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City traffic jam relief by stochastic resonance

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HIGHLIGHTS

- We simulate a row of interacting cars using a cellular automaton model.
- The jammed state dynamics is analyzed in a sequence of synchronized traffic lights.
- Small density jammed states show the expected scaling laws close to the resonance.
- A stochastic resonance-like behavior is found when we include velocity perturbations.

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ABSTRACT

We simulate traffic in a city by means of the evolution of a row of interacting cars, using a cellular automaton model, in a sequence of traffic lights synchronized by a "green wave". When our initial condition is a small density jammed state, its evolution shows the expected scaling laws close to the synchronization resonance, with a uniform car density along the street. However, for an initial large density jammed state, we observe density variations along the streets, which results in the breakdown of the scaling laws. This spatial disorder corresponds to a different attractor of the system. As we include velocity perturbations in the dynamics of the cars, all these attractors converge to a statistically equivalent system for all initial jammed densities. However, this emergent state shows a stochastic resonance-like behavior in which the average traffic velocity increases with respect to that of the system without noise, for several initial jammed densities. This result may help in the understanding of dynamics of traffic jams in cities.

Transport problems represent an interesting field due to its high social impact and its emergent properties [1–4]. Of particular importance are traffic and pedestrian flows, that have been studied extensively in the past [5–10]. In these systems, behaviors such as jamming transitions and chaos have been found to be common [11–13]. Here, we consider the traffic in a city as represented by a number of interacting cars moving through a sequence of traffic lights, a system which has many nontrivial features [14–21].

For example, Varas et al. [21] showed that the critical behavior found around the synchronization resonances [11] is robust, not only with respect to the street length sequence, but also with respect to the car density for an unjammed initial condition. The study of the system was characterized for two traffic light phase behaviors: (a) synchronized phases [14], and (b) phases linked by a green wave [11]. Resonance occurs when (a) the traveling time between traffic signals is the same

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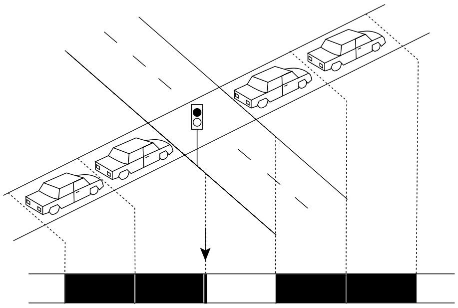

Fig. 1. Possible system state, with a car stopped at a traffic light with a long traffic jam ahead, in which case the car will not move, even if the signal is green. A schematic representation of the state is shown in the lower part of the figure.

as the period of the signals in the synchronized phase strategy, and (b) when the average speed of the car is the same as the green wave velocity in the green wave strategy. In this manuscript we will consider the green-wave synchronization resonances, due to its popular applicability in cities. These resonances are the boundary between two different dynamics, but as was shown numerically and analytically for a single car [11,21], the behavior close to the resonance does not depend on the finite braking and accelerating capabilities of the cars. Recently, the very complex spatiotemporal phenomena of self-organization that occur under green-wave propagation have been studied in detail using a discrete version of the Kerner–Klenov stochastic three-phase microscopic model for large and small flow rates [22]. The complex behavior determined some of the physical effects associated with green-wave propagation. However, that model does not stress explicitly the coupling effect of the traffic light on the car flow, so that the dynamics close to the synchronization resonances was not studied. In a series of published manuscripts, where we have considered the very fine details of the effect of the traffic lights on the car evolution by including continuous accelerating and braking capacities, we have shown that close to the synchronization resonance the car behavior does not depend on the intricacies of the acceleration [11,15]. Hence, the purpose of present manuscript is to investigate how the dynamics close to the synchronization resonance changes as we increase the initial number of interacting cars that jam the system. For that purpose we simulate the dynamics of a number of cars by a simple cellular automaton (CA) model. A large number of CA variants have been proposed to simulate city traffic, including many details of the car dynamics [5]. But, for the purpose of this work, we concentrate on a very simplified CA model since, as mentioned above, the critical behavior close enough to the synchronization resonance should be more or less insensitive to these details (*e.g.*, finite braking and accelerating capabilities, etc. [11,14]).

