

Enhanced distribution of a wave-packet in lattices with disorder and nonlinearity

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Abstract: We show, numerically and experimentally, that the presence of weak disorder results in an enhanced energy distribution of an initially localized wave-packet, in one- and two-dimensional finite lattices. The addition of a focusing nonlinearity facilitates the spreading effect even further by increasing the wave-packet effective size. We find a clear transition between the regions of enhanced spreading (weak disorder) and localization (strong disorder).

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1. Introduction

The simultaneous presence of disorder and nonlinearity in wave systems can give rise to a variety of rich and complex behavior [1]. For a disordered system, it is a widespread belief that an increase in disorder will facilitate localization, that is, the initial spreading of an evolving wave-packet will eventually stop [2, 3]. When - in addition to disorder - a weak nonlinearity is added to the system, it has been predicted that the nonlinearity will either promote or inhibit the wave-packet spreading, depending on the systems details and the relative strength of disorder and nonlinearity [4–8]. Thus, a complete picture of the effects of the interplay between disorder and nonlinearity remains unclear. Disorder-induced localization is based on wave interference and, hence, it is a universal concept applicable to a variety of physical systems [10], such as the transport of acoustic waves, microwaves, spin waves and matter waves [11–15]. In this respect, optical waveguide lattices [16] have emerged as ideal systems in which the interplay of disorder and nonlinearity can be studied by means of simple table-top experiments. After a preliminary experimental study of wave evolution in a weakly-disordered nonlinear 2D fiber array [17], a genuine experimental demonstration of Anderson-localization was published [18] where the averaging over multiple individual realizations of disorder played a key role. In that work it was also shown that focusing nonlinearity indeed facilitates localization. Whereas this work was performed in a two-dimensional (2D) system, in a subsequent experiment [19] the impact of nonlinearity on localization in a one-dimensional (1D) system was experimentally analyzed, culminating in the same result: focusing nonlinearity indeed enhances localization of propagating waves.

In this work we present numerical and experimental evidence that the presence of weak disorder promotes the spreading of a wave-packet, with a more uniform distribution of the energy across the lattice. We focus on a weak disorder regime and finite lattices, and study the average dynamical behavior of the system in terms of the effective size of the output profile. To our knowledge, this is the first systematic and controlled experimental study in waveguide lattices that focusses on the effect of disorder when the localization length (l_c) is larger or equal than the system size. Moreover, in this regime, the spreading effect is further enhanced by the addition of a weak focusing nonlinearity as we will show below.

2. Model

Longitudinal light propagation in weakly-coupled, nonlinear optical, waveguide arrays can be modeled by a set of normalized discrete nonlinear Schrödinger (DNLS) equations [1, 16]:

$$-i \frac{du_{\mathbf{n}}}{dz} = \varepsilon_{\mathbf{n}} u_{\mathbf{n}} + \sum_{\mathbf{m} \neq \mathbf{n}} V_{\mathbf{n},\mathbf{m}} u_{\mathbf{m}} + \gamma |u_{\mathbf{n}}|^2 u_{\mathbf{n}}. \quad (1)$$

Here, $u_{\mathbf{n}}$ is the amplitude of the waveguide mode in the \mathbf{n} -th site. The coordinate \mathbf{n} depends on the dimension and lattice type [for example, $\mathbf{n} = (k, l)$ in a 2D square lattice]. The quantity $\varepsilon_{\mathbf{n}}$ is the propagation constant (i.e., “site energy” in the quantum mechanical context) of the \mathbf{n} -th guide. The hopping between nearest-neighbor lattice sites, \mathbf{n} and \mathbf{m} , is described by the coupling constant $V_{\mathbf{n},\mathbf{m}}$. In our model we impose disorder on both, the propagation ($\varepsilon_{\mathbf{n}} \in [-W_{\varepsilon}/2, W_{\varepsilon}/2]$) and coupling ($V_{\mathbf{n},\mathbf{m}} \in 1 + [-W_c/2, W_c/2]$) constants (on-site and inter-site disorder, respectively); the disorder is therefore characterized by the widths W_{ε} and W_c . Note that W_c is limited to the interval $[0, 2]$, to insure that $V_{\mathbf{n},\mathbf{m}}$ is always positive. The coefficient γ describes the nonlinear response of the optical material, being proportional to the Kerr coefficient n_2 .

