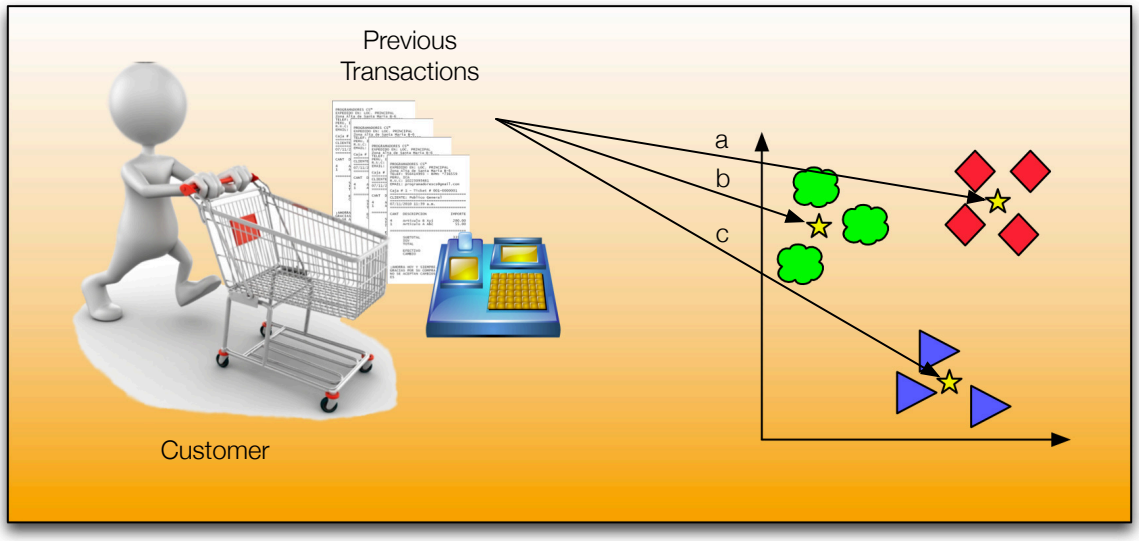


Figure 2.6: Customer belonging to k–means clusters.

2.5.5.2 Self–Organizing Features Maps (SOM)

There is a type of neural network model called the self–organizing map (SOM) that can be employed for a clustering task. Proposed by Kohonen [67], the SOM was originally used for image and sound, and recently applied to clustering people. Like other neural network models, the SOM has an input layer and an output layer. Each unit (or cluster) in the output layer is connected to units (or attributes) in the input layer. The strength of this connection is measured by a weight.

The SOM is fundamentally different from other neural network models in that its goal is to identify no pre–specified "dependent variable" for the output layer. The SOM is looking for unknown patterns in the data. Using the terminology of machine learning, the SOM is developed for unsupervised learning.

SOM represents the result of a vector quantization process as described in [101]. The SOFM maps the input space into a bi–dimensional array of nodes also called neurons. The array's lattice can be defined as rectangular or hexagonal. Every neuron is an array of similar dimensionality than data vectors. Let's call $m_i \in \Re^n$ the neuron i from a SOM. Then the components of every neuron are called the synaptic weights. Figure 2.7 depicts how these neurons and weights are structured.

To begin with the SOFM algorithm all neurons must be initialized. This process is performed creating random-synaptic weights for every neuron. Afterwards, we commence the learning or training phase of the SOM. Let $x_i \in \Re^n$ be an input data vector, where we present x_i to the network, and using a metric, we determine the most similar neuron (center of excitation, winner neuron, best matching unit (BMU) or centroid). This process is performed for every input example x. Once all data vectors have been presented to the network, we say an *epoch* has finished. Next, we must begin another *epoch*, this is done by presenting all data vectors to the network again. Finally, after many epochs, we obtain convergence on the SOM and can finish the training phase.

The best matching unit (BMU) is obtained from equation 2.6, if the SOFM has N neurons, then $m_c(t)$ is the winner neuron defined as the BMU in the whole network for the example x_i in the epoch

$$\|x_i - m_c(t)\| = min\|x_i - m_j(t)\| \text{ or } c = argmin\|x_i - m_j(t)\| \;\; \forall j = 1, \ldots, N \qquad (2.6)$$

Once the winner neuron (BMU) has been identified, we activate it to actually learn the example vector. To do so, we use a learning function as shown in equation 2.7. An

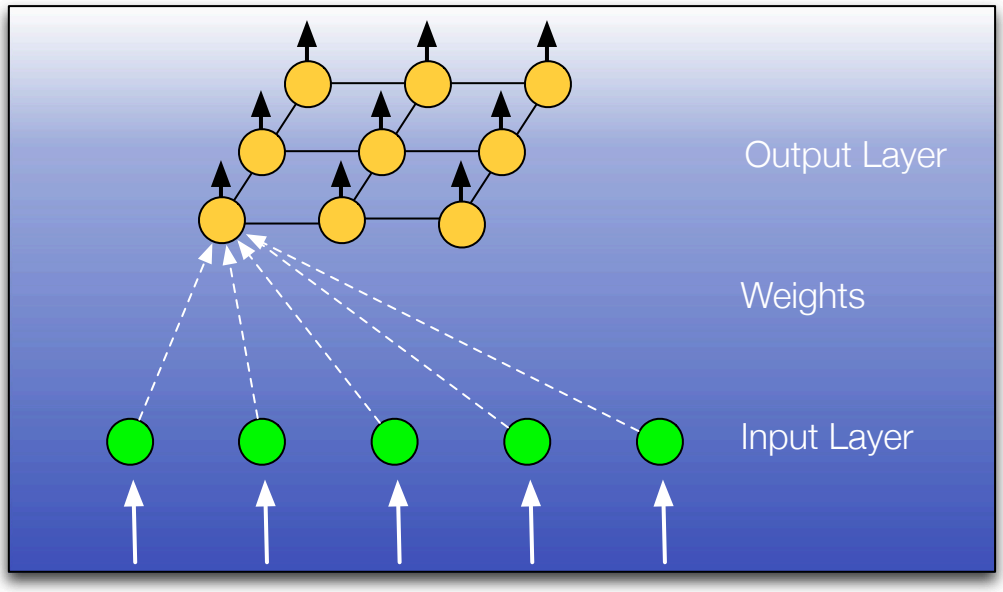


Figure 2.7: Architecture of a SOM.

important factor is that the learning function alters the synaptic weights of the BMU but, it also alters the weights of the surrounding neurons to a lesser degree. This way, the BMU is moved towards the data example and the surrounding neurons also are moved, but less than the BMU. This effect depends on the distance from the BMU (or centroid) as shown in equation 2.7 and is transmitted to all neurons in the network.

$$m_j(t+1) = m_j(t) + h_{cj}(t) * (x_i(t) - m_j(t)) \quad \forall j = 1, \ldots, N \tag{2.7}$$

The equation 2.7 shows the basic weight modification algorithm, where t is an integer and represents the iteration, and the h_{cj} is called "neighborhood kernel". It is a function defined over the lattice points and usually $h_{cj} = h(\|rc - rj\|, t)$, where $r_c, r_j \in \Re$ is the radius between the BMU and another neuron in the array. The expression h_{cj} is so that when $\|r_c - r_j\|$ is increased, $h_{ci} \to 0$. Depending on the points chosen by hci it is possible to define different notions of neighborhood. We are therefore able to define diverse topologies of the SOM (see Fig. 2.7), such as:

- Open topology: The idea is to maintain the bi–dimensional space geometry when producing the learning. The edges of the map are not connected to other neurons,

producing an abrupt end of the learning. This effect is more notorious when the BMU is closer to the edges of the map.

- Tape/cilindrical topology: Using this topology the idea is to connect two borders of the bi–dimensional map. Thus, the learning effect continues to the opposite side of the map, although, the other sides of the map are disconnected.
- Toroidal topology: In this topology we connect all borders of the bi-dimensional grid. This way we never finish the learning effect in either direction. We always transmit the effect to all neurons in the map in a smooth way. We say that this topology helps to maintain the continuity of the space.

The previous topologies presented are depicted in figure 2.8.

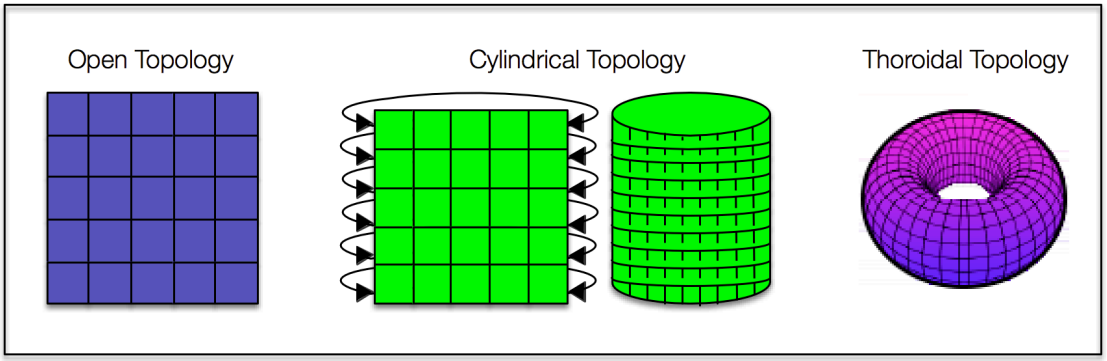


Figure 2.8: SOM possible topologies.

2.5.6 Number of Clusters

Determining the appropriate number of clusters is one of the most difficult problems in clustering. Usually, the criteria chosen tends to be subjective. For example, the relative sizes of the clusters should be large enough to be managerially meaningful. The clusters with few elements (customers) may be treated as outliers and ignored.

The methods for determining the number of clusters depends on the clustering algorithm being used and there are several criteria available. However, Milligan [80] showed that the procedure by Calinski [21] performed the best among 30 different

criteria. The following criterion suggested in [21] is used to determine the number of clusters:

$$G(k) = \frac{(n-k)(T-W)}{(k-1)W} \tag{2.8}$$

where k is the number of clusters, n is the number of customers, W is the square sum of the distances of the customers to the center of its cluster, and T is the square sum of the differences of each customer to the average customer, essentially, the center of the full data. The optimal number of clusters can be determined by selecting k which returns the maximum value for $G(k)$, because in that case, W, or the distances between customers and the center of their clusters, is relatively small compared to T, the distances between customers and the center of the entire data.

2.5.7 Interpreting the results

The results of a cluster analysis are interpreted by examining the means for each cluster of the clustering variables, and also examining the means of any other variable, i.e., "discrimination variables", not included in the clustering routine. For instance, we may cluster based on benefits sought, but have a host of other variables, for example demographics, that we use for interpretation.

Interpreting the clusters is a subjective task that often offers insight into the nature of the market. Also, the interpretations are subjective and make use of both the clustering and discrimination variables. Note also there are clear managerial implications in that companies with particular strengths could plausibly target one of these clusters, but probably not all of them (at least with the same product).

2.6 Market Basket Analysis

Market basket analysis (MBA) tries to examine the products customers usually buy together, and using this information to promote products trying to induce an up-selling or a cross-selling. The first one tries to make customers purchase more expensive items, upgrades, or other add-ons in order to make a more profitable sale. The second one tries to sell an additional product or service to an existing customer. In this chapter we will discuss key concepts of *confidence*, *support*, and *lift*, applied to market basket analysis.

Blattberg et al. [16] show that marketing researchers have been interested in studying product affinity for a long time, because the output of a market basket analysis is a series of rules. These rules are used to improve the efficiency of marketing strategies and tactics. Retail stores learn from the analysis which products are purchased at the same time or in a particular sequence.

Market basket provides valuable information for firms to develop various marketing strategies and tactics. First, association rules from a market basket analysis can be used for a supermarket to manage its space. It may stock the associated items close together such that customers would not forget to purchase both items. On the other hand, it may stock the associated items far apart so that consumers would have to spend more time browsing aisle by aisle.

Second, market basket can be used to design different promotional strategies. It will provide an idea for developing a coupon program where customers purchasing item A get the discount coupon for item B.

Third, market basket, can be very helpful to marketers for selecting cross–selling items; for instance, a customer purchase item A, and the salesperson offers a cross–sell product B.

2.6.1 Definition

Market basket analysis, according to [15] does not refer to a single technique; it refers to a set of business problems related to understanding point–of–sale transaction data. The most common technique is association rules, and will be described in section 2.6.3.

2.6.2 Market Basket Data

Market basket data belong to the family of transactional data (described in section 2.2.4). Basically it is a description and a relationship between three entities:

Customers. Refers to the person that makes a purchase in a particular store or supermarket. This information is not always available, for example, if the customer is not logged in the web site, or if the customer does not belong to a loyalty club.

Orders. The item sets purchased in a single purchase event by a customer. This might correspond to a customer ordering several products on a web site, or a customer purchasing a basket of groceries. This includes the total amount of the purchase, additional shipping charges, payment type, etc.

Item. Refers to the description of a particular product, for example, SKU, product family, price, etc. Usually includes information that might prove valuable for analysis like product hierarchy.

Blattberg [16] describes that the input for a market basket analysis is customer–level transactions data, although it is not necessary that each customer be explicitly identified. For example, grocery stores record each customer's transaction data, with their scanning device even through they do not know the customer's name, address, phone number, e–mail address, etc. Each transaction records the date, the cashier identifier, items purchased, prices of each item, coupons redeemed. Table 2.2 shows an example of the information contained in a grocery store database. There are five transactions and each transaction consists of a set of items. The main focus of market basket analysis is the set of items purchased for each transaction. From this transaction data, market basket analysis provides a series of association rules where we infer which items are purchased together.

Transactions	**Items purchased**
1	Milk, orange juice, ice cream, beer, soap
2	Milk, ice cream, beer
3	Milk, orange juice, detergent
4	Milk, ice cream, pizza
5	Milk, orange juice, soap

Table 2.2: Market basket data from a grocery store.

Each association rule consists of an antecedent and a consequent. For instance, consider the association rule, "if a consumer purchases item A, also tends to purchase item B". In this example, A is the *antecedent* while B is the *consequent*.

2.6.3 Association Rules

Association rules have been widely studied, since the paper by Agrawal [4], within the field of knowledge discovery [5, 18, 76] and is usually called the market basket problem.

2.6.3.1 Definition

Let $I = \{I_1, I_2, \ldots, I_m\}$ be an item(product) set. Let D be a set of a database transactions where each transaction T is a nonempty item set such that $T \subset I$. Each transaction is associated with an identifier, called *transaction id* or TID. Let F be a set of items. A transaction T is said to contain F if $F \subset T$.

An *association rule* is an implication of the form $A \Rightarrow B$, where $A \subset I$, $B \subset I$, $A \neq \emptyset$, $B \neq \emptyset$, and $A \cap B = \emptyset$

2.6.4 Deriving Association Rules

We are going to intuitively obtain purchase patterns, from transactional data of table 2.2. If we look at the data we can check that every purchase includes milk. Milk and orange juice are also purchased together in three out of five transactions. From this observation we can say that there is a cross–selling possibility between milk and orange juice. Ice cream and beer are purchased together in two out of five transactions. If we continue with this process, we can formulate an association rule between orange juice and soap, but we are interested in selecting only "relevant" rules. To do so we need some criteria or a metric to quantify the quality of the rule obtained [13]. Hence, researchers have proposed many different metrics. The three most popular criteria evaluating the quality of an association rule are, support, confidence and lift. Each one of these is described next:

Support is the percentage of transactions containing a particular combination of items relative to the total number of transactions in the data set. Support for an individual item A can be interpreted as the probability a transaction contains item A, or $P(A)$. However, we are interested in associations with multiple items, so the support for the combination A and B would be $P(AB)$. For example, considering the association rule "if milk, then beer" from table 2.2. Support measures how often milk and beer are purchased together. They are purchased together two out of five transactions. Therefore, the support for the association rule is 40%. Support for multiple items can be interpreted as a joint probability.

Measuring the quality of an association rule using support has an issue. In fact, if an element has a high support, then each associated element will have a high support too, this fact is described in table 2.2 where association rule "if beer then milk" has a support of 40%, however as every customers buy milk, the support

for any element purchased together with milk will be high too. We therefore need another measure to avoid this.

Confidence measures how much the consequent (products or items) is dependent on the antecedent (products or items). In other words, confidence is the conditional probability of the consequent given the antecedent, $P(A|B)$. For instance, the confidence for the association rule "if ice cream then beer" is 66% since three transactions contain the antecedent (ice cream) and two among the three transactions contain the consequent (beer). This means that given that the baskets containing ice cream is selected, there is 66% chance that the same basket also contains beer. Confidence, unlike the support, is asymmetric. For example, confidence of "if beer then ice cream" is 100% while the confidence of "if ice cream then beer" is 66%.

The law of conditional probability states that $P(B|A) = P(AB)/P(A)$. That is, confidence is equal to the support of the association rule divided by the probability or the support of the antecedent. For instance, the support of an association rule "if ice cream then beer" is 40% (two of five transactions) while the support of the probability of ice cream is 60% (three out of five). The confidence is $(40\%/60\%) = 66\%$.

Confidence, just like support, has some problems. Consider a rule "if ice cream then orange juice". Its confidence or $P(B|A)$ is 33%, so you may think it is a relevant rule, but there is 60% chance (e.g., $P(B) = 60\%$) that a randomly chosen transaction contains orange juice. Ice cream is therefore not a powerful antecedent for identifying an orange juice purchase. Its confidence is lower than a random chance of identifying an orange juice purchase.

Lift is a measure to surpass the problems with support and confidence. Let us suppose a general association rule "if A then B". The lift for the rule is defined as $P(B|A)/P(B)$ or $P(AB)/(P(A)P(B))$. Lift is symmetric in that the lift for "if A then B" is equal to "if B then A".

$P(B)$ is the probability that a randomly chosen transaction contains item B. It is an unconditional probability of purchasing item B regardless of other items purchased (also known as "expected confidence"). Lift is said to measure the difference between the confidence of a rule and the expected confidence. For instance, the lift of an association rule "if ice cream then beer" is 1.67 because the expected confidence is 40% and the confidence is 67%. This means that consumers who purchase ice cream are 1.67 times more likely to purchase beer than randomly chosen customers. A larger lift means more relevant rules.

Lift equal to 1 has a special meaning. We know that $P(AB) = P(A)P(B)$ if A and

B are independent. Lift greater than 1 indicates that the item A is independent and item B tend to occur together more often than by random chance. On the other hand, if lift is lower than 1, it indicates that the item A and B are purchased together less likely than would be predicted by random chance.

In summary, these are the three most popular criteria for evaluating association rules in market basket analysis, defined as follows:

$$Support = P(A \cup B) \tag{2.9}$$
$$Confidence = P(B|A) \tag{2.10}$$
$$Lift = P(B|A)/P(B) \tag{2.11}$$

Each criteria has its advantages and disadvantages. In general, rules with high support, high confidence and high lift are preferred because they might indicate relevant rules. Rules with very low lift are relevant because it implies that two products "repel" each other. For example, Pepsi and Coke usually tend not to be in the same basket. Knowing which products tend to "repel" each other gives an idea of products that should not be promoted together.

2.6.5 Frequent Itemset Mining Methods

In this subsection we will present the methods for mining the simplest form of frequent patterns such as those described previously in section 2.6.3. We begin our description, with Apriori algorithm (2.6.5.1), the basic algorithm for finding frequent itemsets. Then, we show how to generate strong association rules from frequent itemsets (2.6.5.2).

2.6.5.1 Apriori Algorithm

Apriori is an algorithm proposed by Agrawal et al. [5] for mining frequent itemsets for boolean association rules. The name is based in the fact that the algorithm uses prior knowledge of frequent itemset properties. The apriori algorithm uses an iterative approach known as level–wise search, where k–itemsets are used to explore $(k+1)$–itemsets. First, the set of frequent 1–itemsets is found by scanning the database to accumulate the count for each item, and collecting those items that satisfy minimum

support. The result is named L_1. Next, L_1 is used to find L_2, the set of frequent 2–itemsets, which is used to find L_3, and so on, until there are no more frequent k–itemsets to be discovered. A full scan of the database to discover each set of L_k is necessary.