We will show below that this deterministic CA model displays different behavior close to resonance, depending on the initial jammed density. The different behaviors, representing different attractors of the system, are produced by the spatial variation of the jammed density at each traffic light. As we introduce velocity perturbations in the system, we note that the average velocity increases with respect to the system without velocity perturbations, in a type of stochastic resonance that is produced by the time average homogenization of the spatial variation of the jammed density at each traffic light. This stochastic resonance is similar, although not equal, to the standard stochastic resonance [23,24] observed in the amplification of signals by noise [25], bi-stable nonlinear systems [26,27], climate transitions [28–30], biological systems [31,32], etc.

Following Ref. [21], we consider a street of length L_{tot} with N_s traffic lights. The length L_n between the nth and $(n+1)$th traffic light is divided in $N_{L_n} = L_n/\ell$ cells of length ℓ. The time it takes a car to move to the next cell, namely τ, is the automaton evolution time step. A car will move to the next cell only (a) if no other car is stopped in the next cell; and (b) if the current cell has a traffic light, it must be in green and the next two cells must be empty, so that the drivers avoid stopping at the intersection at the following cell.

The only possible values for the velocity of the cars are $v_{\max} = \ell/\tau$, which corresponds to one cell per time step, and 0. We are also assuming that the cars cannot pass each other. Fig. 1 shows a schematic representation of a state of the system at a particular time. Occupied cells are represented by a black block. An arrow over the cell boundary represents a traffic light.

The switching of the nth traffic light, from green to red and vice-versa, is given by the periodic function $f_n(t) = \sin(\omega_n t + \phi_n)$. When $f_n(t) > 0$ the traffic light is green, and the cars at the intersection can move to the next unoccupied cell. If $f_n(t) < 0$ a red traffic light stops the motion of the vehicles approaching to it. Here, ω_n represents the frequency of the nth traffic light, although for simplicity we are considering that all traffic lights have the same period T, *i.e.* $\omega_n = 2\pi/T\ \forall n$. For the case of the green wave strategy studied here, a green pulse propagates through the sequence of traffic lights with velocity v_{wave}, so that $\phi_n = -(\omega/v_{\text{wave}})\sum_{j=0}^{n} L_j$. We define $\alpha = v_{\max}/v_{\text{wave}}$, to compare the green wave speed with the cars maximum speed. If we take that (a) the distance between traffic lights is about $L = 200$ m, representative of the Alameda Av. in Santiago, Chile; (b) the length of each cell is $\ell = 10$ m; the cruising velocity is $v_{\max} = 10$ m/s (36 km/h); then each time step corresponds to $\tau = 1$ s. Let us note that these values are consistent with the car having equal accelerating and braking capabilities of $a = v_{\max}^2/2\ell = 5$ m/s^2. For the traffic light period we will use $T = 60$ s.

The dynamics described above corresponds to a nontrivial modification of the car model presented in Ref. [21] as we consider an initial jammed situation that shows more complexity than the empty road analyzed in that reference. From

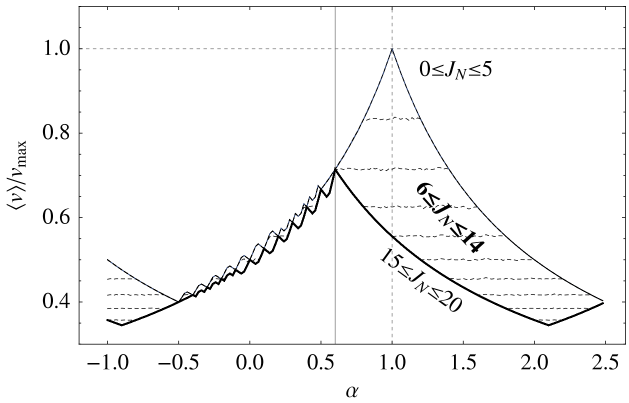

Fig. 2. Averaged speed of the cars normalized to $v_{\max}$ for the full range of initial conditions $J_N \in [0, 20]$ as a function of α for $L = 20\ell$. The thin line corresponds to the $J_N \leq N_L/4$ superimposed curves, and represents the resonant solution with $\langle v\rangle/v_{\max} = 1$ at $\alpha = 1$ studied in Ref. [21]. The dashed lines correspond to $N_L/4 < J_N < 3N_L/4$ with increasing J_N values from top to bottom. The thick line corresponds to the superimposed $3N_L/4 \leq J_N < N_L$ curves.

