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Summary

This thesis studies the effects of credit market imperfections on the aggregate economy and on the different types of firms. I analyze the relationship between financial development, wealth distribution and economic efficiency and the effect of financial development on access to credit and on the performance of SMEs.

I develop a general equilibrium model with heterogeneous agents and non-linear, variable investment decisions which generalizes the fixed investment model of Balmaceda and Fischer (2010). I study an economy which can be either financially open or closed, with a competitive banking system. In the closed economy the equilibrium interest rate is determined endogenously by demand and supply mechanisms, while in the open economy the banking system has unlimited access to capital at the international interest rate. There exists a continuum of entrepreneurs endowed with different amounts of wealth or mobile capital, and with one unit of specific capital (e.g. an idea, an ability, a project). In order to develop their projects, entrepreneurs may require capital loans from the credit market. Due to imperfect creditor protection, credit rationing arises endogenously in the model, limiting access to credit of sub-capitalized firms (SMEs).

Moral hazard (investment decisions are non-verifiable) and credit market imperfections imply that in equilibrium there will exist 2 types of credit constrained agents: those that cannot access to the credit market and do not produce, and entrepreneurs who form a SME which has partial access to credit market and operates inefficiently. Therefore, there exist 2 critical capital levels: a minimum amount of capital is needed to have access to loans and a second capital level allows the firm to borrow up to the optimal investment.

A basic result of the model is that financial development lowers the two critical capital levels and encourages banks to provide more credit. At the aggregate level, the reduction in credit constraints increases the depth of the credit market and economic efficiency.

The model is consistent with several observations of literature. In terms of wealth distribution, the model predicts that greater wealth concentration in (very) poor countries means higher credit penetration and GDP, while in richer economies this result is reversed. Another interesting result is that in closed economies, financial development benefits smaller SMEs, but it harms larger firms (due to the rise in interest rates). On the other hand, in open economies financial development benefits all SMEs and does not have an adverse effect on firms. Thus, larger firms are likely to oppose to financial development in closed economies, but not in open economies. Therefore, financial openness in an important determinant of financial development.

Resumen

El presente trabajo estudia el efecto de las imperfecciones del mercado del crédito sobre la economía agregada y sobre distintas categorías de empresas. Se analiza la relación entre desarrollo financiero, distribución de riqueza y eficiencia económica. En particular, se estudia el efecto del desarrollo financiero sobre el acceso al crédito y el desempeño de las PYMEs.

Se desarrolla un modelo de equilibrio general con agentes heterogéneos y decisiones de inversión variable no-lineal que generaliza el modelo de inversión fija de Balmaceda y Fischer (2010). Se estudia una economía que puede ser financieramente cerrada o abierta, con un sistema bancario competitivo. En la economía cerrada la tasa de interés de equilibrio se determina de manera endógena por mecanismos de demanda y oferta, mientras que en la economía abierta el sistema bancario tiene acceso ilimitado a capital a la tasa de interés internacional. Existe un continuo de empresarios dotados con diferentes niveles de riqueza o capital móvil, más una unidad de capital específico (ej. una idea, una habilidad, un proyecto). Los empresarios pueden acceder al mercado del crédito para obtener préstamos de capital para desarrollar sus proyectos. Debido a que la protección de los acreedores es imperfecta, el racionamiento del crédito surge de manera endógena en el modelo, limitando el acceso a crédito de firmas sub-capitalizadas (PYMEs).

La existencia de riesgo moral (decisiones de inversión no verificables) y las imperfecciones del mercado del crédito implican que en equilibrio existirán 2 tipos de agentes restringidos de crédito: aquéllos sin acceso a crédito y que no producen, y aquellos empresarios que forman una PYME, pero que solo tienen acceso parcial al crédito y operan de manera ineficiente. Por lo tanto, existen 2 niveles críticos de capital: un nivel mínimo de capital para acceder a crédito y otro nivel de capital que permite alcanzar la inversión óptima.

Un resultado básico del modelo es que el desarrollo financiero (o bajas en los costos fijos) reduce los 2 niveles críticos de capital e incentiva a los bancos a otorgar más crédito. A nivel agregado, la reducción de las restricciones de crédito significa un aumento de la profundidad del mercado del crédito y de la eficiencia económica.

El modelo es consistente con varias observaciones de la literatura. En términos de distribución de riqueza, el modelo predice que mayor concentración de riqueza en países (muy) pobres significa mayor penetración del crédito y GDP, mientras que en economías más ricas este resultado se revierte. Otro resultado interesante es que en economías cerradas el desarrollo financiero beneficia a las PYMEs más pequeñas, pero perjudica a las firmas más grandes (debido al alza en la tasa de interés). Por otro lado, en economías abiertas, el desarrollo financiero beneficia a todas las PYMEs y no perjudica a las empresas grandes. Por lo tanto, es probable que las firmas más grandes se opongan al desarrollo financiero en economías cerradas, pero no en economías abiertas. Por lo que, la apertura financiera es un importante determinante del desarrollo financiero.

A mis padres, hermanos y a Rebeca.

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Introduction

The main concerns of this thesis are the effects on the performance of the aggregate economy and sectors of credit market imperfections and barriers to entry, as well as their interaction with different wealth distributions. I examine the impact of changes in parameters describing the quality of financial institutions on credits constraints and on the real economy. I analyze the relationship between wealth distribution, financial development and economic efficiency. In particular, I examine the consequences of financial development and wealth inequality on the performance of Small and Medium Enterprises (SMEs).

This thesis studies the effects of credit market imperfections and barriers to entry (measured by fixed costs of setting up a firm) on the performance of a economy with heterogeneous agents and firms. It generalizes the fixed investment model described in Balmaceda and Fischer (2010) by considering variable investment with non-linear productivity. I use a static general equilibrium model of an economy which can be either financially open or closed. Entrepreneurs are endowed with different amounts of initial wealth or mobile capital, and own one unit of inalienable specific capital (e.g., an idea, an ability, a project, human capital). I assume a competitive banking system which acts as an intermediary for internal transactions between borrowers and lenders. In the case of the financially closed economy, the equilibrium interest rate is determined endogenously via supply and demand mechanisms. The supply for credit comes from agents (entrepreneurs with no access to credit) who save their small wealth on banks, or from wealthy entrepreneurs who deposit their surplus capital in the banks, after investing in their firms. In contrast, in the open economy banks have access to unlimited funds from abroad at the international interest rate. To simplify, I assume that the banking system is costless and competitive and thus, banks will charge to borrowers the same interest rate that they pay for deposits.

In order to introduce credit constraints, I use the mechanism developed in Burkart and Ellingsen (2004): the borrower may choose to abscond in order to finance non-verifiable personal consumption. In that case the legal system is able to recover only a fraction $(1-\phi)$ of the loan. Following Balmaceda and Fischer (2010), the recovery rate $1-\phi$ depends on the quality of *ex ante* creditor protection, i.e. a measure of the extent that creditor rights are enforced by legal system. The inability of financial system to recover all capital loaned leads to credit constraints and credit rationing that restrict some entrepreneurs.

Unlike previous theoretical models, which analyzed the effect of financial market and its imperfections using fixed investment choices (see Hoshi et al., 1993; Holmstrom and Tirole, 1997; Repullo and Suarez, 1995; Balmaceda and Fischer, 2010), my model incorporates non-linear variable investment decisions. This feature allows me to distinguish four types of firms: Microenterprises with no access to credit, Small and Medium size Enterprises (SMEs), Larger medium sized firms and Large Enterprises. The main contribution of this thesis is that variable investment allows me to analyze sub-capitalized firms (SMEs) which have only partial access to the credit market. The fact that they have restricted access to credit means that they cannot achieve the optimal investment and thus operate inefficiently. There is another category of potential entrepreneurs with smaller wealth, in common in the above cited literature. These entrepreneurs do not have access to the credit market and may decide to not form a firm, losing their specific capital. In contrast, firms endowed with higher wealth levels will not see their production activities restricted by credit constraints and will operate efficiently.

Consistent with findings of Ghosh et al. (2000) and Beck et al. (2005), my model predicts that access to credit is especially restricted for poorer firms, since less capitalized entrepreneurs cannot commit to refrain from morally hazardous behavior as effectively as richer agents. Moral hazard and credit market imperfections imply that in equilibrium there will be two types of constrained entrepreneurs: those that cannot access to the credit market and may decide to deposit their wealth in the banking system instead of forming a firm, and those who have partial access to credit and operate an inefficient firm. Therefore, banks will use two levels of credit rationing: a minimum capital stock to have access to the credit market and restrictions on the loan amounts. Similarly, I distinguish between two types of unconstrained agents: those that obtain loans that allow them to attain the optimal investment size, and those who do not need the credit market in order to operate at the efficient level. This last type of agents will loan their surplus mobile capital. In summary in my model the four categories of potential entrepreneurs appear because there is variable investment with a concave productivity function and credit constraints that arise endogenously due to imperfect creditor protection. Tirole (2006) develops a theoretical model with variable investment, but in his case the size of his firms scale linearly. Hence, the implied equilibrium of his model results in a 'bang-bang' solutions, with zero investment or limited by credit constraints. Therefore, variable investment is not a feature that can be exploited to differentiate among types of firms as in my model.

One complication of the use of variable investment with non-linear effects is that I cannot obtain explicit expressions for some important variables in the closed financial economy, such as the equilibrium interest rate and the critical capital levels (such as the minimum wealth required for a loan and the minimum wealth to get an optimal loan). However, I can obtain comparative statics results for many of these variables in the closed economy. Consistent with the findings of Shleifer and Wolfenzon (2002) and Balmaceda and Fischer (2010), I find that improvements in *ex-ante* creditor protection (or a reduction in the cost of setting up firms) leads to higher interest rate when there are constrained entrepreneurs. The explanation is that after creditor rights improvements, credit constraints are eased for sub-capitalized firms (SMEs). Thus, previously constrained agents can now compete more intensively for loans, i.e. the demand for capital increases, but the supply of capital remains constant.[1]

After an improvement in *ex-ante* creditor protection the following effects are observed: the minimum capital level to access the credit market is lower, undercapitalized firms can get bigger loans, the wealth required to receive a loan that allows the optimal investment is lower, SMEs are larger and more efficient. Another important result is that an improvement in creditor protection reduces the range in which inefficient firms can develop. Under certain conditions these results are also satisfied when fixed cost decreases [2]. I show that all these results can be directly translated to a small open economy.

[1]As explained in Balmaceda et al. (2014), the requirement of a totally inelastic supply of capital is not required for the result. So long as the supply of capital is not totally elastic, the result continues to hold.

[2]Except the last result. It is not clear if a decrease in fixed cost reduces the range in which inefficient firms develop.

In the closed economy, I find that smaller SMEs and some previous micro-enterprises stand to benefit the most from financial development, because they will experience an increase in their profits. However, a group of firms will experience a decline in their profits. These are larger SMEs, Larger Medium Enterprises (LMEs) and Large Enterprises (LEs). Thus, the increase in financing costs and the reduction in profits of richer incumbent firms, means that financial development will be opposed by wealthy groups in closed economies with a small and medium-sized business sector (that I interpret as developing or poorer countries in terms of capital stock). These findings are consistent with the theory of opposition of wealthy and politically powerful families to financial development, developed by La Porta et al. (2000b) and Rajan and Zingales (2003). Political opposition of incumbents to financial reform gives an explanation to the fact that underdeveloped countries are more reluctant to develop their financial system (also consistent with Pagano and Volpin, 2005; Hellwig, 1998; Bebchuk and Roe, 1999). In contrast, in the small open economy, I find that financial development will lead to an increase in the profits of all SMEs and some previous micro-entrepreneurs, while profits of LMEs and LEs will be unaltered. Hence, consistent with Rajan and Zingales (2003) and Shleifer and Wolfenzon (2002), opposition from incumbents to financial development is more likely to occur in a closed economy than in a small open economy. Thus, a small open economy is more likely to undertake reforms benefiting financial development. Moreover, financial openness stands to be an important determinant of financial development.

The complexities of the model with variable investment imply that I cannot derive comparative statics results for some important macro variables in the closed economy case, especially for those related to changes in the wealth distribution. For this reason, I simulate the model for a given wealth distribution, degree of creditor protection and fixed costs. The model solves for the fixed point that determines the equilibrium interest rate for the initial setting and then I analyze the effects of changes on parameters on various macroeconomic variables such as GDP, aggregate investment and loans, credit penetration an *ex-post* wealth distribution. I also analyze the response of these variables to changes in the wealth distributions. I use Mean Preserving Spread (MPS) of the initial distribution of wealth in order to isolate the effects of pure wealth redistribution on credit constraints and on economic efficiency.

Similarly as in the model of Balmaceda and Fischer (2010), the implication of my model with regard to different wealth distributions across countries depends on the severity of capital constraints. For the closed economy, I simulated a capital unconstrained country and a strongly capital constrained one (which I interpret as a very poor country). [3] [4] In terms of interest rate, I find that more unequally distributed countries are more likely to have higher interest rates if they are more capital constrained, while this result is reversed in capital unconstrained countries.

Simulations show that in capital constrained countries a more equal wealth distribution leads to lower constraints to access to the credit market, but once an agent is able to receive a loan, restrictions to obtain an optimal loan are higher in the more equally distributed country. These results are reversed in capital unconstrained countries.

Aggregate results relate to basic economic variables: investment, total debt, gross output, GDP, credit penetration and *ex-post* wealth inequality. Based on simulations, in a closed uncon-

[3] I interpret ‘capital constrained’ countries as either poorer or less financial developed ones.

[4] I also simulated a less capital constrained country, obtaining similar results to those observed in the unconstrained country. I do not present these results in the main text.

strained economy, GDP, total debt and credit penetration increase as *ex-ante* protection improves or fixed costs decrease, while in a strongly capital constrained economy these results may be the opposite. This is explained because in a strongly capital constrained economy the decline in the equilibrium interest rate after a decrease in the recovery rate (or an increase in fixed costs) is too high. Hence, under worse legal conditions (or higher barriers to entry), richer entrepreneurs who access the credit market obtain bigger loans, investing more capital in a firm but paying lower financing costs. Therefore, in a strongly capital constrained closed economy, a decline in creditor protection (or an increase in fixed costs) may lead to an increase in total debt, GDP and credit penetration. This effect is new and does not occur when investment is fixed as in Balmaceda and Fischer (2010). Additionally, I find that *ex-post* wealth distribution gets better as *ex-ante* protection improves or barriers to entry are reduced. Therefore, financial development reduces wealth inequality (consistent with theoretical results of Galor and Zeira, 1993; Banerjee and Newman, 1993).

Unlike the case of the closed economy, in the open economy I can derive comparative statics for some macroeconomic variables. I find that in a small open economy, an improvement in *ex-ante* creditor protection or a reduction in fixed cost leads to an increase on: total investment, total output and GDP, total debt and credit penetration (measured as the number, percentage of agents who finance their firms accessing to the credit market). Thus, consistent with findings of Balmaceda and Fischer (2010), financial development leads to a more efficient economy and raises credit penetration (consistent with empirical research of Djankov et al., 2007; La Porta et al., 2008). In terms of wealth distribution [5], I conclude that among highly capital constrained countries, those with a more unequal wealth distribution have higher investment and gross output. As in Balmaceda and Fischer (2010), higher inequality in heavily constrained economies means more agents that access to the credit market, i.e. higher credit penetration measured as the percentage of individuals that finance their firms through banking credit. In contrast, among less capital constrained economies or unconstrained ones, those more equally distributed have higher investment, output and credit penetration. However, if the economy is strongly capital unconstrained (a very rich country), the effect of a wealth redistribution on credit penetration is not clear.

The analysis for the closed and open economy allows to study the effect of financial openness on access to credit and over some macroeconomic indicators. I consider an initial small closed economy with an internal interest rate higher than the international interest rate and with a competitive banking system. I find that if the country decides to open up its financial system then the following effects will be observed: an inflow of mobile capital to the economy, a reduction of credit constraints, an increase in profits of all operating firms, an increase in investment, gross output, GDP, total debt and credit penetration. These results are consistent with the theoretical model of Balmaceda et al. (2014) and empirical research of Fischer and Valenzuela (2013), which suggest that under perfect competition in the domestic credit market prior to liberalization, liberalization leads to lower internal interest rate and to an increase in credit penetration. [6]

Finally, I suggest another possible explanation for the Lucas' puzzle (Lucas, 1990) which tries to understand why capital fail to flow to developing countries (interpreting developing countries

[5] As in the closed economy, I use Mean Preserving Spread (MPS) to derive comparative statics related to changes in wealth distribution.

[6] However, if the initial structure were one of imperfect competition, liberalization could lead to exclusion of less wealthy entrepreneurs from the credit market (as explained in Balmaceda et al., 2014).

as less financially developed ones)[7].

0.1 Literature Review

Research of the last decade or so, has shown that the extent of legal protection of investors and creditors is an important determinant of the development of financial system in countries around the world (see La Porta et al., 2002, 1999; Shleifer and Wolfenzon, 2002). La Porta et al. (1998) documented empirically that legal protection of investors varies systematically among countries depending on legal origins and traditions. The most basic prediction of the literature is that investor protection encourages the development of financial markets. If investors are protected from expropriation, they are willing to pay more for securities, making it more atractive for entrepreneurs to issue these securities (La Porta et al., 1999). This applies to both creditors and shareholders. Creditor rights encourage the development of lending, while shareholders rights encourage the development of equity markets. As La Porta et al. (1999) emphasizes, protection for both creditors and shareholders not only includes how the rights are written into laws, but also the effectiveness of their enforcement. Better investor and creditor protection is related with: the sensitivity of investment opportunities, more valuable stock markets, larger number of listed securities per capita and higher rate of IPO[8] (see La Porta et al., 1997), higher valuation of firms, deeper and broader financial markets (La Porta et al., 2002; Djankov et al., 2007), higher dividend pay-outs (La Porta et al., 2000a), greater proportion of firms that use long-term external financing (Demirgüç-Kunt and Maksimovic, 1998) and lower sensitivity of market value during financial crisis (Johnson et al., 2000).

The effects of credit market imperfections on the performance of the economy is also an important empirical issue. Research of La Porta et al. (2002), La Porta et al. (1999), Shleifer and Wolfenzon (2002) coincide that the degree of investor and creditor protection determinates the development of financial system, and thus the performance of the economy. Through its effect on financial markets, investor protection influences the real economy. As explained by Beck et al. (2000), financial development can accelerate economic growth in three ways: it can enhance savings, it can channel these savings into real investment and foster capital accumulation, and to the extent that financiers exercise some control on investment decision of entrepreneurs, financial development improves the efficiency of resource allocation (capital flows to more productive uses).

A large literature, dating as far back as Schumpeter (1911), have emphasized the positive influence of the development of financial institutions on growth. King and Levine (1993) construct a Schumpeterian model of financial intermediation to show that better financial systems improve the probability of successful innovation and accelerate economic growth. Ghosh et al. (2000) obtain the same conclusion using a model that incoporates moral hazard and credit rationing for the credit market. Empirical research of Rajan and Zingales (1998), Levine and Zervos (1998), Levine et al. (2000), Beck et al. (2005), and more recently of Shen (2013), confirm the hypothesis that financial development influence economic growth. In a related issue, an empirical analysis of Araujo and Funchal (2005) concludes that better legal creditor protection in bankruptcy procedures reduce fraud and results in more efficient outcomes. In addition, Bergoeing et al. (2002) find that better bankruptcy procedures speed the recovery of the economy from a shock.

[7] In Chapter 5 I explain in detail this observation.

[8] Initial Public Offering

In terms of aggregate behavior, many studies have highlighted the importance of credit constraints in explaining fluctuations of economic activity (see Friedman, 1986; Bernanke, 1983; Eckstein and Sinai, 1986). Using a dynamic macro model, Aghion et al. (2004) find that economies at an intermediate level of financial development are more unstable than either very developed or underdeveloped ones. Using a model of the same type, Kiyotaki and Moore (1997) show that the dynamic interaction between credit constraints and asset prices seems to be an important mechanism by which the impacts of shocks persist, amplify and are transmitted to other sectors.

Corporate finance theory suggests that market imperfections caused by underdeveloped of financial and legal systems, constrain entrepreneurs ability to fund investment projects. Empirical research of Demirgüç-Kunt and Maksimovic (1998) suggests that firms with developed financial institutions and efficient legal systems obtain more external financing than firms in less financially developed countries. They also find that improvements in legal rights benefit all firms, although this result is much less significant for the smallest firms, which have limited access to the legal system. Beck et al. (2005) and Ghosh et al. (2000) predict that smallest firms are the most credit constrained ones, since they cannot commit to refrain from hazardous behavior as effectively as larger firms. They find that financial and institutional development weakens the constraining effects of financial and legal obstacles, but in constrast with findings of Demirgüç-Kunt and Maksimovic (1998), it benefits mainly smallest firms. However, since SMEs face tighter constraints to access to the credit market, they restrict their activities, and thus have lower productivity (see empirical research of Bigsten et al., 1998; Muriki, 2008) and higher returns to capital (see Banerjee and Duflo, 2004). On the other hand, larger firms do not restrict their productivity due to credit constraints, as Blinder (1985) emphasizes, the story of firms curtailing their activities due to lack of credit rings true for the small business sector, but not for large enterprises.

The organization of the thesis is as follows. In next chapter I present the basic model. In Chapter 2, I present the basic setup of the simulations that were included in the main text. In the next chapter, I analyze the comparative statics in the closed economy due to changes in *ex-ante* protection, fixed cost and with regard to different wealth distributions. In Chapter 4, I study these effects on a small open economy. Next, I analyze the benefits of opening up to the international financial system under perfect competition in the domestic market prior liberalization. The last chapter presents concluding remarks.

Chapter 1

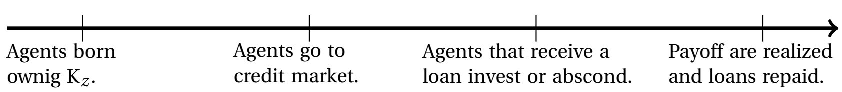
The Basic Model

1.1 Setup

I examine a static model for a closed economy with heterogeneous agents and variable-investment decisions (in Chapter 4 I analyze the case of a small open economy). I divide the single period into four stages (see Figure 1.1). In the first stage, a continuum of agents indexed by $z \in [0,1]$ are born, each endowed with one unit of inalienable specific capital (an idea, an ability or a project) that cannot be transferred or sold. Each entrepreneur owns different amounts of observable wealth or mobile capital K_z. The cumulative wealth distribution among the population of agents is given by $\Gamma(\cdot)$, which has a continuous density and complete support.

During the second stage, agents go to credit market either to loan funds or borrow their mobile capital. In the third stage, agents who receive a loan either invest in a firm or abscond, committing *ex-ante* fraud. As in Burkart and Ellingsen (2004), if the agent abscond, a fraction $1-\phi$ of the loan is recovered by the legal system. Therefore, $1-\phi$ defines *ex-ante* creditor protection or the loan recovery rate. In contrast, agents who do not have access to the credit market may decide to not form a firm and can loan their wealth instead, losing their specific capital. In the last stage, loans are repaid and payoffs are realized.

Figure 1.1: Time line.



In this economy there is produced only one good, with $f(\cdot)$ a production function such that $f'(\cdot) > 0$, $f''(\cdot) < 0$, $f'(0) = +\infty$ and $f(0) = 0$. I suppose there are decreasing returns to capital. In addition, I assume that agents are price takers in the credit and output market. I normalize the output price to one, while in a closed economy the equilibrium interest rate r is endogenously determined via supply and demand mechanisms.

Agents who operate a firm try to maximize their utility function from consumption given by:

$$U(C_z) = U(K_z, D_z) = f(K_z + D_z) - (1+r)D_z - \theta \tag{1.1}$$

The profit owned from operating a firm is defined as:

$$\pi(K_z + D_z) = f(K_z + D_z) - (1+r)(K_z + D_z) - \theta \tag{1.2}$$

Using this definition the utility function can be rewriten as:

$$U(K_z, D_z) = \pi(K_z + D_z) + (1+r)K_z \tag{1.3}$$

where θ is fixed cost of operating a firm, D_z is the amount loaned or borrowed by entrepreneur z, $(1+r)K_z$ is the amount that the entrepreneur would receive if she decided not to form a firm and to deposit all her wealth in the banking system.

Without credit market imperfections, all agents, no matter how small their initial capital level, would have access to the credit market. Thus, all entrepreneurs would be able to borrow as much as they wanted at the interest rate r, and therefore, would be able to operate their firms at the optimal capital level K^*:

$$f'(K^*) = 1 + r \tag{1.4}$$

However, not all entrepreneurs will be able to reach the optimal capital level, because there exist market imperfections and loans are limited by moral hazard. As in Balmaceda and Fischer (2010), the borrower may decide to abscond in order to finance non-verifiable personal consumption. Thus, I assume that investment decisions are non-contractible, and that loans used to finance personal benefits are only repaid to the extent that creditor rights are enforced. As I have mentioned, the legal system is able to recover only a fraction $1-\phi$ of this amount. I interpret an increase in $1-\phi$ as an improvement in *ex-ante* creditor protection or in the loan recovery rate. Therefore, countries with lower (higher) values of ϕ have higher (lower) financial development.

In contrast, those entrepreneurs who decide to invest all their borrowed capital plus their initial wealth in a firm, enjoy returns only after repaying their obligations, i.e. output and sales revenue are verifiable and can be pledged to investors. Furthermore, all these agents would like to operate their firms at the optimal capital level K^*, but due to moral hazard and credit market imperfections, some agents will have partial access to credit market and may decide to operate their business using a lower amount than optimal capital stock. Moreover, poorer agents may not have access to the credit market. In other words, there is credit rationing: a rationed borrower may be willing to pay a higher interest rate to lenders in order to get a loan or a higher loan, but investors do not want to grant such a loan, because they cannot trust the borrower.

Therefore, the model characterizes two types of constrained entrepreneurs: those that do not have enough capital stock to access to the credit market and that may decide to loan their wealth instead of forming a firm (see proposition 1) , and those agents who have partial access to credit market who get a loan that allows them to operate their firms, but at a sub-optimal level. On the other hand, there are two types of unconstrained agents: those who have enough capital stock to get a loan that allows them to operate efficiently, and those richer entrepreneurs who own more than the optimal capital level, who form an efficient firm and decide after to loan their surplus capital. In summary, the model distinguishes between four types of agents.

The demand for credit originates in agents who own less than the optimal capital stock K^*, while the supply of credit is provided by agents who do not have access to credit and decide to not form a firm, and by those richer entrepreneurs who own more than the optimal capital level K^*. In Section 1.7 I show that the demand for credit decreases with the interest rate and that the supply of credit increases with r.

In order to assure that fraudulent behavior never occurs in equilibrium, I define the following incentive compatibility constraint, that must be satisfied for all agents who want to get a loan from the credit market:

$$f(K_z + D_z) - (1+r)D_z - \theta \geq \phi D_z \tag{1.5}$$

Condition (1.5) assures that the utility received by an agent who receives a loan D_z if she decided to not abscond, is at least the same that she would obtain if she did. In addition, this inequality implies that the marginal return for getting a loan is at least $1 + r + \phi$, i.e. returns for borrowers are between this value and $1 + r$. Note that under constant returns to scale all firms are equally profitable and in that case loans are unneeded. In case of having perfect loan recovery, i.e. if $\phi = 0$, then all agents will have access to the credit market.

Additionaly, the following breakeven constraint or participation constraint must be satisfied:

$$\pi(K_z, D_z) \geq 0 \tag{1.6}$$

Condition (1.6) assures that the profit of the firm is not negative. Note that this condition is the same as asking that the utility of the entrepreneur for operating a firm is at least what she will obtain for loaning all her capital:

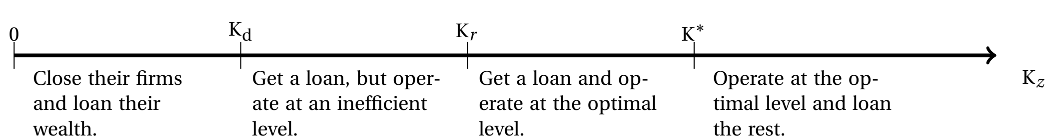
$$U(K_z, D_z) = f(K_z + D_z) - (1+r)D_z - \theta \geq (1+r)K_z \tag{1.7}$$

1.2 Critical capital levels

As in Balmaceda and Fischer (2010), it can be defined a minimum capital level K_d required for a loan. Below I will show that agents who cannot access to the credit market (who owns less than K_d) decide to loan all their wealth instead forming a firm, losing their specific capital. On the other hand, entrepreneurs such that $K_z \geq K_d$ can access to the credit market, however the maximum amount of debt that any of these entrepreneurs can get depends on their wealth . In fact, I define a second critical capital level K_r, such that agents with $K_z \in [K_d, K_r)$ have partial access to credit market and are able to get an inefficient amount of debt which increase non-linearly with K_z , while entrepreneurs such that $K_z \geq K_r$ are able to get a loan that allows them to attain the optimal production level. In summary, agents such that $K_z < K_d$ cannot access to the credit market and do not produce, entrepreneurs with a capital level such that $K_z \in [K_d, K_r)$ have access to the credit market, but operate at an inefficient, sub-optimal level, while entrepreneurs with $K_z \geq K_r$ are able to operate efficiently .

After defining these critical capital levels, I can characterize the set of optimal decisions for any agent just in terms of her wealth K_z (I formally show the set of optimal decisions for each group of entrepreneurs in next section):

Figure 1.2: Entrepreneurs decisions.