The Apriori algorithm used a two step process, **join** and **prune**. *Join* step finds L_k from a set of candidate k–itemsets generated by joining L_{k-1} with itself, generating a set denoted by C_k. It is important to note that C_k is at least equal to or bigger than L_k. The *prune* step is necessary to remove members from C_k that may not be frequent. To do so, a database scan is performed to determine the count of each candidate of C_k that would result in the determination of L_k. As C_k can be huge, a property known as *Apriori property* is used. This property says: *All nonempty subsets of a frequent itemset must also be frequent.* Property is used as follows: any $(k-1)$–itemset that is not frequent cannot be a subset of a frequent k–itemset. Hence, if any $(k-1)$–subset of a candidate k–itemset is not in L_{k-1}, then the candidate cannot be frequent either and so can be removed from C_k. This subset testing can be done quickly by maintaining a hash tree of all frequent itemsets.

Algorithm 2 depicts the Apriori algorithm.

Algorithm 2: The Apriori algorithm.

Input:

D: a database of transactions.

min_sup: the minimum support count threshold

Output:

L: frequent itemsets in D.

Method:

```
1   L_1 = find_frequent_1–itemsets(D)
2   for (k = 2; L_{k-1} ≠ φ; k = k + 1) do
3       C_k = apriori_gen(L_{k-1})
4       foreach transaction t ∈ D do
5           C_t = subset(C_k, t)
6           foreach candidate c ∈ C_t do
7               c.count = c.count + 1
8           end
9       end
10      L_k = {c ∈ C_k | c.count ≥ min_sup}
11  end
12  return  L = ∪_k L_k
```

Definitions:

- l_1 and l_2 are itemsets in L_{k-1}.
- $l_i[j]$, refers to the jth item in l_i (e.g. $l_1[k-2]$ refers to the second to the last item in l_1).

Algorithm 3: Procedures : apriori_gen.

```
apriori_gen(L_{k-1}: frequent (k-1)–itemsets)
for itemset l_1 ∈ L_{k-1} do
    for itemset l_2 ∈ L_{k-1} do
        if (l_1[1] = l_2[1]) & ... & (l_1[k-2] = l_2[k-2]) & (l_1[k-1] < l_2[k-1])
        then
            c = l_1 ⋈ l_2        // Join step
            if has_infrequent_subset(c, L_{k-1}) then
                delete c     // Prune step
            else
                add c to C_k
    end
end
return C_k
```

Algorithm 4: Procedures : has_infrequent_subset.

```
has_infrequent_subset(c: candidate k–itemsets
                      L_{k-1}: frequent (k-1)–itemsets.)
for (k-1)–subset s of C do
    if s ∉ L_{k-1} then
        return TRUE
end
return FALSE
```

2.6.5.2 Generating Association Rules from Frequent Itemsets

Now that we have the frequent itemsets from the transactions database D, it is time to generate strong association rules from them. By "strong" we mean rules that satisfy both minimum support and minimum confidence. These can be done using equation 2.10, for confidence. This equation says:

$$confidence(A \Rightarrow B) = P(B|A) = \frac{support\_count(A \cup B)}{support\_count(A)} \tag{2.12}$$

The conditional probability is expressed in terms of itemset support count, where $support\_count(A \cup B)$ is the number of transactions containing the itemsets $A \cup B$, and $support\_count(A)$ is the number of transactions containing the itemset A. Based on this we can generate the association rules as follows:

- For each frequent itemset l, generate all nonempty subsets of l.
- For every nonempty subset s of l, output the rule "$s \Rightarrow (l - s)$" if $\frac{support\_count(l)}{support\_count(s)} \geq min\_conf$, where min_conf is the minimum confidence threshold.

With this process we can obtain the association rules that are important, meaningful and relevant for the retail. A relevant issue about association rules is that they are difficult to analyze because rules are not grouped and require an expert analysis to be fully understood.

2.7 Social Network Analysis

This section is based on previous work carried out by [24, 32, 84].

The concept of social network analysis, was introduced in 1967. Stanley Milgram conducted an experiment to understand how people are connected with others. The experiment consists in asking people to forward a package to any of their acquaintances who they thought might be able to reach specific target individual [79]. Milgram found that most people were connected by six acquaintances. Backstrom et al. [10] conducted an experiment using the Facebook network of active users, discovering that most people were connected by four acquaintances.

Research on networks typically consists of analyzing observations about the structure and the models giving rise to such structures. This approach aims to discover structural properties or find patterns [24], such as heavy–tailed degree distributions [19, 36], statistical properties [72], local clustering of edges [73], study of network community structure [39, 44], just to name a few.

2.7.1 Networks and Graph Theory

This section will develop basic ideas about graph theory and the study of network structure.

Graphs, nodes and edges According to [24], a graph is a way of specifying relationships among a collection of items, a set of nodes, pairs of which might be connected by edges. This definition applies to a wide array of disciplines. For example, computer networks consists of routers/computers (*nodes*) and the links (*edges*) between them. For example, the graph in Figure 2.9 consists of 5 nodes labelled $1, 2, 3, 4$ and 5, with 3 connected to each of the other four nodes by edges. We say that two nodes are neighbors if they are connected by an edge.

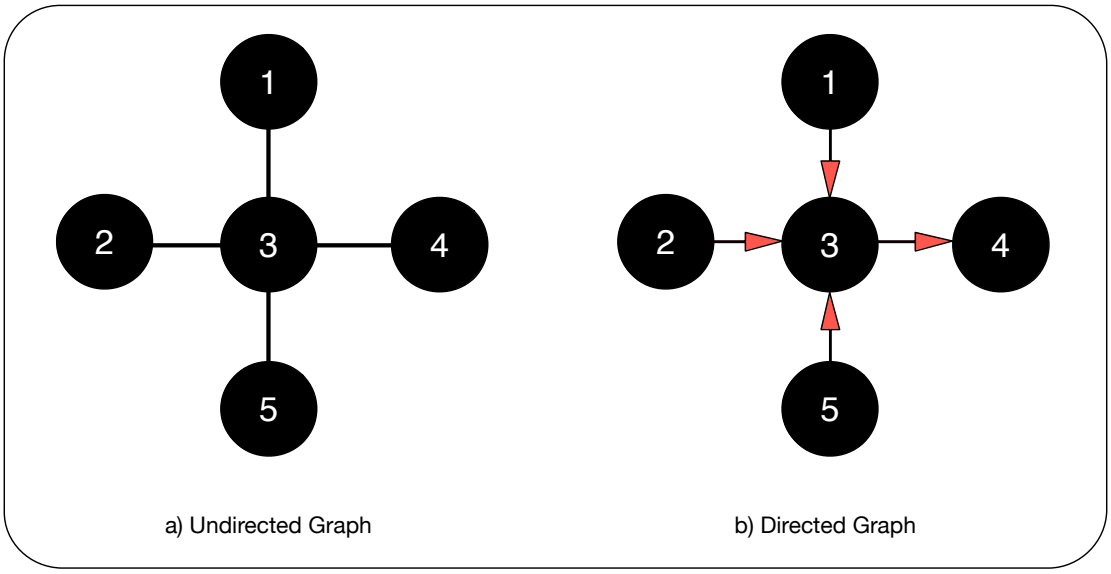


Figure 2.9: Example of graphs: directed and undirected.

In Figure 2.9 a), it is possible to think of the relationship between two ends of an edge as being symmetric; the edge simply connects them to each other. In many settings, however, asymmetric relationships are useful, -for example, where 2 points to 3, but not vice–versa. For this purpose, a *directed graph* is defined as a set of nodes with a set of directed edges; each directed edge is a link from one node to another, in this

case direction is important. Directed graphs are generally drawn as in Figure 2.9(b), with edges represented by arrows. An *undirected graph* is one in which edges have no orientation.

Mathematically, we represent a network by a graph (V, g) which consists of a set of nodes $V = \{1, \ldots, n\}$ and an $n \times n$ matrix $g = [g_{ij}]_{i,j \in V}$ (referred to as an adjacency matrix), where $g_{ij} \in \{0, 1\}$ represents the availability of an edge from node i to node j. The edge weight $g_{ij} > 0$ can also take on non-binary values, representing the intensity of the interaction, in which case we refer to (V, g) as a weighted graph. We refer to a graph as a *directed graph* if $g_{ij} \neq g_{ji}$ and an *undirected graph* if $g_{ij} = g_{ji}\ \forall i, j \in V$.

Another representation of a graph is given by (V, E), where E is the set of edges in the network. For *directed graphs*, E is the set of directed edges, i.e., $(i, j) \in E$ and for *undirected graphs*, E is the set of undirected edges, i.e., $\{i, j\} \in E$.

Also, two nodes are said to be *adjacent*, *neighbors*, or *connected* if there exist an edge between them. If all k nodes in the graph are adjacent, the graph is said to be *k-complete*. A graph is simple, i.e. loop-less and lacks multiple edges, if there is at most one edge between each pair of nodes and no node is a neighbor of itself.

A *walk* is an ordered set of alternating nodes and edges that start in one node i and end in another node j. If the walk only transverses each node at most once, it is called a *path*. A k-*cycle* is a path where the first and last nodes are the same, and the path contains k edges. A graph is *connected*, if there exists a path between any given pair of nodes. The *shortest path* between two nodes is the *geodesic* and the longest geodesic is the diameter of the graph.

A graph without cycles is called a *tree* (or a forest if unconnected). A *subgraph*, G', of a graph G contains all edges that connect a subset of the node set, i.e. $V'(G) \subset V(G)$ such that $E'(G) \subset E(G)$ contains all edges connecting the nodes in $V'(G)$. One says that the edge set is spanned by the set of nodes. Two subgraphs are therefore disjointed and not connected. A k-*clique* is a k-complete sub-graph.

2.7.2 Metrics in social network analysis

Within graph theory and network analysis, there are various measures of the centrality of a vertex within a graph that determines the relative importance of a vertex within the graph. There are four measures of centrality that are widely used in network analysis: degree centrality, closeness, betweenness, and eigenvector centrality.

Degree centrality

The most intuitive measure of centrality of a vertex into a network is called degree centrality. Given a graph $G = (V, E)$ represented by means of its adjacency matrix A, in which a given entry $A_{ij} = 1$ if and only if i and j are connected by an edge, and $A_{ij} = 0$ otherwise, the *degree centrality* $C_D(v_i)$ of a vertex $v_i \in V$ is defined as:

$$C_D(v_i) = d(v_i) = \sum_j A_{ij} \tag{2.13}$$

The idea behind the degree centrality is that the importance of a vertex is determined by the number of vertices adjacent to it, i.e., the larger their degree, the more important the vertex is.

Even though in real world networks only a small number of vertices have high degrees, the degree centrality is a rough measure, but it is often adopted because of the low computational cost required for its computation. As an example of this low cost is found in Rosenthal and Pino [103]. A *normalized* version of the degree centrality exists and is defined as follows:

$$C'_D(v_i) = \frac{d(v_i)}{n-1} \tag{2.14}$$

where n represents the number of the vertices in the network.

Closeness centrality

A more accurate measure of centrality of a vertex is represented by the *closeness centrality* [104]. The closeness centrality relies on the concept of *average distance*, defined as:

$$D_{avg}(v_i) = \frac{1}{n-1} \sum_{j \neq i}^{i} g(v_i, v_j) \tag{2.15}$$

where $g(v_i, v_j)$ represents the geodesic distance between vertices v_i and v_j.

The closeness centrality $C_C(v_i)$ of a vertex v_i is defined as

$$C_C(v_i) = \frac{1}{n-1} \sum_{j \neq i}^{i} g(v_i, v_j) \tag{2.16}$$

In practice, the closeness centrality calculates the importance of a vertex on how close the given vertex is to the other vertices. Central vertices, with respect to this measure, are important as they can reach the whole network more quickly than non-central vertices. Different generalizations of these measures for weighted and disconnected graphs have been proposed in [89].

Betweenness centrality

A more complex measure of centrality is the betweenness centrality [41, 42]. It relies on the concept of shortest paths. In detail, in order to compute the betweenness centrality of a vertex, it is necessary to count the number of shortest paths that pass across the given vertex.

The betweenness centrality $C_B(v_i)$ of a vertex v_i is computed as

$$C_B(v_i) = \sum_{v_s \neq v_i \neq v_t \in V} \frac{\sigma_{st}(v_i)}{\sigma_{st}} \tag{2.17}$$

where σ_{st} is the number of shortest paths between vertices v_s and v_t and $\sigma_{st}(v_i)$ is the number of shortest paths between v_s and v_t that pass through v_i. Vertices with high values of betweenness centrality are important because they maintain an efficient way of communication inside a network and fosters the information diffusion.

Eigenvector centrality

Another way to assign the centrality to a vertex is based on the idea that if a vertex has many central neighbors, it should be central as well. This measure is called eigenvector centrality and establishes that the importance of a vertex is determined by the importance of its neighbors. The eigenvector centrality $C_E(v_i)$ of a given vertex v_i is

$$C_E(v_i) \propto \sum_{v_j \in N_i} A_{ij} C_E(v_j) \tag{2.18}$$

where N_i is the neighborhood of the vertex v_i, being $x \propto Ax$ that implies $Ax = \lambda x$. The centrality corresponds to the top eigenvector of the adjacency matrix A.

For simplicity, we will compute Degree, Betweenness and Closeness Centrality for all networks in this work. However, results for every node of these networks are not shown in this thesis. Results are available online[6].

2.8 Overlapping community detection

Detecting clusters or communities in real-world graphs such as large social networks, web graphs, and biological networks is a problem of considerable practical interest that has received a great deal of attention [29, 40, 44, 49, 61].

The first problem in graph clustering is in finding a quantitative definition of community. According to Fortunato [39] there is no universally accepted definition. As a matter of fact, the definition often depends on the specific system and/or application one has in mind. From intuition we get the notion that there must be more edges *inside* the community than edges linking vertices of the community with the rest of the graph [44]. This is the reference guideline at the basis of most community definitions. But many alternative recipes are compatible with it. Moreover, in most cases, communities are algorithmically defined, i.e., they are just the final product of the algorithm, without a precise a priori definition.

[6] http://users.dcc.uchile.cl/ ividela/resultados/calculos.zip

The problem can be mathematically defined as follows:

Given a network or graph $G = \{E, V\}$, where V is a set of n nodes and E is a set of m edges. For dense graphs $m = O(n^2)$, but for sparse networks $m = O(n)$. The network structure is determined by the $n \times n$ adjacency matrix A for unweighted networks and weight matrix W for weighted networks. Each element A_{ij} of A is equal to 1 if there is an edge connecting nodes i and j; and 0 otherwise. Each element w_{ij} of W takes a non–negative real value representing strength of connection between nodes i and j.

In the case of overlapping community detection, the set of clusters found is called a *cover* $C = \{c_1, c_2, \ldots, c_k\}$ [69], in which a node may belong to more than one cluster. Each node i associates with a community according to a belonging factor (i.e., soft assignment or membership) $[a_{i1}, a_{i2}, \ldots, a_{ik}]$ [85], in which a_{ic} is a measure of the strength of association between node i and cluster c. Without loss of generality, the following constraints are assumed to be satisfied

$$0 \leq a_{ic} \leq 1 \; \forall i \in V, \; \forall c \in C, \tag{2.19}$$

$$\sum_{k=1}^{|C|} a_{ic} = 1, \tag{2.20}$$

where $|C|$ is the number of clusters.

In general, algorithms produce results that are composed of one of two types of assignments, *crisp* (non-fuzzy) assignment or $fuzzy$ assignment [47]. With *crisp* assignment, each node belongs to one or more communities with *equal* strength. The relationship between a node and a cluster is *binary*. That is, a node i either belongs to cluster c or does not. With fuzzy assignment, each node is associated with communities in proportion to a belonging factor. With a threshold, a $fuzzy$ assignment can be easily converted to a *crisp* assignment. Typically, a detection algorithm outputs *crisp* community assignments.

2.8.1 Overlapping Community Detection Algorithms

Numerous algorithms have been developed using a variety of methods; these vary in their effectiveness and time performance for different types of networks. In this section, algorithms for overlapping community detection are reviewed and categorized into five classes which reflect how communities are identified.

2.8.1.1 Clique Percolation

The clique percolation algorithm [91] (CPM) detects communities based on the assumption that a community or k-community is a set of nodes which can be reached through a series of adjacent k-cliques (a k-clique is set of k nodes all connected to each other), where two k-cliques are adjacent if they share $(k-1)$ nodes. The algorithm begins by identifying all cliques of size k in a network. Afterward, the method builds k-communities from k-cliques found. Since a vertex can be in multiple k-cliques simultaneously, overlap between communities is possible. Empirically, $k = 3$ or 4 has been shown to give the best results. CFinder[7] is an implementation of CPM, whose time complexity is polynomial in many applications. Despite conceptual simplicity, one may argue that CPM-like algorithms are more like pattern matching rather than finding communities since they aim to find specific, localized structures in a network.

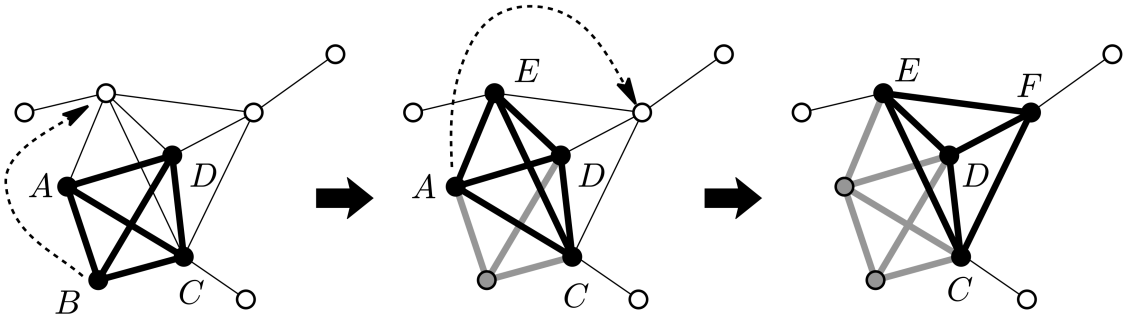


Figure 2.10: Clique Percolation [17]

Figure 2.10 shows the iterative procedure to detected overlapping communities using Clique Percolation methods.

[7] *www.cfinder.org* [online: accessed 06-12-2013]

2.8.1.2 Line Graph and Link Partitioning

The idea of partitioning links instead of nodes to discover community structure has also been explored. A node in the original graph is called overlapping if links connected to it are put in more than one cluster. In [6], links are partitioned via hierarchical clustering of edge similarity. Given a pair of links e_{ik} and e_{kj} incident on a node k, a similarity can be computed via the Jaccard Index [58] defined in Equation 2.21.

$$S(e_{ik}, e_{kj}) = \frac{|N_i \cap N_j|}{|N_i \cup N_j|} \tag{2.21}$$

Where N_i represents the set of nodes reachable from node i including i. Then, the single-linkage hierarchical clustering is used to build a link dendrogram. Cutting this dendrogram at some threshold yields to linked communities. The time complexity is $O(nk_{max}^2)$, where k_{max} is the maximum node degree in the network. Although, the link partitioning for overlapping detection seems conceptually natural, there is no guarantee that it provides higher quality detection than node based detection does [39] because these algorithms also rely on an ambiguous definition of community.

2.8.1.3 Local Expansion and Optimization

Algorithms utilizing local expansion and optimization are based on growing a natural community [69] or a partial community. Most of them rely on a local benefit function that characterizes the quality of a densely connected group of nodes. Baumes et al. [11, 12] proposed a two-step process. First, the algorithm *RankRemoval* is used to rank nodes according to some criterion. Then, the process iteratively removes highly ranked nodes until small, disjoint cluster cores are formed. These cores serve as seed communities for the second step of the process, *Iterative Scan* (IS), that expands the cores by adding or removing nodes until a local density function cannot be improved. The proposed density function can be formally given as

$$f(c) = \frac{w_{in}^c}{w_{in}^c + w_{out}^c} \tag{2.22}$$

where w_{in}^c and w_{out}^c are the total internal and external weight of the community c. The worst-case running time is $O(n^2)$. The quality of discovered communities depends on the quality of seeds.