Ref. [21] we know that the model is quite capable of describing a significant part of the essence of the emergent state that appears in the traffic behavior for large car densities. Hence, even though this model may be simple at first sight, it contains enough complexity to display a type of stochastic resonance capable of increasing the car throughput as velocity perturbations are added to the system. As far as we know, this is a new result that may have important implications for analyzing and controlling city traffic.

As an illustration, we will consider a sequence of $N_s = 100$ equidistant traffic lights, separated by a distance $L_n = L = 20\ell$ (200 m). For the purpose of reproducing an initial jammed state we consider the simplest possibility, namely, an initial condition with a fixed number J_N of cars stopped at each traffic light. Therefore, the initial traffic jam length is $J_N \in [0, N_L]$, such that if $J_n = 0$ we have an empty street, and for $J_n = N_L$ we have a street without empty cells. The cars enter the first simulation cell at a rate $1/f$, that means we inject cars every f time steps, unless the first cell is occupied. We are interested in a jammed situation, so that we will use $f = 1$ throughout, which implies that we inject a new car into the system every time that an empty spot is available. To remove the transient behavior we let the system evolve for a time $10^4\ T$, and to compute the statistics we follow the dynamics for an additional amount of time equal to $10^4\ T$. With these data, we compute the average speed of the cars (total distance traveled divided by total travel time) between the 20th and the $(N_s - 20)$th traffic light, where we are not considering the first and the last 20 traffic lights to avoid boundary effects. It is important to keep in mind that for the given traffic light period T, a traffic light can evacuate at most $T/4 = 15$ cars during a green light period, depending on the number of cars stopped at the following traffic light. The results are shown in Fig. 2, where the normalized average speed is described as a function of the green wave speed parameter $\alpha = v_{\max}/v_{\text{wave}}$.

As shown in Fig. 2, for a range of initial conditions, namely $J_N \leq N_L/4 = 5$, the system converges to the already known dynamics of cars with an empty initial condition ($J_N = 0$). This situation has a resonant average normalized speed $\langle v\rangle/v_{\max} = 1$ at $\alpha = 1$, as was shown in Ref. [21]. In these softly jammed cases, the traffic jam will dissolve after some time. However, there exists initial conditions with $J_N > N_L/4 > 5$ for which the system saturates and it is unable to remove the initial traffic jam. We observe an emergent phenomenon for $5 = N_L/4 < J_N < 3N_L/4 = 15$, where there exists a range of α values in which the average speed is constant, *i.e.*, the car behavior being independent of the green wave speed. This jammed state has its own dynamics and cannot be altered by our control strategy by means of the traffic light phase. This dynamics is expected since the road has a surplus of vehicles which cannot be evacuated due to the limited outflow. For $J_N > 3N_L/4 = 15$, the constant average speed region described above disappears and all these initial conditions converge to the same curve with a resonance at $\alpha \approx 0.6$ with $\langle v\rangle/v_{\max} \approx 0.7$ (true for $J_N \geq 8$). This behavior is quite robust (not shown here) even as we include perturbations to the initial jam length J_N. Furthermore, the three ranges of J_N described above are quite universal even for other values of L, to be analyzed elsewhere.

Let us note that a lower value of α at resonance is expected in a jammed situation, because at a given traffic light we have to wait for the time required to evacuate the cars at the next traffic light before we can make it through. This effect becomes even more apparent in the increase of the average speed close to $\alpha \approx -1$ in Fig. 2, defining an effective anti-greenwave strategy. Hence for some highly jammed situations, it may be convenient to apply an anti-greenwave strategy to increase the traffic flow of the system.