In order to determine the critical capital levels I use the incentive compatibility constraint to define the following auxiliary function:

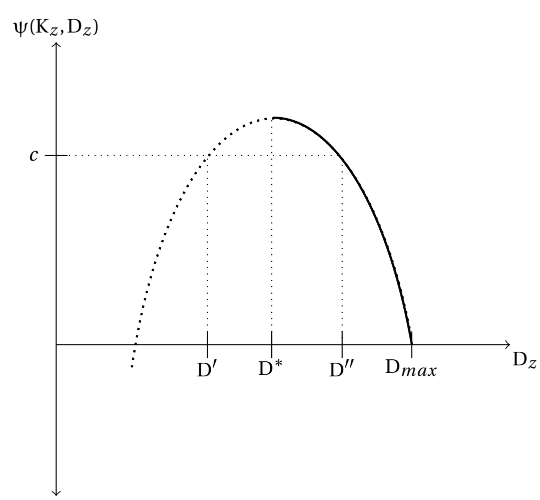
$$\psi(K_z, D_z) \equiv f(K_z + D_z) - (1 + r + \phi)D_z - \theta \tag{1.8}$$

which will be useful to define the minimum capital level to get a loan K_d, the minimum allowable debt D_d and the critical capital level K_r which defines the maximum effective loan. Notice that when $\psi(K_z, D_z) = 0$ it defines the amount of debt D_z that leaves an agent endowed with an initial wealth K_z indifferent between operating a firm or committing *ex-ante* fraud. I will show below that this condition is in fact zero for credit constrained entrepreneurs (agents such that $K_z \in [K_d, K_r)$), i.e. the incentive compatibility constraint is binding for agents who operate an inefficient firm.

Before defining conditions that determinate K_d and D_d, it is important to define the minimum allowable debt that any constrained entrepreneur with $K_z \in [K_d, K_r)$ can get. First, notice that for a fixed wealth K_z, the auxiliary function $\psi(K_z, D_z)$ reaches it maximum in some value $D_z = D^*$, because it is continuous and concave in D_z (note that: $\psi_{DD} = f''(K_z + D_z) < 0$). Secondly, consider that $\psi(K_z, D_z)$ is not negative in some neighborhood of K_d, D_d. Then, if $D' < D^*$ is such that $\psi(K_z, D') = c$, I can find an amount of debt $D'' > D^*$ such that $\psi(K_z, D'') = c$. Hence, at D'' the agent has the same incentives to behave than at D', but the payoff that the lender will receive is higher at D''. Moreover, in a neighborhood of K_d, D_d entrepreneurs are credit constrained and operate an inefficient firm, thus their utility function is increasing in D_z (note that $U_D(K_z, D_z) = f'(K_z + D_z) - (1 + r) > 0$, because $f'(\cdot) > 1 + r$ for constrained agents), i.e. they also prefer to choose D'' instead of D'. Therefore, the effective loan that any constrained entrepreneur may choose and that the lender will offer is to the right of D^*. Thus, the allowable debt curve starts at D^*, which is the minimum allowable debt for any constrained entrepreneur, and ends is some point D_{max} in which $\psi(K_z, D_{max}) = 0$. Using the previous notation, the allowable debt curve for any agent with partial access to credit market is defined by the interval $[D^*, D_{max}]$.

In Figure 1.3 I illustrate these features.

Figure 1.3: Allowable debt curve



Note: continuous line represents $\psi(\cdot)$ evaluated at the allowable debt curve.

Now, in order to define the minimum allowable debt D_d, consider that the value of the minimum capital stock K_d is known. Then, by definition of the minimum allowable debt, an agent who owns K_d cannot receive a higher amount of debt without having incentives to abscond. Hence, the minimum debt is defined as the amount of debt that maximizes the auxiliary function in K_d, subject to the incentive compatibility constraint is satisfied. In addition, the minimum capital stock K_d defines the first agent who is able to get the minimum allowable loan without having incentives to abscond. Therefore, to determine the pair (K_d, D_d), the lender must solve the following *minimax* problem: [1]

$$\min_{K} \max_{D} \quad \psi(K, D) \geq 0$$

To simplify this problem, notice that the minimization of $\psi(K, D)$ will lead to $\psi(K, D) = 0$, which is the minimum value allowed for $\psi(\cdot)$ without violating the compatibility constraint. Therefore, the minimization over K will imply that the incentive compatibility constraint is binding. Then, the equivalent problem that the lender solves is:

$$\begin{aligned} \max_{D} \quad & \psi(K_d, D) \\ \text{s.t.} \quad & \psi(K_d, D) = 0 \end{aligned}$$

Observe that the initial *minimax* problem is translated to a simple maximization problem of a continuous and concave function. Because the incentive compatibility constraint is binding in K_d, D_d and $\psi(\cdot)$ reaches its maximum at that point, the agent who owns the minimum capital stock cannot either get a higher or a lower loan, since in both cases the incentive compatibility constraint will be violated. Thus, the minimum capital stock is such that the minimum allowable debt coincides with the maximum allowable debt at D_d.

Defining λ as the Lagrange multiplier of the compatibility constraint of the previous maximization problem, I obtain the following first order condition:

$$\frac{\partial \mathscr{L}}{\partial D} = (1 + \lambda)\psi_D(K_d, D_d) = 0 \tag{1.9}$$

Notice that equation 1.9 defines an additional condition for the minimum debt D_d, which is that the first derivative of $\psi(K_d, D)$ with respect debt is zero at D_d. Now, I can define conditions that determinate K_d and D_d.

Definition 1. *The minimum allowable debt* D_d *and the minimum capital stock* K_d *are defined by the following two conditions*[2]

$$\psi(K_d, D_d) = f(K_d + D_d) - (1 + r + \phi)D_d - \theta = 0 \tag{1.10}$$

$$\psi_D(K_d, D_d) = f'(K_d + D_d) - (1 + r + \phi) = 0 \tag{1.11}$$

The second condition of definition 1 implies that the marginal return of the first agent with access to credit equalizes the optimal marginal return of capital $(1 + r)$ plus the return that she will obtain if she decides to abscond (ϕ). Since agents such that $K_z < K_d$ decide to not form a firm and because maximum allowable debt increases with K_z (see proposition 2 of next section),

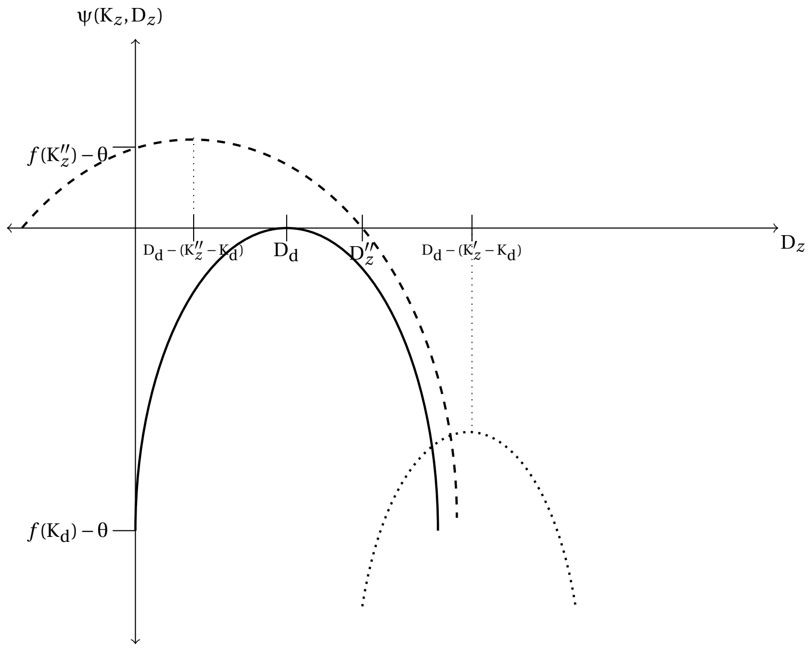
[1] Note that if $\theta = 0$, then $K_d = 0$ and the problem of defining K_d, D_d becomes trivial. I assume that $\theta > 0$ to make things interesting.

[2] I assume that the minimum capital stock to get a loan is positive $(K_d > 0)$.

the maximum return to capital for operating a firm is $1 + r + \phi$, which decreases to the right of K_d and reaches it minimum value $1 + r$ from K_r onwards.

In figure 1.4 I ilustrate the movements of $\psi(K_z, D_z)$ in terms of K_z and D_z. Note that by definition of K_d, D_d this function reaches its maximum (zero) at D_d for $K_z = K_d$, while for $K''_z > K_d$ it reaches its maximum at the left of D_d. As shown in figure 1.4, there exists a debt level D''_z to the right of D_d that solves $\psi(K''_z, D''_z) = 0$ for $K''_z > K_d$, these are the amounts of debt that constrained agents will choose to ask from the credit market (as showed in next section). Agents such that $K'_z < K_d$ do not have access to the credit market, therefore $\psi(K'_z, D_z) < 0 \ \ \forall D_z > 0$. As shown in the figure, the auxiliary function reaches its maximum to the right of D_d for these capital levels and there is not a positive amount of debt such that $\psi(K'_z, D_z) = 0$.

Figure 1.4: Auxiliary function $\psi(K_z, D_z)$ in terms of K_z, D_z.



Note: dotted line represents $\psi(K'_z, D_z)$, continuous line represents $\psi(K_d, D_z)$ and dashed line represents $\psi(K''_z, D_z)$ (with $K''_z > K_d > K'$).

In order to determine the critical capital level K_r, I impose that the maximum allowable debt corresponding to K_r allows the firm to attain exactly the optimal capital level K^*. Therefore, the incentive compatibility constraint binds, and the maximum debt of an entrepreneur who owns K_r is $K^* - K_r$.

Definition 2. *The critical capital level* K_r*, which defines the first agent who is able to get an optimal loan is defined by:*

$$\begin{aligned} \psi(K_r, K^* - K_r) = f(K^*) - (1 + r + \phi)(K^* - K_r) - \theta = 0 \\ \Leftrightarrow \pi(K^*) + (1 + r)K_r = \phi(K^* - K_r) \end{aligned} \tag{1.12}$$

Notice that $K^* - K_r = D_r$ is the maximum effective debt in the economy.

It is important to emphasize that I distinguish between the maximum allowable debt and the effective debt. The first one is the maximum amount of debt that an entrepreneur is allowed to obtain from the bank system. The second one is the amount of debt that the agent decides to ask after solving the maximization problem presented in the following section. In the case of agents such that $K_z \in [K_d, K_r)$, I will show that they decide to ask for the maximum allowable debt which is also their effective debt. In contrast, entrepreneurs such that $K_z \in [K_r, K^*)$ ask for a loan that allows them to produce efficiently. In this case the effective debt is lower than the maximum allowable debt. In Section 1.4 I will discuss in more detail the properties of debt curve.

After defining the minimum capital stock, the minimum debt and the wealth needed to access to an optimal loan, lenders have to determinate the interest rate charged to borrowers and the interest rate payed to deposits. To simplify, I assume that there exists an implicit competitive banking system that intermediates costless transactions between borrowers and lenders [3]. Thus, banks will charge to borrowers the same interest rate that they pay for deposits, i.e. lenders and borrowers will operate at the same equilibrium interest rate r, which is determined endogenously on the closed economy.

1.3 Optimal investment decisions of entrepreneurs

In previous section I have described the problem the lender solves to define the pair (K_d, D_d), as well as the capital needed to get an optimal loan. In this section I characterize the maximization problem of entrepreneurs.

Agents who access to the credit market, i.e. such that $K_z \geq K_d$, will choose to invest the amount of capital that maximizes their utility satisfing the participation and the incentive compatibility constraint. Using the definition of the auxiliary function $\psi(K_z, D_z)$, the maximization problem that solves and agent who owns an initial wealth $K_z \geq K_d$ is defined by:

$$\begin{aligned} \max_{I_z} \quad & U(K_z, D_z) \\ \text{s.t.} \quad & \psi(K_z, D_z) \geq 0 \\ & \pi(K_z, D_z) \geq 0 \end{aligned}$$

Defining the Lagrange multiplier of the participation constraint as μ and the lagrange multiplier of incentive compatibility constraint by λ, the first order condition is:

$$\frac{\partial \mathscr{L}}{\partial D_z} = U_D(K_z, D_z) + \lambda\psi_D(K_z, D_z) + \mu\pi_D(K_z, D_z) = 0$$

Notice that the decision variable is investment of agent z, which is defined by $I_z = K_z + D_z$. However, because each entrepreneur owns a fixed wealth level K_z, the problem is equivalent to find the optimal amount of debt D_z. From condition (1.1) I obtain that $U_D(K_z, D_z) = f'(K_z+D_z) - (1+r)$, the definition of the auxiliary function $\psi(K_z, D_z)$ leads to $\psi_D(K_z, D_z) = f'(K_z + D_z) - (1 + r + \phi)$, while from the definition of firms profits I conclude that $\pi_D(K_z, D_z) = f'(K_z + D_z) - (1 + r)$. Replacing these results in previous condition I obtain an expression for λ:

$$\lambda = -\frac{(1+\mu)[f'(K_z + D_z) - (1+r)]}{f'(K_z + D_z) - (1 + r + \phi)} \tag{1.13}$$

[3] I will discuss in depth the structure of financial market in Section 1.6.

The definition of K_r implies that entrepreneurs such that $K_z < K_r$ cannot receive a loan that would allow them to operate efficiently. Thus, for these agents the marginal return of capital is higher than $1+r$. In addition, note that the Lagrange multiplier of the participation constraint is such that $\mu \geq 0$, therefore the numerator of condition 1.13 is positive for constrained entrepreneurs. In fact $\mu = 0$, because as I will show below there exists a discrete 'jump' in utilities from K_d^- to K_d (see figure 1.6), i.e. utilities from K_d onwards are higher than $(1+r)K_z$, which is the amount that earns an agent with $K_z < K_d$. Since the participation constraint is not binding for $K_z \geq K_d$, the second constraint of previous maximization problem could be omitted.

Now, from the definition of the minimum debt and because agents who cannot access to the credit market do not form a firm, I know that the marginal return of capital cannot be higher than $1+r+\phi$, i.e. the denominator of equation (1.13) is negative. Hence, $\lambda > 0$ for all agents such that $K_z \in [K_d, K_r)$. Then, the incentive compatibility constraint is binding for constrained agents, i.e. they decide to ask for the maximum allowable debt.

In contrast, agents who own more than K_r reach the optimal production level with a marginal return of capital of $1+r$, which implies that $\lambda = 0$, i.e. the compatibility constraint is not binding ($\psi(K_d, D_z) > 0$). Therefore, I conclude that unconstrained agents will not ask for the maximum allowable debt (except for an agent such that $K_z = K_r$), and if they own less than K^* they will decide to ask for the amount of debt that allows them to reach exactly the optimal capital stock. In particular, those richer entrepreneurs who own more than K^* will decide to loan their surplus capital, which is translated in negative values of D_z (notice that those agents choose $D_z = K^* - K_z < 0$).

These previous results are consistent with intuition, because for constrained agents the marginal cost of credit is $1+r$, while the return of capital is higher than $1+r$. Thus, agents with capital levels such that $K_z \in [K_d, K_r]$ must decide to get as much credit as they are allowed to have. Hence, as I showed before, the compatibility constraint is binding :

$$\psi(K_z, D_z) = f(K_z + D_z) - (1+r+\phi)D_z - \theta = 0 \quad \forall K_z \in [K_d, K_r] \tag{1.14}$$

Condition (1.14) defines the amount of debt that any agent with a wealth $K_z \in [K_d, K_r]$ will ask from the banking system. Note that depending on K_z, equation 1.14 may have one or two solutions for agents who access the credit market. But as I have explained in previous section, the allowable debt curve for a particular entrepreneur is defined by the amounts of debt that are to the right of the maximum value of $\psi(\cdot)$, i.e. the optimal debt which gets a constrained entrepreneur is the maximum of the solutions of equation (1.14).

Previously, I have described the maximization problem and optimal investment decisions for agents with access to credit (such that $K_z \geq K_d$). One important question that remains, is whether agents without access to the credit market prefer to form a firm just using their own capital or to loan their wealth instead. I define the wealth level K_θ such that the agent is indifferent between loaning her capital or forming a firm investing all her wealth:

$$f(K_\theta) - \theta = (1+r)K_\theta \tag{1.15}$$

Now, the question that remains is whether K_θ can be lower than K_d, because in that case some agents without access to credit will prefer to form an inefficient firm instead loaning their wealth.

Proposition 1. *Agents with* $K_z \in [0, K_d)$ *prefer to not form a firm and loan their wealth.*

Proof. From condition (1.10) I have that:

$$\psi(K_d, D_d) = f(K_d + D_d) - (1 + r + \phi)D_d - \theta = 0$$

Now consider that $D < D_d$, from condition (1.11) $\psi_D > 0$, because $f'(K_d + D) > 1 + r + \phi$ for $D < D_d$. Therefore, $\psi(K_d, D)$ increases with D to the left of D_d. Thus, because $\psi(K_d, D_d) = 0$ and $\psi(K_d, D)$ decreases for $D < D_d$ when D declines, I conclude that:

$$\psi(K_d, D) = f(K_d + D) - (1 + r + \phi)D - \theta < 0 \quad \forall D \in [0, D_d) \tag{1.16}$$

In particular, at $D = 0$ I have that $\psi(K_d, 0) = U(K_d, 0) = f(K_d) - \theta < 0$. Moreover, $U(K_z, 0)$ decreases when K_z declines (because $U_K(K_z, 0) = f'(K_z) > 0$). Therefore, I conclude that $U(K_z, 0) < 0 \ \ \forall K_z < K_d$. Thus, if agents without access to credit market produced, they would obtain negative utilities. Hence, they prefer to loan all their wealth and earn $(1 + r)K_z$ instead forming a firm.

□

The previous proposition implies that $K_\theta > K_d$. Therefore, in the closed economy the minimum capital level adjusts endogenously so that the agent who owns K_θ prefers to ask for her maximum allowable debt and form a firm, instead loaning her wealth or forming an inefficient firm using her own wealth.

1.4 The debt curve

In previous sections I have characterized critical capital levels (K_d, K_r, K^*) and the maximum allowable debt curve for constrained entrepreneurs. Now it is important to characterize the shape of this curve. The following propositions allows to describe the shape of debt curve.

Proposition 2. *Maximum allowable debt curve* D_z *satisfies the following properties:*

1. $\frac{\partial D_z}{\partial K_z} > 1$ *if* $K_z > K_d$.
2. $D_z > 0$ *and* $D_z > K_z$ *if* $K_z \geq K_d$.
3. *It is concave in* K_z.

Proof. Differentiation of equation (1.14) leads to:

$$\frac{\partial\psi(K_z, D_z)}{\partial K_z} + \frac{\partial\psi(K_z, D_z)}{\partial K_z}\frac{\partial D_z}{\partial K_z} = 0$$

$$\Rightarrow \frac{\partial D_z}{\partial K_z} = -\frac{\psi_K}{\psi_D} \tag{1.17}$$

From conditions that define K_d and D_d I obtain that:

$$\psi_D(K_d, D_d) = f'(K_d + D_d) - (1 + r + \phi) = 0 \tag{1.18}$$

$$\psi_K(K_d, D_d) = f'(K_d + D_d) > 0 \tag{1.19}$$

Note that if $K_z > K_d$, $f'(K_z + D_d) < 1 + r + \phi$ (because $f''(\cdot) < 0$). Then $\psi_D < 0$ to the right of K_d (recall that D_z cannot be lower than D_d, because D_d is the minimum debt). Additionally, replacing (K_z, D_z) in equation (1.19) I obtain that $\psi_K > 0$. Thus using (1.17) I conclude that $\frac{\partial D_z}{\partial K_z} > 0$. Now, replacing the pair (K_z, D_z) in (1.18) and (1.19), equation (1.17) leads to:

$$\frac{\partial D_z}{\partial K_z} = -\frac{f'(K_z + D_z)}{f'(K_z + D_z) - (1 + r + \phi)}$$

$$\Rightarrow \frac{\partial D_z}{\partial K_z} = -\frac{1}{1 - \frac{(1+r+\phi)}{f'(K_z+D_z)}} \tag{1.20}$$

Moreover, because $1 + r < f'(K_z, D_z) < 1 + r + \phi$ for agents with $K_z \in (K_d, K_r)$, from (1.20) I conclude that $\frac{\partial D_z}{\partial K_z} > 1$(note that $\frac{\partial D_d}{\partial K_d} < -1$ at K_d^-).

For the second item, note that if I replace $D_d = 0$ in (1.14) and in (1.11), I will have that $f'(K_d) = 0$ and $f'(K_d) = 1 + r + \phi$ which is a contradiction. Thus, D_d must be positive, and because debt increases with K_z I conclude that $D_z > 0$ for all agents. In order to show that $D_z > K_z$, I equalize incentive compatibility constraint to zero (see equation (1.14)) to define the maximum allowable debt of each agent, and I combine this condition with the participation constraint (see condition (1.7)). The two conditions are:

$$U(K_z, D_z) = \phi D_z$$
$$U(K_z, D_z) \geq (1 + r)K_z$$

Replacing compatibility constraint in participation constraint and using the fact that participation constraint is not binding for agents who access to the credit market:

$$\Rightarrow \phi D_z > (1 + r)K_z$$
$$\Rightarrow D_z > \frac{1 + r}{\phi} K_z \geq K_z \tag{1.21}$$

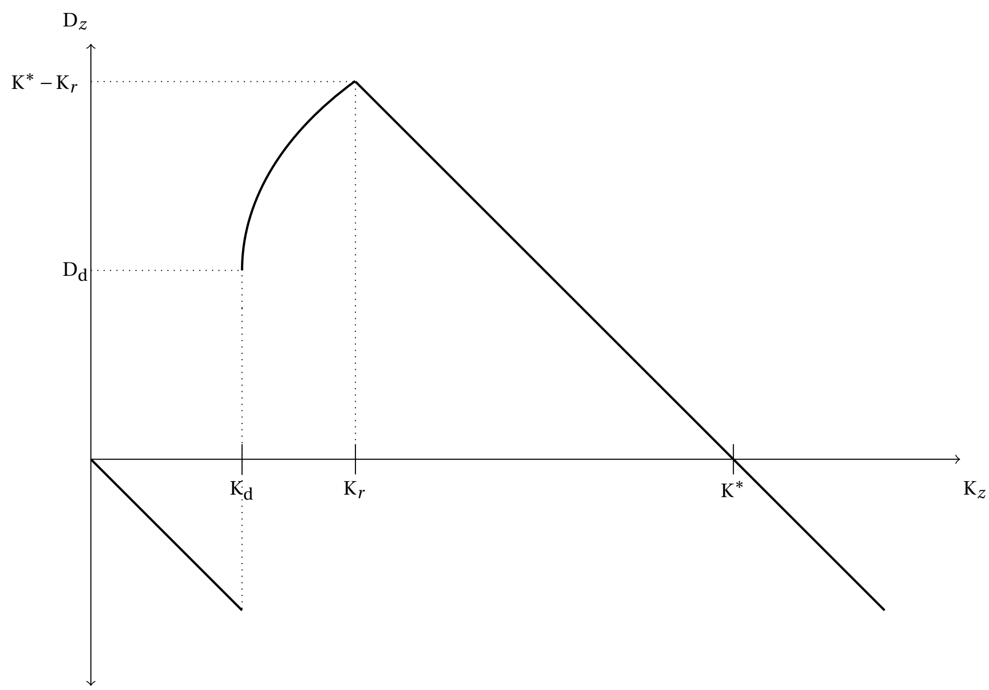
where I have used the fact that $\frac{1+r}{\phi} \geq 1$ (I assume that $\phi \in [0, 1]$ and $r \geq 0$). For the last item, note that maximum allowable loan is concave, because after differentiating (1.17) with respect K_z I obtain that: $\frac{\partial^2 D_z}{\partial K_z^2} = \frac{f''(K_z+D_z)(1+r+\phi)}{(f'(K_z+D_z)-(1+r+\phi))^2} < 0$.

□

Proposition 2 says that agents with higher capital levels are able to get higher loans from the credit market. Moreover, I find that the incentive compatibility constraint allows maximum allowable debt to increase more than 1 with K_z. In addition, from item 2, all agents are allowed to get a positive loan that is higher than the mobile capital they own. Since constrained entrepreneurs decide to ask for the maximum allowable debt, the leverage ratio is higher than 1 for this group. In contrast, because unconstrained entrepreneurs decide to ask for a loan that is lower than the maximum allowable debt or even zero for richer agents, their leverage ratio will decrease from K_r onwards and eventually may become lower than 1. Hence, the leverage ratio of unconstrained firms is more likely to be lower than the leverage ratio of constrained entrepreneurs. Then, I conclude that sub-capitalized firms (such that $K_z \in [K_d, K_r]$) will be the most heavily indebted ones.

The main properties of the maximum allowable debt curve and the optimal decisions of entrepreneurs allow to describe the effective debt curve for all capital levels. I illustrate these results in figure 1.5. First, proposition 1 shows that agents with $K_z < K_d$ will decide to not form a firm, losing their specific capital. As I showed in previous section, the best choice for them is to loan all their wealth, supplying the credit market. Therefore, as showed in figure 1.5, the effective debt curve in negative and linearly decreasing in $[0, K_d)$. In contrast, entrepreneurs such that $K_z \in [K_d, K_r]$ decide to get the maximum allowable loan, because the marginal cost of debt is lower than the marginal return of capital. In contrast, agents such that $K_z \in (K_r, K^*)$ decide to not get the maximum loan, because they can reach the optimal capital level asking for a lower loan $(K^* - K_z)$. Thus, as I show in figure 1.5, the effective loan is concave and increases more than 1 with K_z for $K_z \in [K_d, K_r]$ and is linearly decreasing thereafter. Finally, richer agents of the economy (such that $K_z > K^*$) decide to supply the credit market too, lending their surplus capital after forming an efficient firm. As showed in next figure, debt is negative and linearly decreasing for $K_z > K^*$.

Figure 1.5: Effective loan curve.



Note that negative values showed in figure 1.5 correspond to loaned capital.

1.5 Investment decision patterns and firm size

Notice that the effective loan curve completely defines investment decisions. In the following definition I describe investment decision patterns in terms of capital stock.

Definition 3. *For an entrepreneur with a capital stock* K_z*, the capital invested in a firm* $I(K_z)$ *and the amount loaned or borrowed* $D(K_z)$ *are such that:*

$$(I(K_z), D(K_z)) = \begin{cases} (0, -K_z) & K_z \in [0, K_d) \\ (K_z + D_z, D_z) & K_z \in [K_d, K_r] \\ (K^*, K^* - K_z) & K_z \in (K_r, K^*] \\ (K^*, -(K_z - K^*)) & K_z > K^* \end{cases}$$

where D_z is the maximum of the solutions of condition (1.14) which defines the maximum allowable debt. From definition 3, agents that cannot access to the credit market and entrepreneurs who own more than K^* have linear increasing investment decisions. This last type of agents invest the optimal capital level in a firm and loan the rest of their wealth. It has to be emphasized that the distinctive feature of this model is that it incorporates non-linear variable investment decisions, given by those firms which have to operate at an inefficient capital level due to partial access to credit, i.e. those such that $K_z \in [K_d, K_r)$. Thus, consistent with Ghosh et al. (2000), the model predicts that access to credit is especially restricted for poorer firms. The reason is that less capitalized entrepreneurs cannot commit to refrain from morally hazardous behavior as effectively as richer agents.

I call inefficient firms as Small and Medium Enterprises (SMEs), which due to sub-capitalization have limited access to the credit market. Consistent with studies of Blinder (1985), Bigsten et al. (1998) and Muriki (2008), the model predicts that due to credit limitations these types of enterprises have lower productivity. Therefore, these constrained firms will have higher marginal return of capital (consistent with Banerjee and Duflo, 2004). In contrast, the model predicts that larger firms do not restrict their production due to credit constraints. As Blinder (1985) emphasizes, the story of firms curtailing their activities for lack of credit rings true for the small business sector, but not for larger enterprises.

On the other hand, those firms that cannot access to the credit market, can be understood as Micro-Enterprises (MEs), which have severely restricted their activities due to high sub-capitalization. Those firms that can get a loan to operate at the optimal capital level can be understood as Larger Medium Enterprises (LMEs), which are closer to be a large enterprise. Finally, those firms which are over-capitalized (that produce efficiently and loan capital) can be seen as Large Enterprises (LEs). I must particularly emphasize that the main distinction of this model is that it allows to analyze the consequences of financial development on the performance of SMEs.

The effective debt curve directly determines the profit of each firm in terms of its capital stock.

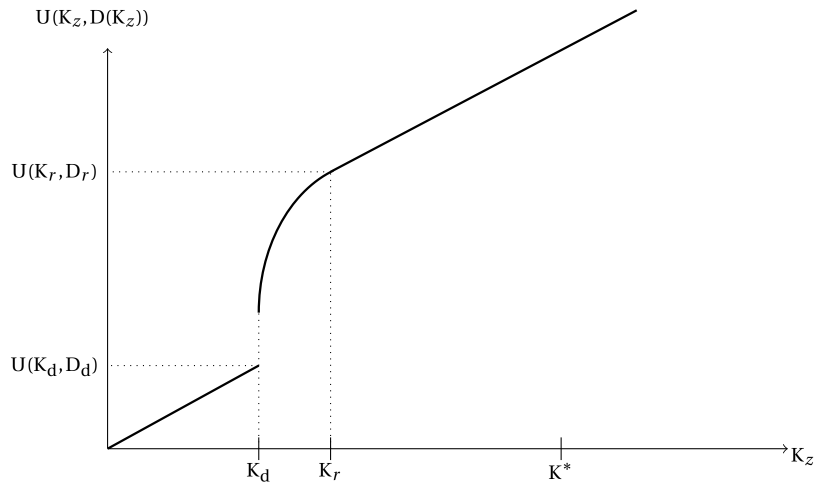
Definition 4. *For an entrepreneur with a capital level* K_z*, the profit function of her firm* $\pi(K_z, D(K_z))$ *is defined as follows:*

$$\pi(K_z, D(K_z)) = \begin{cases} 0 & K_z \in [0, K_d) \\ f(K_z + D_z) - (1+r)(K_z + D_z) - \theta & K_z \in [K_d, K_r] \\ f(K^*) - (1+r)K^* - \theta & K_z > K_r \end{cases}$$

Notice that definition 4 describes the own profit for operating a firm (net of $(1+r)K_z$, which corresponds to the amount received for loaning all mobile capital), i.e. the utility of the entrepreneur is defined by $\pi(K_z, D(K_z)) + (1+r)K_z$.

Definition 4 determines the shape of utility function (which is described in figure 1.6). First, micro-entrepreneurs, i.e. with capital levels such that $K_z < K_d$, will have increasing linear utilities. Moreover, from K_d^- to K_d^+ there is a discrete 'jump' in entrepreneur's utility (see figure 1.6), because at the left of K_d agents cannot access to the credit market and decide to loan their wealth at a marginal return of $1+r$, while to the right of K_d they are able to get a positive loan that allows them to produce at a marginal return higher than $1+r$. As the debt curve, the utility function of entrepreneurs with partial access to credit (with $K_z \in [K_d, K_r]$) will be increasing and concave. Finally, from K_r utilities will be increasing linear in capital stock.