OSLOM [8] [71] is based on the local optimization of a fitness function. This function expresses the statistical significance of clusters with regard to global null model (i.e., the random graph generated by the configuration model). OSLOM can be used alone or as a refinement procedure of partitions/covers delivered by other techniques.

OSLOM consists of three phases:

- First, it looks for significant clusters until convergence is reached
- Second, it analyses the resulting set of clusters, trying to detect their internal structure or possible unions
- Third, it detects found clusters' hierarchical structure

iLCD [9] [23] is capable of detecting communities taking network dynamics into account. Given a set of edges created at some time step, iLCD updates the existing communities by adding a new node if the following rules are satisfied:

- The node is able to access to most of the community (at least as much as the other nodes of the community) easily (in two steps or less)
- The node has a *robust access* to other nodes; which means a node can be reached from, at least, two paths of length two or less.

The complexity of iLCD is $O(nk^2)$ in general, whose precise quantity depends on community structures and its parameters.

First, we need to define the input of the iLCD (intrinsic Longitudinal Community Detection) algorithm. Due to the longitudinal analysis, we will use a list of edges, ordered by their creation time. Those edges could correspond to link creation among existing nodes or could also imply the creation of a new node. As some edge creations can be simultaneous (think of the publication of several articles in a given journal issue), we will use ordered sets of edges, where edges of a given set are created at the same time.

More formally, let us note $G = (V, E)$ the graph that is dynamically built and $C =< C_k >$ the set of communities that is dynamically built. Initially, G and C are

[8] *www.oslom.org* [online: accessed 06-12-2013]
[9] *http://cazabetremy.fr/Cazabet_remy/iLCD.html* [online: accessed 06-12-2013]

empty. We then define E_{in} the set of edges in input as $E_{in} =< E_t >$ i.e. composed by ordered time-stamped sets of edges. (see Algorithm 5 for pseudo-code)

Algorithm 5: iLCD

for *each time-stamped set* E_t **do**
  for *each edge* (u, v) *of the set* E_t **do**
    Add (u, v) **to** E. If u or v is not in V, add it to V
    Determine the updates of existing communities. For each community C_k to which u (respectively v) belongs, try to integrate v (resp. u) to C_k
  end
  Update of previous communities
  If u y v do not already belong to the same community, **Try to create a new community**.
  Merge similar communities.
end

2.8.1.4 Fuzzy Detection

Fuzzy community detection algorithms quantify the strength of association between all pairs of nodes and communities. In these algorithms, a soft membership vector, or belonging factor [46], is calculated for each node. A drawback of such algorithms is the need to determine the dimensionality k of the membership vector. This value can be either provided as a parameter to the algorithm or calculated from the data.

2.8.1.5 Agent Based and Dynamical Algorithms

COPRA[10] [46], is an algorithm based on the label propagation technique of Raghavan, Albert and Kumara; which is able to detect communities that overlap. Like the original algorithm, vertices have labels that propagate between neighbouring vertices so that community members reach a consensus on their community membership, each node updates its belonging coefficients by averaging the coefficients from all its neighbors at each time step in a synchronous fashion.

COPRA algorithm(Community Overlap Propagation Algorithm) keeps two vectors

[10] *www.cs.bris.ac.uk/~steve/networks/software/copra.html* [online: accessed 06-12-2013]

of vertex labels: *old* y *new*; *old.x* (resp. *new.x*) denotes the previous (resp. updated) label for vertex x.

Each vertex label is a set of pairs (c, b), where c is a community identifier and b is the belonging coefficient. $N(x)$ is the set of neighbours of vertex x (see Algorithm 6 for pseudo-code).

Algorithm 6: COPRA

```
foreach vertex x do
    old.x ← {(x, 1)}
end
for each vertex x do
    Propagate(x, old, new).
end

if id(old)=id(new) then
    min ← mc(min, count(new))
. else
    min ← count(new).

if min ≠ oldmin then
    old ← new.
    oldmin ← min.
    Repeat from step 2.
foreach vertex x do
        ids ← id(old.x).
        foreach c in ids do
            if for some g, (c, g) is in coms, (c, i) in sub then
             coms ← coms −{(c, g)} ∪ {(c, g ∪ {x})}.
              sub ← sub −{(c, i)} ∪ {(c, i ∩ ids)}.
            else
                coms ← coms ∪{(c, {x})}.
                sub ← sub ∪{(c, ids)}.
    end
end
foreach (c, i) in sub do
        if i ≠ {} then
         coms ← coms −(c, g).
end
Split disconnected communities in coms.
```

SLPA[11] [122] is a general speaker-listener based information propagation process. It spreads labels between nodes according to pairwise interaction rules. Unlike other

[11] now, SLPA has a new name, GANXiS. https://sites.google.com/site/communitydetectionslpa/ganxis

algorithms, where a node forgets information gained in the previous iterations, SLPA provides each node with a memory to store received information (labels). The membership strength is interpreted as the probability of observing a label in a node's memory. One advantage of SLPA is that it does not require any knowledge about the number of communities. The time complexity is $O(tm)$, linear in the number of edges m, where t is a predefined maximum number of iterations.

SLPA is an extension of the Label Propagation Algorithm (LPA) proposed by Raghavan, Albet and Kumara. In LPA, each node holds only a single label that is iteratively updated by adopting the majority label in the neighborhood. Disjointed communities are discovered when the algorithm converges. One way to account for overlap is to allow each node to possess multiple labels. SLPA follows this idea, but applies different dynamics with more general features.

In the dynamic process, we need to determine 1) how to spread a node's information to others; 2) how to process the information received from others. The critical issue related to both questions is how information should be maintained. A speaker-listener based information propagation process (SLPA) is proposed to mimic human communication behavior.

In SLPA, each node can be a listener or a speaker. The roles are switched depending on whether a node serves as an information provider or information consumer. Typically, a node can hold as many labels as it likes, depending on what it has experienced in the stochastic processes driven by the underlying network structure. A node accumulates knowledge of repeatedly observed labels instead of erasing all but one of them. Moreover, the more a node observes a label, the more likely it will spread this label to other nodes (mimicking people's preference of spreading most frequently discussed opinions).

2.8.2 Evaluation Criteria

Evaluating the quality of a detected partitioning or cover is nontrivial, and extending evaluation measures from disjoint to overlapping communities is rarely straightforward.

In this section we present two well-known quality measures; *Normalized Mutual Information* and *Modularity*. In this thesis we will use *modularity* as a clustering quality measure because the ground-truth is unknown.

2.8.2.1 Normalized Mutual Information

There are many evaluation criteria in the literature (see [47]), but most of them can be used to compare partitions: a *partition* is a union of subsets which are non-overlapping and which cover the whole set; a *cover* is just a collection of(overlapping) subsets.

Although there is currently no consensus on which is the best measure, information theoretic based measures have received increasing attention for their strong theoretical background. Let us first review some of the very fundamental concepts of information theory [31] and then see how those concepts might be used toward assessing clustering agreements.

Definition 1. The information entropy of a discrete random variable X, that can take on possible values in its domain $\chi = \{x_1, x_2, \ldots, x_n\}$ is defined by:

$$H(X) = -\sum_{x \in \chi} p(x)\ log(p(x)) \tag{2.23}$$

Definition 2. The mutual information between two random variables X and Y with respective domains χ and Υ is defined by:

$$I(Y, X) = \sum_{x \in \chi} \sum_{y \in \Upsilon} p(y, x)\ log \frac{p(y, x)}{p(x)p(y)} \tag{2.24}$$

The mutual information (see Figure 2.11) is a symmetric measure that quantifies the mutual dependence between two random variables, or the information that X and Y share. It measures how much knowing one of these variables reduces our uncertainty about the other. This property suggests that the mutual information can be used to measure the information shared by two clusterings, and thus, assess their similarity. Lancichinetti et al. [70] extended the notion of normalized mutual information to account for overlap between communities.

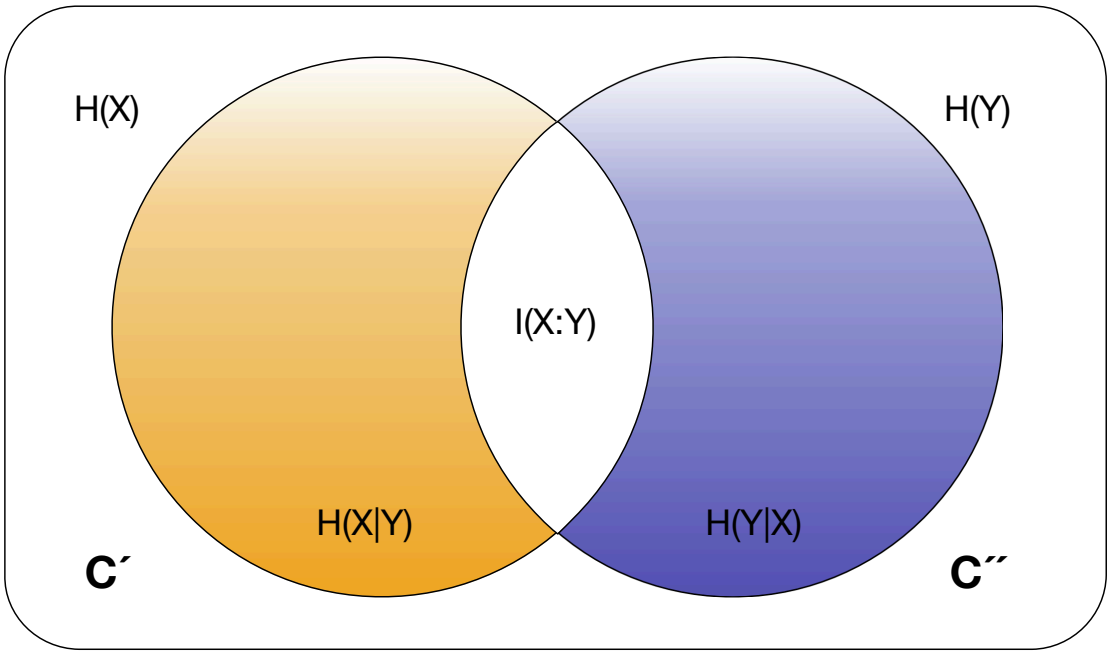


Figure 2.11: Mutual Information

The normalized mutual information is defined as:

$$NMI(X|Y) = \frac{H(X) + H(Y) - H(X,Y)}{(H(X) + H(Y))/2} \tag{2.25}$$

where $H(X)$ $(H(Y))$ is the entropy of the random variable X (Y) associated with the partition C' (C''), whereas $H(X,Y)$ is the joint entropy. This variable is in the range $[0,1]$ and equals 1 only when the two partitions C' and C'' are exactly coincident.

For each node i in cover C', its community membership can be expressed as a binary vector of length $|C'|$ (i.e., the number of clusters in C'). $(x_i)_k = 1$ if node i belongs to the k^{th} cluster C'_k; $(x_i)_k = 0$ otherwise. The k^{th} entry of this vector can be viewed as a random variable X_k, whose probability distribution is given by $P(X_k = 1) = n_k/n$, $P(X_k = 0) = 1 - P(X_k = 1)$, where $n_k = |C'_k|$ is the number of nodes in the cluster C'_k and n is the total number of nodes. The same holds for the random variable Y_l associated with the l^{th} cluster in cover C''. The joint probability distribution $P(X_k, Y_l)$ is defined as:

$$
\begin{aligned}
P(X_k = 1, Y_l = 1) &= \frac{|C'_k \cap C''_l|}{n} \\
P(X_k = 1, Y_l = 0) &= \frac{|C'_k| - |C'_k \cap C''_l|}{n} \\
P(X_k = 0, Y_l = 1) &= \frac{|C''_l| - |C'_k \cap C''_l|}{n} \\
P(X_k = 0, Y_l = 0) &= \frac{n - |C'_k \cup C''_l|}{n}
\end{aligned}
$$

The conditional entropy of a cluster X_k given Y_l is defined as $H(X_k|Y_l) = H(X_k, Y_l) - H(Y_l)$. The entropy of X_k with respect to the entire vector Y is based on the best matching between X_k and any component of Y given by

$$H(X_k|Y) = min_{l \in \{1,2,\ldots,|C''|\}} H(X_k|Y_l)$$

The normalized conditional entropy of a cover X with respect to Y is:

$$H(X|Y) = \frac{1}{|C'|} \sum_k \frac{H(X_k|Y)}{H(X_k)}$$

In the same way, one can define $H(X|Y)$. Finally the NMI for two covers C' and C'' is given by:

$$NMI(X|Y) = 1 - [H(X|Y) + H(Y|X)]/2$$

The extended NMI is between 0 and 1, with 1 corresponding to a perfect matching.

2.8.2.2 Modularity

To develop a method for community identification, one needs an evaluation criteria to judge the quality of the detected community structure. One of such measures was

proposed by Newman and Girvan in [86] and is based on the intuitive idea that random networks do not exhibit (strong) community structure. To measure the quality of a cover produced by overlapping detection algorithms on real-world social networks where the ground truth is usually unknown, most measures extend the framework of modularity Q for a disjoint partition, which is given as:

$$Q = \frac{1}{2m} \sum_{c} \sum_{i,j \in c} \left[A_{ij} - \frac{k_i k_j}{2m} \right],$$

where c is a community, A_{ij} is the element of the adjacency matrix for nodes i and j, $m = \frac{1}{2} \sum_{ij} A_{ij}$ is the total number of edges, and k_i is the degree of node i.

The idea behind modularity of Newman is simple: a subgraph is a community if the number of links among nodes in the subgraph is higher than what would be expected if links were randomly placed. This is exactly what happens in real-world communities, where the number and density of links among people belonging to groups (families, clubs, user groups etc) is higher than expected in a random graph of the same size [87, 92, 110]. This definition of modularity implies the choice of a so-called *null model* [86]; i.e. graph model to which any other graph can be compared in order to assert the existence of any degree of modularity. When testing for modularity of a complex network, the null model used has so far been a random graph with the same number of nodes, the same number of edges and the same degree distribution as in the original graph, but with links among nodes randomly placed.

2.8.2.3 Link Based Modularity

Nicosia et al. [88] proposed an extension to modularity measure based on the belonging coefficients of links to account for overlap between communities. Since each node has a belonging coefficient for each community, it is possible to define this coefficient for incoming or outgoing edges from a node. We can intuitively suppose that the community belonging coefficient c of an edge $l = (i, j)$ which starts at node i and ends at node j can be represented by a certain function of the corresponding belonging coefficients of i and j to community c, in the following equation:

$$\beta_{l(i,j),c} = F(a_{ic}, a_{jc}) \tag{2.26}$$

The expected belonging coefficient of any possible link $l(i,j)$ from node i to a node j in community c can be defined as $\beta^{out}_{l(i,j),c} = \frac{1}{|V|}\sum_{j\in V} F(a_{ic}, a_{jc})$. Accordingly, the expected belonging coefficient of any link $l(i,j)$ pointing to node j in community c is defined as $\beta^{in}_{l(i,j),c} = \frac{1}{|V|}\sum_{i\in V} F(a_{ic}, a_{jc})$. Latter coefficients are used as weights for the probability of an observed link and the probability of a link starting from i to j in the null model. These are used in the new modularity defined as:

$$Q^{Ni}_{ov} = \frac{1}{m}\sum_{c}\sum_{i,j\in V}\left[\beta_{l(i,j),c}A_{ij} - \beta^{out}_{l(i,j),c}\beta^{in}_{l(i,j),c}\frac{k^{out}_i k^{in}_j}{m}\right] \tag{2.27}$$

As we said at the beginning of this section, the evaluation criteria chosen will be *modularity*.

Chapter 3

Methodology

"Live as if you were to die tomorrow. Learn as if you were to live forever."

Mahatma Gandhi

The methodology of this thesis is based on Cross Industry Standard Process for Data Mining (CRISP-DM) [118] which was developed by IBM[1] and is based on the process of *Knowledge Discovery in Databases* (KDD) [38]. KDD is a linear process composed by five continuous stages (Selection, Pre-Processing, Transform, Application of Data Mining Techniques and Interpretation of the Results).

CRISP-DM is a spiral methodology. One of its main characteristics is that it is independent of both the industry sector and technology used. Is intended to be a way, to develop large data mining projects, less costly, more reliable, more repeatable, more manageable, and faster according to [118].

This methodology contains the phases, their tasks, and their outputs. It is based on nine stages, as shown in figure 3.1, that are described as an overview of the life cycle of a data mining project. The sequence describe in figure 3.1 is not strict, and arrows indicate only the most important and frequent dependencies between phases.

[1]Acronym for International Business Machines Corporation

Figure 3.1: CRISP-DM Methodology.

The steps of the methodology and their description are based on [118]. We will add some steps required by the Market Basket Analysis based on Social Network Analysis that we propose.

Business Understanding

First we researched available literature both in Market Basket Analysis and Social Network Analysis. To understand the business is necessary to participate in several meetings with retail analysts.

Data Understanding

After comprehending the business component, it is time to understand how real retail transactional data is structured. It is necessary to understand the meaning of each table in the database. We need to verify if the data available allows us to apply the different algorithms. We have to verify the quality of the data, identifying quality problems.

Data Preparation

Once data is fully understood, it is necessary to prepare data for processing. This stage contains all activities related to data preparation, including data cleaning, removing incorrect values, unknown values or missing values. Data is aggregated according needs of the different models and algorithms that will be applied.

Modeling

At this stage takes place a process of discovery of relationships, patterns and trends, using various techniques such as market basket analysis and graph mining.

Evaluation

Before presenting results to retail analysts is necessary to validate the results in order to check that they are robust solutions and according to the objectives proposed in section 1.4. It is necessary to compare different models in terms of quality and adjustment to the problem. If there is a problem, both in models and considering the whole aspect of the problem, it will be necessary to return to previous stages, reformulate and regenerate the execution and evaluation.

After models are valid in terms of quality, it is time to present them to retail analysts, as they can interpret the results and give a meaning to the solution found. This process is an iterative process, because if they need more information it will be necessary to rebuild the solution according their requirements.

Deployment

After the models are obtained and validated by retail's analysts. It must be encapsulated in a system so that retail analysts can take full advantage of this information.

3.1 Basic Notation

Data from retailers, are obtained over a period of time K. We will partition time equally, for example daily, weekly, monthly, etc., obtaining a set of time periods, $M = \{period_1, period_2, , \ldots, period_{|M|}\}$, where $|M|$ is the total number of periods available.

We have a set of products and transactions. Products are defined formally as $P = \{p_1, p_2, \ldots, p_n\}$ where each p_i represents a specific SKU available. Indeed $|P| = number\ of\ distinct\ SKUs$. Let D_m be a set of transactions occurred during a time period m. A transaction T is defined according to [5] as a set of items (products in this case) purchased in the same buying opportunity, such that $T \subseteq P$. A unique identifier, called its TID is associated with each transaction.

3.2 Data Selection

Data from retailers is usually stored in a relational database management system with a particular structure. Basic data to be stored, in each purchase, is a ticket with particular data to be stored such as: date of purchase, number of products, price, promotions or discount and also, a customer identification number if exists.

This thesis project aims to complete and to characterize customer profiles using graph mining techniques. To meet this objective, it is necessary to recognize at least a subset of customers and their particular purchases. Once we have the products purchased we can obtain a customer characterization and the degree of membership to each discovered community.

3.3 Preprocessing Data

Databases are highly susceptible to noisy, missing and inconsistent data. Low-quality data will generate low-quality results. In this section, we will briefly describe the principal steps necessary to preprocess data.