One of the most commonly used measures for the description of localization phenomena in finite systems is the participation ratio (PR) [22], which gives a rough estimate of the number of sites that are effectively excited by the wave-packet; i.e., where the light has a significant amplitude at the output facet of the sample. For example, a large PR-value implies a smoother distribution of the energy across the lattice. As pointed out in Ref. [22], in order to get meaningful data for finite lattices, one has to average over different realizations of disorder, and thus, work with an averaged PR: $R \equiv \langle \sum_{\mathbf{n}} |u_{\mathbf{n}}|^4 / P^2 \rangle^{-1}$ [23], where $P \equiv \sum_{\mathbf{n}} |u_{\mathbf{n}}|^2$ is a conserved quantity of model (1) and corresponds to the total optical power (sums are made over an array of N sites). We characterized the linear modes of our system by averaging the participation ratio for growing disorder. As expected, we observe a monotonous decrement of the average effective size of the linear modes for an increasing degree of disorder.

Another relevant quantity to study localization problems is the *divergence of the wave packet width* or *second moment* (m_2). This quantity decreases even for weak disorder [24], showing that waves propagate slower and that diffusion is inhibited. In the present work, we are not concerned with diffusion properties and their dependence on disorder and nonlinearity, but rather with the dynamical behavior of the effective area or energy distribution of the profile ($\sim R$), for finite propagation distances in disordered lattices.

3. Numerical results for 1D lattices

We start our analysis by carrying out extended numerical simulations in 1D lattices. We integrate Eq. (1), using a standard 4th-order Runge-Kutta integrator, for several random realizations, and perform an averaging process of the relevant quantities. In the 1D case, we consider an array of $N = 81$ waveguides with an initially localized excitation: $u_n(0) = \sqrt{P} \delta_{n,n_c}$, being

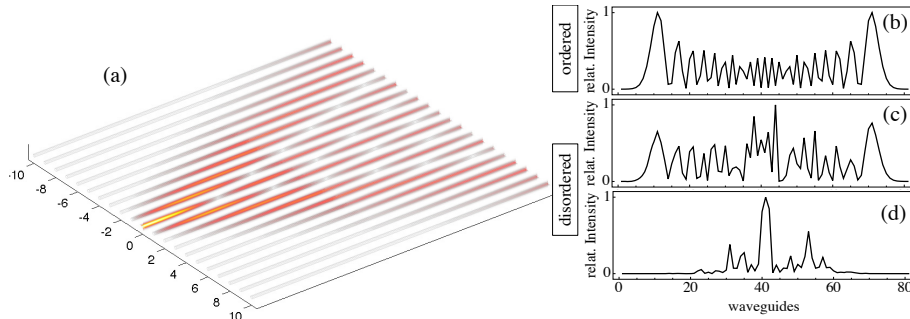


Fig. 1. (a) Sketch of light propagation through a 1D waveguide array, when only the center waveguide is initially excited. (b)-(d) Simulated intensity output distribution for lattices with on-site disorder level $W_\epsilon = 0$ (b), $W_\epsilon = 0.36$ (single realization) (c), and $W_\epsilon = 1$ (single realization) (d), respectively.

n_c the input site. The participation ratio is computed at the propagation distance $z_{max} = 20$ (in normalized units) to avoid reflexions from surfaces. In order to simulate linear propagation, a very small (< 0.01) total optical power P is chosen. Figure 1(a) shows light propagation in an ordered waveguide array. We show in Figs. 1(b)-1(d) output distributions for the ordered lattice and two levels of pure on-site disorder, showing a ballistic spreading for the ordered ($W_\epsilon = 0$) case [Fig. 1(b)]. The diffraction pattern exhibits distinct side lobes and small amplitudes around the initially excited central site. When weak disorder is introduced [Fig. 1(c)], the power contained in these lobes is redistributed to the waveguides close to the center. By increasing the disorder further, a localization around the excited site is observed with exponentially decreasing amplitudes on both sides [Fig. 1(d)]. However, an important aspect here is the occupation of the individual lattice sites, that defines the PR; i.e., the effective area occupied by the diffraction pattern. Our simulations show something very interesting: as the (weak) disorder strength is increased, the average PR increases [black solid line in Fig. 2(a)]. Upon further increment of the disorder, the spatial profile reduces its expansion and the PR decreases. The figure shows a clear maximum that separates regimes of spreading ($l_c \gtrsim N$) and localization ($l_c < N$). We note that, even though this phenomenon was numerically encountered before [24] in the linear regime, it was neither discussed in depth nor experimentally observed.