Figure 1.6: Utility function.



1.6 Financial institutions structure

Another important issue is to define the structure of financial system of the closed economy. First, it can be assumed that there is not financial intermediation. In this case the exchange of capital occurs directly between two entrepreneurs, i.e. agents who need to get a loan directly receive the amount from the lender, and repay later the same amount adjusted by the agreed interest rate. In the aggregate, it will be observed the same equilibrium interest rate r (described in next section). This type of financing will be like a commercial credit between enterprises. On the other hand, one can assume that there exists an implicit competitive bank system, which intermediates costless transactions between borrowers and lenders. As in the model, in this case the interest rate charged to borrowers is the same as the interest rate for deposits, because transactions are costless and there is not bankruptcy risk, i.e. the bank system do not has to cover bankruptcy risk charging higher interest rate to borrowers. I choose this second option, because is easier to think that there exists a competitive banking sector which helps to coordinate the credit market. Note that I assume that banks do not charge to agents an extra intermediation cost.

It is important to note that there is no auction market for credit in this economy, i.e. no commercial paper market. Thus, firms wishing to borrow must borrow from banks. As Blinder (1985) explains, the role of this assumption is to give banks primacy in the credit market, although in a very stark way. This assumption can be justified arguing that it may adequately characterize the availability of credit to small firms whose banks have certain informational advantages over other lenders (see Blinder and Stiglitz, 1983), i.e. this assumption allows to focus the analysis in characterize how credit market imperfections affect smaller firms (like SMEs), but it certainly is not realistic for larger firms. In fact, as Berger and Udell (1995) suggest, large corporations typically obtain credit in the public debt markets, while small firms usually must depend on financial intermediaries, particularly commercial banks.

1.7 Equilibrium

The equilibrium in this economy occurs when supply equal the demand for credit. The supply for credit originates from entrepreneurs who decide to loan their wealth or alternatively to make a deposit in the competitive bank system, while the demand of credit originates from agents which demand a loan.

The supply for credit is defined by:

$$\mathscr{S}_c = \int_0^{\mathrm{K_d}} \mathrm{K}_z \,\partial\Gamma(\mathrm{K}_z) + \int_{\mathrm{K}^*}^{\infty} (\mathrm{K}_z - \mathrm{K}^*)\,\partial\Gamma(\mathrm{K}_z) \tag{1.22}$$

while the demand for credit is:

$$\mathscr{D}_c = \int_{\mathrm{K_d}}^{\mathrm{K}_r} \mathrm{D}_z \,\partial\Gamma(\mathrm{K}_z) + \int_{\mathrm{K}_r}^{\mathrm{K}^*} (\mathrm{K}^* - \mathrm{K}_z)\,\partial\Gamma(\mathrm{K}_z) \tag{1.23}$$

The equilibrium condition for credit market is given by $\mathscr{S}_c = \mathscr{D}_c$. The following proposition verifies the consistency of supply and credit market curves.

Proposition 3. *The supply for credit increases with r and the demand for credit decreases with r.*

Proof. Differentiating (1.22) with respect r:

$$\frac{\partial\mathscr{S}_c}{\partial r} = \mathrm{K_d}\gamma(\mathrm{K_d})\frac{\partial \mathrm{K_d}}{\partial r} - \mathrm{K}^*\gamma(\mathrm{K}^*)\frac{\partial \mathrm{K}^*}{\partial r} + \mathrm{K}^*\gamma(\mathrm{K}^*)\frac{\partial \mathrm{K}^*}{\partial r} - \frac{\partial \mathrm{K}^*}{\partial r}(1-\Gamma(\mathrm{K}^*))$$

$$\Rightarrow \frac{\partial\mathscr{S}_c}{\partial r} = \mathrm{K_d}\gamma(\mathrm{K_d})\frac{\partial \mathrm{K_d}}{\partial r} - \frac{\partial \mathrm{K}^*}{\partial r}(1-\Gamma(\mathrm{K}^*)) \tag{1.24}$$

For the first term, I use (1.14) to obtain the partial derivative of $\mathrm{K_d}$ with respect r:

$$\begin{aligned} f'(\mathrm{K_d}+\mathrm{D_d})(\frac{\partial \mathrm{K_d}}{\partial r} + \frac{\partial \mathrm{D_d}}{\partial r}) - \mathrm{D_d} - (1+r+\phi)\frac{\partial \mathrm{D_d}}{\partial r} = 0 \\ \Rightarrow \frac{\partial \mathrm{D_d}}{\partial r}(f'(\mathrm{K_d}+\mathrm{D_d}) - (1+r+\phi)) + f'(\mathrm{K_d}+\mathrm{D_d})\frac{\partial \mathrm{K_d}}{\partial r} = \mathrm{D_d} \\ \Rightarrow \frac{\partial \mathrm{K_d}}{\partial r} = \frac{\mathrm{D_d}}{f'(\mathrm{K_d}+\mathrm{D_d})} > 0 \end{aligned} \tag{1.25}$$

From (1.4) I obtain that $\frac{\partial \mathrm{K}^*}{\partial r} = \frac{1}{f''(\mathrm{K}^*)} < 0$. Thus, al terms of (1.24) are positive and credit supply increases with r.

Differentiation of (1.23) leads to:

$$\begin{aligned} \frac{\partial\mathscr{D}_c}{\partial r} = \int_{\mathrm{K_d}}^{\mathrm{K}_r} \frac{\partial \mathrm{D}_z}{\partial r}\partial\Gamma(\mathrm{K}_z) + \mathrm{D}_r\gamma(\mathrm{K}_r)\frac{\partial \mathrm{K}_r}{\partial r} - \mathrm{D_d}\gamma(\mathrm{K_d})\frac{\partial \mathrm{K_d}}{\partial r} + \frac{\partial \mathrm{K}^*}{\partial r}(\Gamma(\mathrm{K}^*) - \Gamma(\mathrm{K}_r)) \\ + \mathrm{K}^*(\gamma(\mathrm{K}^*)\frac{\partial \mathrm{K}^*}{\partial r} - \gamma(\mathrm{K}_r)\frac{\partial \mathrm{K}_r}{\partial r}) - \mathrm{K}^*\gamma(\mathrm{K}^*)\frac{\partial \mathrm{K}^*}{\partial r} + \mathrm{K}^r\gamma(\mathrm{K}_r)\frac{\partial \mathrm{K}_r}{\partial r} \end{aligned} \tag{1.26}$$

Rearranging terms of equation (1.26):

$$\begin{aligned} \frac{\partial\mathscr{D}_c}{\partial r} = \int_{\mathrm{K_d}}^{\mathrm{K}_r} \frac{\partial \mathrm{D}_z}{\partial r}\partial\Gamma(\mathrm{K}_z) + \frac{\partial \mathrm{K}_r}{\partial r}\gamma(\mathrm{K}_r)(\mathrm{D}_r + \mathrm{K}_r - \mathrm{K}^*) - \mathrm{D_d}\gamma(\mathrm{K_d})\frac{\partial \mathrm{K_d}}{\partial r} \\ + \frac{\partial \mathrm{K}^*}{\partial r}(\Gamma(\mathrm{K}^*) - \Gamma(\mathrm{K}_r)) \end{aligned} \tag{1.27}$$

For the first term I differentiate equation (1.14) with respect r:

$$f'(K_z + D_z)\frac{\partial D_z}{\partial r} - (1 + r + \phi)\frac{\partial D_z}{\partial r} - D_z = 0$$

$$\Rightarrow \frac{\partial D_z}{\partial r} = \frac{D_z}{f'(K_z + D_z) - (1 + r + \phi)} < 0 \tag{1.28}$$

From equations (1.28) and (1.25) the first and the third term of (1.27) are negative. The last term is also negative (because $\frac{\partial K^*}{\partial r}$ <0). For the second term, note that by definition $D_r + K_r = K^*$, so this term is zero. Thus, I conclude that the demand of credit is downwards sloping in r. □

I can also define the market equilibrium in terms of supply and demand for capital.

Definition 5. *The capital market equilibrium is defined by:*

$$E(K_z) = \int_{K_d}^{K_r} (K_z + D_z)\,\partial\Gamma(K_z) + K^*(1 - \Gamma(K_r)) \tag{1.29}$$

where the left-hand side of equation (1.29) is the supply of capital and the right-hand side is the demand for capital. Moreover, this capital market equilibrium condition can be directly derived from the credit market equilibrium (as showed in the following proposition).

Proposition 4. *Credit market equilibrium is equivalent to capital market equilibrium.*

Proof. From credit market equilibrium:

$$\mathscr{S}_c = \mathscr{D}_c$$

$$\int_0^{K_d} K_z\,\partial\Gamma(K_z) + \int_{K^*}^{\infty} (K_z - K^*)\,\partial\Gamma(K_z) = \int_{K_d}^{K_r} D_z\,\partial\Gamma(K_z) + \int_{K_r}^{K^*} (K^* - K_z)\,\partial\Gamma(K_z) \quad +/\int_{K_d}^{K_r} K_z\,\partial\Gamma(K_z)$$

$$\int_0^{K_d} K_z\,\partial\Gamma(K_z) + \int_{K^*}^{+\infty} K_z\,\partial\Gamma(K_z) + \int_{K_d}^{K_r} K_z\,\partial\Gamma(K_z) + \int_{K_r}^{K^*} K_z\,\partial\Gamma(K_z)$$

$$= \int_{K_d}^{K_r} (K_z + D_z)\,\partial\Gamma(K_z) + \int_{K_r}^{K^*} K^*\,\partial\Gamma(K_z) + \int_{K^*}^{+\infty} K^*\,\partial\Gamma(K_z)$$

$$\Leftrightarrow E(K_z) = \int_{K_d}^{K_r} (K_z + D_z)\,\partial\Gamma(K_z) + K^*(1 - \Gamma(K_r))$$

□

The capital market equilibrium condition shows that the supply of capital is given by total capital available in the economy, while the demand of capital equals to total invested capital. Note that the left-hand side of equation (1.29) depends only of the aggregate capital stock of initial wealth distribution $\Gamma(\cdot)$, i.e. the supply of capital is perfectly inelastic[4]. Therefore, I conclude that total investment is constant for a particular capital distribution, i.e. total investment of the economy does not vary with variations in ϕ and θ. However, I expect that investment decisions vary across different enterprise sizes with variations in parameters.

[4]As explained before, the requirement of a totally inelastic supply of capital is not required to obtain statics of equilibrium interest rate (which are presented in the next chapter). So long as the supply of capital is not totally elastic, those results continue to hold (see Balmaceda et al., 2014).

Proposition 5. *The demand of capital is downwards sloping in r*

Proof. The demand of capital is given by:

$$\mathscr{D} = \int_{K_d}^{K_r} (K_z + D_z)\,\partial\Gamma(K_z) + K^*(1 - \Gamma(K_r)) \tag{1.30}$$

Differentiation of equation (1.30) leads to:

$$\begin{aligned}\frac{\partial\mathscr{D}}{\partial r} = \int_{K_d}^{K_r} \frac{\partial D_z}{\partial r}\partial\Gamma(K_z) + (K_r + D_r)\gamma(K_r)\frac{\partial K_r}{\partial r} - (K_d + D_d)\gamma(K_d)\frac{\partial K_d}{\partial r} \\ + \frac{\partial K^*}{\partial r}(1 - \Gamma(K_r)) - K^*\gamma(K_r)\frac{\partial K_r}{\partial r}\end{aligned} \tag{1.31}$$

Note that the second and last term of right hand side cancel out (because $K_r + D_r = K^*$). From equations (1.28) and (1.25) the first and third terms are negative. For the remaining term, note that $\frac{\partial K^*}{\partial r} < 0$. Therefore, all terms are negative and the demand of capital is downwards sloping in r.

□

Note that proposition 5 implies that there exists an unique fixed point r such that condition (1.29) is satisfied [5].

In order to obtain reasonable results I assume that the variation in equilibrium interest rate is not too large after changes in *ex-ante* protection $1 - \phi$ or in fixed cost θ:

Assumption 1. $|\frac{\partial r}{\partial x}| < 1$ *with* $x = \phi, \theta$.

Hence, interest rate varies less than one when x varies one unit. This assumption will be useful to determinate the sign of statics over critical capital levels, maximum debt and firm profits in the closed economy[6]. In Section 3.1 I analyze the sign of this condition.

[5]The existence of an equilibrium can be deduced from proposition 9 of Chapter 3.

[6]I relax this assumption when I simulate the 'strongly capital constrained' economy (see Section 3.4)

Chapter 2

Simulations Setup

One of the main complications of the use of variable investment with non-linear effects in a closed economy is that I cannot derive comparative statics results for some macroeconomic variables, such as total GDP, total debt and credit penetration. The comparative statics for the case of different wealth distributions are particularly complex, in terms of the effect of wealth redistributions on: equilibrium interest rate, the critical capital levels and the different macroeconomic variables. The impact of wealth redistributions over *ex-post* wealth distribution is even more difficult to examine. In order to understand those effects I simulate the model, defining the production function of the firms, the initial wealth distribution, the degree of *ex-ante* protection and the level of fixed costs.

In this chapter I briefly describe the basic setup used for the simulations presented in the rest of the thesis. The details of the methodology used to find the equilibrium interest rate (a fixed point) and the method used to solve the non-linear equation that determinates the maximum allowable debt curve are described in the Appendix.

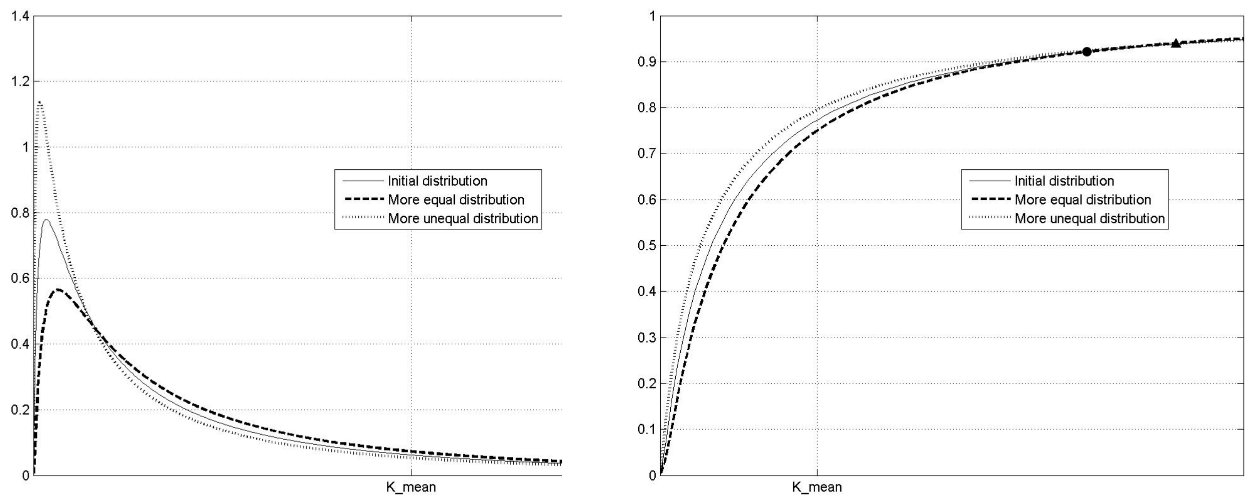
2.1 Wealth distribution setup

Pareto (1897) showed that in the range of high wealth, wealth and income are distributed according to a power-law distribution which is now known as Pareto distribution. Many studies have found an excellent fit to the empirical wealth distribution by using a Pareto distribution (see for e.g. Steindl, 1965; Atkinson and Harrison, 1978; Persky, 1992; Levy and Solomon, 1997). Many others functional forms have been fitted to empirical distributions of wealth/income and some have been proposed solely by their practical bearing upon the encountered empirical distributions: Amoroso (1925) and Salem and Mount (1974) fit the gamma distribution to income data, Dagum (1976) proposes the Dagum distribution to fit income data of four very dissimilar countries, Thurow (1970) and Kakwani and Podder (1976) propose the Beta distribution . In addition, theoretical models have been formulated based on stochastic processes to explain the origins of some of the distributions: Mandelbrot (1960), Levy et al. (2001) and Reed (2003) arrive at a Pareto distribution, Gibrat (1931) arrives at a Log-normal distribution, Champernowne (1953) arrives at the Champernowe distribution, Fisk (1961) arrives at the Fisk distribution or Log-logistic distribution, Rutherford (1955) solves the model to obtain a Gram-Charlier distribution and recently Bose and Banerjee (2005); Brown and Chiang (2012) arrive at a Beta distribution of wealth.

For the simulations in this thesis I use the Log-normal distribution, because it is useful to describe both the low and upper tail of the wealth distribution (unlike the Pareto distribution which has been used to describe the upper tail of wealth/income distribution). Additionally, the Log-normal distribution has been commonly used to represent income/wealth distributions and seems to fit data very well (see Aitchinson and J, 1957; Robinson, 1976). I also ran simulations using Pareto, Fisk, Beta and Dagum distributions, obtaining similar results of those presented in the following sections. I do not incorporate these results in the main text.

In order to isolate the effects of pure wealth redistribution on credit constraints and macroeconomic variables, I use Mean Preserving Spread (MPS) of the initial distribution of wealth. I set the initial distribution of wealth as $LogN(0.05, 1.5)$ which represents the base case for subsequent comparisons. I construct the MPS distributions by varying $\pm 10\%$ the sigma parameter (1.5) and keeping the mean capital of the economy constant (3.24). In the following figure I show the probability density function and cumulative density function for the three wealth distributions, and I use this format in the simulations showed in next chapters.

Figure 2.1: Wealth distributions. MPS curves.



(a) Probability density function

(b) Cumulative density function

As can be observed from figure 2.1a, the more unequal wealth distribution gives larger weights to agents who own lower amounts of capital, while the more equal distribution gives higher weights to richer entrepreneurs. Thus, as showed in figure 2.1b, there exists a threshold (K_z = 10.6, marked by ▲ in figure 2.1b) such that to the left the cumulative density function is higher for the more unequal MPS than for the initial wealth distribution. Similarly, there exists a second threshold (K_z = 8.8, marked by •) such that the more equal cumulative density function dominates the initial wealth distribution at the right. Notice that in order to run the simulations I do not define a particular value for these thresholds[1]. In general terms, a more unequal (equal) MPS of the initial wealth distribution will imply a wealth redistribution that leads to larger (lower) variance, but keeping the aggregate capital stock of the economy constant.

[1]In the case of a small open economy I can derive the effects of an MPS over some important macroeconomic variables. In order to simplify the analysis I set such threshold at $E(K_z)$ (see Chapter 4).

2.2 The production function

I define the production function as a Cobb-Douglas of AK type:

$$f(K) = AK^{\alpha}$$

where A is total factor productivity and α is the ouput elasticity of capital. In order to have decreasing returns to scale I use $\alpha \in (0,1)$, while $A > 0$. In the initial setting I use $\alpha = 0.7$ and $A = 2.5$. The utility of the entrepreneur is written now as: $U(K_z, D_z) = A(K_z + D_z)^{\alpha} - (1+r)D_z - \theta$. Notice that one of the advantages of using this production function is that it can be solved to obtain an explicit expression for the critical capital levels described in Section 1.2 (see the Appendix).

2.3 Parameters setup

In the base case I set $\theta = 1$, $A = 2.5$, $\alpha = 0.7$ and $\phi = 0.6$. Since the recovery rate $1-\phi$ and fixed cost θ are the most interesting parameters in the model, I simulate the equilibrium of the closed economy to changes in ϕ and θ.[2] For the *ex-ante* protection parameter I simulate the scenarios in which $\phi \in [0,1]$, while for θ I simulate $\pm 10\%$ perturbation as sensibility analysis. Thus, for each macroeconomic measure I will have three curves (one for each described MPS wealth distribution). In the abscissa axis I will have the interval for ϕ or θ, while in the ordinate axis I will have the simulated macroeconomic measure for each wealth distribution.

[2] I also simulated the economy for changes in A and α, but I do not present these results in the main text.

Chapter 3

Comparative statics in the closed economy

One main focus of this thesis is to analyze the comparative statics of financial and macroeconomic variables due to variations in parameters. In subsequent sections I examine the effect of variations of ex-ante protection $1-\phi$, fixed cost θ and some wealth distribution statics over equilibrium interest rate[1], critical capital levels and maximum allowable debt. In order to derive statics with regard to different wealth redistributions I use simulations. Moreover, in Section 3.4 I simulate the effects over some financial and macroeconomic measures, like GDP, total debt, credit penetration and ex-post distribution of wealth (which cannot be directly derived from the model).

3.1 Equilibrium interest rate statics

One important issue is to analyze how creditor protection and barriers to entry (measured by fixed cost) affect the equilibrium interest rate in a closed economy. The following proposition characterizes these effects and show some restrictions that wealth distribution must satisfy to assure that there exists a real effect over r.

Proposition 6. *If wealth distribution is such that* $\Gamma(K_r) > 0$*, the equilibrium interest rate* r *increases with improvements of ex-ante creditor protection* $1-\phi$ *or when barriers to entry decrease (i.e. when fixed cost* θ *decreases). Otherwise, the interest rate remains constant.*

Proof. In order to simplify calculations I define $x = \phi, \theta$. Assuming that $\Gamma(K_r) > 0$ and differentiating condition (1.29) with respect to x:

$$\int_{K_d}^{K_r} \left(\frac{\partial D_z}{\partial x} + \frac{\partial D_z}{\partial r}\frac{\partial r}{\partial x}\right)\partial\Gamma(K_z) + K^*\gamma(K_r)\left(\frac{\partial K_r}{\partial x} + \frac{\partial K_r}{\partial r}\frac{\partial r}{\partial x}\right) - (K_d + D_d)\gamma(K_d)\left(\frac{\partial K_d}{\partial x} + \frac{\partial K_d}{\partial r}\frac{\partial r}{\partial x}\right)$$
$$+ \frac{\partial K^*}{\partial r}\frac{\partial r}{\partial x}(1-\Gamma(K_r)) - K^*\gamma(K_r)\left(\frac{\partial K_r}{\partial x} + \frac{\partial K_r}{\partial r}\frac{\partial r}{\partial x}\right) = 0$$

$$\Rightarrow \int_{K_d}^{K_r} \left(\frac{\partial D_z}{\partial x} + \frac{\partial D_z}{\partial r}\frac{\partial r}{\partial x}\right)\partial\Gamma(K_z) - (K_d + D_d)\gamma(K_d)\left(\frac{\partial K_d}{\partial x} + \frac{\partial K_d}{\partial r}\frac{\partial r}{\partial x}\right) + \frac{\partial K^*}{\partial r}\frac{\partial r}{\partial x}(1-\Gamma(K_r)) = 0 \quad (3.1)$$

[1] As I will show in the following section, the analysis that can be done with regard to different wealth distributions is very limited.

In what follows I set $x = \phi$. For the first term of equation (3.1) I differentiate condition (1.14) with respect ϕ:

$$f'(K_z + D_z)\frac{\partial D_z}{\partial \phi} - D_z - (1 + r + \phi)\frac{\partial D_z}{\partial \phi} = 0$$

$$\Rightarrow \frac{\partial D_z}{\partial \phi} = \frac{D_z}{f'(K_z + D_z) - (1 + r + \phi)} < 0 \tag{3.2}$$

For the following term I use equation (1.14) and replace the pair (K_d, D_d) to obtain:

$$f'(K_d + D_d)(\frac{\partial K_d}{\partial \phi} + \frac{\partial D_d}{\partial \phi}) - D_d - (1 + r + \phi)\frac{\partial D_d}{\partial \phi} = 0$$

$$\frac{\partial D_d}{\partial \phi}(f'(K_d + D_d) - (1 + r + \phi)) + f'(K_d + D_d)\frac{\partial K_d}{\partial \phi} = D_d \tag{3.3}$$

$$\Rightarrow \frac{\partial K_d}{\partial \phi} = \frac{D_d}{f'(K_d + D_d)} > 0$$

Now, suppose that $\frac{\partial r}{\partial \phi} > 0$, then from (1.28) and (3.2) the first term of (3.1) will be negative. From (1.25) and (3.3) the second term will be also negative. Thus, because $\frac{\partial K^*}{\partial r} < 0$, all terms will be negative and condition (3.1) will be violated. However, if $\frac{\partial r}{\partial \phi} < 0$ I will have terms with opposite sings. Therefore, the only way to recover the equilibrium is that interest rate falls when ϕ increases.

In what follows I set $x = \theta$. Differentiation of (1.14) in terms of θ leads to:

$$\frac{\partial D_z}{\partial \theta} = \frac{1}{f'(K_z + D_z) - (1 + r + \phi)} < 0 \tag{3.4}$$

Replacing the pair (K_d, D_d) in equation (1.14) and differentiating:

$$f'(K_d + D_d)(\frac{\partial K_d}{\partial \theta} + \frac{\partial D_d}{\partial \theta}) - (1 + r + \phi)\frac{\partial D_d}{\partial \theta} = 1$$

$$\Rightarrow \frac{\partial K_d}{\partial \theta} = \frac{1}{(f'(K_d + D_d) - (1 + r + \phi))\frac{\partial D_d}{\partial \theta} + f'(K_d + D_d)} = \frac{1}{f'(K_d + D_d)} > 0 \tag{3.5}$$

Using the same argument that I used for $x = \phi$, I conclude that if $\frac{\partial r}{\partial \theta} > 0$ then condition (3.1) is violated. Therefore, the only way to recover the equilibrium is that interest rate decreases when fixed cost θ increases.

Now, note that if $\Gamma(K_r) = 0$ then condition (3.1) is:

$$\frac{\partial K^*}{\partial r}\frac{\partial r}{\partial x} = 0 \tag{3.6}$$

Note that in this case the only way to recover the equilibrium is that $\frac{\partial r}{\partial x} = 0$, because $\frac{\partial K^*}{\partial r} < 0$. □

Consistent with findings of Balmaceda and Fischer (2010), and with the results of Shleifer and Wolfenzon (2002) for investor protection, an improvement of *ex-ante* creditor protection leads to higher interest rate in a closed economy, because previously excluded agents compete for access to loans and lenders can assure a higher loan to those who already had access to credit market (see proposition 16 of section 3.3). As I will show in next section, the minimum capital level to receive a loan decreases with improvements in recovery rate. Therefore, some entrepreneurs who were previously excluded now will have access to the credit market and agents who have access to credit (such that $K_z \geq K_d$) will be able to get bigger loans, leading to an increase in the demand for credit. As the demand for capital increases and the supply of capital remains constant, the interest rate increases. Thus, after an increase in the recovery rate, the optimal capital stock decreases, leading to more firms that are able to reach the efficient production without accessing to the bank system. Moreover, as I will show in Section 3.2, the critical capital level which allows to reach an optimal loan decreases, therefore the number of firms that operate efficiently increases(i.e. $1-\Gamma(K_r)$ increases).

On the other hand, an increase in fixed costs leads to higher barriers to entry, i.e. there exist less firms that are able to enter to the output market. Since now abscond is more profitable than forming a firm for some sub-capitalized entrepreneurs who were previously able to access to the credit market, banks will impose tighter restrictions to get a loan, by increasing the minimum capital to get a loan and by reducing the maximum allowable debt (see Section 3.3). Furthermore, as I will show in Section 3.2, under certain conditions the minimum capital stock to get an optimal loan will also increase. Therefore, an increase in fixed costs will lead to a decrease in the demand for capital, and thus equilibrium interest rate will decrease.

Proposition 6 says that if the country does not have SMEs (i.e if $\Gamma(K_r) = 0$), then interest rate remains unchanged after an improvement in *ex-ante* protection or a reduction in θ. I interpret such country as a wealthy one, with a large amount of capital stock. In this economy all agents will produce efficiently and there will not exist constrained entrepreneurs, since the poorest agent of the economy can get an optimal loan. Note that because $\Gamma(K_r) = 0$, and K_d and K_r decrease when ϕ or θ decreases, the demand for credit will remain constant whatever the increase in loan recovery rate $1-\phi$ and whatever the reduction in fixed cost θ, i.e. the equilibrium interest rate will not change after improvements in the strength of creditor rights (or after reductions in barriers to entry). This is consistent with the observation of Shleifer and Wolfenzon (2002), who proposes that higher levels of wealth and capital may exert downward the pressure over interest rates, i.e. the conclusion that higher *ex-ante* protection implies higher interest rate might not hold for richer countries. Nevertheless, observe that if *ex-ante* protection gets too worse or if barriers to entry increase to much, it may occur that $\Gamma(K_r) > 0$, i.e. in that case the economy may be affected by increases in ϕ or θ, despite the fact that it initially satisfied that $\Gamma(K_r) = 0$.

In addition, proposition 6 sheds some light for the observation of La La Porta et al. (2000b) that better creditor protection is opposed by wealthy and politically powerful families in developing countries, because financial development increases interest rates through higher competition (i.e. it raises financing costs to incumbent firms). Furthermore, this result is consistent with the theory of an interest group of financial development proposed by Rajan and Zingales (2003), where incumbents oppose financial development because it breeds competition. In Section 3.3 I present additional evidence to support both observations.

One question that remains is how wealth redistributions will affect the equilibrium interest rate in the closed economy. The implication of the model with regard to different wealth distributions may depend whether a country is 'capital constrained' or not. In the following definition I present four types of economies from the least capital constrained to the most constrained one:

Definition 6.

1. *Any closed economy such that* $E(K_z) = K^*$ *is said to be 'strongly capital unconstrained'.*

2. *Any closed economy such that* $E(K_z) \in [K_r, K^*)$ *is said to be 'capital unconstrained'.*

3. *If* $E(K_z) \in [K_d, K_r)$ *then the economy is said to be 'weakly capital constrained'.*

4. *If* $E(K_z) < K_d$ *then the economy is 'strongly capital constrained'.*

As in Balmaceda and Fischer (2010) I interpret a capital constrained (unconstrained) economy as either a poorer (richer) economy or one that is less (more) financially developed. The first case of definition 6 is the most capital unconstrained closed economy [2], since the average entrepreneur is able to form an optimal firm just investing her own wealth. The second one is also a capital unconstrained country in the sense that the average agent is able to get a loan to produce efficiently. The third one is capital constrained in the sense that the average entrepreneur is able to receive a loan that allows her to produce, but at an inefficient level. The last case is the most restricted economy, because the average agent do not have access to the credit market and may decide to close her firm.