3.3.1 Missing Values

Is important to check that we have all data required to understand the results generated by the community detection process. This implies that we should have all information related to products available, such as, name of the product, SKU, family, line, sub-line, etc. We have to see that tickets are complete, meaning that there are no missing values, and if there are, we have to define a policy, such as, ignore the tuple, fill in the missing value manually, use a global constant to fill in the missing value, use a measure (e.g., mean or median) to fill the missing value, use the most probable value to fill in the missing value.

3.3.2 Noisy Data

Noise is a random error or variance in a measured variable. For example, products with wrong price(USD $100 where it should be USD $10. Common techniques to find noisy data are binning, regression and outlier analysis.

3.3.3 Tuple Duplication

We have to use normalized tables in order to avoid replications or discrepancies. For instance, if a purchase order database contains attributes for the purchaser's name and address instead of a key to this information in a purchaser database, the same purchaser's name can appear with different addresses within the purchase order database.

3.4 Data Reduction

Retail data from a transactional database, where all the purchases are stored, can be a huge source of data. Developing a complex data analysis and mining on huge amounts of data will be difficult to process and can take a long time, making such analyses impractical or infeasible.

Data reduction strategies include dimensionality reduction, numerosity reduction and data compression.

Dimensionality reduction Is the process of reducing the number of random variables or attributes under construction. Dimensionality reduction methods include wavelet transforms (e.g., [8, 63]) and principal components analysis (PCA) (e.g., [60]) which transform or project the original data onto a smaller space. Attribute subset selection is a method of dimensionality reduction in which irrelevant, weakly relevant, or redundant attributes or dimensions are detected and removed (e.g., [50, 116, 123]).

Numerosity reduction These techniques replace the original data volume by alternative, smaller forms of data representation. These can be parametric, where a model is used to estimate the data, so only the data parameters are stored, instead of the actual data. Regression and log–linear models are examples (e.g., [43, 120]). Nonparametric methods for storing reduced representations of the data include histograms, clustering, sampling and data cube aggregation.

Data compression In these methods, transformations are applied to the original data to obtain a reduced or compressed representation. If the original data can be reconstructed from the compressed data without any information loss, the data reduction is called lossless. Instead, if we can only reconstruct an approximation of the original data, then the data reduction is called lossy.

A dimensionality reduction that can be used is, that of considering a transaction to be a set of product families or lines. With the aim of obtaining a small representation that can be manageable and also allows for apply data mining techniques.

Many other ways of organizing methods of data reduction exist. The computational time spent on data reduction should not outweigh the time saved by mining on a reduced data set size.

3.5 Network Configuration

To build the social network graph, tickets must be taken into consideration. We will assume that products purchased in the same buying opportunity are related to each other. Following this assumption, the network will be configured as follows: *nodes* will represent the products present in a specific ticket and *edges* will represent a relationship between them.

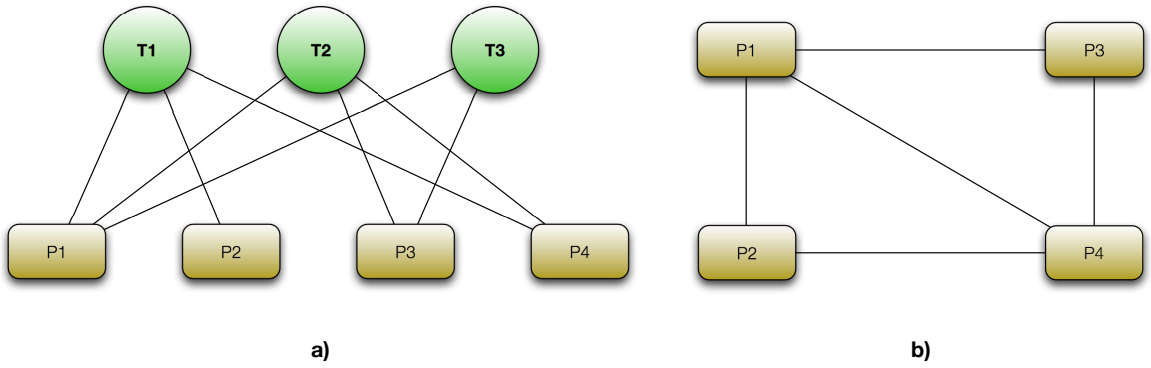


Figure 3.2: Bipartite transaction products network (a) and co–purchased product network (b).

In the literature two approaches to generate a network exist. One is introduced by [64,106–108], where a bipartite customer product network is built. This network links transactions with products, as depicted in 4.3 (a). The second approach is introduced by [97], based only on transactions where each product is linked to others because they appear in the same ticket from the same buyer opportunity. This kind of network is named *co–purchased product network*, depicted in figure 4.3 (b). In this thesis project the *co–purchased product network* will be used.

3.6 Network Construction

The objective of this section is to explain how the network of products are constructed. The network will be built according the description given in section 3.5. Considering T the set of all transactions occurred during a certain period of time k. We will review each transaction t, which will be composed of a subset of products P_{subset}. For each pair of products $(p_i, p_j) \in P_{subset}$ an arc a_{ij} will be created. After this process is done, the resulting graph $G(N, E)$ can be filtered according the process described in section 3.7. This way, we can reduce the size of the graph and the spurious edges, that do not represent powerful relationships between products.

Procedure 7: Procedure to generate the co–purchased product network.

Input:

T_k: set of transactions occurred over a period of time k.

Output:

$G(N, E)$: co–purchased product network.

Method:

$G(N, E)$ = empty graph.
foreach *Transaction* $t \in T_k$ **do**
  foreach *Pair of products* $(p_i, p_j) \in t$ **do**
    $addNode(G(N, E), (p_i, p_j))$
    $addEdge(G(N, E), (p_i, p_j))$
  end
end
return $G(N, E)$

3.7 Network Filtering

The idea is to generate filters over the product network generated in 3.5. Then we will apply a threshold θ that each *edge* has to be, at least bigger or equal to that threshold in order to remain in the product network and not to be removed. The idea behind this process is to remove edges that represent spurious and non–frequent relationships between products, relationships that are not lasting over time.

The process recently described implies the iteration over the complete set of edges looking for those that do not meet the threshold θ.

The process is described in procedure 8.

Procedure 8: Procedure to generate a filtered network.

Input:

$G(N, E)$: a co–purchased product network.

θ: the minimum threshold required.

Output:

$G'(N', E')$: co–purchased product network with spurious edges removed.

Method:

$G'(N', E')$ = empty graph.
foreach $edge(u, v) \in E$ **do**
  if $weight(u, v) \geq \theta$ **then**
    $addEdge(G'(N', E'), (u, v))$
end
return $G'(N', E')$

3.8 Community Detection

Before we explain the overlap community detection process, we have to explain what is understood by *Community Detection* in graphs. It is the process of trying to find a group of strongly connected nodes. According to [39] the first problem is to look for a quantitative definition of community, because the definition usually depends on the specific system or application developed. Basically, the main idea is that there should be more edges *inside* the community than edges linking vertices of the community with the rest of the graph. Moreover, in most of the cases, communities are algorithmically defined without a precise a priori definition. Mathematically the problem of finding communities inside a graph is described as follow:

Given a graph (or network) $G = \{V, E\}$, where V is a set of n nodes and E is a set of m edges, a series of disjoint subgraphs $K = \{K_1, \ldots, K_j\}$ are generated. The number j of subgraphs to find is not known previously and is determined by the algorithm, based on the maximization of a function $f(K)$. Typically, this function is *modularity* [86]. As we mentioned previously, this function measures the quality of the community in terms of the community structure, but its main contribution has to do with the underlying relationships between products.

3.8.1 Overlapping Community Detection

The overlapping community detection problem can be solved using the algorithms exposed in section 2.8.1.5. These algorithms use a topology–based approach. We will specifically use two algorithms: COPRA [46] and SLPA [122]. We based this choice on the results obtained by [84, 121] who show that these two algorithms have the best performance, both in time, and quality of the results.

3.9 Customer Characterization

Once the communities are found, it is time to characterize the customers based on their previous purchases. The idea is to find the belonging degree to each community. In order to do this, we will calculate the belonging degree G_i^k, for each customer i to the community k as:

$$G_i^k = \frac{Q_i^k}{\sum_{k \in K} Q_i^k} \quad , \quad \forall i, k \tag{3.1}$$

where Q_i^k represents the quantity of purchases made by customer i over the period of time, where at least one product belongs to community k. It is clear from equation 3.1 that the degree will be a value between $[0, 1]$. The belonging degree can be recognized as a *community support* because represents the fraction of transactions that belongs to a particular community over the number of transactions made that belongs to the rest of communities in that period of time.

3.10 Evaluation

After the communities are discovered and the customers are characterized, it is necessary to measure the quality of the results. To do so, the communities are measured in terms of the *modularity* [88]. Also, both communities and customer characterization are introduced into analysts. Based on their expert judgement, the quality of the results –both communities and costumer characterization– can be qualified because no measure that can rate it. *Modularity* can measure the quality of the community structure but the meaningfulness. In terms of real benefits for the retailer it can only be given by the analysts.

3.11 Deployment

Finally, once the results are measured, all the information obtained are introduced into a computer system that can help analysts manage both communities and customer characterization. The system will help them handle and improve the results obtained by the whole process.

Chapter 4

Application over real retail data

"When everything seems to be going against you, remember that the airplane takes off against the wind, not with it."

Henry Ford

In this chapter an application over real data from two supermarket chains will be presented. The structure of the chapter will be based on the methodology described in chapter 3. In the first place, we will present the supermarkets which we work with. The characteristics of the data will then be exposed, the cleaning process. Once the data is ready, we will explain how the networks of products are generated, how the algorithms are applied to discover overlap communities as well as the process to characterize customers based on their previous purchases. Next, the developed system to store and to manage the results is presented. Finally, the results are introduced to the retail analysts, in order to measure the usefulness of the work performed.

4.1 Retail Data

The data was obtained from two retail chains in Chile. One is a wholesale supermarket that supplies products to grocery store owners, hereafter, referred to as *Retail A*. The second is member of one of the biggest retail holdings in Chile called *Retail B*.

Our data was gathered over a period of thirty months, around 238 million transactions, approximately 160 thousand clients and over 11 thousand SKUs[1] in the case of *Retail A* chain. For *Retail B*, the studied period was two months, with 128 million transactions, almost 2 million customers and 31 thousand different SKU.

We presented in section 3.1 that data was obtained over a period of time and we denominated it as K. This period is partitioned into smaller sets of time periods. The set of time periods chosen is : $M = \{daily, weekly, monthly, quarterly, biannual, yearly\}$, this set is sub–indexed, in a way that can be manageable, obtaining the following sets:

$$M_a = \{day_1, \ldots, day_{1229}, \ldots, month_1, \ldots, month_{41}, \ldots, year_1, year_2, year_3, year_4\}$$

$$M_b = \{day_1, \ldots, day_{62}, week_1, \ldots, week_9, month_1, month_2\}$$

in the case of *retail A* and *retail B* respectively. We only have three years in M_a because the data is available since 3.5 years. The set of applicable time periods, in the case of *retail B*, are only *daily, weekly* and *monthly*, because we have two month of data available.

According to above, we have subsets of transactions that belong to a particular time partition k, these can be expressed by

$$T_k = \{t_1, t_2, \ldots, t_q\}$$

as an example for a particular day *d1* we will have: $T_{d1} = \{t_{7522}, \ldots, t_{14211}\}$ where t_{7522} and t_{14211} are the first and the last purchase of the day respectively. It is important to remember that each purchase represented by a transaction t like $t_{7522} = \{p_{21}, p_{34}, p_{675}, p_{982}\}$

In the datasets, from these two retailers, products are organized in a three hierarchical level structure. Each level belongs to its predecessor based on an ad–hoc developed taxonomy by each retailer. Figure 4.1 shows a subset of one of our taxonomy and table 4.1 shows an example of product information with its hierarchy.

[1]SKU : Stock Keeping Unit

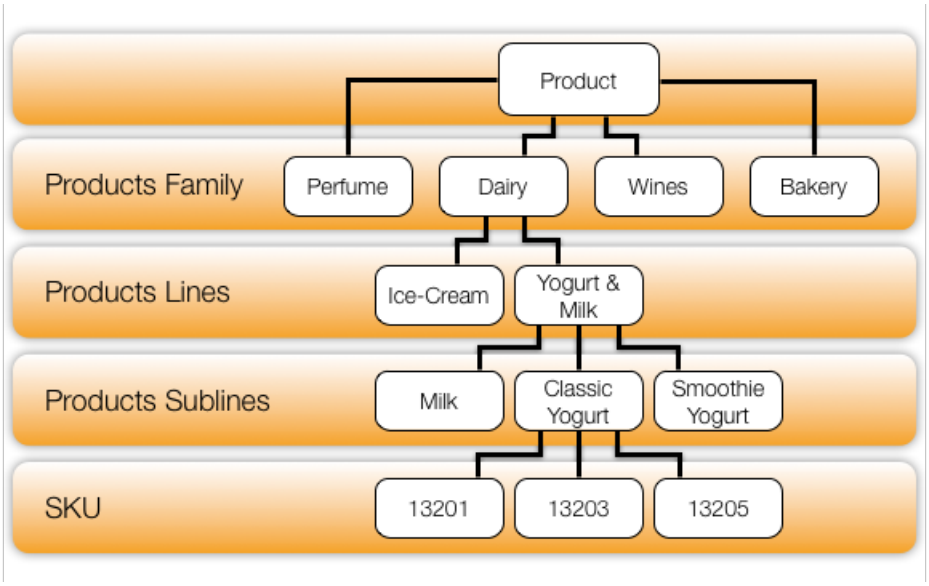


Figure 4.1: Hierarchy of products

Retail A has 23 product families, 150 lines of products and 415 sublines of products. *Retail B* has 50 product families, 287 lines and 1032 sublines of products.

SKU	Product name	Product Family	Product Line	Product Sub-line
13231	Milk "The Happy Cow"	Dairy	Yoghurt & Milk	Milk
13201	Yoghurt "Fancy Yoghurt"	Dairy	Yoghurt & Milk	Classic Yoghurt
13245	Yoghurt "Smoothiest"	Dairy	Yoghurt & Milk	Smoothie Yoghurt

Table 4.1: Products characterization available

In the following sections, we will explain how data is processed, in order to be in a format that enables us to perform our experiments.

4.2 Data Transformation

To develop our proposal, it is important to have data, but not all data is necessary. Retailers have databases with millions or billions of records. For the development of this work, only transactional data is needed, specifically on products purchased in each buying opportunity and who bought the products, if available. Particular information related to each product such as family, line or sub–line can be helpful to analysts' analyses. Our method only requires the products present in each transaction.

We received the data from retailers directly in dump files from their databases. These data, depending on the retailer, have a particular structure. We had to transform this data in order for it to fit into our data model. The data model is described in figure 4.2.

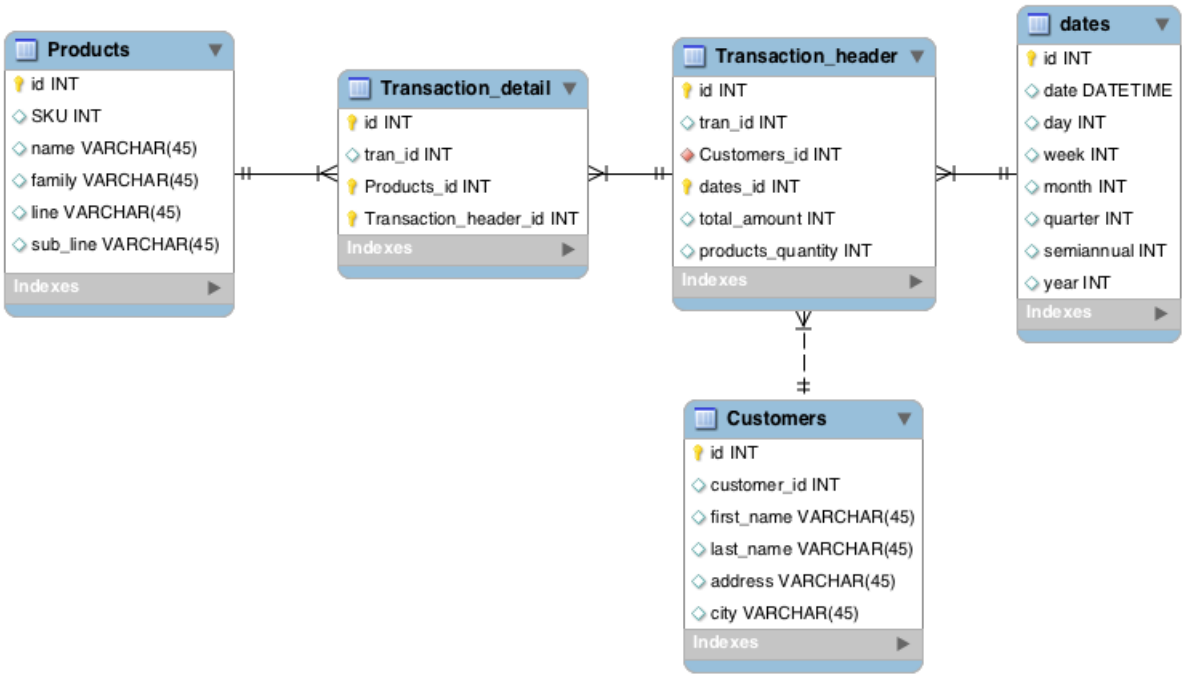


Figure 4.2: Basic Database Model

4.2.1 Data Model Explain

In this subsection, we will describe each of the tables contained in the model of the figure 4.2.

Products is the table used to store the product related information, such as name, family, line, sub–line, etc.

Transaction Header is the table where the main details of each purchase are stored, like the customer id, if available, quantity of products purchased, total amount of the transaction and the date of the purchase.

Transaction Detail is the table used to store the detail of the products purchased in a particular transaction and is a relation between *Products* and the *Transaction Header*.

Dates is a table where a representation of the different dates are organized by different criteria such as day, week, month, etc.

Customers is the table that handles all the information related to customers like first name, last name, address, etc.

To be able to reconstruct a transaction with all the information, it is necessary to make a join between the different tables involved.

Once the data is stored in our system, it is necessary to study the quality of the data, as mentioned in section 3.3, in terms of missing values, noisy data and tuple duplication. The data presents no missing values. Also, as the information comes from data warehouses it does not present noisy data. Finally, we closely studied the data in order to find tuple duplication. No tuples were repeated in the data sets available.

4.3 Classical Approach

In this section, we will explain how we addressed the problem in the first instance. We began with the generation of clusters of products that were purchased together as we mentioned in section 2.5 and in section 2.5.5 we described the two principal algorithms applied over transactional data, K–Means [53] and SOFM [67].

In order to apply K–Means algorithm over transactional data, we had to construct a representation of the data. This is depicted in table 4.2. It is a matrix whose rows are vectors of purchases. Each vector is composed of transactions and the set of products available. The first column stored the transactional id (*TID*) and in the following columns stored a number 1 or 0, which represents whether the product was purchased or not in that particular transaction.

Transaction ID	P1	P2	P3	P4
925	1	1	0	1
926	1	0	1	1
927	1	0	1	0

Table 4.2: Example of a transaction set as a vector of purchase for a particular time partition

Once the data is sorted, according what we need, we proceed to apply the K-Means algorithm over these datasets. The results obtained, independent of the time partition chosen, are far from being useful. From K-means we obtained the same cluster independent of the number of clusters K that we required. To improve the results we applied a dimensionality reduction as we mentioned in 3.4. We constructed a new matrix, but this time, instead of vectors composed of products purchased, we used families of products. The results again, were far from useful. Table 4.3 shows the top 18 categories found by K-means. Each column displayed is sorted by the membership of each product family to the respective cluster.

It is clear from table 4.3 that clusters are very similar between each other and show no real difference among themselves. Also, analysts said that these clusters are meaningless, resulting in no new information from this clusterization. To depict this fact: Cluster 5 and Cluster 6 are very similar. (In fact the top three families are the same in both clusters.). Only Cluster 2 provides information, because the belonging coefficient of *Soft Drinks* is 1.0 and the second and third families have a belonging coefficient of 0.15 and 0.13 respectively. Analysts said that this Cluster represents the transactions where the principal products are members of the *Soft Drinks* family.