We will now characterize some of the complexities of the emergent states (a range of constant $\langle v\rangle/v_{\max}$ as a function of α) that appear in this system for $N_L/4 < J_N < 3N_L/4$. Given that the number of cars flowing out of the system is $T/4$ per period, it leads us to the rate 1/4 cars per time step. Hence, the travel time of the cars between traffic lights, $\Delta\theta$, grows linearly with J_N as $\Delta\theta = 4J_N\tau$ and is independent of L in this range, which leads us to the following relation,

$$\frac{\langle v\rangle}{v_{\max}} = \frac{\tau}{\ell}\frac{L}{\Delta\theta} = \frac{N_L}{4J_N}. \tag{1}$$

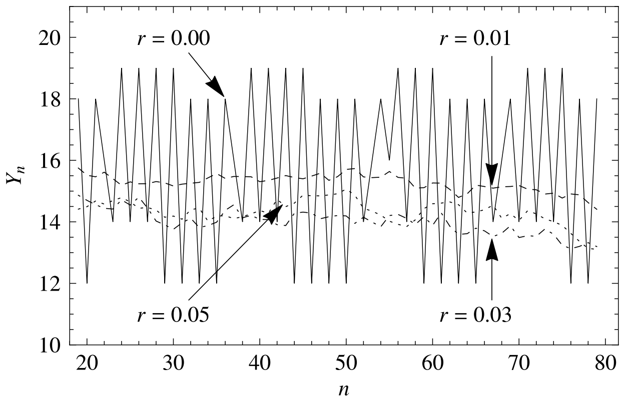

Fig. 3. Traffic light jam number at each traffic light, Y_n, for $J_N = 10$ and $\alpha = 1.6$, as a function of n (traffic light index). We also include the average traffic light jam number at each traffic light for the velocity perturbation levels $r = 0.01, 0.03$ and 0.05 (see below).

Therefore, the average speed in this emergent state is inversely proportional to J_N/N_L, the initial traffic jam fraction. Let us note that we can define the density as the number of cars divided by the street length in cell units, which results in the fraction $\rho = J_N/N_L$ and an average conserved current $\rho\langle v\rangle = v_{\max}/4$.

Another way to characterize the behavior of the traffic jam that occurs in these regions, is to define the traffic jam number Y_n as the number of contiguous cars stopped at the nth traffic light at the moment it switches to green. It is important to note that the maximum normalized length that a traffic light can evacuate, assuming an empty situation ahead, is $Y/L = T/4L = 15/20 = 0.75$. Hence for $J_N > 2N_L/5 = 8$ there are values of α for which the system is completely jammed, in the sense that traffic lights are not able to evacuate all the jammed cars during one light period. The spatial dependence of the traffic jam number at each traffic light, given by Y_n, with n as the traffic light index is shown in Fig. 3 for $J_N = 10$ and a fixed green wave speed $\alpha = 1.6$. In this particular case Y_n has an aperiodic spatial profile.

Hence, for $J_N < N_L/4$ the traffic is under-saturated and the initial jammed condition dissolves as time passes. For $J_N > 3N_L/4$ the traffic is over saturated and there are much more cars trying to cross a traffic light than the ones exiting from the next traffic light, so that all of the initial conditions converge to the same attractor. For $N_L/4 < J_N < 3N_L/4$ there is a range for α values in which the system is not fully saturated and it is unable to dissolve the initial jammed state, leading to constant averaged travel time and averaged speed.

Realistic traffic situations involve a number of uncertain parameters, from city traffic infrastructure to driver's peculiarities. In order to cope with such a randomness we introduce the parameter r, which represents the probability that a car does not move at a given time step, even when all other conditions are satisfied. For example, the case $r = 0.01$ corresponds to having a 1% probability of not advancing in the next time step.

The simulation was done with the same parameters as in Fig. 2, but for $r = 0.01, 0.03, 0.05$. The normalized average speed for the full range of initial jammed conditions is shown in Fig. 4. For $r = 0$, the same curves as in Fig. 2 are obtained, each curve corresponding to an initial jammed state. However, for a given nonzero value of r, all initial conditions yield essentially the same curve. All the attractors for $r = 0$ now converge to a statistically equivalent state when $r \neq 0$ for all initial jam densities. We can still observe an emergent state with constant average speed for a range of α.