The following propositions describe some issues with regard to capital constrained and capital unconstrained economies.

Proposition 7. *Any wealthy economy (such that* $\Gamma(K_r) = 0$*) is strongly capital unconstrained, i.e.* $K^* = E(K_z)$.

Proof. From the equilibrium condition:

$$E(K_z) = \int_{K_d}^{K_r} (K_z + D_z)\,\partial\Gamma(K_z) + K^*(1 - \Gamma(K_r))$$

Using the fact that $\Gamma(K_r) = 0$ I conclude that:

$$E(K_z) = K^*$$

□

Proposition 8. *Any economy such that* $\Gamma(K_r) > 0$ *satisfies that* $E(K_z) < K^*$*. If there exists an economy such that* $\Gamma(K_r) > 0$ *and the aggregate capital stock is the same as the economy described in the previous proposition, then the equilibrium interest rate* r *is lower and the optimal capital level* K^* *is higher than in the case where* $\Gamma(K_r) = 0$.

Proof. Using the equilibrium condition:

$$\begin{aligned} E(K_z) = \int_{K_d}^{K_r} (K_z + D_z)\,\partial\Gamma(K_z) + K^*(1 - \Gamma(K_r)) &< \int_{K_d}^{K_r} K^*\,\partial\Gamma(K_z) + K^*(1 - \Gamma(K_r)) \\ &= K^*(1 - \Gamma(K_d)) < K^* \end{aligned}$$

For the last part, note that because $E(K_z) < K^*$ and $f''(\cdot) < 0$, then $f'(K^*) < f'(E(K_z))$. Using the result of proposition 7 and condition (1.4), I conclude that the interest rate is lower when $\Gamma(K_r) > 0$. Therefore, because $\frac{\partial K^*}{\partial r} < 0$, the optimal capital level is higher. □

Proposition 9. *A capital unconstrained closed economy such that* $E(K_z) > K^*$ *is infeasible.*

[2] One could think that a capital unconstrained economy would be such that $E(K_z) > K^*$, but as I show in proposition 9 this condition is infeasible in a closed economy

Proof. Assuming that an unconstrained closed economy (such that $E(K_z) > K^*$) is feasible and using the equilibrium condition:

$$
\begin{gathered}
E(K_z) > K^* \\
\Leftrightarrow \int_{K_d}^{K_r} (K_z + D_z)\,\partial\Gamma(K_z) + K^*(1-\Gamma(K_r)) > K^* \\
\Rightarrow \int_{K_d}^{K_r} K^*\,\partial\Gamma(K_z) + K^*(1-\Gamma(K_r)) > \int_{K_d}^{K_r} (K_z + D_z)\,\partial\Gamma(K_z) + K^*(1-\Gamma(K_r)) > K^*
\end{gathered}
$$

From this condition I conclude that if $E(K_z) > K^*$ then I will have that $K^* > K^*$, which is a contradiction. Therefore, an unconstrained closed economy such that is $E(K_z) > K^*$ infeasible in this model. □

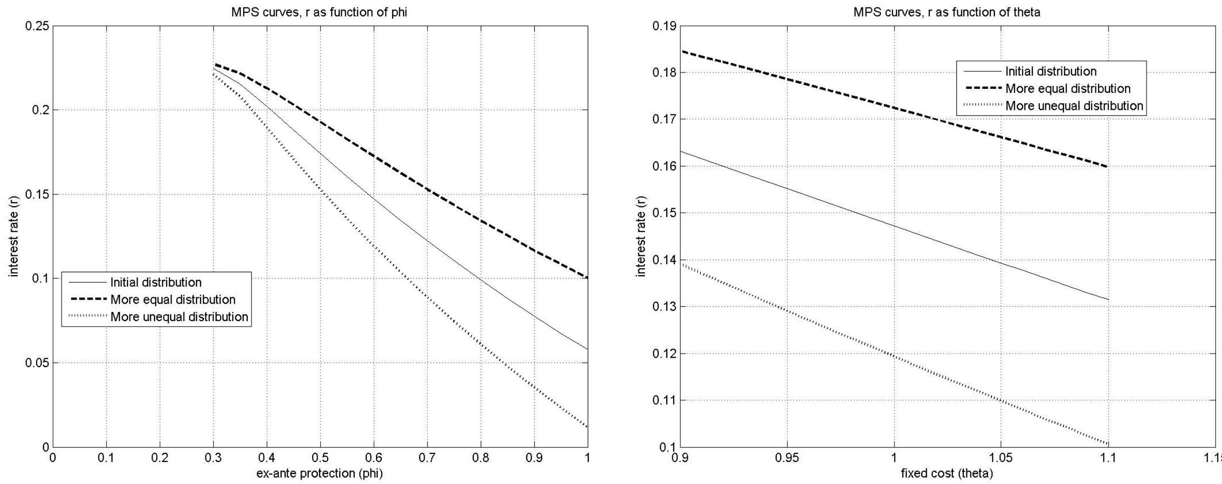
Proposition 7 says that a wealthy economy is always strongly capital unconstrained, while proposition 8 says that a poorer economy such that there exists sub-capitalized firms may be capital constrained. Moreover, if there exists an economy such that $\Gamma(K_r) > 0$ and such that its mean capital is the same amount as the aggregate capital of a wealthy economy (such that $\Gamma(K_r) = 0$)[3], then the interest rate in the economy with SMEs is lower, and its optimal capital stock is higher. Notice that propositions 7 and 8 shed some light on the effects of wealth redistributions over the equilibrium interest rate. One can expect that a more unequal wealth distribution (such that $\Gamma(K_r) > 0$) leads to lower interest than a more equal wealth distribution (such that $\Gamma(K_r) = 0$), consistent with the results showed in figure 3.1. However, to transform a wealthy economy in a poorer economy such that $\Gamma(K_r) > 0$, it may be necessary to change the parameters ϕ and θ. Thus, I cannot analyze the effect of pure wealth redistributions over the interest rate.

Another important issue, described in proposition 9, is that I cannot construct an unconstrained closed economy such that $K^* > E(K_z)$, because the optimal capital level is endogenously adjusted in order to satisfy the equilibrium condition.

Due to the complications explained in Section 2.1, I simulate the closed economy for the different wealth distributions presented in the previous section, and obtain the equilibrium interest rate for each scenario.

[3]Intuitively, is unlikely to find such economy just by wealth redistributions. However, by changing ϕ or θ one may eventually find an economy such that $\Gamma(K_r) > 0$ and with the same aggregate capital stock of the wealthy economy.

Figure 3.1: Interest rate statics with different MPS. Capital unconstrained economy ($E(K_z) \in [K_r, K^*)$)



(a) Variation in r with ϕ

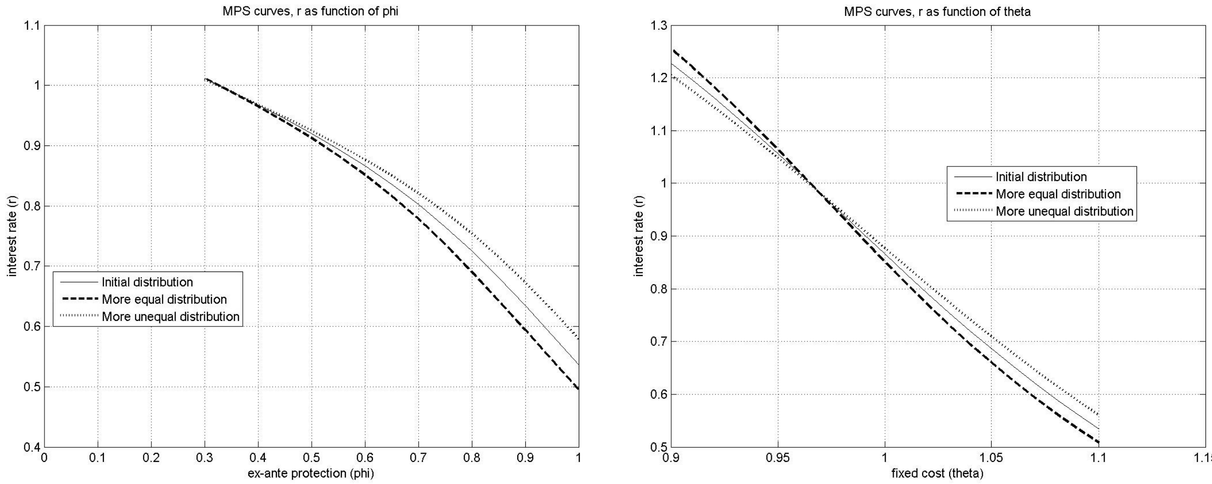
(b) Variation in r with θ

From figures 3.1a and 3.1b, I conclude that higher inequality (equality) leads to lower (higher) interest rate. This may be explained by the fact that the simulations presented in previous figures consider an economy which is capital unconstrained (such that $E(K_z) \in [K_r, K^*]$). Therefore, a more equal wealth redistribution may lead to an increase in the number of agents who access to the credit market, and thus one can expect that the demand for capital is higher in such countries, i.e. the equilibrium interest rate is higher.

Now, in order to determinate how wealth redistributions may affect the interest rate in a strongly capital constrained economy, I simulate an economy such that $K_d > E(K_z)$ [4].

[4] Note that I do not know *ex-ante* whether an economy is capital constrained or not. To find such economy I solve the equilibrium condition for different configurations of the parameters of wealth distribution until the condition of a capital constrained country is satisfied (see the Appendix for further detail) .

Figure 3.2: Interest rate statics with different MPS. Capital constrained economy ($K_d > E(K_z)$)



(a) Variation in r with ϕ

(b) Variation in r with θ

Unlike the case of a capital unconstrained economy, in a capital constrained economy the interest rate seems to be higher in the more unequally distributed country. In contrast with the previous case, I expect that a more concentrated wealth distribution will lead to a higher proportion of agents that have access to the credit market. Therefore, the demand for capital may be higher in a poorer capital constrained country with a worse wealth distribution, leading to a higher interest rate. Notice that from figure 3.2b, one can conclude that there exists a threshold (≈ 0.97) such that interest rate becomes higher in a more unequal wealth distributed country[5] , i.e. the conclusion that a capital constrained economy will have higher interest rate when there exists a worse wealth distribution may not always hold (see the points at the left of such threshold in figure 3.2b) . However, as the economy becomes more capital constrained (as ϕ or θ increases), one can expect that more unequal distributions lead to higher interest rates than more equal distributions. From figures 3.1a and 3.2b, I conclude that as ϕ increases the difference in the interest rate between the more unequal and more equal economy becomes higher.

It is important to emphasize that this last economy is in fact poorer than the economy used in the base case. The aggregate capital stock of the constrained economy is about 0.26, much lower than the mean capital of the simulated unconstrained economy (3.25).

I also simulated a wealthy economy ($\Gamma(K_r) = 0$) for different wealth redistributions. I found that interest rate remains unchanged either under a more equal or a more unequal wealth distribution. I conclude that a wealthy economy cannot be transformed in an economy such that $\Gamma(K_r) > 0$ just by wealth redistributions (that is why interest rate does not change after wealth redistributions).

In conclusion, theoretical results predict that interest rate varies after changes in the recovery rate $1-\phi$ in countries with credit constrained firms (such that $\Gamma(K_r) > 0$). The model recovers the result obtained by Balmaceda and Fischer (2010) that interest rate increases in a closed economy

[5]Notice that if I had simulated a weakly capital constrained economy, then I would may not have found such threshold. This is why I present the results for a strongly capital constrained economy.

when *ex-ante* protection gets better. It is also consistent with the observation of Shleifer and Wolfenzon (2002) that this result may not hold for a wealthy country (such $\Gamma(K_r) = 0$), which is interpreted in the model as an economy with too much capital stock, such that all firms are able to produce efficiently. The model also suggests novel results on the effect of barriers to entry over equilibrium interest rate in a closed economy. I find that a reduction in the recovery rate (or in fixed costs) encourages banks to ease credit constraints by lowering the minimum wealth to receive a loan and by increasing the allowable amount of debt that any firm can get. The demand of capital will increase after a decline in ϕ (or in θ), and thus the equilibrium interest rate will also increase. In a wealthy economy, an increase in the recovery rate or a reduction in barriers to entry will not have any impact on the equilibrium interest rate.

From simulations I conclude that countries with worse wealth distributions are more likely to have higher interest rates if they are more capital constrained (i.e. if *ex-ante* creditor protection is worse or fixed cost is higher). In contrast, more equal wealth distributions may lead to higher interest rates in capital unconstrained countries. Similarly as in Balmaceda and Fischer (2010), the implication of the model with regard to different wealth distributions across countries depends on the severity of capital constrains.

3.2 Critical capital levels statics

In this Section I examine critical capital variations (K_d, K_r, K^*) due to improvements in ex-ante creditor protection or declines in barriers to entry.

First, I examine the effect of variations in $1-\phi$ over all critical capital stocks of the economy. The assumption 1 allows to have reasonable results, because it assures that the direct effect of an improvement in the recovery rate over K_d exceeds its indirect effect[6].

Proposition 10. *If* $\Gamma(K_r) > 0$*, then the optimal capital stock* K^* *decreases with improvements in ex-ante protection* $1-\phi$*. Otherwise,* K^* *remains constant.*

Proof. Differentiation of condition (1.4) leads to:

$$f''(K^*)\frac{\partial K^*}{\partial \phi} = \frac{\partial r}{\partial \phi} \Rightarrow \frac{\partial K^*}{\partial \phi} = \frac{\frac{\partial r}{\partial \phi}}{f''(K^*)} \tag{3.7}$$

Note that if $\Gamma(K_r) > 0$, then from proposition 6 $\frac{\partial r}{\partial \phi} < 0$, i.e. from equation (3.7) and because $f''(\cdot) < 0$ I conclude that $\frac{\partial K^*}{\partial \phi} > 0$. Else, if $\Gamma(K_r) = 0$, then $\frac{\partial r}{\partial \phi} = 0$, i.e. ϕ does not impact K^*. □

Proposition 11. *The minimum capital level* K_d *and the capital stock required for an optimal loan* K_r *decrease with improvements in the loan recovery rate* $1-\phi$*.*

Proof. For the minimum capital level I use (1.25) and (3.3) to obtain:

$$\frac{\partial K_d}{\partial \phi} = \frac{(1+\frac{\partial r}{\partial \phi})D_d}{f'(K_d + D_d)} > 0 \tag{3.8}$$

[6] The indirect effect on K_d occurs due to the fall in the equilibrium interest rate (see equations (3.3) and (1.25) to verify the direct and indirect effect respectively).

where I have used the fact that $|\frac{\partial r}{\partial \phi}| < 1$, i.e. $(1+\frac{\partial r}{\partial \phi}) > 0$. Differentiating condition (1.12) with respect ϕ:

$$\underbrace{\frac{\partial \pi(K^*)}{\partial K^*}}_{=0}\frac{\partial K^*}{\partial \phi} + \underbrace{\frac{\partial \pi(K^*)}{\partial r}\frac{\partial r}{\partial \phi}}_{=-\frac{\partial r}{\partial \phi}K^*} + \frac{\partial r}{\partial \phi}K_r + (1+r)\frac{\partial K_r}{\partial \phi} = (K^* - K_r) + \phi(\frac{\partial K^*}{\partial \phi} - \frac{\partial K_r}{\partial \phi})$$

$$(1+r+\phi)\frac{\partial K_r}{\partial \phi} = (K^* - K_r) + \frac{\partial r}{\partial \phi}(\frac{\phi}{f''(K^*)} + K^* - K_r)$$

$$\Rightarrow \frac{\partial K_r}{\partial \phi} = \frac{(1+\frac{\partial r}{\partial \phi})D_r + \frac{\partial r}{\partial \phi}(\frac{\phi}{f''(K^*)})}{1+r+\phi} > 0 \tag{3.9}$$

where I have used the fact that condition (3.9) is positive independently if $\Gamma(K_r) > 0$ or $\Gamma(K_r) = 0$, i.e. independently if $\frac{\partial r}{\partial \phi} < 0$ or $\frac{\partial r}{\partial \phi} = 0$. □

Proposition 12. *The distance between* K_r *and* K_d *decreases with improvements of ex-ante protection.*

Proof. Using equations (3.8) and (3.9) I compute the variation in the distance between K_r and K_d in terms of ϕ:

$$\begin{aligned}\frac{\partial(K_r - K_d)}{\partial \phi} = \frac{\partial K_r}{\partial \phi} - \frac{\partial K_d}{\partial \phi} &= \frac{(1+\frac{\partial r}{\partial \phi})D_r + \frac{\partial r}{\partial \phi}(\frac{\phi}{f''(K^*)})}{1+r+\phi} - \frac{(1+\frac{\partial r}{\partial \phi})D_d}{f'(K_d + D_d)} \\ &= \frac{(D_r - D_d)(1+\frac{\partial r}{\partial \phi}) + \frac{\partial r}{\partial \phi}(\frac{\phi}{f''(K^*)})}{1+r+\phi}\end{aligned} \tag{3.10}$$

where I have used the fact that $f'(K_d + D_d) = (1+r+\phi)$. Note that the first term of the numerator of equation (3.10) is positive, because $D_r > D_d$ and $1+\frac{\partial r}{\partial \phi} > 0$. For the second term, note that $f''(\cdot) < 0$ and $\frac{\partial r}{\partial \phi} < 0$ if $\Gamma(K_r) > 0$, thus this term is positive too. Moreover, note that from proposition 6 condition (3.10) is still positive if $\Gamma(K_r) = 0$. Therefore, I conclude that $\frac{\partial(K_r - K_d)}{\partial \phi} > 0$. □

Consistent with Balmaceda and Fischer (2010), proposition 11 says that as *ex-ante* credit protection improves, the minimum wealth level K_d to get a loan falls, since opening a firm becomes more attractive than absconding for some previous constrained agents. Moreover, because the agency costs of fraudulent behavior are lower when creditor protection improves, lenders are willing to trust in entrepreneurs with lower capital stocks at a higher interest rate (see proposition 6). This result is also consistent with the assumption used by Aghion et al. (2004) in their model: that firms experience tighter (loose) constrains at lower(higher) financial development, i.e. at lower $1-\phi$.

In addition, proposition 11 shows that improvements in the recovery rate leads to less restrictions to attain the optimal production level, because the capital stock required to get an optimal loan decreases. Furthermore, proposition 10 says that the optimal capital level is lower for higher values of creditor protection $1-\phi$ (if there exists SMEs), i.e. is easier for smaller firms to receive a loan that allows them to produce efficiently, and the capital needed to reach this production level is also lower. Consequently, consistent with results of Beck et al. (2005), a marginal development in the financial and legal system helps to relax the financial constraints on SMEs, reducing the capital that allows to start a firm without losses and easing restrictions that allow to be a larger medium firm.

Additionally, proposition 12 implies that an improvement in *ex-ante* creditor protection will lead to a decline in the range where inefficient, sub-capitalized firms operate. This result also implies that the reduction in K_r will be higher than the reduction in K_d after an increase in the recovery rate. Moreover, because maximum allowable debt increases (see next section) and optimal capital level decreases when the recovery rate increases, sub-capitalized firms will be less inefficient than in the previous state, i.e. the difference of capital needed to attain the optimal production level will be lower. Therefore, financial development mainly benefits SMEs or sub-capitalized firms (I will present additional support to this observation in Section 3.3).

Note that proposition 12 also implies that as the loan recovery rate $1-\phi$ increases, K_d and K_r are increasingly closer, i.e. there exists a point at which these critical capital levels collapse. This point is reached when there are not credit market imperfections due to moral hazard, i.e. when *ex-ante* protection $1-\phi$ is exactly 1. In fact, if I replace $\phi = 0$ in equation (1.9), I will obtain that $f'(K_d + D_d) = 1 + r$. Thus, from condition (1.4) that defines K^*, the debt corresponding to K_d will allow to attain the optimal capital level, i.e. by definition K_d will coincide with K_r [7].

Another important issue is that in countries that do not have SMEs, i.e. countries such that $\Gamma(K_r) = 0$, the effect of an improvement in loan recovery rate is null. Thus, for wealthy countries, it is not worth making an effort to improve creditor legal protection, because the implied fall in K_d and K_r will not have any positive effect in the performance of current firms. As I showed in proposition 6, the interest rate will remain constant, and therefore K^* will also do so. However, if *ex-ante* protection gets worse, there will be a point in which $\Gamma(K_r) > 0$. In that case, some firms that were operating at an efficient level, now may have to produce at an inefficient level. Furthermore, if the decline in the recovery rate is too high, then it may occur that $\Gamma(K_d) > 0$, leading to a higher lost of efficiency due to some agents that will close their firms, losing their specific capital. Hence, rich countries should not concern about improving their legal *ex-ante* creditor protection, but they should care if it gets too worse.

Finally, I can summarize six sources of inefficiency in a country with a poorer *ex-ante* creditor protection and where there exists sub-capitalized firms (SMEs):

1. There are more agents that cannot operate a firm. Therefore, total specific capital lost is higher.

2. Firms having partial access to credit market are allowed to get a lower loan. Thus, restrictions to credit constrained firms are higher.

3. SMEs are more inefficient in relative and absolute terms (see proposition 21).

4. The range in which can operate an inefficient firm is higher.

[7] In fact, when $\phi = 0$ the legal system is able to recover all capital loaned and credit market imperfections disappear. Therefore, the bank system is willing to lend K^* to entrepreneurs without own capital, i.e. $K_d = K_r = 0$ and $D_d = K^*$.

5. There are more restrictions to get a loan that allows to attain the optimal capital stock. Thus, the number of firms that operate efficiently is lower.

6. Unconstrained firms need to invest too much capital.

Another important issue is to study the effects of barriers to entry over critical capital levels. Assumption 1 implies that $|\frac{\partial r}{\partial \theta}| < 1$. I impose this condition to try to assure that the direct effect of an increase in θ over K_d is higher than the indirect effect due to a reduction in the equilibrium interest rate. However, as I show below this condition may not be enough to assure an unambiguous effect of θ over K_d.

Proposition 13. *If* $\Gamma(K_r) = 0$ *or* $D_d \leq 1$*, the minimum capital level* K_d *to get a loan and the critical capital level* K_r *increase when barriers to entry increases. Otherwise, the effect of* θ *over* K_d *and* K_r *is ambiguous.*

Proof. From equation (1.8), differentiation of K_d leads to:

$$f'(K_d + D_d)(\frac{\partial K_d}{\partial \theta} + \frac{\partial D_d}{\partial K_d}\frac{\partial K_d}{\partial \theta}) - \frac{\partial r}{\partial \theta}D_d - (1 + r + \phi)\frac{\partial D_d}{\partial K_d}\frac{\partial K_d}{\partial \theta} = \frac{\partial r}{\partial \theta}$$
$$\Rightarrow \frac{\partial K_d}{\partial \theta} = \frac{1 + \frac{\partial r}{\partial \theta}D_d}{f'(K_d + D_d)} \tag{3.11}$$

Thus, because $1 + \frac{\partial r}{\partial \theta} > 0$, from (3.11) I conclude that K_d increases with fixed cost θ if $D_d \leq 1$. Otherwise, note that the sign of the numerator of condition (3.11) is ambiguous.

Differentiating condition (1.12) I obtain that:

$$\frac{\partial K_r}{\partial \theta} = \frac{1 + \frac{\partial r}{\partial \theta}(D_r + \frac{\phi}{f''(K^*)})}{1 + r + \phi} = \frac{(1 + \frac{\partial r}{\partial \theta}D_r) + \frac{\partial r}{\partial \theta}\frac{\phi}{f''(K^*)}}{1 + r + \phi} \tag{3.12}$$

From proposition 6, $\frac{\partial K_r}{\partial \theta} > 0$ if $\Gamma(K_r) = 0$. Additionally, assumption 1 implies that $1 + \frac{\partial r}{\partial \theta} > 0$. Therefore, if $D_r \leq 1$ the first term of the numerator of equation (3.12) will be positive. Thus, because $f''(\cdot) < 0$ and $\frac{\partial r}{\partial \theta} < 0$, condition (3.12) will be positive. However, if $\Gamma(K_r) > 0$ and $D_r > 1$ the effect of θ over K_r will be ambiguous. □

Proposition 14. *If* $\Gamma(K_r) > 0$*, then an increase in barriers to entry (an increase in* θ*) increases the optimal capital level. Otherwise,* K^* *remains constant.*

Proof. Proposition (1.4) implies that:

$$\frac{\partial K^*}{\partial \theta} = \frac{\frac{\partial r}{\partial \theta}}{f''(K^*)} \tag{3.13}$$

which from proposition 6 is positive if $\Gamma(K_r) > 0$ and zero if $\Gamma(K_r) = 0$. □

Notice that the effect of barriers to entry over all critical capital levels depends on certain conditions. First, as showed in proposition 13, the minimum capital level to receive a loan increases when fixed costs increase if $D_d \leq 1$, which implicitly implies that I have to restrict the initial wealth distribution. In fact, one can assure this condition by defining the support of the wealth distribution in $[0,1]$. In order to satisfy this, I could normalize the initial wealth distribution to $[0,1]$ or I could initialy choose a wealth distribution supported in the interval $[0,1]$ (like the Beta distribution). Indeed, recent papers of Bose and Banerjee (2005), and Brown and Chiang (2012), obtain a Beta distribution of wealth after solving theoretical economic models.

Assuming that $D_d \leq 1$ or if $\Gamma(K_r) = 0$, it will be observed that an increase in barriers to entry leads to higher restrictions to access the credit market. This is because fraudulent behavior becomes more profitable than operating a firm for some previous constrained agents who had access to the credit market (due to higher fixed costs). Thus, lenders will raise the minimum capital level to grant a loan and they will also limit the allowable amount of debt for constrained entrepreneurs (see next section). On the other hand, if $D_d > 1$, it may occur that the decrease in interest due to higher barriers to entry is higher than the increase in fixed costs. In that case there could exist previously constrained agents who will prefer to form a firm instead absconding, i.e. lenders will trust in some previous restricted agents and will reduce K_d. Note that I alternatively assure that this last case never happens if $|\frac{\partial r}{\partial \theta}| \leq \frac{1}{D_d}$, condition which again implicitly restricts wealth distribution.

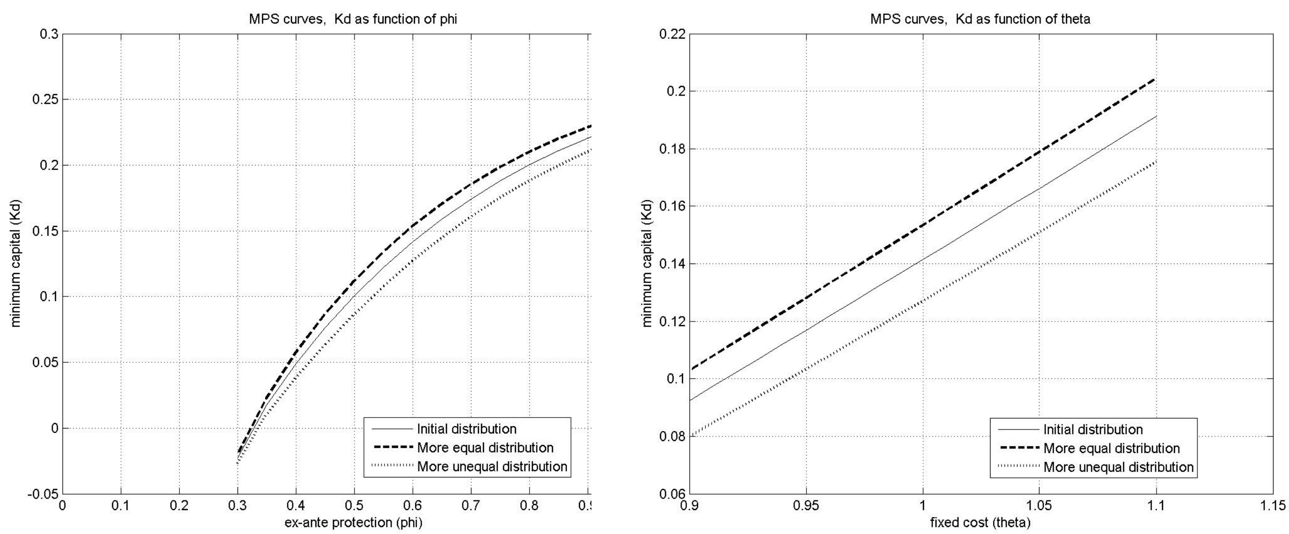
Similarly, in order to obtain an unambiguous effect of θ on K_r, I have to restrict wealth distribution to $[0,1]$ to assure that $D_r \leq 1$. If this last condition is satisfied (or if $\Gamma(K_r) = 0$), then lenders will raise K_r after an increase in fixed costs, because some previous unconstrained entrepreneurs now may have incentives to abscond (if they received the same amount of debt). Hence, firms experience higher (lower) financial constraints when fixed costs increase (decrease), since banking system raises critical capital levels K_d and K_r, and restricts the amount of debt that sub-capitalized firms can get. However, if $D_r > 1$ and $\Gamma(K_r) > 0$ the effect of a reduction in barriers to entry is ambiguous. In that case, it may occur that the direct effect of an increase in fixed cost is lower than the indirect reduction in K_r due to a decline in the interest rate, i.e. K_r may decrease with θ. Furthermore, from proposition 14, note that higher barriers to entry lead to an increase in K^* in economies with sub-capitalized firms, because agents must compensate the increase in fixed costs by investing higher capital in a firm, and as I showed in Section 3.1, the interest increases.

In summary, if I restrict the analysis to wealth distributions supported in $[0,1]$, then the critical capital levels K_d and K_r will increase after an increase in the fixed cost of operating a firm (assuming an economy with SMEs). Therefore, after an increase in barriers to entry, banks will impose tighter credit constraints by restricting the minimum wealth to access the credit market and by raising the amount of wealth to get an optimal loan. In addition, as I will show in next section, lenders will also decide to restrict maximum allowable debt to SMEs. Thus, if I restrict the economy to wealth distributions supported in $[0,1]$, an increase in fixed costs has the same effects on credit constraints that I have explained for reductions in *ex-ante* protection. On the other hand, in wealthy economies I find that a reduction in θ has no effect, while an increase in θ eventually may lead to credit constraints in rich countries.