Clusters	**Cluster 1**	**Cluster 2**	**Cluster 3**	**Cluster 4**	**Cluster 5**	**Cluster 6**	**Cluster 7**
% Transactions Involved	5,1	10,8	46,8	5,5	11,4	13,4	7,0
Families	Yoghurt	Soft Drinks	Cigarettes	French Fries	Milk	Yoghurt	Long Noodles
	Milk	Nectars	Milk	Cookies	Yoghurt	Frozen Desserts	Sugar
	Sugar	Milk	Cheese	Snacks	Frozen Desserts	Milk	Rice
	Toilet Paper	Cookies	Cookies	Milk	Nectars	Cookies	Tomato Sauce
	Vegetable Oil	Yoghurt	Nectars	Souffle	Cheese	Nectars	Vegetable Oil
	Tomato Sauce	Mineral Water	Toilet Paper	Yoghurt	Margarine	Cheese	Short Noodles
	Margarine	Beers	Margarine	Nectars	Butter	Milk	Toilet Paper
	Juice (Powder)	Cheese	Sugar	Soft Drinks	Cookies	Margarine	Yoghurt
	Cheese	Toilet Paper	Vegetable Oil	Biscuits	Soft Drinks	Sausage	Milk
	Short Noodles	Sausage	Sausage	Soft Candy	Juice (Powder)	Toilet Paper	Tea
	Rice	Wine	Juice (Powder)	Pretzels	Milk	Juice (Powder)	Margarine
	Long Noodles	French Fries	Beers	Cookies	Toilet Paper	Biscuits	Salt
	Cookies	Juice (Powder)	Frozen Desserts	Frozen Desserts	Sausage	Pretzels	Juice (Powder)
	Bleach	Sugar	Wine	Hard Candies	Sugar	Soft Drinks	Sausage
	Frozen Desserts	Vegetable Oil	Paper Towels	Chocolate	Sausage	Butter	Bleach
	Nectars	Biscuits	Flour	Juice (Powder)	Vegetable Oil	Sugar	Detergent
	Mayonnaise	Margarine	Chocolate	Cheese	Bleach	Cereals	Cookies
	Tea	Pretzels	Pretzels	Toilet Paper	Delicacy	Sausage	Mayonnaise

Table 4.3: Top 18 families per cluster after executing k-means algorithm with $K = 7$

In the case of SOM we obtained basically meaningless information. The main problem with these results is that it is impossible to clusterize customers with these meaningless groups of products.

These facts motivated us to propose a new methodology to generate clusters of products related among themselves as we describe in section 4.4. This methodology tries to find a meaningful item set with an approach based on product network with overlapping community detection.

We generated sets of *association rules* (explained in 2.6.3) where we obtained a set of rules of the type $A \rightarrow B$, where A and B are products purchased in the same buying opportunity. These rules can be ordered by their *support, confidence* or *lift*. However, the underlying relationship between these products is not clear in a simple way, after a deep analysis, analysts can recognize groups of rules that are related, for example, the *soft drinks* group could be recognized after studying a set with more than 500 rules, but other rules present no meaning for analysts. Instead, our methodology gave the group of soft drinks, as an output, with no effort for the analysts. Groups that were not discovered by association rules were found by our methodology.

4.4 Proposed Methodology

Previously, in section 4.3, we depicted the problems in our first attempt to address the problem. In this section we will show how these products networks are generated and will show our proposal of a *Temporally Transactional Weighted Products Networks*. We will later present what is understood as *community* and its relation with frequent item sets.

We present in this section our approach that is a novel way of generating frequent itemsets through community discovery. From now on we will refer to frequent itemsets as community.

4.4.1 Product Network and Graph Construction

We used a network as a way to represent sets of elements interconnected with one another. As shown in 2.7.1, a common way to represent a network is using a *graph*. A *graph* is used to specify relationships between sets of items. Formally, a graph consists of a set of objects, called *nodes* with some pairs connected by links called *edges*.

A *product network* is defined as a network where nodes represent products and edges represent relationships between them. We have to define what kind of relationship is represented by an edge. In this case, an edge between two products represents that both products are present in the same ticket from the same buyer opportunity.

We use a network representation based on transactional data shown in figure 4.2. In this subsection we will show how we build our product network following the methodology proposed in 3.6. We start building our transactional product bipartite network where each transaction is linked to the products that are purchased in that particular transaction (as in figure 4.3 (a)). Here, the set of transactions T and products P represents the two disjointed sets required to build a bipartite graph. Finally, we move from that bipartite network to the co-purchased product network as shown in figure 4.3 (b).

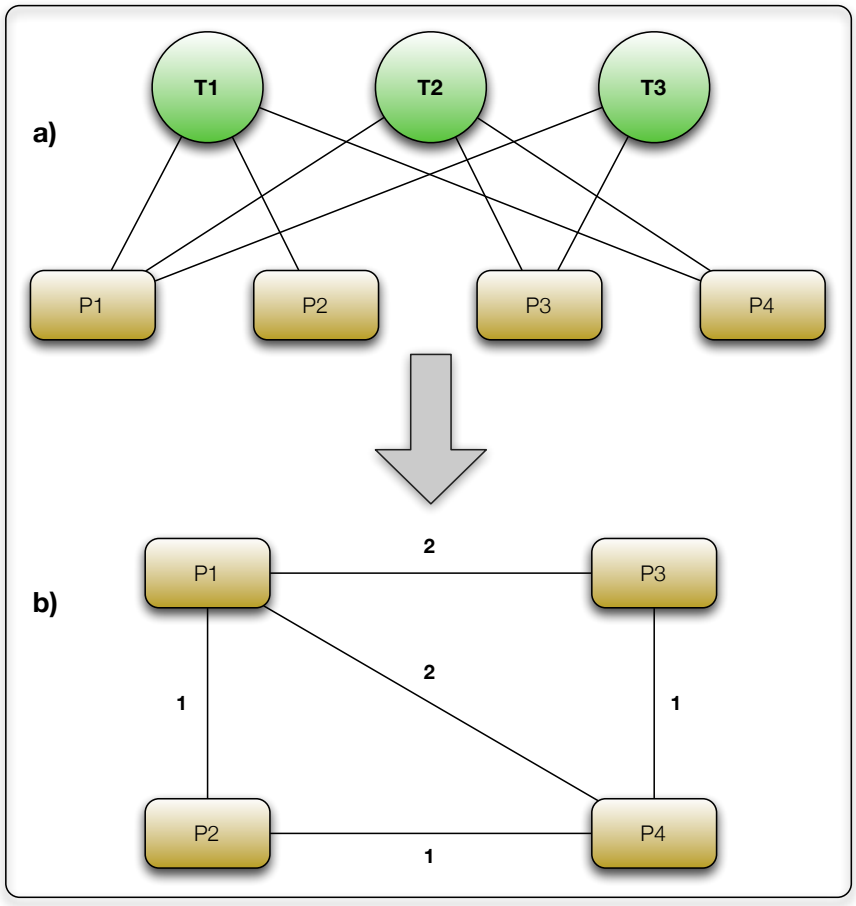


Figure 4.3: From Bipartite Transaction Products Network (a) to Product-to-Product undirected weighted network (b)).

After this processing we obtained a product–to–product weighted network. These networks can be represented by an adjacency matrix showing the weight between each pair of products. The weight is the number of tickets, in which a couple of products are present simultaneously. In table 4.4 we show this representation.

	P1	P2	P3	P4
P1	–	1	2	2
P2	1	–	0	1
P3	2	0	–	1
P4	2	1	1	–

Table 4.4: Adjacency matrix representing a product-to-product weighted network.

Similarly to [97] we found the same problems, such as very dense product networks and high degree nodes. Most of the time these edges make no sense and represent spurious associations. For example, a month of data has 485,136 transactions, the adjacency matrix of a product network obtained has 5,359 products with 5,362,906 edges. As the adjacency matrix is symmetrical we only consider one half of it. The most heavy weighted edge is 31,224. From those edges, 4,235,093 have a weight lower than 10. This should not be considered because it represents only a sporadic and not frequent relation between products.

4.5 Network Filtering

We mentioned in section 3.7 that it is important to apply a filter over the networks in order to remove spurious edges that contribute nothing. We will explain in the following subsections our proposed methodology.

4.5.1 Threshold Setup Methodology

We showed that product networks present high degree nodes with spurious edges between them. To remove spurious edges, a threshold θ has to be defined, then the graph is fully revised in the search of edges with a weight θ' lower than θ ($\theta' \leq \theta$). The edges that match with this criteria are removed. Raeder et al., [97] decided to filter those edges that have a weight lower than 10. Kim et al., [64] filter the *co–purchased* network by choosing a threshold θ equal to the average value of all links.

We found that there is no common criteria to choose a threshold. This makes it highly necessary to remove these spurious edges in an objective way, because it is clear that a particular number (constant) like 10 or the average value of all links are very particular thresholds that apply to particular instances or certain data. For instance, in our case, 10 is not a good threshold because the network obtained after applying this threshold still contains spurious edges and isolated nodes that do not produce communities of good quality.

We generate this threshold based on a process denominated as *top three heavy edges threshold* (tthet) which was used in both retailer data, proving its effectiveness. This approach consists in ranking the edges $E = \{E_1, E_2, ..., E_m\}$ based on the weight of these in a descendant order. Then *tthet* is equal to the average of the top three edges.

$$tthet = \frac{E_{max} + E_{2nd\ max} + E_{3rd\ max}}{3} \tag{4.1}$$

where E_{max} makes reference to the heaviest edge, $E_{2nd\ max}$ and $E_{3rd\ max}$ to the second and third heaviest edges respectively.

4.5.2 Network Filter Methodology

In the case of *Retail A* we obtained 1,492 *Temporally Transactional Weighted Product Networks* and in the case of *Retail B* we obtained over 12,000. To each one of these

Temporally Transactional Weighted Product Networks we computed a threshold using equation 4.1.

If we apply the obtained *tthet* to its corresponding network, only one or two elements would satisfy the minimum edge weight imposed by the threshold. Since tthet allows us to keep the most relevant part of the *Temporally Transactional Weighted Product Network*, rendering the analysis useless.

This fact prompted us to generate a set of filters using the tthet, that allows for gradually incorporating relevant edges and nodes into our analysis. These filters are a proportion of the *top three heavy edges threshold* (proportion is a percentage of the threshold). The percentages are: $Percentage = \{5\%, 10\%, \ldots, 95\%, 100\%\}$; these percentages give 20 filters (or new thresholds), as a result of a dot product between *percentage* and *tthet* resulting in:

$$filters = percentage \otimes tthet \tag{4.2}$$

equal to:

$$filters = \{0.05 * tthet, 0.1 * tthet, \ldots, 0.95 * tthet, tthet\} \tag{4.3}$$

It is clear that for each threshold we have a set of twenty filters associated with the same *Temporally Transactional Weighted Product Network*.

We applied immediately these filters to *Temporally Transactional Weighted Product Networks* resulting in a new set that we named *Filtered Temporally Transactional Weighted Product Networks*, composed of a 29,500 filtered networks in *Retail A* and over 200,000 in the case of *Retail B*.

Figure 4.4 depicts the number of nodes and edges from one month of transactional data after applying *filters* obtained from the *top three heavy edges threshold*. Is clear that when the percentage goes up; the number of nodes and edges goes down in a power law figure. This fact *power law figure* is important, because it allows us to apply the overlapping community discovering techniques.

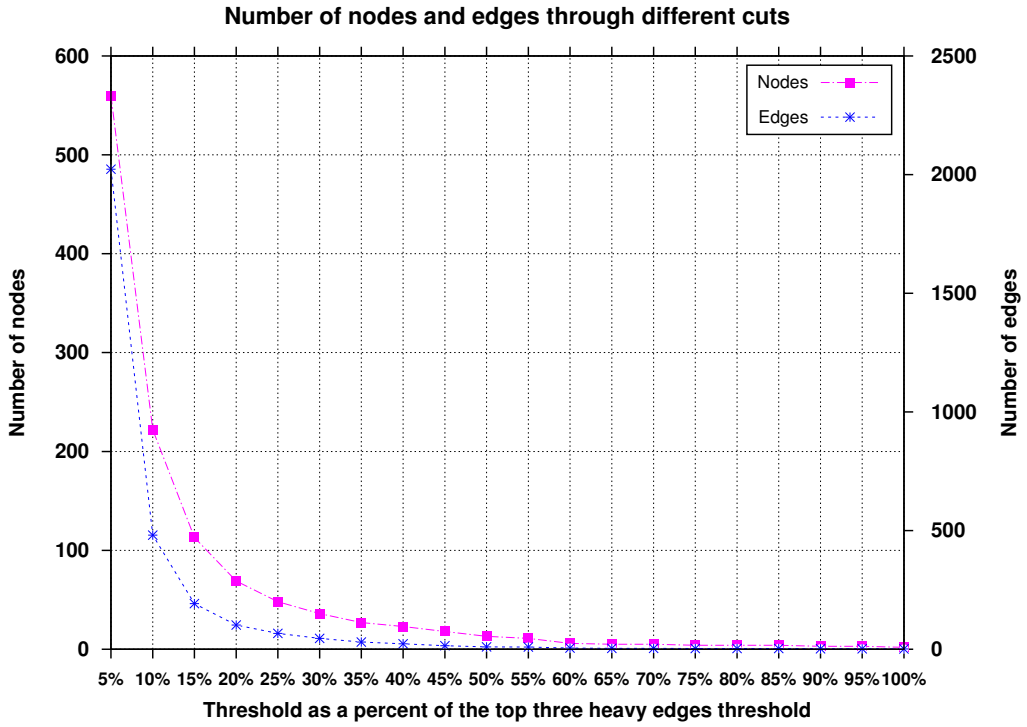


Figure 4.4: Graph depicting the number of nodes and edges through different cuts over the period of a month.

One of the main advantages of choosing a threshold this way is their independence of the underlying data. This means that our threshold and filters are independent of the quantity of nodes and work very well with big networks both in number of nodes or edge weight. It is also objective because it only depends on the data and requires no intervention from the analyst or a thorough understanding of the business, which is desirable, but not a prerequisite. Thus, our methodology can be reproduced by other works, giving the possibility to benchmark their results with ours.

4.6 Overlapping Communities of Products

Graph theory is applied in many fields including analysis of social networks [102], the world wide web [36], epidemiology [83], scientific collaboration [74, 98].

Figure 4.5 depicts a product-to-product network for *Retail A* on a particular day before any filter was applied. As we can see, this network is meaningless because it has many edges with little value (as explained in section 4.5.1).

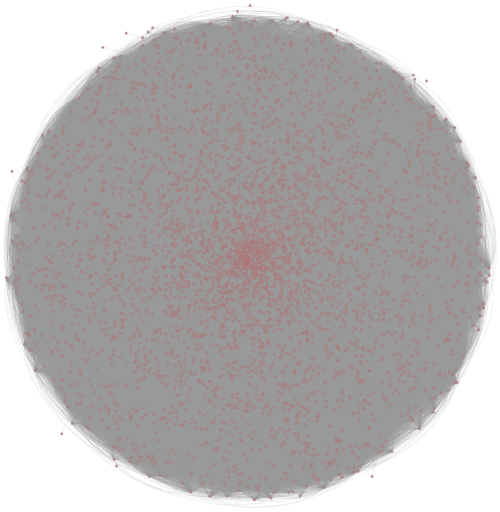


Figure 4.5: Visualization of the product-to-product network without filters.

Now, when a filter is applied, the network shows meaningful zones as we can see in figures 4.6 and 4.7. These figures were obtained after we applied a filter equal to the 5% and 10% of the *top three heavy edges threshold*.

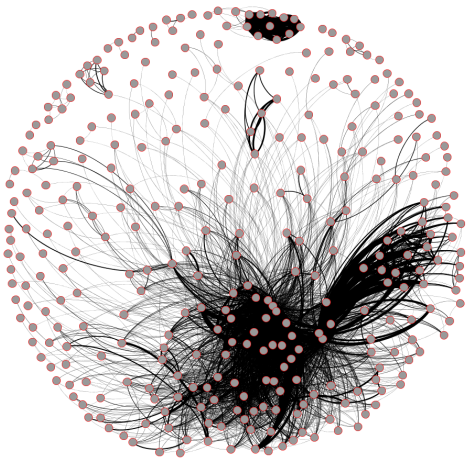


Figure 4.6: Product–to–Product Network with a 5% filter

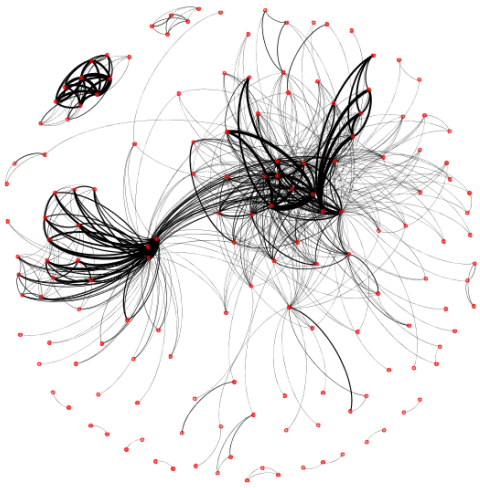


Figure 4.7: Product–to–Product Network with a 10% filter

These zones describe products with a powerful relationship between them. To understand these relationships and giving them a meaningful interpretation, we applied overlapping community detection algorithms, because this process, with overlapping, allows that a *product* (node) may belong to more than one community,

After we have generated our set of *Filtered Temporally Transactional Weighted Products Networks* it is time to apply the algorithms for overlapping community detection, described previously in section 2.8.1. We apply to each of the *Temporally Transactional Weighted Product Networks* (over 29,000 and 12,000 for *Retail A* and *Retail B* respectively) COPRA and SLPA algorithms.

Both algorithms take a network as an input and generate a file as an output with the communities discovered inside. COPRA generates only one file with the communities and SLPA generates a number of files that are the product of the number of repetitions that a user selects –10 in our case– and a threshold r, which takes values in $[0, 1]$ ($r \in [0, 1]$ specifically (0.01;0.05;0.1;0.15;...;0.5)). Threshold r is used as a filter of the number of labels that a particular node can have, checking if the probability of find a particular label during the whole process is lower than r. If that occurs, this label is removed from node's memory. When $r \to 1$, the algorithm tends to find disjointed communities. This is because only nodes in one community can have a high probability that can overcome the threshold represented by r. After the entire process, the nodes are grouped into communities with the same label. If a node has more than one label it is grouped into

several communities.

SLPA is executed in a series of runs. A run is an execution of an SLPA algorithm. In each run different values of r ($r \in \{0.01, \ldots, 0.5\}$) are used. Every time the algorithm is executed, a new node is chosen randomly making it necessary to run the SLPA algorithm several times to avoid that results were improved because SLPA started from a '*good*' node. We want to find the results that are maintained over time or between executions, to do so the entire set of results is stored and it is checked the quality of the results with retailer analysts.

Once the *Filtered Temporally Transactional Weighted Product Networks* are processed by the algorithms we have as result over 3.2 million files in *Retail A* and over 18 million files in *Retail B*.

4.7 Parallel Processing

Previously, we presented the process developed in order to generate product networks (4.4) and also how a filter was generated to reduce the number of spurious edges (4.5) and the application of the techniques to discover overlapping communities (4.6). The large amount of files involved make sequential processing unfeasible because of the time required. This prompted us to develop a new way of addressing the problem through parallel processing techniques. In this section, we show the techniques applied to address this objective.

To process the tasks in parallel, we used a dynamic mapping, specifically a centralized schema as depicted in [68]. This schema is based on the idea that all executable tasks are maintained in a common central data structure or are maintained by a special process or subset of processes. When a special process is designated to manage the pool of available tasks, then it is often referred to as the *master* and the other processes that depend on the master to obtain work are referred to as *slaves*. Whenever a process has no work, it takes a portion of available work from the central data structure or the master process. Whenever a new task is generated, it is added to this centralized data structure or reported to the master process.