We also investigate the role of mixed disorder by increasing the amount of the inter-site disorder W_c . Results for $W_c = W_\epsilon/3$, $W_\epsilon/2$, and W_ϵ , are shown in Fig. 2(a) with red-dashed, blue-dotted, and pink-dash-dotted lines, respectively. From our simulations it is evident that, as the inter-site disorder increases, the initial growth of the PR is reduced, and eventually vanishes (i.e. $R/R_0 < 1$ for all disorder levels) for sufficiently high inter-site disorder.

Next, we analyze the impact of nonlinearity by increasing the power of the input beam (it is easy to show that an increment of P with a fixed value of γ is equivalent to fixing the power and increasing the nonlinearity). In general, one expects that a focusing nonlinear term [$\gamma > 0$ in Eq. (1)] facilitates the self-focusing of the excitation around the initially excited site [25]. We find that, in the weak-disorder regime the increase of the PR is enhanced in the presence of a small amount of nonlinearity. Figure 2(b) shows the averaged participation ratio of the profile after propagating a distance z_{max} in the presence of on-site disorder W_ϵ and power P in a 1D lattice. Interestingly, when disorder is switched off and only nonlinearity remains, the picture is similar: the PR increases up to some maximum value due to the redistribution of the power in the waveguides, and then drops as the wave-packet self-traps [26–28]; i.e., below a critical

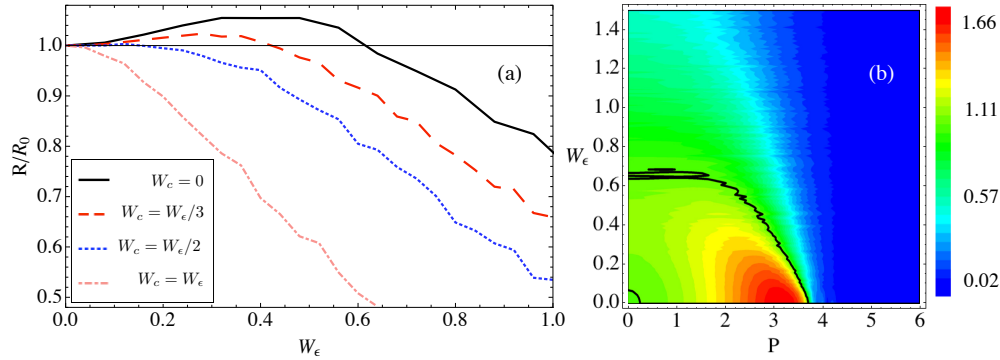


Fig. 2. (a) Simulated disorder-averaged PR vs. on-site disorder width for different levels of inter-site disorder after linear (low power) propagation. (b) Simulated disorder-averaged PR vs. power P and onsite-disorder W_ϵ after nonlinear propagation (thick contour denotes $R = R_0$). In all cases, the PR is normalized to $R_0 \equiv R(W_\epsilon = W_c = 0)$.

value ($P \sim 4$) nonlinearity facilitates the delocalization process. Therefore, when weak disorder and small nonlinearity are simultaneously present, a larger number of excited sites is observed [a larger R -value in Fig. 2(b)] and the spreading/distribution of the wave-packet is enhanced.