As shown in Ref. [21] the introduction of this noise changes the average velocity of an isolated car as $v_{\max} \to v_{\max}(1-r)$, assuming an empty road ahead without cars or traffic lights. Thus, to compare the different curves produced by the different results of r, we use this normalization for $v_{\max}$ in both axes of Fig. 4. The fact that the curves remain different for different values of r, suggests that the resulting dynamics is a consequence of collective effects. By analyzing the car motion in the emergent state, it is possible to see that it represents the dynamics of a traffic jam, with well defined pulses of about $T/4$ cars propagating with velocity $v_{\max}(1-r)$, and in which a given pulse drops a number of cars at a given traffic light, and picks up the cars left by the pulse propagating ahead at the next traffic light. The pulses are produced by the traffic lights, but there is a range of α ($0.5 \leq \alpha \leq 1.0$) values in which the average speed does not change with α, hence the traffic lights just generate the pulses, but do not seem to affect the dynamics in other ways [21]. This resembles a classical gas, where collisions establish and maintain an equilibrium state, but do not otherwise affect macroscopic thermodynamic quantities.

In Fig. 4 we notice a very counter intuitive effect of the velocity perturbations. We see that for a range of α values the introduction of velocity perturbations increases the averaged speed relative to the case $r = 0$, when $J_N > 3N_L/4$, *e.g.*, for the emergent state discussed above. The velocity perturbations not only add stability to the system, but in some cases, improve the car flow. To understand this behavior, we have included in Fig. 3 the average value of Y_n for a given perturbation level. We note that in general the number of cars stopped at a traffic light has a smooth spatial dependence with an average value that is smaller than for the $r = 0$ case. For $r = 0$ there is a significant spatial variation, reaching sometimes a completely jammed situation between traffic lights with $Y_n \sim L$, which is impossible to evacuate in just one traffic light. In essence these traffic lights strongly inhibit the flow of cars through the system, acting as a clog. For the $r > 0$ case, now we have an average value $\langle Y_n\rangle_T \sim T/4$, so that in principle we could evacuate all the cars in one traffic light cycle if the next street

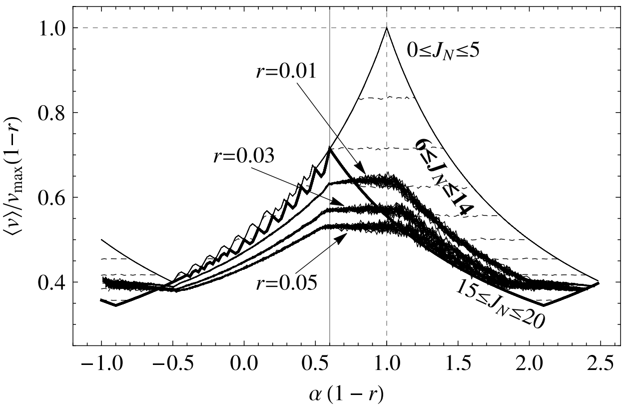

Fig. 4. Averaged speed of the cars normalized by $v_{\max}$ for the full range of initial jammed conditions $J_N \in [0, 20]$ as a function of α. We show a comparison between $r = 0$, $r = 0.01$, $r = 0.03$, and $r = 0.05$. The labels of the curves follow the description used in Fig. 2, with the dashed lines increasing in J_N value from top to bottom.

between lights were empty. Of course this does not occur, and in general not all cars are able to get to the next traffic light in one light cycle as the next traffic light has cars waiting. This can be intuited by the variation of the average velocity with r in Fig. 4.

Hence, this behavior is similar to a stochastic resonance in which the average traffic speed increases with respect to that of the system without noise for some initial jammed densities, *i.e.*, the noise is able to unjam the system, and increases the average flow. The resonant behavior reduces the average number of cars at a given traffic light, partially liberating the clog that inhibits the flow of cars through the system. This counter-intuitive result may be relevant in the understanding of dynamics of traffic jams in cities. In this respect, it is important to realize that all models are restricted approximations of the real city traffic dynamics, and each model has its own strength and range of validity. In the case of our CA model, we expect it to be a good representation close to the synchronization resonance (e.g. $v_{\text{wave}} = v_{\max}$) where the flux is expected to be controlled mainly by the traffic light and noise level, and not by the acceleration details. And it is precisely close to this synchronization resonance where the stochastic resonance-like behavior occurs, as can be observed in Fig. 4. Hence, this process may have practical implications for city traffic, and should be considered as more complexities are introduced into the modeling and studies of city traffic.

Acknowledgments

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