The adequacy of the scheme made by us to execute parallel is: first, generate a *master* process that will fill an empty queue of jobs with the tasks to be done. Second, create a set of *slave workers* that will carry out those tasks. It is important to note that, each worker immediately –as soon as they are created– will start executing a task. The number of concurrent jobs K that can be executed in parallel is limited by the writing capacity of the hard disk drive in this case, because as we have to write the results to files. This was our bottleneck. We try with several values of K, but the best results were obtained with $K = 2P - 1$, where P is the total number of cores availables in the CPU (including virtual cores).

First, we decided on the time window to be applied over the transactional data. Then, we generated product networks obtaining the results from the database and generating the product networks for the time period desired. This process can be done sequentially. After this, we generate and apply the filter over the product networks.

Next, it is necessary to generate a set of new product networks in what we can *Filtered Temporally Transactional Weighted Product Networks*. To do so, we generated the twenty filters as a proportion of the threshold θ, and then review the set of edges, checking in parallel if the value of the edge exceeds an element from the set of filters. If it does, both nodes and edges are written into the respective files.

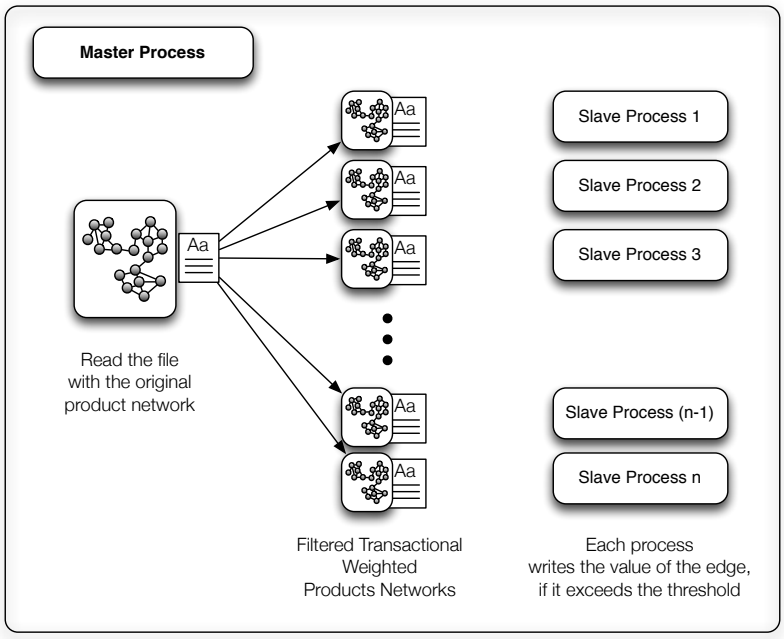


Figure 4.8: Master–Slave process to filter in parallel the *Temporally Transactional Weighted Product Networks*.

Figure 4.8 presents a graphical representation of the parallel execution using the master–slave scheme for the process of filtering a *temporally transactional weighted product network*.

After the filtered networks are generated, it is time to run the algorithms to detect overlap communities. As we mentioned previously, the filtered networks are over 3.2 million files in retail A and over 18 million in retail B. We processed these files sequentially and obtained results after five days of processing, i.e., a process that took over 120 hours, then with the parallel version of the execution we obtained results in 20 hours. This is a quantitative reduction of the processing time.

Algorithm 9 shows the pseudo–code of the procedure applied to discover overlapped communities.

Algorithm 9: Master–Slave scheme selected to discover communities in parallel.

Input:

F: set of files to be processed.

Output:

P: set of files with the overlapping communities discovered.

Method:

Master Process	***Slave Process***
$Q = \{empty\ queue\}$	
$P = \{empty\ set\ of\ files\}$	
$inicialize\_K\_Workers()$	$discoverCommunities()$
foreach *File* $f \in F$ **do**	$writeResultsToDisk(P)$
$addToQueue(f, Q)$	$notifyMasterProcess()$
$notifyWorkers()$	
end	
$waitEndOfAllProcess()$	
return P	

This reduction –in the processing time– becomes relevant, because retailers would like to run the developed methodology and its algorithms with a frequency that would be unreachable with sequential processing.

4.8 Communities Results

Having 101 files for each *Filtered Temporally Transactional Weighted Product Networks* is necessary for finding a way to discover which file (from 101 available) has the best community representation based on a criteria. As explained in section 2.8.2.2, this criteria is *modularity*.

To obtain modularities we apply a program to each file with their corresponding *Filtered Temporally Transactional Weighted Product Network* and obtained an equal number of modularity text files. These were parsed and inserted into a column oriented database so we could filter by different criteria such as time window, number of nodes, number of edges and modularity value.

As a manner to depict the process we show in table 4.5 the metadata of a Temporally Transactional Weighted Product Network for November, 2012 for a set of supermarket stores from *Retail B*.

In table 4.6 the metadata of one of the twenty filtered networks obtained after applying a threshold is shown. The threshold applied is equal to 1,286, equivalent to the 5 % of the *top three heavy edges threshold*. Finally, in table 4.7, the information of the results gathered from the appliance of SLPA and COPRA algorithms are presented.

Network	# of Nodes	# of Edges	Period	Begin Date	End Date
graph_month_201211	23,615	24,153,638	Monthly	2012-11-01	2012-11-30

Table 4.5: Metadata of a Temporally Transactional Weighted Product Network for November, 2012.

Network	**# of Nodes**	**# of Edges**	**Threshold**	**Density**	**Degree**
filtered_graph_month_201211	356	1,133	1,286	0.0179	6,365
	Cluster Coefficient	**Barycenter Scorer**	**Betweenness Centrality**	**Closeness Centrality**	**Degree Scorer**
	0,529	0,078	404,60	0,473	0.078

Table 4.6: Metadata of a Filtered Temporally Transactional Weighted Product Network for November, 2012.

We present a subset of the results from SLPA (10 from 101 available) and the result obtained from COPRA in table 4.7. This table shows the results for the SLPA algorithm after one run.

Network	Modularity	# of communities	# of products	# overlaps	r
SLPA_1	0,586	28	358	2	0.01
SLPA_2	0,602	29	360	4	0.05
SLPA_3	0,607	29	358	2	0.10
SLPA_4	0,607	29	358	2	0.15
SLPA_5	0,607	29	358	2	0.20
SLPA_6	0,607	29	358	2	0.25
SLPA_7	0,607	29	358	2	0.30
SLPA_8	0,610	29	357	1	0.35
SLPA_9	0,617	29	356	0	0.40
SLPA_10	0,617	29	356	0	0.45
SLPA_11	0,607	28	356	0	0.50
COPRA_1	0,394	45	361	5	N/A

Table 4.7: Results from one iteration of SLPA and COPRA algorithms.

As we can see from table 4.7, both algorithms utilize all products available, 356 in this case. For example, in SLPA_7 the number of products is 358, because 2 are overlapped. The COPRA algorithm discovered more communities than the SLPA algorithm, but modularity obtained from COPRA is worse than SLPA due to the fact that 30 communities from COPRA are singletons. On the other hand, the larger number of overlapped products found by COPRA is explained by the fact that COPRA found similar communities that only differed in one product. For example, for a pair of communities found there is $a = \{269324, 901093, 901095\}$ and $b = \{269324, 901096, 901095\}$, which are almost the same community except, for the middle product. A product identified by SKU equal to 901096 is missing in the case of a and SKU equal to 901093 in the case of b.

Analysts said that overlapped nodes discovered are relevant, because they expected products (nodes) present in more than one community. They referred to these products as *nexus*.

We also found a relationship that is present in almost all results obtained from SLPA, that the higher number of overlapped products is found when $r \leq 0.2$. This is because when r is small ($r \to 0$), too many nodes can overcome this threshold, as explained previously.

Once the communities are identified it is time to analyze them. In order to give a description of each one of them based on the products contained.

4.8.1 Discovered Communities

Once the algorithms were applied, we had as a result, a set of communities of products, where each product is related to each other. In this section we will describe the communities found, in terms of the meaning of these products within the community.

Table 4.8 depicts the top ten communities ordered by the number of products inside each community. We also provide a description from the analysts from *Retail B* who study the products involved.

Community	# of products	Description
1	242	Grocery
2	15	Soft Drinks & Beers
3	10	Convenience Food
4	6	Juice Powder Brand A
5	6	Juice Powder Brand B
6	6	Liquid Juice Brand C
7	5	Yoghurt Brand D
8	5	Yoghurt Brand E
9	5	Liquid Juice Brand F
10	5	Cookies Brand G

Table 4.8: 10 largest communities discovered (ordered by number of products inside) which account for 85% of the products in the network.

It is very important to note that these results changed the opinion of the business analysts because they believed that people were not *loyal* to a specific brand, in the sense of buying products of the same brand. However, the results showed that people are loyal to a brand in most cases, with the only exception being *Drinks & Beers*.

The average number of products within a community is 7. This number gives to the analyst a manageable, interpretable and characterizable number of products. This is one of the benefits of our work, because in order to get a lower quality result from the association rules, a deep and complex analysis must be performed.

We previously present in section 4.6 how a graph looks after we apply our *top three heavy edges threshold*. Now in figure 4.9 we show how the graph looks with each node colored according to its corresponding community. We found a big community of groceries according to an analyst description depicted in green, and the rest of communities in different colors.

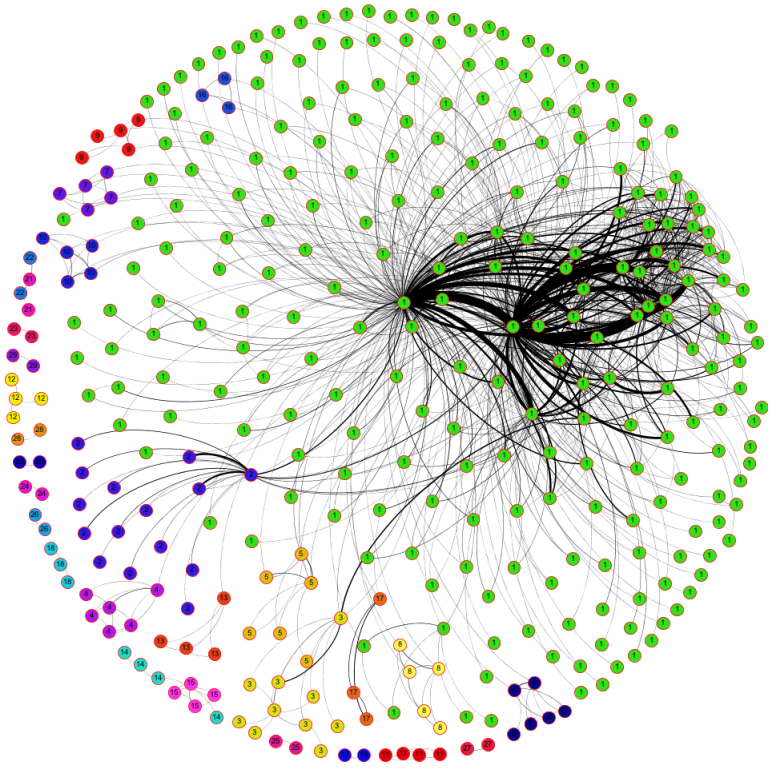


Figure 4.9: Visualization of the product-to-product network with each product associated with their corresponding community.

We found a property, also present in [9, 29], demonstrating that when a network is partitioned in such a way to maximize modularity, the community size q appears to have a power-law form $P(q) \sim q^{-w}$ for a constant w. In this case this constant is $w \simeq 1.3$ considering all the communities available and $w \simeq 0.63$ if we exclude the first community. The number of products in each community is depicted in figure 4.10. The respective power-law function is also plotted.

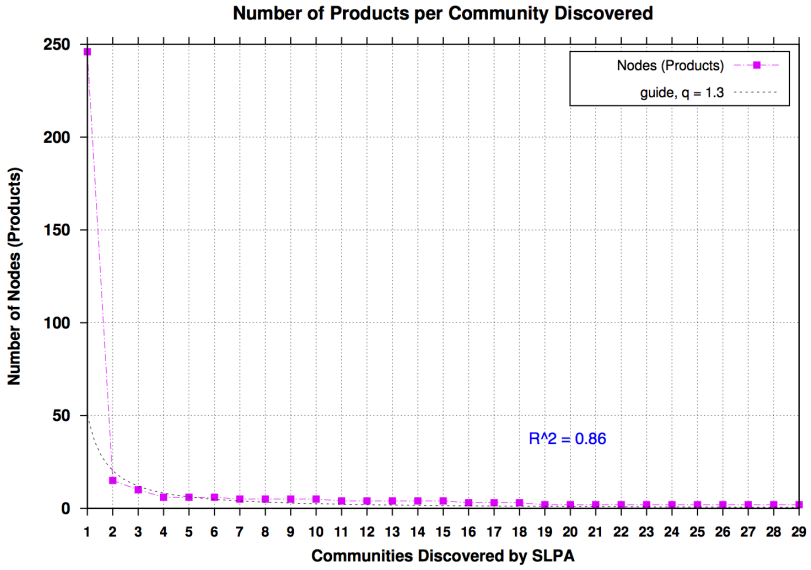


Figure 4.10: Number of products per community discovered by SLPA.

Figure 4.11 is a zoom of the plot presented in figure 4.10. It leaves the first community out of the plot and only shows the 28 remaining communities.

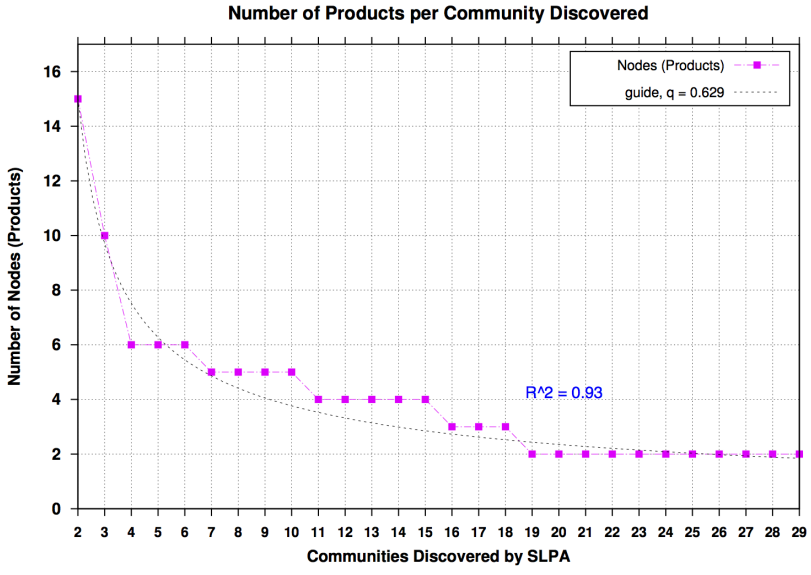


Figure 4.11: Zoom of communities discovered by SLPA.

The *coefficient of determination* denoted by R^2 is 0.86 considering all the communities and 0.93 if we remove the first community. From both figures and the value of R^2, it is clear that power-law function fits very well with the number of elements found in each community.

4.9 Communities Stability

One important aspect when communities are studied is their stability over time. To do so, we analyze the communities discovered during the period of one year, comparing the similarity between different periods of time over this year. This information will help us determine which period of time is representative, containing communities that do not vary over time.

The process followed for analyzing the stability was that of taking a particular *Temporally Transactional Weighted Product Network* from a specific time window (TW_1), for example a *day*, then we took another time window, for instance a *week* (TW_2), then we iterate over different communities from TW_1 and TW_2 searching for the most similar community c_i, which contains the bigger number of products both in c_1 and c_2. This process is repeated for all the time windows considered –for example–, day, week, month, quarter, semester and year.

Mathematically, we have a set $TW = \{tw_1, tw_2, \ldots, tw_n\}$ of *Temporally Transactional Weighted Product Network* indexed by a particular time window. Each tw_i contains a set C_i of communities and each community C_i^j contains a set P_j of associated products.

We calculated the Jaccard Index [57] between the common products from two communities C_i^j and C_r^q. We then defined the similarity between those two communities $C_i^{j'}$ and $C_r^{q'}$ as the maximum Jaccard Index. Is important to note that community $C_r^{q'}$ is the most similar to community $C_i^{j'}$.

To obtain the most representative community over different time windows, it is necessary to calculate the similarity between communities. Figure 4.12 depicts the similarity of three communities from a particular day $d1$ in comparison with the rest of the periods.

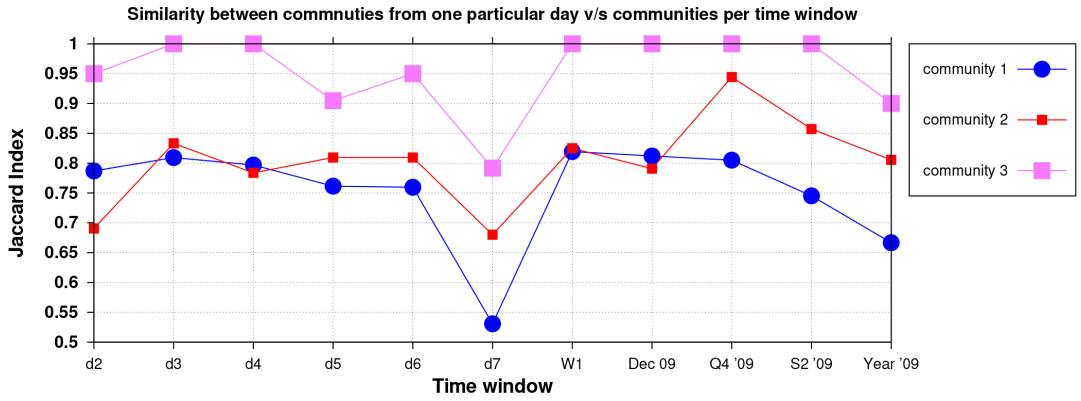


Figure 4.12: Similarity between three communities from day $d1$ compared to different time windows.

We then repeated the process for the each time window. Comparing the three most important communities with the three most relevant communities of the following periods. Figure 4.13 shows the similarity between a particular week (November 7th, 2009 to November 13th, 2009) and the rest of the time window. Figure 4.14 and Figure 4.15, depicts the information from a month (November 2009), a quarter (October, 2009 to December, 2009), a semester (July, 2009 to December, 2009) and finally, a year (January, 2009 to December, 2009) as the pivot to make comparisons respectively.

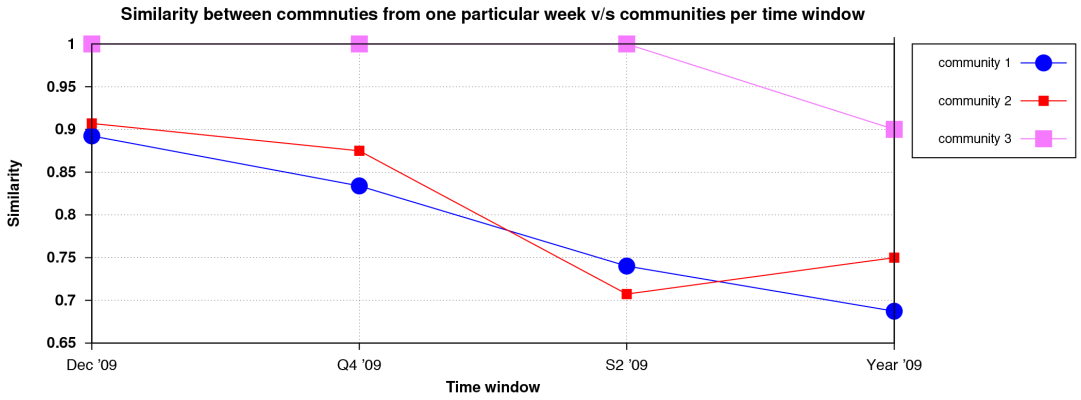


Figure 4.13: Similarity between three communities from week $w1$ compared to a different time window.