4. Experimental results for 1D lattices

The above numerical predictions are nicely confirmed by our experiments (see Fig. 3). The experiments are carried out in waveguide arrays fabricated by the direct-writing laser technology [29] in polished bulk fused silica wafers. The dimensions of each guide are $4 \times 12 \mu\text{m}^2$ with a refractive index contrast of $\approx 5 \times 10^{-4}$ and a propagation length of 100 mm. For the analysis of 1D samples, we fabricated several arrays with $N = 81$ sites each: one ordered lattice and nine with different degrees of disorder (we considered 30 realizations for each degree of disorder). Disorder was created by varying the spacing between the guide centers: $d = (23 \pm \delta_d)[\mu\text{m}]$ with $\delta_d = (0, 0.25, 0.5, 0.75, 1, 1.5, 2, 3, 4, 6)$. A microscope image of the front facet of an ordered and a disordered 1D lattice is shown in Fig. 3(a). For pure inter-site disorder no spreading-peak was observed numerically, we therefore conclude that the experimental variation of the inter-site spacing results in a mixed disorder in the fabricated samples. The fluctuations in the overlap of the individual waveguide modes generate a statistic detuning of the guides and, therefore, create an additional on-site disorder. At the input facet, light at $\lambda = 800 \text{ nm}$ from a Ti:Sapphire laser was launched into a single waveguide at the center of the array using a standard microscope objective. At the end facet of the sample, the intensity patterns were recorded with a CCD camera. Fig. 3(b) shows different propagation patterns of an ordered array (upper row), weak-disorder (middle row), and a strong-disorder level realization (lower row). Whereas the linear regime (P_{lin}) was measured by using the continuous wave-mode of the laser (i.e., the laser power was only a few mW), the nonlinear case was studied in the pulsed regime with pulse peak power of $P_{nl1} \simeq 1\text{MW}$ and $P_{nl2} \simeq 2.7\text{MW}$. The participation number R and its standard deviation were computed for all the different settings, and normalized to the value of the linear single-site excitation in an ordered array ($R_{0,lin}$). From each realization, the PR was computed and then averaged. Fig. 3(c) shows that, for the linear regime (black curve), there is a delocalization tendency of the wave-packet when disorder increases, with a clearly defined spreading-peak at a disorder level of $\delta_d = 1.5 \mu\text{m}$. When nonlinearity comes into play, the spreading-peak shifts towards smaller disorder levels, and its

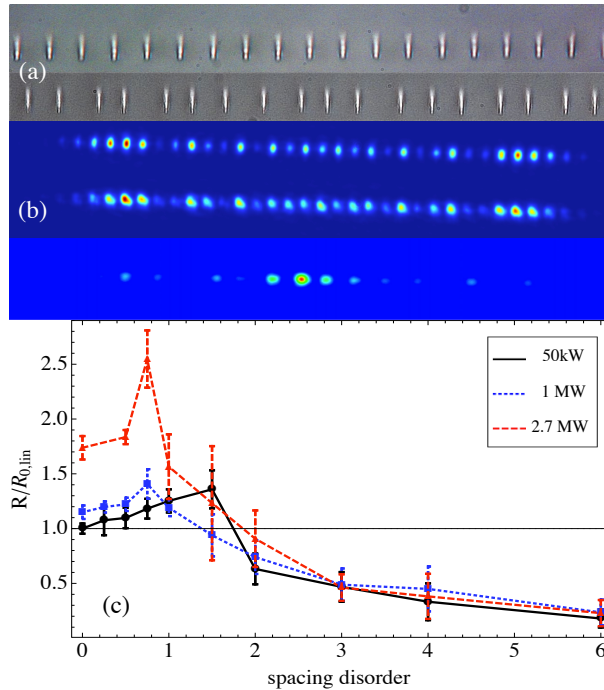


Fig. 3. (a) Microscope images of an ordered (upper row) and disordered (bottom row) 1D array. (b) Experimental linear intensity output patterns: ordered array - averaged (upper row), weak disorder $\delta_d = 1.5\mu m$ - averaged (middle row), and strong disorder $\delta_d = 6\mu m$ - single realization (bottom row). (c) Experimental averaged R versus disorder strengths for different levels of input power, normalized to $R_{0,lin}$. Errors bars indicate the standard deviation of R .

height increases significantly [see red-dashed and blue-dotted lines in Fig. 3(c)]. Therefore, the delocalization (distribution) of the wave-packet is indeed enhanced in the presence of disorder and nonlinearity, as our numerical results have predicted. If we further increase the nonlinearity, the delocalization effect vanishes as the self-focusing is sufficiently strong to inhibit any diffusion/expansion process [26–28]. Errors bars in Fig. 3(c) and Fig. 4(d) indicate the standard deviation of $R/R_{0,lin}$. Close to the region of enhanced spreading (peak), the average value and error bars are always above $R_{0,lin}$ showing that this effect is robust and well defined.