In order to analyze statics on critical capital levels with regard to different wealth distributions, I use the simulations. Notice that any wealth redistribution will impact the critical capital levels through changes in the demand of capital, that will affect the equilibrium interest rate. Simulations of the previous section showed that countries with more unequal distributions have higher interest rates if they are more capital constrained. Therefore, it is expected that in such countries the minimum capital to get a loan is higher, while this result may be reversed in capital unconstrained countries. The effect on K_r of a wealth distribution is not clear, since changes in the interest rate affect K_r through a direct effect and a negative indirect effect (due to a reduction in K^* which reduces K_r).

For the capital unconstrained economy (such that $E(K_z \in [K_r, K^*))$ the following results are obtained:

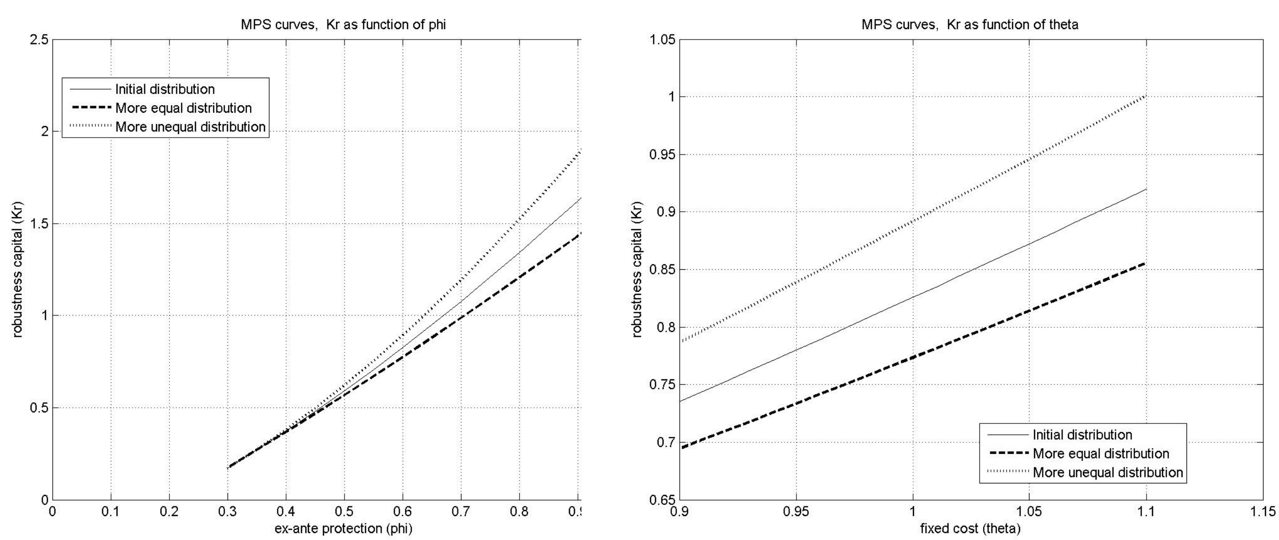
Figure 3.3: K_d statics with different MPS. Capital unconstrained economy ($E(K_z) \in [K_r, K^*)$)



(a) Variation in K_d with ϕ

(b) Variation in K_d with θ

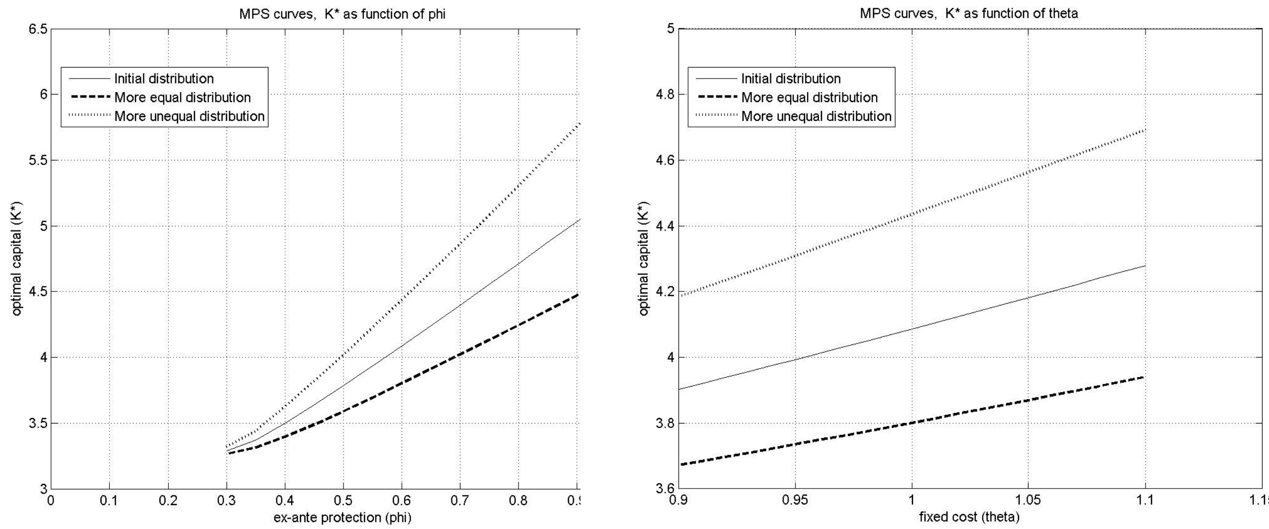
Figure 3.4: K_r statics with different MPS. Capital unconstrained economy ($E(K_z) \in [K_r, K^*)$)



(a) Variation in K_r with ϕ

(b) Variation in K_r with θ

Figure 3.5: K^* statics with different MPS. Capital unconstrained economy ($E(K_z) \in [K_r, K^*)$)



(a) Variation in K^* with ϕ

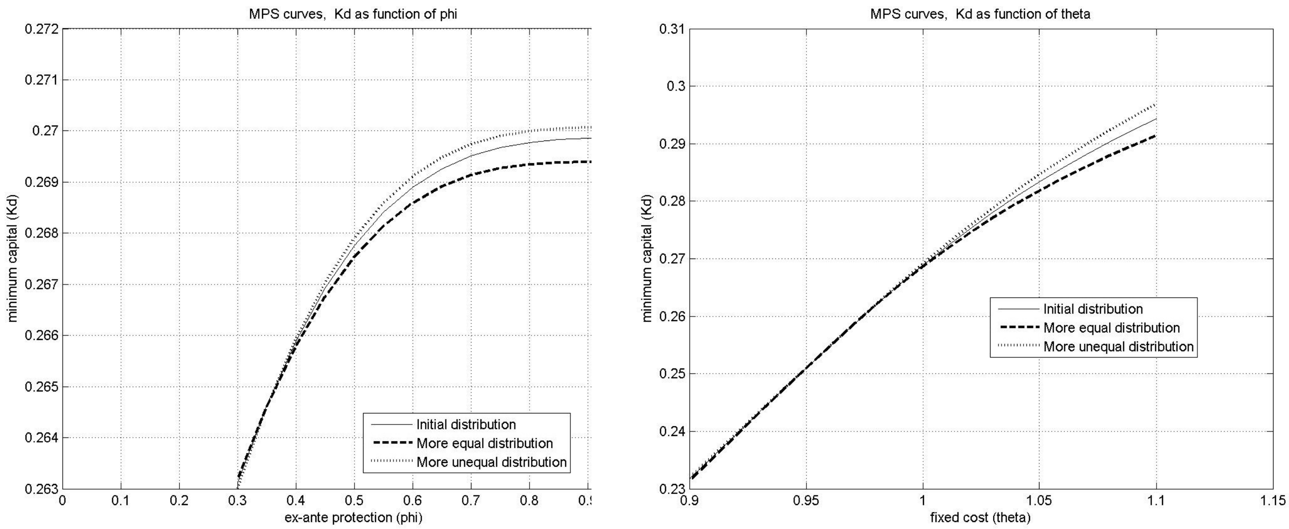
(b) Variation in K^* with θ

First, notice that as I showed in previous propositions, all critical capital levels increase when *ex-ante* protection and fixed costs increase. From figures 3.3a and 3.4a, note that K_d is concave in ϕ, while K_r seems to be convex in ϕ[8]. Therefore, as showed in proposition 12, the distance between K_d and K_r increases when the recovery rate gets worse. In addition, the minimum capital level to receive a loan seems to be higher under more equal wealth distributions, while K_r seems to be higher in more unequal wealth distributed countries. The optimal capital stock is higher in the more unequal economy, consistent with the fact that the interest rate is lower (see previous section). Therefore, in capital unconstrained economies, if the wealth distribution is more equal there exists tighter restrictions to access to the credit market (K_d is higher). However, once an agent is able to receive a loan, restrictions to get an optimal loan are lower in the more equally distributed country (K_r is lower). Finally, I conclude that the distance between K_d and K_r increases more when ϕ increases in the more unequally distributed economy.

Now, in order to determine whether these previous results depend on the aggregate capital stock of the economy, I study the simulations for a capital contrained economy (such that $K_d > E(K_z)$).

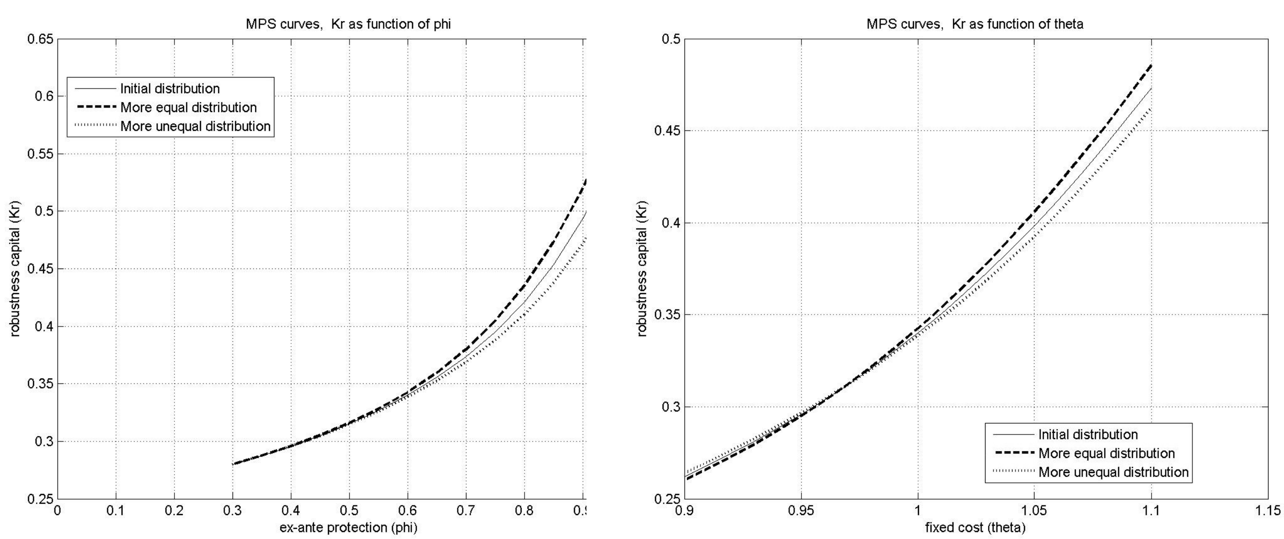
[8]The concavity of K_d and the convexity of K_r may hold because of the form of production function used in the simulations. Note that $f(K) = AK^\alpha$ satisfies that $f'''(K) > 0$, a condition that would be useful to demonstrate that K_d is concave in ϕ and K_r is convex in ϕ. However, note that the result of proposition 12 holds regardless if this condition is satisfied.

Figure 3.6: K_d statics with different MPS. Capital constrained economy ($K_d > E(K_z)$)



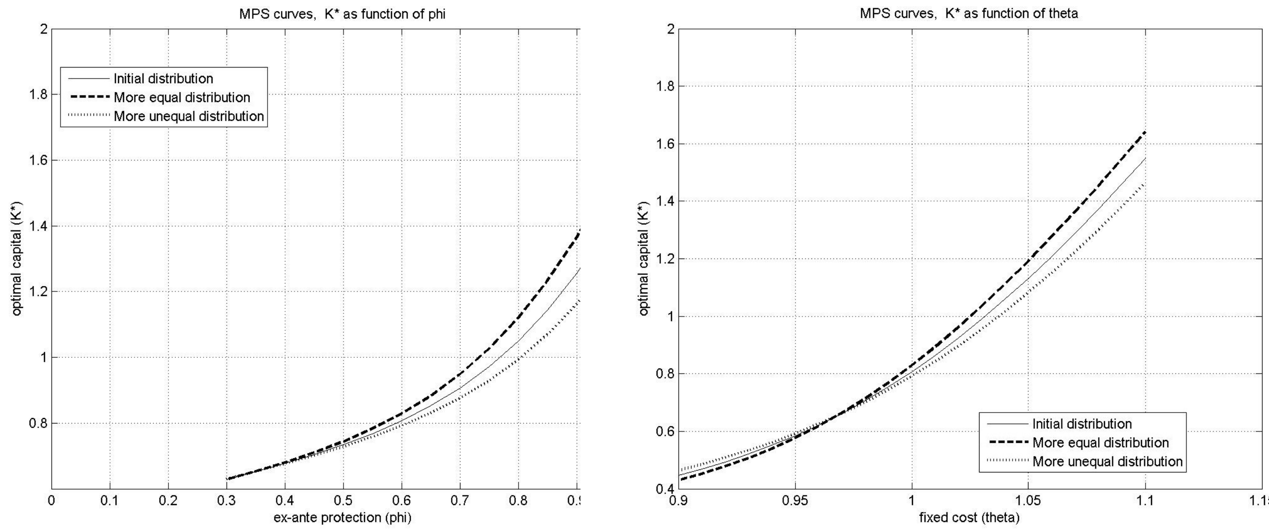
(a) Variation in K_d with ϕ (b) Variation in K_d with θ

Figure 3.7: K_r statics with different MPS. Capital constrained economy ($K_d > E(K_z)$)



(a) Variation in K_r with ϕ (b) Variation in K_r with θ

Figure 3.8: K^* statics with different MPS. Capital constrained economy ($K_d > E(K_z)$)



(a) Variation in K^* with ϕ

(b) Variation in K^* with θ

Figures 3.6a and 3.7a, confirm the hypothesis that K_d is concave in ϕ and K_r is convex in ϕ[9]. However, in the case of a capital constrained economy the statics over critical capital levels with regard to different wealth redistributions are just the opposite: restrictions to access to the credit market are higher in the more unequal country (K_d is higher), while the conditions to get an optimal loan are tighter in the more equal country (K_r is higher). In addition, the optimal capital stock is higher (lower) in the more equal (unequal) country, consistent with the fact that it has lower (higher) interest rate (see figure 3.2). In contrast with the previous case, the increase in the distance between K_d and K_r after an increase in ϕ is higher in the more equally distributed country.

In summary, I conclude that in capital constrained (unconstrained) countries a more equal (unequal) wealth distribution leads to lower (higher) constraints to access the credit market, but once an agent is able to receive a loan, restrictions to obtain an optimal loan are lower (higher) in the more equally (unequally) distributed country. In a capital constrained (unconstrained) economy, the range in which an inefficient firm can develop increases more after an improvement in *ex-ante* protection in more equally (unequally) distributed countries.

3.3 Firm size and maximum allowable debt statics

In this section I characterize the variation in firm size in terms of total firm output or sales, due to improvements in the loan recovery rate or in the fixed cost. In addition, I examine how maximum allowable debt varies through SMEs.

Proposition 15. *The smallest firm in operation is larger when fixed cost* θ *increases or ex-ante protection* $1-\phi$ *increases.*

[9] In the case of using an AK production function.

Proof. Differentiating condition (1.18) with respect θ I obtain that:

$$\begin{aligned} f''(K_d + D_d)\frac{\partial(K_d + D_d)}{\partial\theta} - \frac{\partial r}{\partial\theta} = 0 \\ \Rightarrow \frac{\partial(K_d + D_d)}{\partial\theta} = \frac{\frac{\partial r}{\partial\theta}}{f''(K_d + D_d)} > 0 \end{aligned} \tag{3.14}$$

Similarly, for ϕ I obtain:

$$\begin{aligned} f''(K_d + D_d)\frac{\partial(K_d + D_d)}{\partial\phi} - (1 + \frac{\partial r}{\partial\phi}) = 0 \\ \Rightarrow \frac{\partial(K_d + D_d)}{\partial\phi} = \frac{1 + \frac{\partial r}{\partial\phi}}{f''(K_d + D_d)} < 0 \end{aligned} \tag{3.15}$$

□

Proposition 16. *Maximum allowable debt increases with improvements in ex-ante creditor protection.*

Proof. From (1.28) and (3.2), differentiation of D_z with respect ϕ leads to:

$$\frac{\partial D_z}{\partial\phi} = \frac{(1 + \frac{\partial r}{\partial\phi})D_z}{f'(K_z + D_z) - (1 + r + \phi)} < 0 \tag{3.16}$$

where I have used the fact that $|\frac{\partial r}{\partial\phi}| < 1$ and that $f'(K_z + D_z) < 1 + r + \phi$ for $K_z > K_d$. Now, equations (3.15) and (3.8) lead to:

$$\begin{aligned} \frac{\partial D_d}{\partial\phi} &= \frac{1 + \frac{\partial r}{\partial\phi}}{f''(K_d + D_d)} - \frac{\partial K_d}{\partial\phi} \\ &= \frac{1 + \frac{\partial r}{\partial\phi}}{f''(K_d + D_d)} - \frac{(1 + \frac{\partial r}{\partial\phi})}{f'(K_d + D_d)} < 0 \end{aligned} \tag{3.17}$$

Thus, maximum allowable debt increases when $1 - \phi$ increases. □

Proposition 17. *If* $D_r \leq 1$*, then maximum allowable debt decreases for* $K_z \in (K_d, K_r]$ *when barriers to entry increase. Otherwise, the effect is ambiguous.*

Proof. From equations (1.28) and (3.4) I obtain that:

$$\frac{\partial D_z}{\partial\theta} = \frac{1 + \frac{\partial r}{\partial\theta}D_z}{f'(K_z + D_z) - (1 + r + \phi)} \tag{3.18}$$

Notice that the minimum value of the numerator is reached when $D_z = D_r$, if $D_r \leq 1$ I assure that the numerator will be positive for all capital levels, because D_r is the maximum allowable debt and $1 + \frac{\partial r}{\partial\theta} > 0$. Thus, if $D_r \leq 1$, the maximum allowable debt decreases in $K_z \in (K_d, K_r]$, otherwise the effect of θ is ambiguous. In addition, the effect of fixed cost over the minimum debt is ambiguous, regardless if $D_r \leq 1$. In fact, from equations (3.14) and (3.5) I have that:

$$\frac{\partial D_d}{\partial\theta} = \frac{\frac{\partial r}{\partial\theta}}{f''(K_d + D_d)} - \frac{1 + \frac{\partial r}{\partial\theta}D_d}{f'(K_d + D_d)} \tag{3.19}$$

Observe that the first term is positive and the second term is negative, thus the sign of $\frac{\partial D_d}{\partial\theta}$ is ambiguous. □

Proposition 18. *If* $K_z \in [K_d, K_r)$, *then the fraction of the optimal capital level* K^* *that can be financed only by debt decreases when ex-ante protection gets worse.*

Proof. Differentiating the ratio $\frac{D_z}{K^*}$ with respect ϕ:

$$\frac{\partial(\frac{D_z}{K^*})}{\partial\phi} = \frac{\frac{\partial D_z}{\partial\phi}K^* - \frac{\partial K^*}{\partial\phi}D_z}{(K^*)^2} \tag{3.20}$$

From proposition 16, notice that the first term of the numerator is negative. For the second term, note that from equation (3.7) I have that $\frac{\partial K^*}{\partial\phi} > 0$, i.e. $\frac{\partial(\frac{D_z}{K^*})}{\partial\phi} < 0$. □

Proposition 19. *If* $D_r \leq 1$ *and* $K_z \in (K_d, K_r)$, *then the fraction of the optimal capital stock* K^* *that can be financed by debt decreases when fixed cost increases.*

Proof. Similarly as in previous proposition I obtain:

$$\frac{\partial(\frac{D_z}{K^*})}{\partial\theta} = \frac{\frac{\partial D_z}{\partial\theta}K^* - \frac{\partial K^*}{\partial\theta}D_z}{K^{*2}} \tag{3.21}$$

From proposition 17 and from condition (3.13) I have that $\frac{\partial(\frac{D_z}{K^*})}{\partial\theta}$ is negative if $D_r \leq 1$. □

Proposition 15 suggests an opposite effect of increases in θ and ϕ over the size of the smallest firm. First, the smallest firm in operation would like to invest more when fixed cost increases, in order to compensate the direct reduction in profits due to higher barriers to entry. In fact, lenders will raise K_d to assure that the smallest firm does not have incentives to commit fraud, i.e. the smallest firm will have more own mobile capital available to invest. However, from equation (3.19), note that the effect over the minimum allowable debt D_d is ambiguous. But if this effect is negative, condition (3.14) assures that the reduction in D_d cannot be higher than the increase in K_d. Thus, the net effect over $(K_d + D_d)$ is positive if barriers to entry increases.

In contrast, the effect of a reduction in *ex-ant*e protection over the smallest firm in operation is negative. When the recovery rate $1-\phi$ is reduced, agency costs increase and lenders are willing to loan lower amounts to these firms, as showed in equation (3.17). On the other hand, the minimum capital to grant a loan K_d must be higher to avoid fraudulent behavior. However, condition (3.15) says that the increase in the critical capital level K_d is lower than the reduction in the minimum allowable debt D_d. Therefore, the smallest firm is smaller (larger) when ex ante protection $1-\phi$ is improved (worsened).

In addition, notice that either a reduction in the recovery rate or an increase in barriers to entry has two opposite effects over maximum allowable debt. First, it has a negative direct effect over maximum debt, because some previous agents with access to credit may now have incentives to abscond. Thus, lenders will impose tighter restrictions in terms of the amount loaned. Secondly, worse loan recovery rate or higher fixed cost leads to a lower interest rate, therefore lenders are willing to loan more (see equation (1.28)).

Furthermore, as I showed in proposition 16, a decline in creditor protection leads to a reduction in the maximum debt that any constrained entrepreneur can get. Because the direct effect of a reduction in $1-\phi$, that leads to tighter restrictions in the amount loaned, is higher than the indirect effect of having a lower interest rate, that incentives lenders to loan more. Conversely, and consistent with La Porta et al. (1999), an improvement in creditor rights will encourage the development of lending, by raising the amount loaned to each entrepreneur and by lowering the minimum capital needed to ask for a loan. On the other hand, proposition 17 shows that an increase in barriers to entry measured by fixed costs, will reduce the amount loaned to each agent if the maximum effective debt is lower than 1 (again I have to restrict wealth distribution to [0,1]).

Note that the limitation of the maximum allowable debt is another mechanism of credit rationing. When there exists a decline in creditor protection or and increase in barriers to entry, lenders have to impose tighter restrictions in the credit amount in order to assure that the sub-capitalized borrower will behave. Therefore, worse recovery rate or higher barriers to entry leads to higher credit constraints by two ways: by increasing the minimum wealth level to access to the credit market, and by limiting even more the amount of credit that sub-capitalized firms can get.

Another important issue is to analyze the variation in maximum allowable debt relative to optimal capital stock. As I showed in propositions 18 and 19, when ϕ or θ increases the ratio between D_z and K^* is lower, i.e. the portion of the optimal capital stock that any constrained agent can finance with debt is lower. Therefore, I conclude that previous agents with partial access to credit will be more rationed in relative terms after an increase in the fixed cost or a reduction in the loan recovery rate.

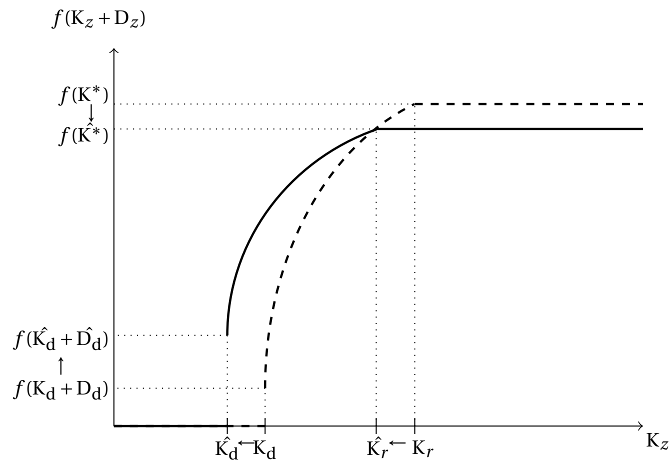
In the following proposition I describe the effect of variations in *ex-ante* creditor protection over the size of firms such that $K_z > K_d$.

Proposition 20. *Output of firms such that* $K_z \in (K_d, K_r)$ *increases with improvements in ex-ante creditor protection, while it decreases for enterprises such that* $K_z \geq K_r$.

Proof. Proposition 10 says that K^* decreases when $1-\phi$ increases, i.e. the investment curve in $[K_d, K_r]$ ends in a lower point when ex-ante protection is improved, and remains constant thereafter. However, from proposition 15, I have that $K_d + D_d$ increases with $1-\phi$, i.e. the starting point of investment curve increases. Moreover, from proposition 16 I conclude that $\frac{\partial K_z + D_z}{\partial \phi} < 0$, i.e. the capital invested by the rest of constrained enterprises is higher too. Therefore, the *ex-post* investment curve (after the reduction in ϕ) cuts the initial investment curve in K_r, because K_r is the first point in which investment is reduced. Thus, SMEs' output is higher when $1-\phi$ increases, while the output of larger firms is lower. Figure 3.9 shows this result.

□

Figure 3.9: Output curve movement after an improvement in recovery rate $1-\phi$.



Note: dashed line represents output curve previous the improvement in $1-\phi$.

Proposition 21. *SMEs are more efficient in absolute terms after an improvement in ex-ante creditor protection, i.e. the distance between* $I_z = K_z + D_z$ *and* K^* *declines for firms such that* $K_z \in [K_d, K_r)$.

Proof. Differentiation of $K^* - (K_z + D_z)$ with respect ϕ leads to:

$$\frac{\partial(K^* - [K_z + D_z])}{\partial\phi} = \frac{\partial K^*}{\partial\phi} - \frac{\partial(K_z + D_z)}{\partial\phi} \tag{3.22}$$

From proposition 16 and because $\frac{\partial K^*}{\partial\phi} > 0$, I have that condition (3.22) is positive. Thus, the distance between K^* and $K_z + D_z$ for $K_z \in (K_d, K_r)$ decreases with improvements in $1-\phi$. □

Proposition 20 describes another important issue. Larger firms invest less when there is an increase in the recovery rate $1-\phi$. Assuming that production technology remains constant, larger firms will be smaller in terms of output after an improvement in the *ex-ante* creditor protection. On the other hand, smaller firms are larger after finance development, because some previous micro-entrepreneurs will be able to open a firm and SMEs will have access to higher loans. These last type of firms will combine their own wealth with a higher amount of debt. Thus, as showed in proposition 21, their total investment will be closer to the optimal capital stock (that decreases with $1-\phi$). Therefore, previous inefficient SMEs will be more efficient in absolute and relative terms after an improvement in creditor protection, because they will be able to command more capital to try to attain a lower optimal stock.

Additionally, as can be inferred from figure 3.9, the 'jump' in utility between the last micro-entrepreneur that decide to not open a firm and the first SME that is able to get a loan, is higher when the recovery rate is improved. This means that under better legal conditions the opportunity cost of not producing is higher for micro-entrepreneurs.

Although proposition 20 can shed some light on the theory of a group of interest that opposes financial development, is not sufficient to allow firm conclusions. Higher recovery rate reduces the output of Larger Medium Enterprises and Larger Enterprises, that may lead to a decrease in their profits. However, due to the increase in the interest rate is not clear if all SMEs will experience an increase in their profits. In the following proposition I analyze the variation in firm profits due to an improvement in $1-\phi$.

Proposition 22. *If* $\Gamma(K_r) > 0$*, then if ex-ante protection* $1-\phi$ *increases there exists a critical capital level* K_c *such that all firms with* $K_z \in [K_d, K_c)$ *experience and increase in their profits, while profits fall for enterprises such that* $K_z > K_c$*. Otherwise, profits remain constant.*

Proof. Differentiating the profits function for firms which produce at the optimal capital level:

$$\begin{aligned}\frac{\partial \pi(K^*)}{\partial \phi} &= f'(K^*)\frac{\partial K^*}{\partial \phi}\frac{\partial r}{\partial \phi} - K^*\frac{\partial r}{\partial \phi} - (1+r)\frac{\partial K^*}{\partial \phi}\frac{\partial r}{\partial \phi} \\ &= (f'(K^*) - (1+r))\frac{\partial K^*}{\partial \phi}\frac{\partial r}{\partial \phi} - K^*\frac{\partial r}{\partial \phi} \\ &= -K^*\frac{\partial r}{\partial \phi} > 0\end{aligned}$$

where I have used the fact that $f'(K^*) = 1+r$ and $\frac{\partial r}{\partial \phi} < 0$. Now, for constrained firms which receive profits $\pi(K_z + D_z)$ I obtain that:

$$\begin{aligned}\frac{\partial \pi(K_z + D_z)}{\partial \phi} &= f'(K_z + D_z)\frac{\partial D_z}{\partial \phi} - \frac{\partial r}{\partial \phi}(D_z + K_z) - (1+r)\frac{\partial D_z}{\partial \phi} \\ &= [f'(K_z + D_z) - (1+r)]\frac{\partial D_z}{\partial \phi} - \frac{\partial r}{\partial \phi}(K_z + D_z)\end{aligned} \tag{3.23}$$

Replacing condition (3.16) in (3.23) leads to:

$$\begin{aligned}\frac{\partial \pi(K_z + D_z)}{\partial \phi} &= \frac{[f'(K_z + D_z) - (1+r)](1 + \frac{\partial r}{\partial \phi})D_z}{f'(K_z + D_z) - (1+r+\phi)} - \frac{\partial r}{\partial \phi}(K_z + D_z) \\ &= \frac{[f'(K_z + D_z) - (1+r)]((1 + \frac{\partial r}{\partial \phi})D_z - \frac{\partial r}{\partial \phi}[f'(K_z + D_z) - (1+r+\phi)](K_z + D_z)}{f'(K_z + D_z) - (1+r+\phi)}\end{aligned} \tag{3.24}$$

Note that the first term of the numerator is positive (because $|\frac{\partial r}{\partial \phi}| < 1$ and $\frac{\partial r}{\partial \phi} < 0$) and the second term is negative. The denominator is negative for all capital levels except for K_d. In particular, condition (3.24) is positive for K^* (because $f'(K^*) = 1+r$). While, at K_d^+ after simplification I obtain:

$$\frac{\partial \pi(K_d^+ + D_d^+)}{\partial \phi} = \frac{\phi D_d^+(1 + \frac{\partial r}{\partial \phi})}{f'(K_d^+ + D_d^+) - (1+r+\phi)} \tag{3.25}$$

Note that at K_d^+ I have that $\frac{\partial \pi(K_d^+ + D_d^+)}{\partial \phi} < 0$. Thus, by continuity of $f'(\cdot)$, there exists a $K_c \in [K_d, K_r)$ such that profits decreases to the left of K_c and increases to the right when ϕ increases (note that agents with $K_z < K_d$ are worse because the interest rate is lower when ϕ increases). On the other hand, from proposition 6, notice that if the economy is such that $\Gamma(K_r) = 0$ then $\frac{\partial r}{\partial \phi} = 0$. Therefore, in that case profits remain constant for all enterprises after an improvement in the recovery rate.