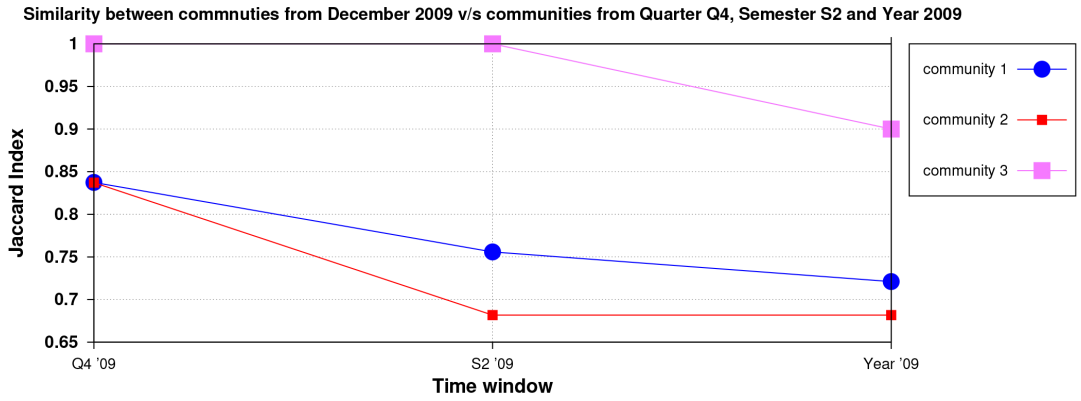
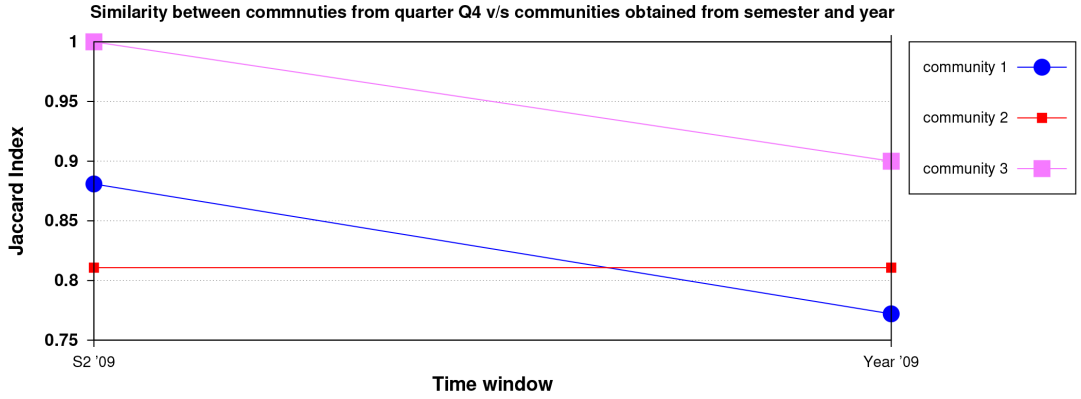
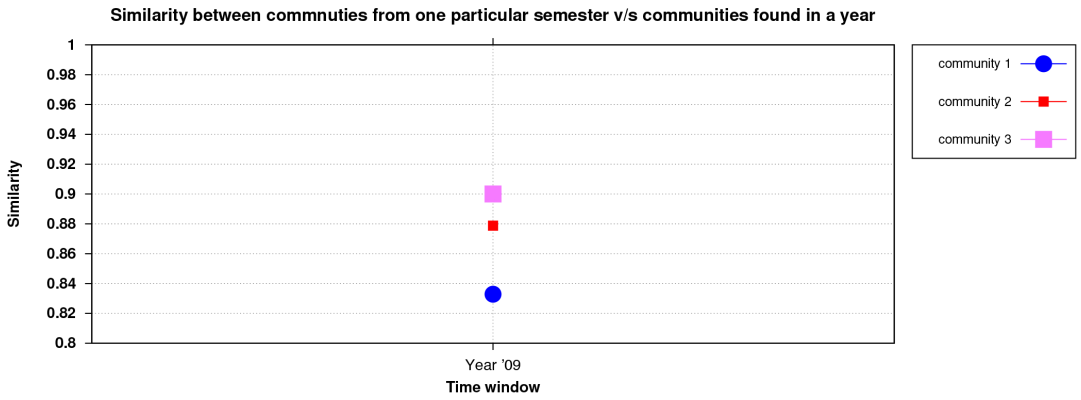


Figure 4.14: Similarity between three communities from month $m1$ compared to a different time window.



(a) Quarter: October to December 2009



(b) Semester: July to December 2009

Figure 4.15: Similarity between three communities over different time windows.

We presented this information to analysts who studied the results and concluded that the best time window found is a *month*, because the percentage is relatively similar over time. It is also a very manageable time window and aligned with the time window used by the retail. Every model used by the retail is re–calibrated each *month*.

4.10 Customer Characterization

We described in section 3.9 how customers will be characterized. In this section we will show the results from that characterization.

Once the communities are found we can begin to characterize customers based on their previous purchases. To do so, we first need to identify a period of time to be analyzed. Next, we choose a particular customer, then select the purchases made over that time period for that particular customer. After that, it is necessary to choose the *filtered temporally transactional weighted products network* to be used as well as the algorithm used to find communities.

To depict the process we will present an example for two customers identified by the *ids* equal to 7343 and 56181. The period to be considered will be one month of March 2010. We chose the specific *filtered product network* with a threshold of $3,030$. The algorithm will be SLPA and the output results of SLPA chosen is the one with a higher value of *modularity*. Table 4.9 depicts the quantity of purchases to each one of the communities identified by the customers selected. The three highest values are bolded.

Community (k)	**Number of purchases Q^k_{7343}**	**Number of purchases Q^k_{56181}**
Groceries	**142**	**45**
Soft Drinks	**11**	**12**
Cigarettes	0	**63**
Yoghurts Brand A	4	2
Jelly Brand B	3	7
Nectar Juice Brand C	**10**	0
Juice Brand D	7	3
Nectar Juice Brand E	5	8
Nectar Juice Brand F	4	0
Compote Brand B	3	6
Powder Juice Brand G	6	8
Jelly Brand H	2	0
Semolina with Milk Brand B	1	5
Yoghurt Brand E	0	0
Cookies Brand J	7	6

Table 4.9: Communities identified and the number of purchases Q^k_i. For customer $i = 7343$ and $i = 56181$.

The total number $\sum_{k \in K} Q_i^k$ is equal to 205 and 165 for the customers 7343 and 56181 respectively. Table 4.10 shows the belonging degree to each one of the respective communities discovered.

Community (k)	**Belonging Degree Customer** $id = 7343$	**Belonging Degree Customer** $id = 56181$
Groceries	**0,692**	**0,272**
Soft Drinks	**0,0536**	**0.0727**
Cigarettes	0	**0,381**
Yoghurts Brand A	0,019	0,012
Jelly Brand B	0,0146	0,042
Nectar Juice Brand C	**0,048**	0
Juice Brand D	0,034	0,018
Nectar Juice Brand E	0,024	0,048
Nectar Juice Brand F	0,019	0
Compote Brand B	0,014	0,036
Powder Juice Brand G	0,029	0,048
Jelly Brand H	0,00975	0
Semolina with Milk Brand B	0,004	0,03
Yoghurt Brand E	0	0
Cookies Brand J	0,034	0,036

Table 4.10: Belonging degree of customer 7343 and 56181 to each one of the communities discovered.

We can say that costumer ($i = 7343$) prefers to buy *Groceries*, *Soft Drinks* and *Nectar Juice Brand C*, but does not purchase *Cigarettes*. On the other hand, customer $i = 56181$ prefer to purchase *Cigarettes*, *Groceries* and *Soft Drinks* as depict its main communities. This method allows analysts to examine the preferred groups of purchases for each customer, also complementing the already existing profiles. We can generate groups of customers that match particular criteria, for example, all the customers that have a belonging degree of *Soft Drinks* community over a certain threshold θ. Combinations of these criteria are allowed too.

4.11 Software

In this section, we will show the system built to manage the communities discovered as well as the customer profiles.

4.11.1 Database

First, we will show the tables added to the data model depicted in figure 4.2. The new data model with its new components is presented in figure 4.16.

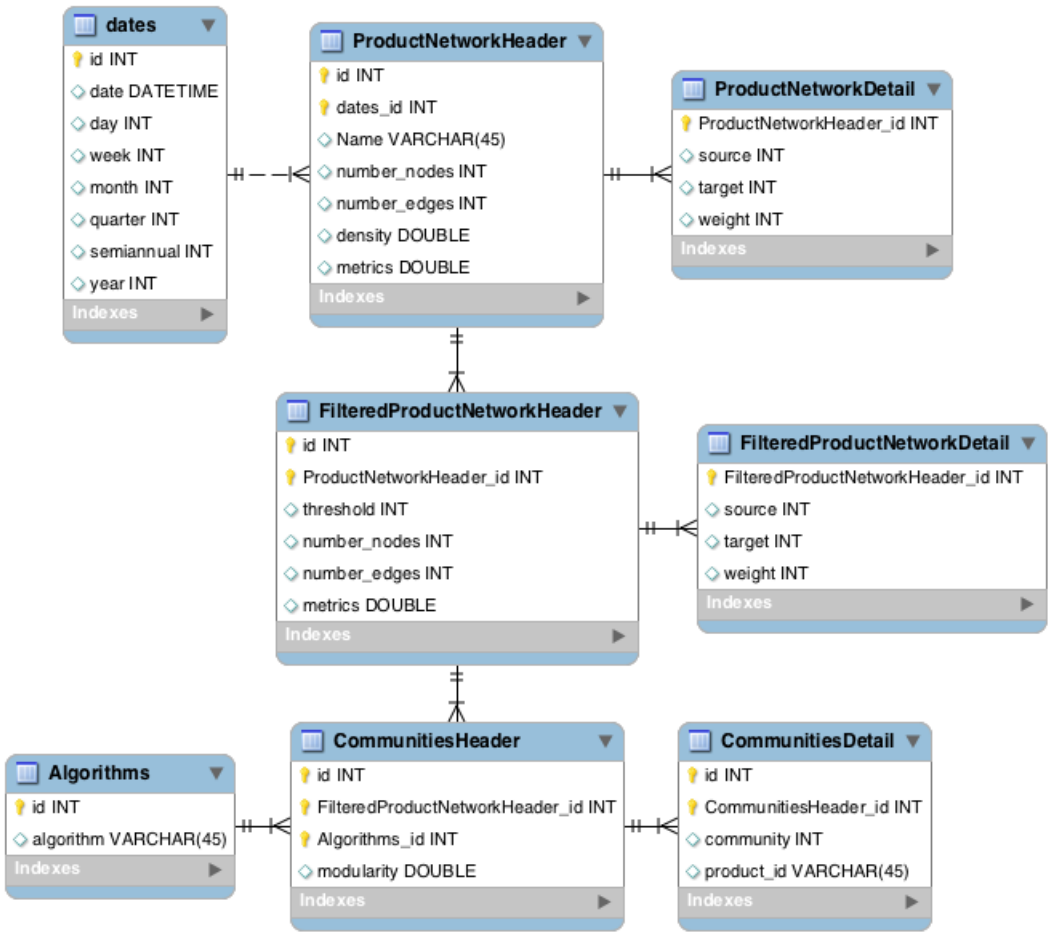


Figure 4.16: Full Database Model

Tables added are used to store the *Temporally Transactional Weighted Products Networks*, the *Filtered Temporally Transactional Weighted Products Networks* and also the communities discovered.

ProductNetworkHeader is used to store the product networks. The data stored includes the time window, the number of nodes and edges inside the product network. Is depicted a field *metrics* which is a representation of the several metrics that can be stored.

ProductNetworkDetail is used to store the graph. Their fields are an abstraction of the nodes represented by *source* and *target*. *Weight* represents the value of the edge between them.

FilteredProductNetworkHeader is used to store the filtered networks. The representation is basically the same as the previous one.

CommunitiesHeader stores the results of the application of a particular algorithm over a filtered product network

CommunitiesDetail the products associated with a particular community are stored.

4.11.2 Graphical User Interface

The system is designed to help analysts manage the information obtained. This information, as we mentioned previously in this thesis project, can be filtered in several ways. According to the analysts, the main way that the information can be filtered is by choosing the *time window* and the minimum number of *nodes* that a particular *filtered temporally transactional weighted product network* should contain. In figure 4.17 a screenshot of the system introducing this information to the user is shown.

Figure 4.17: System search by time window filter or minimum number of edges.

The system, after we have selected a *time window* and the minimum number of *nodes*, presents the results of this search as depicted in figure 4.18. All columns can be sorted, ascending or descending, by clicking in the header of the particular column.

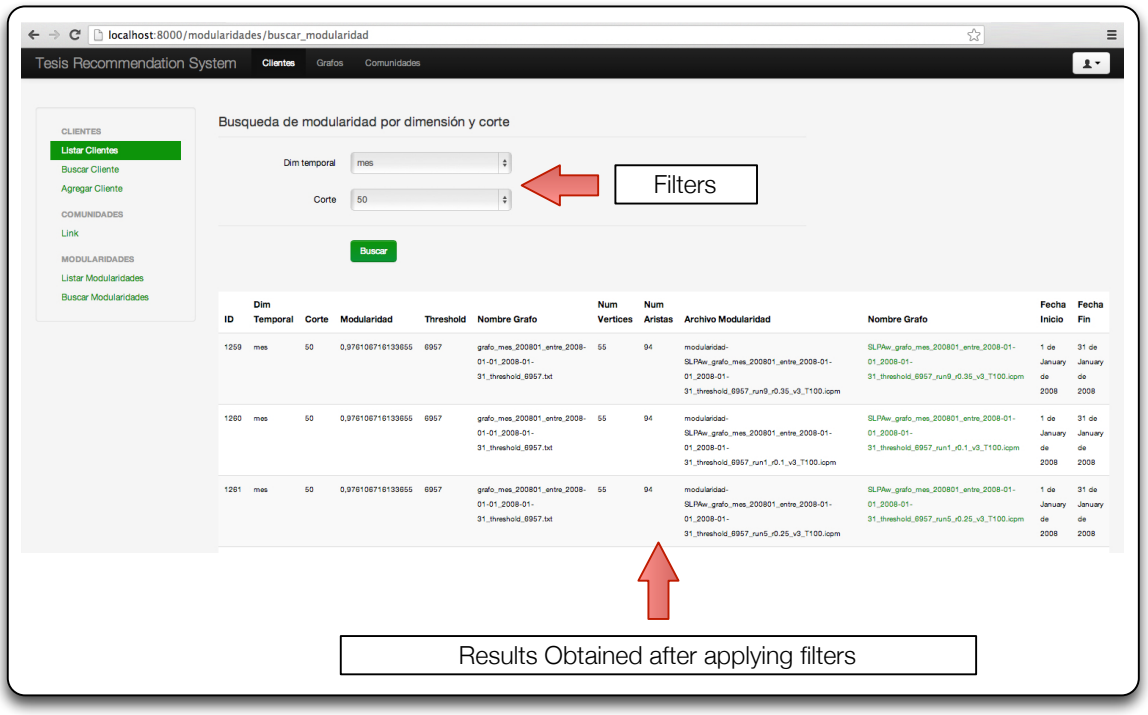


Figure 4.18: Results presented by the system after the internal search conducted.

After the system presents the results obtained, an analyst can select a particular *filtered temporally transactional weighted product network* in order to study the obtained communities according the processing described in section 4.6.

At the beginning of the page, the system presents an aggregate summary that only shows the number of communities and the number of products. In figure 4.19 this aspect is represented.

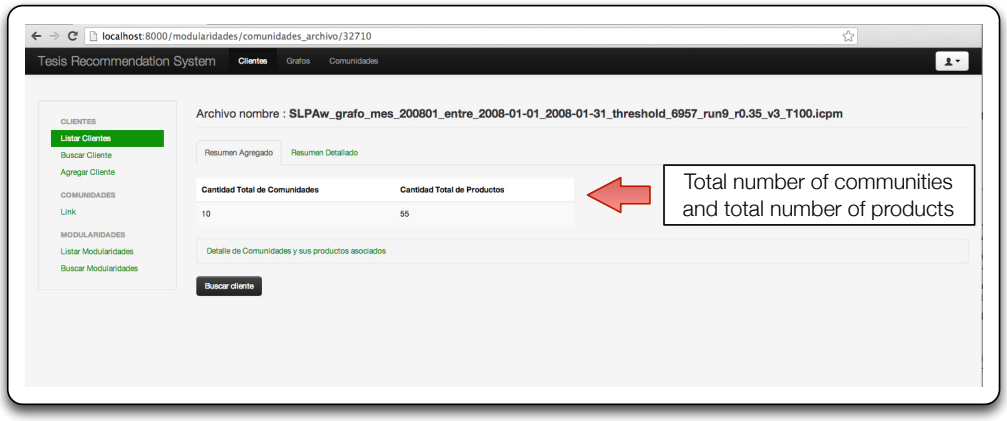


Figure 4.19: Aggregate summary of the communities found.

If we click on the next menu, we can see the communities with their number of products associated. These columns can be sorted in ascending or descending order. Figure 4.20 depicts this fact.

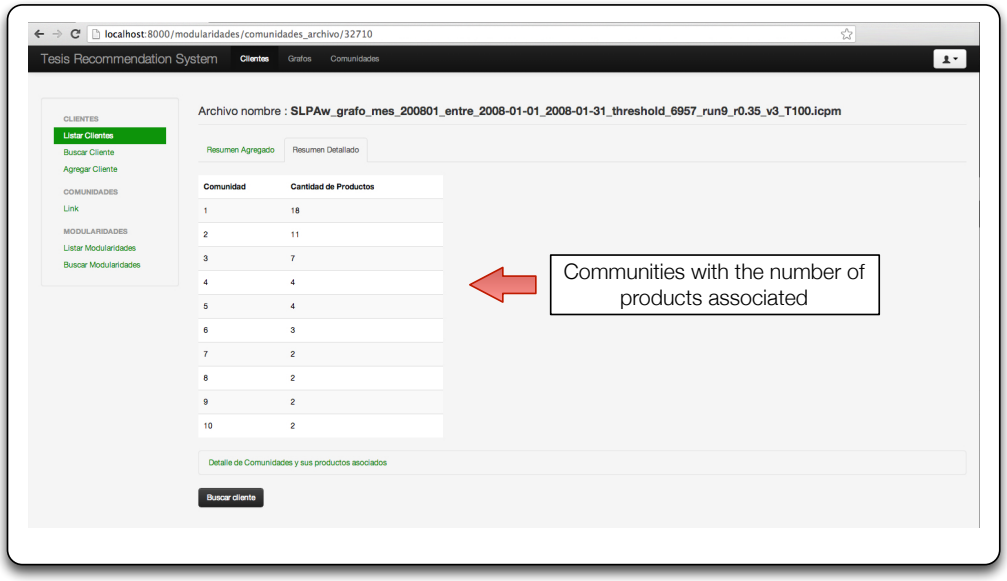


Figure 4.20: Set of communities with their respective number of products.

Analysts can then study the communities with their products in detail. Figure 4.21 shows how the products in each community are presented, which are highlighted by a color. This fact is noted as very valuable by analysts, because they can easily study the products. Also, they can move products from one community into another, just choosing the target community and pressing the button next to the selection. Finally, the system offers the possibility of mixing two communities. To do so, an analyst only needs to select the source community and the target community that will house the result of the mix.

Figure 4.21: Set of products with their respective community.

Once the analyst has obtained a community characterization, proceed to obtain a customer characterization. To do so, he/she chooses a particular customer and generates its belonging degree according its previous purchases, just as we explained in section 3.9. To prevent that the system lists all the customers already registered on it, we added a previous filter by the business category which is pointed out when the customer is registered as a costumer by the retail. In figure 4.22 we showed the filter described.