5. Numerical and experimental results for 2D lattices

One of the simplest two-dimensional structures is the square lattice, where each site is coupled to four nearest-neighbors only. We fabricated lattices with 21×21 sites, one ordered array and three with different levels of disorder. Disorder was implemented by varying the mean distances $d_{hor} = (17 \pm \delta_h)[\mu m]$, with $\delta_h = (0, 2, 4, 6)$. (Although the vertical distance was kept fixed at $d_v = 23[\mu m]$, this setting has shown to be equivalent to a 2D fully disordered lattice [30]). Figure 4(a) shows a microscope image of an ordered square waveguide lattice, while Fig. 4(b) shows its experimental output intensity pattern for a single-guide input excitation, exhibiting four distinct side lobes. Figure 4(c) shows our numerical results for the averaged PR versus the degree of disorder. They are qualitatively similar to the ones obtained in 1D lattices, with the corresponding “spreading-peak” and the decreasing tendency of delocalization

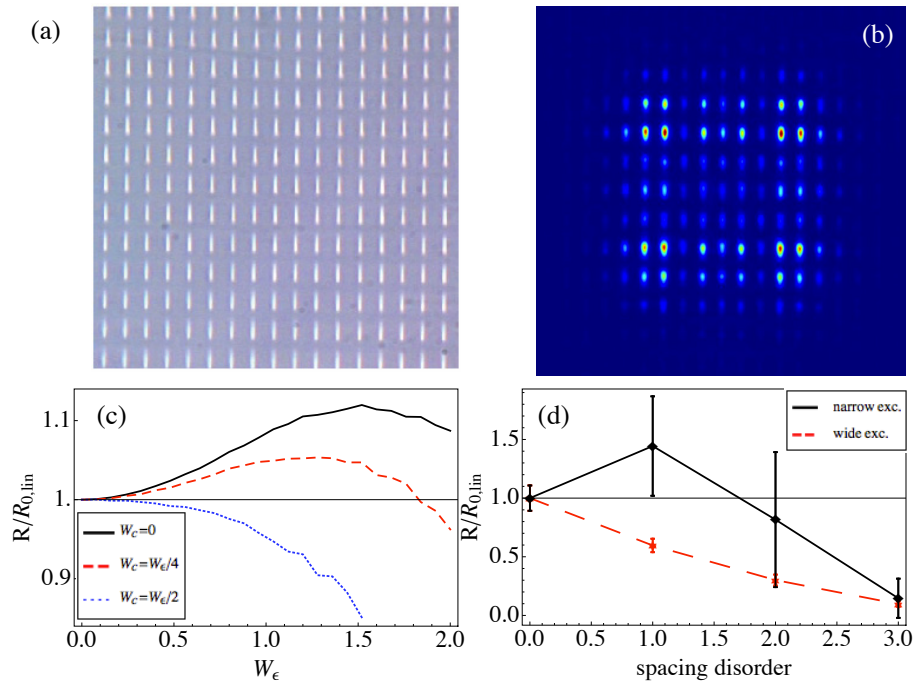


Fig. 4. (a) Microscope image of the fabricated ordered square lattice. (b) Output intensity profile for a single-site excitation. (c) Numerical simulation of the PR for different mixed disorder widths: $W_c = 0$ (black-solid line), $W_c = W_\epsilon/4$ (red-dashed line), $W_c = W_\epsilon/2$ (blue-dotted line). (d) Experimental results for single-site (black-solid line) and gaussian (red-dashed line) input excitations, averaged over 25 realizations. All data is normalized to $R_{0,lin}$. Errors bars indicate the standard deviation of R .