□

I illustrate the results of the previous proposition in the following simulation[10]:

Figure 3.10: Firms profits in terms of own capital K_z and variations in ϕ

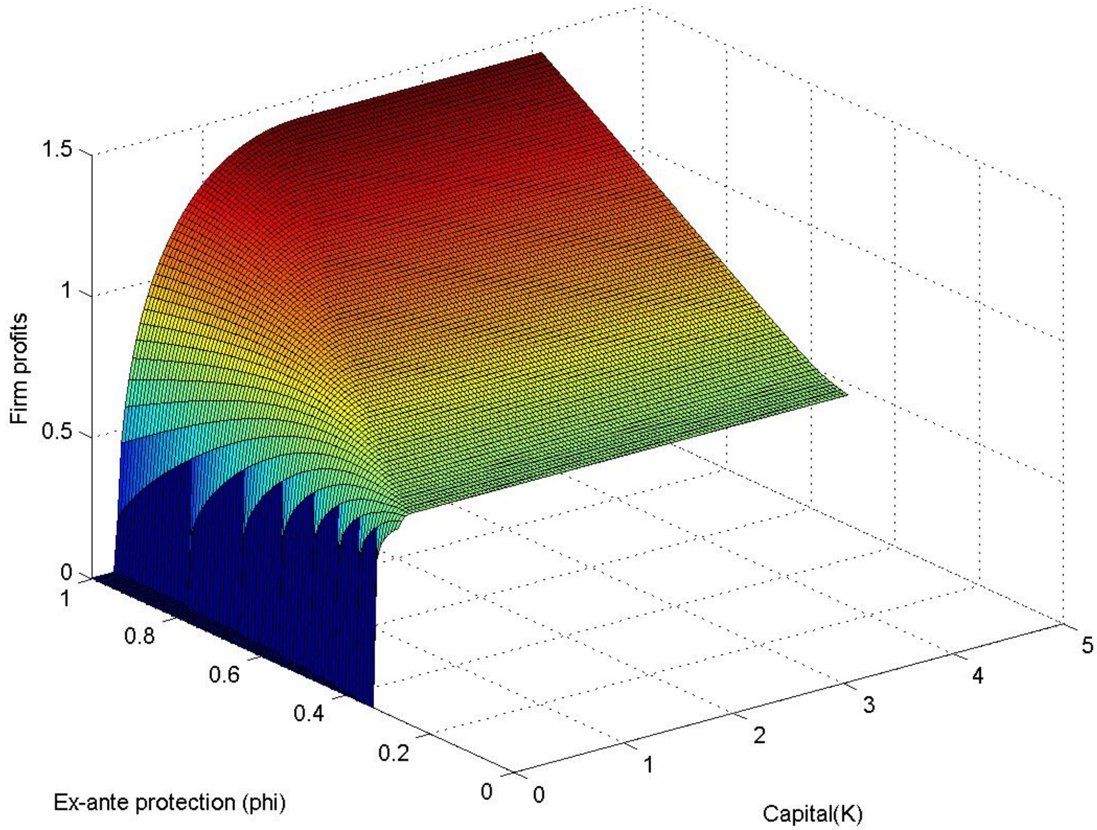


Figure 3.10 shows that as *ex-ante* protection gets worse firms with higher amounts of capital have higher profits, while less capitalized firms experience a decrease in their profits. The figure shows that for each level of *ex-ante* protection, there exists a threshold such that profits decreases to left and increases to the right. Therefore, simulations confirm the result proposed in previous proposition.

From proposition 22, I conclude that there will exist a conflict of interest between larger firms and more constrained SMEs and Micro-Enterprises. Larger Medium Enterprises (such that $K_z \in [K_r, K^*)$) will oppose to financial development because it raises competition by relaxing credit constraints to smaller firms, leading to a higher equilibrium interest rate and raising financing costs. In addition, the model predicts that there exists a group of wealthier SMEs (such that $K_z \in (K_c, K_r)$) that will be worse after an improvement of *ex-ante* creditor protection, because the increase in the maximum allowable debt, and thus in their invested capital and output, cannot compensate the increase in financing costs.

On the other hand, some previous Micro-Enterprises and smaller SMEs will benefit most from a development in creditor rights. Since it allows to some micro-enterprises to enter to the out-

[10]These results were obtained using the initial setup described in Chapter 2.

put market and became a SME, and because smaller SMEs are able to reach a production level closer to K^*. This is consistent with empirical findings of Beck *et al.* (2004; 2005), which suggest that smaller firms stand to benefit the most from improvements in financial development. Conversely, they show that smallest enterprises are consistently the most adversely affected by all these obstacles (Micro-Enterprises and smaller SMEs in the case of my model).

In addition, notice that proposition 22 says that in wealthy countries, such that there not exist SMEs and Micro-Enterprises (i.e. such that $\Gamma(K_r) = 0$), incumbent firms will be indifferent to an improvement in the recovery rate, because as I showed in proposition 22 their profits will remain constant. Thus, it is expected that resistance against financial development is more likely to occur in developing countries (such that $\Gamma(K_r) > 0$, that I interpret as poorer countries in terms of aggregate capital stock). This seems to be an explanation to the fact that some countries are reluctant to develop their financial markets.

Recent literature has provided a political and economic explanation to this fact. As I have mentioned in Section 3.1, La Porta et al. (2000b) and Rajan and Zingales (2003) showed that better investor or creditor legal rights is opposed by politically and wealthy groups in developing countries, because it breeds competition and leads to higher interest rates, increasing financing cost to incumbent firms. In addition, research of Pagano and Volpin (2005), Hellwig (1998) and Bebchuk and Roe (1999) also suggests that political opposition to reform from incumbent entrepreneurs is an important issue to explain why underdevelopment countries do not improve legal protection. Moreover, the model developed by Balmaceda and Fischer (2010) also provides a framework that helps to interpret that interest group politics is an important factor in financial development across countries.

3.4 Macroeconomic variables statics in the closed economy

In this section I present statics of some important macroeconomics variables of the closed economy (such GDP, total debt and credit penetration) due to different degrees of *ex-ante* protection and variations in fixed costs. In addition, I analyze the effect on these macroeconomic variables with regard to wealth redistributions. The complexity of obtaining these results theoretically comes from the fact that most of these effects are ambiguous, and may eventually depend on the form of the initial wealth distribution. As I show in the simulations presented below, the direction of the effect of each parameter on macroeconomic variables may change depending whether the country is capital constrained or not.

First, I analyze theoretically the effect of $x = \phi, \theta$ on total debt to clarify why this effect may depend on the aggregate capital stock of each country.[11] Total debt is defined by:

$$D_T = \int_{K_d}^{K_r} (K_z + D_z)\, \partial\Gamma(K_z) + \int_{K_r}^{K^*} (K^* - K_z)\, \partial\Gamma(K_z) \tag{3.26}$$

To determinate the effect of $x = \phi, \theta$ on total debt I take the first derivative:

[11] Notice that total debt may determinate the effects on GDP and credit penetration. It is expected that higher debt leads to higher GDP and credit penetration.

$$
\begin{aligned}
\frac{\partial D_T}{\partial x} &= \int_{K_d}^{K_r} \frac{\partial D_z}{\partial x} \partial\Gamma(K_z) + D_r \frac{\partial K_r}{\partial x}\gamma(K_r) - D_d \frac{\partial K_d}{\partial x}\gamma(K_d) \\
&\quad \int_{K_r}^{K^*} \frac{\partial K^*}{\partial x} \partial\Gamma(K_z) - (K^* - K_r)\frac{\partial K_r}{\partial x}\gamma(K_r) \\
\Rightarrow \frac{\partial D_T}{\partial x} &= \int_{K_d}^{K_r} \frac{\partial D_z}{\partial x} \partial\Gamma(K_z) - D_d \frac{\partial K_d}{\partial x}\gamma(K_d) + (\Gamma(K^*) - \Gamma(K_r))\frac{\partial K^*}{\partial x}
\end{aligned} \tag{3.27}
$$

Note that from equations (3.2) and (3.4), the first term of (3.27) is negative. From results of Section 1.2, the second term is negative and the last term is positive (because $\frac{\partial K_d}{\partial x} > 0$ and $\frac{\partial K^*}{\partial x} > 0$). Thus, the sign of condition (3.27) is ambiguous. In order to simplify this expression, I differentiate the equilibrium condition of capital market (see condition (1.29)) with respect x. This allows to obtain an expression for the first term of (3.27):

$$
\begin{aligned}
0 &= \int_{K_d}^{K_r} \frac{\partial D_z}{\partial x} \partial\Gamma(K_z) + K_r\gamma(K_r)\frac{\partial K_r}{\partial x} - (K_d + D_d)\gamma(K_d)\frac{\partial K_d}{\partial x} - K^*\gamma(K_r)\frac{\partial K_r}{\partial x} + (1 - \Gamma(K_r))\frac{\partial K^*}{\partial x} \\
&\Rightarrow \int_{K_d}^{K_r} \frac{\partial D_z}{\partial x} \partial\Gamma(K_z) = (K_d + D_d)\gamma(K_d)\frac{\partial K_d}{\partial x} - (1 - \Gamma(K_r))\frac{\partial K^*}{\partial x} < 0
\end{aligned} \tag{3.28}
$$

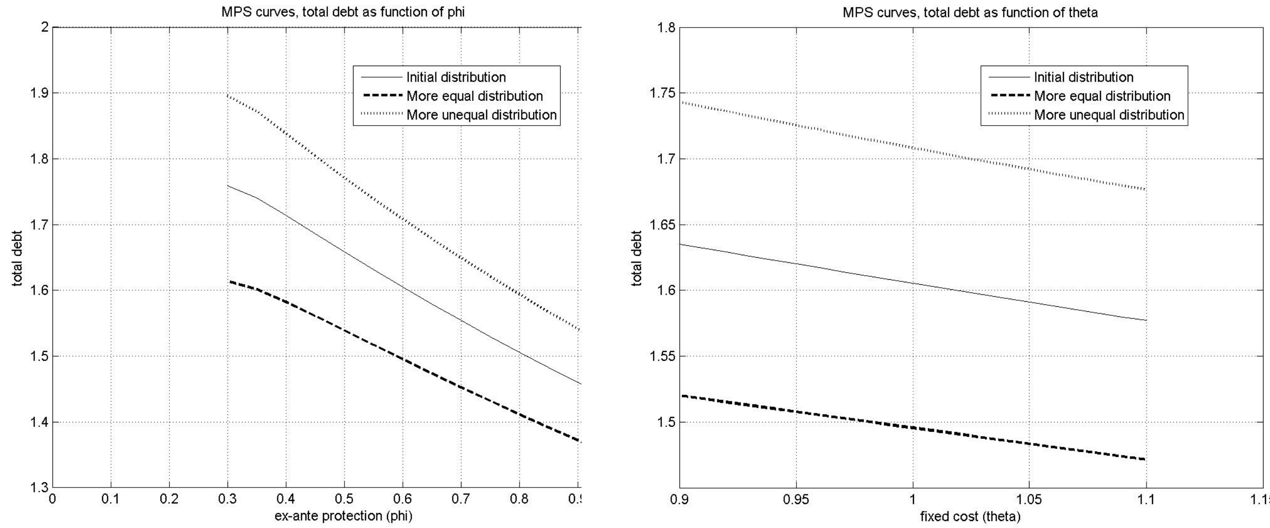
Replacing condition (3.28) in equation (3.27) I obtain:

$$
\frac{\partial D_T}{\partial x} = K_d\gamma(K_d)\frac{\partial K_d}{\partial x} - \frac{\partial K^*}{\partial x}(1 - \Gamma(K^*)) \tag{3.29}
$$

Therefore, I conclude that if $\frac{\partial K^*}{\partial x}(1 - \Gamma(K^*)) > K_d\gamma(K_d)\frac{\partial K_d}{\partial x}$ then the effect of x on total debt is positive. However, the opposite may also occur, and in that case total debt will increase when *ex-ante* protection gets worse or fixed cost increases. Notice that if the country is strongly capital contrained, then $1 - \Gamma(K^*)$ is more likely to be close to zero, since the expected fraction of agents who owns less than K^* is likely to be high. Thus, in a strongly capital constrained economy, the effect of x in total debt is more likely to be positive than negative. In contrast, if the country is close to be strongly capital unconstrained (a wealthy economy), one can expect that $1 - \Gamma(K^*)$ is likely to be positive, because the average agent owns almost the optimal capital level (see definition 6), while the first term of (3.29) is more likely to be zero, since I expect that $\gamma(K_d)$ is close to zero (assuming that such wealth distribution gives low weights to poorer agents) . Hence, the effect of x in total debt is more likely to be negative as the economy becomes capital unconstrained (and the same intuition applies for GDP and credit penetration)[12]. In order to test these hypothesis, I simulate the effects of ϕ and θ on total debt, GDP and credit penetration, for both the capital unconstrained economy and the strongly capital constrained one.

[12]If the country is such that $\Gamma(K_r) = 0$, there exists a range such that changes in x do not affect total debt, since interest rate remains constant (K^* does not change) and $\gamma(K_d) = 0$.

Figure 3.11: Total debt statics with regard to different MPS. Capital unconstrained economy ($E(K_Z) \in [K_r, K^*)$)

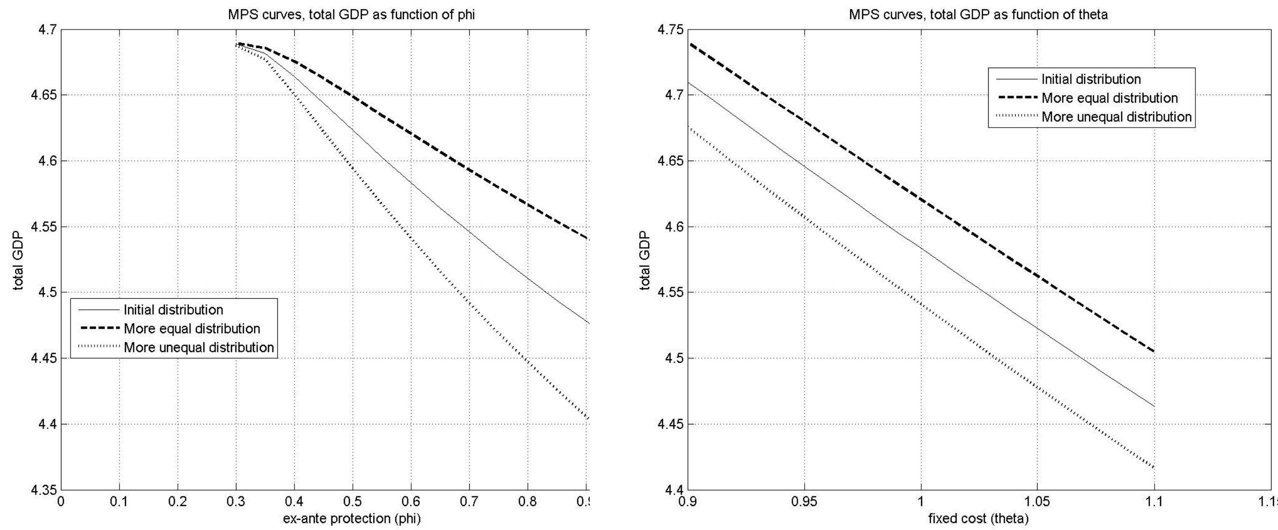


(a) Variation in total debt with ϕ

(b) Variation in total debt with θ

Results of figure 3.11a are consistent with empirical findings of La Porta et al. (1997) which suggest that countries with lower protection of legal rights have significantly lower debt. Additionally, I find that more unequal (equal) wealth distributions lead to higher (lower) total debt in capital unconstrained countries. It is observed that an increase in barriers to entry also leads to lower total debt. These results can be explained by the fact that worse *ex-ante* protection or higher fixed cost leads to tighter constraints to access to the credit market. In fact, banks will increase the minimum capital to receive a loan, and will reduce the amount of debt that any rationed entrepreneur can get, therefore the aggregate debt of the economy is likely to decrease.

Figure 3.12: GDP statics with regard to different MPS. Capital unconstrained economy ($E(K_Z) \in [K_r, K^*))$

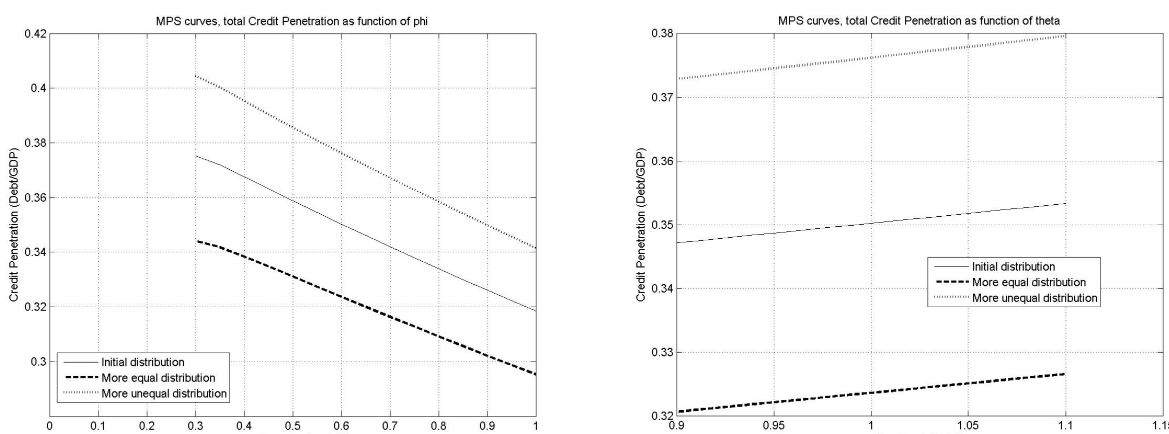


(a) Variation in total GDP with ϕ

(b) Variation in total GDP with θ

From figure 3.12 I conclude that in a capital unconstrained economy total GDP increases as *ex-ante* protection and fixed costs increase. This result is basically explained because as x increases the reduction in interest rate that lowers financing costs cannot compensate the decline in the allowable debt and the increase in the number of agents who do not produce. Moreover, figure 3.12a shows that GDP is higher in more equally capital unconstrained countries, result which is consistent with findings of Balmaceda and Fischer (2010). The same result is valid for fixed costs.

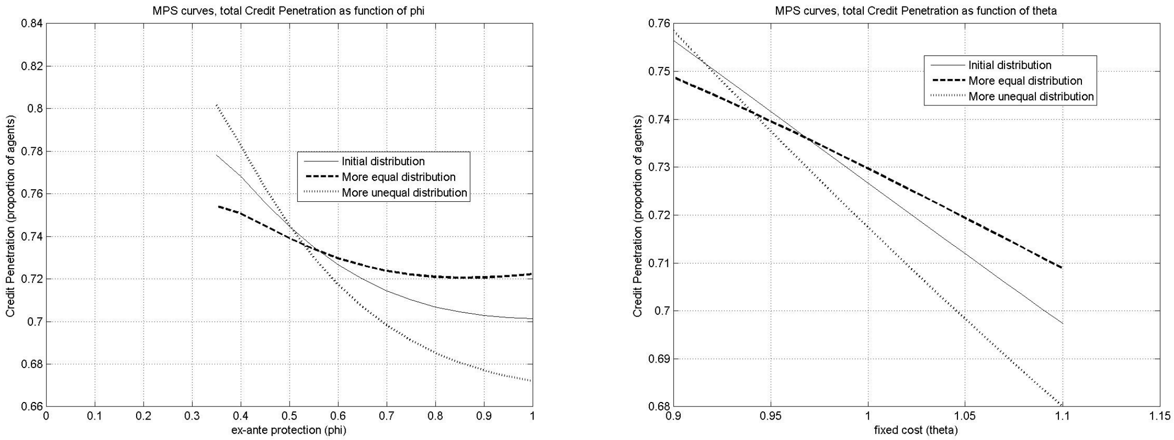
Figure 3.13: Credit Penetration (Debt/GDP) statics with regard to different MPS. Capital unconstrained economy ($E(K_Z) \in [K_r, K^*))$



(a) Variation in credit penetration (Debt/GDP) with ϕ

(b) Variation in credit penetration (Debt/GDP) with θ

Figure 3.14: Credit Penetration (fraction of agents) statics with regard to different MPS. Capital unconstrained economy ($E(K_Z) \in [K_r, K^*)$)



(a) Variation in credit penetration (fraction of agents) with ϕ

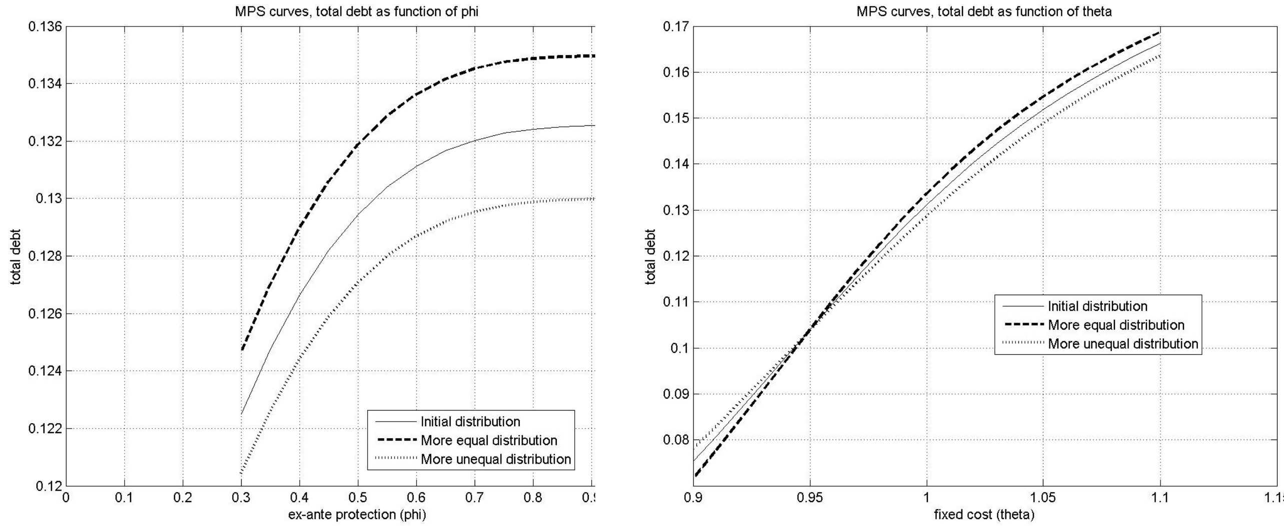
(b) Variation in credit penetration (fraction of agents) with θ

Note that I have used two measures of credit penetration. The first one, used in figure 3.13, is the ratio of debt to GDP, and the second one is the number, proportion of entrepreneurs who borrow from the bank system. Consistent with findings of Djankov et al. (2007) and La Porta et al. (2008), I find that credit penetration measured as percentage of debt on GDP increases as creditor protection improves. Contrary to intuition, figure 3.13b shows that as fixed cost increases there is a small increase in credit penetration. This may be explained because in the simulated economy GDP decreases much more than total debt with θ (compare figure 3.12b with 3.15b), i.e. the ratio of total debt to GDP may increase. On the other hand, it is observed that an increase in x leads to a lower proportion of agents which borrow from the bank system. This result comes from the fact that as x increases banks impose tighter restrictions to access to the credit market and also constraint the maximum allowable loan.

With regard to wealth redistributions, I find that credit penetration measured as debt over GDP is higher in the more unequal country, since total debt is higher and GDP is lower than in the more equally distributed country. In contrast, wealth statics over the fraction of agents which access to the credit market may depend on the value of x. In fact, it is observed that there exists a threshold (≈ 0.55) such that the fraction of agents who finance their firms with loans is higher (lower) for more unequal (equal) countries to the left, while it is lower (higher) to the right. Thus, as the economy becomes more capital constrained (as x increases) credit penetration, measured as the fraction who finance their projects with credit, is more likely to be higher in more equally distributed countries.

In what follows I present simulations for the capital constrained economy.

Figure 3.15: Total debt statics with regard to different MPS. Capital constrained economy ($K_d > E(K_z)$)



(a) Variation in total debt with ϕ

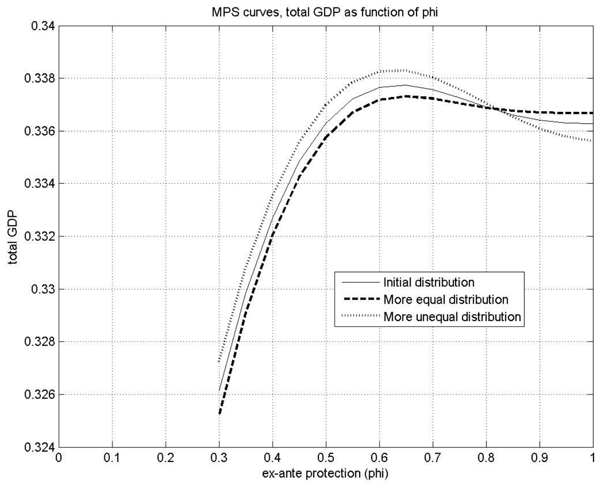
(b) Variation in total debt with θ

Contrary with initial intuition and with literature, I find that in the capital constrained economy total debt increases when ϕ and θ increase. This may be explained because the variation of r due to changes in x seems to be larger than 1 in the simulated constrained economy (see figure 3.2).

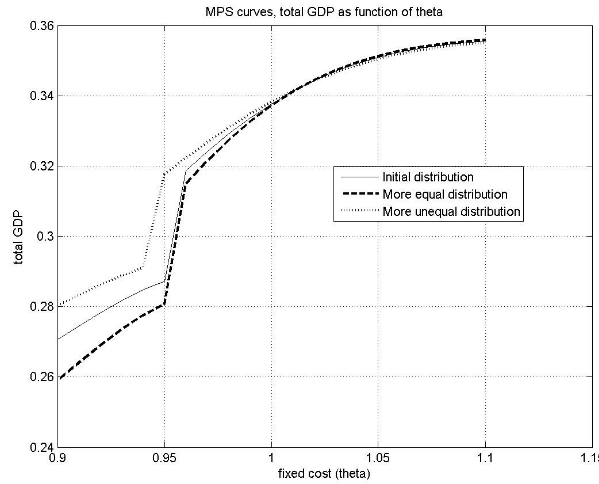
Recall that an increase in x has two opposite effects on the maximum allowable debt. First, there exists a direct negative effect that encourages banks to restrict the amount of debt that any agent is able to get. Secondly, there is an indirect positive effect due to a reduction in the equilibrium interest rate (that encourage banks to loan more). Since the decline in the interest rate after an increase in x is too large (in the strongly capital constrained country), it is expected that the indirect positive effect compensates the direct negative effect. Hence, maximum allowable debt may increase with x and total debt may also do so. Thus, richer entrepreneurs (who access to the credit market) benefit the most from a poor recovery rate or higher barriers to entry, because they can get bigger loans and operate larger firms. In contrast, the number of agents who cannot access to the credit market and that decide to close their firms is larger after an increase in ϕ or in θ, leading to a higher lost of specific capital.

Notice that the simulated strongly capital constrained economy violates assumption 1, thus all expected results may not hold. In fact, as I show below, total GDP and credit penetration measured as Debt/GDP may also increase with x in the simulated constrained economy.