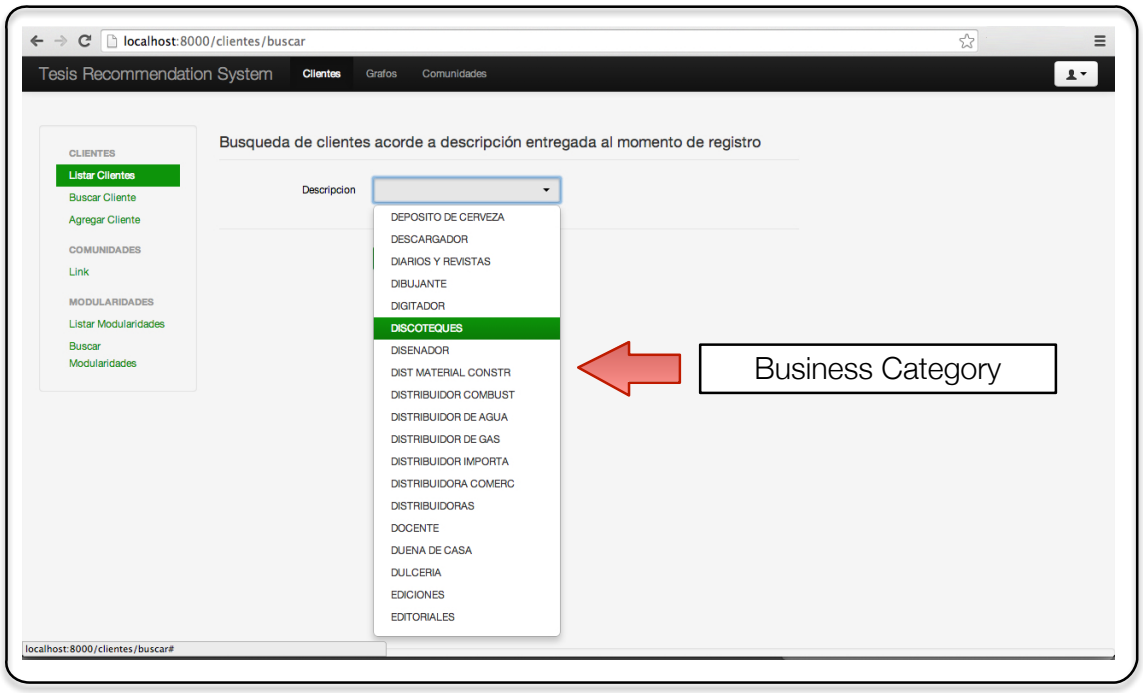


Figure 4.22: Set of business categories available.

Figure 4.23 shows all the customers that belong to the same business category, with the number of previous purchases made.

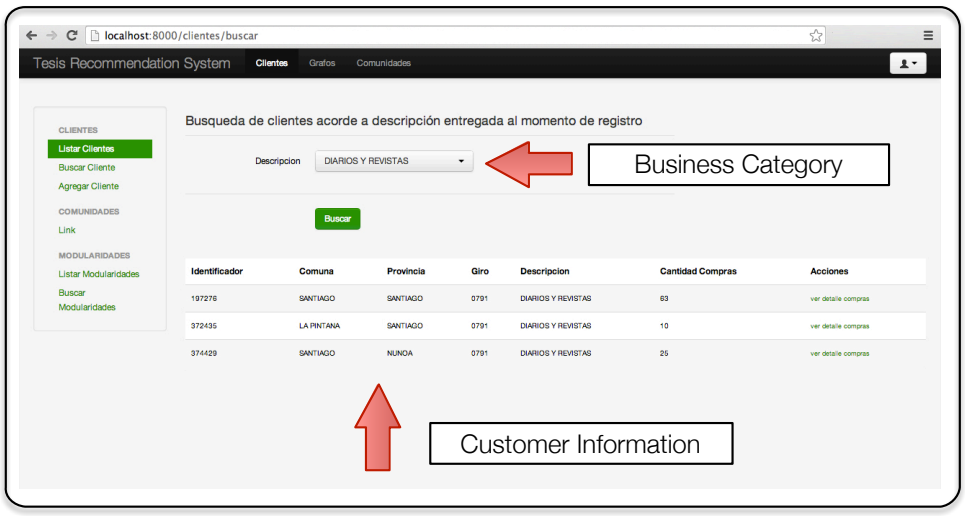


Figure 4.23: Set of business categories available.

The analyst proceeds to choose a customer, then the system will present the set of products purchased previously, highlighting the products that belong to one of the communities depicted in figure 4.21. In figure 4.24 the main information of the purchases made by a specific customer is depicted.

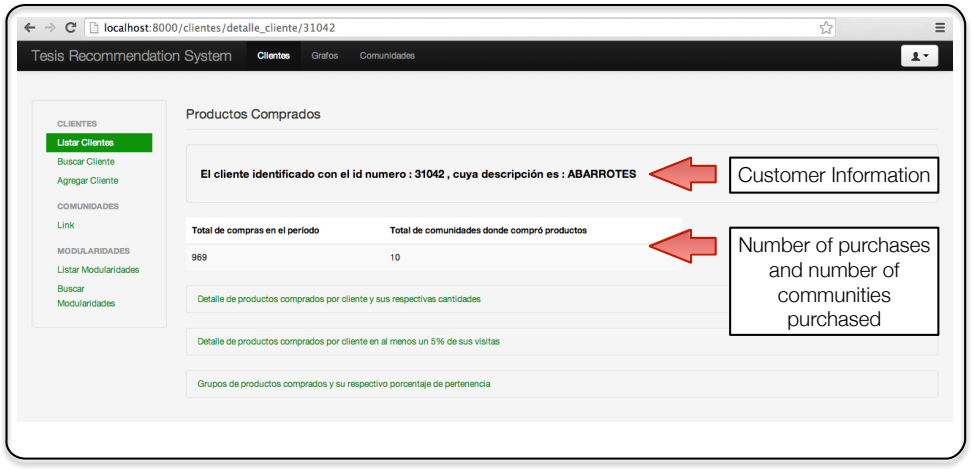


Figure 4.24: Customer information of previous purchases.

The system also depicts the products purchased with the communities that are associated. This information is highlighted in order to make this process easier to the analyst. Figure 4.25 shows how this information is presented to the analysts.

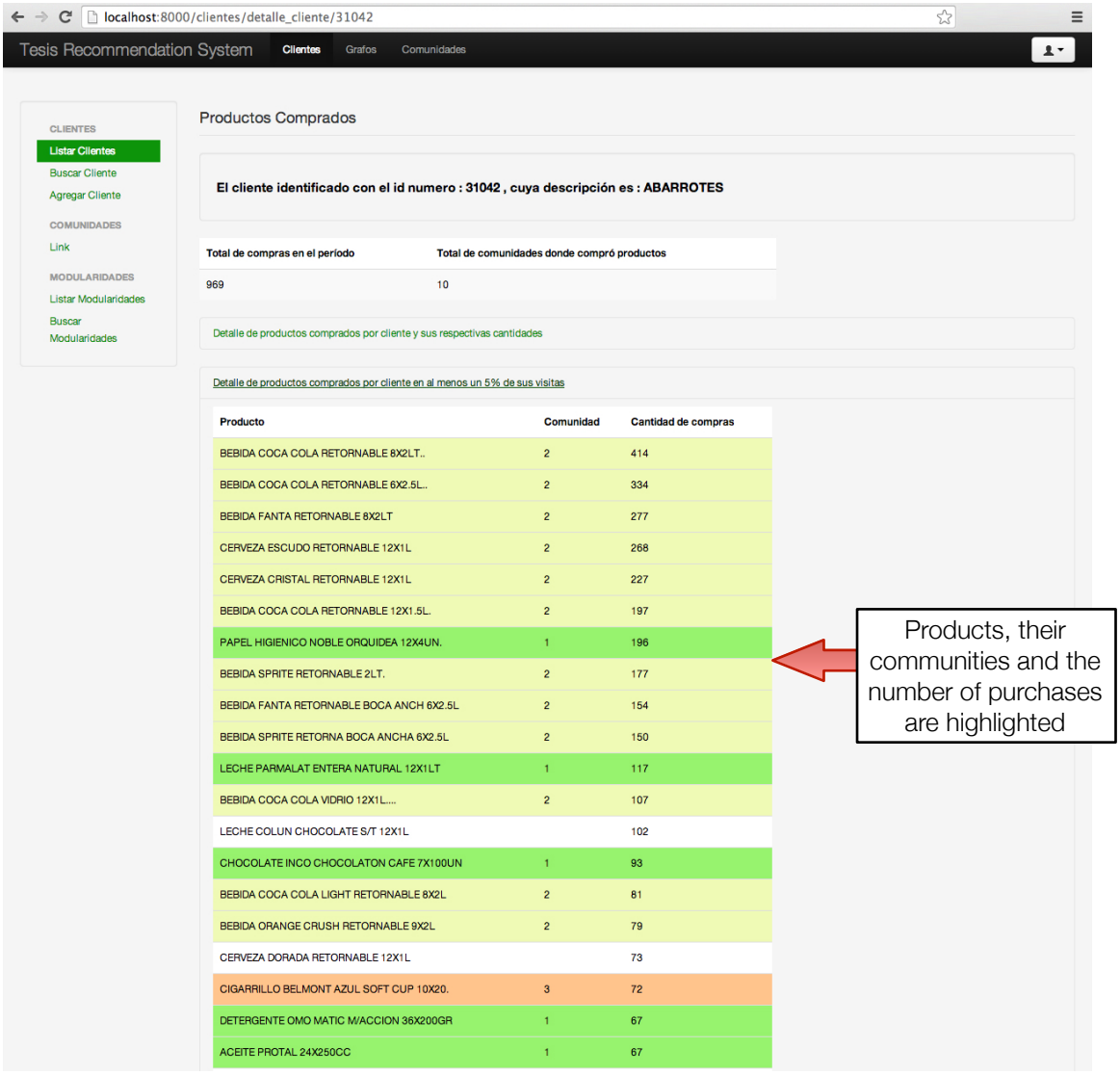


Figure 4.25: Previous purchases with the communities associated.

Finally, the information of purchases grouped by communities is presented to the

analyst. The customer characterization that we presented in section 4.10 is also depicted. Figure 4.26 depicts an example from a real customer.

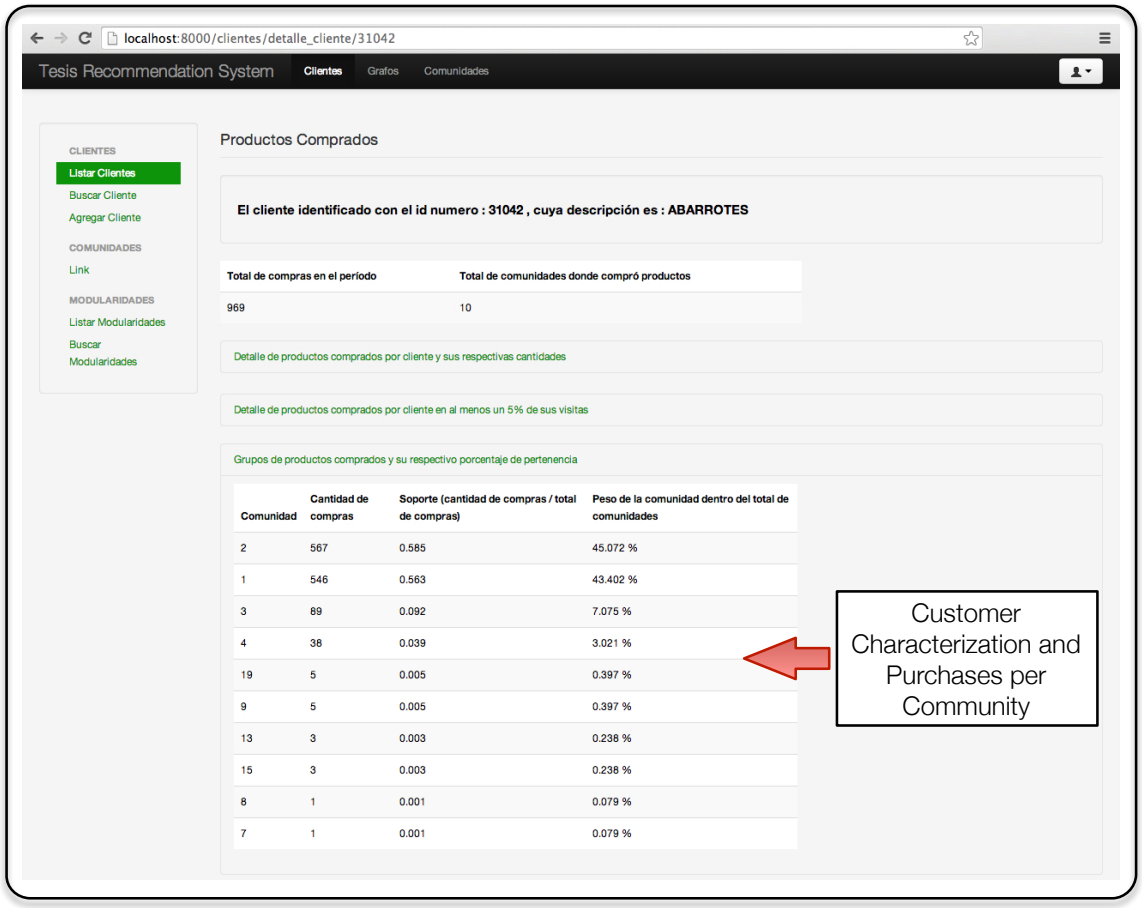


Figure 4.26: Customer characterization based on communities.

In this chapter we have presented an application of our novel methodology, in order to develop market basket analysis and customer characterization. This process allows analysts to generate groups of products that are related each other because they are purchased in the same buying opportunity. It also offers a better understanding of the customer, based on their previous purchases. We presented the results obtained from the analysis carried out by analysts that indicate that this methodology produces more relevant results than those obtained with traditional techniques.

Chapter 5

Conclusions and future work

"In literature and in life we ultimately pursue, not conclusions, but beginnings."

Sam Tanenhaus

It is commonly known that retaining a customer is cheaper than acquiring a new one. This fact motivates both retailers and companies to develop new ways of interacting with their clients in order to get a better understanding of them. Chilean retailers currently cluster consumers based in the amount they expend within a certain period of time. On the other hand, literature recommends generating groups of products found in market baskets, then associate customers to groups according to their previous purchases. We addressed this problem using traditional techniques, such as K-Means, SOM and market basket analysis, but results were impractical.

Our first approach did not go well, therefore a novel approach was developed. The method included graph mining techniques, for performing market basket analysis, and overlapping community detection algorithms as a frequent item set discovery technique. It also presents a way to extract useful information from millions of product sales transactions.

We introduce the concept of *Temporally Transactional Weighted Product Network* which allows carrying different types of information needed by retailers and other organizations. These networks can be divided by a mix of different criteria, such as

time window (daily, weekly, monthly, etc), a particular store, a cluster of stores or the entire *retail* transactions.

We propose an objective methodology for thresholds and filters setup in order to reduce noisy data (independent from the nodes and edges quantity). Firstly, we proposed the *top three heavy edges threshold* method. An unsupervised threshold that only depends on the data available. Secondly, we defined filters which are a proportion of the *top three heavy edges threshold*. Subsequently, we propose the application of overlapping community detection algorithms as a manner to generate frequent item sets.

Our approach has shown that the method presented can be used as a valid technique to discover frequent item sets present in transactional data. It is important to remark that the information delivered is very useful for retail, depicting relationships that were not obvious for the analyst of the *retail*. For example, people are loyal to a particular brand instead of a mix of brands as expected.

The main contribution is a methodology that can be used as a framework for completing customer profiles based on transactional data. This methodology not only applies to retailers. It can also be applied in other industries that need to generate groups built from transactional data. The steps depicted in chapter 3 can be modified according the specific needs.

The main objective of this thesis, generating customer profiles from transactional data of a supermarket retail chain using graph mining techniques, was achieved. The methodology required was introduced and developed in this work. In section 4.10 we present a real case of customer profiling using the proposed techniques, which allows for effectively characterizing customers based on their previous purchase.

We have several specific objectives that were also completed. In fact, we will explain each one of them showing that they were totally fulfilled and indicate their contribution to this work.

In section 2.6, research on the state–of–the–art in market basket analysis and its techniques applied over transactional data was shown. We introduced a definition of the concept, the data used to apply market basket analysis and the meaning of association rules. We presented the most common techniques to generate frequent item sets, such as apriori algorithms and the principal measures to evaluate the quality of rules generated.

In section 2.8.1 we presented the algorithms related to overlapping community detection. Existing algorithms were classified in five major classes. The importance of this section is that we determined the best two algorithms to be used in the rest of this thesis.

Chapter 4 depicts the application of the methodology described in chapter 3 over real retail data. Section 4.3 shows traditional clustering and market basket analysis techniques, the results obtained by this classic approach made the generation of meaningful groups of products unfeasible, forcing us to generate a novel methodology to solve this problem. Then, in section 4.4 we described the application of our methodology, depicting each step performed over the process.

We benchmarked using state–of–the–art algorithms COPRA and SLPA. We also applied state–of–the–art frequent item set algorithms such as K-means, SOM and Apriori. The benchmark was done using data of a wholesale supermarket chain and a retail supermarket chain, processing over 400 million transactions.

However, with our own methodology we were able to produce meaningful and useful frequent item sets. For example, using K-means we obtained mainly 2 representative clusters and one of these concentrating 93% of the products (around 14,000), versus our method, which found –in the same data– 30 clusters (communities) with an average of 7 products. It is clear that 7 products is a manageable number for any analyst. Another example is the Apriori algorithm results which gave several rules with very low support and confidence, making the generation of groups of products that can be easily labeled an unfeasible task.

We asked the *retail* business experts to compare our methodology results with traditional market basket analysis algorithms like Apriori, K-Means and SOM. We discovered that results from traditional techniques were far from being useful, because clusters were formed by huge amounts of mixed products, thus, a segmentation based on these results was meaningless. The main reason was the sparse nature of supermarket data and the large size of information involved.

We completed the customer profile based on mixing previous purchases and the communities obtained from the developed methodology. We characterized the customer based on the belonging degree to each community as explained in section 4.10.

Finally, in section 4.11 we depict the system developed and built, which allows analysts to handle communities obtained and the customer characterization based on the communities found. We presented the software to retail analysts in order that they help us measure the utility of the software. They tested the software and expressed their satisfaction about the system and the possibilities that are being supplied with, such as community management and customer profiles. The satisfaction of the analysts complete the last of the objectives proposed at the beginning of this work.

Future Work

Several directions for future work can be considered and noted. In this thesis we made a characterization and completion of the customer data based on previous purchases and overlapped communities found from industry transactional information.

In future work, one could try to use these communities of products as a useful source of information for generating personalized recommendations of products and offers. Existing products in a community could be used as a basis of the most likely products to be recommended according a recently discovered customer profile.

In this work, we used undirected weighted networks. But for some retailers it could be useful to use directed weighted or directed unweighted networks. Along the same line, non-overlapped communities could work. Benchmarking the results obtained with overlapped communities in terms of number of communities, number of products, stability, just to name a few.

Detecting key elements inside communities is a topic for future work. This could be done through the application of algorithms for discovering key members, such as HITS [65], PageRank [90] or following the approach presented in [78], to detect products that are relevant inside a community.

It could be important to study the relation between relevant products inside a community and breaks of stock. Evaluating the effect on related products that may not be purchased because the main product was sold out. It would be useful to investigate this fact and quantify it in economic terms. The idea of applying the proposed methodology to another retailer such as a supermarket, convenience store or home improvement, in order to try the proposed methodology with a new case of study, is significant.

From the computational point of view, it would be very relevant to put the software developed in production, facing all the requirements from architecture to usability of the system. Indeed, it is very motivating to continue improving the parallelization challenge, trying to enhance time and the number of transactions processed as was done in this thesis project.

Acknowledgments

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Journal Publications

[1] Videla-Cavieres, I. F., & Ríos, S. A. (2014). Extending market basket analysis with graph mining techniques: A real case. *Expert Systems with Applications*, 41(4), 1928-1936.

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