when mixed disorder is increased. The overall expansion tendency is enhanced for 2D-lattices because there are more escaping possibilities for the wave-packet. Our experimental results, summarized in Fig. 4(d) (black-solid line), validate this scenario: for weak disorder the PR significantly grows, and for further increase of the disorder the PR drops, resulting into localization of the wave-packet. Importantly, distinct side lobes in the diffraction pattern only occur when a single waveguide is excited [1, 16]. Numerical simulations show that a wide Gaussian beam creates a propagation pattern with a single lobe at the center, whose height (width) decreases (increases) continuously during propagation. Its PR decreases monotonically with increasing disorder. This was experimentally verified by using a Gaussian input beam that covered ≈ 9 sites [see Fig. 4(d), red-dashed line]. We clearly see that there is no delocalization enhancement for such initial condition.

6. Discussion

Most explanations concerning diffusion processes in disordered lattices resort to band structure changes. Such is the case of disordered quasicrystal structures [22]. However, in that case the reduction of the pseudo-gaps due to disorder does not necessarily imply that a particular excitation located at any input position will be able to excite more states. For weak disorder it is not possible to claim that, by considering a single-site excitation, a particular state will get excited; this may be only possible for strong disorder when all states are very localized. Therefore, in

our view it is not useful to discuss a precise excitation of states, as this would be certainly a matter of probability [21] and it will not correspond to a representative case. Instead, we rely on standard localization length arguments [31]: In the absence of disorder, during propagation the wave-packet forms the typical side lobes of the diffraction pattern [see Fig. 1(b)], and most of the propagating power is concentrated in these lobes. For weak disorder, the initial excitation can be decomposed into a superposition of localized eigenmodes with different localization lengths, many of which will be larger than the dimensions of the finite lattice. These will continue to support the side-lobe structure, while the rest will begin to concentrate energy around the vicinity of the initial site. As a consequence, a more uniform power distribution will form, joining the side-lobe and the center site [see Fig. 1(c)]. This is the regime in which the participation ratio R grows. As the disorder grows further, the number of modes whose localization length is smaller than the dimensions of the lattice will increase, causing power to concentrate more and more in the vicinity of the initial site, causing R to stop growing (see Fig. 1(d) and Refs. [17–19]). Finally at strong disorder levels, the localization length of all modes will be smaller than the system, and R will decrease steadily with the strength of the disorder.

It is important to clarify that this process does not have implications in the *edge-to-edge-diameter* (m_2) of the wave-packet, which decreases steadily for growing disorder [24]; it rather implies a smoother distribution of the light and, therefore, a larger PR.

In the absence of disorder, the side lobe-dominated energy distribution mechanism is also found in higher dimensional lattices: 2D square and honeycomb, and 3D lattices - all of these structures will therefore exhibit the initial increase of the PR for small disorder and, therefore, a spreading-peak. One exception is the triangular lattice which, due to its high coordination number (6), possesses an unusual discrete diffraction pattern with no distinct lobes, resembling the diffraction pattern of a continuous medium. Our numerical simulations of these lattices found no enhancement of the PR, even at weak disorder, in agreement with recent experiments [18].

For a disorder level in the vicinity of the spreading-peak, the addition of a weak nonlinearity will increase the (random) refractive index at each site, in an amount proportional to the light intensity on the site. This causes the high-intensity side lobes to be affected the most, whereas the waves in the center are not affected much. Thus, the nonlinear deepening of the random potential wells, renormalizes the disorder strength of the individual lattice sites. This, in turn, causes an increment of the participation ratio which occurs now at *smaller* values of disorder strength than in the linear case, in agreement with our experimental and numerical results.

7. Conclusions

In summary, we have shown, both numerically and experimentally, that the presence of weak disorder can lead to a more uniform distribution/spreading of an initially localized optical beam in 1D and 2D waveguide arrays. Moreover, we found that the addition of a focusing nonlinearity facilitates this effect even further. The regions separating enhanced spreading (weak disorder) and localization (strong disorder) are clearly identified and explained in terms of standard localization length arguments. Our findings apply to any lattice that displays discrete diffraction features - with distinct side lobes - in the ordered regime.

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