Figure 3.16: GDP statics with regard to different MPS. Capital unconstrained economy ($E(K_Z) \in [K_r, K^*)$)

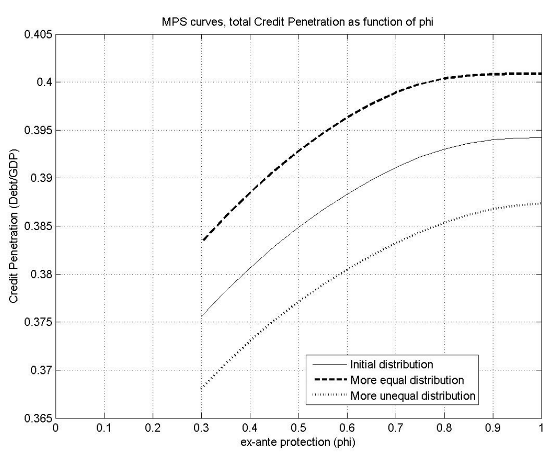


(a) Variation in total GDP with ϕ

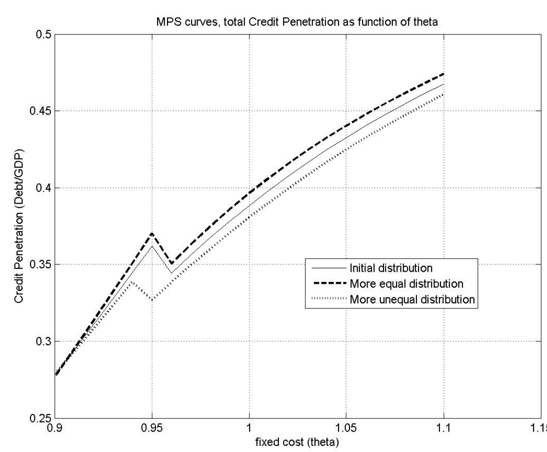


(b) Variation in total GDP with θ

Figure 3.17: Credit Penetration (Debt/GDP) statics with regard to different MPS. Capital unconstrained economy ($E(K_Z) \in [K_r, K^*)$)

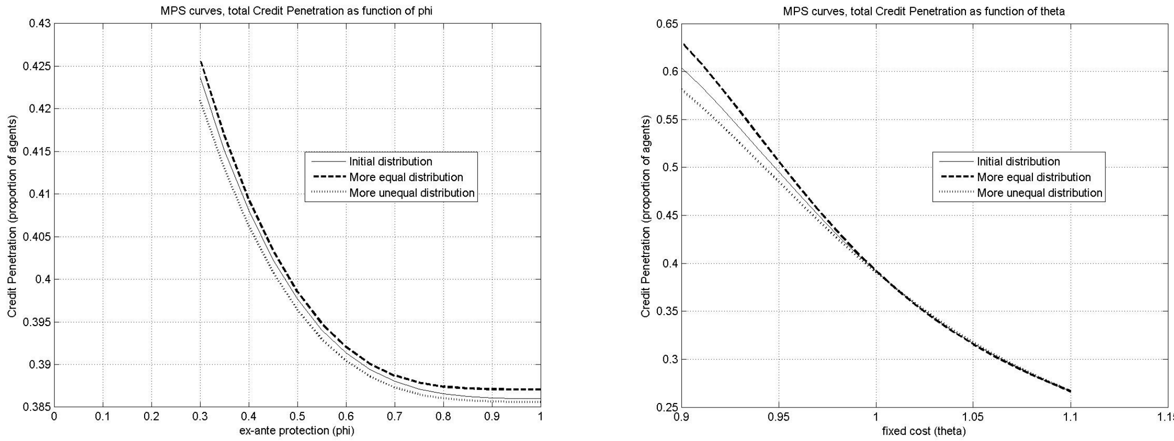


(a) Variation credit penetration (Debt/GDP) with ϕ



(b) Variation credit penetration (Debt/GDP) with θ

Figure 3.18: Credit Penetration (fraction of agents) statics with regard to different MPS. Capital unconstrained economy ($E(K_Z) \in [K_r, K^*)$)



(a) Variation in credit penetration (fraction of agents)with ϕ

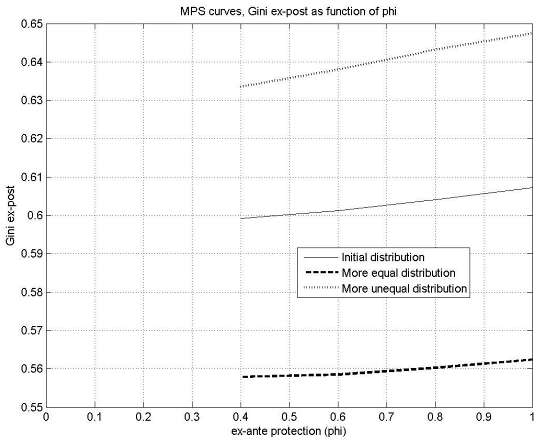
(b) Variation in credit penetration (fraction of agents) with θ

From figure 3.16a, I conclude that there exists a range in which GDP increases with ϕ. Similarly, I find that GDP increases with θ. This is explained because richer entrepreneurs who access to the credit market are able to receive a higher loan and to form larger firms when ϕ and θ increase. However, as showed in figure 3.16a, there exists a point (≈ 0.82) such that GDP becomes lower with ϕ, because the increase in the invested capital of larger firms cannot compensate the increase in the number of agents who do not produce. From figure 3.16, I conclude that as the strongly capital constrained economy becomes more heavily constrained (as x increases), more equal wealth distributions lead to higher GDP.

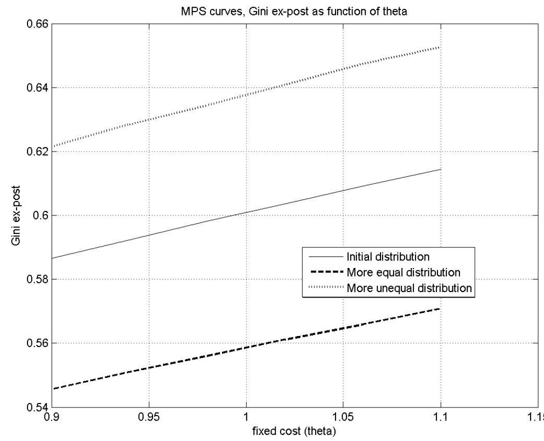
The increase in Debt with x implies that credit penetration measured as Total debt to GDP increases with x (see figure 3.17). The fraction of agents who finance their firms with a loan is reduced when x increases, consistent with the fact that credit constraints (measured by K_d and K_r) increases.

Another important issue is to determinate the effect of financial development and barriers to entry on *ex-post* distribution of wealth. I use Gini index to measure the *ex-post* inequality of the wealth distribution. The following figure presents the simulations of *ex-post* Gini due to changes of x in the capital uncostrained economy.

Figure 3.19: Gini statics due to variations in x



(a) Variation in Gini *ex-post* with ϕ



(b) Variation in Gini *ex-post* with θ

Figure 3.19 shows that as *ex-post* creditor protection gets better or fixed costs are reduced, the Gini index of *ex-post* wealth distribution is lower, i.e. the *ex-post* economy is more equally distributed. As expected, financial development has redistributive effects, because it benefits mainly smaller firms (see proposition 22) leading to a more equally distributed country. This result is consistent with recent empirical analysis of Clarke et al. (2006) and Batuo et al. (2010), and with theoretical results of Galor and Zeira (1993) and Banerjee and Newman (1993), which suggest that financial development have a significant effect in reducing the inequality.

Another important result is that the *ex-post* wealth distribution is more equal than the initial wealth distribution [13], i.e. wealth distribution is improved after firms are set up.

[13]The Gini index of initial wealth distribution is 0.71. The Gini index of the more unequal wealth distribution is 0.76, while the Gini index of the more equal MPS is 0.66

Chapter 4

The Open Economy

In this section I study the case of an open economy. I assume that the equilibrium interest rate r is set in the rest of the world. Thus, adjustments occur through variations in the aggregate capital stock of the economy. As explained in Section 1.6, I assume that there exists an implicit competitive bank system without any additional intermediation cost. But, unlike the closed economy, in the open economy banks have unlimited access to the international credit market at a fixed interest rate r^*. Since transactions are costless, banks will charge to borrowers the same interest rate charged in the rest of the world.

In addition, investment decision patterns will remain unchanged, i.e. micro-entrepreneurs will deposit all their capital at the interest rate $r = r^*$, while SMEs will be able to get a loan that allows them to operate a firm, but at an inefficient level. As in the closed economy, Larger Medium Enterprises and Large Enterprises will operate at the optimal capital stock. These last type of firms will also invest their remaining capital at the international interest rate.

Notice that in this section I use r^* to symbolize the international interest rate, while r is the internal interest rate of the economy (which in equilibrium is equal to r^*).

4.1 Capital flows and critical capital levels statics

The main difference with the closed economy is that variations in ex-ante protection $1-\phi$ or in barriers to entry measured by θ, will be translated in outflows or inflows of capital stock. In order to keep the internal interest rate constant, the aggregate capital stock of the economy will be adjusted through flows of mobile capital with the rest of the world. Notice that here I have implicitly assumed that the economy is not large enough to affect the world interest rate, i.e. the following results are valid for a small open economy.

In order to determine the way in which variations in parameters affect the capital stock of the economy, I analyze how these variations affect critical capital levels. As I show in the following proposition, the results obtained for the closed economy can be directly translated to the open economy.

Proposition 23. *In a small open economy, an improvement in ex-ante protection or a reduction in barriers to entry will reduce* $\mathrm{K_d}$ *and* K_r*.*

Proof. Using previous results of the closed economy and the fact that interest rate remains constant (i.e. $\frac{\partial r}{\partial x} = 0$ for $x = \phi, \theta$) in the open economy:

$$\frac{\partial \mathrm{K_d}}{\partial \phi} = \frac{\mathrm{D_d}}{1 + r^* + \phi} > 0 \tag{4.1}$$

$$\frac{\partial \mathrm{K}_r}{\partial \phi} = \frac{\mathrm{D}_r}{1 + r^* + \phi} > 0 \tag{4.2}$$

$$\frac{\partial \mathrm{K_d}}{\partial \theta} = \frac{\partial \mathrm{K}_r}{\partial \theta} = \frac{1}{1 + r^* + \phi} > 0 \tag{4.3}$$

□

Note that in this case statics over $\mathrm{K_d}$ and K_r were obtained without restricting the initial wealth distribution. Unlike the closed economy, in the open economy an improvement in *ex-ante* protection or in barriers to entry will affect $\mathrm{K_d}$ and K_r only through its direct effect. Better legal conditions or lower fixed costs will discourage some previous constrained entrepreneurs to commit fraudulent behavior. Therefore, banks will lower the minimum capital level and K_r, because they can trust in those borrowers. On the other hand, the indirect effect that these improvements would have through changes in the interest rate is vanished by adjustments in the aggregate capital stock of the economy, that keep the internal interest rate constant.

Proposition 24. *The reduction in* $\mathrm{K_d}$ *after an improvement in ex-ante protection or in barriers to entry is higher in a small open economy than a in closed economy which initial interest rate is the same as the international interest rate.*

Proof. Note that in the closed economy $\frac{\partial r}{\partial x} < 0$ for $x = \phi, \theta$. Thus, from equations(3.8) and (3.11) and using the results of previous proposition, I conclude that $\mathrm{K_d}$ decreases more in the small open economy after an increase in $1 - \phi$ or a decrease in θ. □

The result of proposition 24 suggest that the reduction in credit constraints after an improvement in the recovery rate or a reduction in barriers to entry will be higher in an open economy (compared to a closed economy with an initial internal interest rate $r = r^*$). Thus, under the same initial interest rate r^*, efforts to ease credit constraints may be greater in a closed economy, because changes in legal protection and fixed costs raise the internal interest rate, and partially mitigate the direct reduction in $\mathrm{K_d}$. Moreover, openness to international credit market seems to be important when countries are seeking to improve access to credit, because it allows to keep their internal interest rate constant after financial development (or a decrease in fixed costs), avoiding an indirect negative impact on access to credit (due to an increase in r). However, if financial protection or barriers to entry are getting worse, then the negative impact on credit constraints may be lower in a closed economy.

Notice that is not clear if the effect of an increase in $1 - \phi$ or a decline in θ over K_r is higher or lower in an open economy, because in the closed economy there will exist an additional negative indirect effect over K_r due to a reduction in K^* (see equation (3.9) and (3.12)).

Proposition 25. *In a small open economy, the distance between* K_r *and* $\mathrm{K_d}$ *is reduced after an improvement in the recovery rate* $1 - \phi$.

Proof. Using results of proposition 23:

$$\frac{\partial(K_r - K_d)}{\partial\phi} = \frac{D_r - D_d}{1 + r^* + \phi} > 0 \tag{4.4}$$

□

Proposition 26. *In a small open economy, the distance between* K_r *and* K_d *remains constant after a reduction in barriers to entry.*

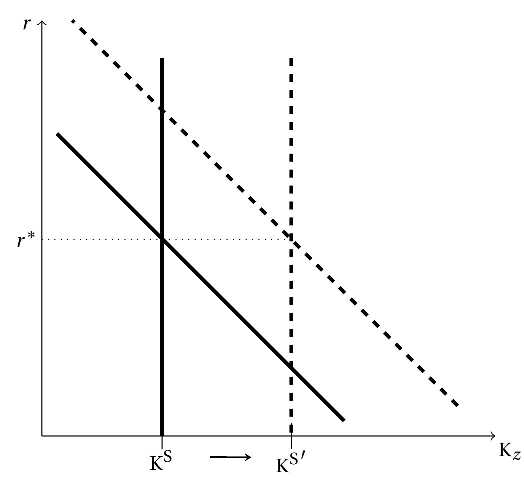
Proof. From equation (4.3) I have $\frac{\partial K_r - K_d}{\partial\theta} = 0$. □

As I showed for the closed economy, proposition 25 says that an improvement in ex-ante protection will reduce the range in which inefficient firms develop in an open economy. On the other hand, an improvement in barriers to entry keeps constant the distance between K_r and K_d.

Proposition 27. *In a small open economy, an improvement in ex-ante protection or a reduction in barriers to entry leads to an inflow of mobile capital to the economy.*

Proof. First, notice that as in a closed economy, in a small open economy the maximum allowable debt decreases with $x = \phi, \theta$, because $\frac{\partial D_z}{\partial\phi} = \frac{D_z}{f'(K_z+D_z)-(1+r^*+\phi)} < 0$ and $\frac{\partial D_z}{\partial\theta} = \frac{1}{f'(K_z+D_z)-(1+r^*+\phi)} < 0$. Thus, from proposition 23 I have that demand for capital is increased after a decline of x, because the maximum allowable debt increases and the number of agents with access to credit increases (note that $\Gamma(K^*) - \Gamma(K_d)$ is higher, since K_d is reduced and K^* remains constant), i.e. there are more entrepreneurs getting higher loans. Therefore, the only way to keep the internal interest rate constant is by increasing the aggregate capital stock of the economy, which is translated in inflows of capital to the economy. The result of this proposition is illustrated in figure 4.1. □

Figure 4.1: Variation in capital stock after an improvement in $x = \phi, \theta$.



Note: dashed lines represent demand and supply of capital after an improvement in x.

Notice that if recovery rate or barriers to entry gets worse, then there will be observed an outflow of mobile capital to the rest of the world, i.e. a contraction in the supply of capital.

4.2 Firm profits statics

An important result of Chapter 3 is that an improvement in ex-ante protection will be opposed by incumbents firms in a closed economy with a small and medium size sector (such that $\Gamma(K_r) > 0$), because it breeds competition and raises financing costs to incumbent firms. However, as I show in the following proposition, one may not expect the same opposition in a small open economy.

Proposition 28. *In a small open economy, an improvement in ex-ante protection will raise profits of firms such that* $K_z \in [K_d, K_r)$*, while profits of the rest of the firms (such that* $K_z \geq K_r$*) will remain constant.*

Proof. Using equations of proposition 22 and the fact that $\frac{\partial r}{\partial \phi} = 0$:

$$\frac{\partial \pi(K^*)}{\partial \phi} = 0$$

Thus, profits remain constant with ϕ for firms that operate at the optimal capital stock. Moreover, differentiation of profits of the rest of the firms leads to:

$$\frac{\partial \pi(K_z + D_z)}{\partial \phi} = \frac{\partial D_z}{\partial \phi}(f'(K_z + D_z) - (1 + r^*)) \tag{4.5}$$

which is negative for firms such that $K_z \in [K_d, K_r)$, i.e. their profits decrease with ϕ. □

Proposition 28 shows that an improvement in ex-ante protection will not have negative effects over firms profits. Unlike the closed economy, in the open economy all SMEs will be better off. Due to a reduction in agency costs, SMEs can access to higher loans (at the same interest rate), while Larger Medium sized firms and Large Enterprises will be not affected. Therefore, one can expect that the wealthy sector of the economy will be indifferent to financial development, since in a small open economy development does not raise financing costs (measured by r), and thus does not affect the optimal capital stock K^*. On the other hand, if ex-ante protection gets worse then small and medium size sector will be negatively affected. However, larger firms (those that remain operating at K^*) will not oppose to underdevelopment.

Therefore, from propositions 22 and 28, I conclude that opposition from incumbents to financial development is more likely to occur in a closed economy than in a small open economy. This result is consistent with Rajan and Zingales (2003), who suggest that opposition of incumbents will be weaker if the economy allows both cross-border trade and capital flows. Therefore, an open economy is more likely to undertake reforms benefiting financial development. Moreover, openness stands to be an important determinant of financial development.

Previous result is also consistent with findings of Shleifer and Wolfenzon (2002) for investor protection: in a country with perfect capital mobility, all entrepreneurs setting up firms are better off after financial development. In contrast, in the case of no capital mobility, there is a group of entrepreneurs who are worse off.

Another important issue is to analyze the effect of variations in barriers to entry over firms profits. Notice that in Chapter 3 I have omitted this effect for the closed economy, because in that case it is ambiguous.

Proposition 29. *In a small open economy, an improvement in barriers to entry leads to an increase in profits of all operating firms.*

Differentiating profits of enterprises that operates efficiently:

$$\frac{\partial \pi(K^*)}{\partial \theta} = \frac{\partial K^*}{\partial \theta}(f'(K^*) - (1 + r^*)) - 1 = -1 \tag{4.6}$$

Differentiation of profits of the rest of the firms leads to:

$$\frac{\partial \pi(K_z + D_z)}{\partial \theta} = \frac{\partial D_z}{\partial \theta}[f'(K_z + D_z) - (1 + r^*)] - 1 < 0 \tag{4.7}$$

Thus, all enterprises will be benefit from a reduction in θ.

Proposition 29 says that all firms that produce will be benefit from a reduction in barriers to entry. First, there will be a direct increase in profits of larger firms (those that operate at K^*) due to a reduction in fixed costs. On the other hand, SMEs will also benefit from an increase in their maximum allowable debt that allows them to produce more, but at a lower operating cost. Therefore, in an open economy, SMEs stand to benefit the most from a reduction in barriers to entry.

4.3 Macroeconomic variables statics

In this section I analyze the effect of improvements in the recovery rate or in barriers to entry over some important macroeconomic variables. The main advantage of assuming a small open economy is that, unlikely a closed economy, these effects are unambiguous.

Proposition 30. *In a small open economy, an improvement in ex-ante protection or a decrease in barriers to entry result in an increase in the following macroeconomic variables:*

1. *Gross output and GDP.*
2. *Total investment.*
3. *Total debt and credit penetration.*

Proof. The gross output is defined by:

$$GO = \int_{K_d}^{K_r} f(K_z + D_z)\,\partial\Gamma(K_z) + f(K^*)(1 - \Gamma(K_r)) \tag{4.8}$$

Differentiating equation (4.8) with respect $x = \phi, \theta$:

$$\begin{aligned} \frac{\partial GO}{\partial x} &= \int_{K_d}^{K_r} f'(K_z + D_z)\frac{\partial D_z}{\partial x}\,\partial\Gamma(K_z) - f(K_d + D_d)\frac{\partial K_d}{\partial x}\gamma(K_d) + f'(K^*)(1 - \Gamma(K_r))\underbrace{\frac{\partial K^*}{\partial x}}_{=0} \\ \Rightarrow \frac{\partial GO}{\partial x} &= \int_{K_d}^{K_r} f'(K_z + D_z)\frac{\partial D_z}{\partial x}\,\partial\Gamma(K_z) - f(K_d + D_d)\frac{\partial K_d}{\partial x}\gamma(K_d) < 0 \end{aligned} \tag{4.9}$$

where I have used the fact that $\frac{\partial D_z}{\partial x} < 0$ and $\frac{\partial K_d}{\partial x} > 0$. I define GDP as follows:

$$GDP = \int_{K_d}^{K_r} [f(K_z + D_z) - (1 + r^*)D_z - \theta]\,\partial\Gamma(K_z) + \int_{K_r}^{K^*} [f(K^*) - (1 + r^*)(K^* - K_z) - \theta]\,\partial\Gamma(K_z) + (f(K^*) - \theta)(1 - \Gamma(K^*) \tag{4.10}$$

Differentiating this condition in terms of x:

$$\frac{\partial GDP}{\partial x} = \int_{K_d}^{K_r} \left([f'(K_z + D_z) - (1 + r^*)] \frac{\partial D_z}{\partial x} - \frac{\partial \theta}{\partial x} \right) \partial \Gamma(K_z) - U(K_d, D_d) \frac{\partial K_d}{\partial x} \gamma(K_d) - \frac{\partial \theta}{\partial x} [1 - \Gamma(K_r)] - U(K^*, 0) \frac{\partial K_r}{\partial x} < 0 \tag{4.11}$$

For the second item, I differentiate total investment with respect x:

$$\frac{\partial I}{\partial x} = \int_{K_d}^{K_r} (K_z + D_z) \frac{\partial D_z}{\partial x} \partial \Gamma(K_z) - \frac{\partial K_d}{\partial x} (K_d + D_d) \gamma(K_d) < 0 \tag{4.12}$$

For the last result I use that total debt is given by:

$$D_T = \int_{K_d}^{K_r} D_z \, \partial \Gamma(K_z) + \int_{K_r}^{K^*} (K^* - K_z) \, \partial \Gamma(K_z) \tag{4.13}$$

Differentiating condition (4.13) with respect x:

$$\frac{\partial D_T}{\partial x} = \int_{K_d}^{K_r} \frac{\partial D_z}{\partial x} \partial \Gamma(K_z) - D_d \frac{\partial K_d}{\partial x} \gamma(K_d) < 0 \tag{4.14}$$

Finally, I define credit penetration as follows:

$$CP = \Gamma(K^*) - \Gamma(K_d) \tag{4.15}$$

Differentiating this condition:

$$\frac{\partial CP}{\partial x} = -\frac{\partial K_d}{\partial x} \gamma(K_d) < 0 \tag{4.16}$$

where I have used the fact that optimal capital stock remains constant and $\frac{\partial K_d}{\partial x} > 0$. □

The first item of proposition 30 implies that a small open economy becomes more efficient in terms of gross output and GDP after an improvement in ex-ante protection or a reduction in barriers to entry. Better legal conditions or lower fixed costs help to relax credit constraints by lowering the minimum capital stock to access to the credit market and by increasing total amount of capital that each restricted entrepreneur can invest in a firm. Because larger enterprises keep operating at the same level, the increase in gross output and GDP is mainly explained by an increase in productivity of sub-capitalized firms (such that $K_z \in [K_d, K_r)$), given by an increase on investment of small and medium size sector that leads to an increase in total investment. The results of items 1 and 2 are consistent with Balmaceda and Fischer (2010), who show that an improvement in creditor protection (or in the efficiency of bankruptcy procedures) result in higher GDP and total investment.

Moreover, the third item of proposition 30 is consistent with empirical studies of La Porta et al. (1997), which suggest that countries with a poor legal protection have significantly lower debt. In addition, notice that condition (4.15) defines credit penetration as the number or percentage of agents that finance their firms borrowing from the bank system. Using this measure, the last part of the proposition shows that improvements in creditor protection or reductions in barriers to entry will result in higher credit penetration. Thus, after an increase in recovery rate or a reduction in fixed cost, more entrepreneurs will have access to credit and the amount loaned will increase. This result is consistent with empirical research of Djankov et al. (2007) and La Porta et al. (2008), which suggest that better creditor rights is associated with higher credit penetration measured as private credit to GDP. This is also consistent with theoretical results of Balmaceda and Fischer (2010), who show that an improvement on creditor protection (or in the efficiency of bankruptcy procedures) leads to higher private credit to GDP ratio.

It is important to emphasize, that in literature there are two broad views on the determinants of the amount of private credit that a financial system will extend to firms. The first one, formalized for some authors as Aghion and Bolton (1992), and Hart and Moore (1994, 1998), proposes that what matters for private credit viability is the strength of creditor rights (known as the 'power' theory). In countries where lenders are more protected against fraudulent behavior or against another payment defaults, lenders are more willing to extend credit, since repayment can be more easily forced. On the other hand, the second theory, developed by Jaffee and Russell (1976) and Stiglitz and Weiss (1981), emphasizes that what matters for lending is access of lenders to information ('information' theory). When countries have developed information institutions or lenders have access to credit history, they can loan to more reliable firms, and thus extend more credit.

In the case of my model, what determinates how much credit the bank system will extend to enterprises is creditor protection, i.e. the result of last part of proposition 30 is consistent with the 'power' theory. Better legal conditions discourage entrepreneurs to commit fraudulent behavior, and therefore banks ease credit constraints by lowering $\mathrm{K_d}$ and K_r and by increasing maximum allowable debt D_z. Thus, improvements in *ex-ante* creditor protection lead to more firms accessing to higher amounts of credit, that results in higher credit penetration and debt.

4.4 Wealth distribution changes

In this section I analyze the effect of inequality of initial wealth distribution on some important macroeconomic variables of a small open economy. In order to isolate the effects of wealth redistributions I use a Mean Preserving Spread (MPS) instead of First-Order Stochastic Dominance (FOSD). The main advantage of using a MPS is that it does not impact the aggregate capital of the economy, that may affect exogenously interest variables.

Recall that a MPS of any distribution function implies a single-crossing property at the mean of the distribution.

Definition 7. *The distribution* $\Gamma_1(\mathrm{K}_z)$ *is said to be a MPS of the initial wealth distribution* $\Gamma_0(\mathrm{K}_z)$*, if the following both conditions are satisfied:*

1. $\Gamma_1(\mathrm{K}_z) > \Gamma_0(\mathrm{K}_z)$ *if* $\mathrm{K}_z < \mathrm{E}(\mathrm{K}_z)$
2. $\Gamma_1(\mathrm{K}_z) < \Gamma_0(\mathrm{K}_z)$ *if* $\mathrm{K}_z \geq \mathrm{E}(\mathrm{K}_z)$

In order to facilitate calculations, as in Balmaceda and Fischer (2010), I define $\Gamma_\lambda = \lambda\Gamma_1 + (1-\lambda)\Gamma_0$, where $\lambda \geq 0$ and Γ_1 is a MPS of Γ_0. Notice that as λ increases there are riskier distributions that transform Γ_0 into Γ_1.

Proposition 31. *Consider a small open economy such that* $\mathrm{K_d} > \mathrm{E}(\mathrm{K}_z)$ *and with an initial wealth distribution* $\Gamma(\mathrm{K}_z)$*. Suppose that* $\Gamma'(\mathrm{K}_z)$ *is a Mean Preserving Spread (MPS) of* $\Gamma(\mathrm{K}_z)$*, then the following macroeconomic variables will increase:*

1. *Gross Output.*

2. *Total investment.*

3. *Credit penetration.*

Otherwise, if $E(K_z) > K_d$*, gross output and total investment will decrease. If* $E(K_z) \in (K_d, K^*)$ *then credit penetration will decrease.*

Proof. Differentiating gross output with respect λ and using Γ_λ as defined above:

$$\frac{\partial GO}{\partial \lambda} = \int_{K_d}^{K_r} f(K_z + D_z)\,(\partial\Gamma_1 - \partial\Gamma_0) - f(K^*)(\Gamma_1(K_r) - \Gamma_0(K_r)) \tag{4.17}$$

Suppose that $K_d > E(K_z)$, then from the definition of MPS $\Gamma_1(K_r) - \Gamma_0(K_r) < 0$ and $\Gamma_1(K_d) - \Gamma_0(K_d) < 0$. Now, notice that I can find a lower bound for the expression of equation (4.17):

$$\begin{aligned}
\frac{\partial GO}{\partial \lambda} &> \int_{K_d}^{K_r} f(K_d + D_d)\,(\partial\Gamma_1 - \partial\Gamma_0) - f(K^*)(\Gamma_1(K_r) - \Gamma_0(K_r)) \\
&= \underbrace{(\Gamma_1(K_r) - \Gamma_0(K_r))}_{<0}\underbrace{(f(K_d + D_d) - f(K^*))}_{<0} - \underbrace{(\Gamma_1(K_d) - \Gamma_0(K_d))}_{<0} f(K_d + D_d) > 0 \\
&\Rightarrow \frac{\partial GO}{\partial \lambda} > 0
\end{aligned}$$

Thus, I conclude that $\frac{\partial GO}{\partial \lambda} > 0$ if $K_d > E(K_z)$. Else if $K_d < E(K_z)$, I can find an upper bound for equation (4.17):

$$\begin{aligned}
\frac{\partial GO}{\partial \lambda} &< \int_{K_d}^{K_r} f(K^*)\,(\partial\Gamma_1 - \partial\Gamma_0) - f(K^*)(\Gamma_1(K_r) - \Gamma_0(K_r)) \\
&= -f(K^*)\underbrace{(\Gamma_1(K_d) - \Gamma_0(K_d))}_{>0} < 0
\end{aligned}$$

Therefore, I conclude that $\frac{\partial GO}{\partial \lambda} < 0$ if $E(K_z) > K_d$. Differentiation of total investment with respect λ leads to:

$$\frac{\partial I}{\partial \lambda} = \int_{K_d}^{K_r} (K_z + D_z)\,(\partial\Gamma_1 - \partial\Gamma_0) - K^*(\Gamma_1(K_r) - \Gamma_0(K_r)) \tag{4.18}$$

Similarly as I did for gross output, I can find a positive lower bound for $\frac{\partial I}{\partial \lambda}$ if $E(K_z) < K_d$ and a negative upper bound for $\frac{\partial I}{\partial \lambda}$ when $E(K_z) > K_d$. Finally, if $K_d > E(K_z)$, for credit penetration I obtain that:

$$\frac{\partial CP}{\partial \lambda} = \underbrace{(\Gamma_1(K^*) - \Gamma_0(K^*))}_{>0} - \underbrace{(\Gamma_1(K_d) - \Gamma_0(K_d))}_{<0} > 0 \tag{4.19}$$

Notice that if $E(K_z) \in (K_d, K^*)$ I assure that this condition is negative. However, if $K^* < E(K_z)$ then the sign of equation (4.19) is ambiguous. □

Proposition 31 suggests a theoretical relationship between wealth distribution and financial development. Moreover, the way in which wealth distribution affects some macroeconomics indicators, as gross output, total investment and credit penetration, depends on the level of financial development. As I have shown in proposition 23, in countries with better creditor protection measured by $1 - \phi$, there is a lower threshold for an entrepreneur to access to the credit market (K_d is lower). In addition, in more financially developed economies, there is also a lower threshold for accessing to the amount of credit needed to produce efficiently (K_r is lower). Thus, economies with poor protection of creditor rights or less financially developed, are more likely to be capital constrained, because as $1-\phi$ decreases both minimum capital levels increases (K_d and K_r), i.e. it is more likely to have that $K_d > E(K_z)$.

Another important issue of my model is that barriers to entry also help to determinate whether a country is capital constrained or not. In proposition 23 I showed that higher fixed costs leads to an increase in both critical thresholds. Thus, countries with higher barriers to entry are more likely to be capital constrained, while economies with lower fixed costs to operate a firm are more likely to be weakly capital constrained.

Notice that the definition of a 'capital constrained economy' is similar to the one used for the closed economy. In the open economy I can distinguish between two types of capital constrained countries and two types of capital unconstrained countries.

Definition 8.

1. *A small open economy such that* $E(K_z) \geq K^*$ *is said to be 'strongly capital unconstrained'.*
2. *A small open economy such that* $E(K_z) \in [K_r, K^*)$ *is said to be 'capital unconstrained'.*
3. *If* $E(K_z) \in [K_d, K_r)$*, then the economy is said to be 'weakly capital constrained'.*
4. *If* $K_d > E(K_z)$*, then the economy is said to be 'strongly capital constrained'.*

Unlike the closed economy, in the open economy it may occur that $E(K_z) \geq K^*$, I define this country as a 'strongly capital unconstrained' one, because the average entrepreneur can attain the optimal production level without accessing to the bank system, and in addition, she may be able to invest her surplus capital at the interest rate r[1] . It is important to note that countries such that $E(K_z) \in [K_r, K^*)$ cannot become a strongly capital unconstrained economy just by improving their *ex-ante* protection or by reducing barriers to entry (assuming that the mean capital stock $E(K_z)$ remains constant) [2]. Therefore, under the assumption of a small open economy, the only way to have that $E(K_z) > K^*$ is through an exogenous increase of international interest rate (that would reduce K^*), or by changing the own aggregate capital stock of the economy.

From proposition 31 I conclude that among strongly capital constrained countries or less financial developed economies (such that $K_d > E(K_z)$), those with a worse wealth distribution in the sense of a MPS have higher gross output and investment. This result is consistent with Balmaceda and Fischer (2010) and is explained basically because higher inequality in heavily constrained economies means more agents that have access to credit, i.e higher credit penetration measured as the percentage of of individuals that finance their firms through banking credit. On the other hand, among less capital constrained economies (such that $E(K_z) \in [K_d, K_r)$) or among

[1]Note that the strongly capital unconstrained economy exports mobile capital to the rest of the world, despite the fact that constrained entrepreneurs (without access to credit) may choose to close their firms. However, if the economy decided to close to capital flows, then the internal interest rate would be lower than the international interest rate and credit penetration would be higher than in the initial open economy. Thus, in that case there would be less entrepreneurs who do not produce and lose their specific capital.

[2]Note that as in Balmaceda and Fischer (2010), I define the level of capital constraints of the economy in terms of its own aggregate capital stock $E(K_z)$ that depends on initial wealth distribution. In definition 8, I do not consider the outflows or inflows of capital that may change the aggregate capital stock (K^S).

capital unconstrained ones (such that $E(K_z) \in [K_r, K^*)$ or $E(K_z) > K^*$), those with a better wealth distribution in terms of MPS have greater investment and output. Therefore, in heavily capital contrained countries, a more unequal wealth distribution results in a more efficient economy in terms of gross output. In contrast, weakly capital constrained and unconstrained countries are more efficient under a more equal wealth distribution.

Additionally, in a strongly capital constrained country, higher wealth inequality leads to higher credit penetration, while in weakly capital constrained and weakly unconstrained economies, a better wealth distribution in the sense of MPS leads to higher credit penetration. However, in strongly capital unconstrained countries is not clear if greater equality results in higher credit penetration, because in average all agents can finance an optimal firm on their own, regardless of wealth distribution. In general terms, I conclude that inequality is positively related with credit penetration in poorer or less financial developed countries.

The relationship between rising inequality and increasing credit penetration has been widely studied in literature. The Great Depression of 1929 and the recent Great Recession starting at 2007 were both preceded by a sharp increase in inequality and by a credit boom. This recent global crisis rejuvenated the interest in studying the relationship between inequality, credit expansion and financial crisis. In his book 'Fault Lines' Rajan (2011) argues that rising inequality combined with political pressure for solution conspired to ballon the credit market. In constrast, Acemoglu (2011) argues that it was politics the drivers of both the rise of inequality and the financial crisis. Theoretical dynamic-stochastic models have also been developed to show that rising inequality (triggered by idiosyncratic shocks) leads to a expansion of credit (see Ranciere and Kumhof, 2011; Kumhof et al., 2012; Iacoviello, 2008; Krueger and Perri, 2006). However, empirical evidence is sparse and contradictory. Bordo and Meissner (2012) do not find evidence that income concentration leads to credit booms. In contrast, Perugini et al. (2013) find empirical evidence that support a positive relationship between inequality and private sector indebtedness.

This thesis contributes to the above cited literature based on a different argument: in more heavily capital constrained countries, rising inequality leads to an increase in the mass of entrepreneurs that access to the credit market, which means higher credit penetration in poorer or less financial developed countries. In contrast, this effect is reversed in less capital constrained countries (or in capital unconstrained ones).

4.5 Changes in the international interest rate

In this section I study the effects of variations in the international interest rate on the main variables of an open economy. It is important to emphasize that I assume that the economy is not large enough to impact the world interest rate (small open economy). In practice, variations in the international interest rate will occur due to changes in global macroeconomic and financial conditions or through monetary policies conducted by institutions of the international financial system. In my model, I consider that the international interest rate is determined by the aggregate capital stock of the rest world (which is assumed to be exogenous). Therefore, changes in the global interest rate may be attributed to variations in the aggregate world's capital stock (as well as changes in the global demand for capital).

In this section I use r^{**} to symbolize the *ex-post* global interest rate, while r^* is the previous international interest rate.

Proposition 32. *The minimum capital level* K_d *and the maximum allowable debt* D_z *decrease when international interest rate increases.*

Proof. See equations (1.25) and (1.28). $\square$

From proposition 32 I conclude that credit constraints will increase in two ways after an increase in the world interest rate: the minimum capital stock required for a loan will be higher and the maximum allowable debt that any restricted entrepreneur can get will be lower. The rise in the world interest rate will increase financing cost to the bank system, which will be directly transferred to borrowers. Moreover, banks will raise credit constraints because some previous unconstrained agents will have incentives to commit *ex-ante* fraud. On the other hand, the effect of an increase in the world interest rate on K_r will be not clear, because there will exist an indirect effect due to a reduction in the optimal capital stock. In fact using condition (1.12) I obtain that $\frac{\partial K_r}{\partial r} = \frac{\phi \frac{\partial K^*}{\partial r} + (K^* - K_r)}{1 + r^{**} + \phi}$, which has an ambiguous sign.

Another important issue is that changes in the international interest rate will be translated in flows of capital with the rest of the world. In order to keep internal interest rate equal to the *ex-post* international interest rate r^{**}, there will exist some adjustments in the aggregate capital stock of the economy.

Proposition 33. *An increase in the international interest rate will be translated in outflows of mobile capital with the rest of the world.*

Proof. Proposition 5 says that demand for capital decreases with r. Thus, the only way to reach a higher internal interest rate is by reducing the aggregate capital stock, i.e. through outflows of capital with the rest of the world. $\square$

Proposition 34. *An increase in the world interest rate will reduce profits of all firms.*

Proof. For firms that operate at the optimal capital stock I have:

$$\frac{\partial \pi(K^*)}{\partial r} = \underbrace{[f'(K^*) - (1 + r^{**})]}_{=0} \frac{\partial K^*}{\partial r} - K^* = -K^* < 0 \tag{4.20}$$

For sub-capitalized firms I obtain that:

$$\frac{\partial \pi(K_z + D_z)}{\partial r} = (f'(K_z + D_z) - (1 + r^{**})) \frac{\partial D_z}{\partial r} - (K_z + D_z) < 0 \tag{4.21}$$

$\square$

Proposition 34 shows that all operating firms will be negatively affected by an increase in the international interest rate. Larger Medium firms and Large Enterprises will produce less (because K^* is reduced) and will also face higher capital costs. On the other hand, SMEs will have access to lower amounts of loans at a higher interest rate, that will restrict even further their productivity, and therefore their profits.

In addition, as in the previous section, I can obtain statics of some important macroeconomic variables that are showed in the following propositions.

Proposition 35. *An increase in the world interest rate leads to a decrease in the following macroeconomic variables:*

1. *Gross output and GDP.*
2. *Total investment.*
3. *Total debt and credit penetration.*

Proof. Differentiating condition that defines gross output:

$$\begin{aligned}\frac{\partial \mathrm{GO}}{\partial r} &= \int_{\mathrm{K_d}}^{\mathrm{K}_r} f'(\mathrm{K}_z+\mathrm{D}_z)\frac{\partial \mathrm{D}_z}{\partial r}\,\partial\Gamma(\mathrm{K}_z) + f(\mathrm{K}^*)\gamma(\mathrm{K}_r)\frac{\partial \mathrm{K}_r}{\partial r} - f(\mathrm{K_d}+\mathrm{D_d})\gamma(\mathrm{K_d})\frac{\partial \mathrm{K_d}}{\partial r}\\ &\quad + f'(\mathrm{K}^*)(1-\Gamma(\mathrm{K}_r))\frac{\partial \mathrm{K}^*}{\partial r} - f(\mathrm{K}^*)\gamma(\mathrm{K}_r)\frac{\partial \mathrm{K}_r}{\partial r}\\ &= \int_{\mathrm{K_d}}^{\mathrm{K}_r} f'(\mathrm{K}_z+\mathrm{D}_z)\frac{\partial \mathrm{D}_z}{\partial r}\,\partial\Gamma(\mathrm{K}_z) - f(\mathrm{K_d}+\mathrm{D_d})\gamma(\mathrm{K_d})\frac{\partial \mathrm{K_d}}{\partial r} + f'(\mathrm{K}^*)(1-\Gamma(\mathrm{K}_r))\frac{\partial \mathrm{K}^*}{\partial r} < 0\end{aligned} \tag{4.22}$$

For GDP I obtain that:

$$\begin{aligned}\frac{\partial \mathrm{GDP}}{\partial r} &= \int_{\mathrm{K_d}}^{\mathrm{K}_r}\left({}'(\mathrm{K}_z+\mathrm{D}_z)-(1+r^{**})]\frac{\partial \mathrm{D}_z}{\partial r}-\mathrm{D}_z\right)\partial\Gamma(\mathrm{K}_z) - \mathrm{U}(\mathrm{K_d},\mathrm{D_d})\frac{\partial \mathrm{K_d}}{\partial r}\gamma(\mathrm{K_d}) - \int_{\mathrm{K}_r}^{\mathrm{K}^*}(\mathrm{K}^*-\mathrm{K}_z)\,\partial\Gamma(\mathrm{K}_z)\\ &\quad + f'(\mathrm{K}^*)\frac{\partial \mathrm{K}^*}{\partial r}\Gamma(\mathrm{K}^*)) < 0\end{aligned} \tag{4.23}$$

Differentiating the condition that defines total investment:

$$\frac{\partial \mathrm{I}}{\partial r} = \int_{\mathrm{K_d}}^{\mathrm{K}_r}\frac{\partial \mathrm{D}_z}{\partial r}\,\partial\Gamma(\mathrm{K}_z) - \frac{\partial \mathrm{K_d}}{\partial r}(\mathrm{K_d}+\mathrm{D_d})\gamma(\mathrm{K_d}) + \frac{\partial \mathrm{K}^*}{\partial r}(1-\Gamma(\mathrm{K}_r)) < 0 \tag{4.24}$$

For total debt I obtain that:

$$\frac{\partial \mathrm{D_T}}{\partial r} = \int_{\mathrm{K_d}}^{\mathrm{K}_r}\frac{\partial \mathrm{D}_z}{\partial r}\,\partial\Gamma(\mathrm{K}_z) - \frac{\partial \mathrm{K_d}}{\partial r}\mathrm{D_d}\gamma(\mathrm{K_d}) + \int_{\mathrm{K}_r}^{\mathrm{K}^*}\frac{\partial \mathrm{K}^*}{\partial r}\,\partial\Gamma(\mathrm{K}_z) < 0 \tag{4.25}$$

Finally, differentiation of the credit penetration condition leads to:

$$\frac{\partial \mathrm{CP}}{\partial r} = \frac{\mathrm{K}^*}{\partial r}\gamma(\mathrm{K}^*) - \frac{\partial \mathrm{K_d}}{\partial r} < 0 \tag{4.26}$$

□

In conclusion, an increase in the international interest rate will have the expected effects of a contractive policy. The increase in world interest rate will be translated in outflows of capital with the rest of the world, which means a contraction of the supply of capital or a decline in the amount of mobile capital available for loans, that will raise internal interest rate. Thus, the bank system will face higher financing costs that will be transferred to the borrower. Banks will impose tighter conditions to get a loan and will also offer lower amounts of capital to each entrepreneur. Therefore, as showed in the last item proposition 35, total debt and credit penetration will be reduced. This will have direct negative effects on total investment, that will be translated in a less efficient economy in terms of gross output and GDP.

Chapter 5

Financial Openness

In this chapter I study the effects of financial liberalization on access to credit and over some important macroeconomic indicators. Initially, I assume a small closed economy with an internal interest rate r, such that r is higher than the international interest rate r^*. Then, I study the impact on the economy of open up its financial market to the rest of the world. In this analysis I maintain the assumption of a economy with a perfectly competitive banking system.

Proposition 36. *Consider an initial small closed economy with an internal interest rate r and a competitive banking system. If the country decide to opening up to the international financial system, that charges a fixed interest rate $r^* < r$, then the following effects will be observed:*

1. *An inflow of mobile capital to the economy.*

2. *A reduction of credit rationing.*

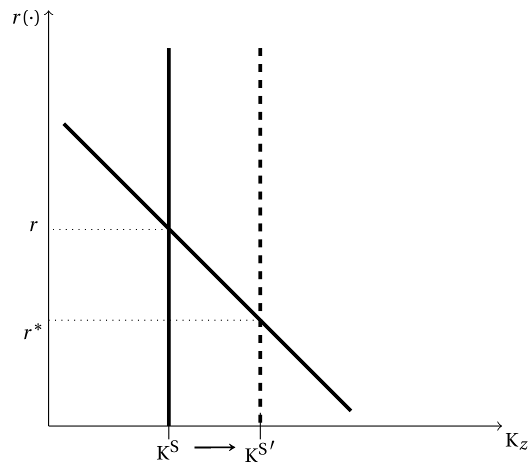
3. *An increase in profits of all operating firms.*

4. *An increase in total investment, gross output and GDP.*

5. *An increase in total debt and credit penetration.*

Proof. For the first item note that after financial liberalization banks will have access to capital at a lower interest rate r^*, thus banks will be willing to extend more credit to firms. Entrepreneurs will ask for higher amounts of debt and the demand for capital will increase. Hence, the only way to keep internal interest rate equal to the lower international interest rate is with an expansion of the supply of capital, i.e. through inflows of mobile capital from the rest of the world (figure 5.1 illustrates this effect). For the last items see propositions of previous chapter. $\square$

Figure 5.1: Variation in capital stock after financial liberalization (assuming that $r > r^*$).



Note: dashed line represent the supply of capital after financial liberalization.

Proposition 36 shows that financial openness will benefit the economy by allowing the bank system to have access to a cheaper credit. Under the assumption of a competitive banking sector prior to liberalization, all the reduction in financing costs of banks will be transferred to borrowers. In addition, because the internal interest rate decreases, banks will ease credit constraints by lowering the minimum capital stock to get a loan and by increasing the amount of debt that any small and medium size firm can get. Thus, banks will loose both credit rationing mechanisms. Moreover, all firms in operation will benefit from an increase in their profits, because SMEs will be able to invest more at a lower financing cost, while Larger Medium Enterprises and Large Enterprises will reach a higher optimal production level. However, notice that micro-entrepreneurs will face a decrease in their utilities due to a reduction in the interest rate, i.e. financial liberalization will have a negative impact on the *ex-post* micro-entrepreneurial sector.

In macroeconomic terms, financial liberalization will allow all operating firms to invest more, thus total investment of the economy will increase. Therefore, as showed in proposition 36, gross output and GDP will increase, i.e. financial liberalization will raise the economy efficiency. Another important issue is that financial liberalization will also increase credit penetration and total debt. In what follows I discuss in greater detail this last result.

Many authors have study the effectiveness of financial openness on the way through it affects growth and financial deepening without having robust conclusions (see Detriagache et al., 2008; Prati et al., 2012; Gormley, 2010; Eichengreen et al., 2011; Mian, 2006; Giannetti and Ongena, 2009). On the other hand, some researchers have proposed different conditions in which financial liberalization is beneficial for countries: Chinn and Ito (2006) suggest that financial openness contribute to equity market development only if there exists a certain level of legal institutions, Martell and Stulz (2003) argue that the capacity of countries to benefit from financial liberalization depends on the protection of investor rights and corporate governance. Recently Balmaceda et al. (2014) and Fischer and Valenzuela (2013) have shown that the effects of financial liberalization depend on the *ex-ante* credit market structure.

Consistently with the model developed by Balmaceda et al. (2014), I find that financial liberalization results in more entrepreneurs that access to the credit market, only if there exists an *ex-ante* competitive bank system and if the previous internal interest rate of the closed economy is higher than the international interest rate. As showed in the first item of proposition 36, the model also replicates the result that under these conditions financial liberalization leads to an inflow of mobile capital from the rest of the world. On the other hand, if the internal interest rate of the initial closed economy is lower than the international interest rate, then financial liberalization will result in outflows of capital with the rest of the world. In that case the *ex-post* economy will be less efficient, access to credit will be more restricted, and thus total debt and credit penetration will decline. However, previous micro-entrepreneurs will benefit from the increase in the interest rate, while profits of operating firms will be reduced.

I conclude that financial openness is related to: larger and more efficient firms, to deeper financial markets and more efficient economies. Recall that results of this chapter are valid only under an *ex-ante* competitive banking system, if I assumed imperfect competition prior to liberalization then these results may not hold. In the case of imperfect competition, liberalization could lead to exclusion of less wealthy entrepreneurs from the credit market (see Balmaceda et al., 2014).

Proposition 6 and 36 suggest another possible explanation of the Lucas puzzle (Lucas, 1990), which tries to explain why capital fail to flow from rich to poor countries. Lucas proposes three explanations to this puzzle: capital market imperfections, differences in human capital and external benefits of human capital. On the other hand, both propositions suggest another explanation: that lower creditor protection leads to lower interest rate, eliminating the incentives for capital to flow to a country with worse creditor protection.

Consider two countries that differ in their strength of creditor protection and that are initially closed to the international financial market. Country A has a very poor legal protection against *ex-ante* fraud, while country B has a very developed financial market (*ex-ante* protection is high). Then, proposition 6 implies that country A will have a very low interest rate, while the interest rate in country B will be high. Therefore, from proposition 36, if both countries decide to open up their financial markets it will be observed an inflow of capital to country B, and an outflow of capital from country A to the rest of the world [1]. Therefore, capital is expected to flow to more financially developed countries, while it will fail to flow to financial underdeveloped countries. [2]

[1]Note that I have implicitly assumed that the international interest rate is higher than the interest rate of the country A and lower than the interest rate of the country B.

[2]In the model I interpret developing countries as either poorer (in term of their aggregate capital stock) or as less financially developed ones. Note that here I describe capital flows based on the second definition.

Chapter 6

Conclusion

In this thesis I present a static model with heterogeneous entrepreneurs and non-linear variable investment decisions. In order to setup their firms, entrepreneurs require capital loans from the credit market. Due to imperfect creditor protection of the legal system, credit rationing arises endogenously in the model restricting some firms activities. I examine entrepreneurs' decisions and market equilibrium in an environment with imperfect creditor protection. When the model is solved, I distinguish four types of entrepreneurs and investment patterns: entrepreneurs who do not produce, lend and then consume their wealth, entrepreneurs with Small and Medium size Enterprises (SMEs) which have partial access to the credit market and operate inefficiently, entrepreneurs with Large Medium sized Enterprises (LMEs) which attain the optimal size and entrepreneurs with Large Enterprises (LEs) who not only do not need loans, but supply capital to the credit market. I compare the performance of the economy and of the four different entrepreneurial sectors changing the fundamental parameters of the model. The parameters are the degree of *ex-ante* creditor protection and barriers to entry measured by the fixed cost of operating a firm, as well as the parameters that define the initial wealth distribution. The model replicates basic theoretical and empirical predictions of the literature on financial constraints, financial development and economic performance, while it suggests novel results which appear to be untested.

The results show that reforms to credit protection and reductions in barriers to entry encourage the banking system to loosen credit constraints (in both the closed and the small open economy). The conditions for access to the credit market are eased, banks are willing to extend more credit to sub-capitalized firms and the capital required to get an optimal loan is reduced. However, in the closed economy, financial development (or a reduction of fixed costs) raises the demand for credit that results in a higher interest rate (if there exist a small and medium sized entrepreneurial sector)[1], i.e. financial development increases financing costs of incumbent firms.

Simulations of the closed economy suggest that countries with worse wealth distributions in the sense of MPS are more likely to have higher interest rate if they are more capital constrained (or less financially developed)[2]. In capital constrained countries a more equal wealth distribution

[1]Consistent with the observation of Shleifer and Wolfenzon (2002), the model predicts that in wealthy countries (such that all firms produce efficiently, i.e. without SMEs) financial development (or reducing fixed cost) will not impact the equilibrium interest rate.

[2]As I said before, I interpret a capital constrained country as either a poorer (which has lower aggregate capital stock) or less financially developed one.

results in lower constraints to access the credit market, but once an agent is able to receive a loan, restrictions to get an optimal loan are higher in the more equally distributed country.

My model predicts that under better creditor protection (and lower fixed costs) SMEs are larger and more efficient. In the closed economy, smaller SMEs stand to benefit the most from financial development, since they experience an increase in their profits. However, there exist a group of wealthier firms (larger SMEs, LMEs and LCs) that are negatively affected from financial development, since it reduces their profits. This result has some important political implications with regard to financial reforms. In a closed economy, it is expected that reforms which promote the development of legal institutions that protect creditors rights are likely to be opposed by the wealthy entrepreneurial sector. The theory of opposition of wealthy and politically powerful families to financial development has been suggested in literature as a possible explanation to the fact that underdevelopment countries are more reluctant to undertake financial reforms (see La Porta et al., 2000b; Rajan and Zingales, 2003). In contrast, in a small open economy, financial development benefits all SMEs and do not affect profits of larger firms. Therefore, consistent with findings of Rajan and Zingales (2003) and Shleifer and Wolfenzon (2002), I conclude that opposition of incumbent firms to financial reforms is more likely to occur in a closed economy than in a small open economy. Therefore, openness stands to be an important determinant of financial development.

Aggregate results relate to basic economic variables: investment, total debt, gross output, GDP, credit penetration and *ex-post* wealth inequality. In general terms, I find that financial development and reductions in barriers to entry lead to higher total debt, investment, gross output, GDP and credit penetration. This results are satisfied for both the open and the closed economy [3], and are verified empirically in literature. A very particular result of the model is that in heavily capital constrained closed economies an increase in creditor protection (or in fixed costs) may lead to an increase in GDP, total debt and credit penetration. This result is explained because in the strongly capital constrained country the decrease of interest rate when creditor protection worsens is too high. Therefore, richer entrepreneurs (who access to the credit market) of a heavily capital constrained economy stand to benefit the most from financial underdevelopment, because they can get bigger loans and are able to form larger firms, which leads to a more efficient economy in terms of GDP [4]. This result is novel and does not appear in the fixed investment model developed by Balmaceda and Fischer (2010) or in the empirical related literature. In terms of wealth inequality, for the closed economy, the model predicts that *ex-post* wealth distribution becomes more equal as *ex-ante* creditor protection gets better (and barriers to entry are reduced). Since financial development benefits mainly smaller firms, it is expected to have redistributive effects. This last result is consistent with theoretical findings of Galor and Zeira (1993) and Banerjee and Newman (1993).

I also analyze the effects of different wealth redistributions among countries. Based on the simulations of the closed economy, I find that more unequal distributed countries are more efficient in terms of GDP as they become more heavily capital constrained, while the opposite is satisfied in capital unconstrained countries. Theoretical results for the small open economy suggest that a more unequal wealth distribution results in higher credit penetration, investment and GDP as it becomes more heavily capital constrained. These results are reversed in less capital constrained countries.

[3] Except the result for total investment, which remains constant in the closed economy. Aggregate results for the closed economy are based in simulations

[4] This result is explained in detail in the main text.(see Section 3.4 of Chapter 3)

Finally, I analyze the effects of financial openness on credit constraints and economic performance. I consider an initial closed economy with an internal interest rate higher than the international interest rate and a competitive banking system prior to liberalization. The model predicts that if the economy decides to open up its financial market then the following effects will be observed: an inflow of capital from the rest of the world, an easing of credit rationing mechanisms, an increase in profits of all firms, an increase in total debt, investment, credit penetration and gross output. Therefore, I conclude that financial openness in countries with an *ex-ante* competitive banking system is related to deeper financial markets and more efficient economies in terms of gross output. This result is consistent with findings of Balmaceda et al. (2014). In addition, based on the analysis of the closed and the open economy, I suggest another possible explanation for the Lucas' puzzle (Lucas, 1990) which tries to understand why capital fail to flow to developing countries (interpreting developing countries as less financially developed ones)[5].

This thesis generalizes the fixed investment model described in Balmaceda and Fischer (2010) by considering variable investment with non linear productivity. The main contribution of this thesis is that it analyzes the effects of credit market imperfections on sub-capitalized firms (SMEs) which have only partial access to the credit market. The model is consistent with the basic empirical regularities concerning the relationship between creditor protection and financial constraints. In addition, the model makes a number of general equilibrium predictions of the literature concerning the relationship between wealth distribution, financial development and economic performance, as well as the politics of reform of creditor protection. These predictions are consistent with recent developed models and empirical evidence. Moreover, this thesis also presents novel results which appear to be untested.

[5] In Chapter 5 I explain in detail this observation.

Appendix A

A.1 Simulations methodology

Critical capital levels and debt curve

One of the advantages of using the Cobb-Douglas production function of AK type is that it can be solved to obtain an explicit expression for the critical levels in terms of the parameters described in Chapter 2:

$$
\begin{aligned}
K_d &= \left(\frac{A\alpha}{1+r+\phi}\right)^{\frac{1}{1-\alpha}} - D_d \\
D_d &= \frac{A(\frac{A\alpha}{1+r+\phi})^{\frac{\alpha}{1-\alpha}} - \theta}{1+r+\phi} \\
K_r &= \frac{(1+r+\phi)K^* - AK^{*\alpha} + \theta}{1+r+\phi} \\
K^* &= \left(\frac{A\alpha}{1+r}\right)^{1-\alpha}
\end{aligned}
$$

The non-linear equation that determinates the maximum allowable loan which is the effective loan for capital constrained entrepreneurs is as follows:

$$A(D_z + K_z)^{\alpha} - (1+r+\phi)D_z - \theta = 0$$

Notice that the analytic solution of the previous equation may depend on the value of $\alpha \in (0,1)$. Thus, I use the Newton's Method to find the value of D_z that solves the debt curve for each $K_z \in (K_d, K_r)$. The solution depends on the equilibrium interest rate, which is endogenously determined in the closed economy. Recall that this equation may have one or two solutions for entrepreneurs with partial access to credit market. As explained in section 1.3, the effective loan that chooses any constrained agent is the maximum of these solutions.

Equilibrium condition

To obtain the equilibrium interest rate $r = \hat{r}$ I have to solve numerically the equilibrium capital market condition:

$$
\begin{aligned}
E(K_z) &= \int_{K_d(\hat{r})}^{K_r(\hat{r})} (K_z + D_z(\hat{r}))\, \partial\Gamma(K_z) + K^*(\hat{r})(1 - \Gamma(K_r(\hat{r}))) \\
&\Leftrightarrow \mathscr{S} = \mathscr{D}(\hat{r})
\end{aligned}
$$

To compute numerically the integral of the right hand side, I use the Composite Simpson's rule. To find the equilibrium interest rate (a fixed point) I use the Quasi Newton's Method. I approximate the first derivative of $\mathscr{D}(r)$ at $r = \hat{r}$ using that $\left.\frac{\partial \mathscr{D}(r)}{\partial r}\right|_{r=\hat{r}} \approx \frac{\mathscr{D}(\hat{r}+\varepsilon)-\mathscr{D}(\hat{r})}{\varepsilon}$, for $\varepsilon \approx 0$. Afterwards I follow the same procedure of the Newton's Method. I solve the equilibrium condition for all scenarios described in section 2.3.

Mean Preserving Spreads (MPS)

In order to obtain statics with regard to different wealth distributions I vary the parameter σ of the initial Log-normal distribution, but keeping the mean capital $\mathrm{E}(\mathrm{K}_z)$ unaltered. I start with an initial distribution defined by $\mathrm{L}og\mathrm{N}(\mu = 0.05, \sigma = 1.5)$. Then, I simulate two scenarios: $\sigma' = \sigma \cdot (1 \pm 10\%)$. To keep the aggregate capital stock constant, I use that $\mu' = \frac{\sigma^2 - \sigma'^2}{2} + \mu$. The two MPS of initial wealth distribution are defined by $\mathrm{L}og\mathrm{N}(\mu', \sigma')$.

The 'strongly capital constrained' economy

To find the strongly capital constrained economy ($\mathrm{E}(\mathrm{K}_z) < \mathrm{K_d}$) presented in the main text I solve the following equation:

$$h(r(\mu,\sigma)) \equiv \mathrm{K_d}(r(\mu,\sigma)) - \mathrm{E}(\mathrm{K}_z) - \delta = 0$$

where δ defines the severity of capital constraints of the economy. Higher values of δ mean that the difference between $\mathrm{K_d}$ and $\mathrm{E}(\mathrm{K}_z)$ is higher and that the economy is more heavily capital constrained(I set $\delta = 0.01$). Notice that the equilibrium interest rate depends on the wealth distribution which is defined by μ and σ. Therefore, I have to find the values of μ and σ that solves $h(r(\mu,\sigma)) = 0$.

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